Aspects of Hilbert space fragmentation

K. Sengupta

IACS, Kolkata

Collaborators: *Bhaskar Mukherjee, UCL, UK.*
Anwesha Chattopadhyay, RKMVU, Belur
Arnab Sen, IACS, Kolkata.
Somsubhra Ghosh, IACS, Kolkata.
Indranil Paul, LPMQ, Paris, France.

Refs:
Phys. Rev. B 105, 155117 (2023)
Scipost Phys. 14, 146 (2023).
Phys. Rev. Lett. 130, 120401 (2023)
arXiv: 2307.05679 (PRB, in press).

Outline

1. *Eigenstate thermalization hypothesis (ETH): Introduction*
2. *Hilbert space fragmentation (HSF)*
3. *Floquet signatures of HSF in driven Fermi chain*
4. *Non-Hermitian quantum model*
5. *Reality of eigenspectrum of non-Hermitian systems via HSF.*
6. *Conclusion*

Introduction to ETH

Consider a generic state of quantum non-integrable many-body system

$$|\psi(t)\rangle = \sum_m C_m \mathrm{e}^{-iE_m t}|m\rangle,$$

D'Alessio et. al
Adv. Phys. 65, 239 (2016)

The time evolution of a generic operator for this state is given by

$$O(t) \equiv \langle\psi(t)|\hat{O}|\psi(t)\rangle = \sum_{m,\,n} C_m^* C_n e^{i(E_m - E_n)t} O_{mn}$$

$$= \sum_m |C_m|^2 O_{mm} + \sum_{m,\,n\neq m} C_m^* C_n e^{i(E_m - E_n)t} O_{mn}$$

$$O_{mn} = \langle m|\hat{O}|n\rangle.$$

Issues with long-time behavior:

a) *The steady state value of O(t) depends on the overlap coefficients: no thermalization (in the sense that the value does not agree with standard ME prediction)*

a) *It takes an incredibly long time to reach the steady state (predicts a very large relaxation time).*

Invoking random matrix theory remedies these problems since within RMT O_{mm}= O' and O_{mn}=0. However it provides an energy independent answer which does not agree with standard numerical results.

Eigenstate Thermalization Hypothesis

Generalization of the RMT result for the matrix elements of a "typical" operator

$$O_{mn} = O(\bar{E})\delta_{mn} + \mathrm{e}^{-S(\bar{E})/2} f_O(\bar{E}, \omega) R_{mn},$$

$$\bar{E} \equiv (E_m + E_n)/2,$$

Both O and f_O are smooth functions of their arguments, S is the entropy, and R is a gaussian random number.

It states that for a large-enough system, the answer is nearly identical to that obtained using a microcanonical ensemble at the average energy.

$$\bar{O} \equiv \lim_{t_0 \to \infty} \frac{1}{t_0} \int_0^{t_0} \mathrm{d}t O(t) = \sum_m |C_m|^2 O_{mm} = \mathrm{Tr}[\hat{\rho}_{\mathrm{DE}} \hat{O}],$$

$$O_{\mathrm{ME}} = \mathrm{Tr}\left[\hat{\rho}_{\mathrm{ME}} \hat{O}\right]$$

$$\bar{O} \simeq O(\langle E \rangle) \simeq O_{\mathrm{ME}}.$$

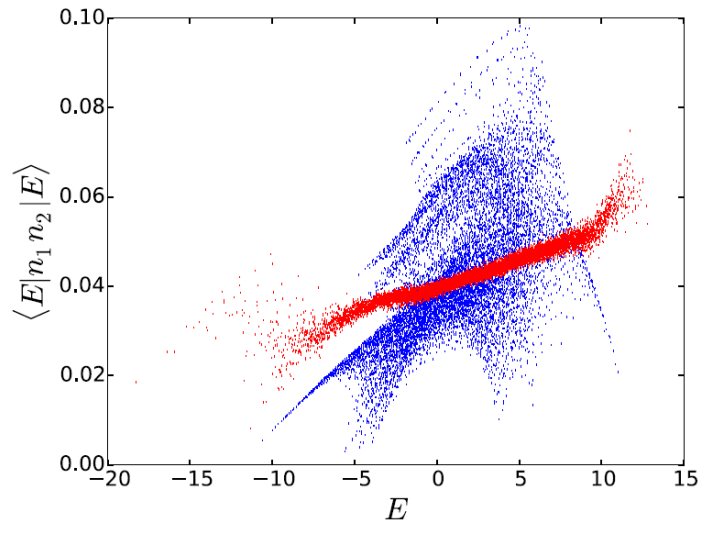
This relies on the fact that energy fluctuations in a many-body system are subextensive.

$$O_{mm} \approx O(\langle E \rangle) + (E_m - \langle E \rangle) \left.\frac{dO}{dE}\right|_{\langle E \rangle} + \frac{1}{2}(E_m - \langle E \rangle)^2 \left.\frac{d^2 O}{dE^2}\right|_{\langle E \rangle},$$

$$\overline{O} \approx O(\langle E \rangle) + \frac{1}{2}(\delta E)^2 O''(\langle E \rangle) \approx O_{\mathrm{ME}} + \frac{1}{2}\left[(\delta E)^2 - (\delta E_{\mathrm{ME}})^2\right] O''(\langle E \rangle),$$

Violation of ETH

1. ***Integrable models: Presence of large number of conserved quantities lead to loss of ergodicity and prevents realization of long-time thermal steady states.***



2. ***Many-body localization: The system becomes non-ergodic due to strong disorder leading to localization of all states in its Hilbert space.***

3. ***Violation of ETH due to presence of a special class of eigenstates in its Hilbert space leading to long-time coherent oscillations: Quantum scars.***

4. ***Violation of ETH due to fragmentation of Hilbert space leading to loss of ergodicity: Strong Hilbert space fragmentation.***

Strong Hilbert space fragmentation

Hilbert space fragmentation: Introduction

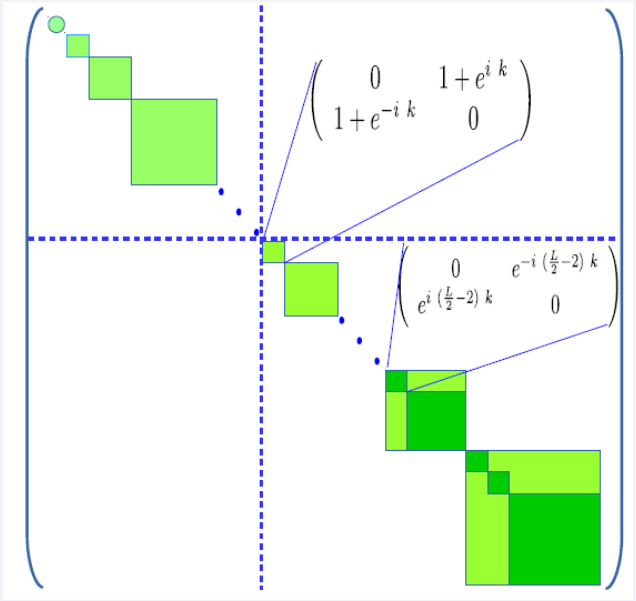
Breakdown of the Hilbert space into an exponentially large number of dynamically disconnected sectors.

The fragmentation is usually observed in the computational basis; classical Fock states such as number basis states $|n_1,n_2, \ldots n_j \ldots n_L\rangle$

Such a separation of Hilbert space in dynamically disconnected sectors is different from those due to global symmetries; in the latter case number of sectors scale algebraically with L.

For strong Hilbert space fragmentation (HSF), with n being the largest fragment and N being the total Hilbert space dimensions, $n/N \sim e^{-L}$

Most of the model exhibiting strong HSF are 1D models; More recently a few higher dimensional models have been put forth (see for example, Scipost Phys. 14, 146 (2023)).



