Emergent strong zero mode through local Floquet engineering.

Bhaskar Mukherjee

UCL, London

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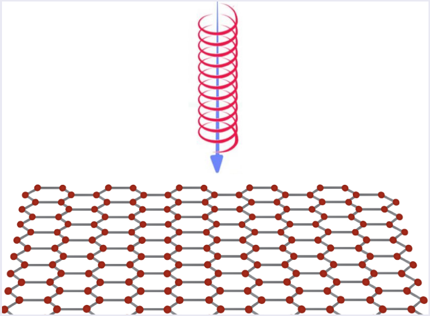

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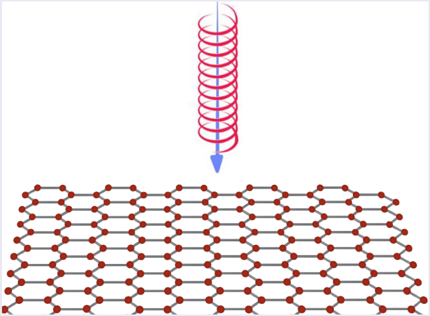

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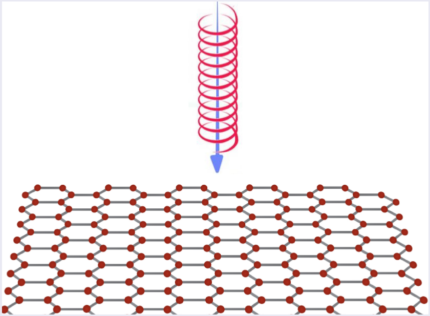
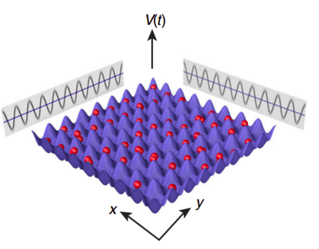


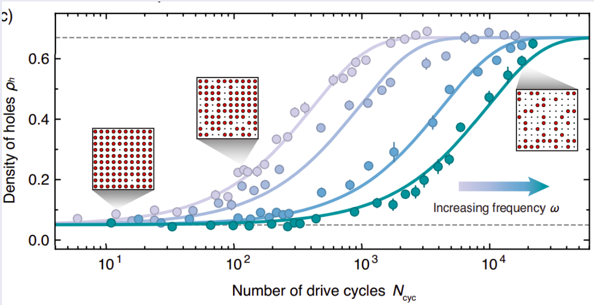
- Photovoltaic Hall effect in graphene, Oka et al, PRB (2009).

- Floquet-Bloch states in Bi_2Se_3, Science (2013).

Floquet prethermalization

- $e^{-iH_F T} \simeq e^{-iH_F^{(n)} T}$ where $H_F = \sum_n H_F^{(n)} = \sum_n T^n \Omega_n$ (FM expansion)
- Exponentially slow heating.
- Abanin et al, PRL (2015), T Mori et al, PRL (2016).

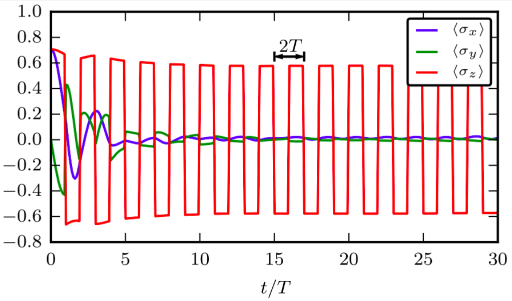




Experiment in Bose-Hubbard system, PRX (2020).

Floquet time crystals (FTC)

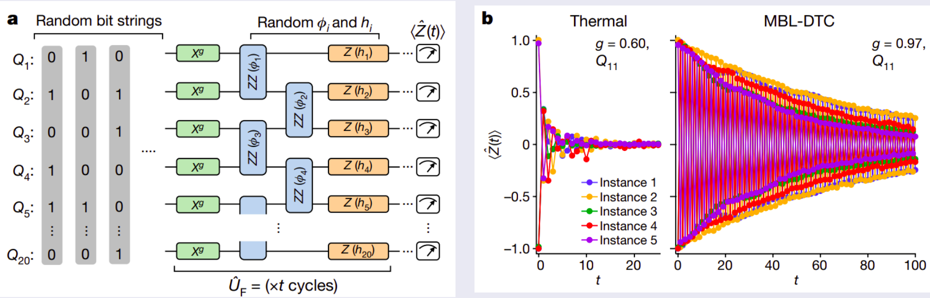
- $\langle\psi(t)|O|\psi(t)\rangle \neq \langle\psi(t+T)|O|\psi(t+T)\rangle$



- Khemani et al, PRL(2016), V. Else et al, PRL(2016).

