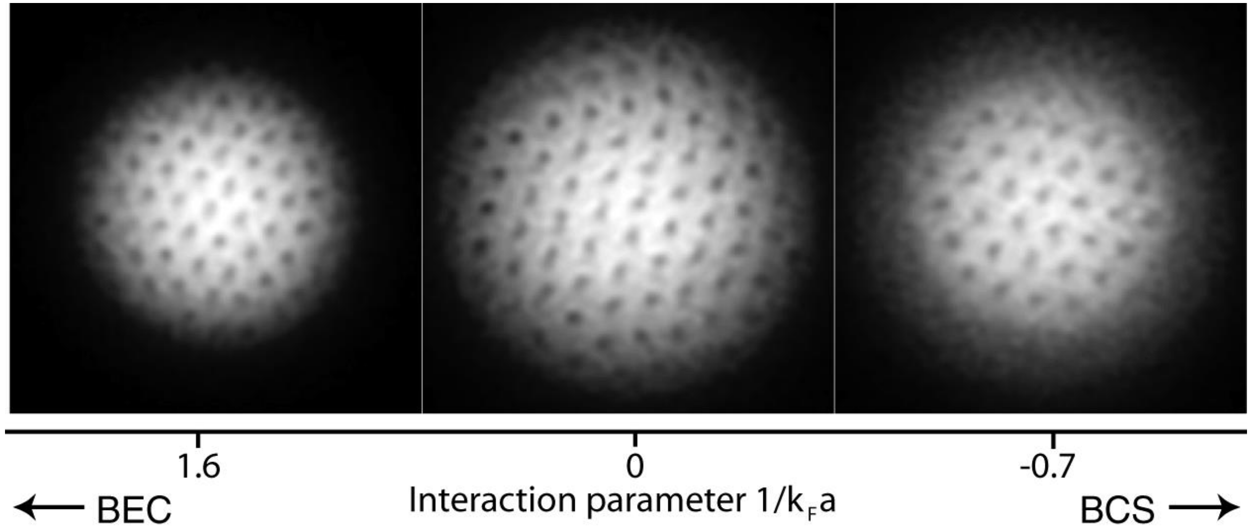
ICTS Summer School, May 14th, 2021

Superfluid Fermi Gases

Martin Zwierlein

Massachusetts Institute of Technology

Center for Ultracold Atoms



GORDON AND BETTY MOORE FOUNDATION

Some reviews

Varenna Notes:
on Fermi Gases:
Ketterle, Zwierlein, *Making, Probing and Understanding Ultracold Fermi Gases*
http://arxiv.org/abs/0801.2500
Zwierlein, *Thermodynamics of Strongly Interacting Fermi Gases*,
Varenna School of Physics “Enrico Fermi” 2014, vol 191

Stefano Giorgini, Lev P. Pitaevskii, Sandro Stringari
The theory of Fermi gases
http://arxiv.org/abs/0706.3360

Immanuel Bloch, Jean Dalibard, Wilhelm Zwerger
Many-Body Physics with Ultracold Gases:
http://arxiv.org/abs/0704.3011

Lecture Notes “The BEC-BCS crossover and the Unitary Fermi Gas”
Edited by W. Zwerger, Springer, 2012

How cold is ultracold?

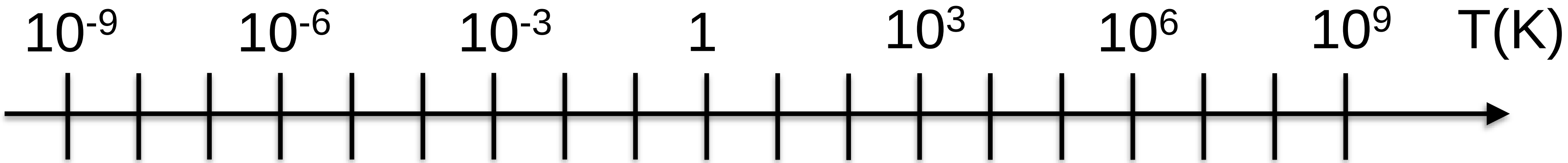
Atoms move at:

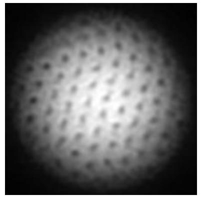
~ 1 mm/s

~ 100 m/s

~ 10^6 m/s

NY – Paris in 10 seconds

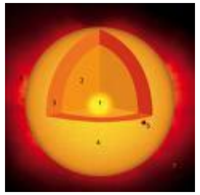




Ultracold **atom experiments**

Outer space

Your living room



Center of the sun

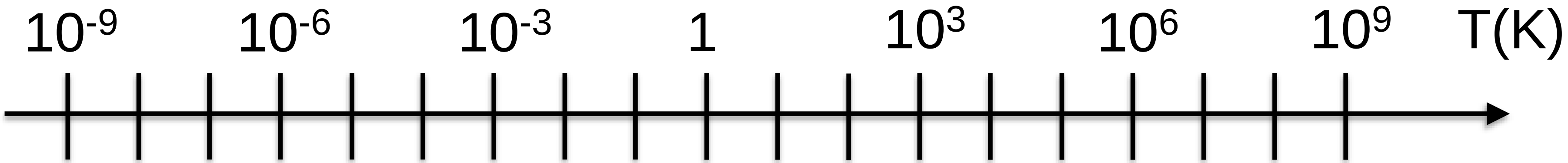
Supernova explosion

Particles behave as waves

Louis de Broglie

$$\lambda = \frac{h}{mv}$$

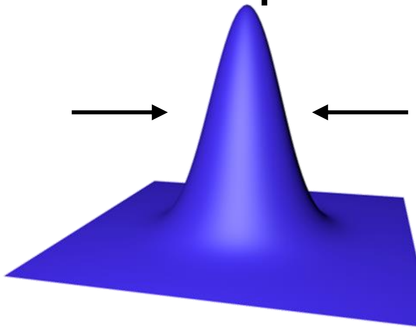
Planck's constant (h), Mass (m), Velocity (v)



