

Problem 1 Find all positive integers x, y satisfying

$$\frac{1}{x} + \frac{1}{y} = \frac{1}{6}.$$

Problem 2 Show that the equation

$$x^2 + y^2 = 3z^2$$

has no solution in positive integers x, y, z .

Problem 3 Consider the equation

$$x^2 + y^2 + 1 = 3xy.$$

1. Find the first several positive integer solutions.
2. Guess a rule which produces new solutions from old ones.
3. Prove your rule, and show that your rule produces all positive integer solutions.

Problem 4 Let a, b be positive integers such that

$$ab + 1 \mid a^2 + b^2.$$

Prove that

$$\frac{a^2 + b^2}{ab + 1}$$

is a perfect square.

Problem 5 Prove Fermat's classical $n = 4$ theorem in the following form: there are no positive integers x, y, z satisfying

$$x^4 + y^4 = z^2.$$

Problem 6 Prove that the area of a right triangle whose side lengths are positive integers cannot be a perfect square.