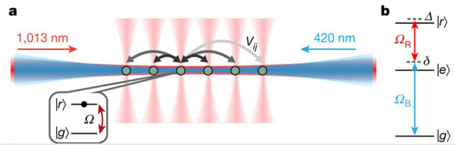
- Long-range order in Floquet eigenstates (needs MBL).

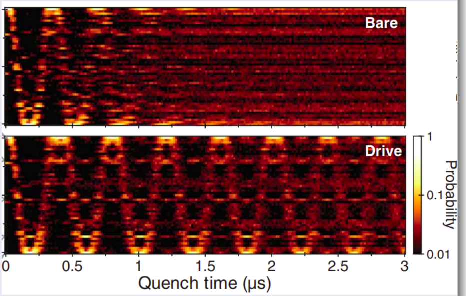
Experiments (MBL-FTC in Quantum processor)



Xiao Mi et al, Nature (2022).

Experiments (no disorder)





- driven Rydberg atom array, Bluvstein et al, Science (2021).
- non-perturbative parameters (Floquet ETH should hold).
- only two Floquet eigenstates are cat like
- A. H., J. Y.D., **BM**, GX Su, J. C. H., Z. P., PRB (2022)

6. QUANTUM COMPUTING

Unpaired Majorana fermions in quantum wires

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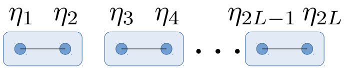

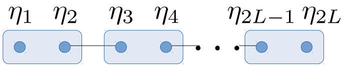

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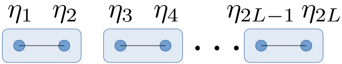

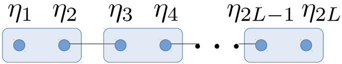

$$= \frac{1}{2} \sum_i [(w + \Delta)\sigma_i^x \sigma_{i+1}^x + (w - \Delta)\sigma_i^y \sigma_{i+1}^y - \mu \sigma_i^z]$$ (**ordered phases**)

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- **Non-integrable model ???**

Outline

- Dynamic localization in globally driven systems.
- Local driving
- Emergence of SZM
- Entanglement Structure

- Dynamical localization of single electron (Dunlap & Kenkre, PRB (1986))

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PHYSICAL REVIEW B **82**, 172402 (2010)

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- Bukov et al, Advances in Physics, 2015.

  Rydberg atom array, **BM** et al, PRB (2020)

  Emergent conservation laws, Asmi Haldar et al, PRX (2021).

Local driving

Staggered Heisenberg model

- $H_0 = \sum_i (-1)^i S_i . S_{i+1} - h \sum_i S_i^z$.

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Haldane phase
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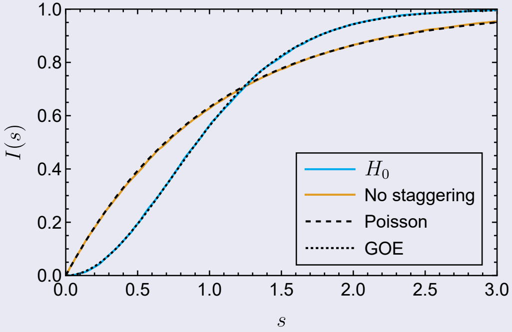

