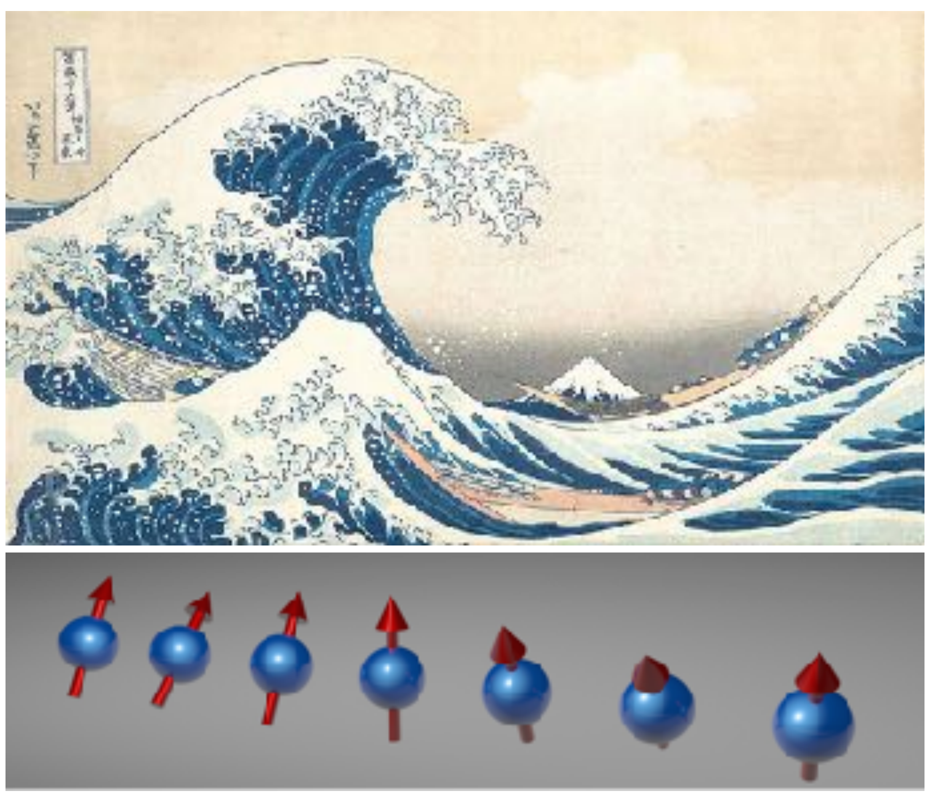
DYNAMICS CLOSE TO INTEGRABILITY: A HYDRODYNAMIC PROSPECTIVE



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NON-EQUILIBRIUM QUANTUM DYNAMICS

$$e^{-iHt}|\Psi_0\rangle$$

$$\langle\Psi_0(t)|O|\Psi_0(t)\rangle \qquad ?$$

QUANTUM GASES IN LOW DIMENSION

MENU

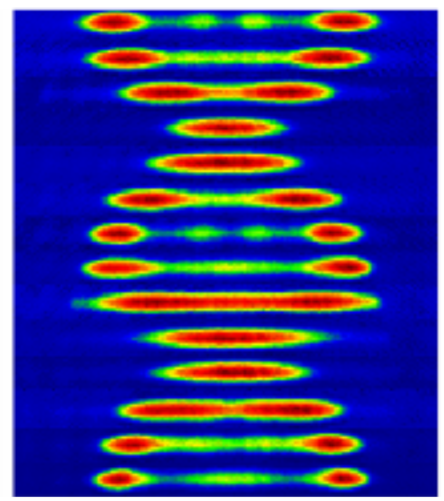
nature

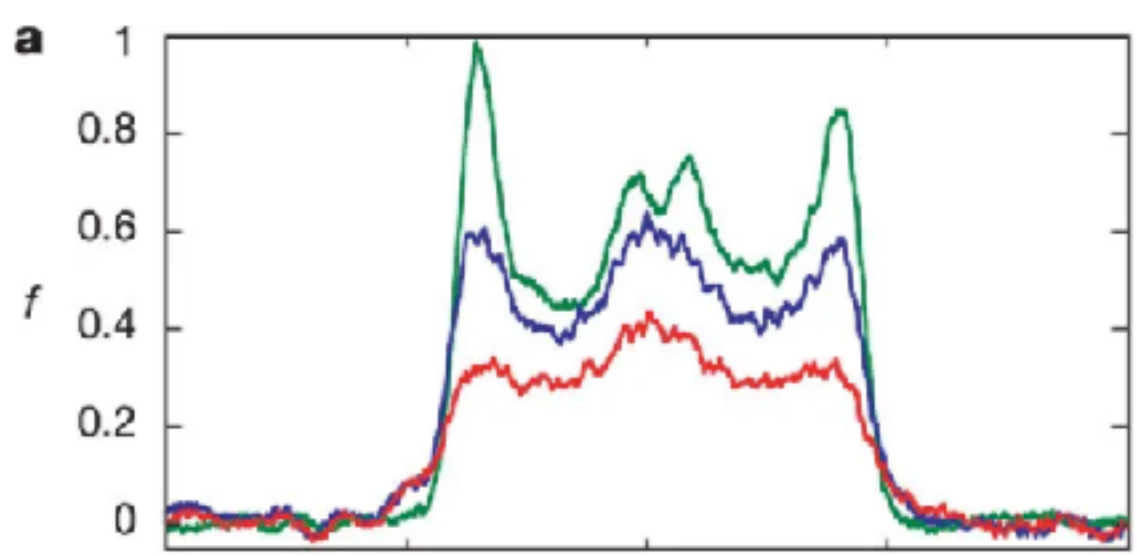
Letter | Published: 13 April 2006

A quantum Newton's cradle

Toshiya Kinoshita, Trevor Wenger & David S. Weiss

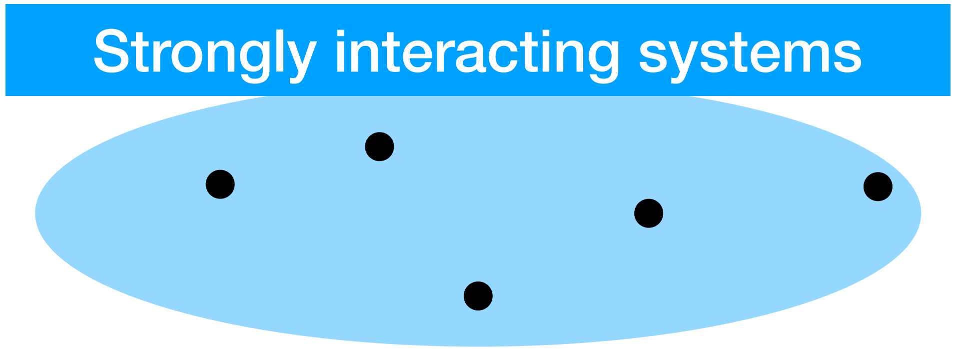
Nature **440**, 900–903(2006) | Cite this article





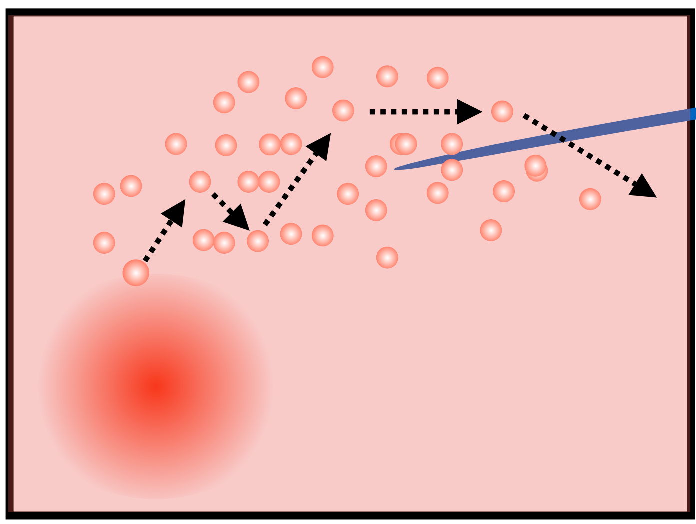
QUANTUM INTEGRABLE SYSTEMS

$$\hat{H}_{\mathrm{LL}} = \int dx \Big[\frac{\hbar^2 \partial_x \psi^\dagger(x) \partial_x \psi(x)}{2m} + c\ \psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x) \Big]$$



$$\Psi_{\mathrm{Bethe}} \sim \sum_P f(\{\theta_j, \theta_{P_j}\})$$

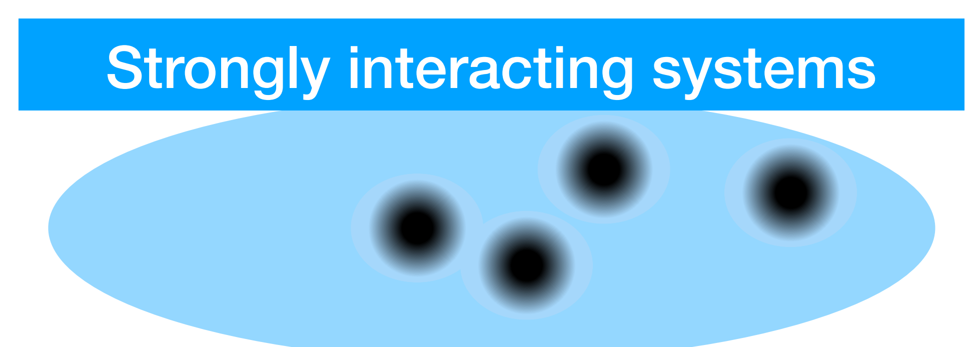
COARSE GRAINING: HYDRODYNAMICS



$$\partial_t \rho = \mathfrak{D} \partial_x^2 \rho$$

?