Signatures of strong HSF

1. ***Memory retainment leading to finite value of the autocorrelation function at long times***

2. ***Deviation of the entanglement entropy from its symmetry resolved Page value.***

A model of strong HSF

Tomasi et. al PRB 100, 214313 (2019)

Fermionic model with correlated hopping

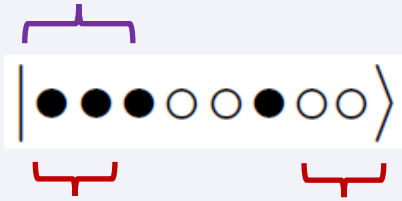
$$\hat{H}_\infty = -t \sum_x \hat{P}_x (\hat{c}^\dagger_{x+1} \hat{c}_x + h.c.) \hat{P}_x$$

$$\hat{P}_x = 1 - (\hat{n}_{x+2} - \hat{n}_{x-1})^2, \qquad \hat{P}_x^2 = \hat{P}_x,$$

Conservation: The conserved quantities are
i) the fermionic charge N and
ii) the number of bonds N_b with two occupied sites

The largest symmetry sector corresponds to N=L/2 and N_b= L/4

Adjacent particle defects



Particle defect

Hole defect

The presence of these conserved quantities leads to exponentially many frozen states: $N_f \sim (1.62)^L/L^{1/2}$

L=8, largest sector

There are (dynamical) states in this sector which has no overlap with a frozen states. Thus even within a symmetry sector, states are dynamically disconnected.

Number of particle and hole defects are separately conserved.
The parity of the starting defect sites are conserved.

Counting of fragmentation

A: Number of states in the largest symmetry sector:

We need to fill L sites with L/2 particles and L/4 bonds

To do this first fill L/2 sites with particles keeping L/2 bonds

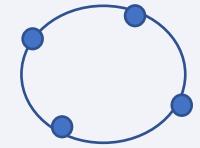
Next we insert L/4 empty sites so as to break L/4 of these L/2 bonds. This can be done in ${}^{L/2}C_{L/4}$ ways

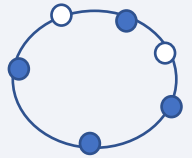
Next, one needs to insert L/4 more empty sites without breaking any bonds. This can be done in ${}^{L/2-1}C_{L/4}$ ways.

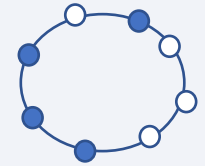
Finally one gets a factor of 2 due to particle-hole inversion

$$N_1 = 2\ {}^{L/2-1}C_{L/4}\ {}^{L/2}C_{L/4} = ({}^{L/2}C_{L/4})^2 \sim 2^L/L$$

Example for L=8

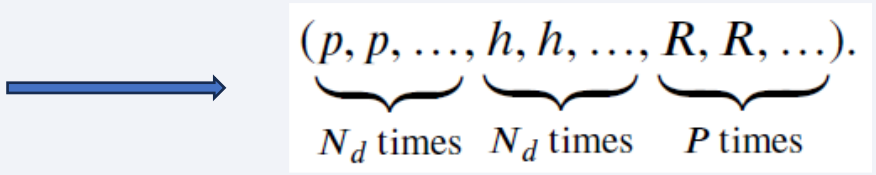




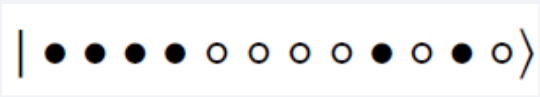


Dimension of the largest fragment

Start from an initial state for which $N_b = L/4$ such that there are $N_d = L/4$ particle and hole defects. This also leaves $P = L/4-1$ particle-hole pairs.

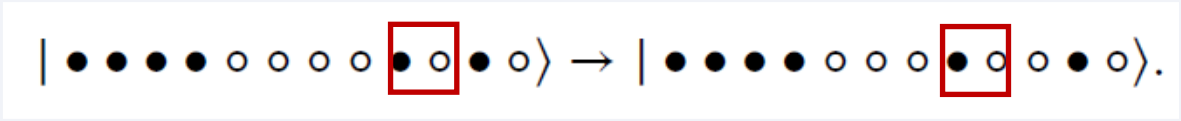


For half-filling one has $P = L/2 - (N_d+1)$



L=12 N_d=3 P=2

Starting from this initial state the action of H leads to diffusion of particle from R to the sequence of hole defects. Such diffusion needs an accompanying hole motion and thus leads to pair movement.



The number of possible configurations obtained by action of H is the number of ways P pairs can be distributed among 2 N_d empty space between particle and hole defects.

Counting of number of ways P balls can be distributed between $2N_d$ -1 bars (partitions for $2N_d$ spaces)

Number of distinct configurations

$N_0 = (2N_d+P-1)!/[P!\,(2N_d-1)!]$

Total number of such configurations is $N_t = L\,N_0/2$.

For large L and L/4 particle and hole defects, one has

$N_t/N_1 \sim (0.8)^L$

Prethermal signature of strong fragmentation

Driving a spinless fermion chain: Numerics

We start from a fermion chain and drive the interaction term

$$H_0(t) = V(t) \sum_{j=1..L} \hat{n}_j \hat{n}_{j+1}$$

$$H_1 = \sum_{j=1..L} -J(c_j^\dagger c_{j+1} + \text{H.c.}) + \hat{n}_j(V_0 \hat{n}_{j+1} + V_2 \hat{n}_{j+2})$$

For a square pulse protocol

$$[V(t) = -(+)V_1 \quad \text{for} \quad t \leq (>)T/2]$$

The time evolution operator

$$U^s(T,0) = e^{-iH_+T/2\hbar} e^{-iH_-T/2\hbar}$$

Can be expressed in terms of eigenvalues and eigenfunctions of H_+ and H_-

$$U^s(T,0) = \sum_{m,n} c_{mn}^{+-} e^{-i(\epsilon_m^+ + \epsilon_n^-)T/2\hbar} |\xi_m^+\rangle\langle\xi_n^-| \quad c_{mn}^{+-} = \langle\xi_m^+|\xi_n^-\rangle.$$

For a continuous protocol

$$[V(t) = V_1 \cos\omega_D t]$$

The time evolution operator requires Suzuki-Trotter decomposition of U

$U = \Pi_j \exp[-i H_j \Delta t/h]$ $\Delta t = T/N$

U can be expressed in terms of eigenvalues and eigenfunctions of H_j

$$U^c(T,0) = \prod_{j=1..N} \sum_n e^{-i\epsilon_n^j T/\hbar} |\xi_n^j\rangle\langle\xi_n^j|$$

This procedure allows one to numerically compute the exact Floquet Hamiltonian for the system

$$U = \exp[-iH_F T/\hbar]$$

Perturbative Analytics: Floquet perturbation theory

We consider a Hamiltonian H(t)= $H_0(t)$ + V(t) and construct the evolution operator U_0 corresponding to the largest term of the Hamiltonian [$H_0(t)$]

$$i\hbar\frac{\partial U_0(t,0)}{\partial t} = H_0(t)U_0(t,0).$$

Next, we construct states in the interaction picture and construct the corresponding Schrodinger equation

$$\psi^I(t) = U_0(0,t)\psi(t). \qquad i\hbar\frac{\partial \psi^I}{\partial t} = V^I(t)\psi^I(t),$$

$$V^I(t) = U_0(0,t)VU_0(t,0).$$

The evolution operator in the interaction picture reads

$$U^I(t,0) = \mathcal{T}e^{-(i/\hbar)\int_0^t dt' V^I(t')}, \qquad i\hbar\frac{\partial U^I(t,0)}{\partial t} = V^I(t)U^I(t,0).$$

U^I has the solution

$$U^I(t,0) = I + \left(\frac{-i}{\hbar}\right)\int_0^t dt' V^I(t') + \left(\frac{-i}{\hbar}\right)^2 \int_0^t dt_1 V^I(t_1)\int_0^{t_1} dt_2 V^I(t_2) + ...$$

The perturbative evolution operator is given by

$$U(t,0) = U_0(t,0)U^I(t,0).$$

The method reduces to the usual rotating wave approximation when the drive term is the one with largest amplitude

Driven Fermi chain

Consider a chain of spinless fermions with nearest neighbor hopping and density-density interactions

$$H = -J\sum_j \left(c_j^\dagger c_{j+1} + \text{h.c.}\right) + V_1 \sum_j \hat{n}_j \hat{n}_{j+1} + V_2 \sum_j \hat{n}_j \hat{n}_{j+2}$$