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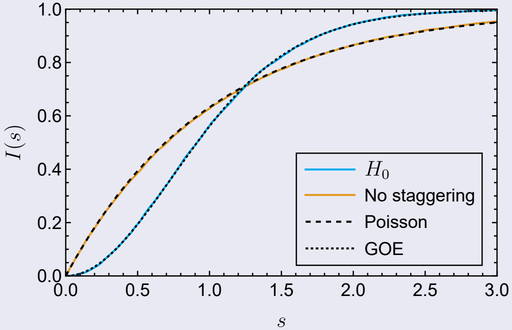
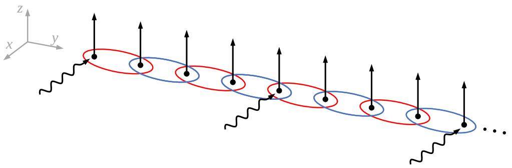
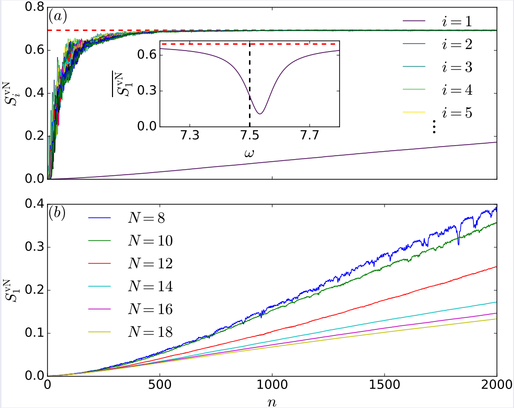


- $H_D(t) = \gamma Sgn(\sin(\omega t)) \sum_{i \in s_d} S_i^x$; $H(t) = H_0 + H_D(t)$.



boundary site driving (Absence of freezing)

- $H_F = H_F^{(0)}[\mathcal{O}(\frac{1}{\gamma})] + H_F^{(1)}[\mathcal{O}(\frac{1}{\gamma^2})] + H_F^{(2)}[\mathcal{O}(\frac{1}{\gamma^3})] + \cdots$
- $[S_1^x, H_F^{(0)}] = 0$ for $\omega = \frac{\gamma}{2n}$ **Dynamical freezing**
- Exact numerics ($\gamma = 15,\ h = 1,\ \omega = 7.53.$)



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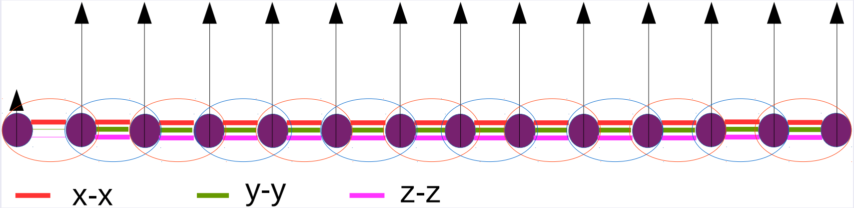

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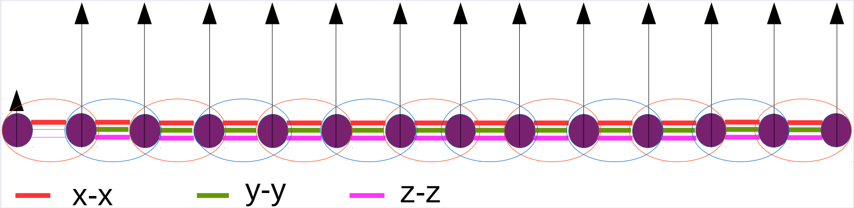
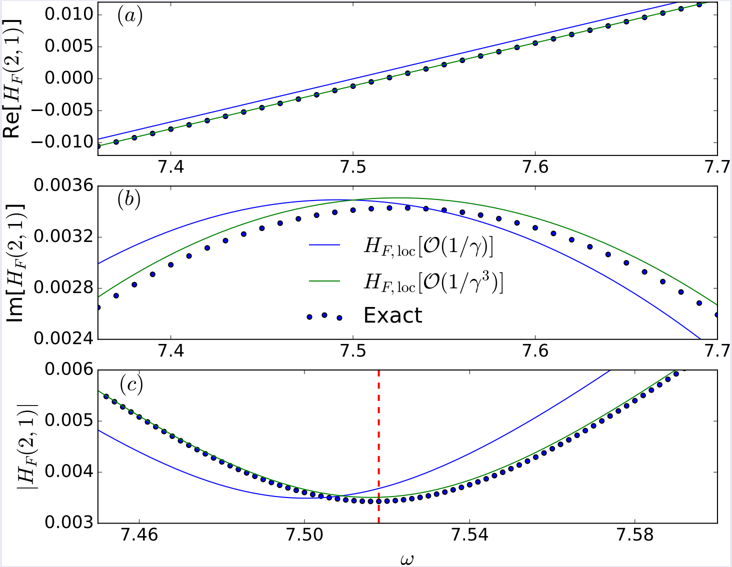


- $[S_1^x, H_{F,\text{loc}}] \neq 0$ (Boundary site is not even perturbatively frozen).