Werner Heisenberg

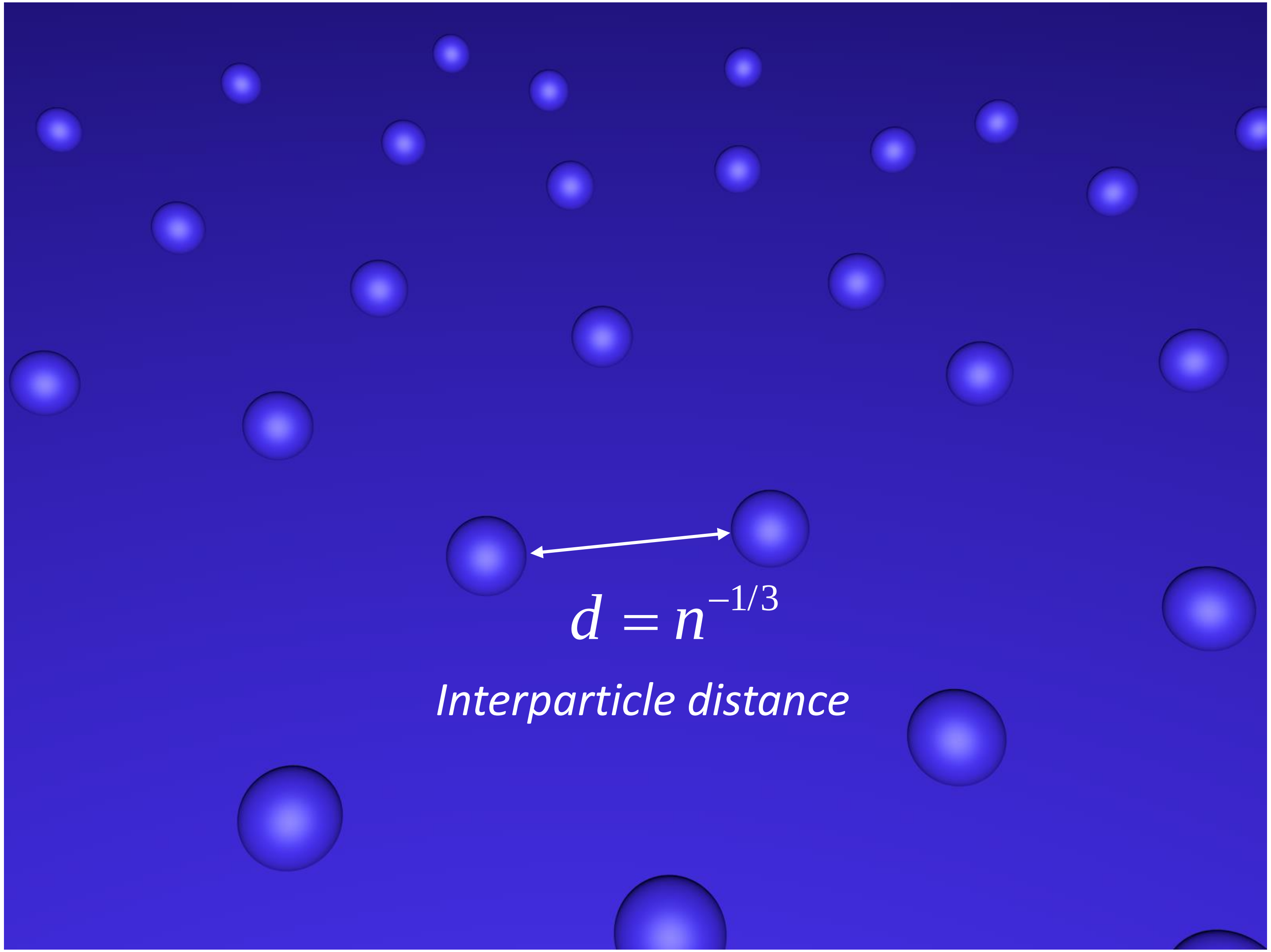
Temperature = Uncertainty of velocity^2

Size of a wave packet = Uncertainty of position

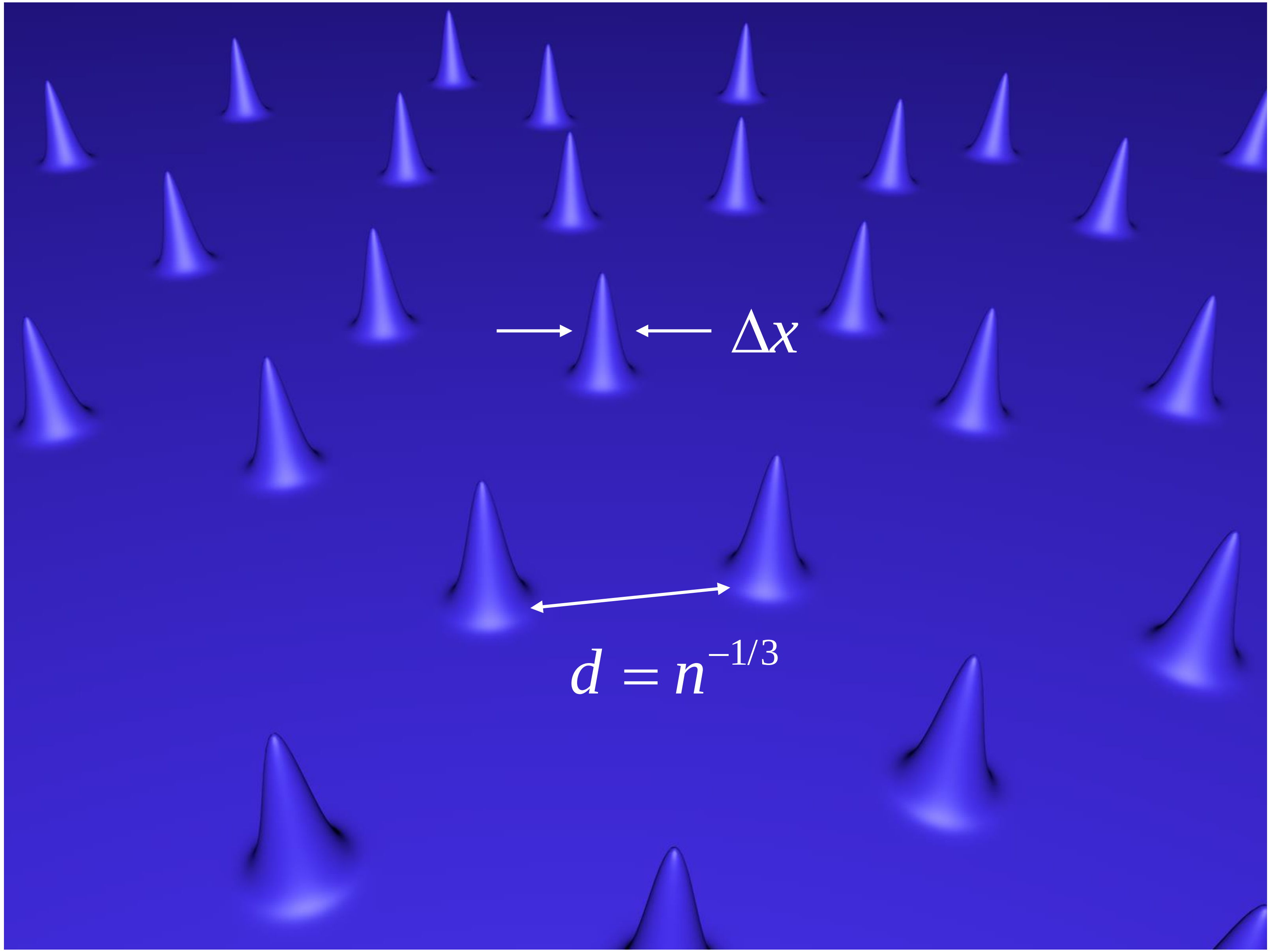


$$\Delta x \approx \frac{\hbar}{\Delta p} \approx \frac{\hbar}{\sqrt{mT}}$$

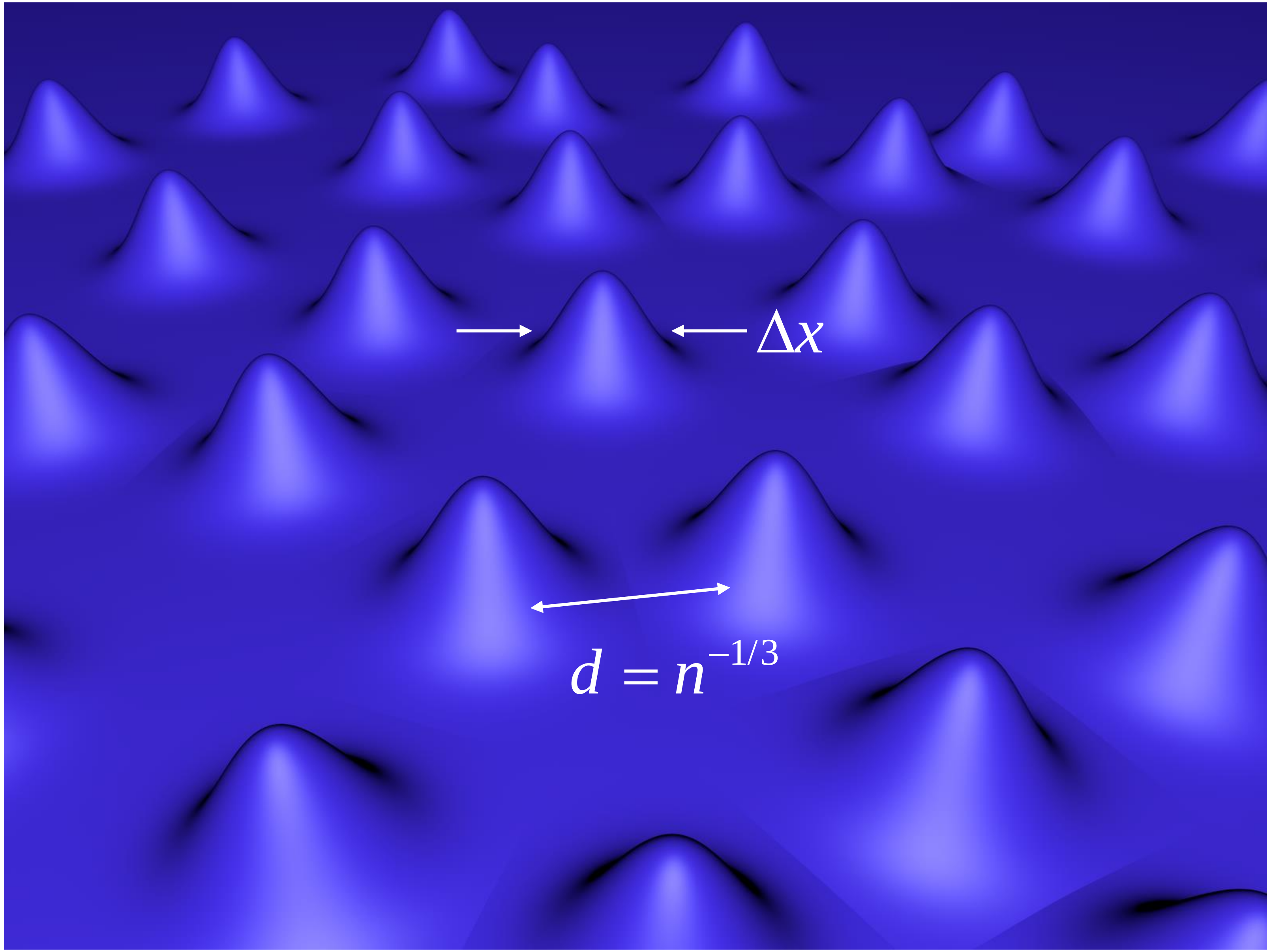
Particles...



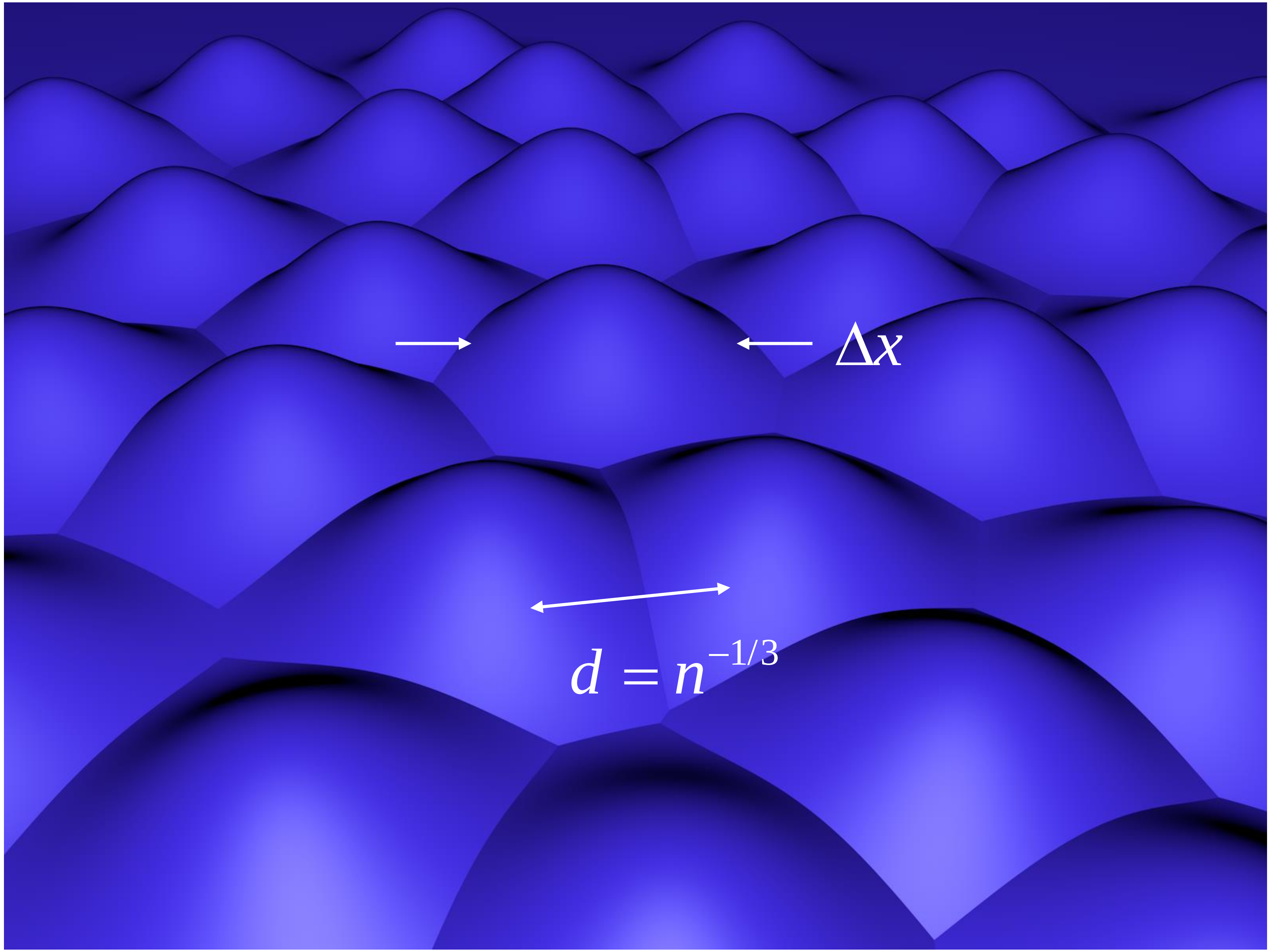
...behave as waves



When does wave mechanics matter?



When does wave mechanics matter?



Bosons versus Fermions

Fermions (unsociable):

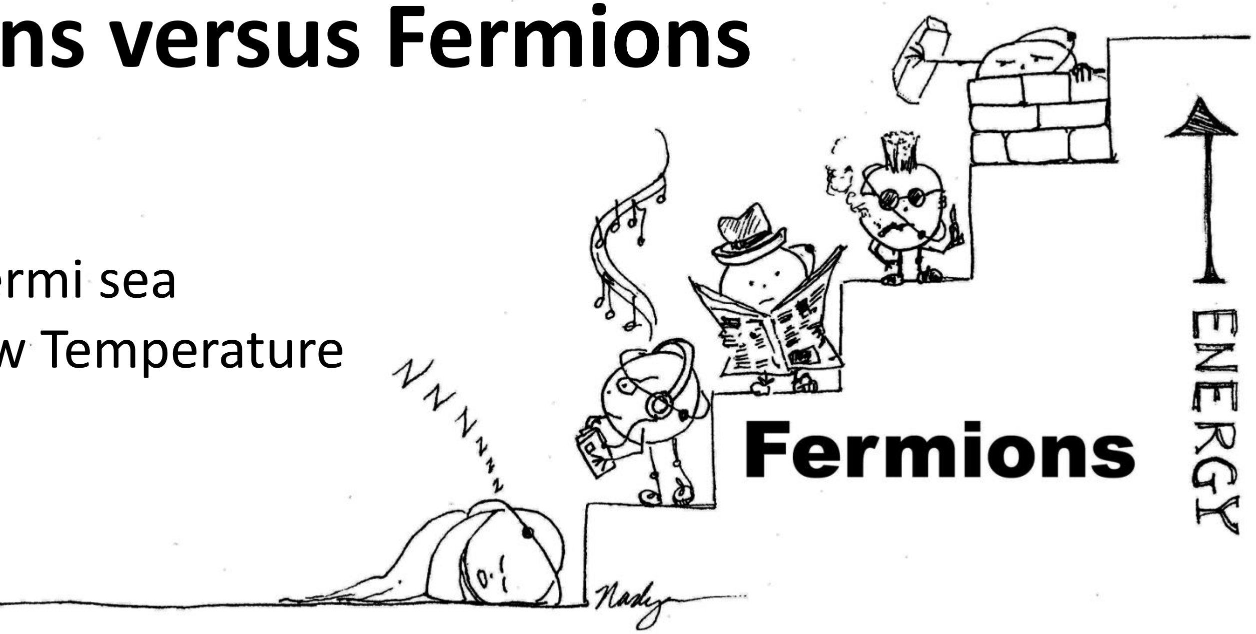
Half-Integer Spin

Pauli blocking → Form Fermi sea

No phase transition at low Temperature

Fermi Feb. 1926

Dirac Oct. 1926



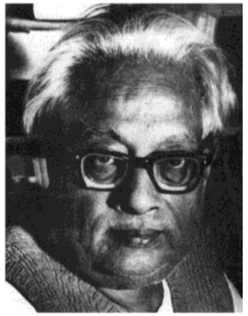
Bosons (sociable):

Integer Spin

Can share quantum states

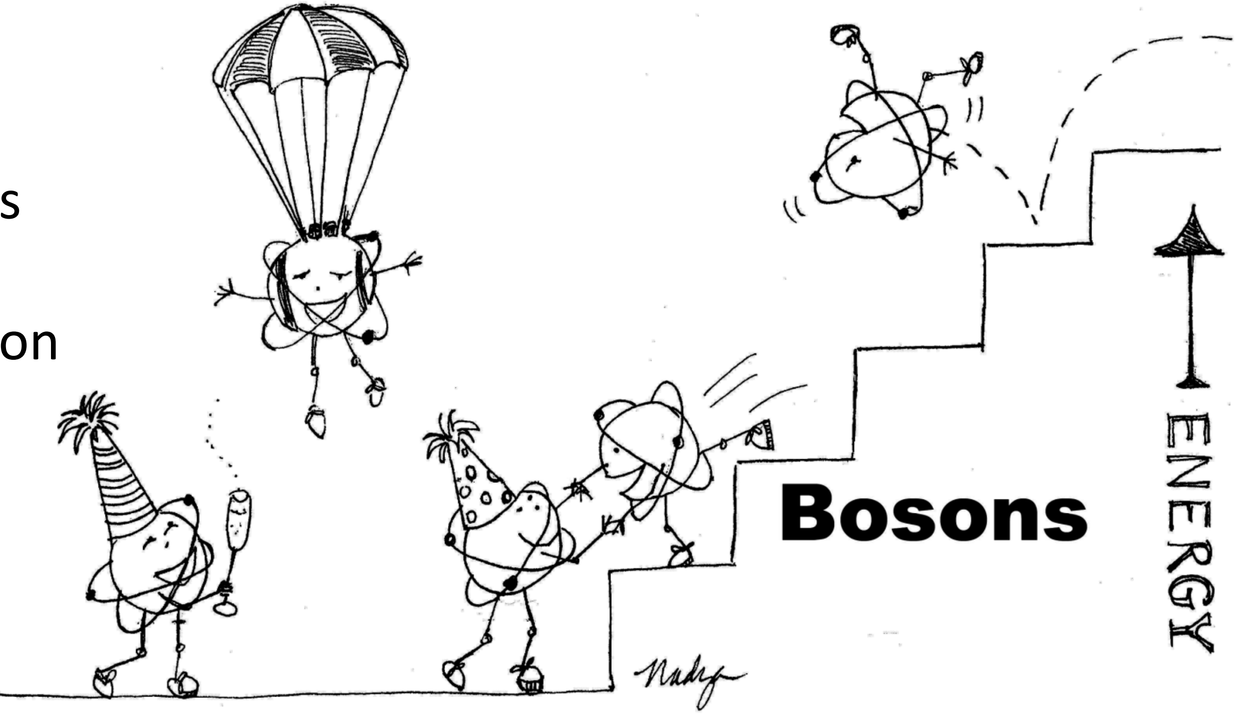
At low temperatures:

Bose-Einstein condensation



Bose 1924

Einstein 1924/25



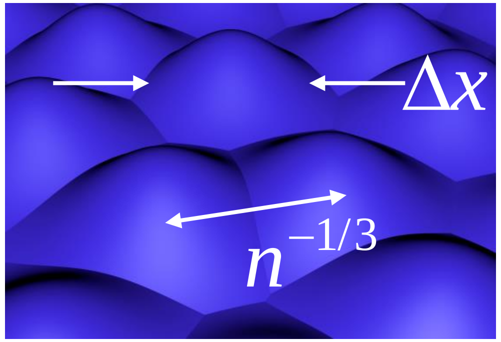
Bosons


N bosons sharing one and the same macroscopic matter wave
(Artist’s conception)

