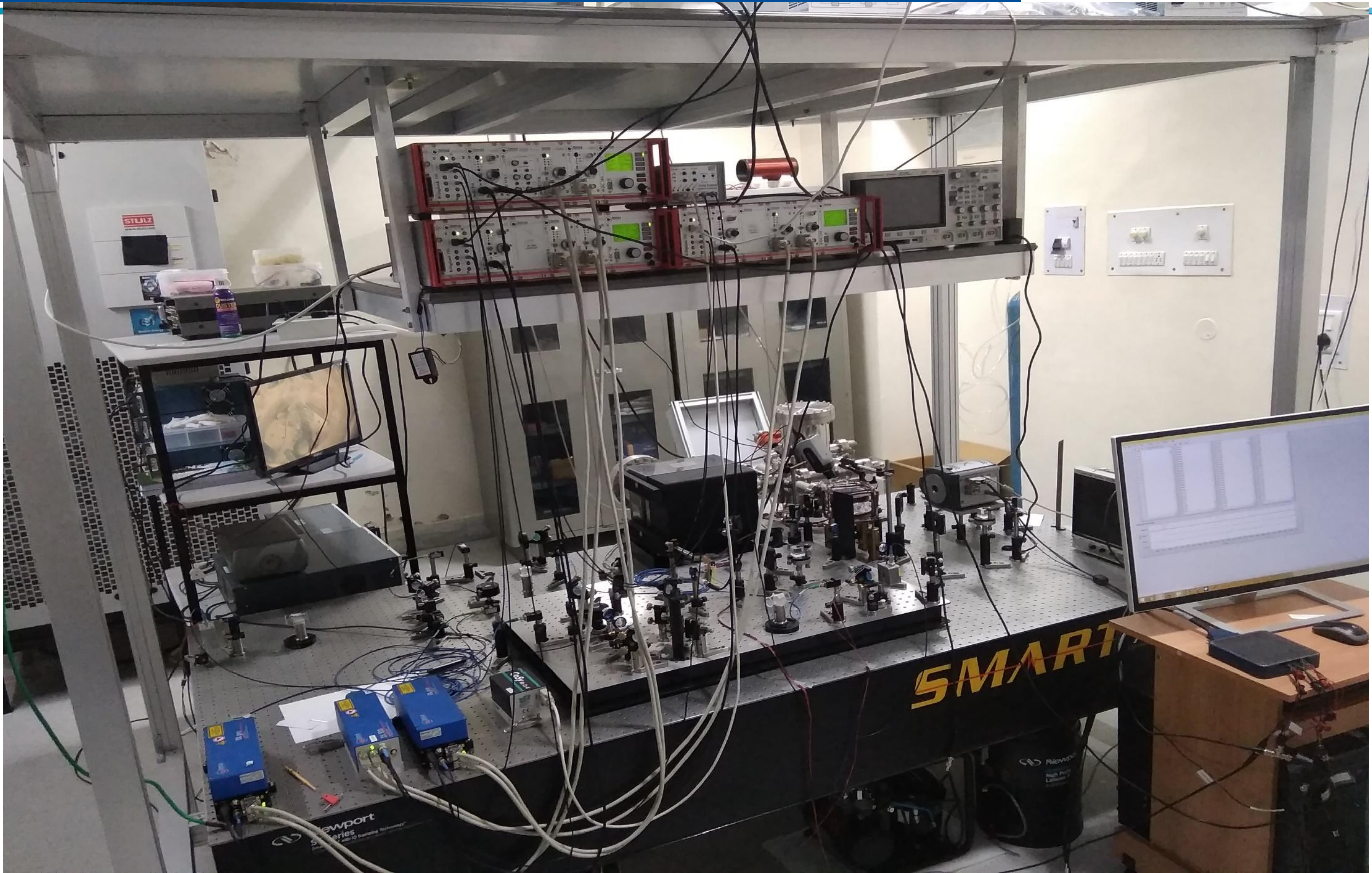
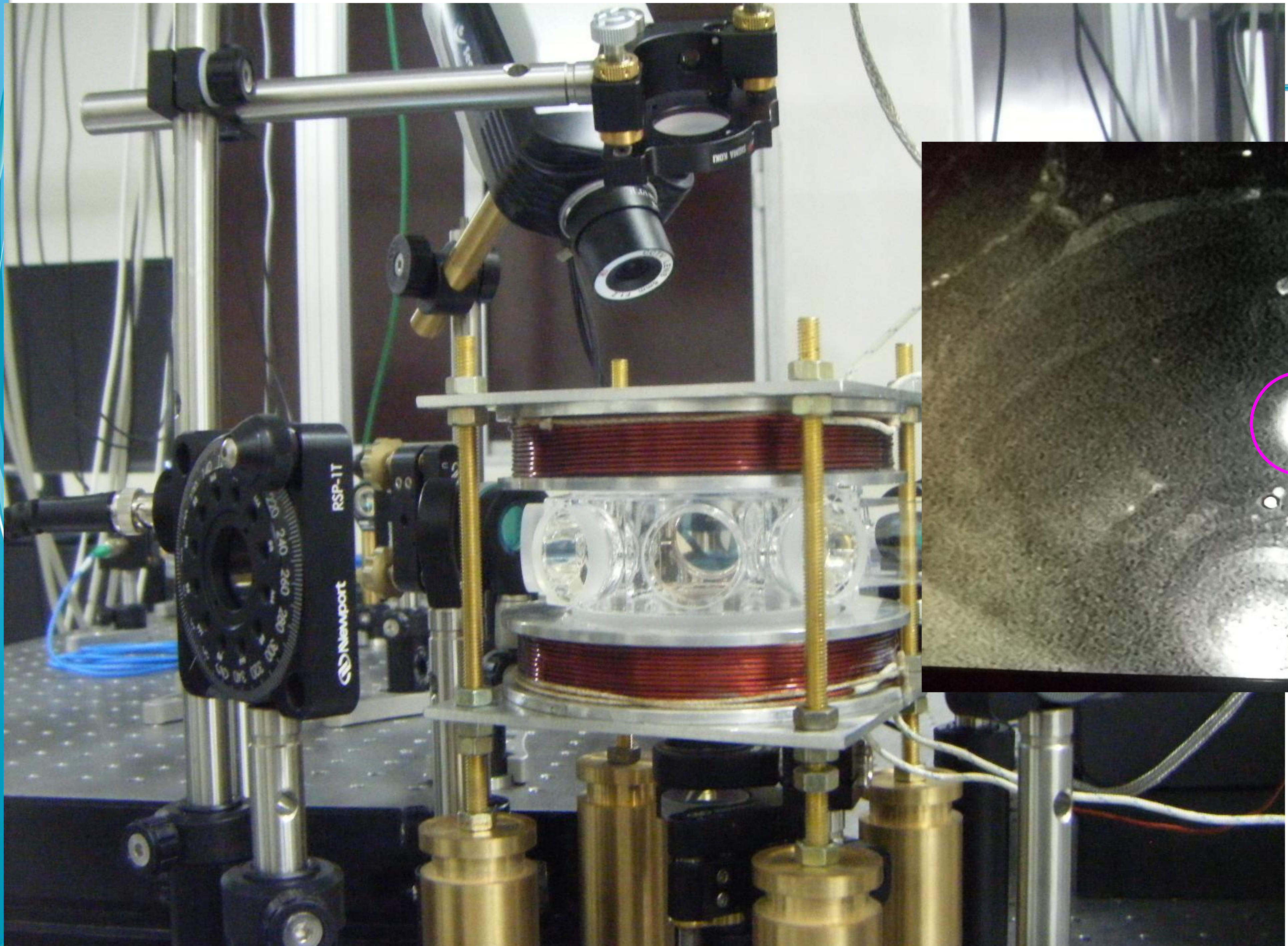
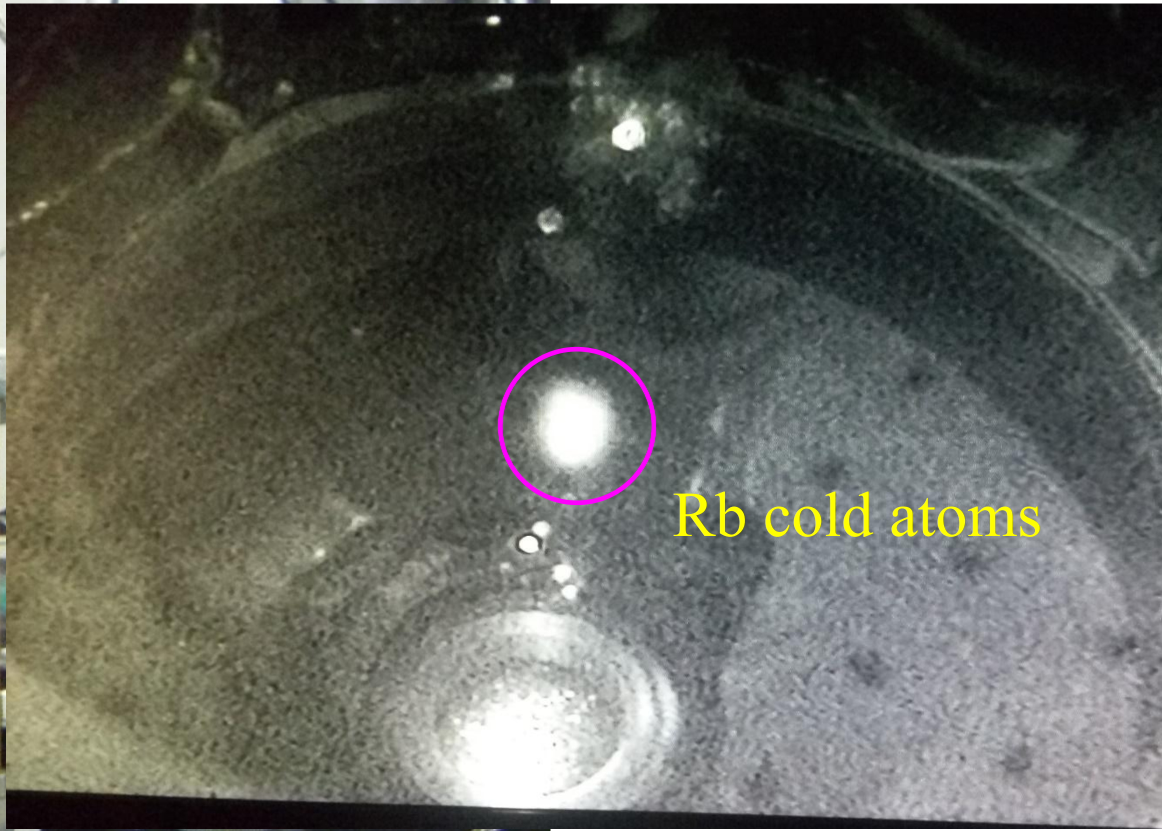
Quantum computers - Physical realization

YEARS OF
IIT ROORKEE
Estd. 1847

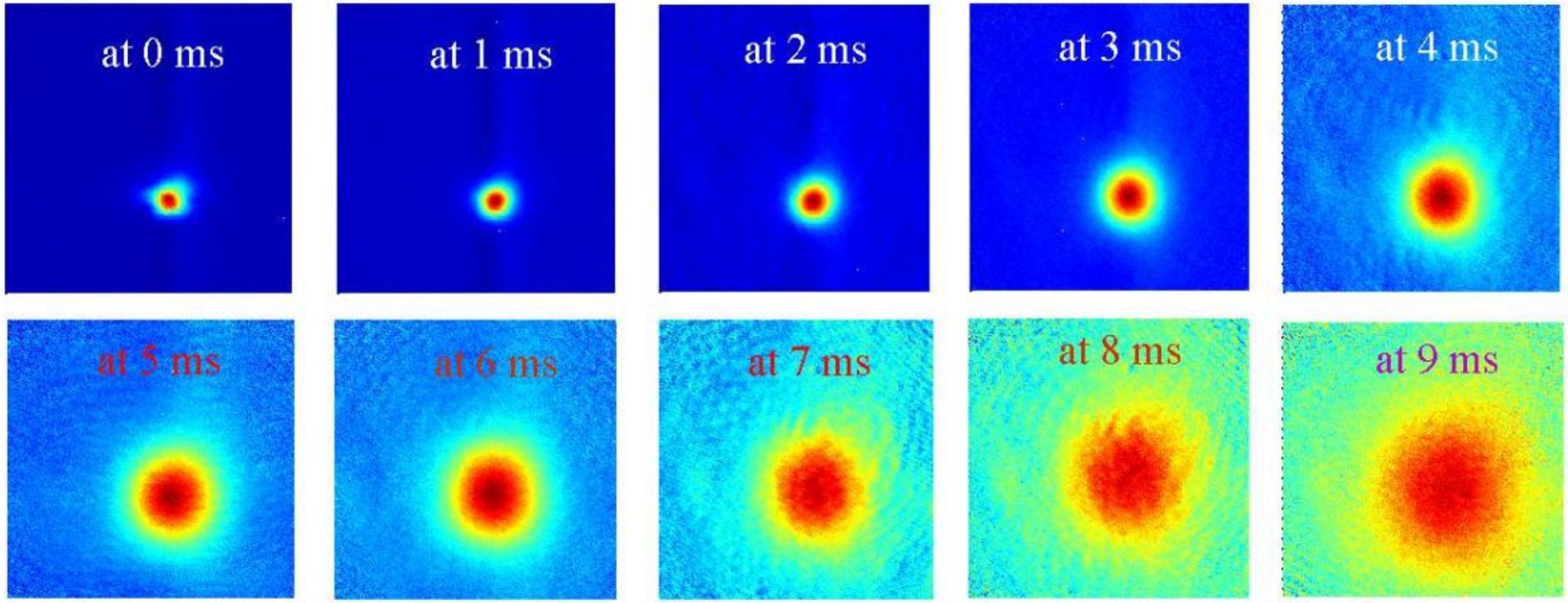
Ajay Wasan
Department of Physics


SMART


RSP-1T
Newport


Rb cold atoms

MOT's cold atom images

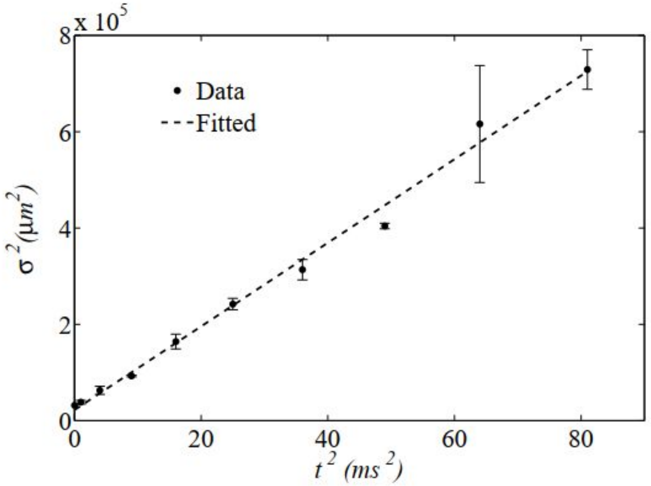


[redacted]

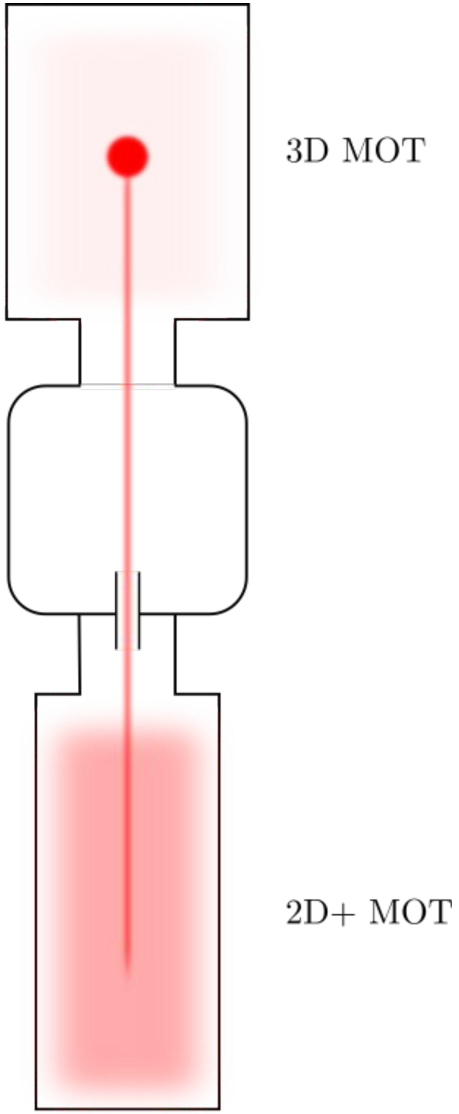
$$\sigma_x^2(t) = \sigma_x^2(0) + \sigma_v^2 t^2$$

[redacted]

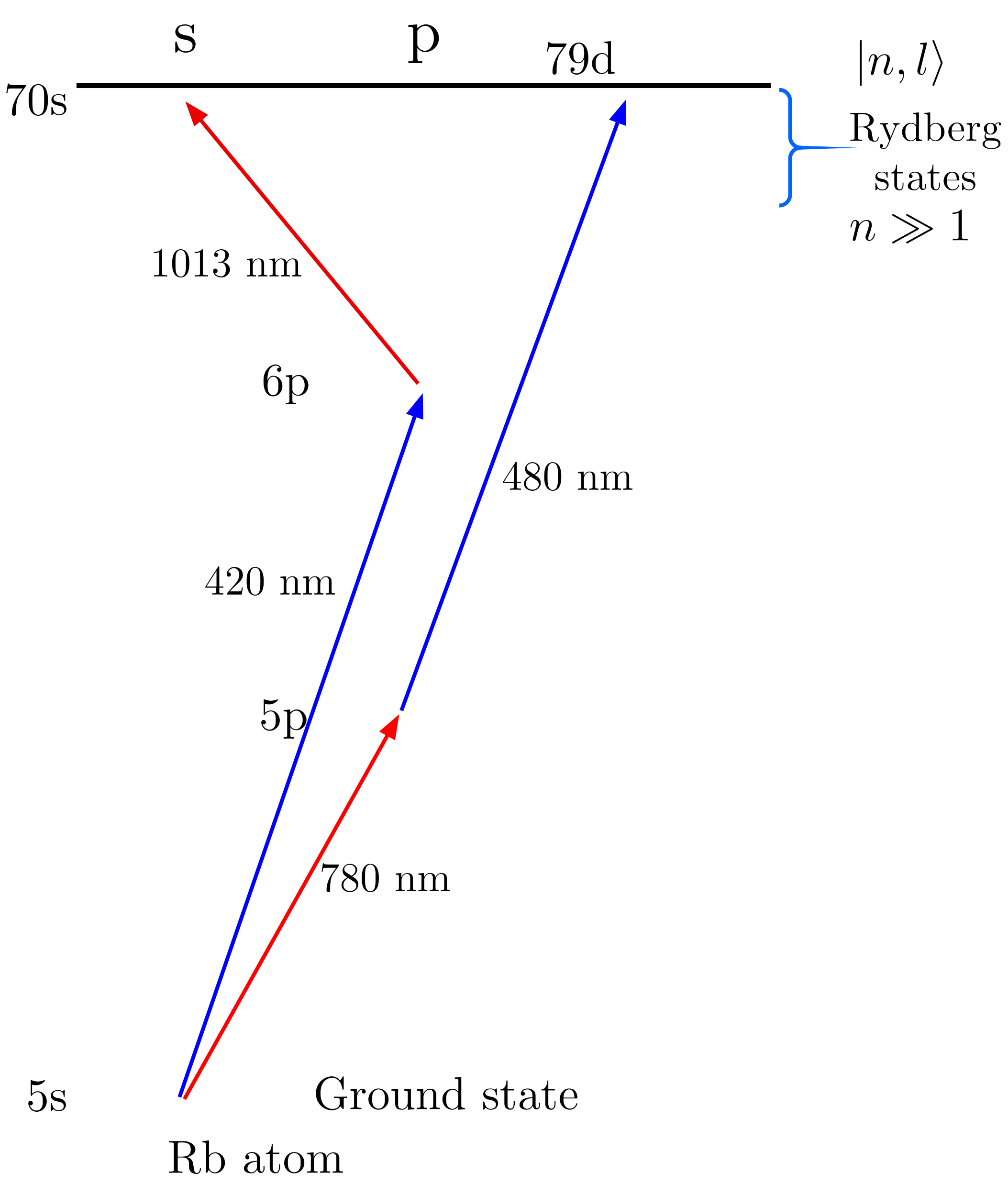
$= 88.87 \pm 6.07\ \mu\text{K}$



New experimental setup

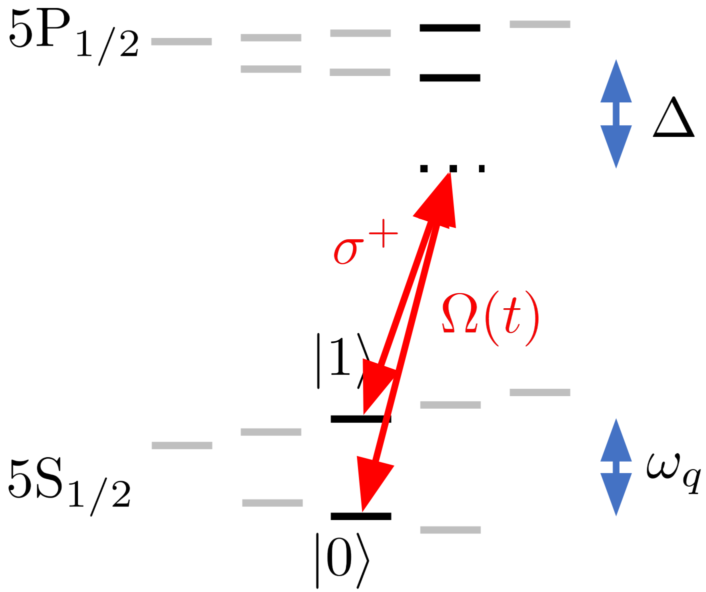


- Two Glass Cells for double MOT system (ColdQuanta)
- 1064 nm cw high-power (50 W) laser system (Azur Light System)
- Tapered Amplifier Laser System at 795 nm (2 W) (TOPTICA Photnoics AG)