We drive the chain by making V_1==$V_1(t)$ a periodic function of time characterized by an amplitude V_1 and frequency $\omega_D = 2\pi/T$, Where T is the time period of the drive

$V_1(t) = V_1 \cos \omega_D t$ *for cosine drive*

$V_1(t) = +(-)\ V_1$ *for* $t <=(>)\ T/2$ *for square pulse*

In the high drive amplitude regime, one can obtain the Floquet Hamiltonian using FPT

$$H_F^{(1)} = \sum_{j=1..L} \hat{n}_j (V_0 \hat{n}_{j+1} + V_2 \hat{n}_{j+2}) - J\sum_{j=1..L} [(1-\hat{A}_j^2) + f(\gamma_0)\hat{A}_j^2] c_j^\dagger c_{j+1} + \text{H.c.},$$

$$\hat{A}_j = (\hat{n}_{j+2} - \hat{n}_{j-1}),\ \gamma_0 = V_1 T/(4\hbar)$$

$f(\gamma_0) = J_0[2\gamma_0/\pi]$ *Cosine protocol*

$f(\gamma_0) = \gamma_0^{-1} \sin\gamma_0 \exp[i\gamma_0 A_j]$ *Square Pulse protocol*

Realization of a Hamiltonian hosting HSF within first order Floquet at frequencies for which f(T)=0

$$\omega_m^* = V_1/(\hbar\zeta_m) \quad \text{for cosine protocol}$$
$$= V_1/(2\hbar m) \quad \text{for square-pulse protocol.}$$

$$H_F^{(1)} = \sum_{j=1..L} \hat{n}_j (V_0 \hat{n}_{j+1} + V_2 \hat{n}_{j+2})$$
$$- J \sum_{j=1..L} [(1 - \hat{A}_j^2)\, c_j^\dagger c_{j+1} + \text{H.c.},$$

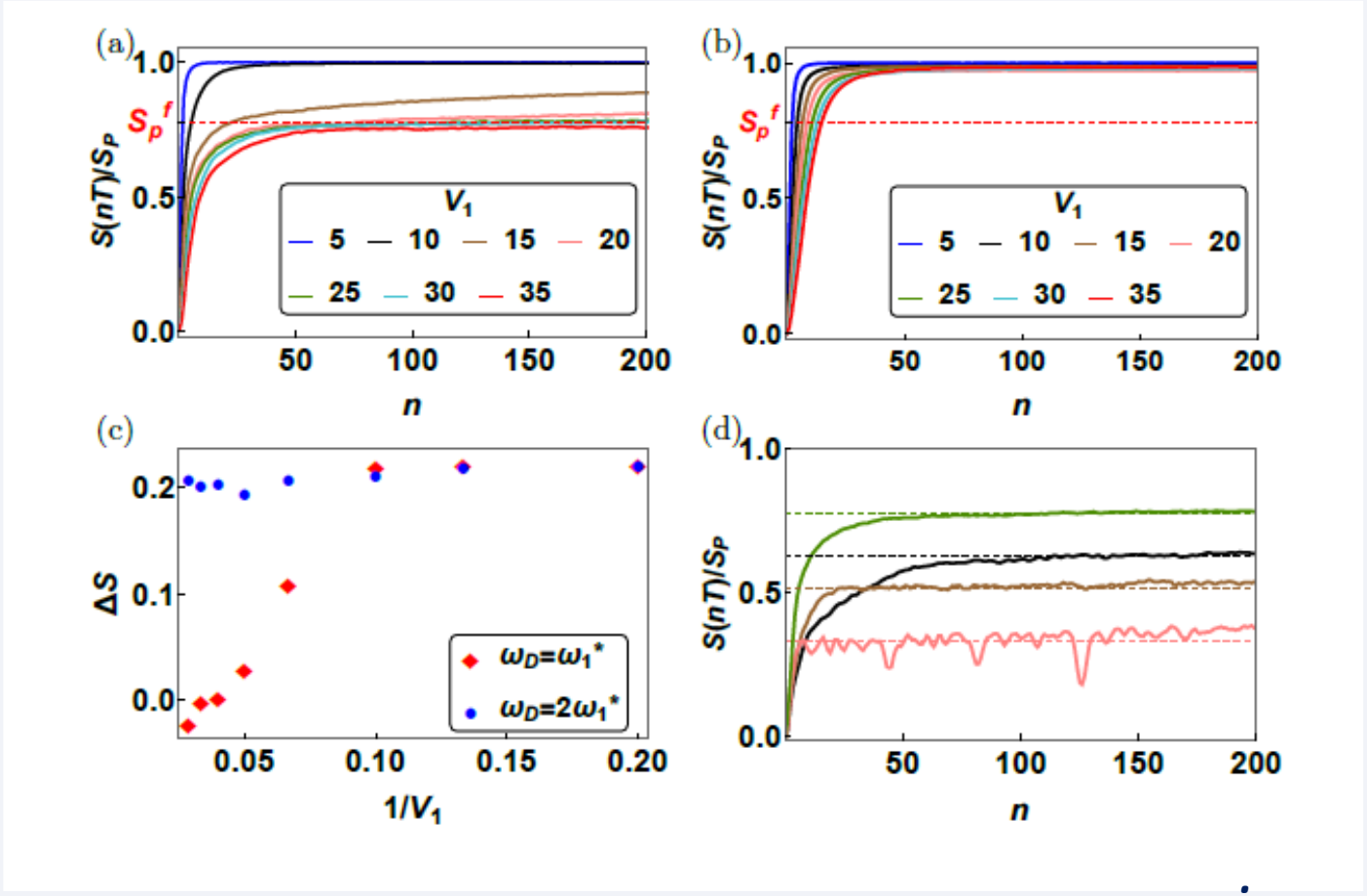
Higher order terms is expected to destroy the fragmentation.

However for large drive amplitudes such terms are small.

This leads to a large prethermal region where signatures of HSF can be seen.

The extent of the prethermal regime showing signatures of HSF can be controlled by tuning drive parameters

Entanglement entropy of the driven chain: Square-pulse protocol



$S_p = \ln D_{system} - 1/2$

$S_p^f = \ln D_{fragment} - 1/2$

(a bit more complicated for symmetry-resolved sectors)

Entanglement entropy saturates to the Page value of the sector (S_p^f) instead of that of the system (S_p) at special drive frequencies for an exponentially large prethermal timescale