$$\Psi_{\text{Bethe}} \sim \sum_P f(\{\theta_j, \theta_{P_j}\})$$



COARSE GRAINING: HYDRODYNAMICS

Bertini, De Nardis, Collura, Fagotti, 2016

Castro-Alvaredo, Yoshimura, Doyon, 2016

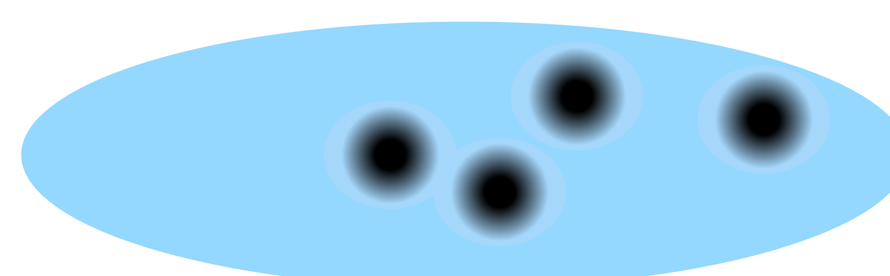
Bulchandani, Vasseur, Karrasch, Moore, 2017

Doyon, Yoshimura, 2017

…

GHD

$$\Psi_{\mathrm{Bethe}} \sim \sum_P f(\{\theta_j, \theta_{P_j}\})$$



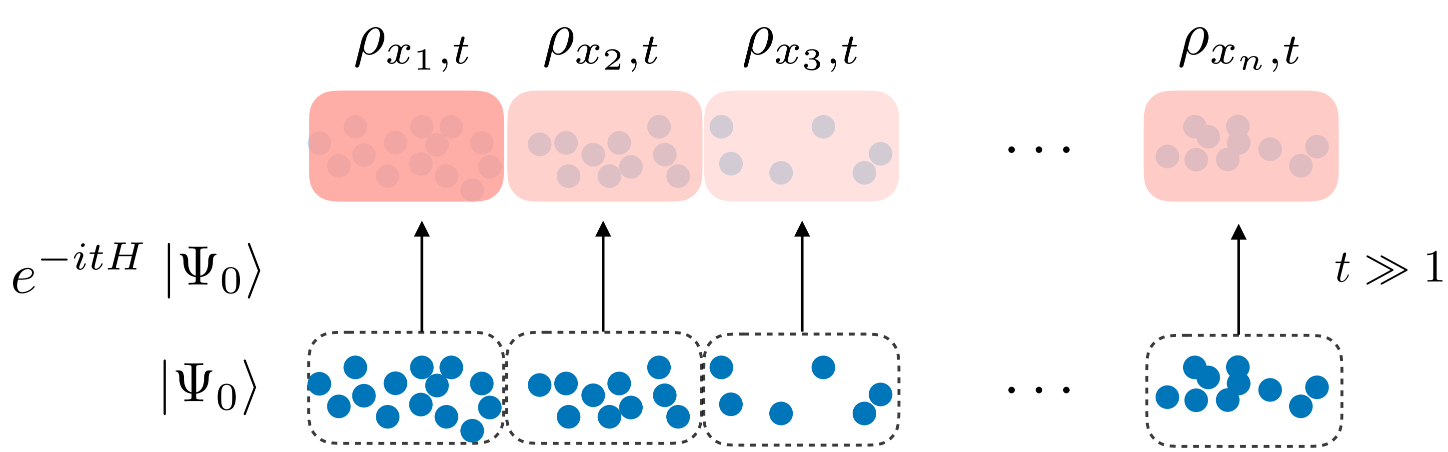
HYDRODYNAMICS

$$Q_i = \int dx \; q_i(x) \qquad\qquad [Q_i, Q_j] = 0$$

$$\partial_t q_i(x,t) + \partial_x j_i(x,t) = 0$$

Hydrodynamic expectation values $\langle \cdot \rangle^{\mathrm{hydro}}_{x_0,t_0}$

Durnin, De Luca, De Nardis, Doyon, 2021



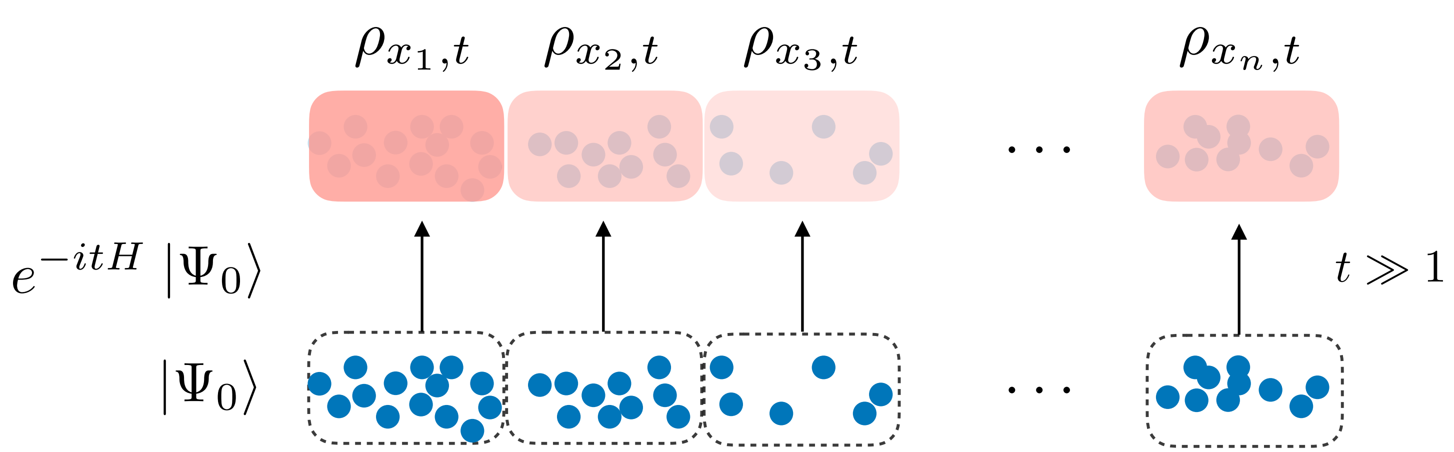
HYDRODYNAMICS

$$Q_i = \int dx\ q_i(x) \qquad [Q_i, Q_j] = 0$$

$$\partial_t q_i(x,t) + \partial_x j_i(x,t) = 0$$

a) Separation into local fluid cells

$$\tilde{\rho}_{x_0,t_0} \propto e^{-\beta^l(x_0,t_0)Q_l - \partial_{x_0}\beta^l(x_0,t_0)\int dy\,(y-x_0)\,q_l(y)}$$



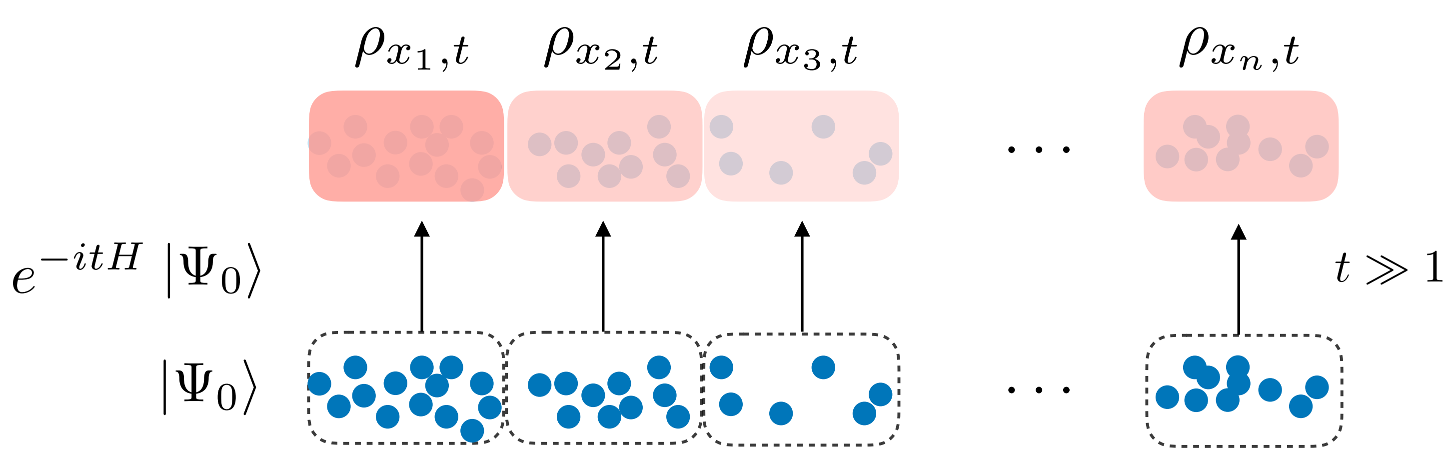
HYDRODYNAMICS

$$Q_i = \int dx \; q_i(x) \qquad\qquad [Q_i, Q_j] = 0$$

$$\partial_t q_i(x,t) + \partial_x j_i(x,t) = 0$$

b) Local relaxation (irreversibility)

$$\tilde{\rho}_{x_0,t_0} \propto e^{-\beta^l(x_0,t_0)Q_l - \partial_{x_0}\beta^l(x_0,t_0)\int dy\,(y-x_0)\,q_l(y)}$$



$$\langle j\rangle^{\text{hydro}}_{x_0,t_0} = \lim_{t\to\infty}\Big[\mathrm{Tr}(\tilde{\rho}_{x_0,t_0}\, j(x_0,t))\ \text{expressed in terms of hydrodynamic variables}\Big]$$