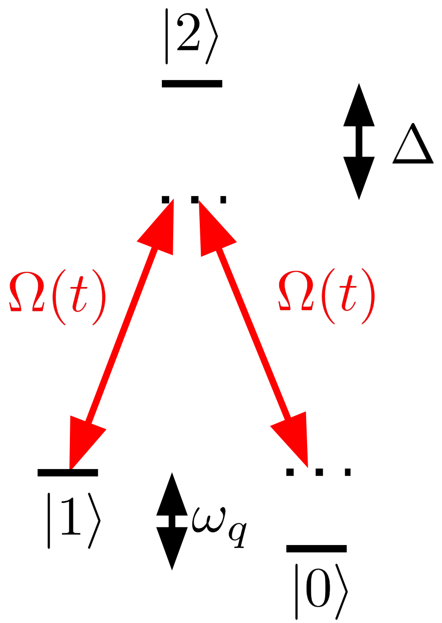
s
p
79d
$|n,l\rangle$
70s
Rydberg
states
$n \gg 1$
1013 nm
6p
480 nm
420 nm
5p
780 nm
5s
Ground state
Rb atom

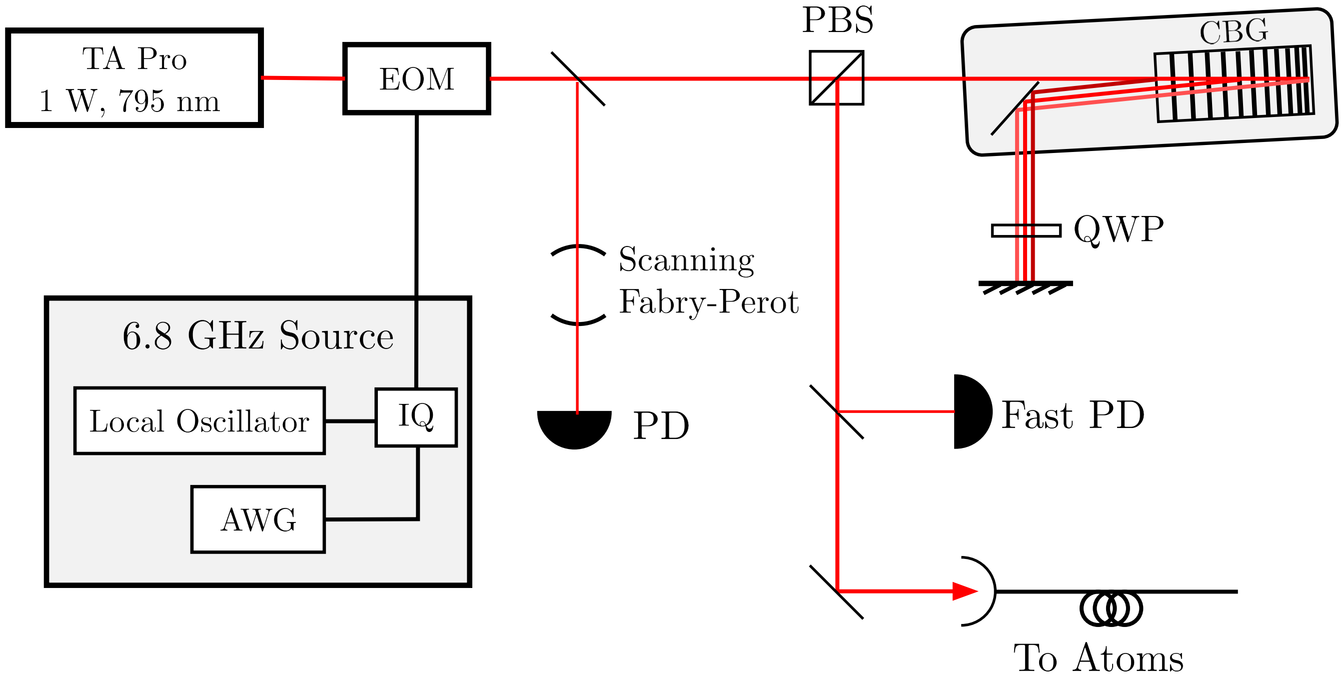
Raman Laser



$5P_{1/2}$
Δ
σ^+
$\Omega(t)$
$|1\rangle$
$5S_{1/2}$
ω_q
$|0\rangle$

Raman Laser





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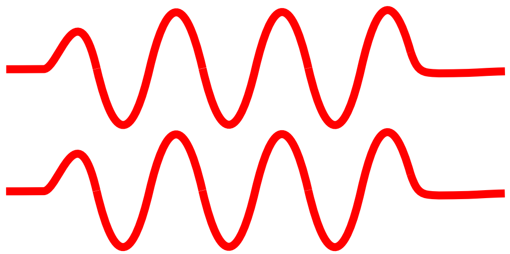
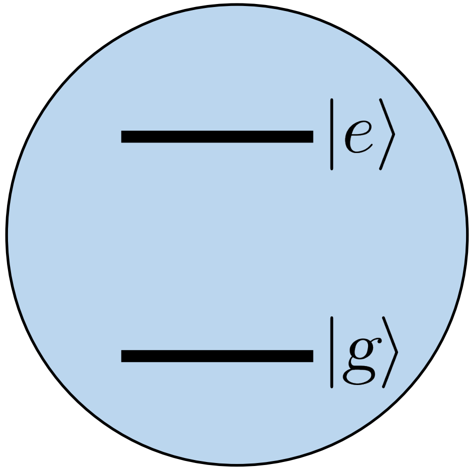
Interaction of light with atom

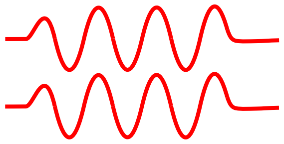


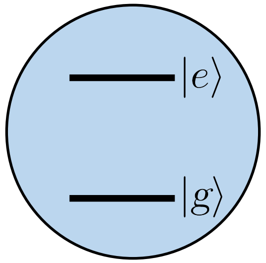
E.M. field
(Photons)

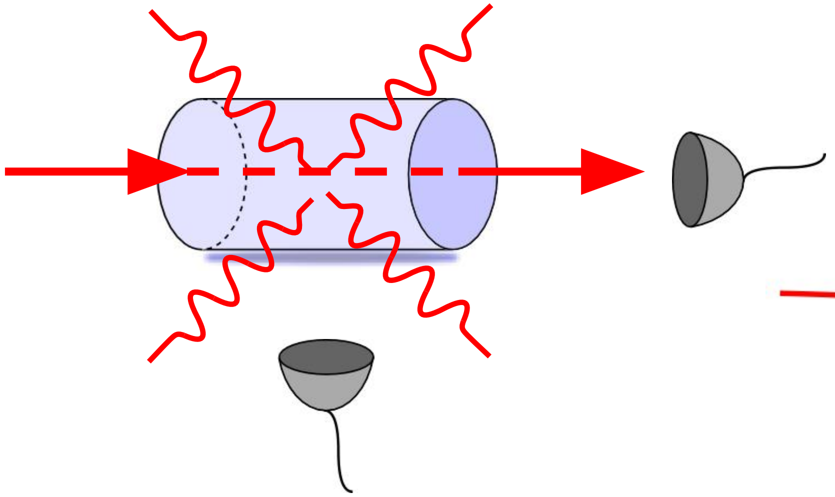


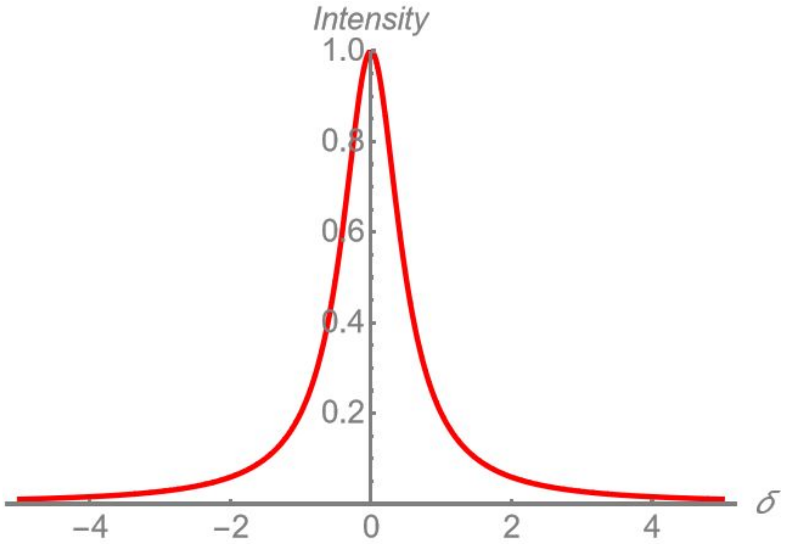
Atom

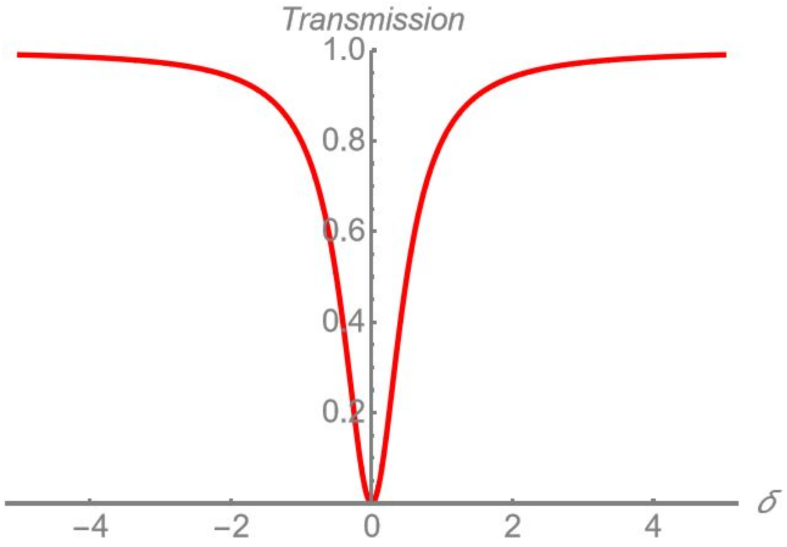
Interaction of light with atom









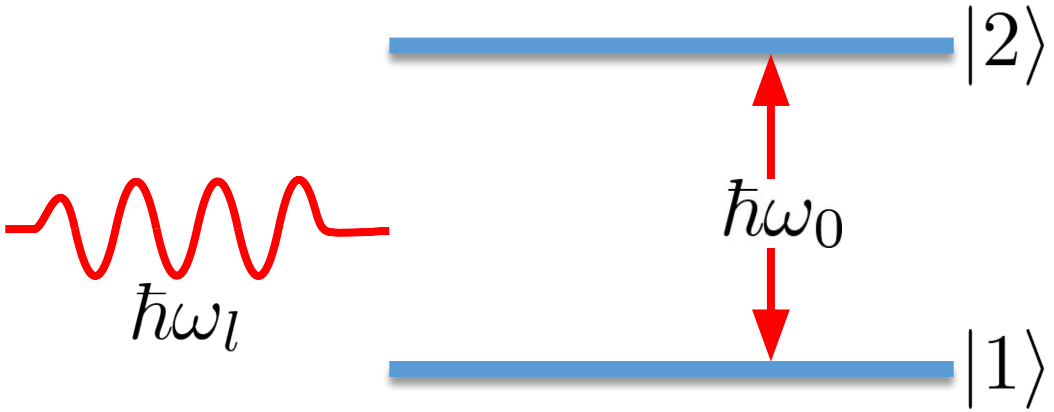


Physics of Two Level Atom (TLA)

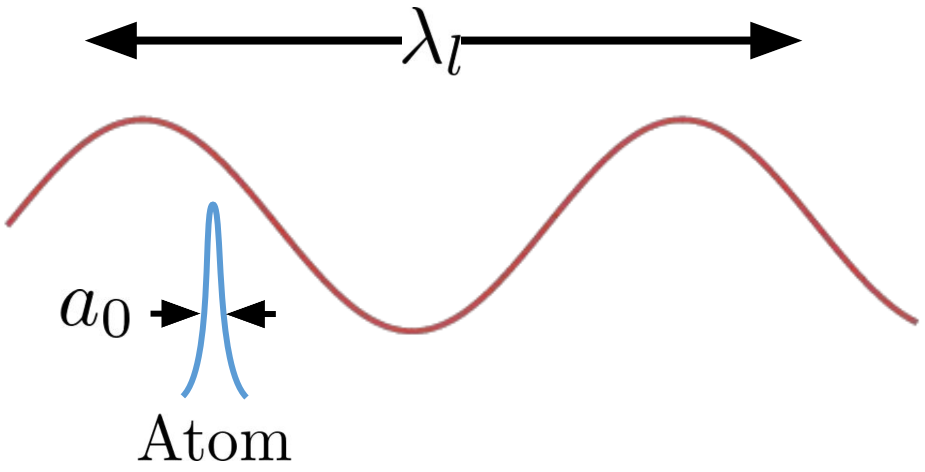
Questions:

- What governs the interaction of the photon with TWA?
- What is the time evolution of the two states?
- What are the properties of the states?

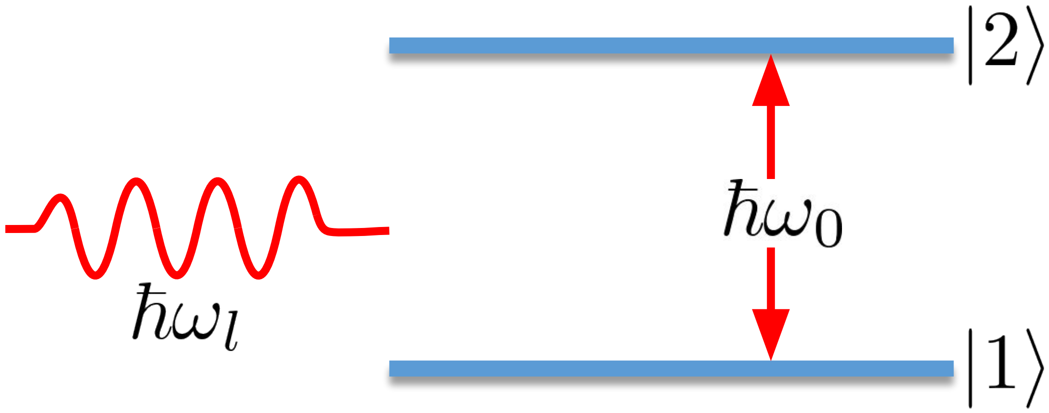
Two level atomic system



Light: Classical plane wave
Atom: Quantum Mechanically



Two level atomic system



Light: Classical plane wave
Atom: Quantum Mechanically

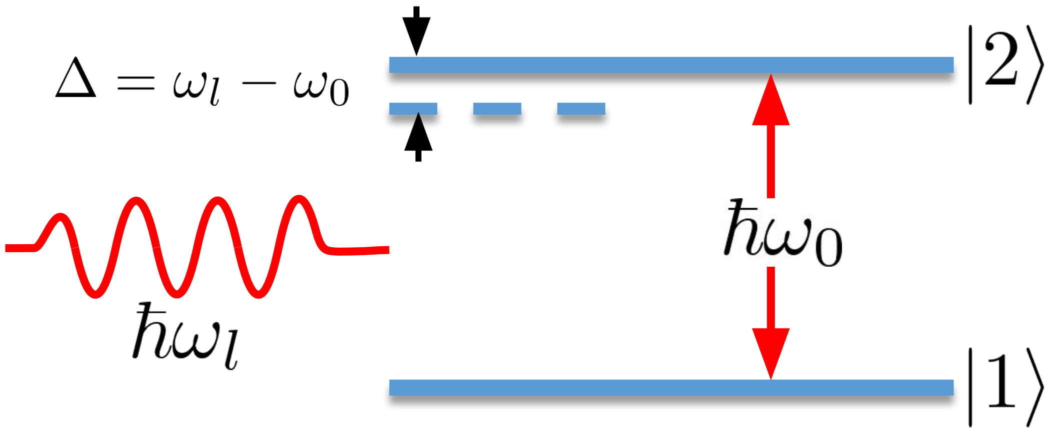
Strength of interaction:

The coupling between field and atom is given by resonant Rabi frequency,

$$\Omega_{12} = \frac{\mu E_0}{\hbar}$$

where $\mu = -e\langle 1|\hat{r}|2\rangle$

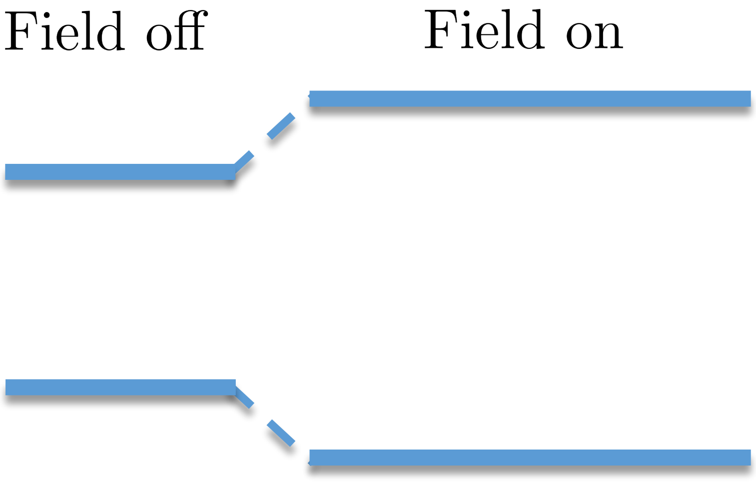
AC stark shift



Interaction Hamiltonian: $$H = \frac{\hbar}{2}\begin{bmatrix} 0 & \Omega_{12} \\ \Omega_{12}^* & -2\Delta \end{bmatrix}$$