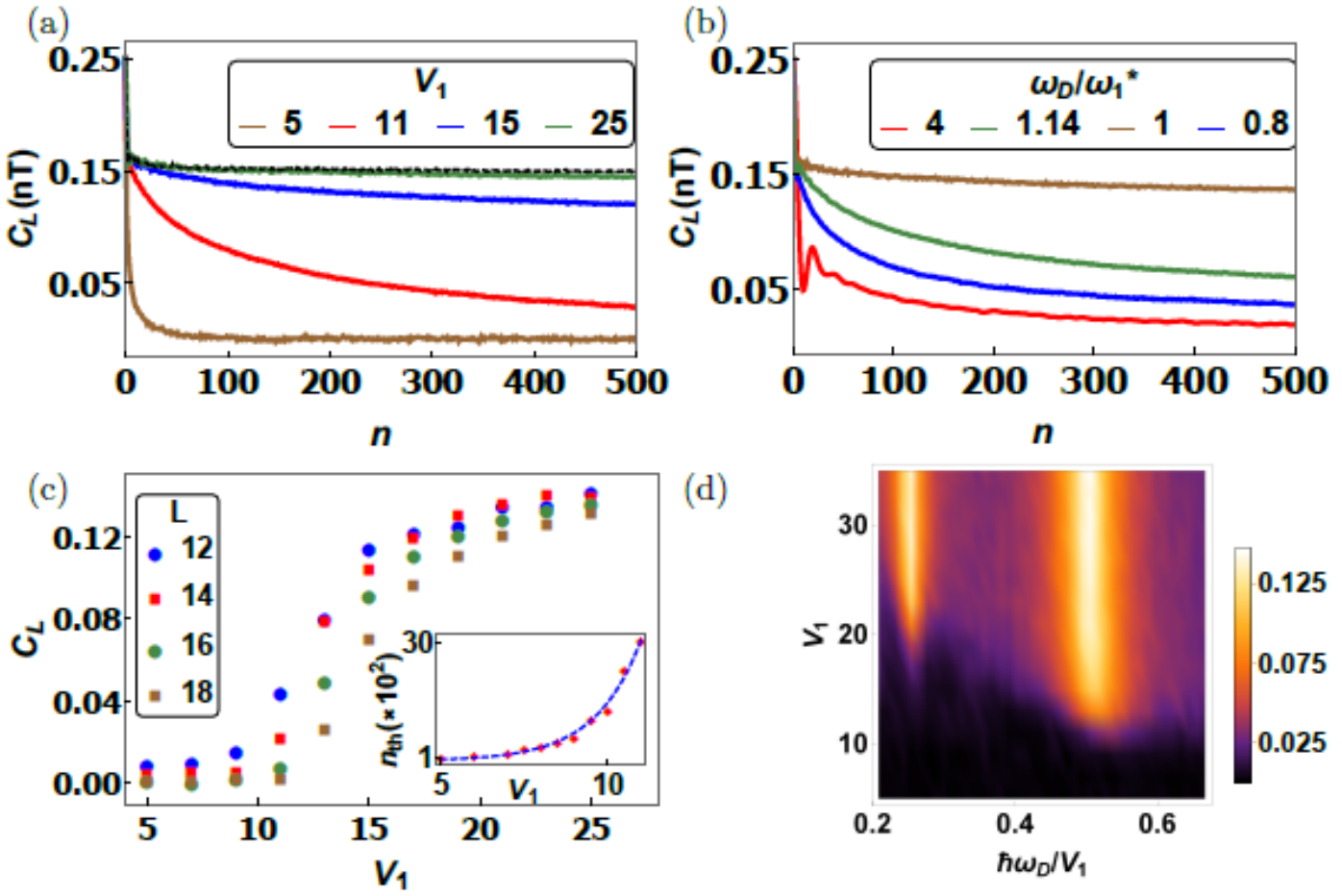
Entanglement entropy saturates to an Initial state dependent value.

Signature of prethermal HSF.

Fermion density-density autocorrelators: Square pulse protocol

$C_L(nT) = \langle (n_{L/2}(nT) - 1/2)\,(n_{L/2}(0) - 1/2) \rangle$

L=16



The system retains memory of the initial state for the large number of drive cycles

Thermalization time and hence the extent of the prethermal regime increases exponentially as a function of the drive amplitude showing stability of prethermal fragmentation.

Near the threshold value, the extent of the prethermal regime grows exponentially with drive amplitude

Dynamics of the frozen state: Continuous protocol

$$\chi_j(nT) = \langle \psi_f(nT) | \hat{n}_j \hat{n}_{j+2} | \psi_f(nT) \rangle$$

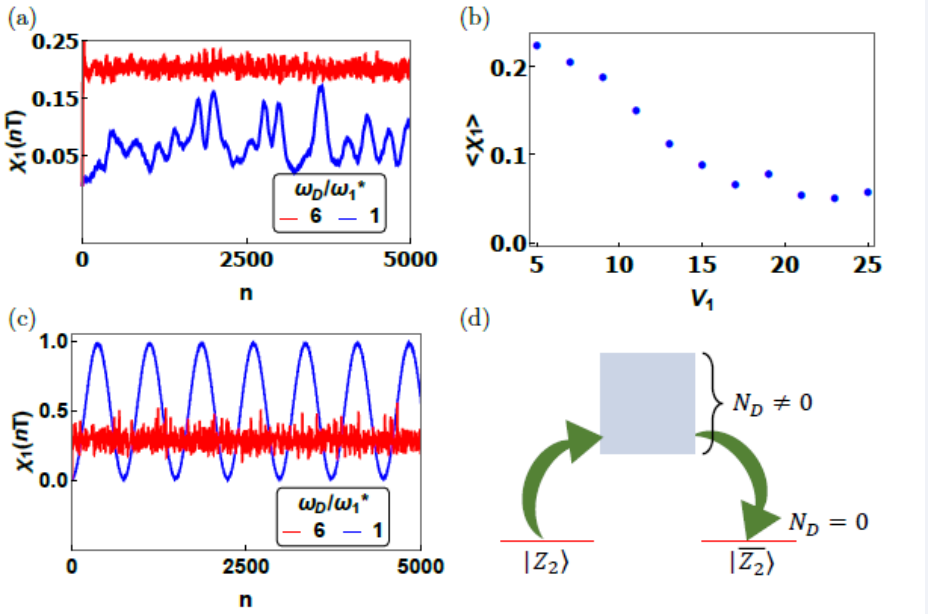


FIG. 3: (a) Plot of $\chi_1(nT)$ as a function of n for $V_1 = 19$ at ω_1^* (blue curve) and $6\omega_1^*$ (red curve) starting from a random frozen state showing lack of ETH predicted thermalization at ω_1^*. (b) Plot of $\langle \chi_1 \rangle$ as a function of V_1 at ω_1^*; $\langle \chi_1 \rangle$ stays close to its initial value for large V_1 which is consistent with prethermal HSF. (c) Same as in (a) but for initial $|Z_2\rangle$ state showing slow oscillations at ω_1^*. (d) Schematic diagram for the Floquet quasienergies showing doubly degenerate $|Z_2\rangle$ and $|\bar{Z}_2\rangle$ with $N_d = 0$ and other states with $N_d \neq 0$. The arrows indicate transition to $|\bar{Z}_2\rangle$ from $|Z_2\rangle$ using intermediate states with $N_d \neq 0$ leading to slow oscillations. For all plots $V_0 = 10V_2 = 2$, $L = 14$, and all energies are scaled in units of J.

Oscillatory dynamics of frozen states due to residual terms in H_F beyond $H_F^{(1)}$

This requires Z_2 symmetry. Two states with N_d=0 which are eigenstates of H_F^1 with same quasienergy.

In addition, it requires fragmentation so that starting from the Z_2 state (which correspond to N_d=0), the system does not spread out in Hilbert space; the dynamics receives most significant contribution from states with N_d = 1.

Since χ_1=0 (1) for Z_2 and Z'_2, the oscillations occur between 0 and 1.

The oscillation time scale is determined by higher-order terms in H_F and is the energy split between bonding and antibonding states due to tunneling to N_d = 1 sector.

$$|\psi_{B,A}\rangle \equiv |\mathbb{Z}_2\rangle \pm |\bar{\mathbb{Z}}_2\rangle. \qquad H_F|\psi_{B,A}\rangle \approx \hbar(\alpha_s \pm \alpha_d)|\psi_{B,A}\rangle$$

$$\chi_1(nT) \approx \sin^2(\alpha_d nT)$$

This dynamics of frozen states has no analogue in standard HSF in equilibrium

Out-of-time correlators

$C(mT) = \langle [n_j(mT), n_i(0)]^2 \rangle = 2(1 - F(mT))$

$F(mT) = \langle n_j(mT)\, n_i(0)\, n_j(mT)\, n_i(0) \rangle$

Provides information about spread of correlation between sites of the driven chain

Generic properties

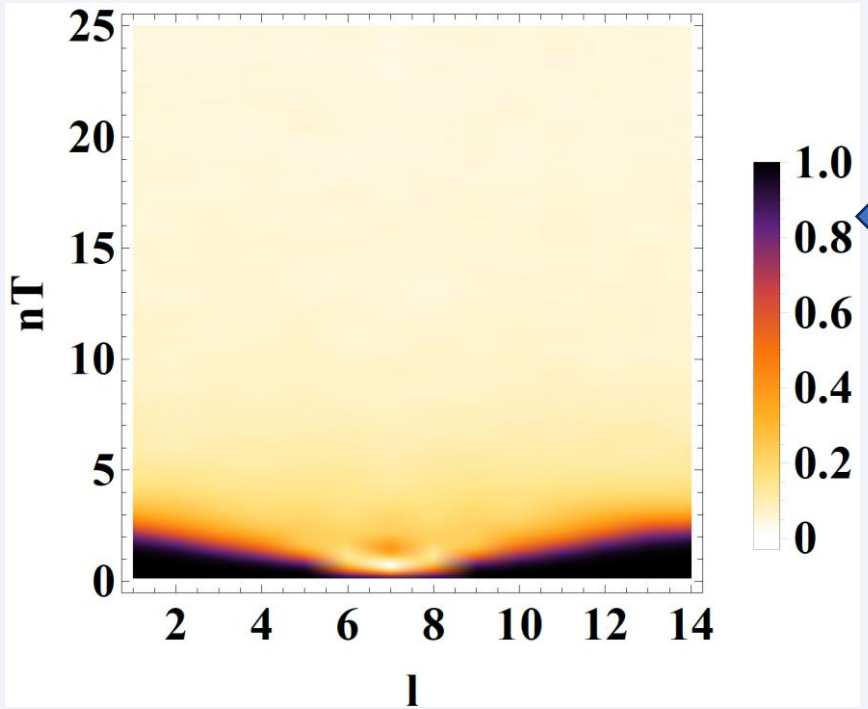
At the initial time, C=0 and F=1.