HYDRODYNAMICS

$$\mathsf{q}_i(x,t) = \langle q_i(x,t)\rangle$$

Hydrodynamic variables

$$\partial_t \mathsf{q}_i(x,t) + \partial_x \langle j_i \rangle_{x,t}^{\text{hydro}} = 0$$

$$\tilde{\rho}_{x_0,t_0} \propto e^{-\beta^l(x_0,t_0)Q_l - \partial_{x_0}\beta^l(x_0,t_0)\int dy\,(y-x_0)\,q_l(y)}$$

Expand in each fluid cell

Express in terms of hydrodynamic variables

Compute hydrodynamic values of currents in the large time limit (local relaxation: irreversibility)

HYDRODYNAMIC EXPANSION

$$\tilde{\rho}_{x_0,t_0} \propto e^{-\beta^l(x_0,t_0)Q_l - \partial_{x_0}\beta^l(x_0,t_0)\int dy\,(y-x_0)\,q_l(y)}$$

Expand in each fluid cell

$$\langle q_i(0,t)\rangle_{\tilde{\rho}_{x_0=0,t_0=0}} = \langle q_i\rangle - \partial_x\beta^k \int dy\, y\,(q_k(y), q_i(0,t))$$

$$\mathsf{q}_i = \langle q_i\rangle + t\,\partial_x\beta^k (J_k, q_i)$$

$$\langle j_i(0,t)\rangle_{\tilde{\rho}_{x_0=0,t_0=0}} = \langle j_i\rangle - \partial_x\beta^k \int dy\, y\,(q_k(y), j_i(0,t))$$

$$\langle j_i\rangle \equiv \langle j_i\rangle[\langle q\rangle] = \langle j_i\rangle[\mathsf{q}] - t\,\partial_x\beta^k \frac{\delta\langle j_i\rangle}{\delta\langle q_l\rangle}(J_k, q_l) + \ldots$$

Express in terms of hydrodynamic variables

HYDRODYNAMIC EXPANSION

$$\tilde{\rho}_{x_0,t_0} \propto e^{-\beta^l(x_0,t_0)Q_l - \partial_{x_0}\beta^l(x_0,t_0)\int dy\,(y-x_0)\,q_l(y)}$$

$$\langle j_i(0,t)\rangle_{\tilde{\rho}} = \mathtt{j}_i + \frac{\partial_x \beta^k}{2}\int dy \int_{-t}^{t} ds \left(j_k(y,s), j_i(0,0) - \frac{\delta\langle j_i\rangle}{\delta\langle q_l\rangle} q_l(0,0)\right)$$

Local relaxation

$$\lim_{t\to\infty}$$

$$\partial_t \mathtt{q}_i + \partial_x \left[\mathtt{j}_i + \frac{1}{2}\partial_x \boldsymbol{\beta}^k \mathfrak{L}_{k,i}\right] = 0 \qquad \mathsf{C}^{i,k}\partial_x\boldsymbol{\beta}^k = -\partial_x \mathtt{q}_i$$

Euler current

Onsager matrix

INTEGRABLE MODELS: QUASIPARTICLE BASIS

$$\partial_t \mathtt{q}_i + \partial_x \left[\mathtt{j}_i + \frac{1}{2} \partial_x \boldsymbol{\beta}^k \mathfrak{L}_{k,i} \right] = 0$$

quasiparticle density

quasiparticle filling

$$\mathtt{q}_i(x,t) = \int d\theta \rho_{\rm p}(\theta; x, t) h_i(\theta)$$

$$n = \frac{\rho_p}{k'/2\pi}$$

INTEGRABLE MODELS: GHD

$$\partial_t \mathtt{q}_i + \partial_x \left[\mathtt{j}_i + \frac{1}{2} \partial_x \boldsymbol{\beta}^k \mathfrak{L}_{k,i} \right] = 0$$

quasiparticle density

quasiparticle filling

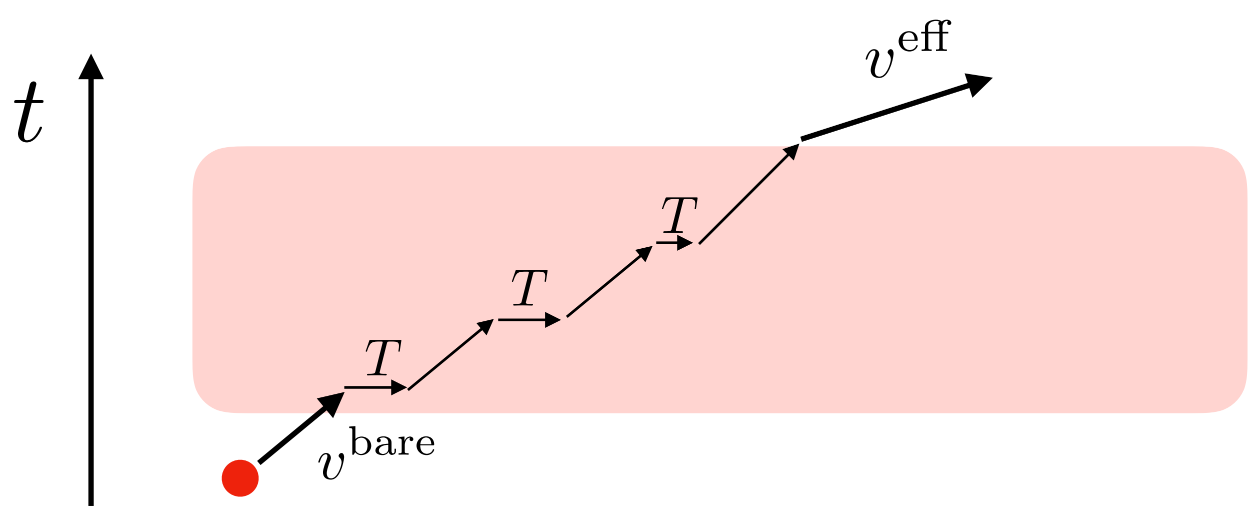
$$\mathtt{q}_i(x,t) = \int d\theta \rho_{\mathrm{p}}(\theta; x, t) h_i(\theta)$$

$$n = \frac{\rho_p}{k'/2\pi}$$

$$\mathtt{j}_i(x,t) = \int d\theta \rho_{\mathrm{p}}(\theta; x, t) v^{\mathrm{eff}}(\theta; x, t) h_i(\theta)$$

$$h^{\mathrm{dr}} = (1 - Tn)^{-1} \cdot h = h + Tnh + (Tn)^2 h + (Tn)^3 h + \dots$$

dressing

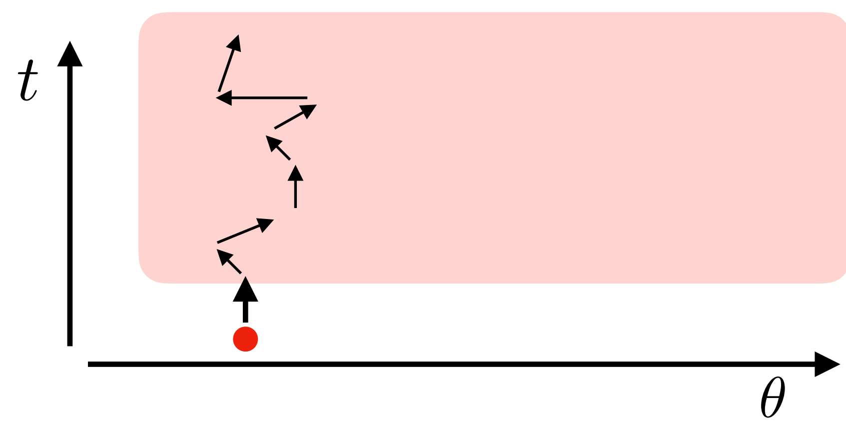


Effective velocity

INTEGRABLE MODELS: GHD

$$\mathfrak{L}_{ik} = h_i \cdot \mathfrak{D}\mathsf{C} \cdot h_k$$

$$\mathfrak{D}\mathsf{C} = (1-nT)^{-1} \cdot \frac{\delta_{\theta_1,\theta_2}\int d\alpha\,\kappa_{\mathfrak{D}}(\theta_1,\alpha) - \kappa_{\mathfrak{D}}(\theta_1,\theta_2)}{k'(\theta_1)k'(\theta_2)} \cdot (1-Tn)^{-1}$$



fluctuating effective velocity: quasiparticle diffusion

$$\partial_t \rho_{\mathrm{p}} + \partial_x \left(v^{\mathrm{eff}} \rho_{\mathrm{p}}\right) = \frac{1}{2}\partial_x(\mathfrak{D} \cdot \partial_x \rho_{\mathrm{p}})$$