Energy Eigen values:

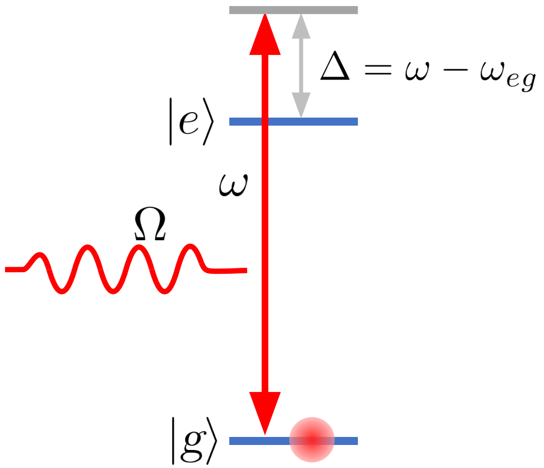
$$E_\pm = \frac{\hbar}{2}(-\Delta \pm \sqrt{|\Omega_{12}|^2 + \Delta^2})$$



$$E_+ = \frac{\hbar}{2}(-\Delta + \sqrt{|\Omega_{12}|^2) + \Delta^2})$$

$$E_- = \frac{\hbar}{2}(-\Delta - \sqrt{|\Omega_{12}|^2) + \Delta^2})$$

Coherent interaction of atoms and light



Two level atomic system

$$|\psi(t)\rangle = c_g(t)e^{-i\omega_g t}|g\rangle + c_e(t)e^{-i\omega_e t}|e\rangle$$

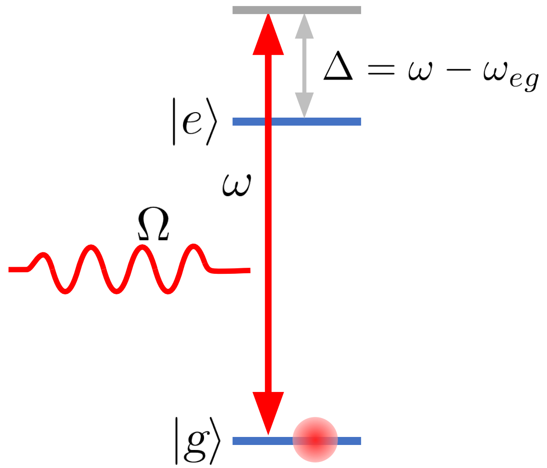
$$\begin{bmatrix} c_g(t) \\ c_e(t) \end{bmatrix} = U(t, t_0) \begin{bmatrix} c_g(t_0) \\ c_e(t_0) \end{bmatrix}$$

[redacted]

where $\Omega' = \sqrt{|\Omega|^2 + \Delta^2}$

[redacted]

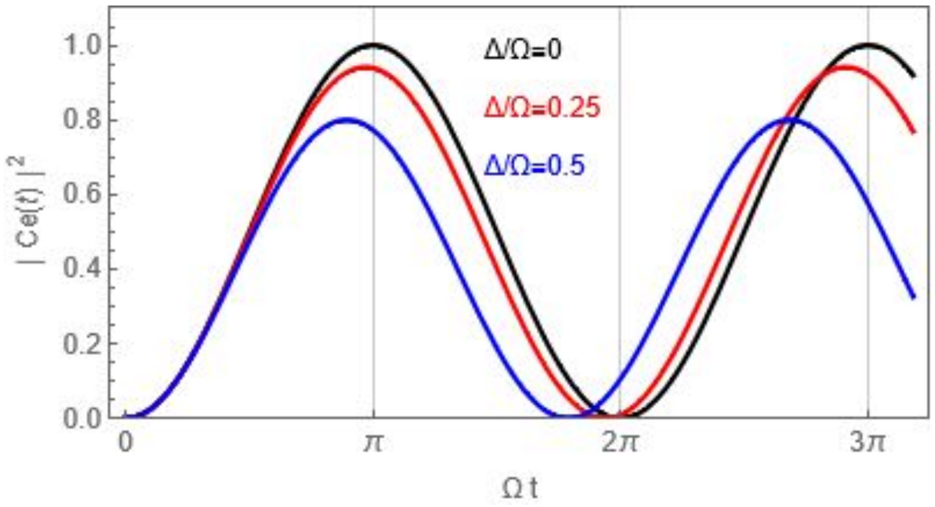
Rabi Oscillations



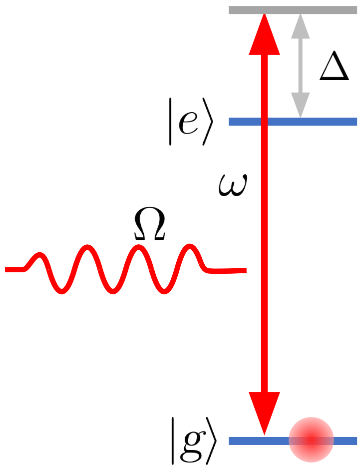
When the driving field is resonant,

$\Rightarrow \quad \Delta = 0, \quad$ and setting $t_0 = 0$

$$U(t) = \begin{bmatrix} \cos\left(\frac{|\Omega|t}{2}\right) & i\frac{\Omega^*}{|\Omega|}\sin\left(\frac{|\Omega|t}{2}\right) \\ i\frac{\Omega}{|\Omega|}\sin\left(\frac{|\Omega|t}{2}\right) & \cos\left(\frac{|\Omega|t}{2}\right) \end{bmatrix}$$



Rabi Oscillations

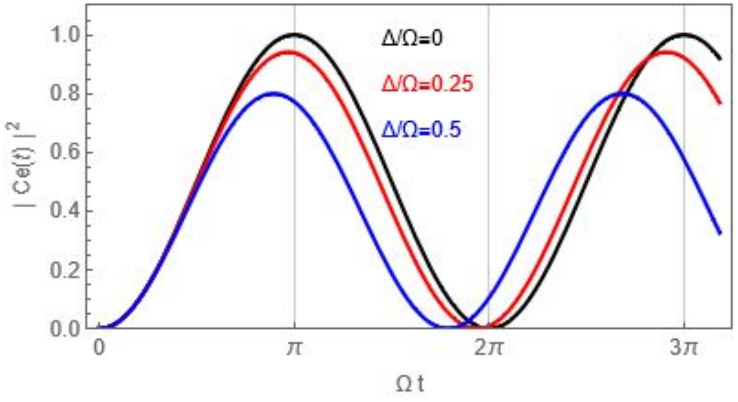


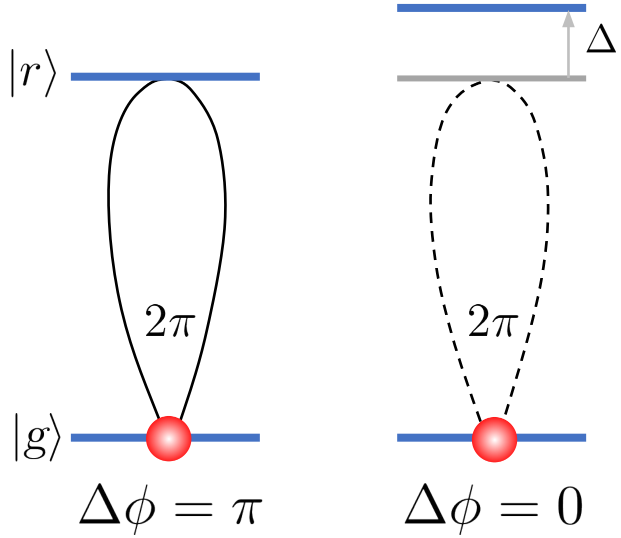
[redacted]

$$U(t-t_0) = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix}$$

which results in the transformation

$$|\psi\rangle = -|\psi\rangle = e^{i\pi}|\psi\rangle$$





Rotation Operators

In quantum computing the *rotation operators* about the $\hat{x}$, $\hat{y}$, $\hat{z}$ axes are defined as:

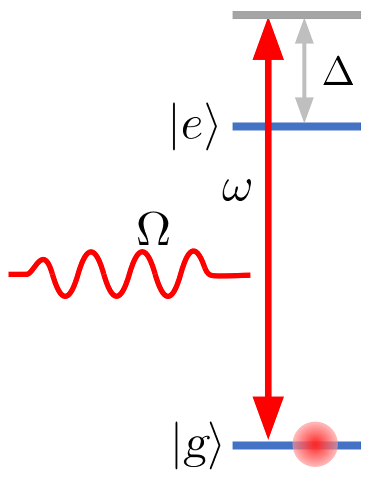
$$R_x(\theta) = e^{-i\theta X/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} X = \begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_y(\theta) = e^{-i\theta Y/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} Y = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_z(\theta) = e^{-i\theta Z/2} = \cos\frac{\theta}{2} I - i\sin\frac{\theta}{2} Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} = e^{-i\theta/2} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

$$X = \sigma_x = iR_x(-\pi) \qquad Y = \sigma_y = iR_y(-\pi) \qquad Z = \sigma_z = iR_z(-\pi)$$

One-Qubit Gates



$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$U(t) = \begin{bmatrix} \cos\left(\frac{|\Omega|t}{2}\right) & i\frac{\Omega^*}{|\Omega|}\sin\left(\frac{|\Omega|t}{2}\right) \\ i\frac{\Omega}{|\Omega|}\sin\left(\frac{|\Omega|t}{2}\right) & \cos\left(\frac{|\Omega|t}{2}\right) \end{bmatrix}$$

[redacted]

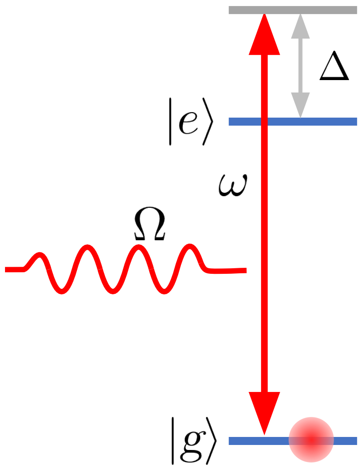
$$U(t) = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} = R_x(\theta)$$

$$R_x(\pi) = \begin{bmatrix} 0 & -i \\ -i & 0 \end{bmatrix}$$

$$= -iX$$

X gate up to a global phase.

One-Qubit Gates



$$X = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \qquad Y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$R_x(\theta) = \begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_y(\theta) = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$U(t) = \begin{bmatrix} \cos\left(\frac{|\Omega|t}{2}\right) & i\frac{\Omega^*}{|\Omega|}\sin\left(\frac{|\Omega|t}{2}\right) \\ i\frac{\Omega}{|\Omega|}\sin\left(\frac{|\Omega|t}{2}\right) & \cos\left(\frac{|\Omega|t}{2}\right) \end{bmatrix}$$

[redacted]

$$U(t) = \begin{bmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{bmatrix} = R_y(\theta)$$

$$R_\pi(\pi) = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$

$$= -iY$$

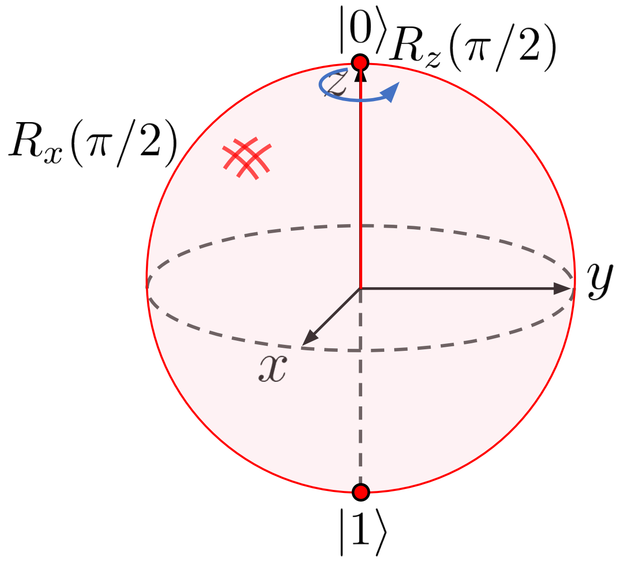
Y gate up to a global phase.

Rotation Operators

- A Hadamard gate H can be expresses as a product of R_x and R_z rotations.

$$H = R_z(\pi/2)R_x(\pi/2)R_z(\pi/2)$$

Showing $H|0\rangle$ on Bloch sphere.



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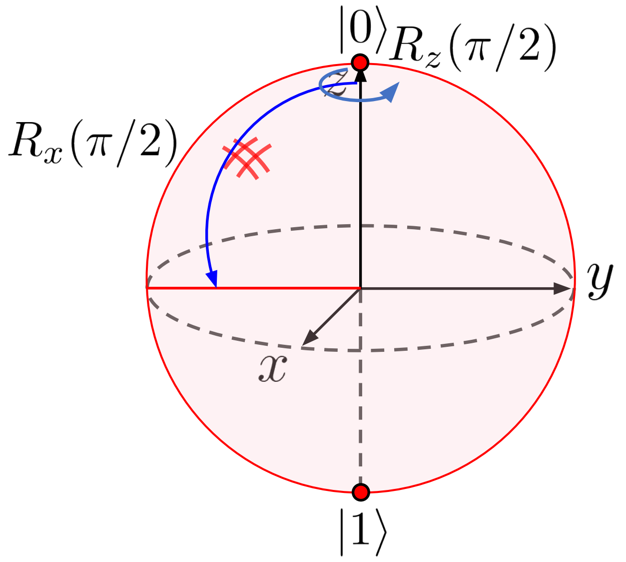

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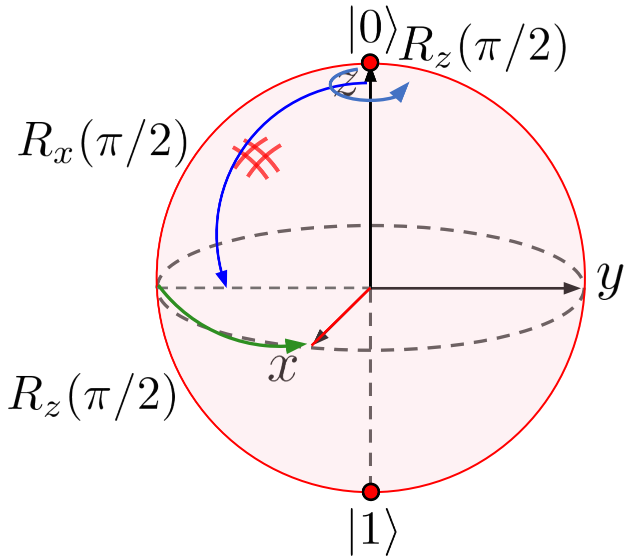
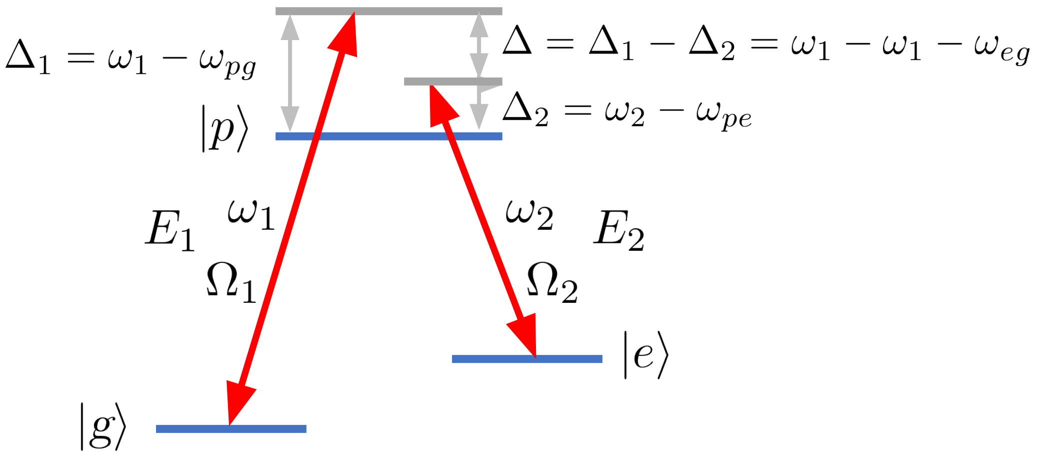


Two-photon Raman transitions



$$\begin{bmatrix} c_g(t) \\ c_e(t) \end{bmatrix} = U(t,0) \begin{bmatrix} c_g(t_0) \\ c_e(t_0) \end{bmatrix}$$

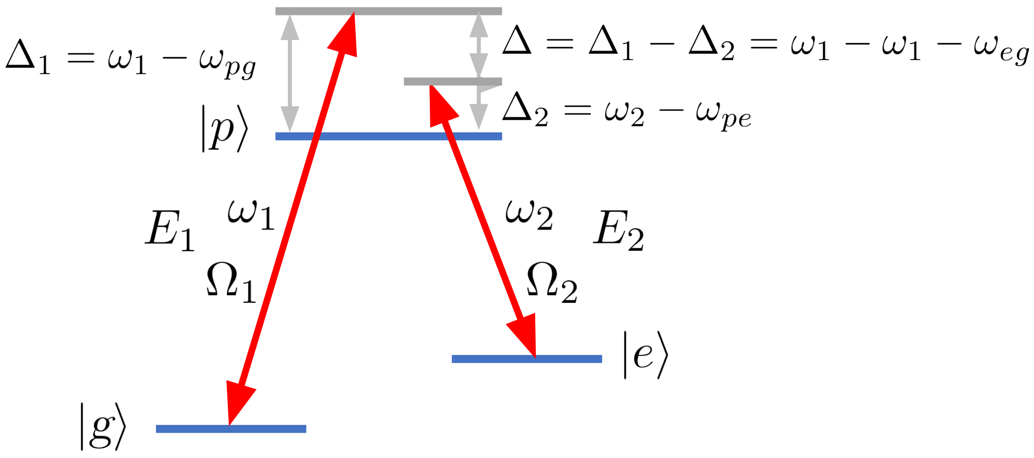
$$\Delta_1, \Delta_1 \gg \Omega_1, \Omega_1$$

[redacted]

where $\delta_p = \frac{\Delta_1 + \Delta_2}{2}$, $\Omega_R^* = \frac{\Omega_1^* \Omega_2}{2\delta_p}$, $\Omega' = \sqrt{4|\Omega_R|^2 + \Delta'^2}$,

$$\Delta' = 2\Delta + \frac{(|\Omega_1|^2 - |\Omega_2|^2)}{2\delta_p}$$

Two-photon Raman transitions



$$\begin{bmatrix} c_g(t) \\ c_e(t) \end{bmatrix} = U(t,0) \begin{bmatrix} c_g(t_0) \\ c_e(t_0) \end{bmatrix}$$

$$\delta_p = \frac{\Delta_1+\Delta_2}{2}, \quad \Omega_R^* = \frac{\Omega_1^*\Omega_2}{2\delta_p}, \quad \Omega' = \sqrt{4|\Omega_R|^2 + \Delta'^2},$$

