Maths Circle Explorations: Session 5

TIFR, Mumbai

24th December 2021

Problem 1

Metrics and Distances

In a piece of paper ($\mathbb{R}^2$) one measures distance between two points (x_1, y_1) and (x_2, y_2) by the "length" of the straight line segment joining the two points. By Pythagoras Theorem this is given by the formula

$$\sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}.$$

Now, say, one defines in a colony, two people are neighbors if the distance between their houses is less than 2 units.

1. Are you your neighbor ?
2. If I am your neighbor then are you my neighbor ?
3. Is there anyone other than you who is at a distance 0 from you ?
4. Can you have a group of neighbors who are neighbor to each other ? Can you 'describe' this collection ?

This idea will be further developed during the session.

Problem 2

There are 100 coins weighing 1, 2, or 3 grams that are visually alike. If there is at least one coin of each type, explain how to make three piles each of different type using using only a balance scale. What is the least number of trials of the balance scale required to do so?

Instead of 100 coins, generalize the problem for n coins. Make a conjecture for least number of trials of balance scale.

Problem 3

1. Choose $n+1$ integers from the set $\{ 1, 2, \cdots 2n \}$. Then there will be two which are co-prime. Prove or disprove.
2. Choose $n+1$ integers from the set $\{ 1, 2, \cdots 2n \}$. Then there will be two numbers such that one divides the other. Is it true ?
3. Suppose every point in $\mathbb{R}^2$ is colored with one of the 3 colors Red, Green and Blue. Then in at least one of the colors (depends on the coloring) for any given distance d there are two points x and y with distance d between them. Is it always true ?

Announcement

Writing formal proofs is an important part of doing Mathematics. During the session follow up questions and new ideas come up. The sessions are designed and conducted such that this happens. So it is very important (and we encourage you) to write formal statements and proofs that arose from your session and send it to us. We will go through it and send you the feedback or take it up in future sessions. In the subject line write the Session number and date. Please send emails with a copy to your legal guardian or from their account.

The email to submit your write up for TIFR, Mumbai sessions is

`mumbaimathcircle at gmail.com`

Any email you send, may be shared with the TIFR team organising the Math Circles.