quasiparticle current

Diffusive spreadings

De Nardis, Bernard, Doyon, 2018

De Nardis, Bernard, Doyon, 2019

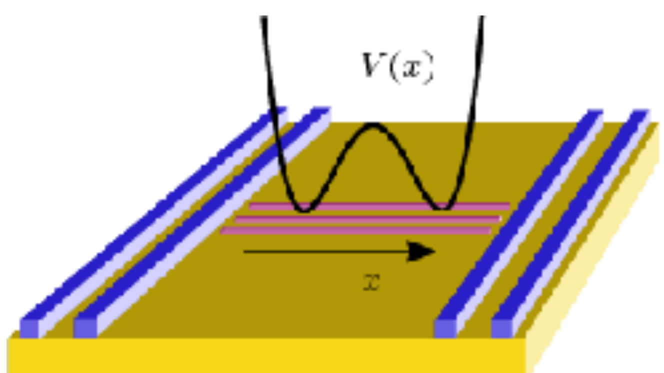
Gopalakrishnan, et al, 2018

Gopalakrishnan, Vasseur, 2019

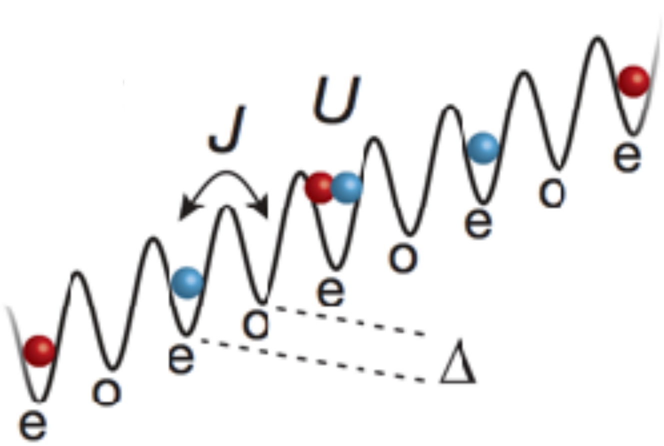
INHOMOGENEOUS HAMILTONIANS

$$Q_i = \int dx\ q_i(x) \qquad [Q_i, Q_j] = 0$$

$$H = \sum_{i=1}^{n} \int dx\ w^i(x) q_i(x)$$



$$\hat{H}_{\mathrm{LL}} = \int dx \Big[\frac{\hbar^2 \partial_x \psi^\dagger(x) \partial_x \psi(x)}{2m} + c\ \psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x) \Big] + \int dx V(x) \psi^\dagger(x)\psi(x)$$



$$H = \sum_{j,\sigma} c^\dagger_{j+1,\sigma} c_{j,\sigma} + h.c. + U \sum_j n_{j,\uparrow} n_{j,\downarrow} + \sum_{j,\sigma} \delta_{j,\sigma} n_{j,\sigma}$$

INHOMOGENEOUS HAMILTONIANS

$$Q_i = \int dx\ q_i(x) \qquad [Q_i, Q_j] = 0$$

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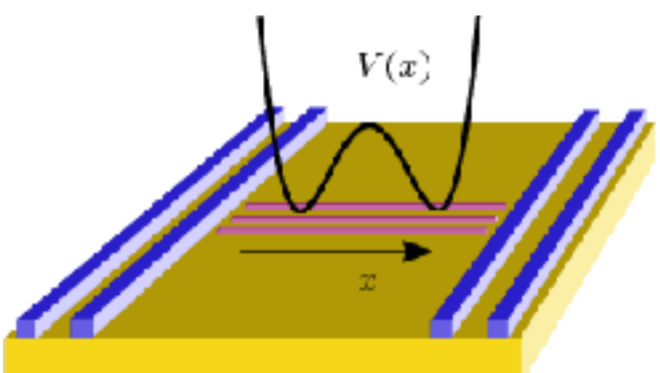

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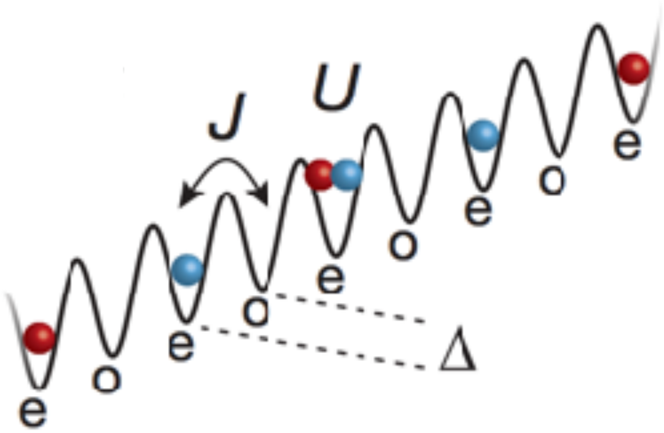

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thermalisation?

INHOMOGENEOUS HAMILTONIANS

$$Q_i = \int dx \; q_i(x) \qquad\qquad [Q_i, Q_j] = 0$$

$$H = \sum_{i=1}^{n} \int dx \; w^i(x) q_i(x)$$

Doyon, Yoshimura, 2017

Euler scale

$$\partial_t \rho_{\rm p} + \partial_x \left(v^{\rm eff}_{\boldsymbol{w}} \rho_{\rm p} \right) + \partial_\theta \left(a^{\rm eff}_{\mathfrak{f}} \rho_{\rm p} \right) = 0$$

$$v^{\rm eff}_{\boldsymbol{w}}(\theta; x, t) = \frac{w^i(x) (h_i')^{\rm dr}(\theta; x, t)}{k'(\theta; x, t)}$$

$$a^{\rm eff}_{\mathfrak{f}}(\theta; x, t) = \frac{\mathfrak{f}^i(x) (h_i)^{\rm dr}(\theta; x, t)}{k'(\theta; x, t)}$$

Caux et al, 2018

Schemmer, Bouchoule, et al, 2018

Cao, Bulchandani, Moore, 2018

Scopa, Calabrese, Piroli, 2021

INHOMOGENEOUS HAMILTONIANS

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Caux et al, 2018

Schemmer, Bouchoule, et al, 2018

Cao, Bulchandani, Moore, 2018

Scopa, Calabrese, Piroli, 2021

Diffusive corrections?

INHOMOGENEOUS HAMILTONIANS

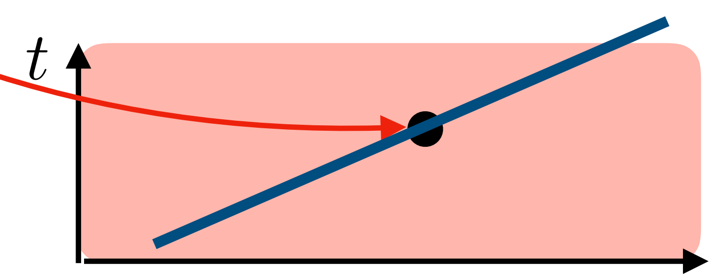
$$Q_i = \int dx \; q_i(x) \qquad\qquad [Q_i, Q_j] = 0$$

$$H = \sum_{i=1}^{n} \int dx \; w^i(x) q_i(x)$$

local perturbation to the Hamiltonian

$$H_{x_0} = w^i(x_0) Q_i - \mathfrak{f}^i(x_0) \int dx (x - x_0) q_i(x) + \ldots$$

$$\mathfrak{f} = -\frac{\partial w^i(x)}{\partial x}\Big|_{x=x_0}$$



$$\boldsymbol{w} = w^i(x_0)$$

INHOMOGENEOUS HAMILTONIANS

$$Q_i = \int dx \; q_i(x) \qquad\qquad [Q_i, Q_j] = 0$$

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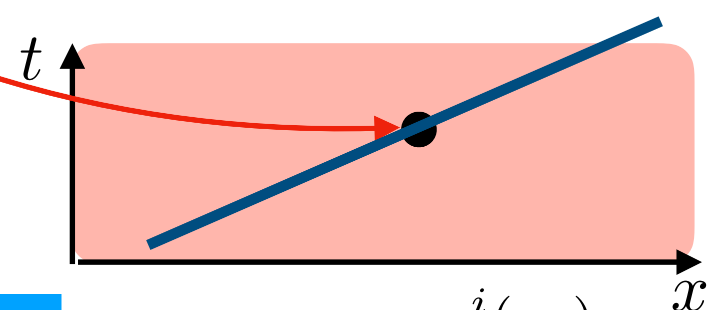


$$\boldsymbol{w} = w^i(x_0)$$

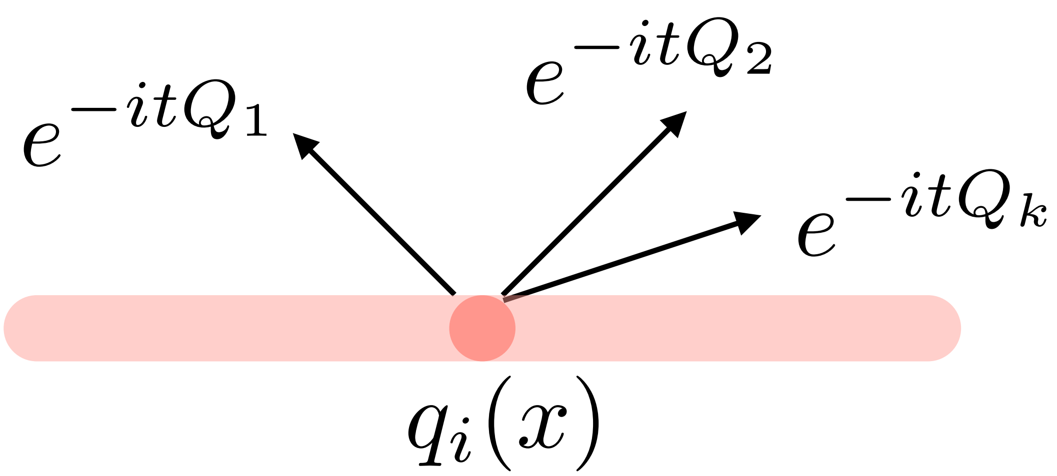
evolution respect to local Hamiltonian

1st order perturbation theory

$$o(0,t) = o(0, \boldsymbol{w}t) - i \int_0^t ds \int dx \; x \; \mathfrak{f}^i(0) \; [q_i(x, \boldsymbol{w}s), o(0, \boldsymbol{w}t)] + \ldots$$