Comparison with exact numerics

Check $H_F(1,2)$; $|1\rangle = |\rightarrow\rightarrow\rightarrow\cdots\rangle$ and $|2\rangle = |\leftarrow\rightarrow\rightarrow\cdots\rangle$



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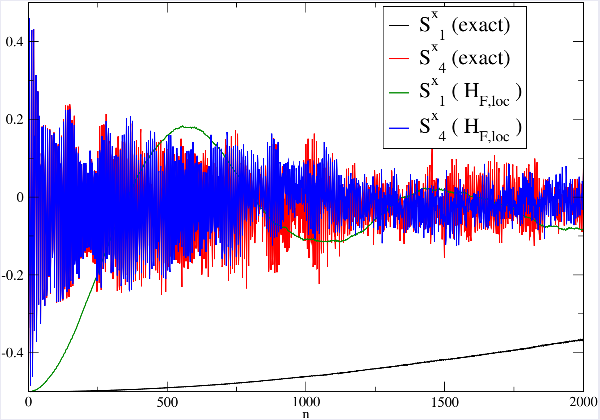

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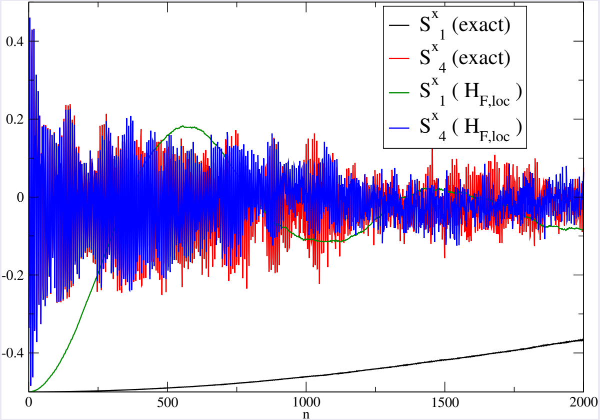


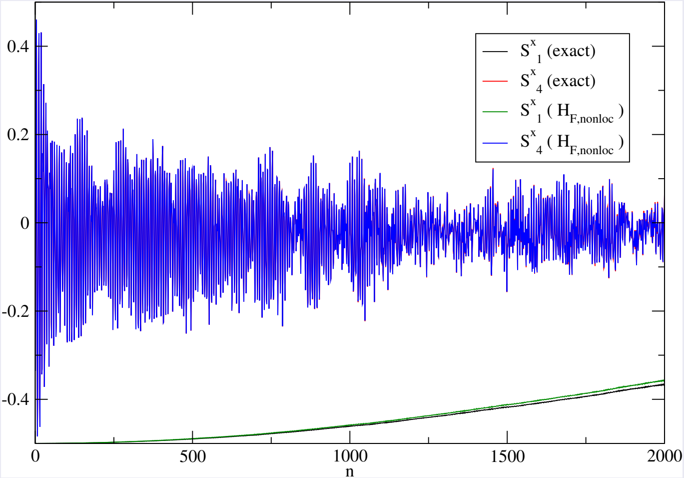
- Why the boundary relaxation is so slow ?

Role of non-local terms : $H^{(2)}_{F,\ nonloc}$

$$
\begin{aligned}
H^{(2)}_{F,\mathrm{nonloc}} = \beta(&3S_1^z S_3^z + 4S_1^z S_2^y S_3^y S_4^z - 4S_1^z S_2^y S_3^z S_4^y - 4S_1^z S_2^x S_3^z S_4^x \\
&+ 4S_1^z S_2^x S_3^x S_4^z + 3S_1^y S_3^y + 12hS_1^y S_2^y S_3^z S_4^y - 4S_1^y S_2^z S_3^z S_4^y \\
&- 12hS_1^y S_2^z S_3^y - 4S_1^y S_2^z S_3^y S_4^z - 4S_1^y S_2^x S_3^y S_4^x + 4S_1^y S_2^x S_3^x S_4^y) \\
&+ \eta(3S_1^z S_3^y - 12hS_1^z S_2^y S_3^z - 4S_1^z S_2^z S_3^z S_4^y + 12hS_1^z S_2^z S_3^y \\
&+ 4S_1^z S_2^z S_3^y S_4^z + 4S_1^z S_2^x S_3^y S_4^x - 4S_1^z S_2^x S_3^x S_4^y + 3S_1^y S_3^z \\
&+ 4S_1^y S_2^y S_3^y S_4^z + 4S_1^y S_2^y S_3^z S_4^y + 4S_1^y S_2^x S_3^z S_4^x - 4S_1^y S_2^x S_3^x S_4^z) \\
&+ \cdots ,
\end{aligned}
$$