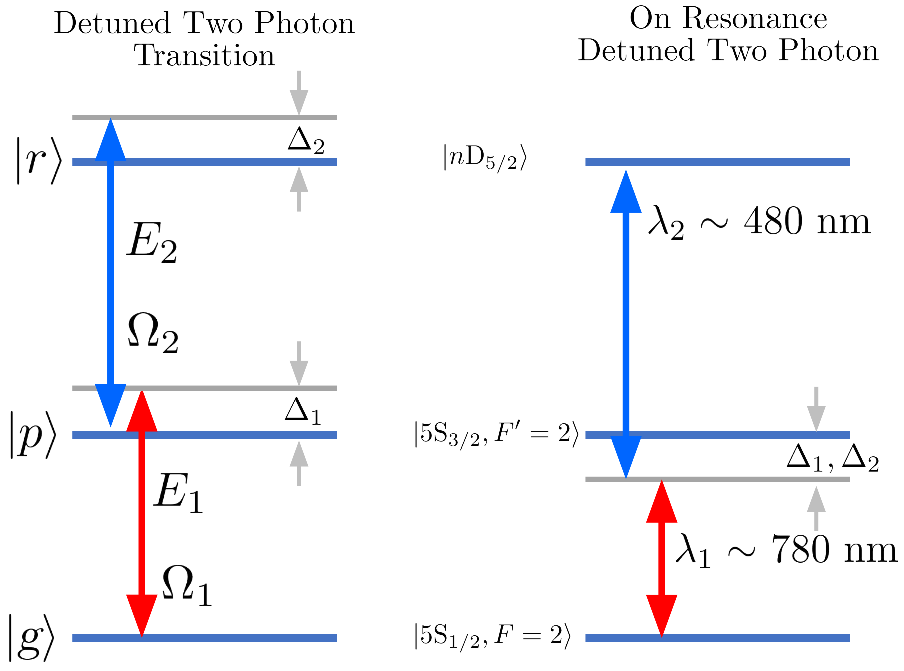
$$\Delta' = 2\Delta + \frac{(|\Omega_1|^2 - |\Omega_2|^2)}{2\delta_p}$$

[redacted]

Assume $\Delta' = 0$

$$U(t,0) = \underbrace{e^{-i\frac{|\Omega_1|^2+|\Omega_2|^2}{8\delta_p}t} \times e^{i\frac{\Delta t}{2}}}_{\text{a global phase shift}} \times \underbrace{\begin{bmatrix} 1 & 0 \\ 0 & e^{-i\Delta t} \end{bmatrix}}_{\text{a single qubit phase gate with } \phi = -\Delta t} \times \underbrace{\begin{bmatrix} \cos(\frac{\Omega_R t}{2}) & -i\sin(\frac{\Omega_R t}{2}) \\ -i\sin(\frac{\Omega_R t}{2}) & \cos(\frac{\Omega_R t}{2}) \end{bmatrix}}_{\text{a rotation about an equitorial axis of } \theta = \frac{\Omega_R t}{2}}$$

Two-photon coherent transitions



$$\begin{bmatrix} c_g(t) \\ c_r(t) \end{bmatrix} = U(t,0) \begin{bmatrix} c_g(t_0) \\ c_r(t_0) \end{bmatrix}$$

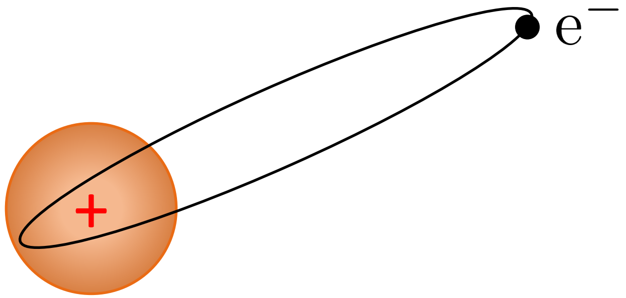
[redacted]

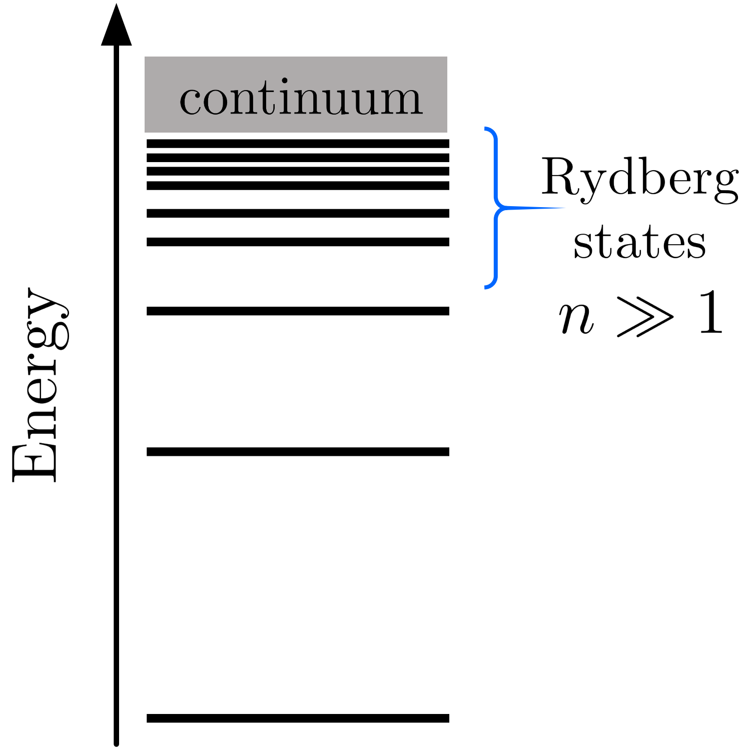
where $\delta_p = \frac{\Delta_1 - \Delta_2}{2}$, $\Omega_R = \frac{\Omega_1 \Omega_2}{2\delta_p}$, $\Omega' = \sqrt{4|\Omega_R|^2 + \Delta'^2}$,

$$\Delta' = 2\Delta + \frac{(|\Omega_1|^2 - |\Omega_2|^2)}{2\delta_p}$$

Rydberg atoms (alkali case)

Johannes Rydberg
1854-1919





Rydberg atoms (alkali case)

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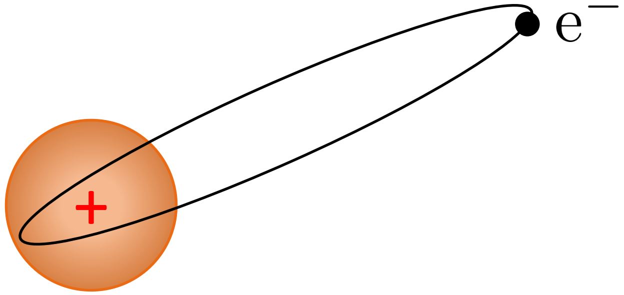
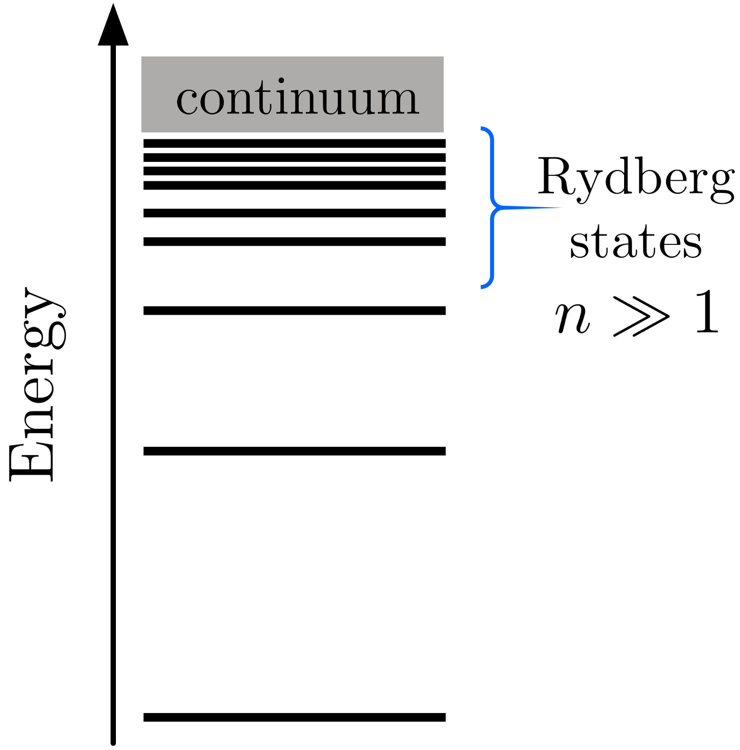


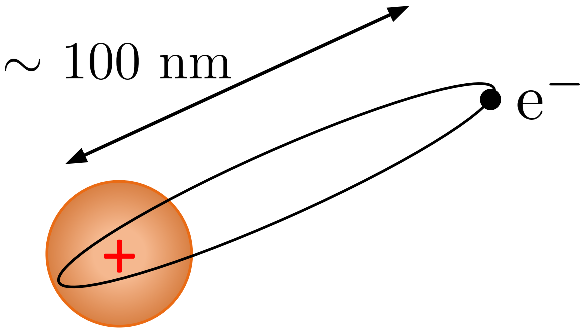


Alkali atoms (Rb, Cs) $\Rightarrow$ hydrogenoid $|n, l, j, m\rangle$

$$E_{n,l} = -\frac{13.6}{n^{*2}}\ \mathrm{eV}$$

$n - \delta_l$

Properties of Rydberg atoms



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T. F. Gallagher "Rydberg Atoms" (2005)

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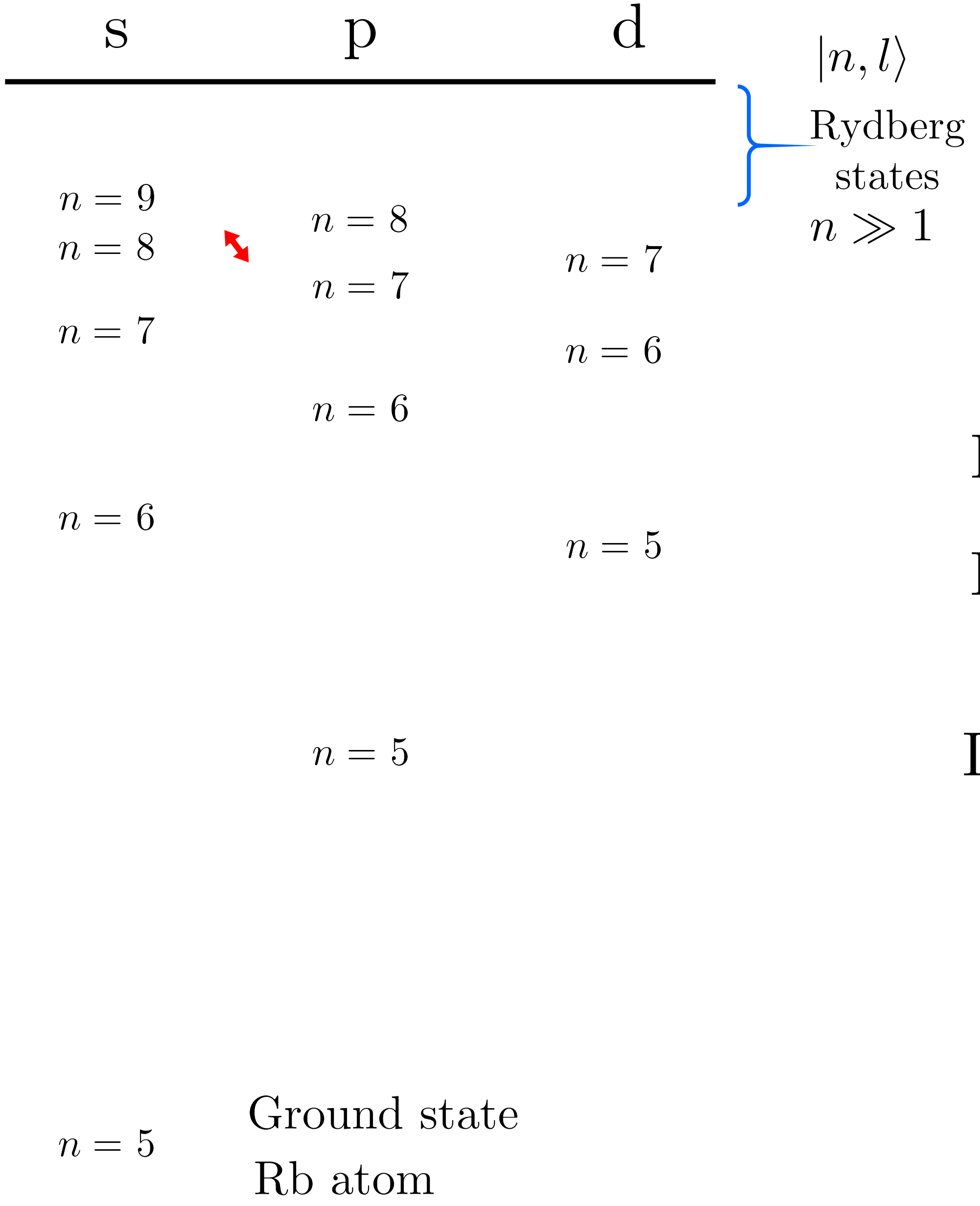
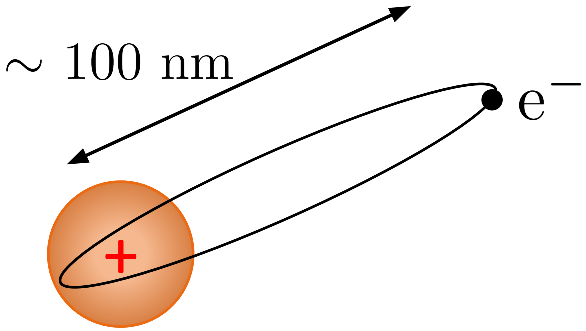
Rydberg atoms





Bohr model: size $\langle r \rangle \sim n^2 a_0$

Long lifetime $\tau \sim n^3$

$$\Rightarrow n > 60, \tau > 100\mu s$$

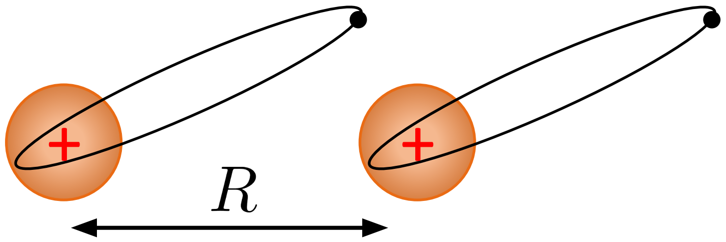
Large dipole elements:

$$(n, l) \rightarrow (n', l \pm 1)$$

$$d = \langle n, l | \hat{D} | n', l' \rangle \approx n^2 e a_0 \text{ for } n \approx n'$$

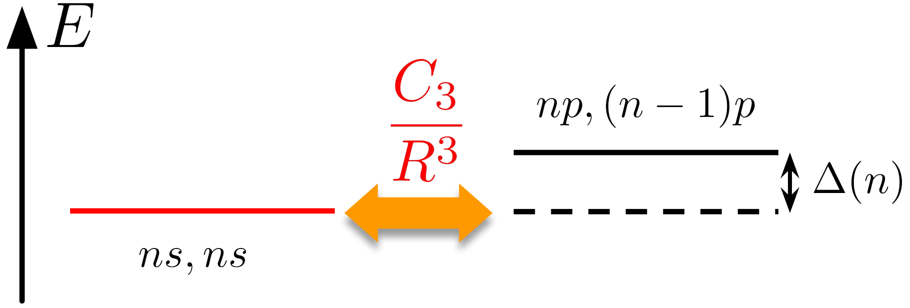
$\Rightarrow$ Large dipole-dipole interactions

Interaction between Rydberg atoms



[redacted]

Two atom basis: $\{|n,l\rangle \otimes |n',l'\rangle\}$



$$\hat{H} = \begin{pmatrix} 0 & \frac{C_3}{R^3} \\ \frac{C_3}{R^3} & \Delta \end{pmatrix}$$

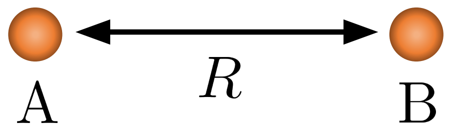
Van der Waals: $\frac{C_3}{R^3} \ll \Delta \quad \Rightarrow \Delta E_{ss} \approx \frac{1}{2\Delta}\left(\frac{C_3}{R^3}\right)^2 = \frac{C_6}{R^6}$

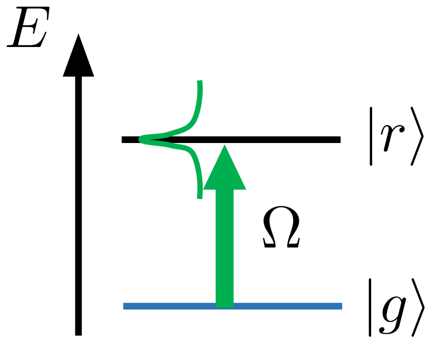
Scaling: $C_6 \propto n^{11}$

Resonant regime: $\frac{C_3}{R^3} \gg \Delta \quad \Rightarrow \Delta E_{\pm} \approx \pm\frac{C_3}{R^3}$

Browaeys lecture (2021)

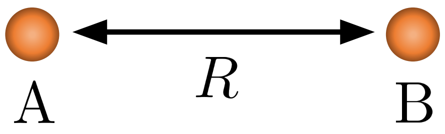
Rydberg blockade

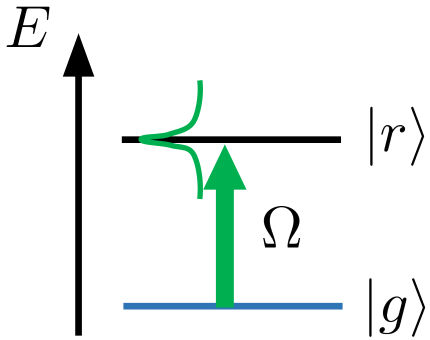


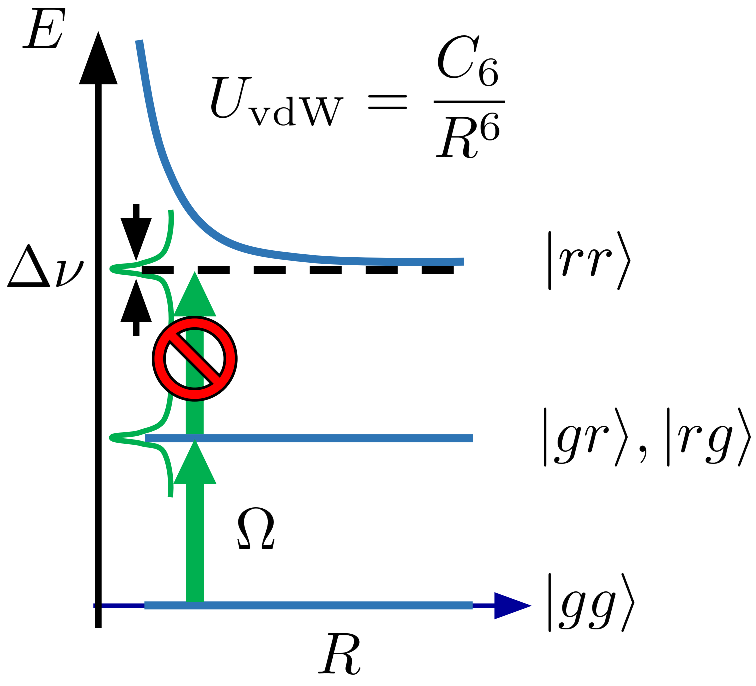


[redacted]

Rydberg blockade





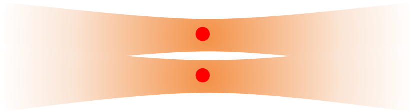


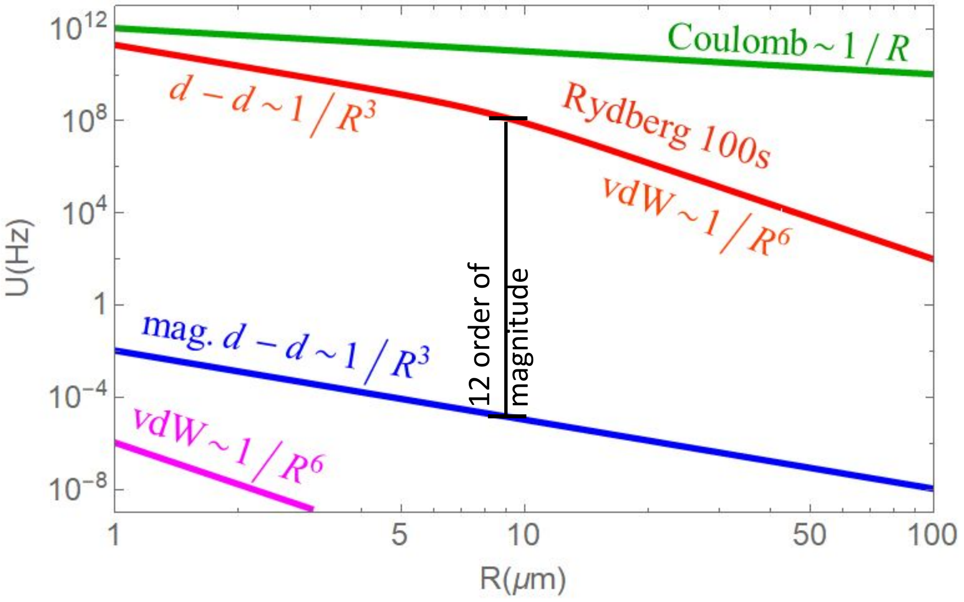
If $\hbar\Delta\nu \ll U_{\text{vdW}}$: no excitation of $|rr\rangle \Rightarrow$ BLOCKADE