EXTENDED SPACE OF CURRENTS

Currents of charge respect to the flow of any charge



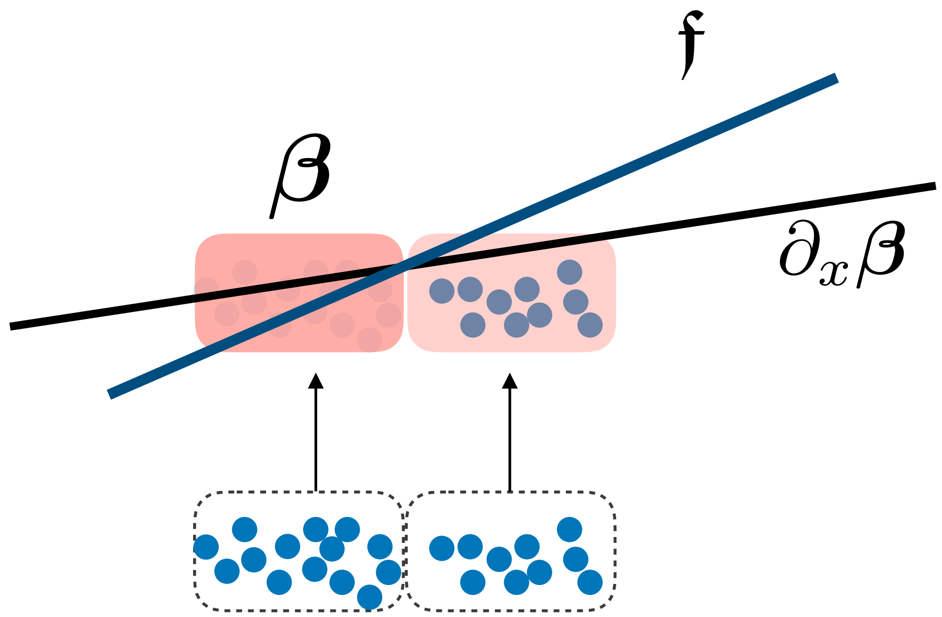
$$i[Q_k, q_i(x)] + \partial_x j_{k,i}(x) = 0$$

$$j_{\boldsymbol{a},\boldsymbol{b}} = a^k j_{k,i} b^i$$

$$\mathfrak{L}[\boldsymbol{a}, \boldsymbol{b}; o] = \int_{-\infty}^{\infty} ds \, (J_{\boldsymbol{a},\boldsymbol{b}}(s\boldsymbol{w}), o)^C$$

Extended Onsager coefficients

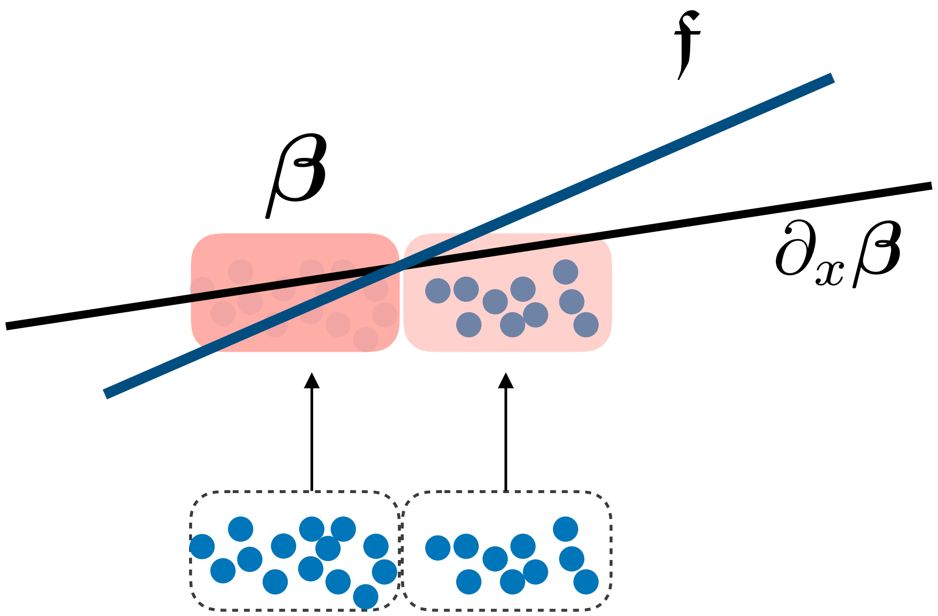
DIFFUSIVE HYDRODYNAMICS IN INHOMOGENOUS HAMILTONIANS



$$\mathfrak{L}[\boldsymbol{a}, \boldsymbol{b}; o] = \int_{-\infty}^{\infty} ds\, (J_{\boldsymbol{a},\boldsymbol{b}}(s\boldsymbol{w}), o)^C$$

$$\begin{aligned}\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta}, \boldsymbol{\mathfrak{f}}; j_{\boldsymbol{w},i}]\right) \\ = \mathtt{j}_{i,\mathfrak{f}} + \frac{1}{2}\left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \boldsymbol{\mathfrak{f}}; j_{i,\mathfrak{f}}]\right)\end{aligned}$$

DIFFUSIVE HYDRODYNAMICS IN INHOMOGENOUS HAMILTONIANS



$$\mathfrak{L}[\boldsymbol{a}, \boldsymbol{b}; o] = \int_{-\infty}^{\infty} ds \, (J_{\boldsymbol{a},\boldsymbol{b}}(s\boldsymbol{w}), o)^C$$

current of local Hamiltonian

its diffusive spreading due to spatial unbalance

its diffusive spreading due to forces

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},i}]\right)$$

$$= \mathtt{j}_{i,\mathfrak{f}} + \frac{1}{2}\left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{i,\mathfrak{f}}]\right)$$

accelleration

its diffusive spreading due to spatial unbalance

its diffusive spreading due to forces

INHOMOGENEOUS HAMILTONIANS

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},i}] \right)$$
$$= \mathtt{j}_{i,\mathfrak{f}} + \frac{1}{2} \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{i,\mathfrak{f}}] \right)$$

Self-conserved charge

$$H = \bar{w}^i Q_i + \int dx w^0(x) q_0(x) \qquad j_{k,0} \quad \text{conserved}$$

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \cancel{\mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},i}]} \right)$$
$$= \mathtt{j}_{i,\mathfrak{f}} + \cancel{\frac{1}{2} \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{i,\mathfrak{f}}] \right)}$$

INHOMOGENEOUS HAMILTONIANS

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},i}]\right)$$
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Self-conserved charge

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$$= \mathtt{j}_{i,\mathfrak{f}} + \cancel{\frac{1}{2}\left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{i,\mathfrak{f}}]\right)}$$

Ultra-local charge

$$H = \bar{w}^i Q_i + \int dx w^0(x) q_0(x) \qquad [Q_0, q_k(x)] = 0$$

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\bar{\boldsymbol{w}},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\bar{\boldsymbol{w}}, \partial_x \boldsymbol{\beta}; j_{\bar{\boldsymbol{w}},i}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\bar{\boldsymbol{w}},i}]\right)$$
$$= \mathtt{j}_{i,\mathfrak{f}} + \frac{1}{2}\left(\mathfrak{L}[\bar{\boldsymbol{w}}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{i,\mathfrak{f}}]\right)$$

ENTROPY INCREASE AND THERMALISATION

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},i}]\right)$$
$$= \mathtt{j}_{i,\mathfrak{f}} + \frac{1}{2}\left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{i,\mathfrak{f}}]\right)$$

$$\partial_t S = \frac{1}{2}\int_{-\infty}^{\infty} dx \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},\partial_x \boldsymbol{\beta}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},\partial_x \boldsymbol{\beta}}] + \mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{\beta},\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{\beta},\mathfrak{f}}]\right) \;\; \geq 0$$

ENTROPY INCREASE AND THERMALISATION

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w},\partial_x\boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta},\mathfrak{f}; j_{\boldsymbol{w},i}]\right)$$
$$= \mathtt{j}_{i,\mathfrak{f}} + \frac{1}{2}\left(\mathfrak{L}[\boldsymbol{w},\partial_x\boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta},\mathfrak{f}; j_{i,\mathfrak{f}}]\right)$$

$$\partial_t S = \frac{1}{2}\int_{-\infty}^{\infty} dx\, \Big(\mathfrak{L}[\boldsymbol{w},\partial_x\boldsymbol{\beta}; j_{\boldsymbol{w},\partial_x\boldsymbol{\beta}}] + \mathfrak{L}[\boldsymbol{\beta},\mathfrak{f}; j_{\boldsymbol{w},\partial_x\boldsymbol{\beta}}] + \mathfrak{L}[\boldsymbol{w},\partial_x\boldsymbol{\beta}; j_{\boldsymbol{\beta},\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta},\mathfrak{f}; j_{\boldsymbol{\beta},\mathfrak{f}}]\Big) \geq 0$$