Role of non-local terms : $H^{(2)}_{F,\ nonloc}$

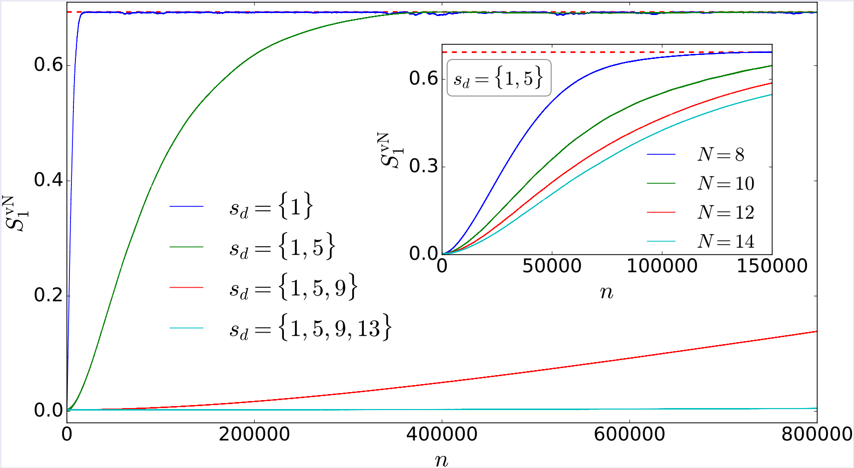
- Nonlocal terms play a pivotal role to freeze the edge spin.



Absence of freezing, Verdeny et al, PRL (2013).

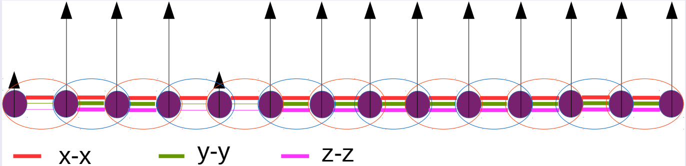
Multisite driving : optimal protocol

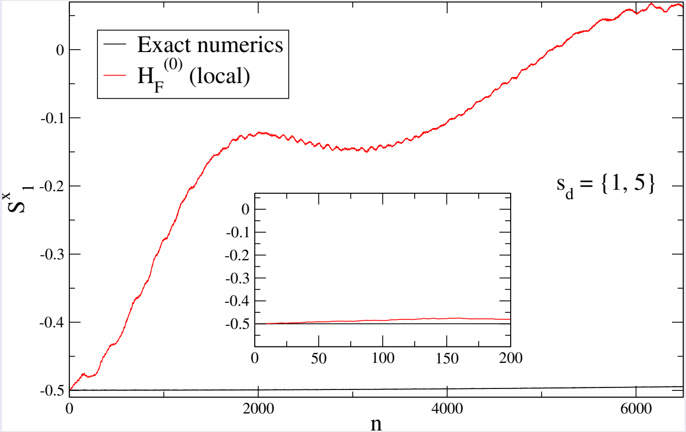
- Drive every fourth site with resonant frequency.



- $\gamma = 15$, $h = 1$, $\omega = 7.53$

Multisite driving : nonlocal terms are important again





Emergence of strong zero mode

SZM : Formal definition

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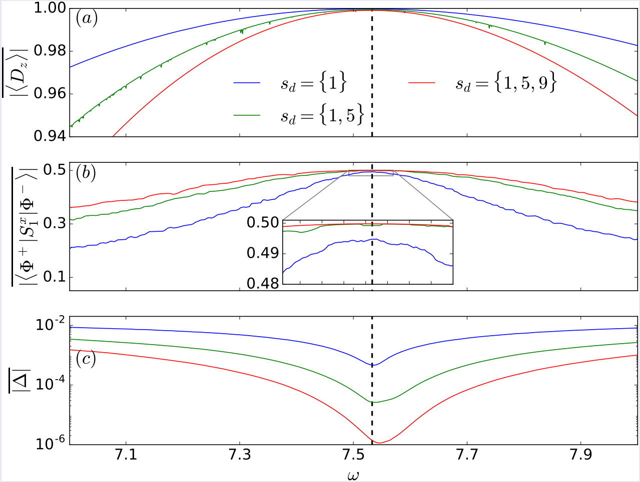
- An operator Ψ : $\Psi^2 = \mathbb{I}$, $\{\Psi, D\} = 0$.
- Ψ satisfies : $||[\Psi, H]|| = \exp(-\alpha L)$
- Consequence : $\Delta \sim \exp(-\alpha L)$.