As the operator spreads, F -> 1 and C->0

This spread of this correlation at initial times is linear and is bounded by the Lieb-Robinson velocity.

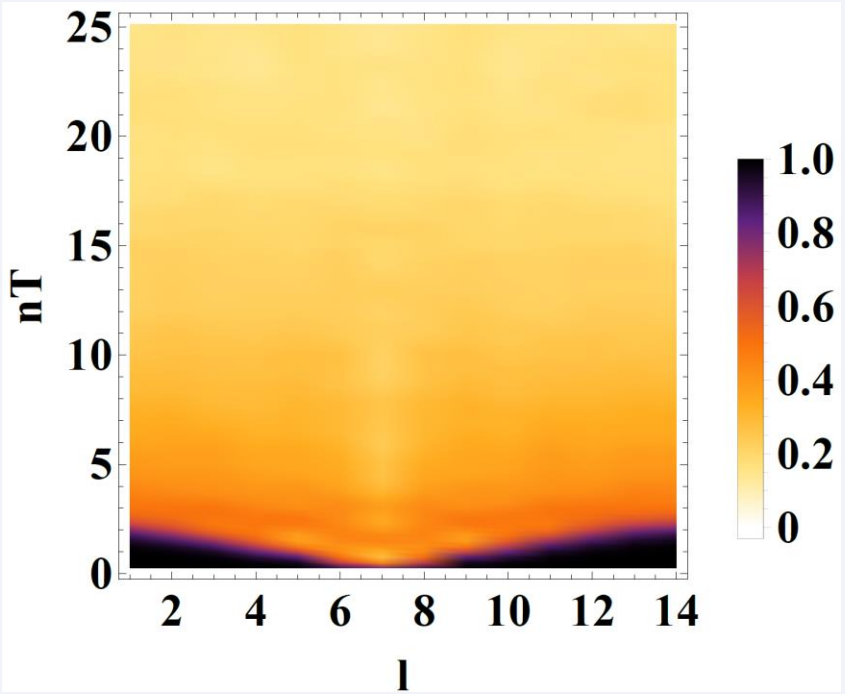
For ergodic systems, F shows a rapid convergence to its steady state value (F ~ 1).

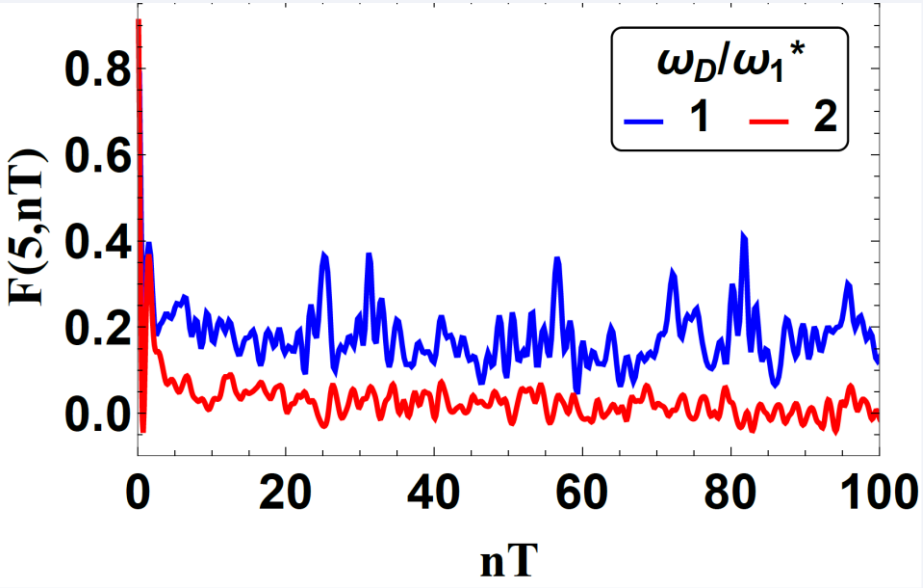
For fragmented systems, the late time values of F depends on the initial condition.



Away from special frequency

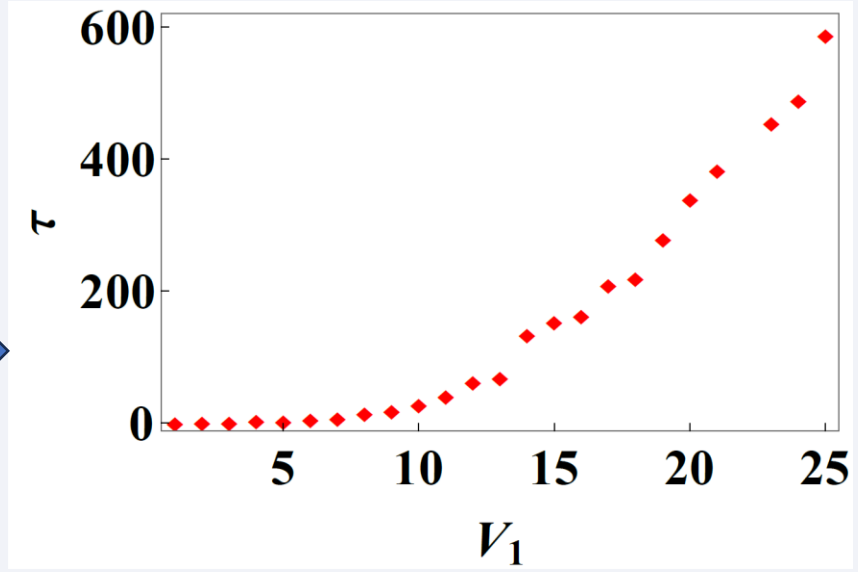
At a special frequency





Dynamics of F

No. of drive cycles after which F reaches near-zero (0.01) value on the farthest site.



A non-Hermitian quantum many-body model

Non-Hermitian quantum systems

It is expected that a generic non-Hermitian Hamiltonian would yield complex eigenvalues.

Is there a sufficient condition which guarantees reality of its eigenspectra

Global symmetries (such as PT) are not enough; they ensure that eigenvalues are either real or occur in complex pairs.