Stationary entropy states

$$\boldsymbol{\beta}(x) = \beta(\boldsymbol{w}(x) - \mu_0(x)\boldsymbol{e}_0 - \mu_1\boldsymbol{e}_1)$$

ultra-local & self-conserved

ultra-local

ENTROPY INCREASE AND THERMALISATION

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},i}]\right)$$
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$$\partial_t S = \frac{1}{2}\int_{-\infty}^{\infty} dx \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},\partial_x \boldsymbol{\beta}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},\partial_x \boldsymbol{\beta}}] + \mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{\beta},\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{\beta},\mathfrak{f}}]\right) \geq 0$$

Stationary entropy states

$$\boldsymbol{\beta}(x) = \beta(\boldsymbol{w}(x) - \mu_0(x)\boldsymbol{e}_0 - \mu_1 \boldsymbol{e}_1)$$

ultra-local & self-conserved

ultra-local

Stationary states

$$\partial_x \mathtt{j}_{\boldsymbol{w},i} = \mathtt{j}_{i,\mathfrak{f}} \to \mu_0(x) = \mathrm{const}$$

INTEGRABLE SYSTEMS: QUASIPARTICLE BASIS

$$\partial_t \mathtt{q}_i + \partial_x \mathtt{j}_{\boldsymbol{w},i} + \frac{1}{2}\partial_x \left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{\boldsymbol{w},i}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{\boldsymbol{w},i}]\right)$$
$$= \mathtt{j}_{i,\mathfrak{f}} + \frac{1}{2}\left(\mathfrak{L}[\boldsymbol{w}, \partial_x \boldsymbol{\beta}; j_{i,\mathfrak{f}}] + \mathfrak{L}[\boldsymbol{\beta}, \mathfrak{f}; j_{i,\mathfrak{f}}]\right)$$

convective flow

acceleration

$$\partial_t \rho_{\rm p} + \partial_x \left(v^{\rm eff}_{\boldsymbol{w}} \rho_{\rm p}\right) + \partial_\theta \left(a^{\rm eff}_{\mathfrak{f}} \rho_{\rm p}\right) =$$
$$\frac{1}{2}\partial_x(\mathfrak{D} \cdot \partial_x \rho_{\rm p}) + \frac{1}{2}\partial_x(\mathfrak{D}_{\mathfrak{f}} \cdot \partial_\theta \rho_{\rm p}) + \frac{1}{2}\partial_\theta(\mathfrak{D}_{\mathfrak{f}} \cdot \partial_x \rho_{\rm p}) + \frac{1}{2}\partial_\theta(\mathfrak{D}_{\mathfrak{f}^2} \cdot \partial_\theta \rho_{\rm p})$$

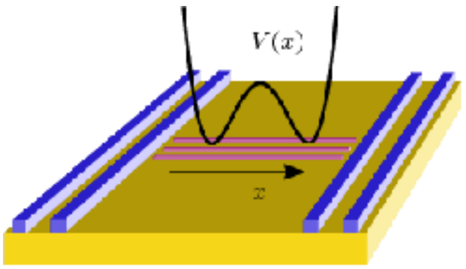
diffusion in real and momentum space

$$\partial_t S = \frac{1}{2}\begin{pmatrix}\Delta_x n & \Delta_\theta n\end{pmatrix} \cdot \begin{pmatrix}\mathfrak{D}\mathsf{C} & \mathfrak{D}_{\mathfrak{f}}\mathsf{C} \\ \mathfrak{D}_{\mathfrak{f}}\mathsf{C} & \mathfrak{D}_{\mathfrak{f}^2}\mathsf{C}\end{pmatrix} \cdot \begin{pmatrix}\Delta_x n \\ \Delta_\theta n\end{pmatrix}$$

$$\Delta_i n = (1 - Tn) \cdot \frac{\partial_i n}{n f(n)}$$

THERMALISATION OF A 1-D BOSE GAS

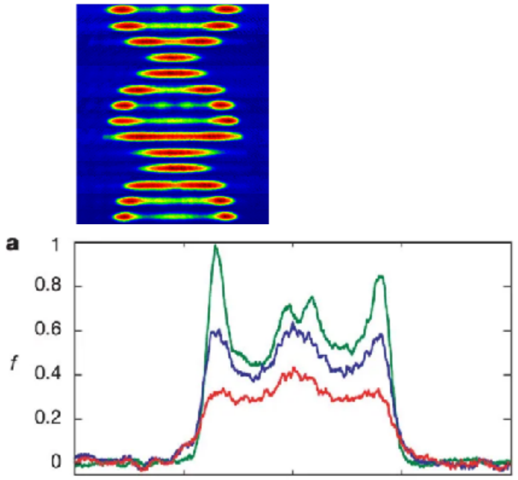
$$\hat{H}_{\rm LL} = \int dx \Big[\frac{\hbar^2 \partial_x \psi^\dagger(x)\partial_x\psi(x)}{2m} + c\,\psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x)\Big] + \int dx V(x)\psi^\dagger(x)\psi(x)$$



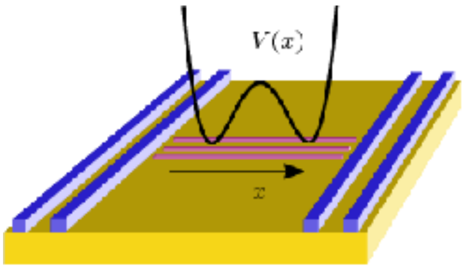
$$\partial_t\rho_{\rm p} + \partial_x\left(v^{\rm eff}_{\boldsymbol{w}}\rho_{\rm p}\right) + \partial_\theta\left(a^{\rm eff}_{\mathfrak{f}}\rho_{\rm p}\right) = \frac{1}{2}\partial_x(\mathfrak{D}\cdot\partial_x\rho_{\rm p}) + \frac{1}{2}\partial_x(\mathfrak{D}_{\mathfrak{f}}\cdot\partial_\theta\rho_{\rm p}) + \frac{1}{2}\partial_\theta(\mathfrak{D}_{\mathfrak{f}}\cdot\partial_x\rho_{\rm p}) + \frac{1}{2}\partial_\theta(\mathfrak{D}_{\mathfrak{f}^2}\cdot\partial_\theta\rho_{\rm p})$$

$$\partial_t\rho_{\rm p} + \partial_x\left(v^{\rm eff}\rho_{\rm p}\right) - V'\partial_\theta\rho_{\rm p} = \frac{1}{2}\partial_x(\mathfrak{D}\cdot\partial_x\rho_{\rm p})$$

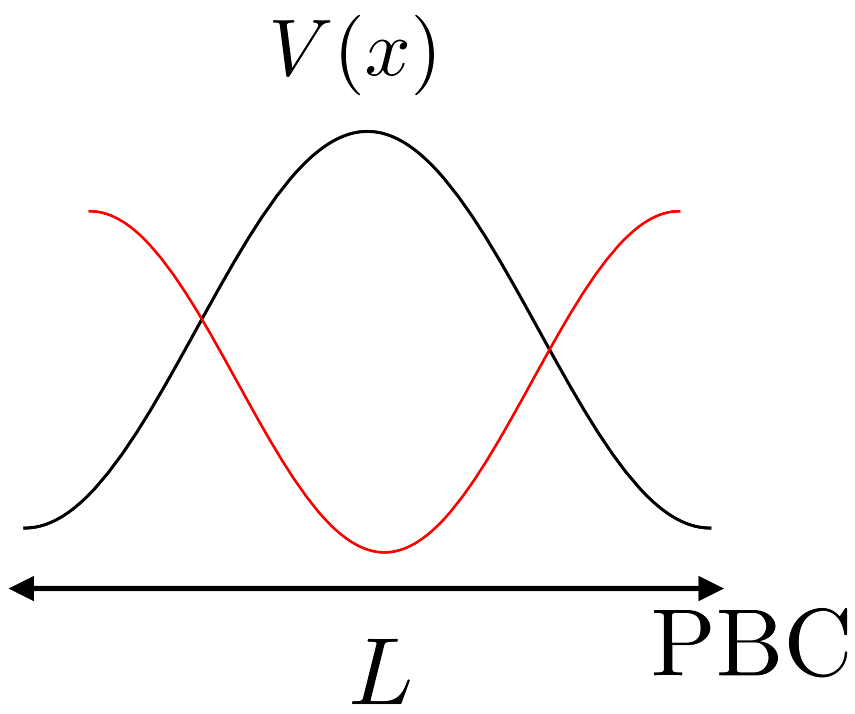
Density in Galileian systems is a self-conserved & ultra-local charge