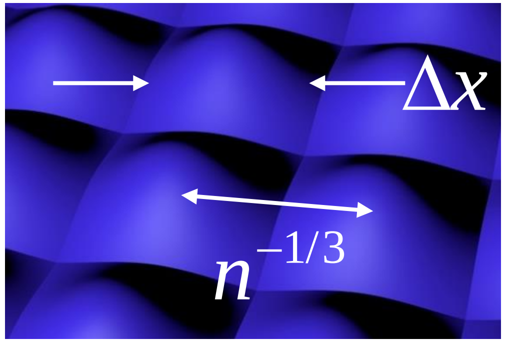
Fermions

Condition for quantum degeneracy

Position uncertainty ~ Interparticle spacing

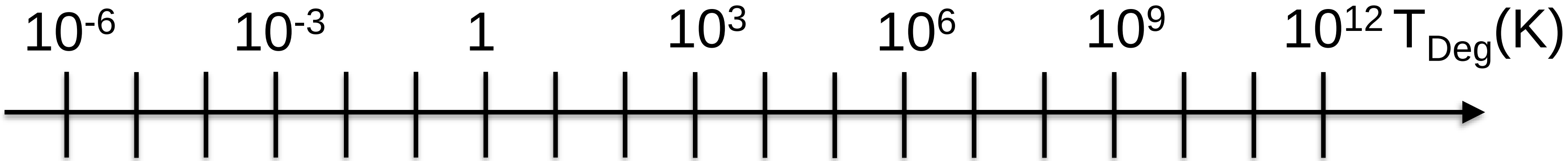


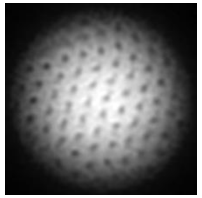
$$\Delta x = \frac{\hbar}{\Delta p} \approx n^{-1/3}$$



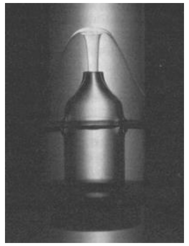
$$k_B T_{\mathrm{Degeneracy}} \approx \frac{\hbar^2}{m} n^{2/3} \approx E_F$$