We study the finite Nelson-Hatano chain at half filling and try to provide an answer to this question

$$\mathcal{H} = \sum_{i=1}^{L} \left[(J-\gamma) c_i^\dagger c_{i+1} + (J+\gamma) c_{i+1}^\dagger c_i + V_1 n_i n_{i+1} \right]$$

Global symmetries at half-filling

Symmetry	Fermion operators	Hamiltonian
Parity(P)	$c_j \dashrightarrow c_{L-j+1}$	$P^{-1} H P = H^+$
Particle-Hole (C)	$c_j \dashrightarrow c_j^+ (-1)^j$	$C^{-1} H C = H^+$
Translation (R)	$c_j \dashrightarrow c_{[(j+1) \bmod L]}$	$R^{-1} H R = H$

$[PC, H]=0$

Requires periodic(anti-periodic) boundary condition for L=4n+2 (4n)

Properties of the model at V/J, V/γ>>1

The effective model reduces to non-Hermitian version of the constrained hopping model.

$$\mathcal{H} \approx \mathcal{H}_f = \sum_{i=1}^{L} \hat{P}_i \left((J-\gamma) c_i^\dagger c_{i+1} + (J+\gamma) c_{i+1}^\dagger c_i \right) \hat{P}_i$$

$$\hat{P}_i \equiv 1 - (\hat{n}_{i-1} - \hat{n}_{i+2})^2$$

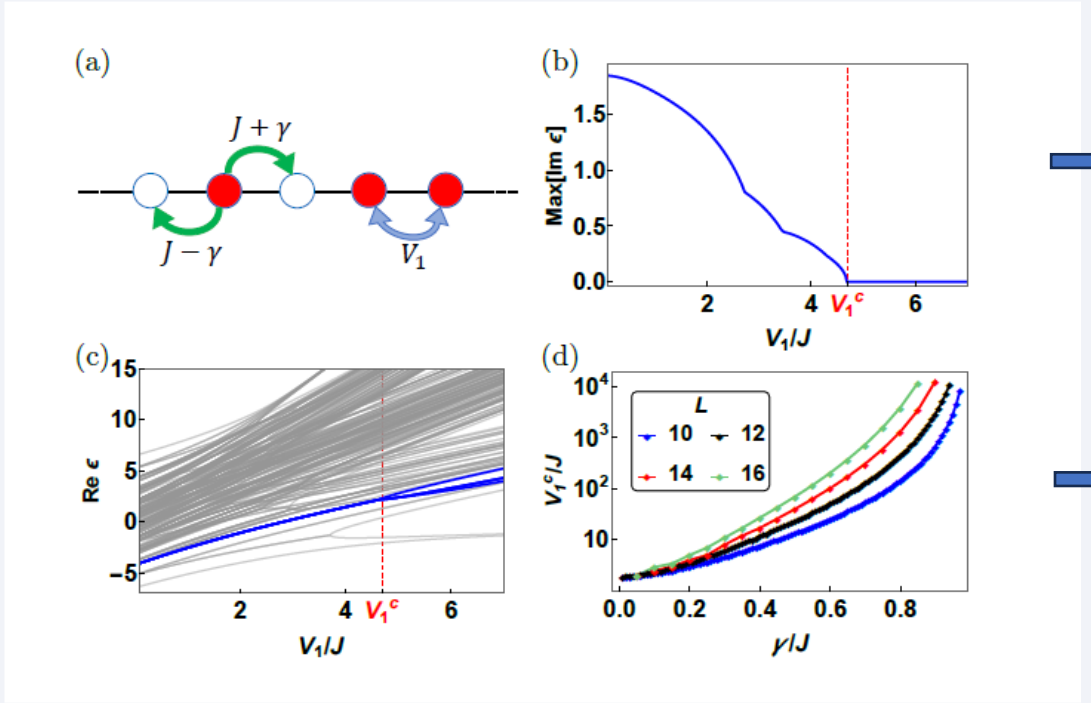
Hopping is allowed only if the total number of neighboring particles, N_d, does not change.

The model exhibits HSF in the same way as its Hermitian counterpart.

Claim: In the fragmented limit, all the eigenvalues of the fragmented Hamiltonian are real for J>γ

Numerical results (See also Zhang et al PRB 106 L121102 (2022))

Pictorial representation of the Hamiltonian



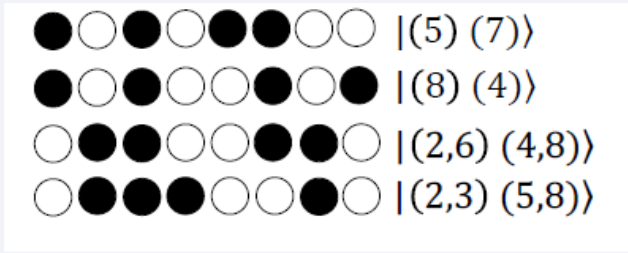
Existence of V_1^c above which all eigenvalues are real

Appearance of the first exceptional point in the spectrum making two eigenvalues complex for $V<V_1^c$

Behavior of V_1^c as a function of γ/J and L

Understanding the reality of eigenspectrum: fragmented limit

In this limit, the Fock states in the Hilbert space are conveniently labelled by the position of their particle and hole defects: $|(i_1,i_2\ldots i_n);(j_1,j2\ldots j_n)>$



Due to half filling there are equal number of particle and hole defects.

Moreover two adjacent particle/hole defects can only have (i_1,i_2) or (j_1,j_2) = (even,odd) or (odd,even) while adjacent particle and hole defects must have (i_1,j_1)= (odd, odd) or (even, even).

The action of the Hamiltonian which changes the position of the defects can be viewed as a many-body hopping problem in the Fock space.

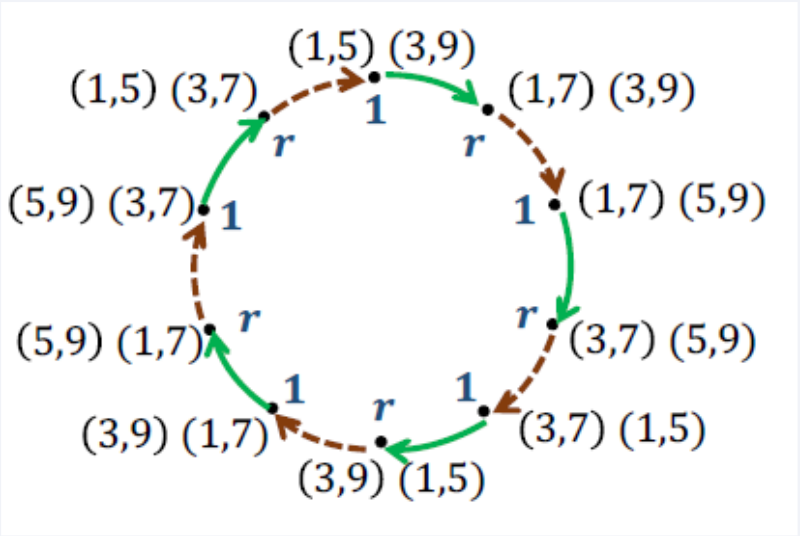
The defects can not move through each other and their number N_d and parity of their site coordinates remain conserved.

(1,3) to (1,5) hop for L=6

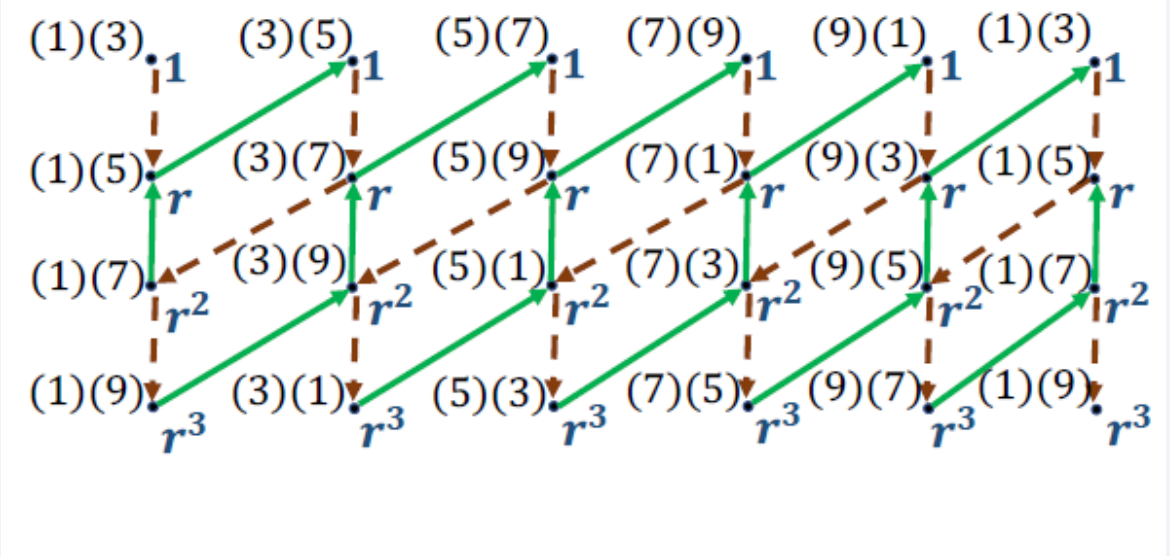
The positions of the defects changes by 2 mod L due to any hop.