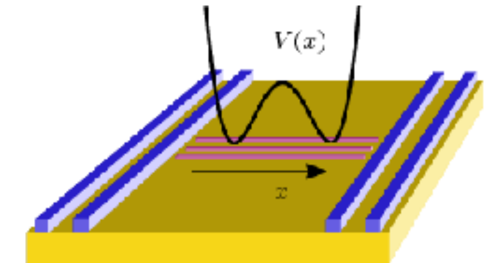
THERMALISATION OF A 1-D BOSE GAS



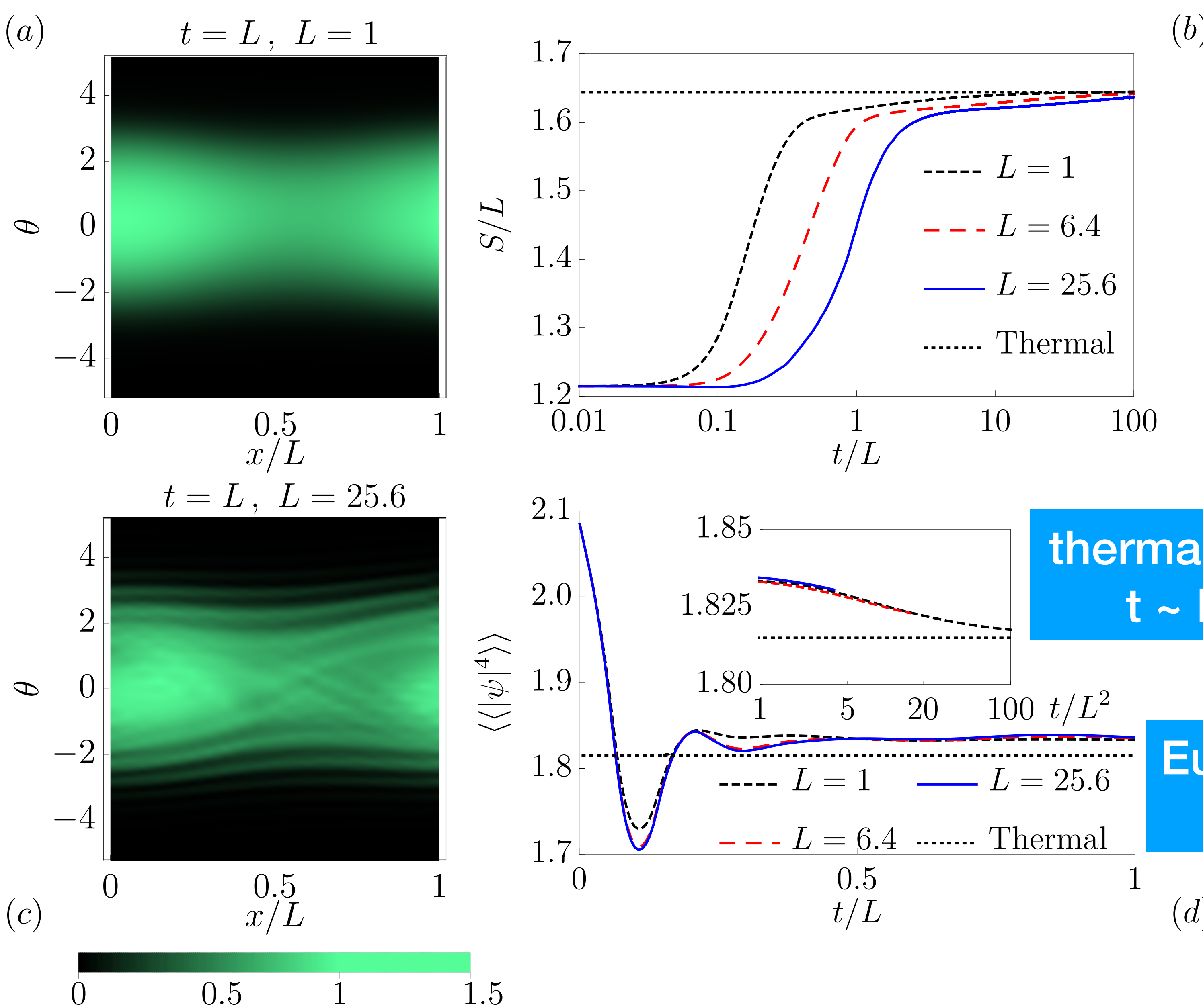
$$\partial_t \rho_{\mathrm{p}} + \partial_x \left(v^{\mathrm{eff}} \rho_{\mathrm{p}} \right) - V' \partial_\theta \rho_{\mathrm{p}} = \frac{1}{2} \partial_x (\mathfrak{D} \cdot \partial_x \rho_{\mathrm{p}})$$

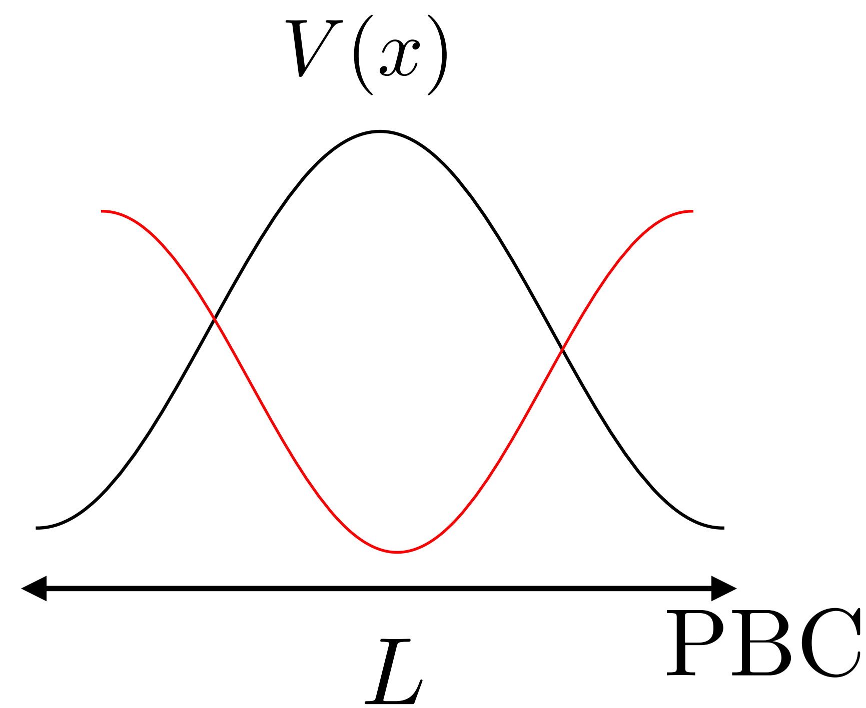


THERMALISATION OF A 1-D BOSE GAS



$$\partial_t \rho_{\rm p} + \partial_x \left(v^{\rm eff} \rho_{\rm p} \right) - V' \partial_\theta \rho_{\rm p} = \frac{1}{2} \partial_x (\mathfrak{D} \cdot \partial_x \rho_{\rm p})$$





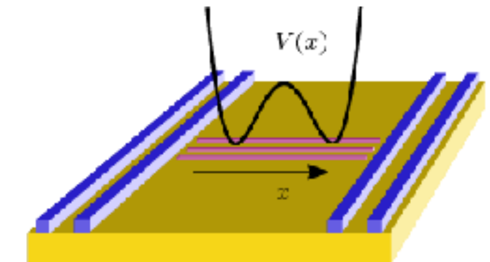
thermalisation
t ~ L^2

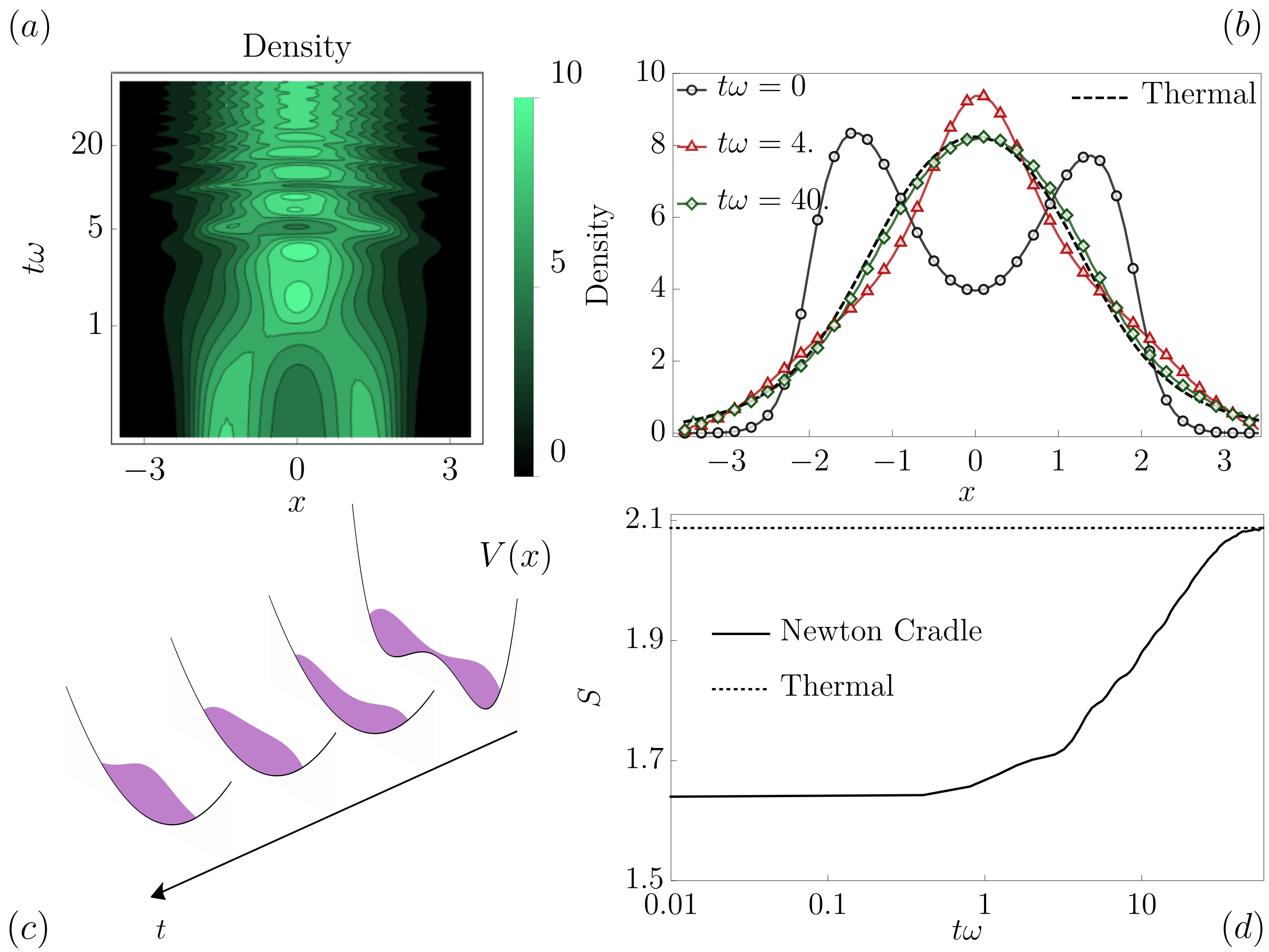
Euler transient
t ~ L

Approach to thermalisation is on diffusive time-scales

THERMALISATION OF A 1-D BOSE GAS

$$\hat{H}_{\mathrm{LL}} = \int dx \Big[\frac{\hbar^2 \partial_x \psi^\dagger(x)\partial_x \psi(x)}{2m} + c\, \psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x)\Big] + \int dx V(x)\psi^\dagger(x)\psi(x)$$





