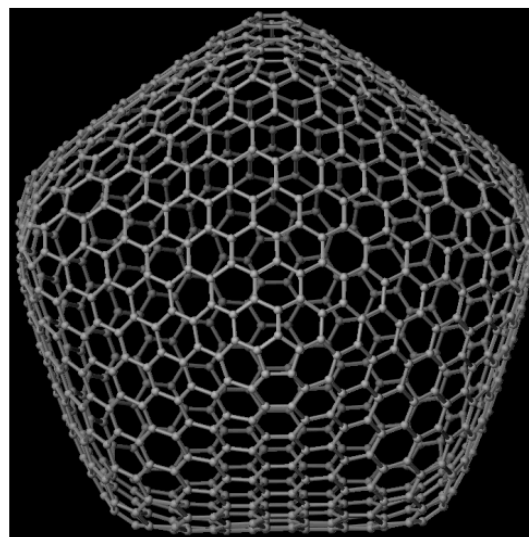
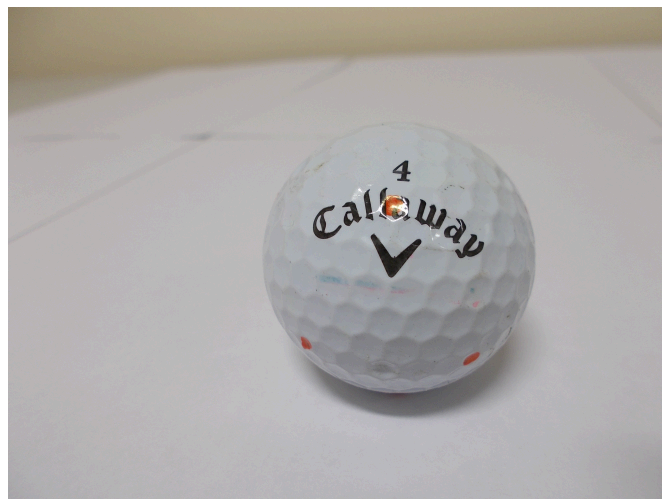
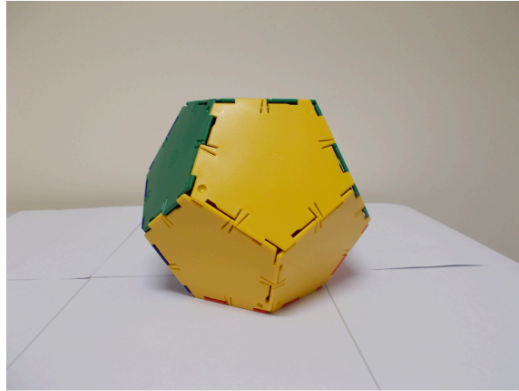
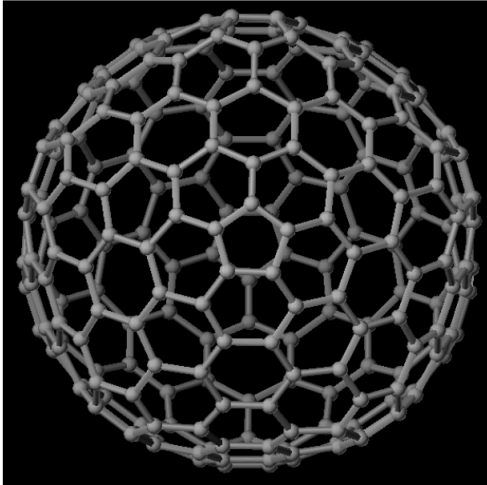


What do a football, golf ball, buckyball - a C₆₀ carbon atom and a dodecahedron have in common?





What is the mathematics of balls made from hexagons and pentagons? How many pentagons and hexagons are required to make a closed ball like structure?

- If we want to make a larger football, do we need more polygonal patches or larger polygonal patches?
- Can we make balls with only hexagons?
- How about other polygons? Can they be used to make balls?
- In how many different ways can we make balls with polygons?

We will need five definitions:

- faces: the number of faces is f .
- edges: the number of edges is e .
- vertices: the number of vertices is v .
- vertex degree. This is the number of edges radiating from each vertex.
- face degree. This is the number of sides each face has

| Polyhedron | Faces | Edges | Vertices | Face Degree | Vertex Degree |
|---------------|-------|-------|----------|-------------|---------------|
| Tetrahedron | | | | | |
| Cube | | | | | |
| Octahedron | | | | | |
| Dodecahedron | | | | | |
| Icosahedron | | | | | |
| Football | | | | | |
| Hex Golf ball | | | | | |

Can you make a conjecture on the number of hexagons and pentagons used to make a ball structure? Try and prove that with the help of the above definitions and the basic formula of polyhedrons.

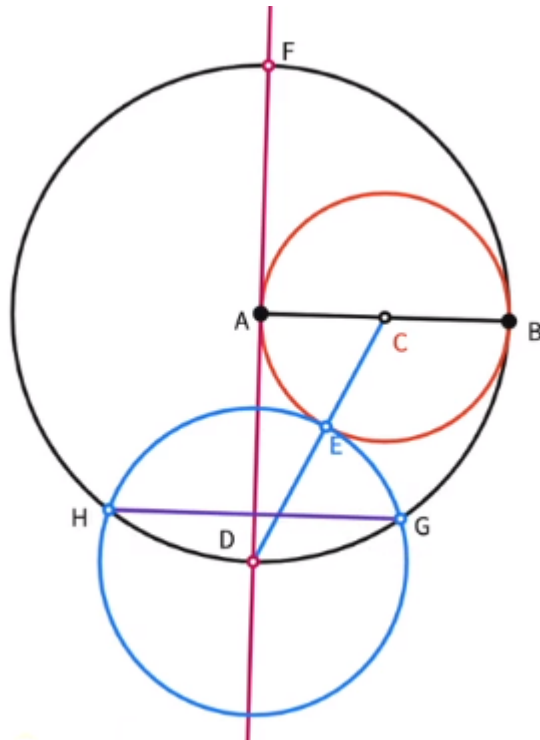
Part II

Construct the following polygons using only a straightedge and compass. You cannot use the ruler to measure lengths or the protractor to measure the angles.

- An equilateral triangle
- A square
- A regular pentagon
- A regular hexagon

There are different ways of constructing a regular pentagon. Hirano's construction method of a regular pentagon is one of them.

- Start with drawing a circle with the radius AB , where A is the center of the circle.
- Mark the midpoint of radius AB with C and draw another circle with center C .
- Draw a perpendicular line to radius AB through the center A .
- Mark and label the intersection of the circle with the perpendicular line as D .
- Connect D and C with a line segment.
- Mark the intersection of the circle with center C and the line segment CD , and label as E .
- Draw another circle with center D and the radius DE .
- Now Mark the intersection points of the circles centered A and D , and label them as H and G .
- Place the compass on H and arrange its width as HG to draw arcs on the circle. Repeat three more times until you mark five vertices of the pentagon.



Can you write the proof for this construction?

Can you find the balance point of all the polygons you've constructed ?

Use the pentagons and hexagons to make a ball-like structure that doesn't have 20 hexagons.