Chat Time Sam!

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Exploratory Sheet for the Math Circles Session

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Have Fun!

Domino Tilings

Suppose, one tiles a rectangular $m \times n$ grid using dominos. For a $2 \times n$ grid, it is very easy to determine the number c_n of domino tilings.

Question 0. Find c_n.

Question 1. For a $3 \times 2n$ grid, can you find a recursive formula in terms of n for the number of domino tilings?

Question 2. What about an $4 \times n$ grid, for general n?

Collatz's Friendly Cousin

Question 3. Consider the transformation $T(n) = n+1$ or $T(n) = \frac{n}{2}$ according as to whether n is odd n is even. Let us write $T^2(n) = T(T(n)), T^3(n) = T(T(T(n)))$ etc. Prove that for each n, there exists k for which $T^k(n) = 1$. Can we determine the number a_k of natural numbers n which satisfy $T^k(n) = 1$ (note that we are not asking for the smallest k)?

More on Permutations

Question 4. (Preliminary) Why is the parity of a permutation well-defined? Can we give a combinatorial interpretation?

Question 5. A well-known puzzle considers the picture here of a 4×4 square on which 15 coins have been placed leaving out the last square empty.

1	2	3	4
5	6	7	8
9	10	11	12
13	14	15	

The idea is to slide the coins utilizing the empty square and to find out what kind of arrangements are possible. Can you generalize this for a general $n \times n$ square?

(Upping the ante) **Question 6.** Start with a deck of $3n$ cards, out of which n each are colored red, green and blue, in the denominations 1 to n. Select a subset S of the denominations and partition all the cards of these denominations into a list of three equal size sets such that the first set contains no red cards, the second no green cards, and the third no blue cards. Equivalently, deal all cards of the chosen denominations into three equal-size hands to players designated red, green and blue in such a way that no player receives a card of her own color. Let T_n denote the set of all triples obtained in this way.

Write down the 15 deals in T_2. Find the number of ways to deal all $3n$ cards so that no player receives a card of her own color. Deduce that the number of deals in T_n with S having size k equals $\binom{n}{k}\sum_{j=0}^{k}\binom{k}{j}^3$. Similarly, determine the number of deals in T_n with k distinct denominations in red's hand. Finally, deduce the identity

$$\sum_{k=0}^{n}\binom{n}{k}\sum_{j=0}^{k}\binom{k}{j}^3=\sum_{k=0}^{n}\binom{n}{k}^2\binom{2k}{k}.$$