[redacted]

Far Off Resonance Dipole Traps

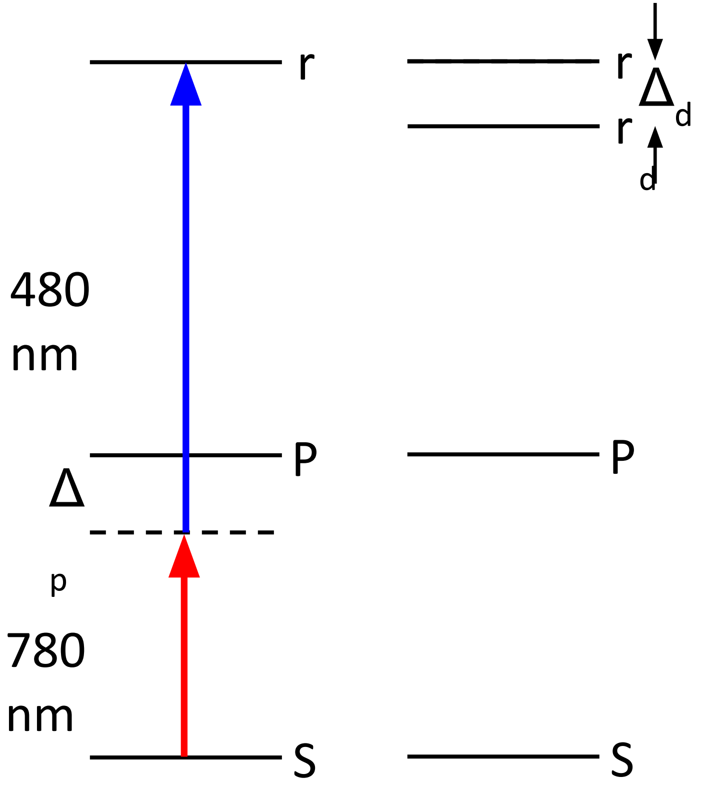
(FORT)



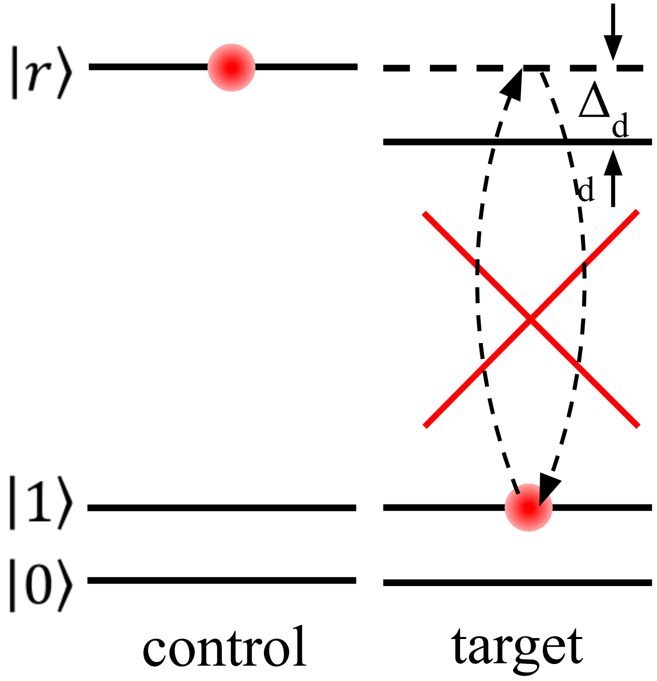


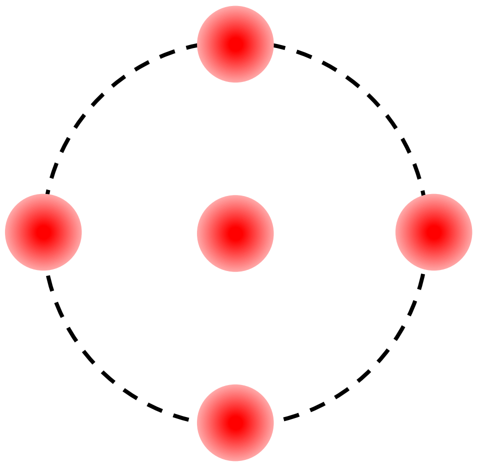
[redacted]

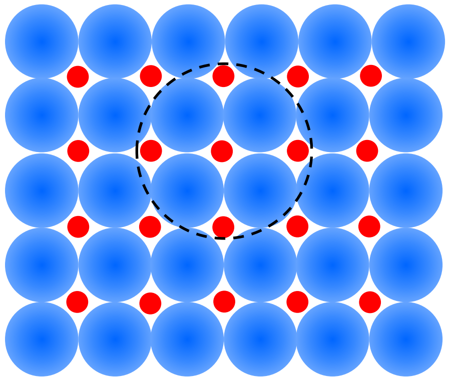
Rydberg excitations and blockade



Rydberg Atoms

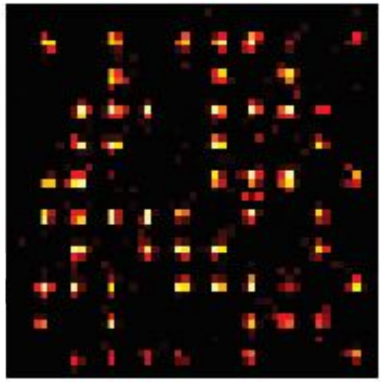


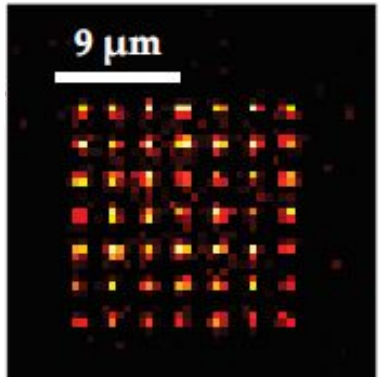


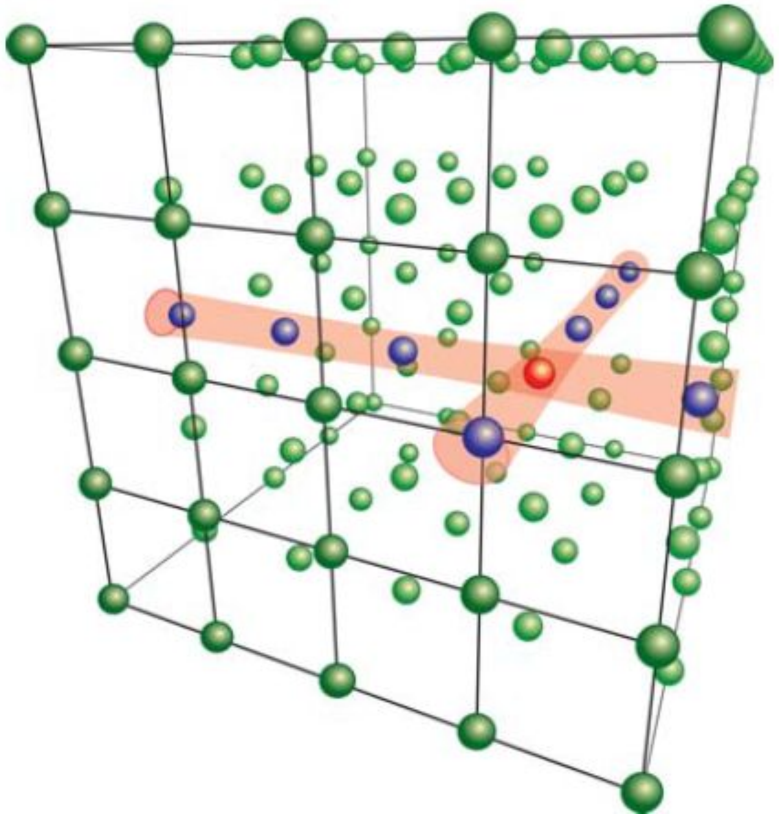


2D beam array pattern

Arrays of single atoms trapped by light





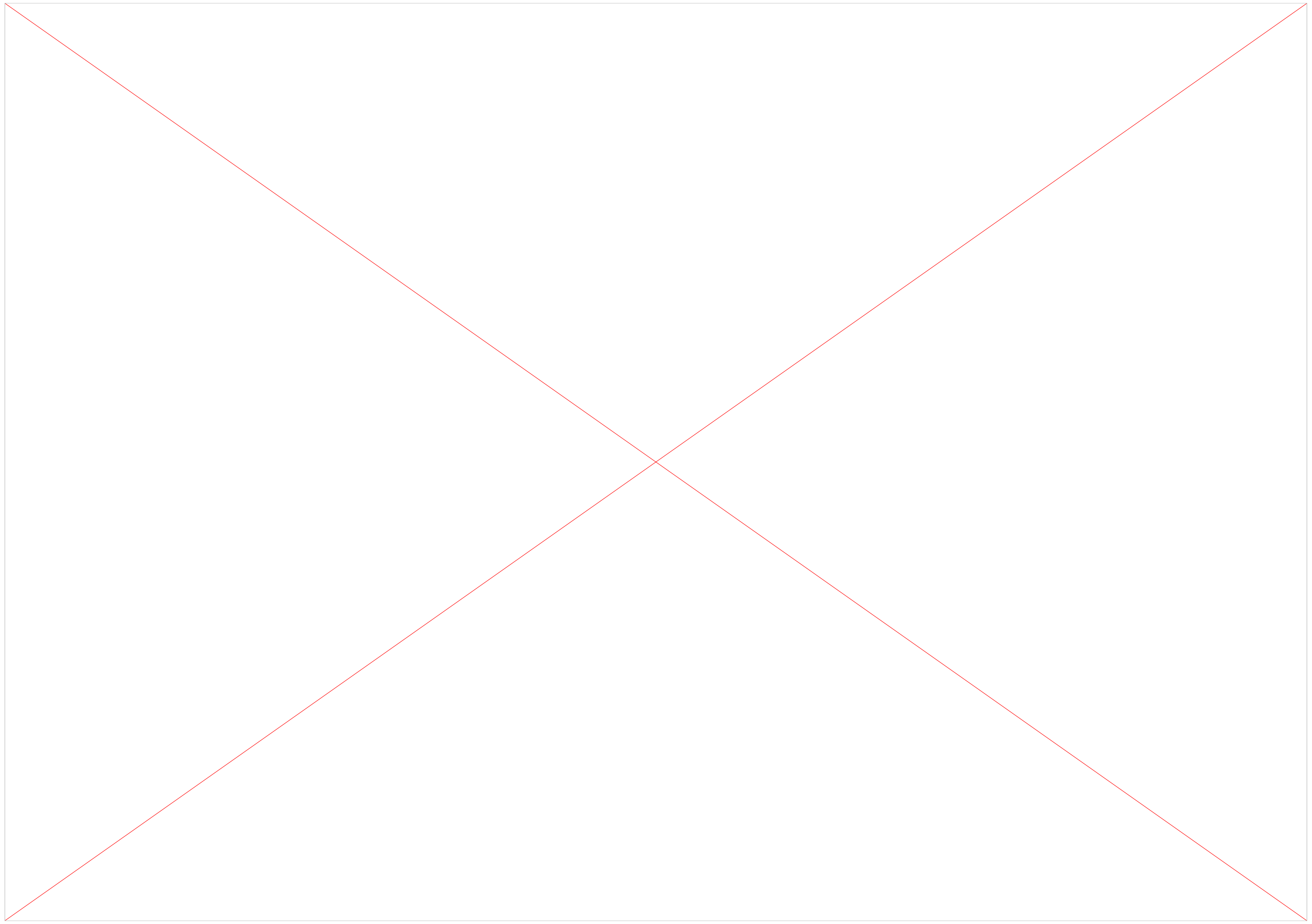


Physics Today **70**, 44 (2017)

Arrays of single atoms trapped by light

[redacted]

Browaeys group, Nature **561**, 79 (2018)



[redacted]

Arrays of single atoms trapped by light

Browaeys group, Nature **561**, 79 (2018)

Arrays of single atoms trapped by light

Browaeys group,
Nature **561**, 79 (2018)

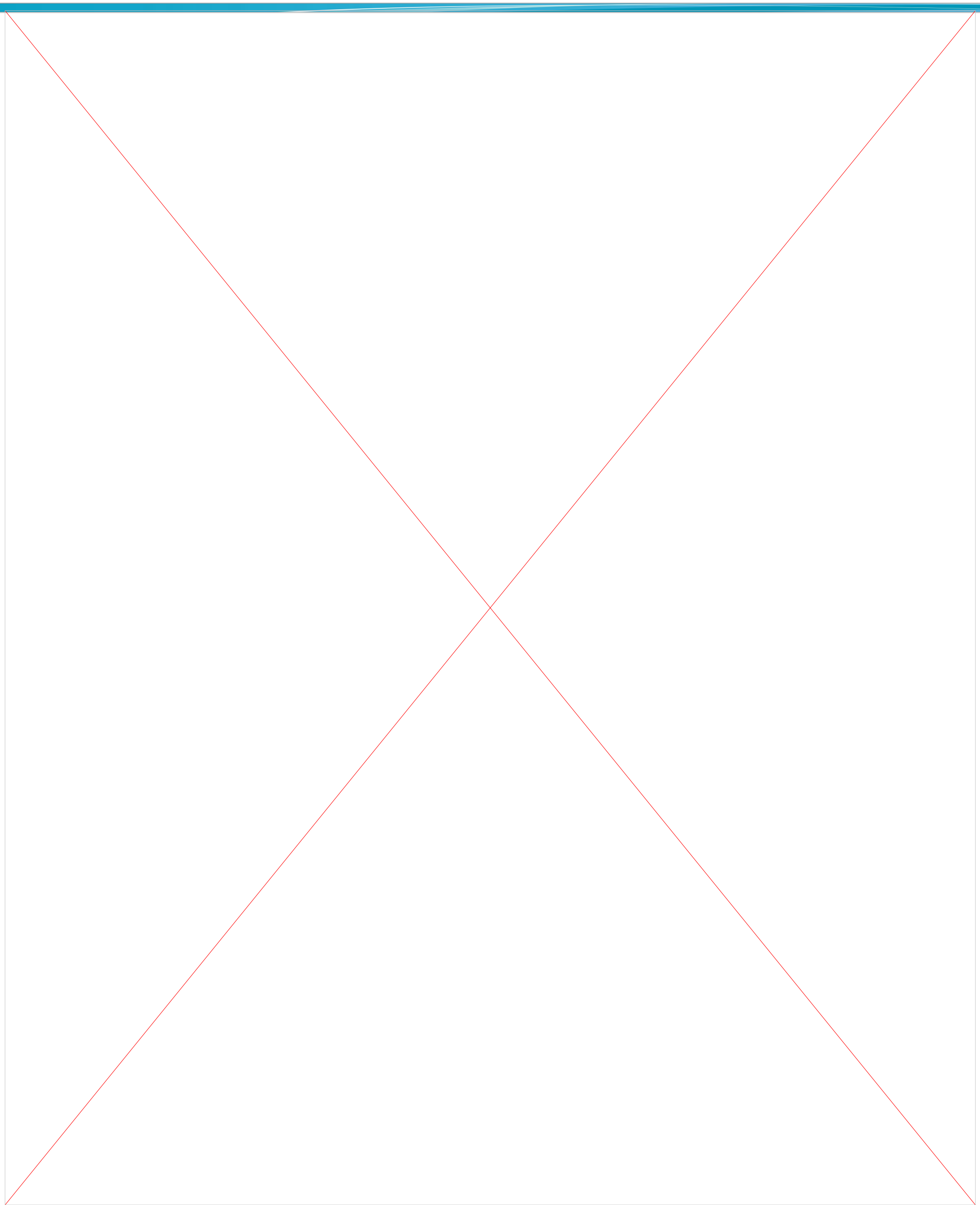
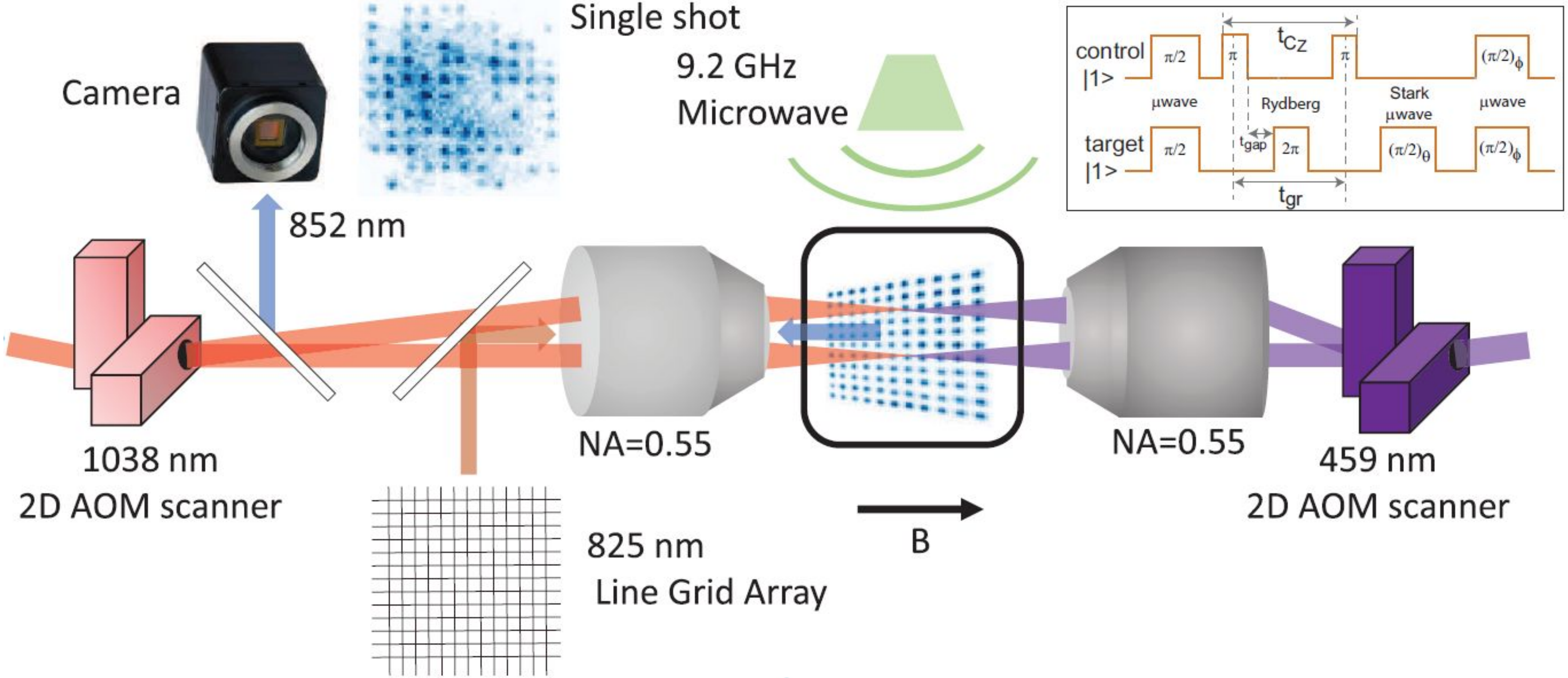
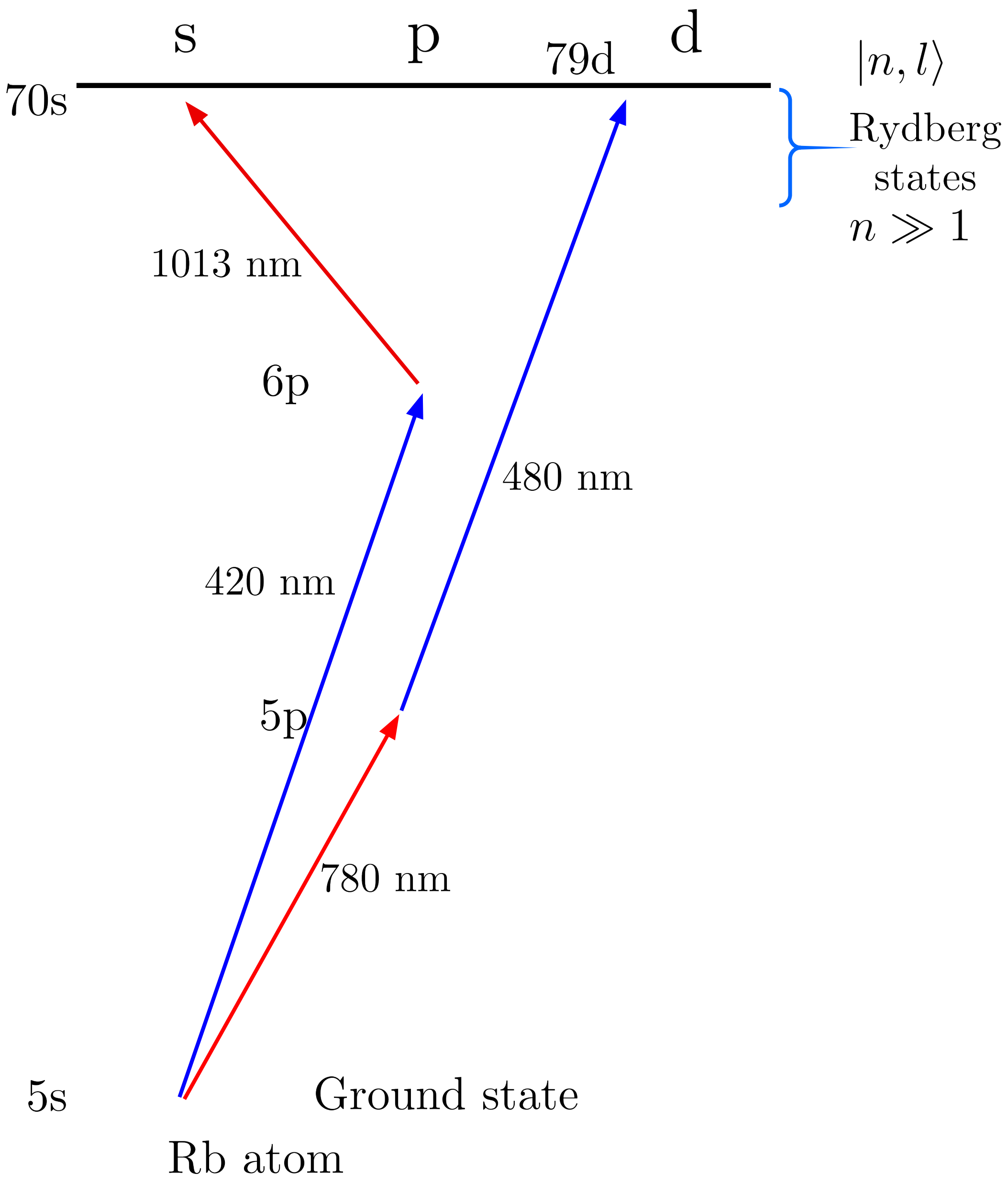