Fermi energy





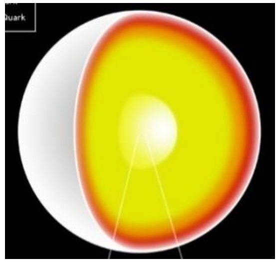
Ultracold
atomic gases



Liquid Helium

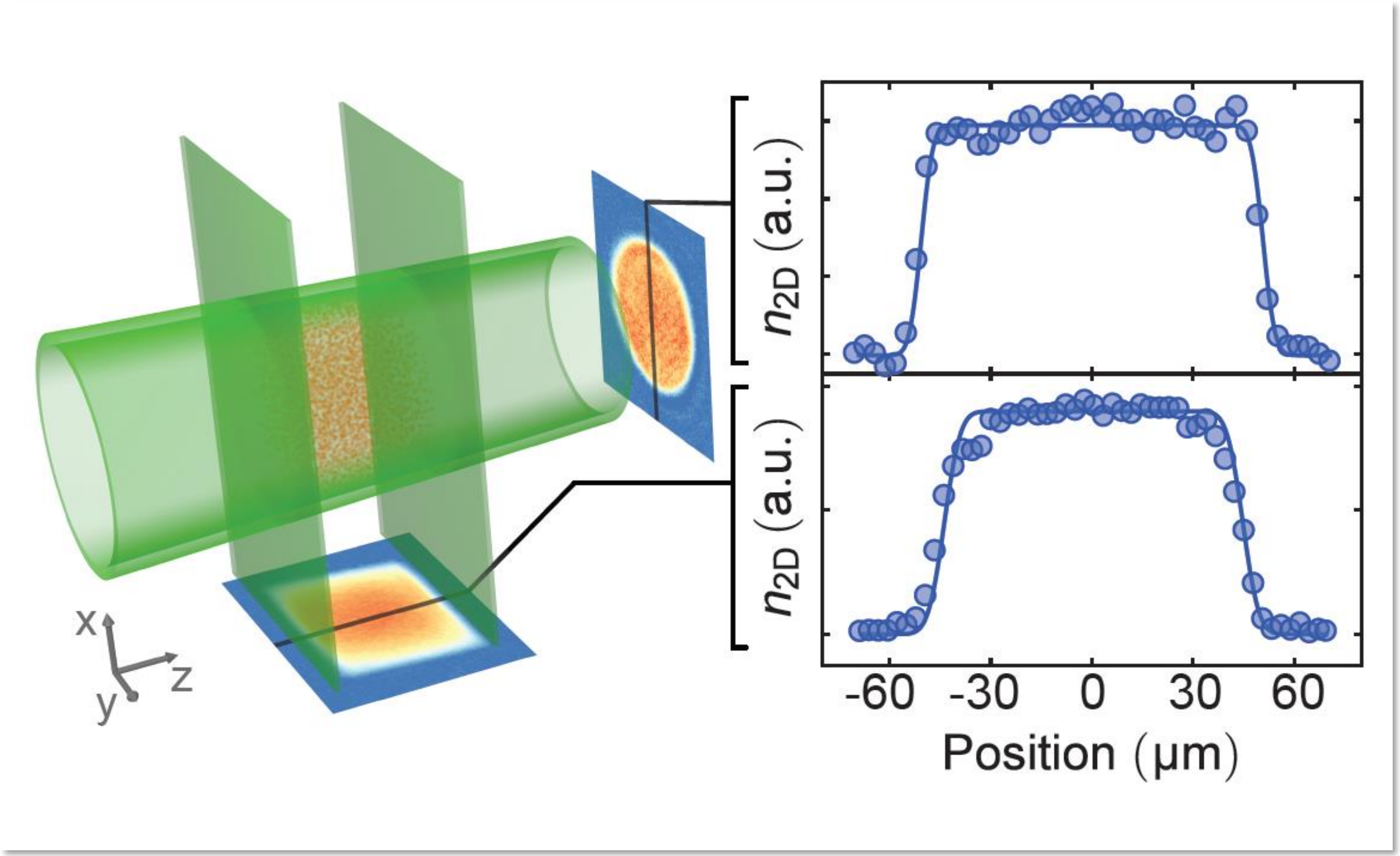
Metals

White dwarf



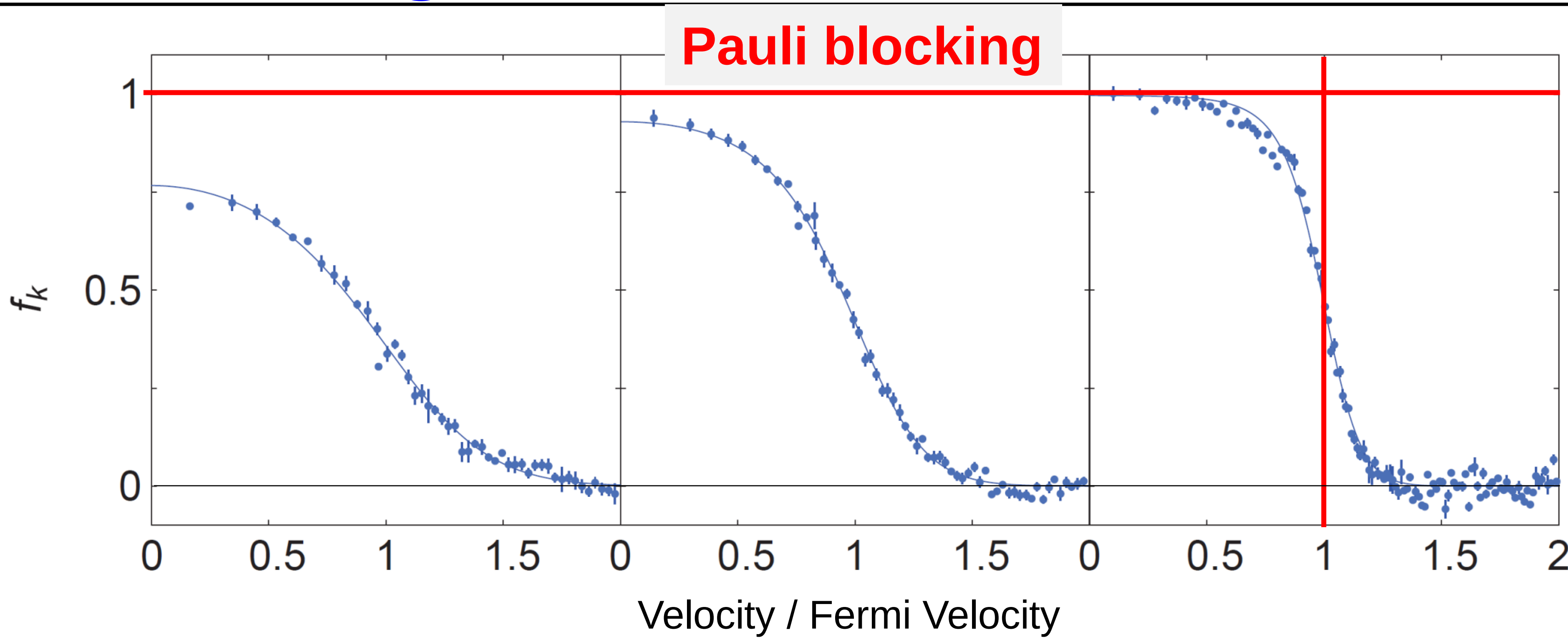
Neutron star

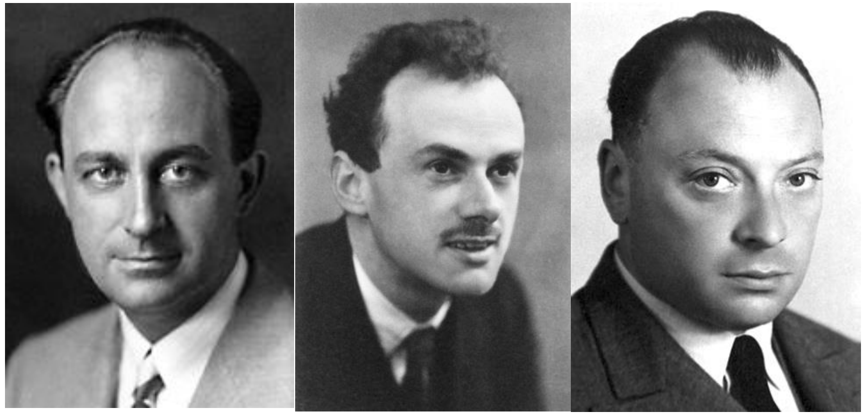
Fermions in a Box



Z. Yan, P. Patel, B. Mukherjee, Z. Hadzibabic, T. Yefsah, J. Struck, MWZ, PRL 2017

Measuring the Fermi-Dirac distribution





$$f_k = \frac{1}{e^{\beta(\varepsilon_k - \mu)} + 1}$$

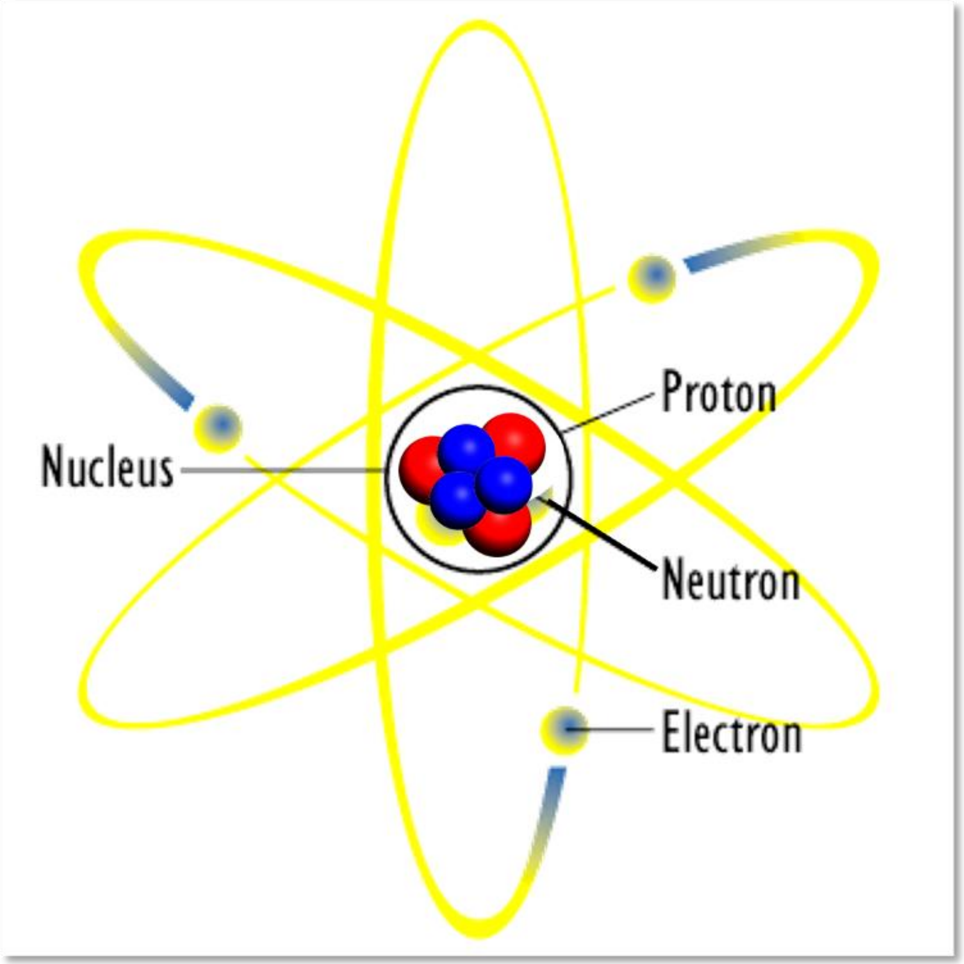
90th anniversary

Fermi surface formation

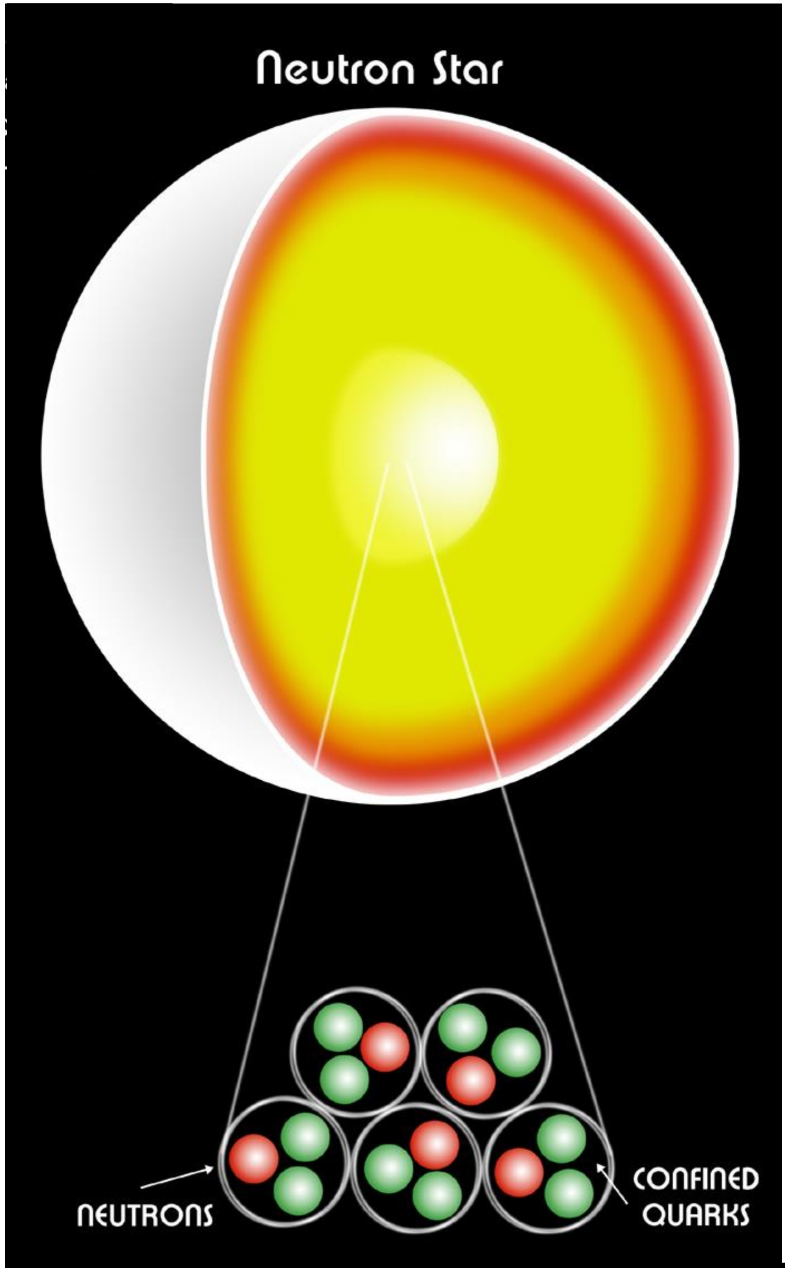
See also: Drake et al., PRA 2012, selectively probe the central portion of an inhomogeneous gas.

Z. Yan, P. Patel, B. Mukherjee, Z. Hadzibabic, T. Yefsah, J. Struck, MWZ, PRL 2017

Fermions – The Building Blocks of Matter

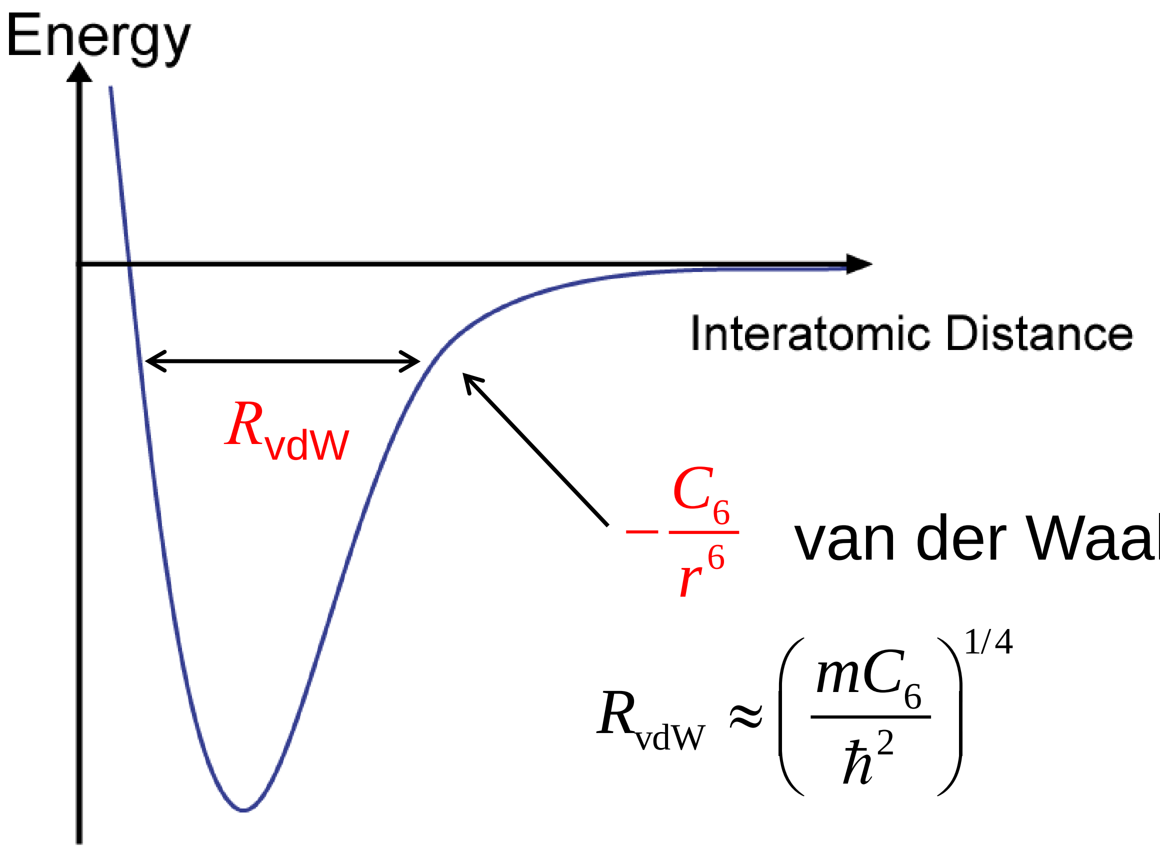


Lithium-6



Harvard-Smithsonian Center for Astrophysics

Interatomic interactions



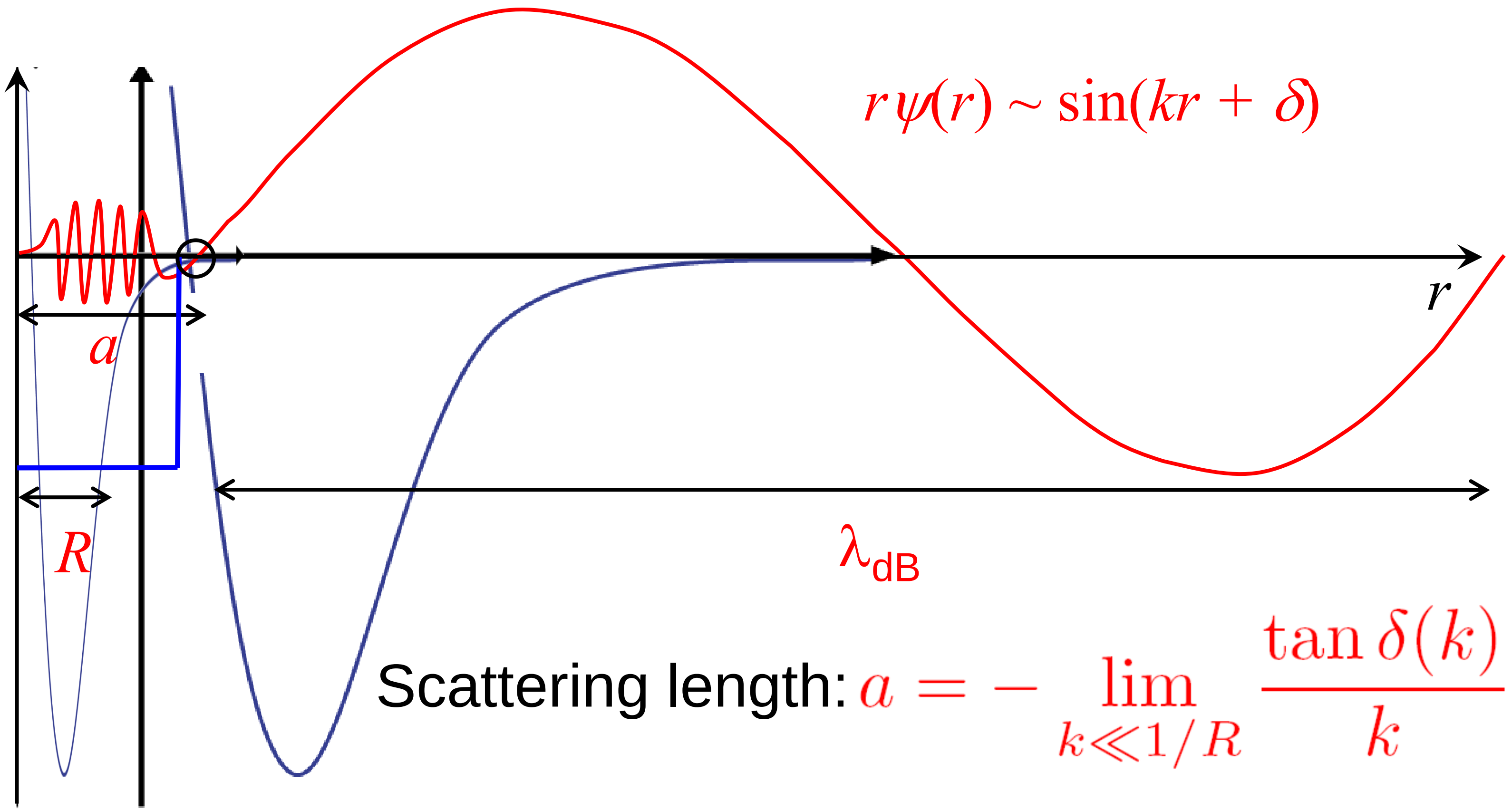
$-\frac{C_6}{r^6}$ van der Waals potential

$$R_{\mathrm{vdW}} \approx \left(\frac{mC_6}{\hbar^2} \right)^{1/4}$$

- For Alkali atoms: R_{vdW} ~ 50-200 a_0
- Ultracold collisions: $\lambda_{dB} \approx 1 \mu m >> R$

→ atoms do not probe the details of the potential

Interatomic interactions

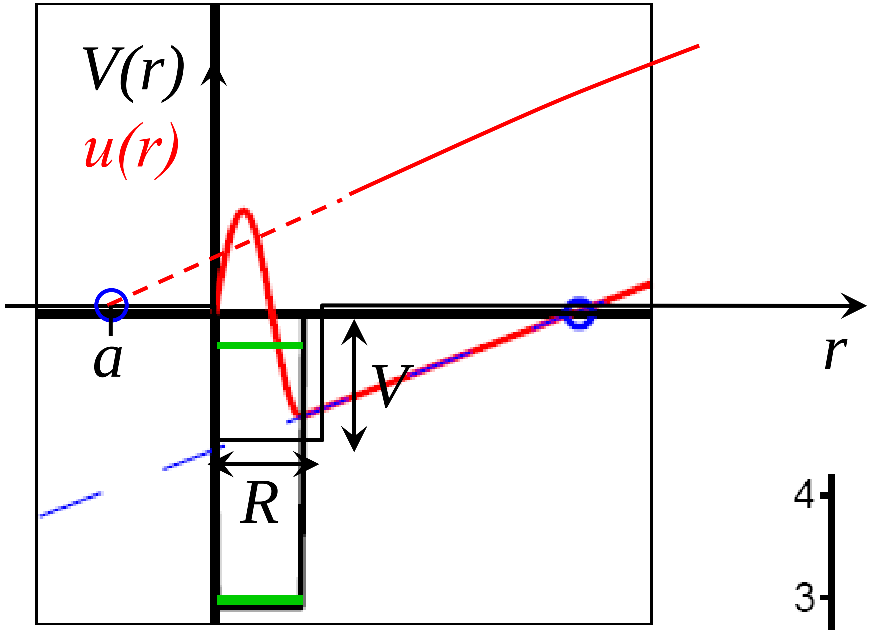


Scattering length: $a = -\lim_{k \ll 1/R} \frac{\tan\delta(k)}{k}$

- For Alkali atoms: R_{vdW} ~ 50-200 a_0
- Ultracold collisions: $\lambda_{dB} \approx 1\mu m >> R$