The right [left] hops have amplitude $J_2= J+\gamma$ [$J_1= J-\gamma$].

Many body hopping problems: Connectivity diagram



L=10 N_d=2



L=10 N_d=1

The action of the Hamiltonian generates closed loops in Fock space as shown above.

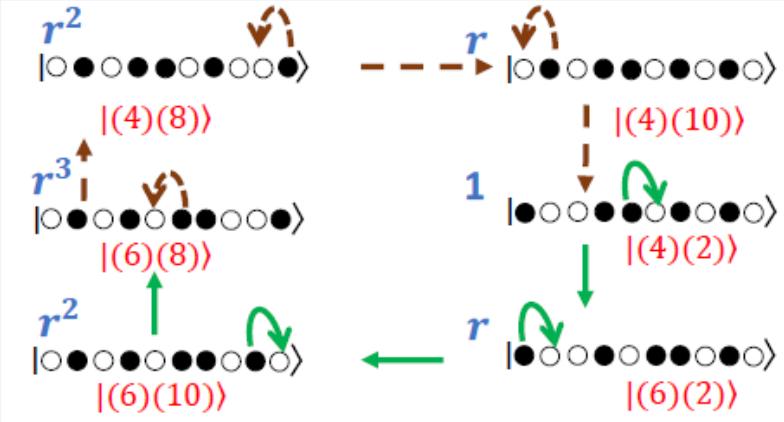
The many-body hopping problem is an example of geometry with periodic boundary condition.

For all closed loops, in the fragmented limit, there are equal number of right and left hops.

What do the defects do as one moves through a closed loop

Possibility 1: The particle/hole defects retraces their path; for example

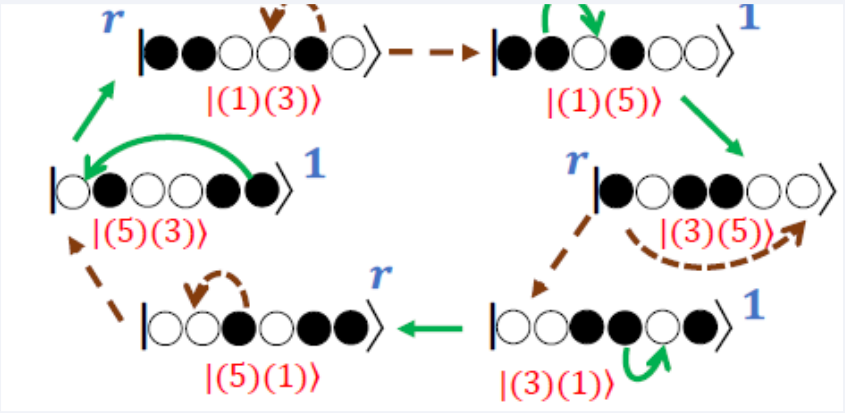
$i_1 \rightarrow i_2 \rightarrow i_3 \ldots\ldots i_3 \rightarrow i_2 \rightarrow i_1$



Since the reverse of a right hop is a left hop and vice-versa, one must have equal number of J_2 and J_1.

Possibility 2: The defects move across the chain and gets back to its original position using PBC.

$i_1 \rightarrow i_2 \rightarrow i_3 \ldots\ldots i_{L-1} \rightarrow i_L \rightarrow i_1$



This can only happen if other defects perform the same circular motion (defects can not cross)

Particle (hole) defect moving to the right is a J_2(J_1)

There must be equal number of J_2 and J_1.

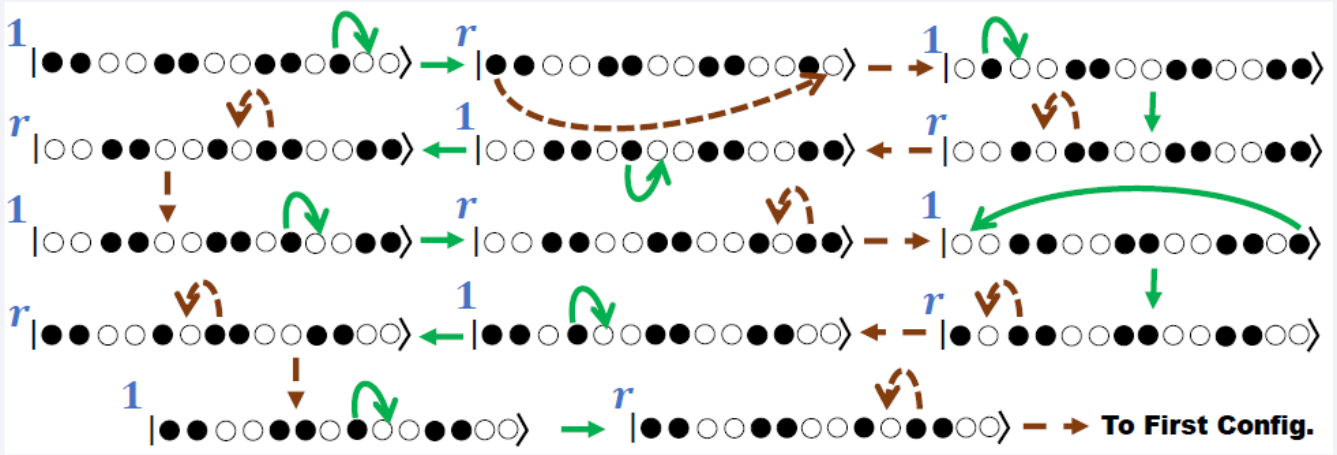
Possibility 3:

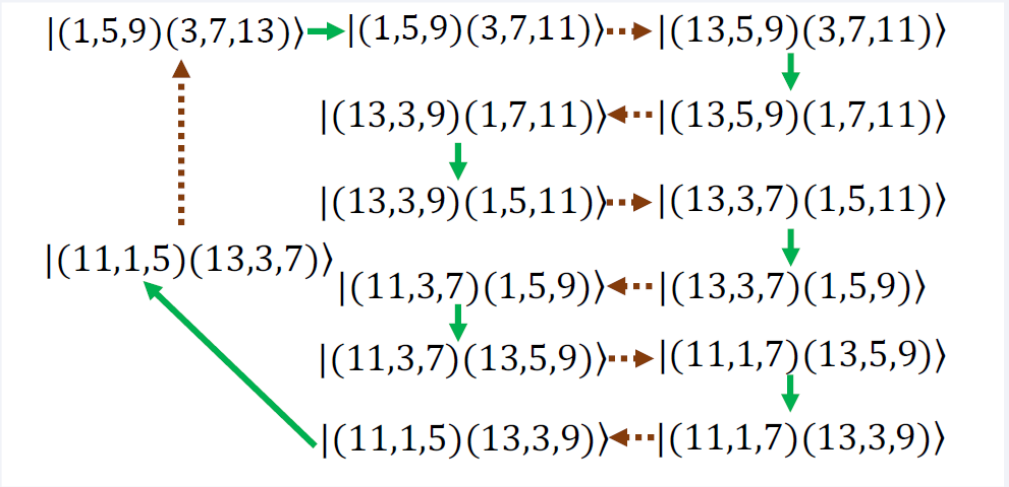
Any single particle/hole defects do not retrace their paths or move through an entire loop.

Permutation of indices of defects leads to original configuration.

The sum of the sites traversed by particle and hole defects are L and they move in the same direction

Once again, one finds equal number of right and left hops.