Fig. 2: Single-atom fluorescence in 3D arrays.

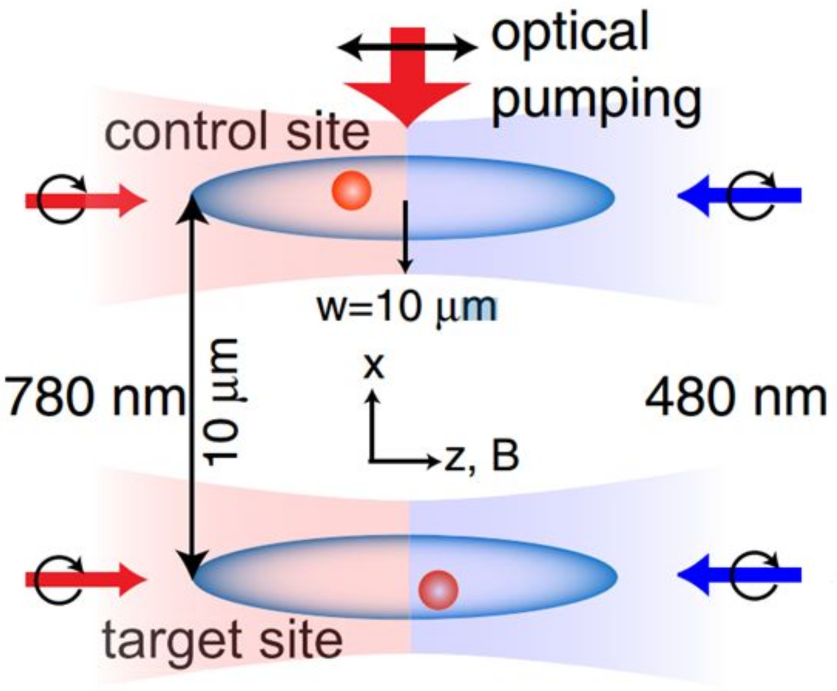
Arrays of single atoms trapped by light



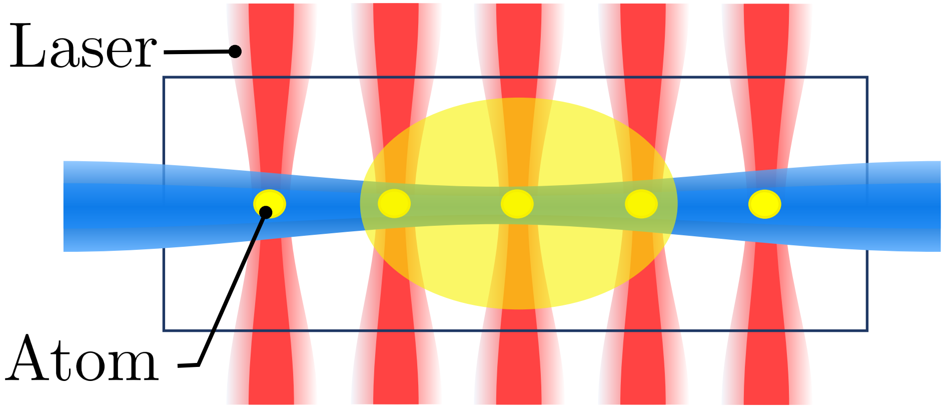
Saffman group, PRL **123**, 230501 (2019)



s
p
79d
d
70s
$|n,l\rangle$
Rydberg states
$n \gg 1$
1013 nm
6p
480 nm
420 nm
5p
780 nm
5s
Ground state
Rb atom



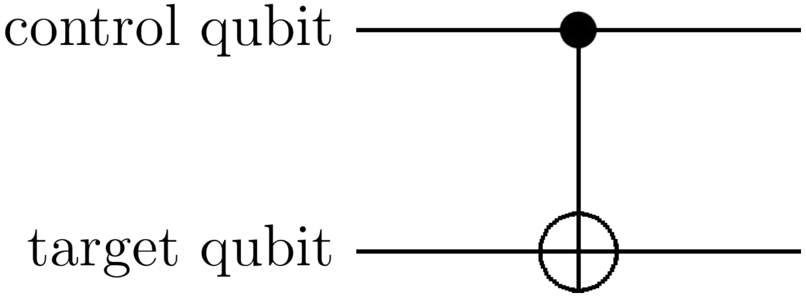
optical pumping
control site
w=10 μm
780 nm
10 μm
x
z, B
480 nm
target site



Laser
Atom

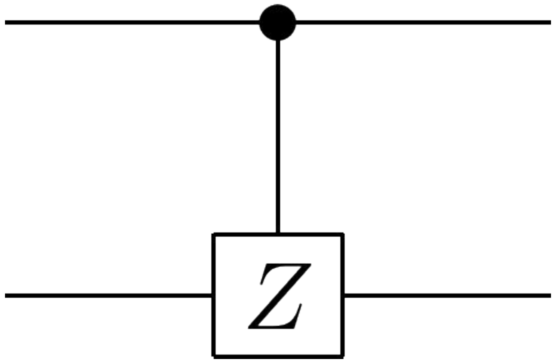
Controlled Operations

CNOT gate



$$|c\rangle|t\rangle \to |c\rangle|t \oplus c\rangle$$

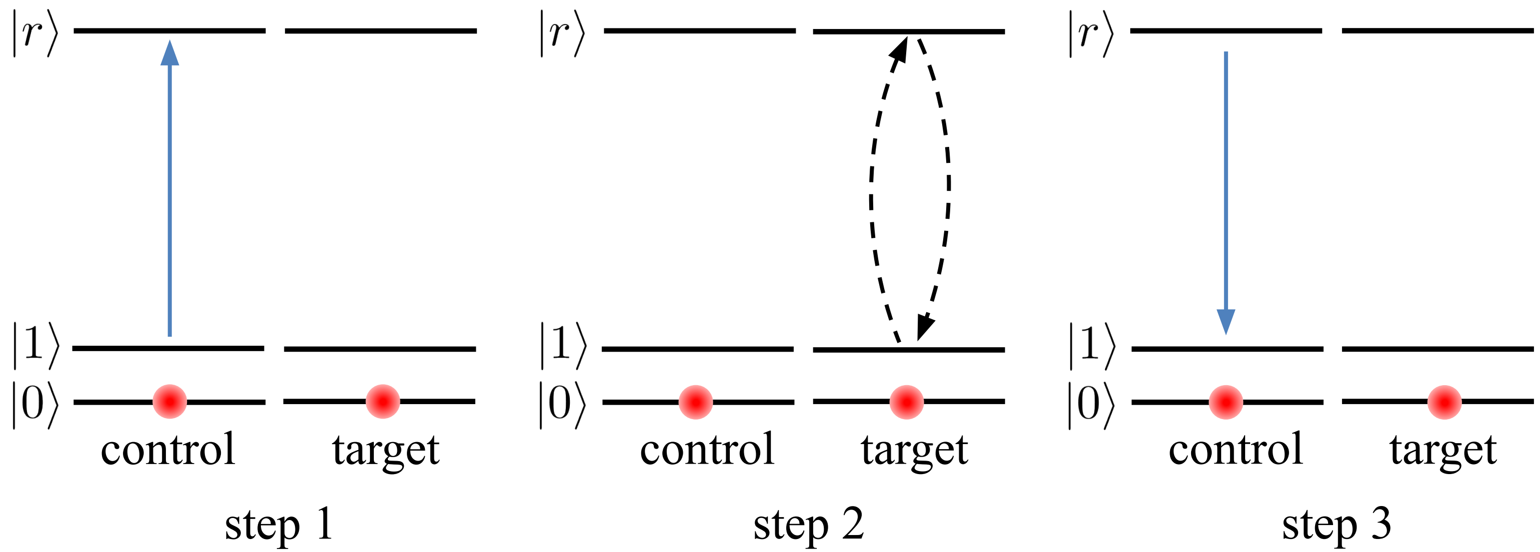
Controlled-Z operation



$$|c\rangle|t\rangle \to |c\rangle Z|t\rangle$$

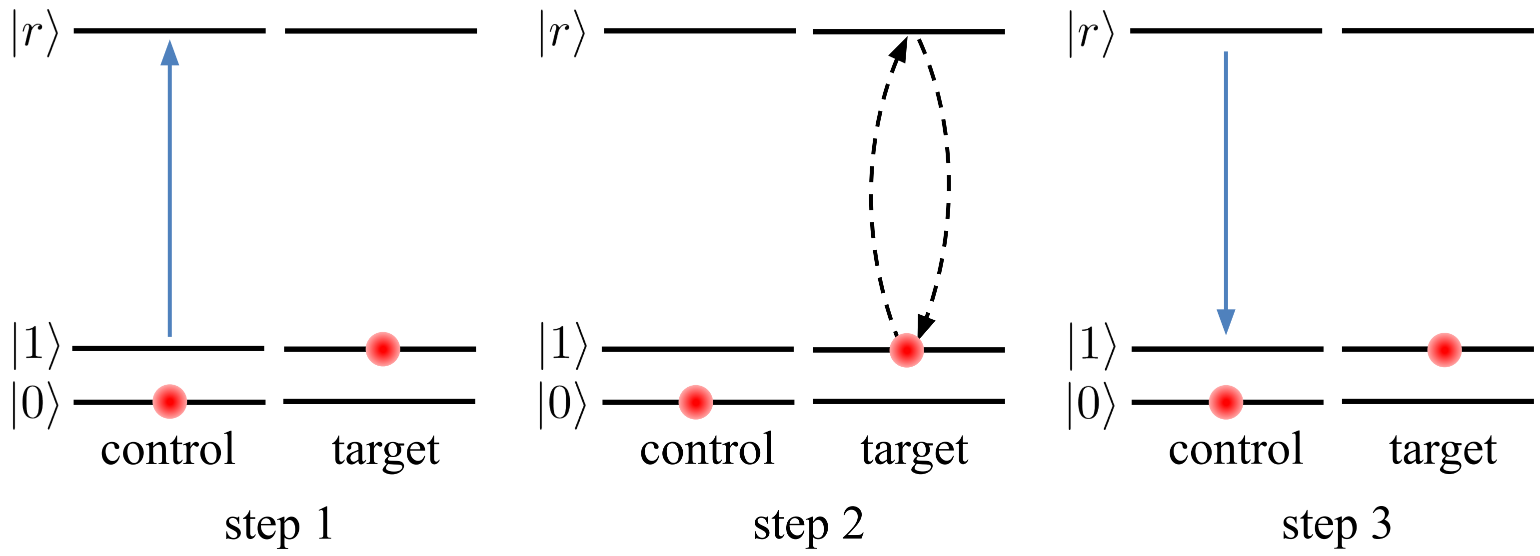
Controlled phase gate

Controlled-phase gate $\mathcal{CZ}$



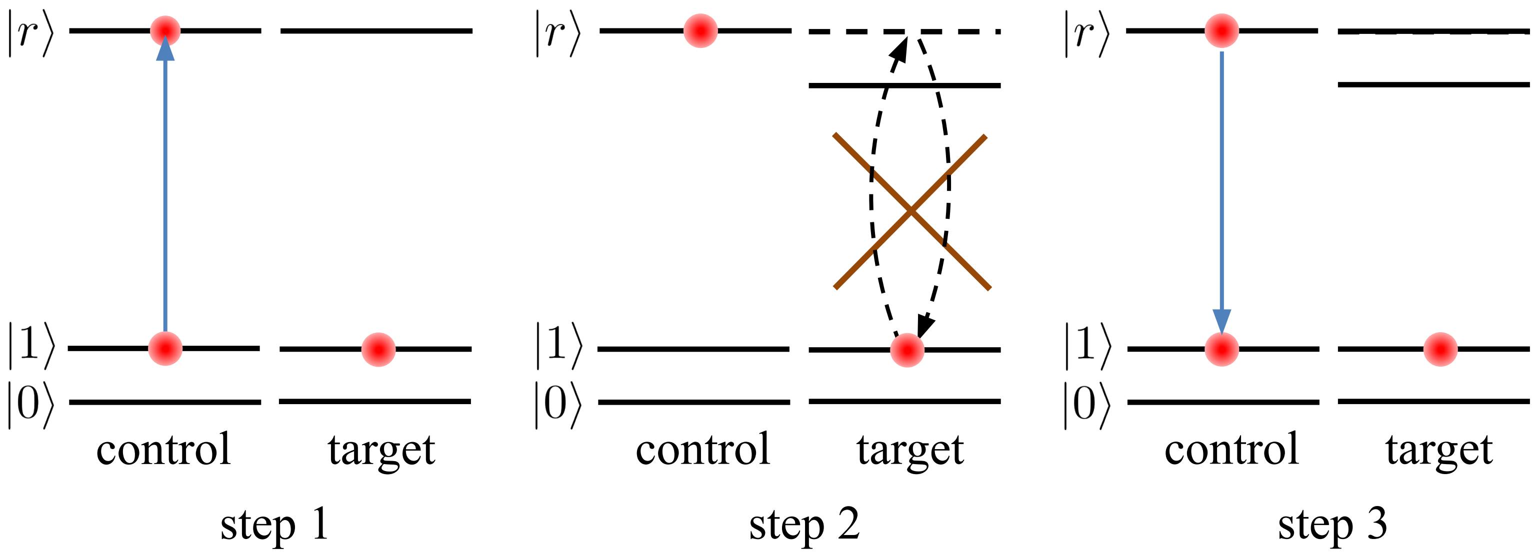
Input	Step 1	Step 2	Step 3	Output
$\lvert C, T\rangle$	π pulse on C	2π pulse on T	π pulse on C	$\lvert C, T\rangle$
$\lvert 0, 0\rangle$	$\lvert 0, 0\rangle$	$\lvert 0, 0\rangle$	$\lvert 0, 0\rangle$	$\lvert 0, 0\rangle$

Controlled-phase gate $\mathcal{CZ}$



Input $\lvert C, T\rangle$	Step 1 π pulse on C	Step 2 2π pulse on T	Step 3 π pulse on C	Output $\lvert C, T\rangle$
$\lvert 0,0\rangle$	$\lvert 0,0\rangle$	$\lvert 0,0\rangle$	$\lvert 0,0\rangle$	$\lvert 0,0\rangle$
$\lvert 0,1\rangle$	$\lvert 0,1\rangle$	$e^{i\pi}\lvert 0,1\rangle$	$e^{i\pi}\lvert 0,1\rangle$	$-\lvert 0,1\rangle$
$\lvert 1,0\rangle$	$e^{i\pi/2}\lvert r,0\rangle$	$e^{i\pi/2}\lvert r,0\rangle$	$e^{i\pi}\lvert 1,0\rangle$	$-\lvert 1,0\rangle$

Controlled-phase gate $\mathcal{CZ}$



Input	Step 1	Step 2	Step 3	Output
$\vert C,T\rangle$	π pulse on C	2π pulse on T	π pulse on C	$\vert C,T\rangle$
$\vert 0,0\rangle$	$\vert 0,0\rangle$	$\vert 0,0\rangle$	$\vert 0,0\rangle$	$\vert 0,0\rangle$
$\vert 0,1\rangle$	$\vert 0,1\rangle$	$e^{i\pi}\vert 0,1\rangle$	$e^{i\pi}\vert 0,1\rangle$	$-\vert 0,1\rangle$
$\vert 1,0\rangle$	$e^{i\pi/2}\vert r,0\rangle$	$e^{i\pi/2}\vert r,0\rangle$	$e^{i\pi}\vert 1,0\rangle$	$-\vert 1,0\rangle$
$\vert 1,1\rangle$	$e^{i\pi/2}\vert r,1\rangle$	$e^{i\pi/2}e^{i\frac{\pi\Omega}{2\Delta}}\vert r,1\rangle$	$e^{i\pi}e^{i\frac{\pi\Omega}{2\Delta}}\vert 1,1\rangle$	$\approx -\vert 1,1\rangle$