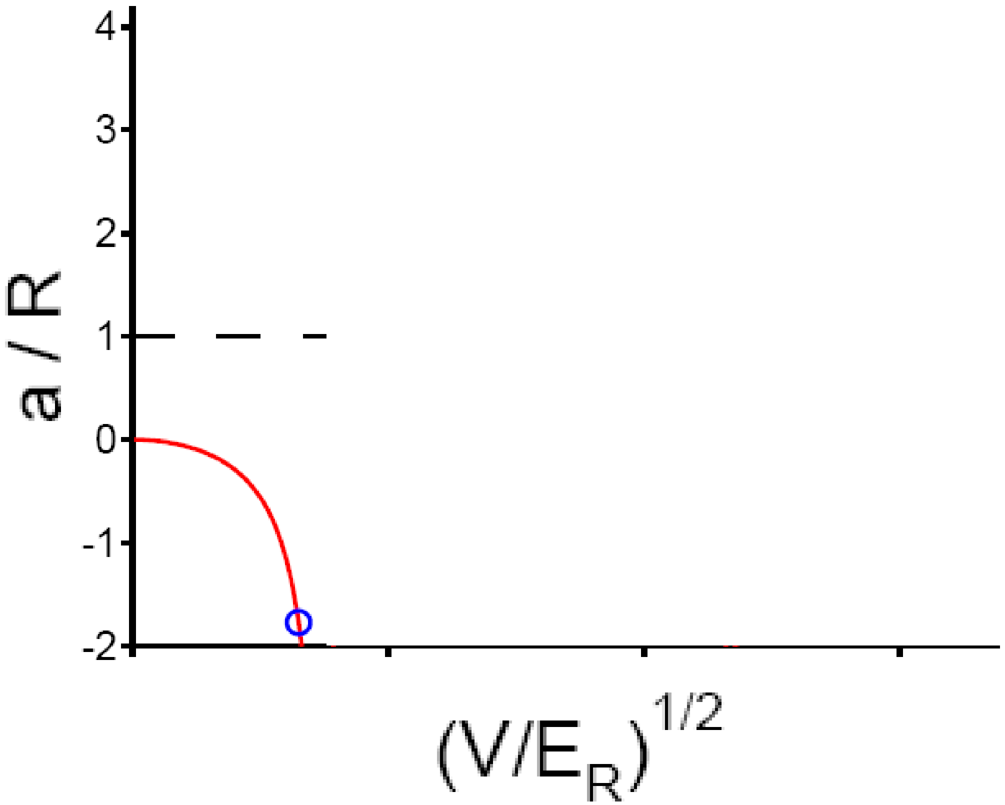
→ atoms do not probe the details of the potential

Scattering Resonances



$$u(r) \overset{r>R}{=} \sin(k(r-a))$$

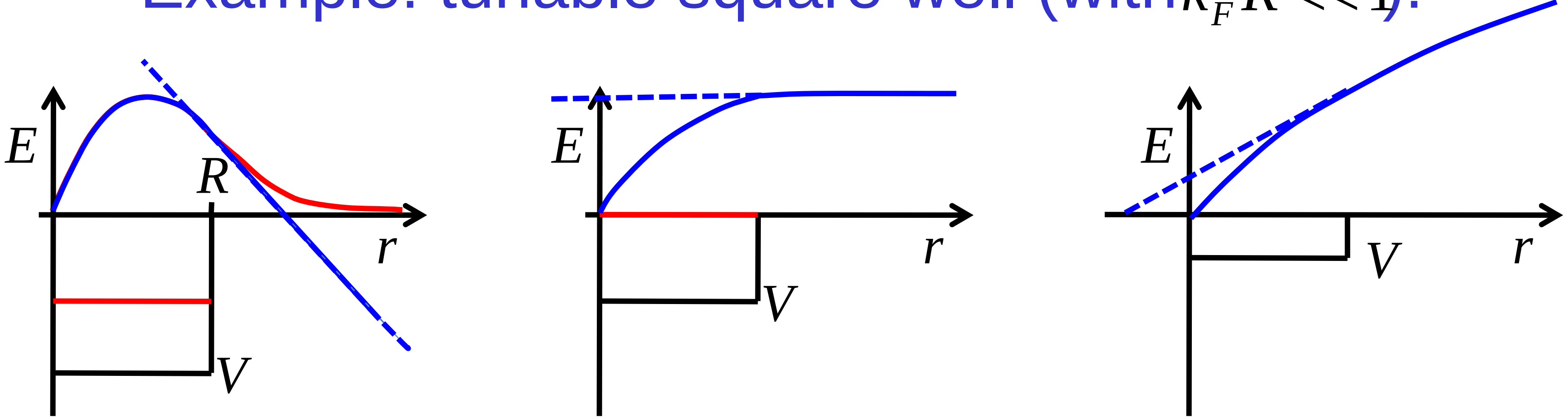
(s-wave) scattering length



Tunable Interactions

Vary interaction strength between spin up and spin down

Example: tunable square well (with $k_F R << 1$):



strong attraction	Resonance	weak attraction
deep bound state	*bound state appears*	*no bound state*

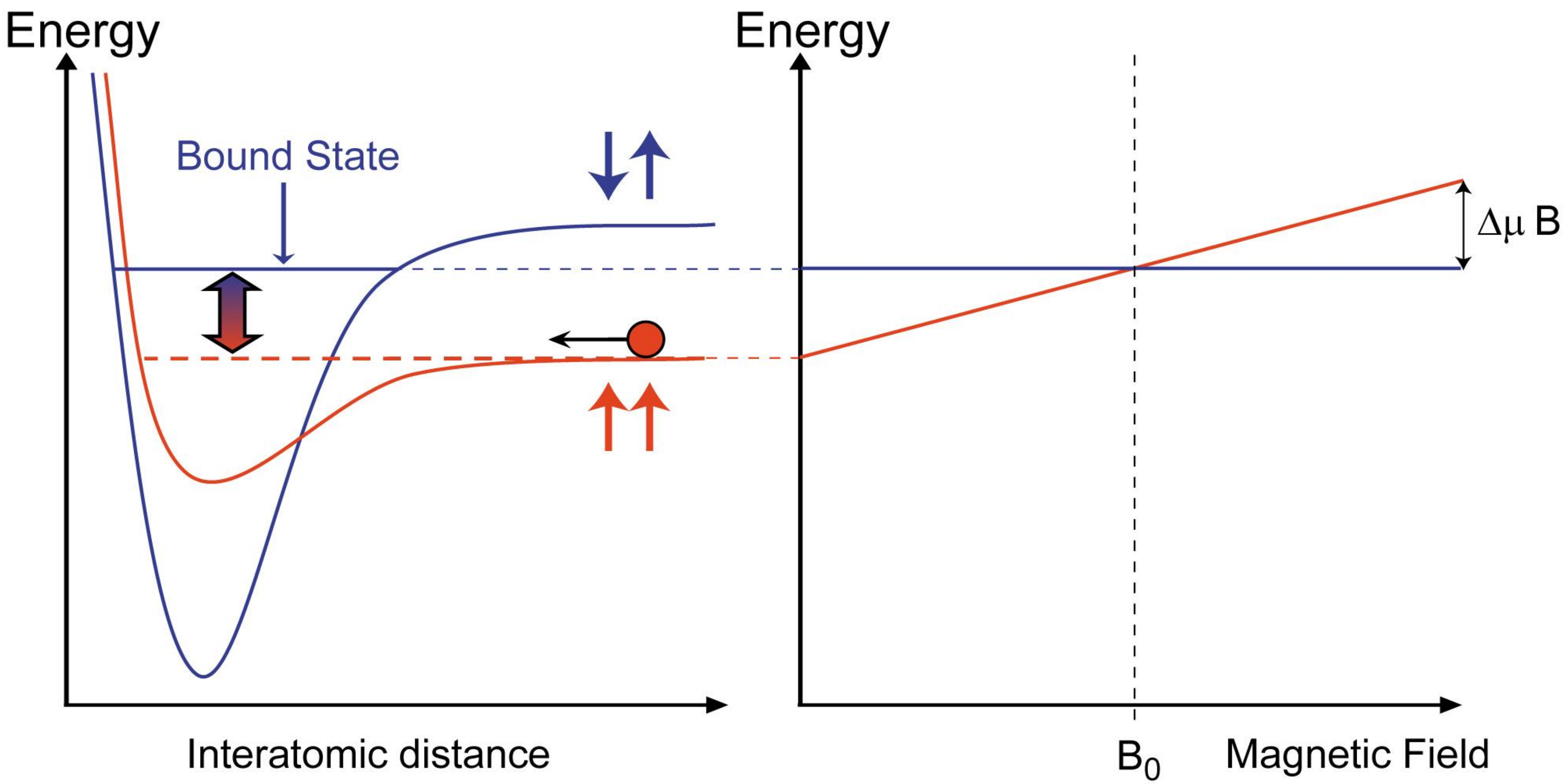
scattering length

$a > 0$	$a \to \pm\infty$	$a < 0$

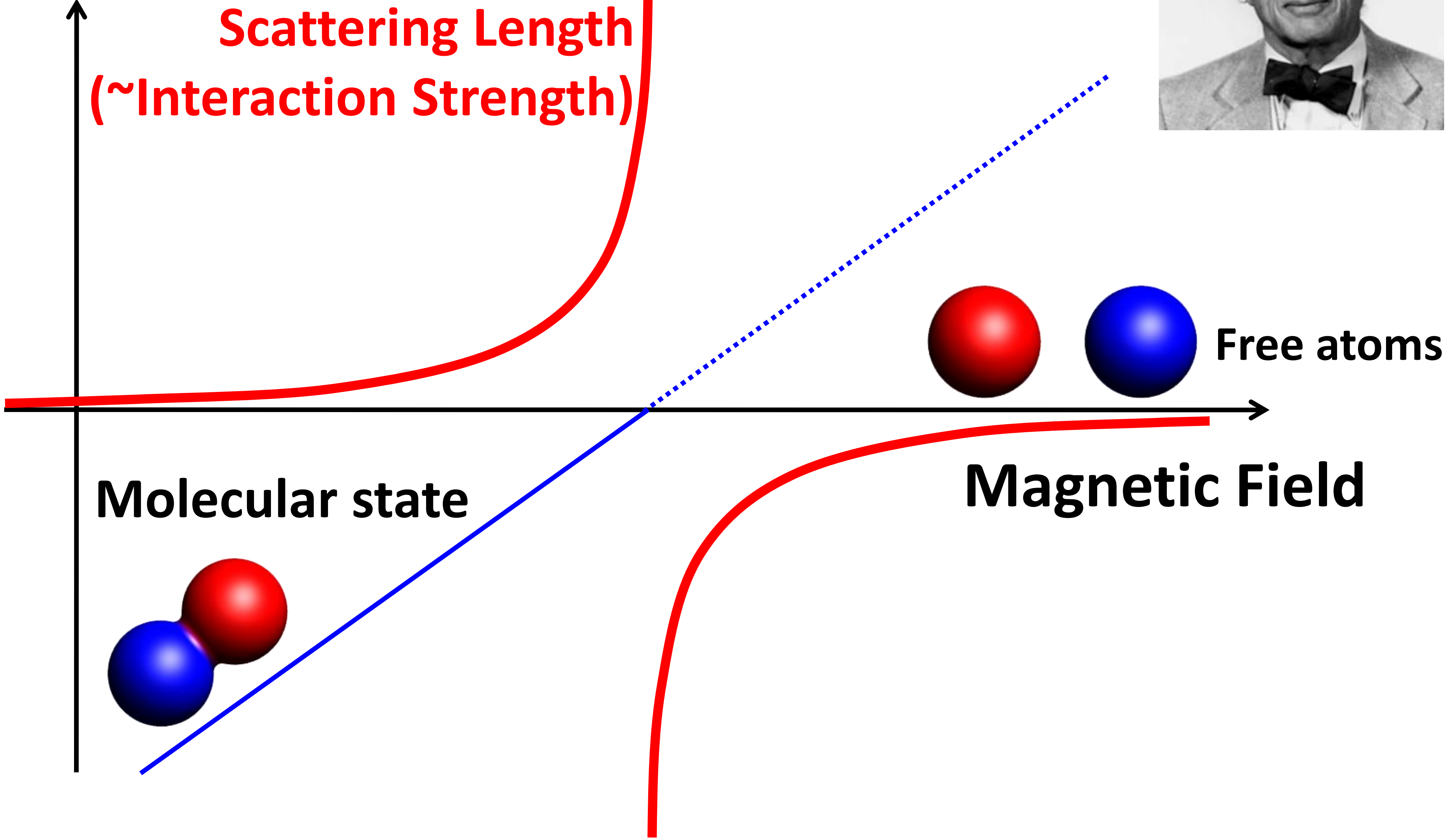
Important for Many-Body Physics:

$$\frac{\text{Interparticle distance}}{\text{Scattering length}} = \frac{1}{k_F a}$$

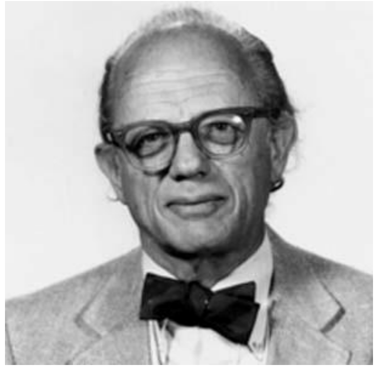
Feshbach resonances: Tuning the interactions



Feshbach resonances: Tuning the interactions



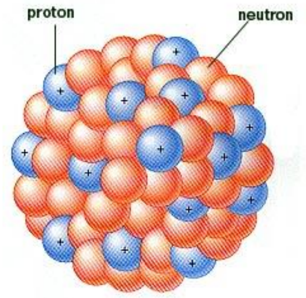
Herman Feshbach

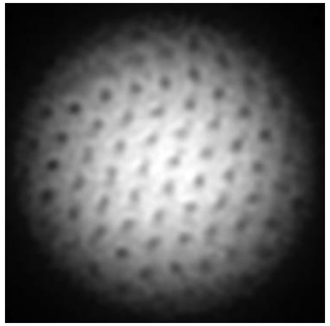


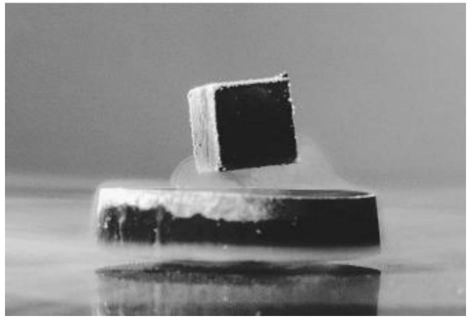
Strongly Interacting Fermi Systems

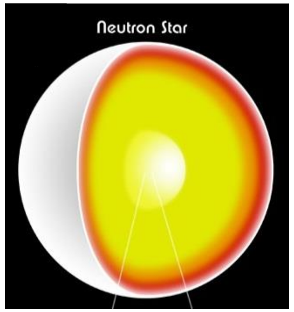
Length scales

10^{-15} m | 1 mm | 1 m | 10^{4} m | 10^{7} m

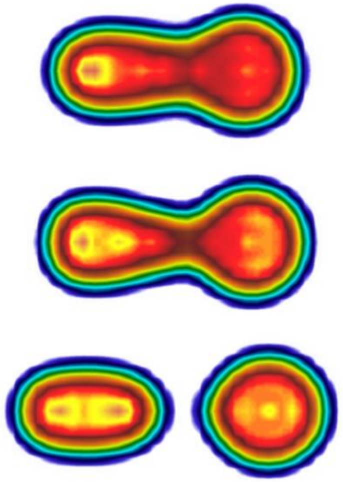






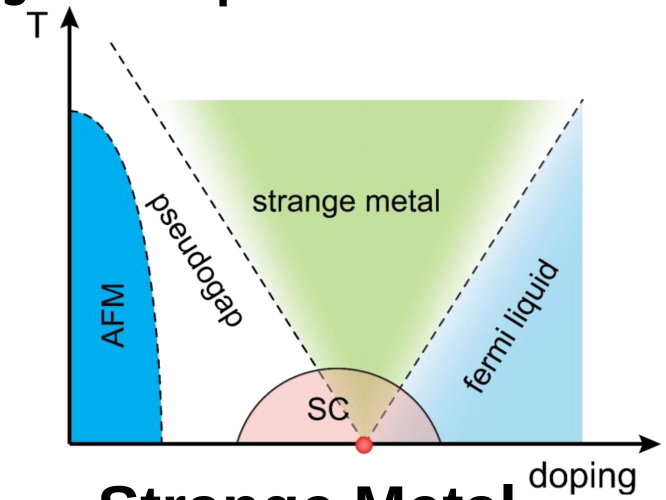


Nuclei | **Ultracold Gases** | **High-T_c Superconductors** | **Neutron Star** | **White dwarf**