Signature of SZM in our locally driven system

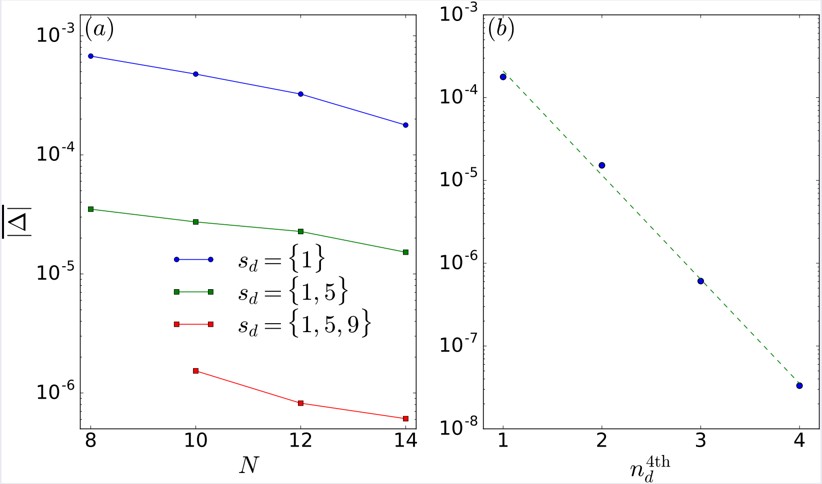
$H_F|\Phi^\pm\rangle = \epsilon^\pm|\Phi^\pm\rangle$, $\quad D_z = \prod_i \sigma_i^z$.



$D^z|\Phi^\pm\rangle = \pm|\Phi^\pm\rangle$ near the dynamic freezing frequencies.

Signature of SZM in our locally driven system

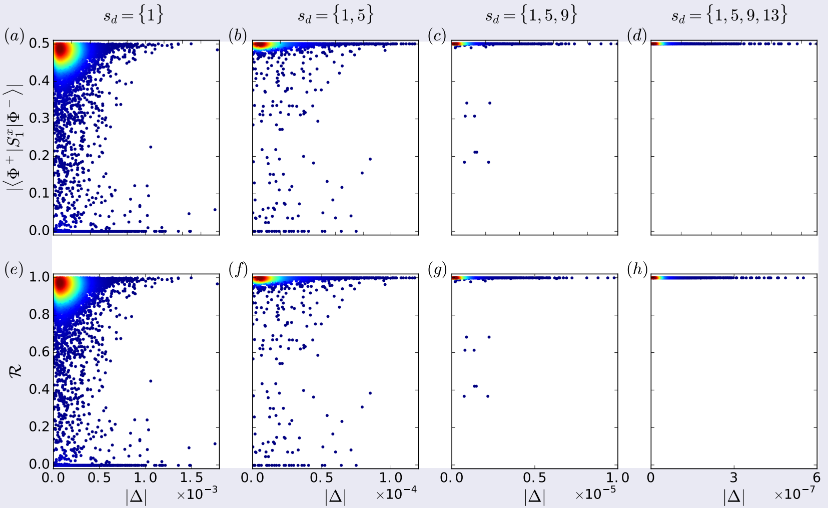
$\Delta = \epsilon^{+} - \epsilon^{-}$



$\Delta \sim \exp(-2.9 n_d^{4th})$.

