Notes and Problems for lectures on the Theory of Games

We learn about games as children, and enjoy playing them. It turns out that there is a rather abstract mathematical theory of games, that was originally developed by the mathematician John von Neumann, and an economist Oskar Morgenstern in 1944, which has many new insights, and has subsequently been developed by many people, and been applied in modelling stock markets, and simulations of wars between countries, and national economic planning etcetera.

In these lectures, I will try to introduce you to some of the basic concepts in this theory.

I. MANY TYPES OF GAMES

One can think of many types of games.

1. Games of skill : running, shooting, cricket(?)...
2. Games of luck: dice, roulette..
3. Games of strategy: Chess, checkers, go, 20 questions, hangman, solitaire
4. Mixture of these: Ludo, card games like bridge, monopoly...

Or one can classify differently as 1-person/ 2-persons / many-persons games.
Games of perfect information/ imperfect information/ no information.
Repeated or non-repeated games...

In team games like cricket, or football, one may think of these as two-person games, with each team as a person.

II. ACTIVITY 1

Make a list of 10 games that you are familiar with, and discuss under which group they belong to in classification schemes discussed above.

III. ACTIVITY 2

Take a familiar game like snakes and ladders. To describe the game in a way that makes it easier to see the similarities, we formally describe it by specifying the setting : What is the board, how many people, what are the allowed moves, when does the game stop, what are the payoffs.

Do the same for bridge/ poker/ chess.

IV. ACTIVITY 3

Consider the simple coin-toss game: Two players A and B. A tosses a coin, and B has to guess the outcome. If B guesses correctly, he gets a payoff +1, if wrong, the payoff is -1. For A, the payoffs are negatives of those for A. They play this game repeatedly.

What is the best strategy for playing this game for A? for B?

We will assume that players play to colorred Maximize the Expected Value of their Total Payoff. Such players are called Rational Players. Rational players are assumed to be as intelligent as possible, and always act using the best strategy available.

V. ACTIVITY 4

The decision tree for a game: The game rules determine all possible plays in the form of a decision tree.

Construct the decision tree for the tic-tac-toe game.

VI. ACTIVITY 5

In two betting games A and B, the entree fee is Re. 1000.
In game A, you have a probability 1/5 of winning Rs. 5000, and else you get nothing.
In game B, you have a probability 1/100 of getting Rs. 200000, and else nothing.
Which game will you choose to play?

VII. ACTIVITY 6

Can you think of a game that does not fit well into any of the classifications described above? (This question is deliberately ill-posed.)

Can the analysis techniques of game theory be used to design interesting games? For example can you imagine a better variation of snakes and ladders, e.g. bigger board/ more ladders/ more pieces per person?

Would a T30 cricket match be better than T20? How do you decide which game is better?