Controlled-phase gate $\mathcal{CZ}$

Theoretically $$S_t = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

Experimentally $$S_e = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix}$$

$$|00\rangle \rightarrow |00\rangle$$

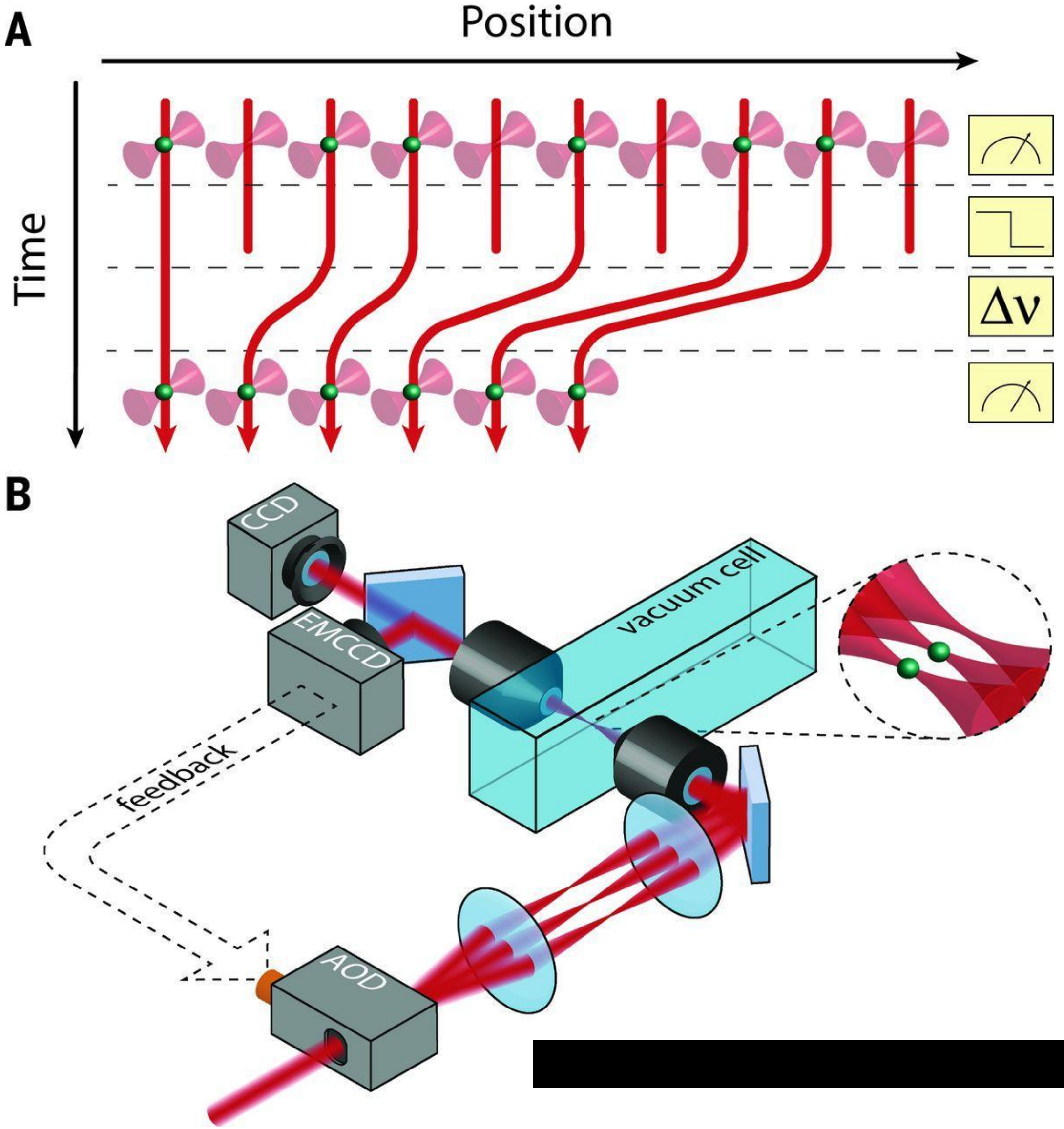
$$|01\rangle \rightarrow |01\rangle e^{i\phi}$$

$$|10\rangle \rightarrow |10\rangle e^{i\phi}$$

$$|11\rangle \rightarrow |11\rangle e^{i(2\phi-\pi)}$$

[redacted]

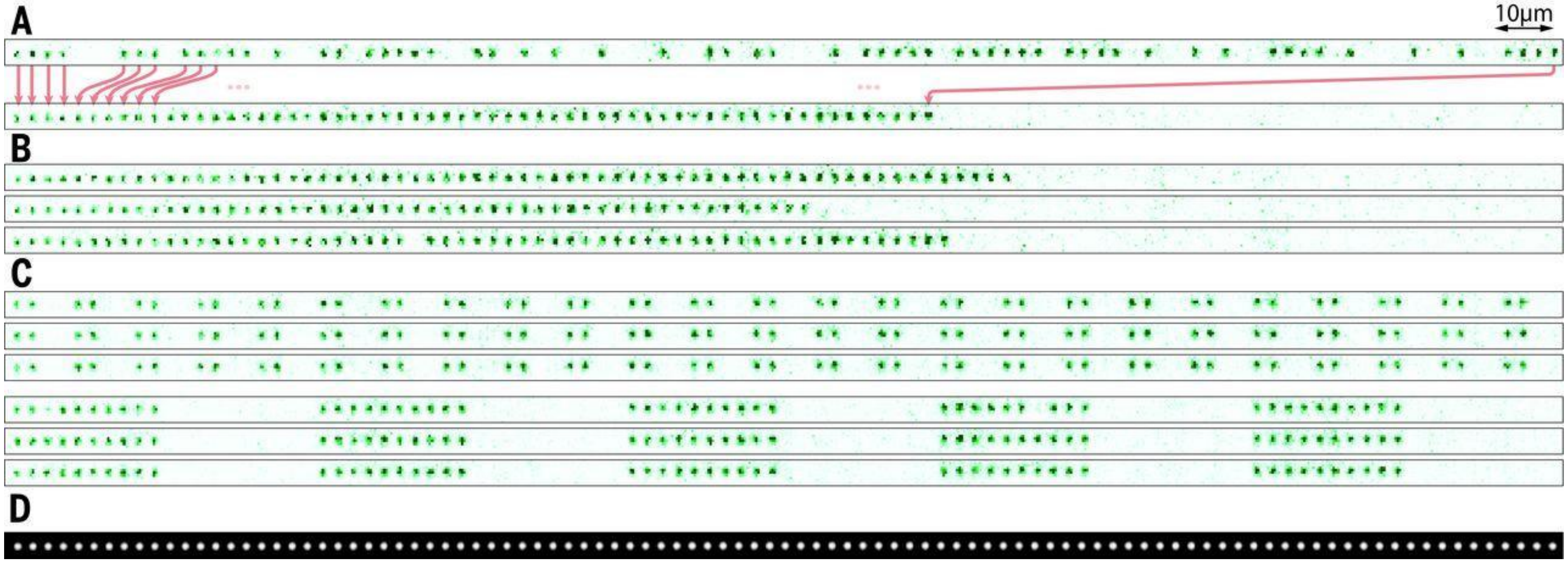
Lukin group, Science **354**, 1024 (2016)



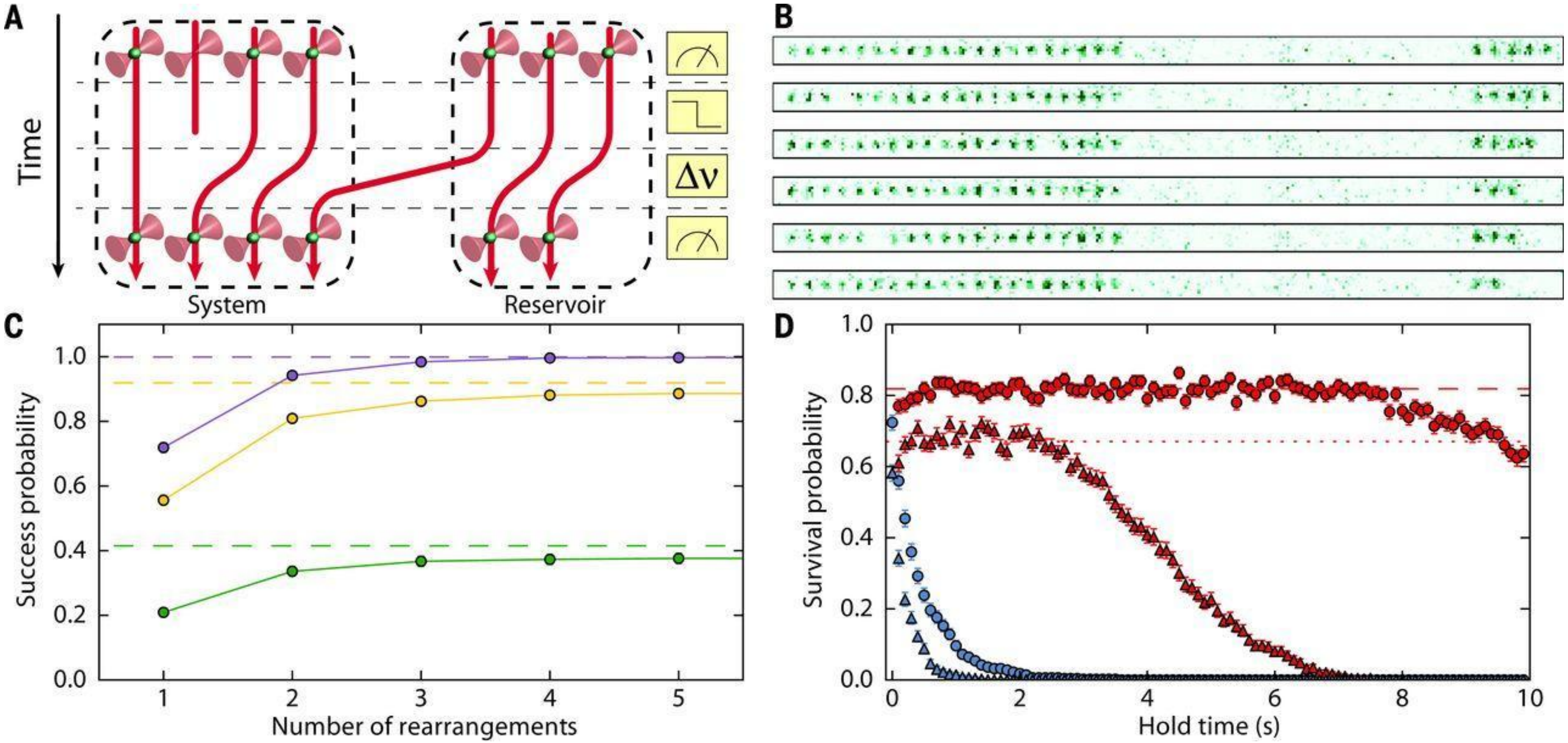
[redacted]

Lukin group, Science **354**, 1024 (2016)

Fig. 2 Assembly of regular atom arrays.



Lukin group, Science **354**, 1024 (2016)



[redacted]

Lukin group, PRL **123**, 170503 (2019)

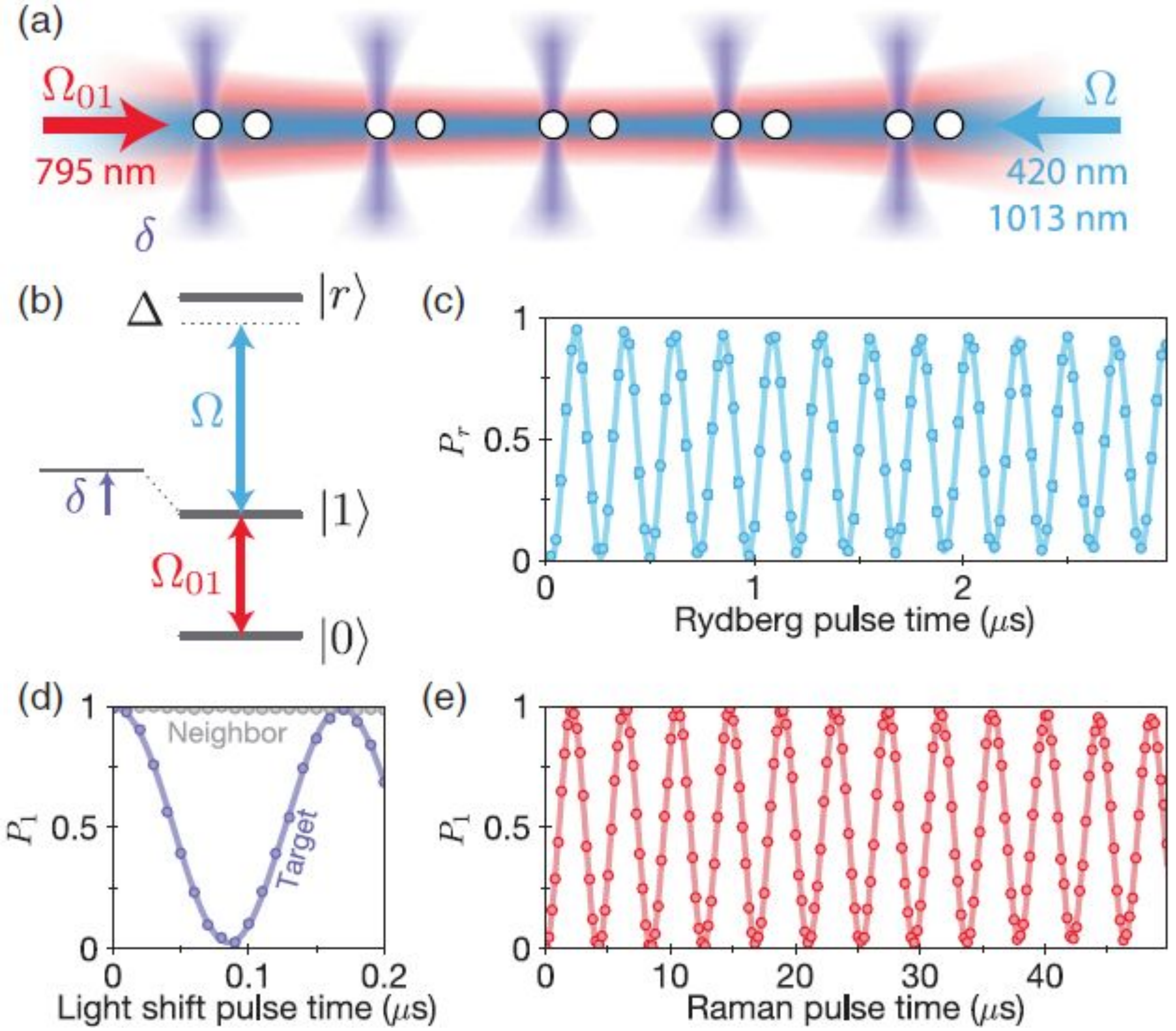
FIG. 1. Control of individual qubits in atom arrays.

$|70S_{1/2}, m_j = 1/2\rangle$

$|5S_{1/2}, F = 2, m_F = 0\rangle$

Rb

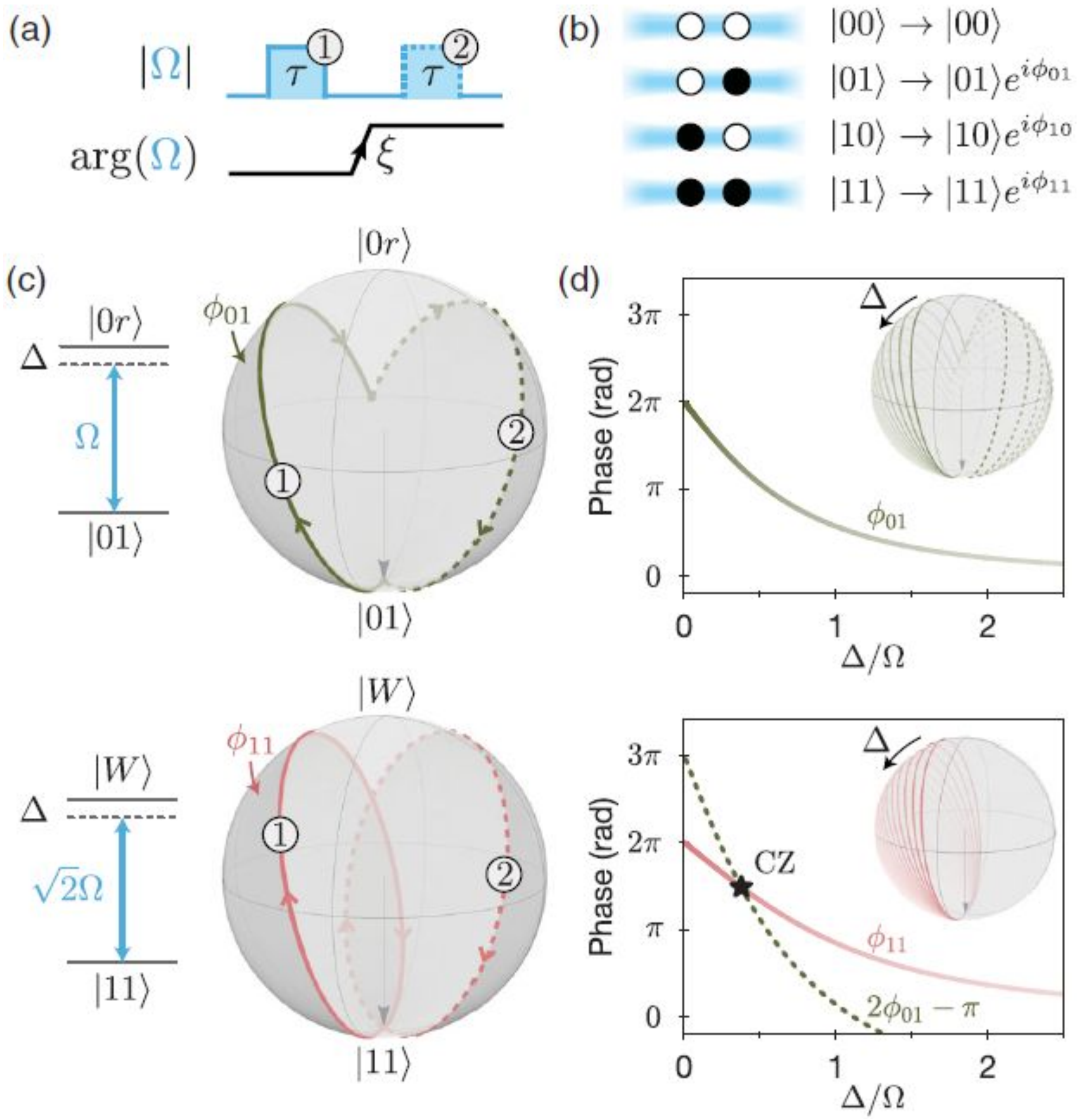
$|5S_{1/2}, F = 1, m_F = 0\rangle$



Rabi Oscillations

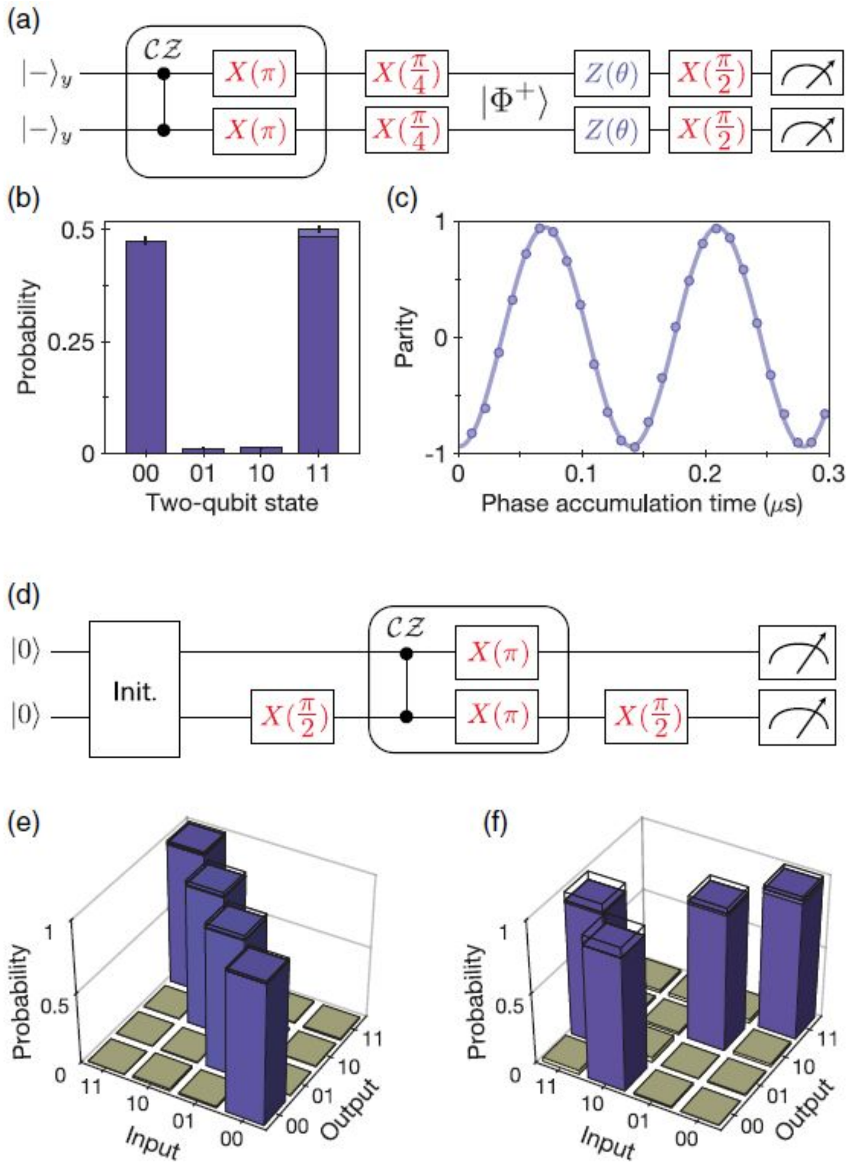
Lukin group, PRL **123**, 170503 (2019)

FIG. 2. Controlled-phase (CZ) gate protocol.