Signature of SZM in our locally driven system

$$|\Phi^{\pm}\rangle = \frac{1}{\sqrt{2}}(|\rightarrow, \xi^{\pm}\rangle \pm |\leftarrow, \bar{\xi}^{\pm}\rangle) \; ; \; \mathcal{R} = |\langle \xi^{+}|\xi^{-}\rangle|$$



Entanglement structure

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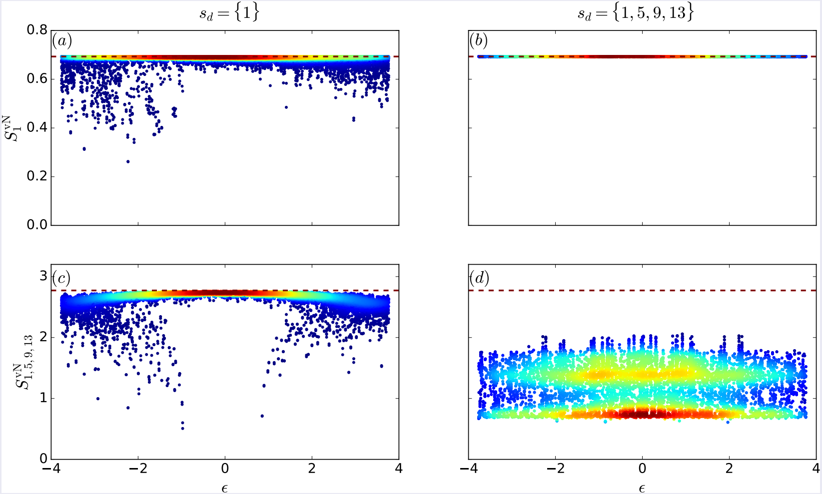

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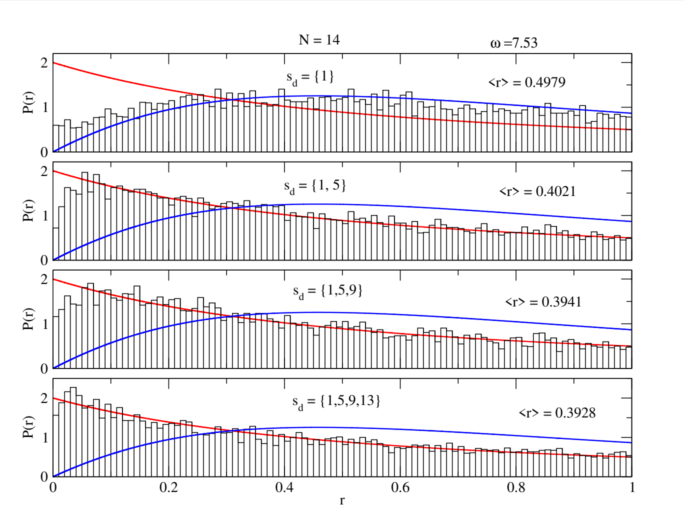
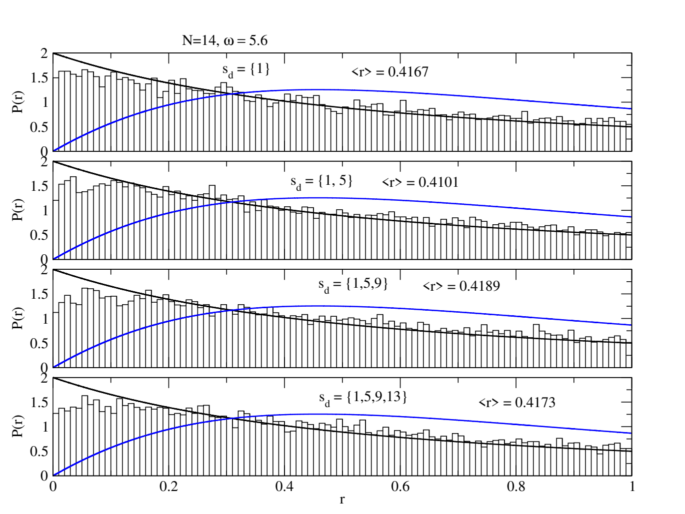
Thank you for your attention !!!

Level statistics



$\omega = 7.53,\ N = 14.$

Level statistics



$\omega = 5.6,\ N = 14$

Level statistics

- an approximate conserved quantity at any λ.

$$\mathcal{M} = 2^N \prod_{i \in s_d} [|\cos\frac{\lambda}{2}|(S_1^z + \tan\frac{\lambda}{2} S_1^y)] \prod_{i \notin s_d} S_i^z$$

- $[\mathcal{M}, H_F^{(0)}] = 0$ at any λ. $\mathcal{M}$ converts to D_z at $\lambda = 2\pi k$.
- Such quantities may remain nearly conserved and give rise to Poissonian statistics.