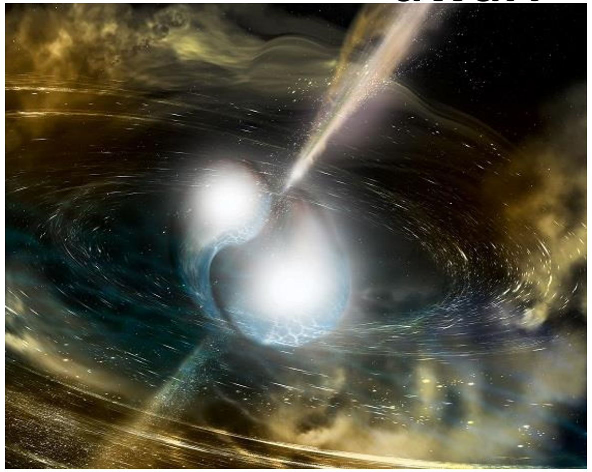


Nuclear Fission

Bulgac et al., PRL 116, 122504 (2016)



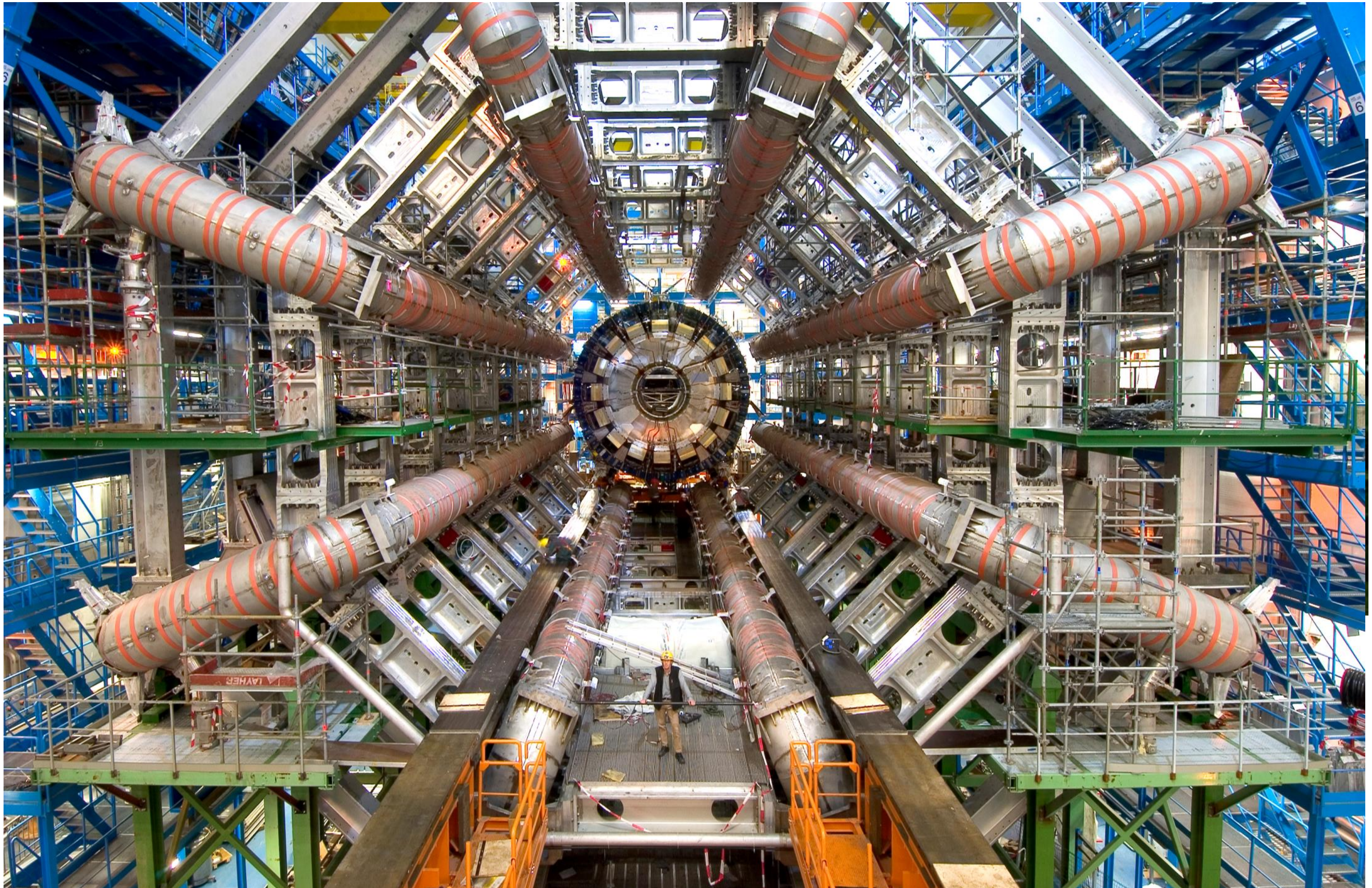
Strange Metal

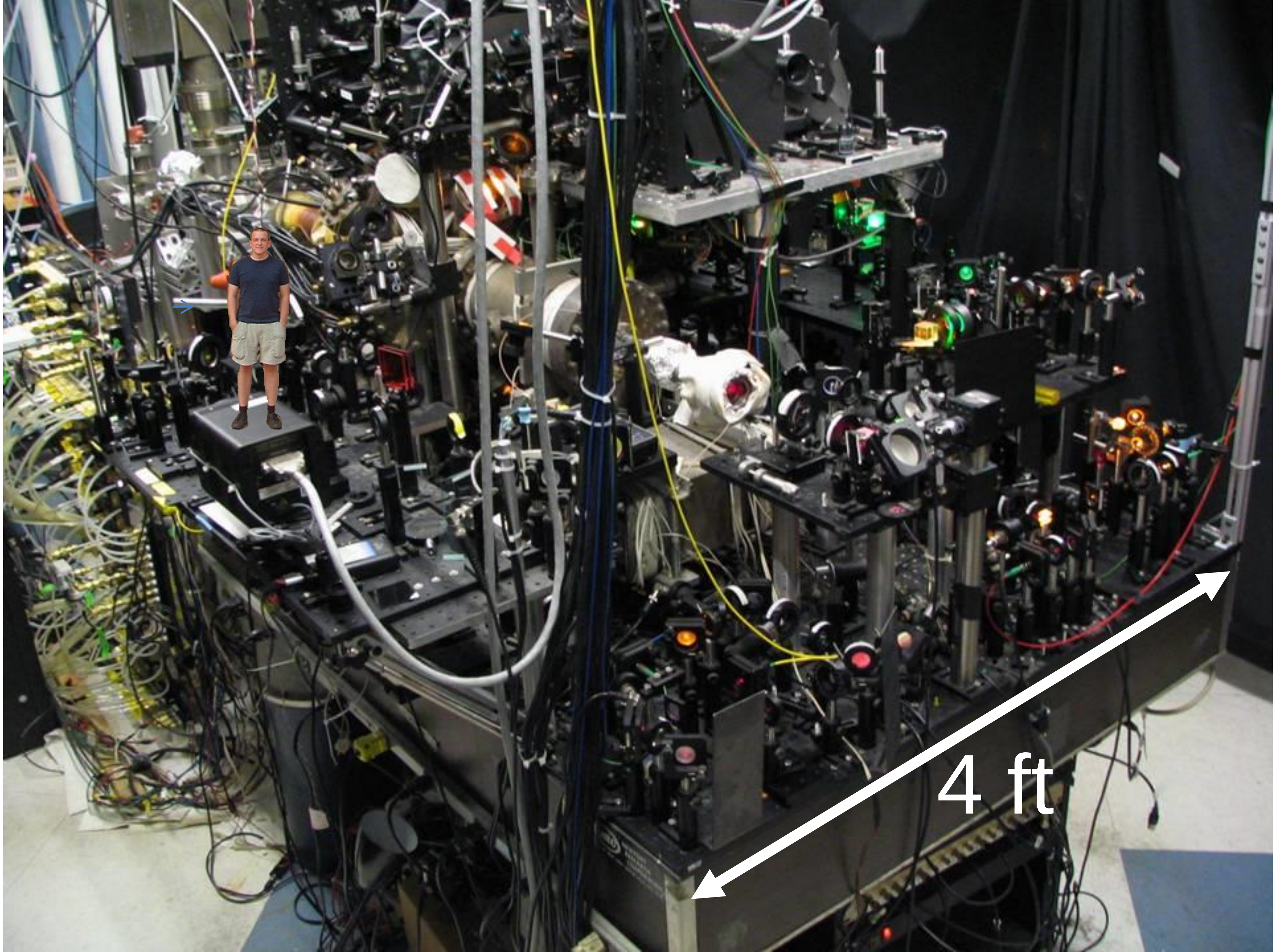


Neutron Star Merger

NSF/LIGO/Somona/A. Simonnet

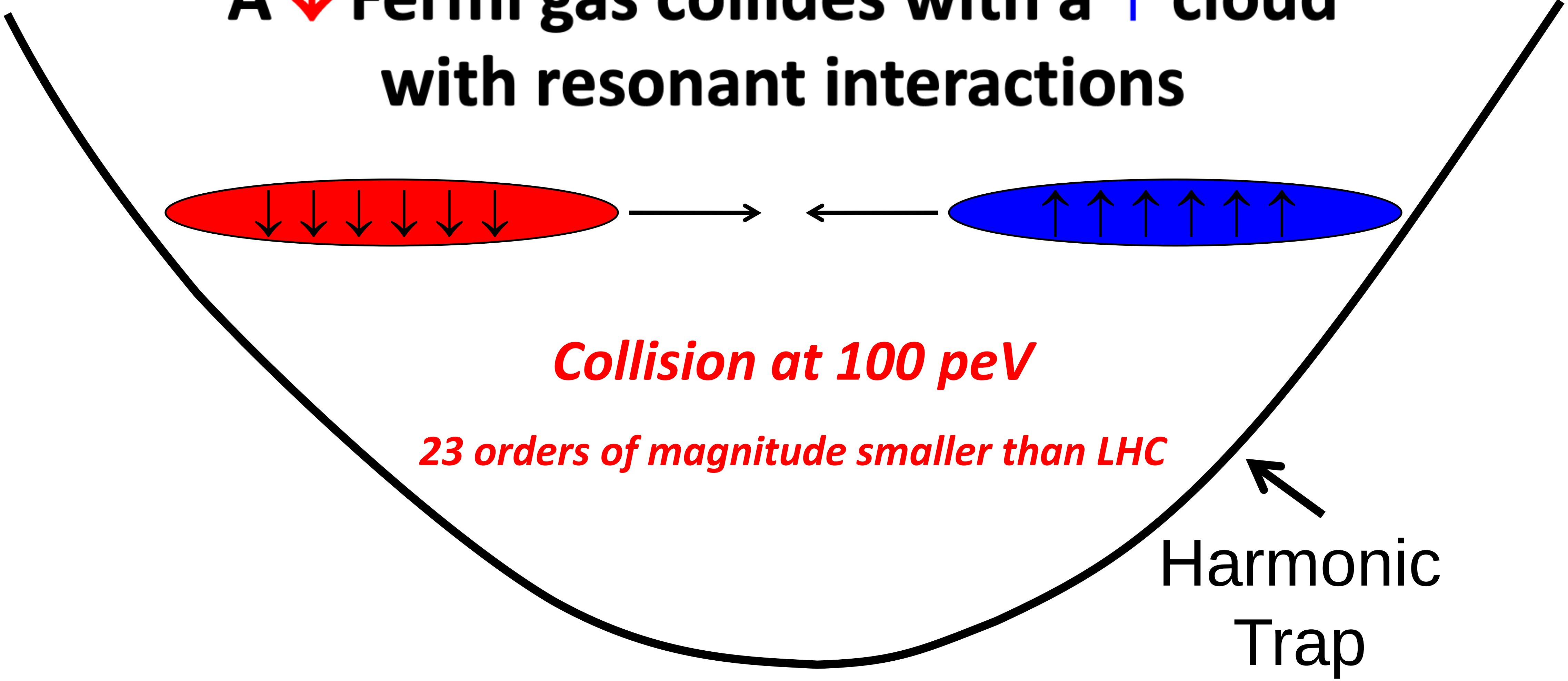
Large Hadron Collider (LHC)




4 ft

Little Fermi Collider (LFC)