Lukin group, PRL **123**, 170503 (2019)

FIG. 3. Bell state preparation and controlled-NOT (CNOT) gate.



Using this, we can express the *rotation operators* about the $\hat{x}$, $\hat{y}$, $\hat{z}$ axes

$$R_x(\theta) = e^{-i\theta X/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}X = \begin{bmatrix} \cos\theta/2 & -i\sin\theta/2 \\ -i\sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_y(\theta) = e^{-i\theta Y/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Y = \begin{bmatrix} \cos\theta/2 & -\sin\theta/2 \\ \sin\theta/2 & \cos\theta/2 \end{bmatrix}$$

$$R_z(\theta) = e^{-i\theta Z/2} = \cos\frac{\theta}{2}I - i\sin\frac{\theta}{2}Z = \begin{bmatrix} e^{-i\theta/2} & 0 \\ 0 & e^{i\theta/2} \end{bmatrix} = e^{-i\theta/2}\begin{bmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{bmatrix}$$

VOLUME 74, NUMBER 20 PHYSICAL REVIEW LETTERS 15 MAY 1995

Quantum Computations with Cold Trapped Ions

J. I. Cirac and P. Zoller*

Institut für Theoretische Physik, Universiät Innsbruck, Technikerstrasse 25, A-6020 Innsbruck, Austria
(Received 30 November 1994)

A quantum computer can be implemented with cold ions confined in a linear trap and interacting with laser beams. Quantum gates involving any pair, triplet, or subset of ions can be realized by coupling the ions through the collective quantized motion. In this system decoherence is negligible, and the measurement (readout of the quantum register) can be carried out with a high efficiency.

PACS numbers: 89.80.+h, 03.65.Bz, 12.20.Fv, 32.80.Pj

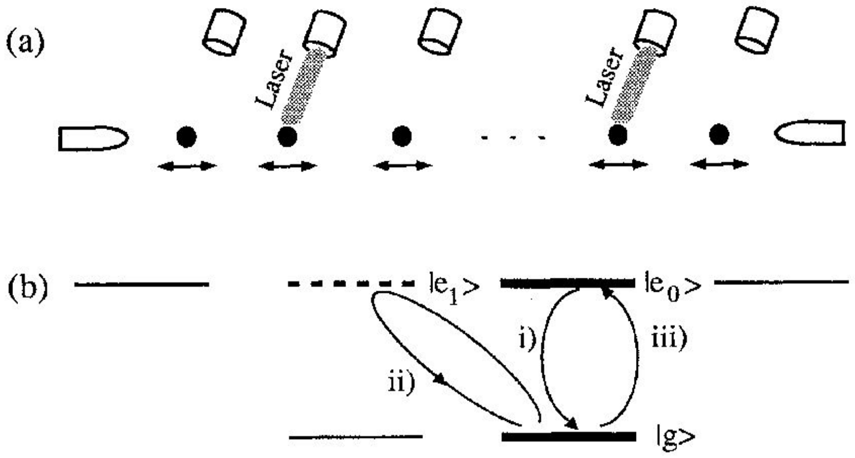


FIG. 1. (a) N ions in a linear trap interacting with N different laser beams; (b) atomic level scheme.

Controlled – NOT :

$$|\varepsilon_1\rangle|\varepsilon_2\rangle \rightarrow |\varepsilon_1\rangle|\varepsilon_1 \oplus \varepsilon_2\rangle$$

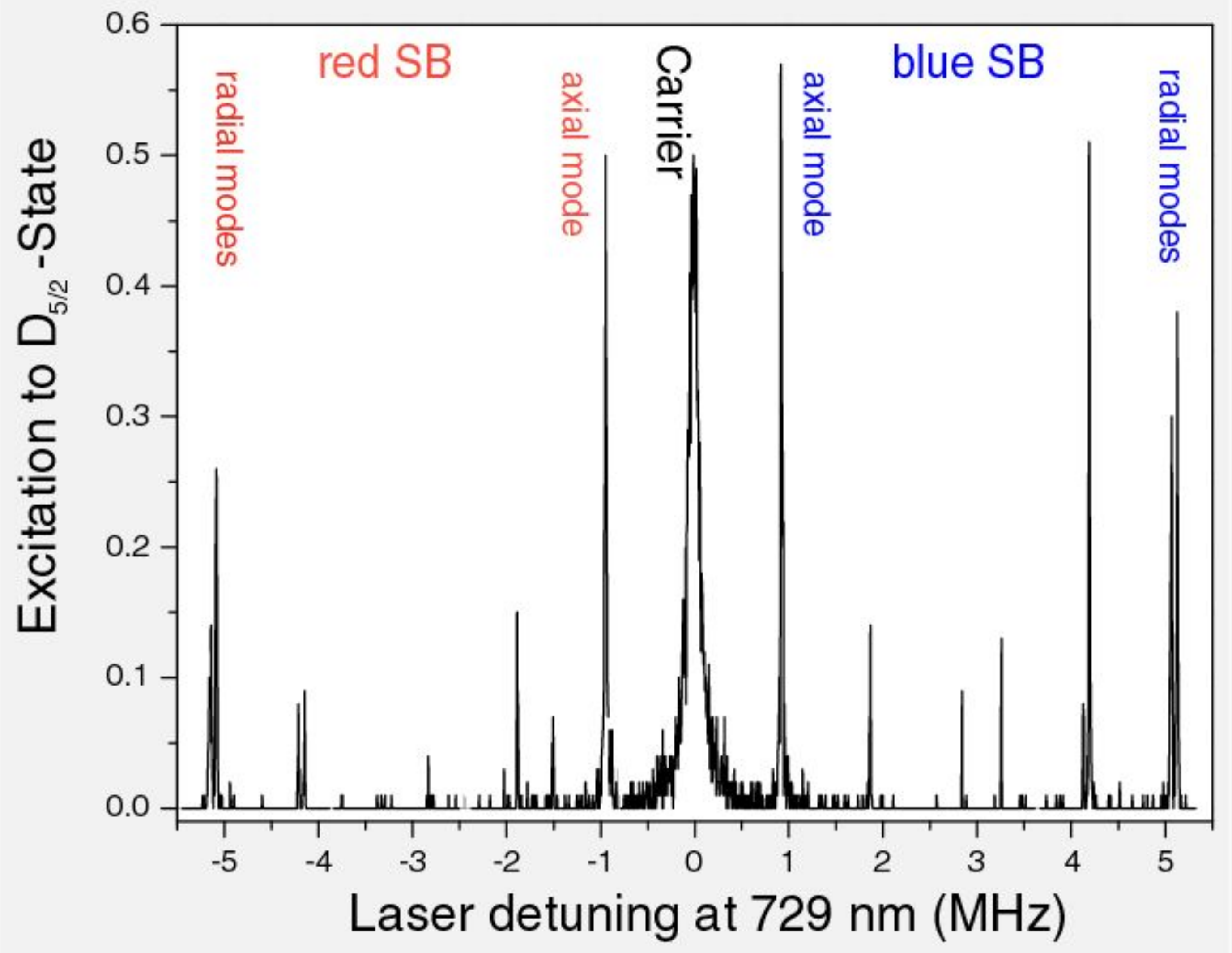
$$|0\rangle|0\rangle \rightarrow |0\rangle|0\rangle$$
$$|0\rangle|1\rangle \rightarrow |0\rangle|1\rangle$$
$$|1\rangle|0\rangle \rightarrow |1\rangle|1\rangle$$
$$|1\rangle|1\rangle \rightarrow |1\rangle|0\rangle$$

control bit target bit

other gate proposals (and more):

- Cirac & Zoller
- Mølmer & Sørensen, Milburn
- Jonathan, Plenio & Knight
- Geometric phases
- Leibfried & Wineland

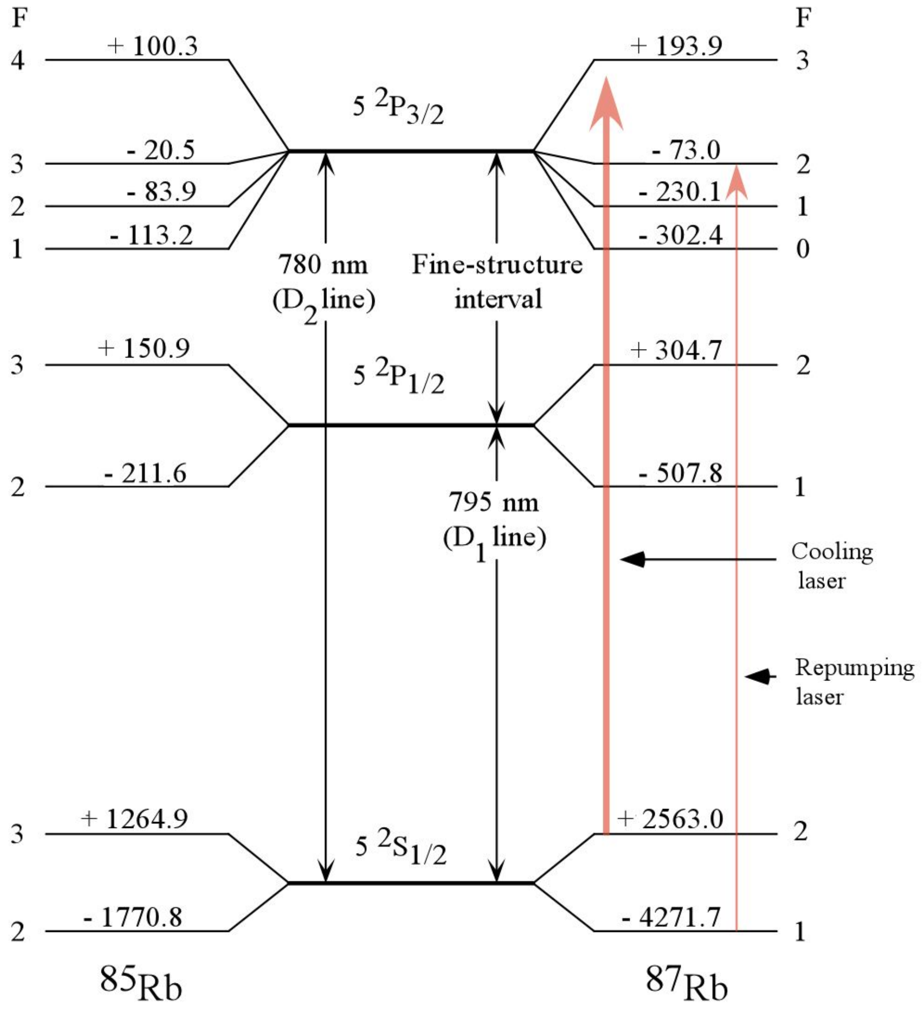
Excitation spectrum of single ion in linear trap



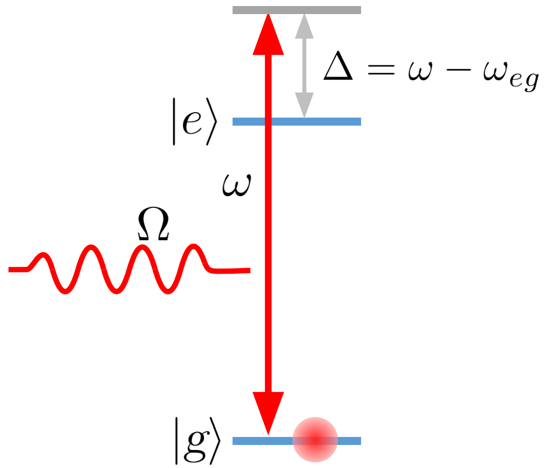
w_{ax} = 1.0 MHz

w_{rad} = 5.0 MHz

Energy levels of Rb



Two level atomic system

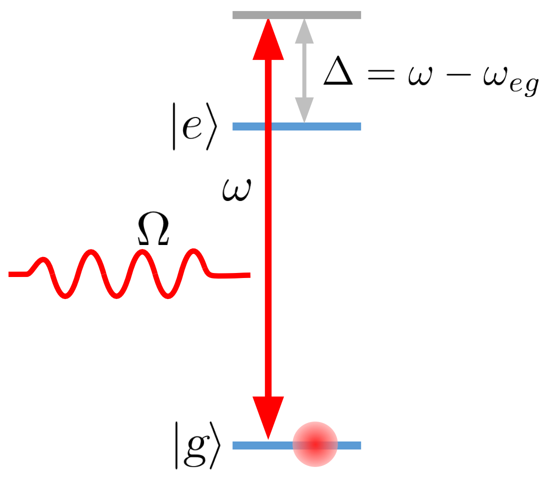


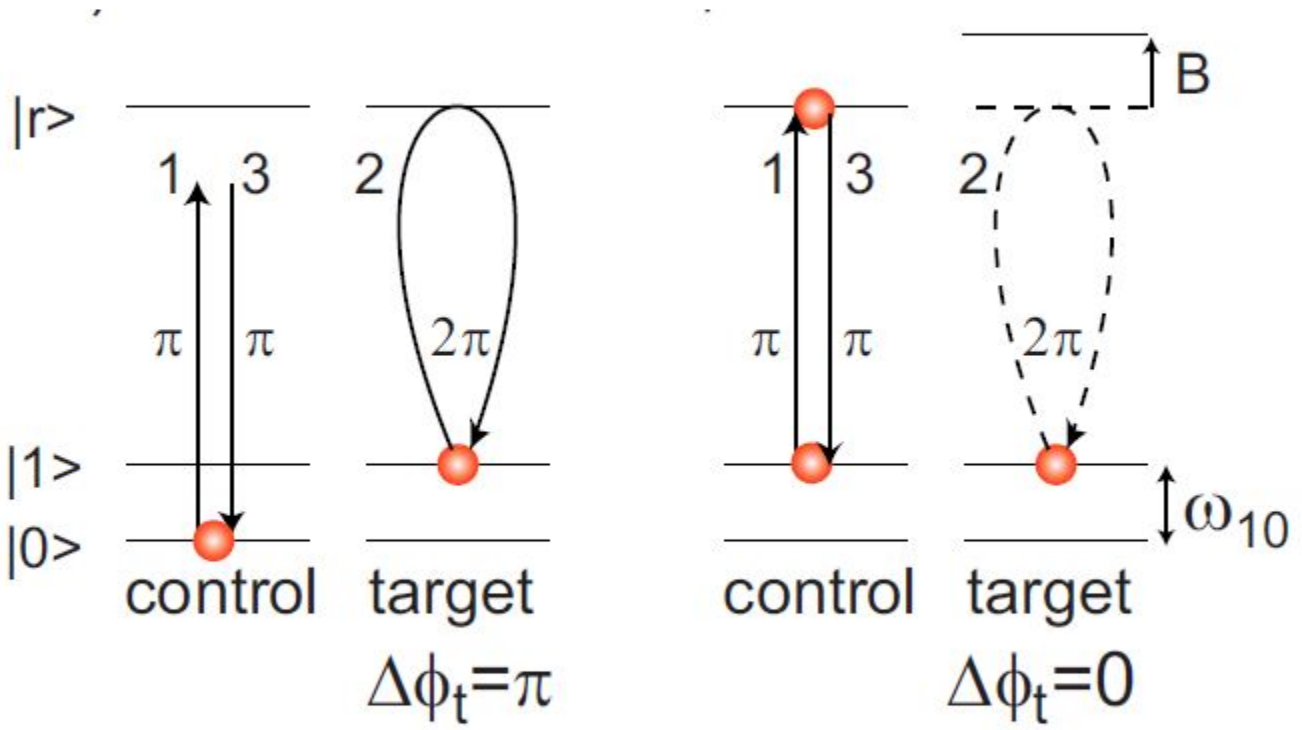
$$|\psi\rangle = c_g e^{-i\omega_g t} t + c_e e^{-i\omega_e t} t|e\rangle$$

$$U(t, t_0) = \begin{bmatrix} c_g(t_0) \\ c_e(t_0) \end{bmatrix}$$

$$U(t, t_0) = \begin{bmatrix} e^{i\frac{\Delta t}{2}}[\cos(\frac{\Omega' t}{2}) - i\frac{\Delta}{\Omega'}\sin(\frac{\Omega' t}{2})] & ie^{i\frac{\Delta t}{2}}\frac{\Omega^*}{\Omega'}\sin(\frac{\Omega' t}{2}) \\ ie^{-i\frac{\Delta t}{2}}\frac{\Omega}{\Omega'}\sin(\frac{\Omega' t}{2}) & e^{-i\frac{\Delta t}{2}}[\cos(\frac{\Omega' t}{2}) + i\frac{\Delta}{\Omega'}\sin(\frac{\Omega' t}{2})] \end{bmatrix}$$

Two level atomic system

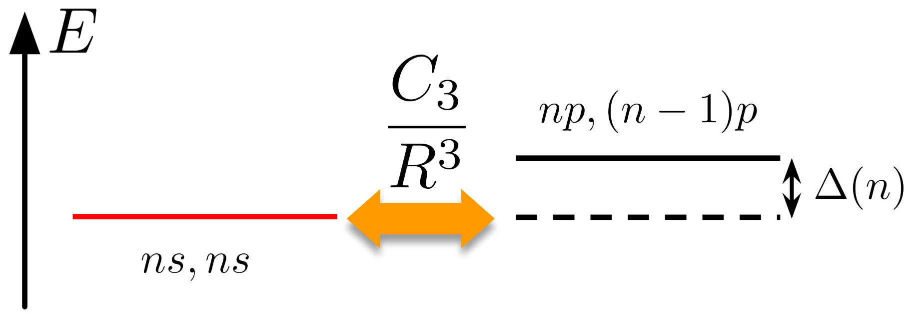




DiVincenzo Criteria

[redacted]

[redacted]



Tune $\Delta(n)$: Stark shift

[redacted]