Thermalisation in a trap

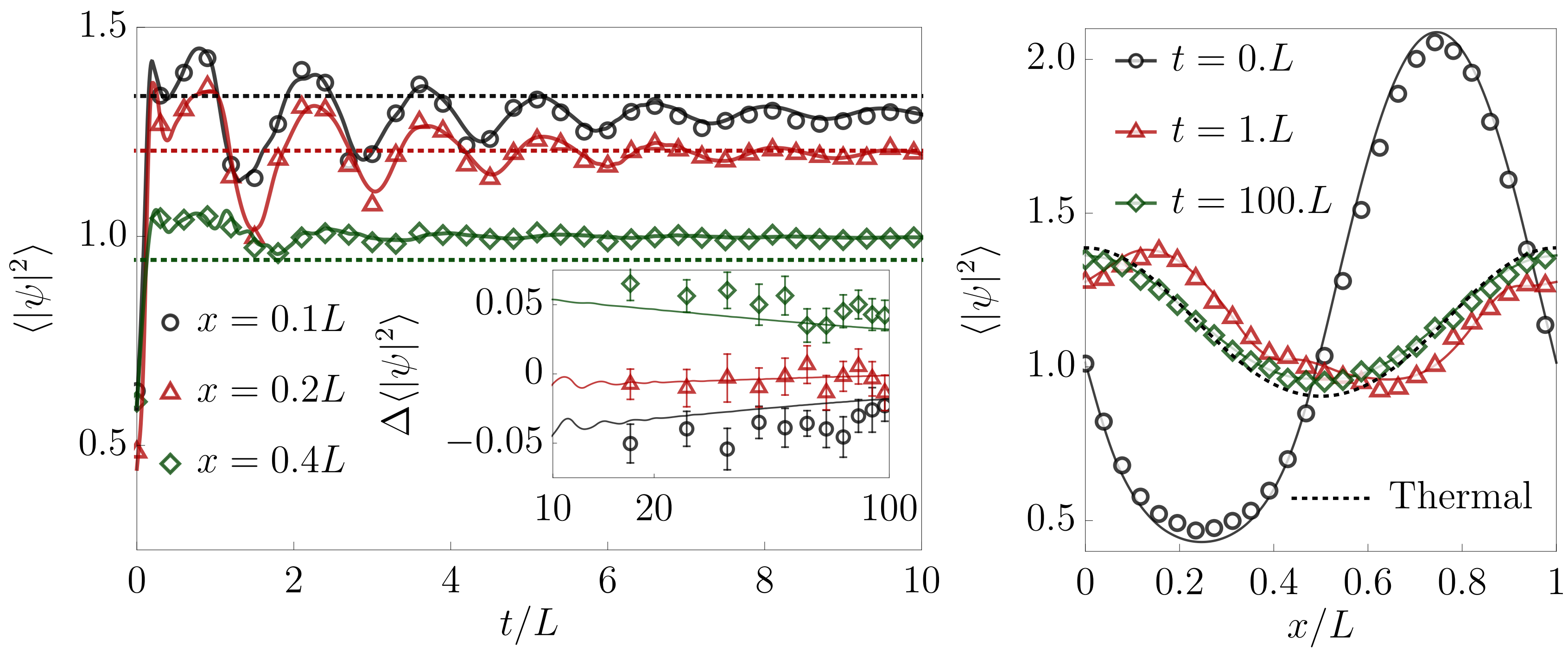
Bastianello, De Luca, Doyon, De Nardis, 2020

THERMALISATION OF A 1-D BOSE GAS

$$\hat{H}_{\rm LL} = \int dx \Big[\frac{\hbar^2 \partial_x \psi^\dagger(x) \partial_x \psi(x)}{2m} + c\ \psi^\dagger(x)\psi^\dagger(x)\psi(x)\psi(x) \Big] + \int dx V(x) \psi^\dagger(x)\psi(x)$$

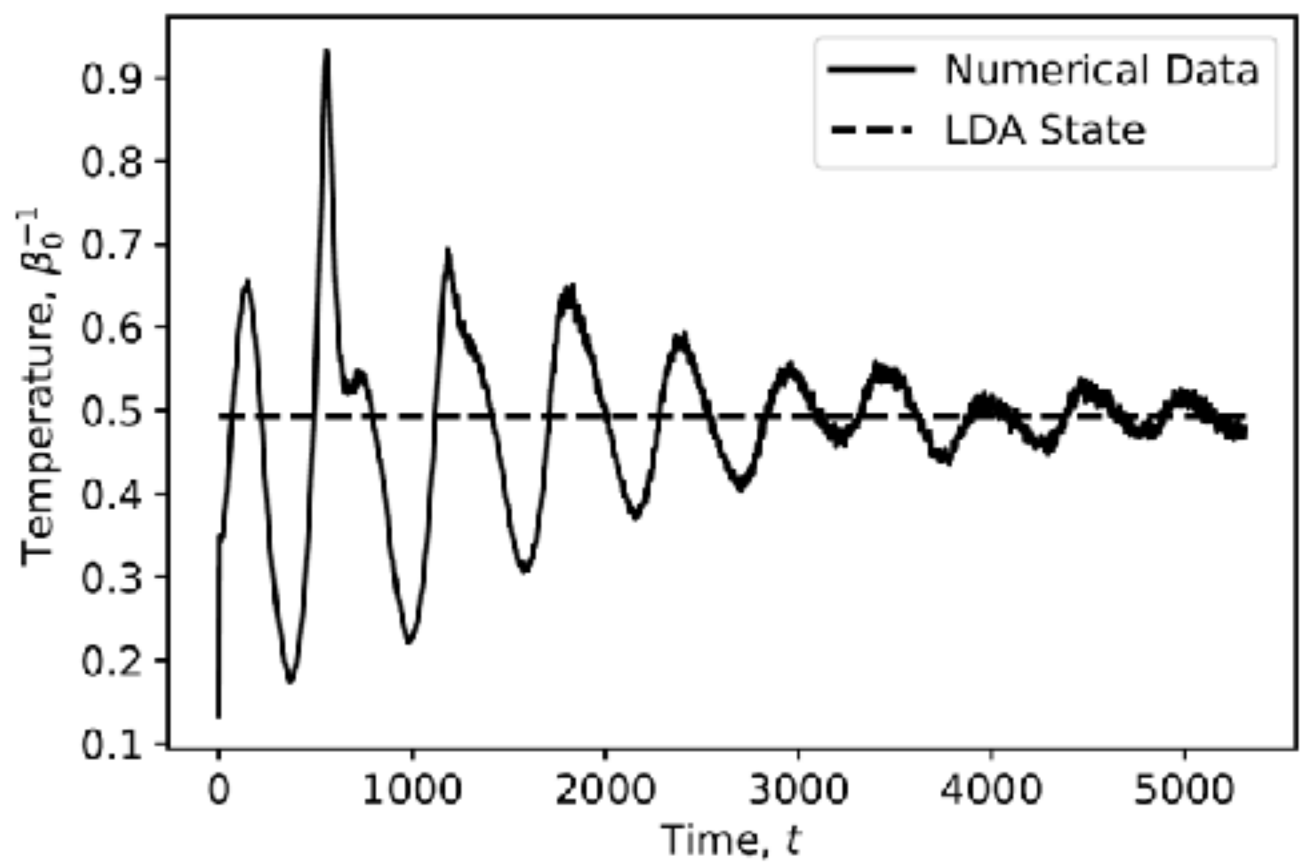
$$c \to 0$$

$$T \simeq 1/c$$



THERMALISATION IN THE TODA GAS

$$H = \sum_{i=1}^{N} \frac{V(x_i)}{2} \left(p_i^2 + e^{-(x_{i+1}-x_i)} + e^{-(x_i - x_{i-1})} \right)$$



$$\beta_0^{-1} = N^{-1} \sum_{i=1}^{N} V(x_i) p_i^2$$

CONCLUSIONS AND WORK IN PROGRESS

A precise understanding of hydrodynamic expansion

A precise understanding of diffusive hydrodynamics for inhomogeneous Hamiltonians

Inhomogeneous Hamiltonians lead to thermal stationary states

The diffusive terms in the hydrodynamics are responsible for thermalisation
(on the time scales necessary for diffusion)

Going to higher orders in the hydrodynamic expansion?

Different types of thermalisation depending on type of external fields?