A ↓ Fermi gas collides with a ↑ cloud with resonant interactions



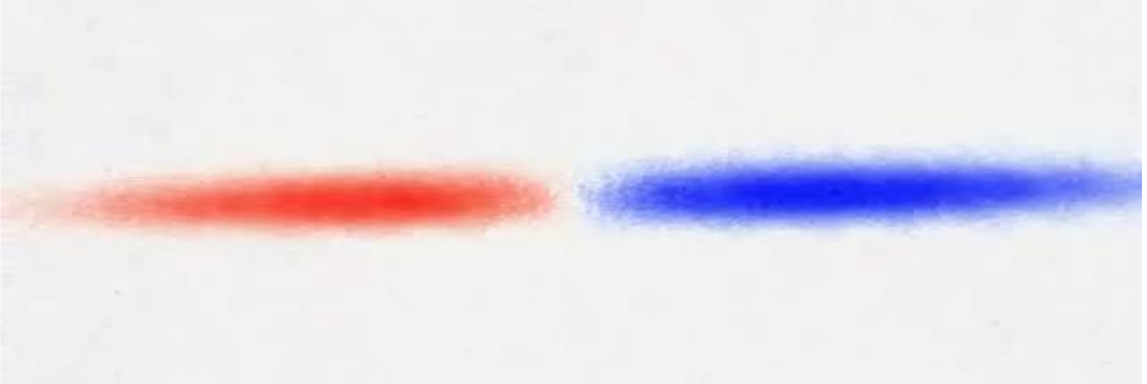
Collision at 100 peV

23 orders of magnitude smaller than LHC

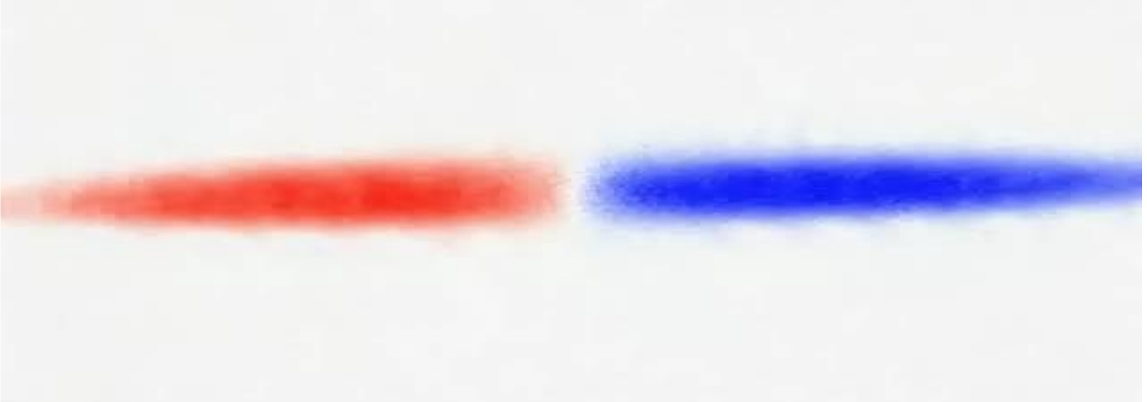
A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Little Fermi Collider (LFC)

Without Interactions



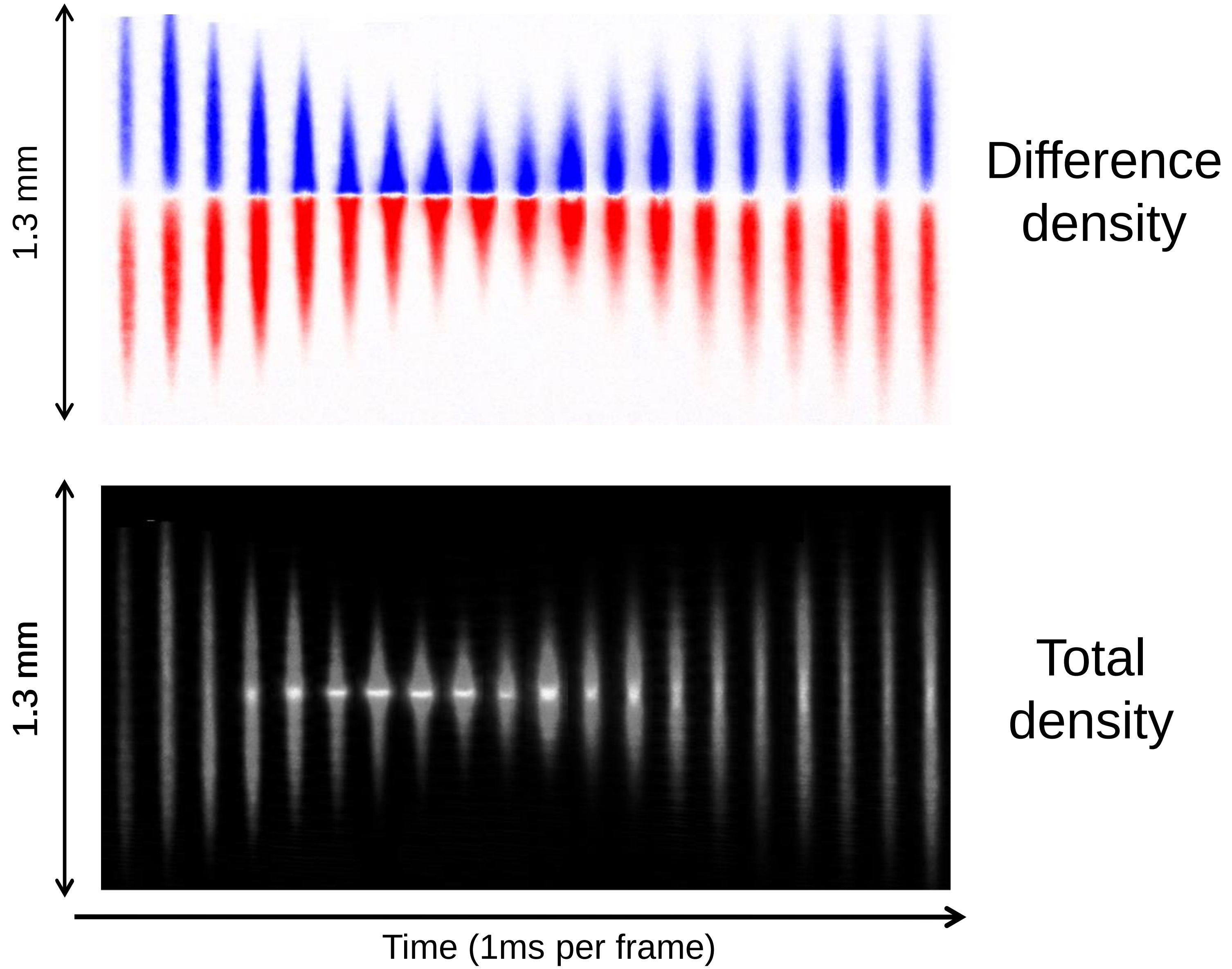
Resonant Interactions



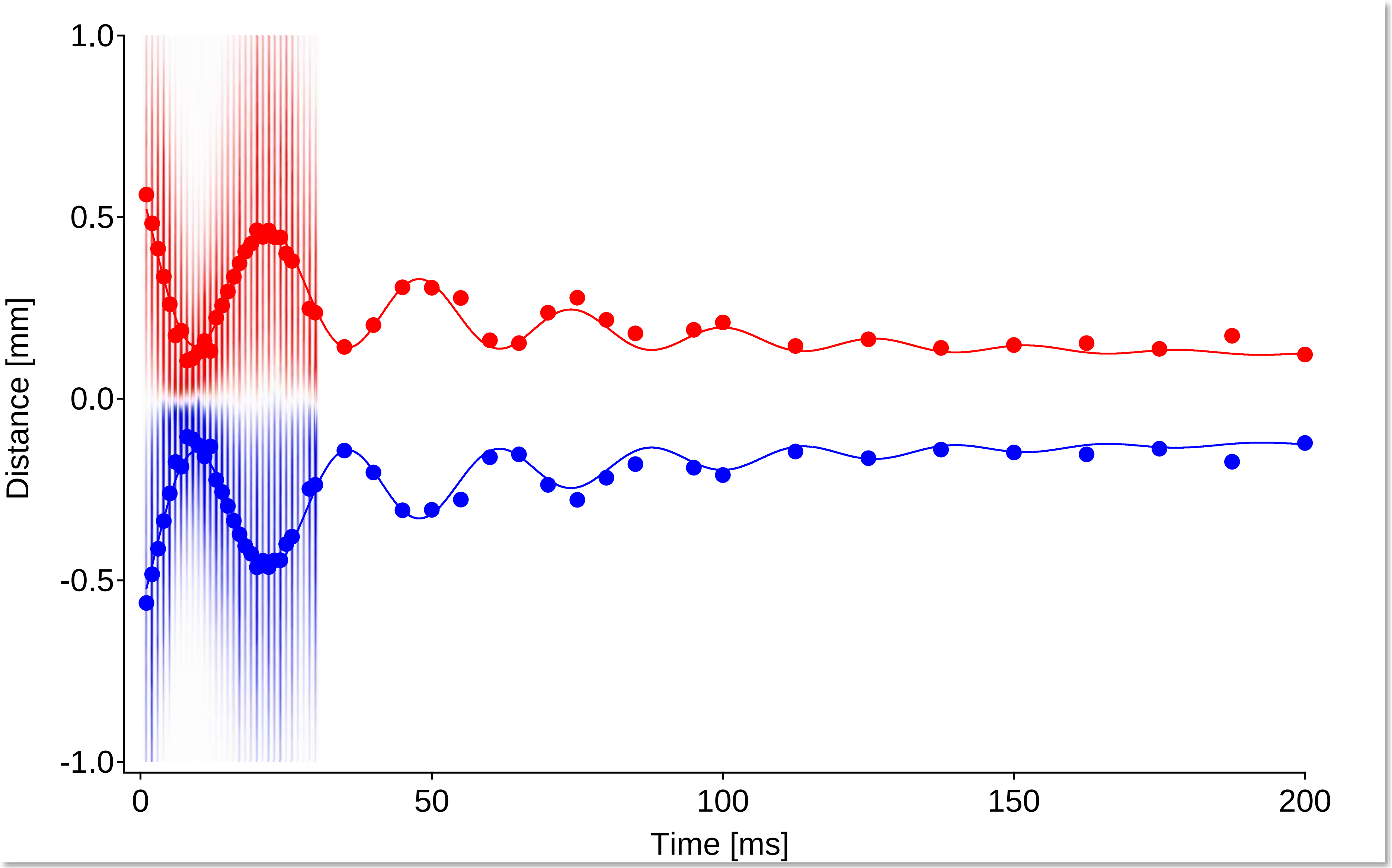
A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

The bouncing gas

First collision

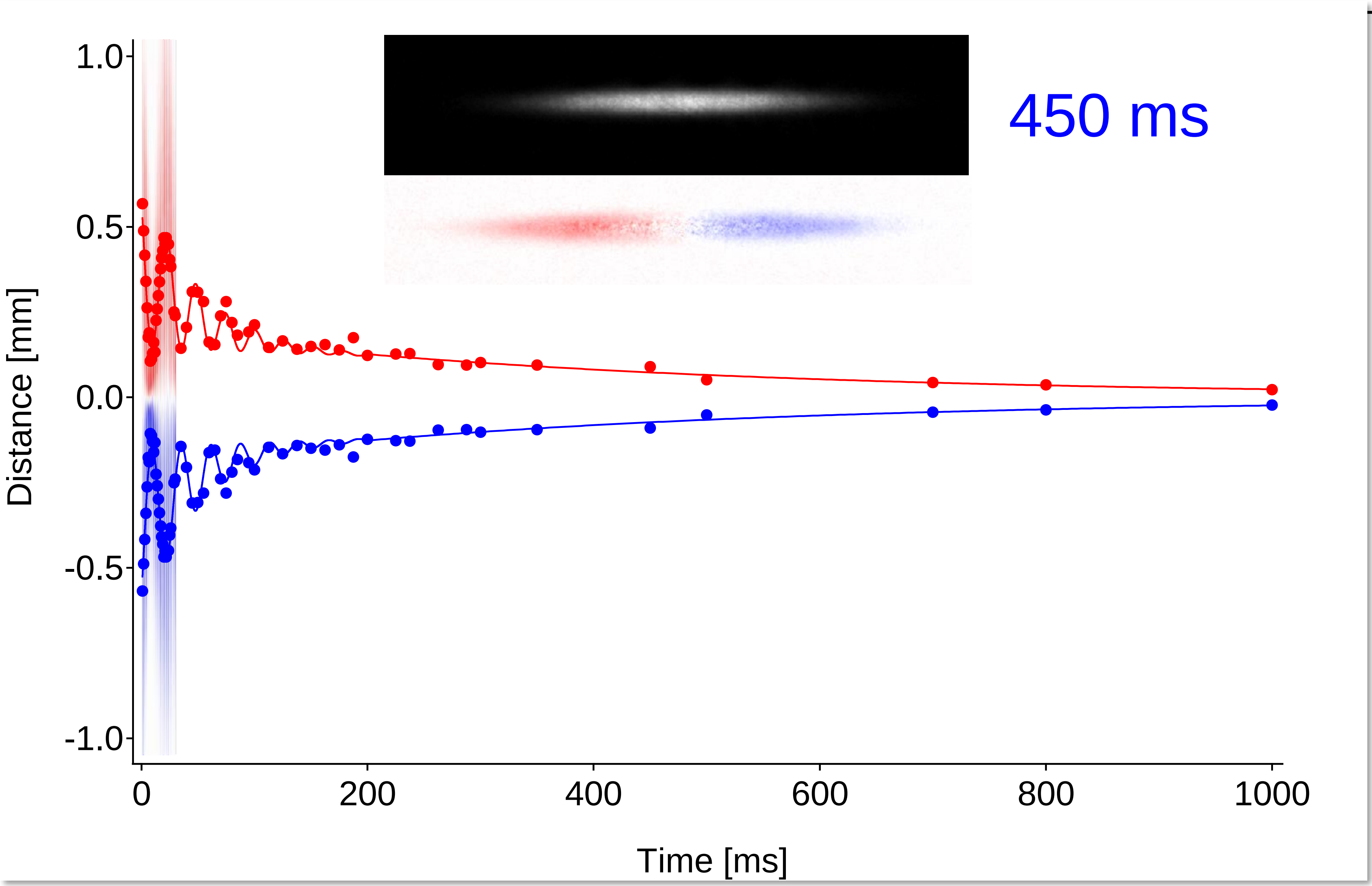


Later times



A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Much later times



A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Quantum limit of spin diffusion