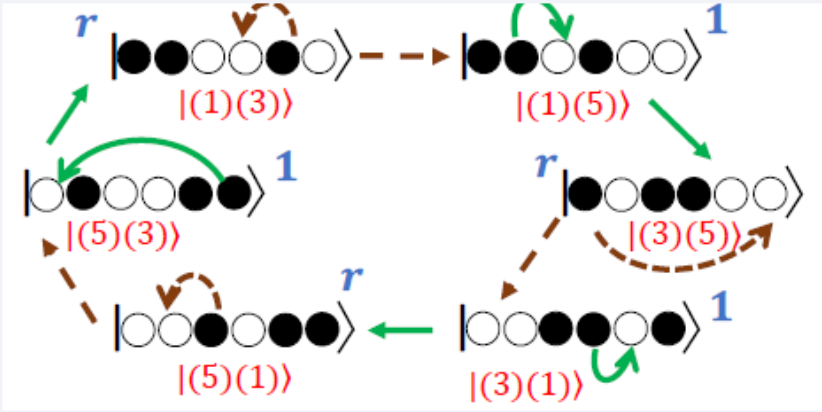




L=14 N=7

Reality of eigenspectrum

*In the fragmented limit, since there equal number of right and left hops, one can think of a **many-body similarity transformation** **even in a periodic geometry**.*

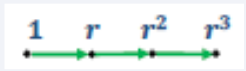


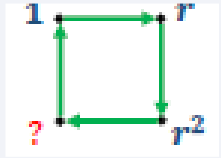
Step1: Think of Fock states as sites on a lattice.

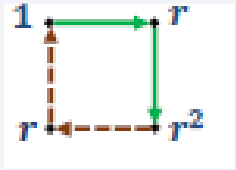
Step2: If a state |2> is connected to a state |1> by a right particle hop assign a factor r; for left hops assign 1/r.

$$|i\rangle \to \lambda_i |i\rangle, \quad \langle i| \to (1/\lambda_i)\langle i|,$$

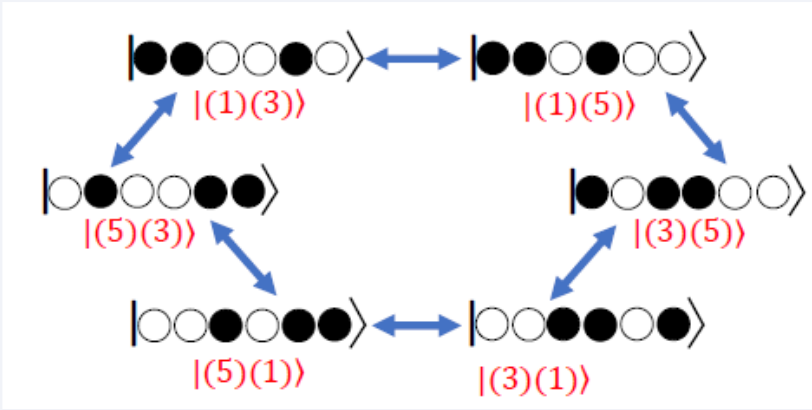
Step3: Note that step 2 can be carried out keeping PBC if and only if there are equal number of right and left hops.







Step4: Choose r= $(J_2/J_1)^{1/2}$. One can show that this maps the many-body hopping problem to a Hermitian one with $J_{eff} = (J^2-\gamma^2)^{1/2}$



Reality of eigenspectrum in the fragmented limit

Lowering the interaction strength

For typical finite sized system, one expects a finite gap between any two eigenstates so that the reality is protected.

$$\mathcal{H}_2 = \begin{pmatrix} l & m_1 + m_2 \\ m_1 - m_2 & -l \end{pmatrix}$$

$$m_{1,2} \sim \mathcal{O}((J,\gamma)^2/V_1) \qquad l \sim \sqrt{J^2 - \gamma^2}/e^{cL}$$

Typically the exception to the above situation occurs due to degeneracies induced by global symmetries such PC, T == G. For these states l=0.

→

So let us consider two such degenerate states related by symmetry: $|\psi\rangle$ and $|\varphi\rangle = G|\psi\rangle$

↓

Let us consider the matrix element of H between two such states

$$\langle\langle\psi|\mathcal{H}|\phi\rangle = \langle\langle\psi|\mathcal{H}\mathcal{G}|\psi\rangle = \langle\langle\psi|\mathcal{G}\mathcal{H}|\psi\rangle = \langle\langle\phi|\mathcal{H}|\psi\rangle$$

←

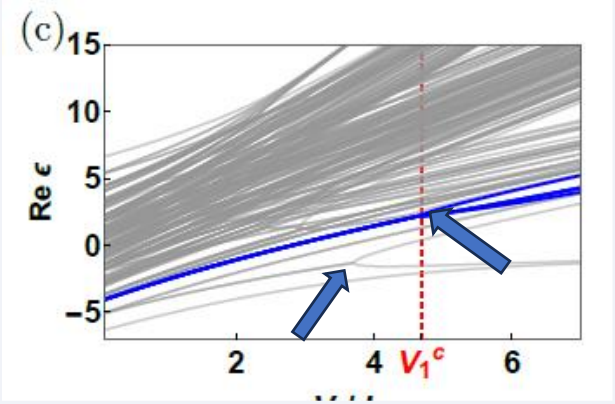
This Hermitian structure induced by global symmetries protect the reality of eigenspectrum at large V

Existence of a finite critical V for translationally invariant half-filled chains with periodic (anti-periodic) boundary for L=4n+2(4n)

Detection of critical V

At critical interaction strength, typically two states which are not related by any symmetry coalesce at an exceptional point.

Upon further lowering of V, more such points appear and there is a cascade of transitions across each of which two real eigenvalues coalesce and give rise to two complex eigenvalues

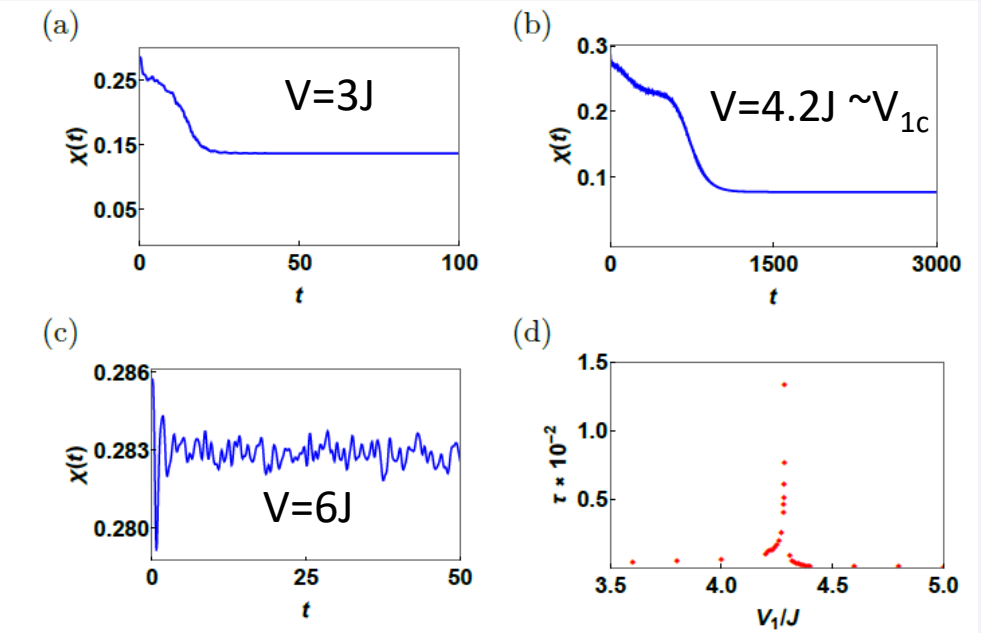


The first exceptional point at critical value of V can be detected by measuring equal time correlation functions starting from a random Fock state $|\psi(0)\rangle = \sum_m c_m |\phi_m\rangle$

The correlation function can be written as

$$\chi(t) = \langle\psi(t)|\hat{N}_d|\psi(t)\rangle/L$$

$$|\psi(t)\rangle = \frac{e^{-i\mathcal{H}t/\hbar}|\psi(0)\rangle}{||e^{-i\mathcal{H}t/\hbar}|\psi(0)\rangle||} = \frac{\sum_m c_m(t)|\phi_m\rangle}{\sqrt{\sum_{m,n} c_m^*(t)c_n(t)\langle\phi_m|\phi_n\rangle}}$$



γ=0.2 J and L=14

The time taken by the equal time correlation to reach its steady state diverges at the first exceptional point.