ICTS-RRI Math Circle, Saturday 28th January 2023

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For the 28th January session, we have planned two activities: the first is a game and the second is a puzzle. Both are open ended and get you to think creatively. The puzzle is a repeat of one of the math challenge problems. Very few tried it and no one seems to have made a dent in it. Let us see what we can do by working together and pooling our knowledge and resources. At the end of this note you will find a write-up on how we learn to solve problems. This is a condensation of some ideas due to the mathematician George Polya. Read through this as a preparation for the session on Saturday. Also please bring along some loose sheets of paper, pencils or pens and a ruler.

First Activity: (about an hour) Games are fun. They are also a serious business. There is a whole branch of mathematics called Game Theory. Every game has rules and the players are bound by them. The objective is to 'win' the game following the rules. The game we will play is a simple card game and the rules will be explained to you during the session. The mathematical part arises when we ask: what is the best strategy to use to win this game? You will see that the game has many layers and we will peel them one by one.

Small break (fifteen minutes)

Second Activity (about 100 minutes) 10 cables were laid across the Zambezi river in a remote part of Africa. After laying the cables, the engineer realised that he had forgotten to label them, so he didn't know which of the 10 ends on one bank corresponded to the 10 ends on the other. He had at his disposal a multimeter to test continuity, some copper wire to connect the ends on each bank and a boat to cross the river with. As you may know, the Zambezi river has herds of hippos and these animals can be dangerous to crossing boats. It would be wise to minimise the number of river crossings.

How would the engineer figure out the corresponding cables with a minimum number of crossings?

How to Solve it? by George Polya

This writeup explains in simple language Polya's method for solving problems.

All of us are faced with problems in everyday life and we use a number of tricks to solve them. For instance, you come home from school and find the front door locked and nobody home. What do you do? Think hard. Is there a friendly neighbour? Could there be a window open? Would they have left the key in a hiding place? Can I reach them on the phone? As we get older we get better and better at solving life's problems. We learn from past experience and develop a bag of tricks.

The same is true for problems in mathematics or physics. Many of us use these tricks without being aware that we are using them. For those of you who are learning to solve problems, it is useful to make these rules explicit. This is what George Polya, a Hungarian Mathematician did in a book called "How to solve it?". What follows is a simplified and condensed version of Polya's ideas.

The process is divided into four steps: Understand, Plan, Solve and Review. When you start, you will slowly and consciously go through these steps. As you get more experienced, you will use this method without even thinking about it.

1. *Understand the Problem:* It goes without saying that in order to solve a problem we have to understand it. Here are questions that you ask yourself in order to do this:

   - What form will the answer take? Is it a number, a length? Or is it a logical argument? A proof? An algorithm? A strategy?
   - What is the data that is given to me? Scan the statement of the problem to isolate the data.
   - What are the conditions of the problem? For instance, the answer may have to be a whole number. Or, in a logical proof, the statement may apply only to polygons.

2. *Planning an attack on the Problem:* Ask yourself these questions:

   - Have I seen a related problem before? A problem with a similar unknown?
   - How is the data related to the unknown? Is there too little data? Too much data?
   - Can I simplify the problem by considering a special case? an extreme case?
   - Can I simplify the problem by making it more general?
   - Can I give up a condition? Eg. give up the condition that the answer has to be a whole number and solve it with real numbers.
   - In some problems (a maze for example), it is advantageous to work backwards: start from the end, assuming we have reached our goal. Would this work in our problem?
   - Are there obvious symmetries in the problem?

3. *Solve* In this step, we implement the plan devised in the last section. This may involve calculation or developing the logical steps of a proof. It may be that the first attempt does not succeed. If so we go back to the planning stage and refine it.

4. *Review* This step is important for you to develop problem solving skills for the future. Don't regard a solved problem as dead. You can learn a lot from problems you have already solved and use this knowledge in the future. Ask yourself

   - is there a way to check the solution?
   - Can the solution be generalised?
   - Can I use the solution to devise new problems?

   The next time you are faced with a problem, (even a simple one) try to go through these steps and learn from them. With practice you will find your skills improving. Good luck!