Mean free path ~ Interparticle spacing d

Diffusion constant:

$$D \sim \text{mean free path} \times \text{average velocity}$$

$$\cancel{d} \times \frac{\hbar}{m\cancel{d}}$$

$$D \sim \frac{\hbar}{m} = \frac{\text{Planck's constant}}{\text{Particle mass}} = \frac{(0.1\text{ mm})^2}{1\text{s}}$$

→ Quantum Limit of Diffusion

In a hot relativistic fluid (e.g. Quark-Gluon Plasma): $D \sim \frac{\hbar c^2}{T}$

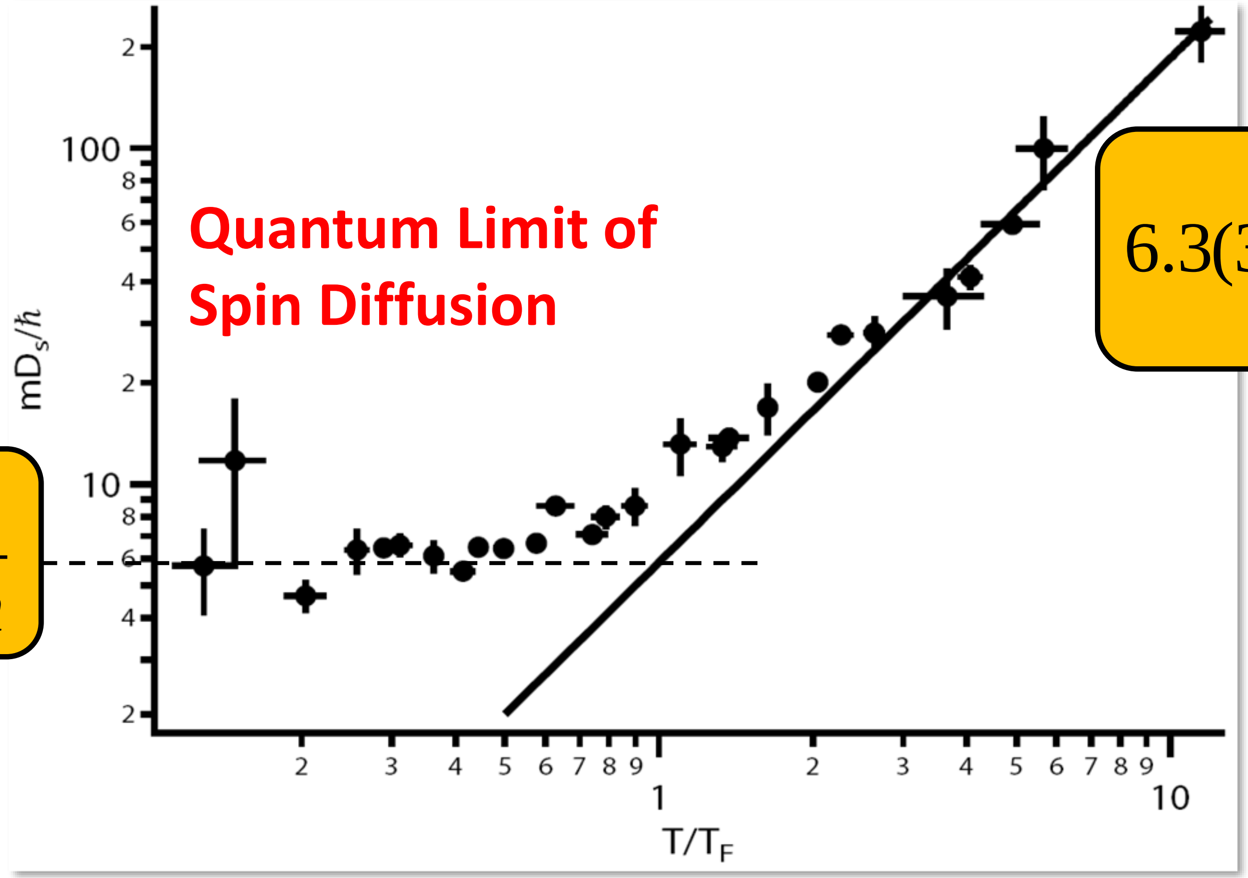
$$mc^2 \to T$$

A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Spin Diffusion vs Temperature

Spin current = -D · Spin density gradient

Universal high-T behavior:

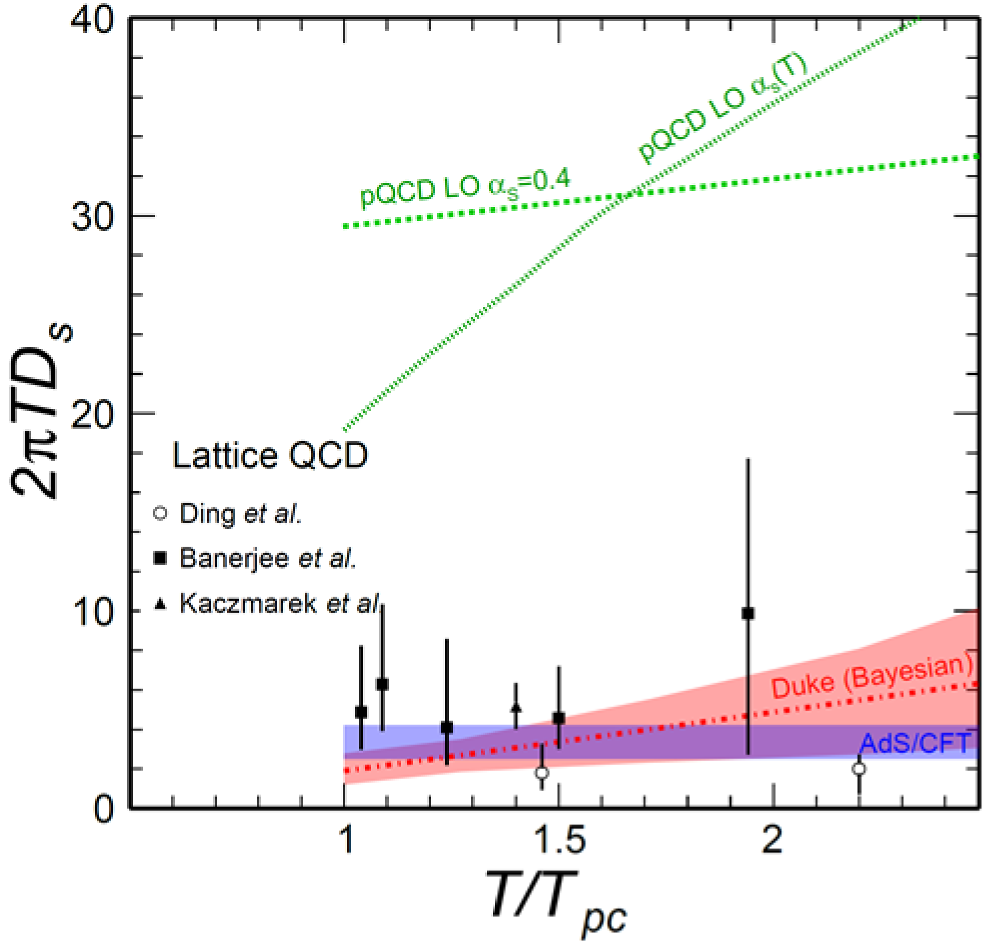


$$6.3(3)\,\frac{\hbar}{m}\left(\frac{T}{T_F}\right)^{3/2}$$

$$5.8\,\frac{\hbar}{m}$$

A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Diffusivity in the Quark-Gluon Plasma



Can Fermi Gases become superfluid?

Superconductivity

Electrons are Fermions

Discovery of superconductivity 1911

Heike
Kamerlingh-Onnes
1911

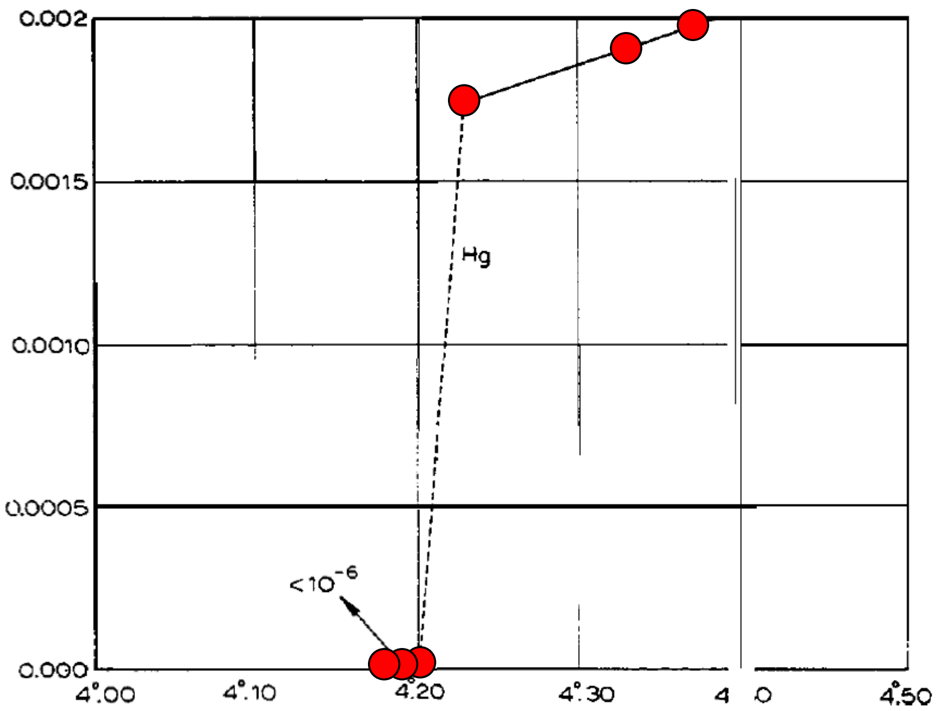


Fig. 17.

Fermionic Superfluidity

Condensation of Fermion Pairs

- Helium-3 (Lee, Osheroff, Richardson 1971)
- Superconductors: *Charged* superfluids of electron pairs

Frictionless flow ⇔ Resistance-less current

John Bardeen

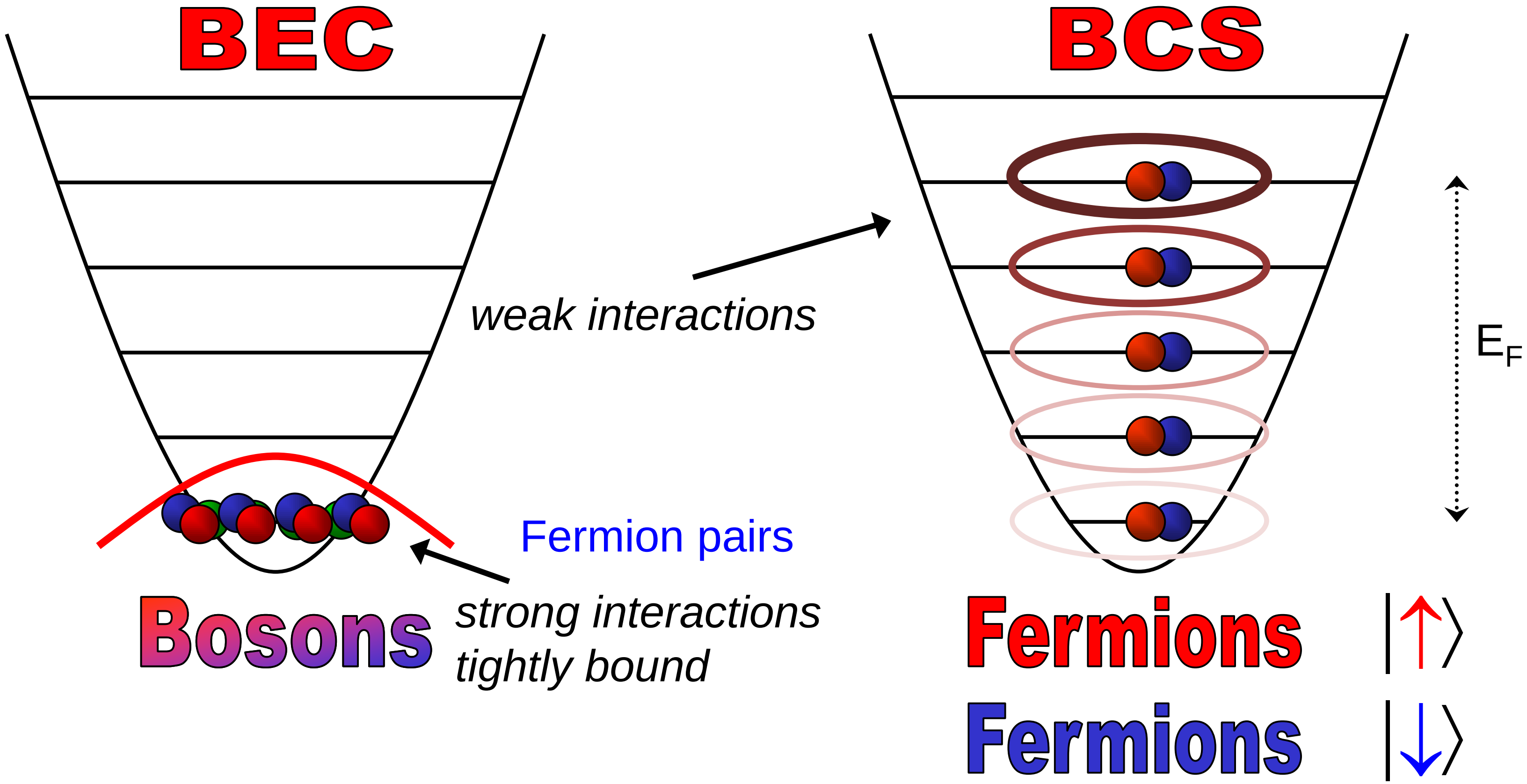
Leon N. Cooper

John R. Schrieffer

- Neutron stars

In the core: Quark superfluid

Bosons vs Fermions



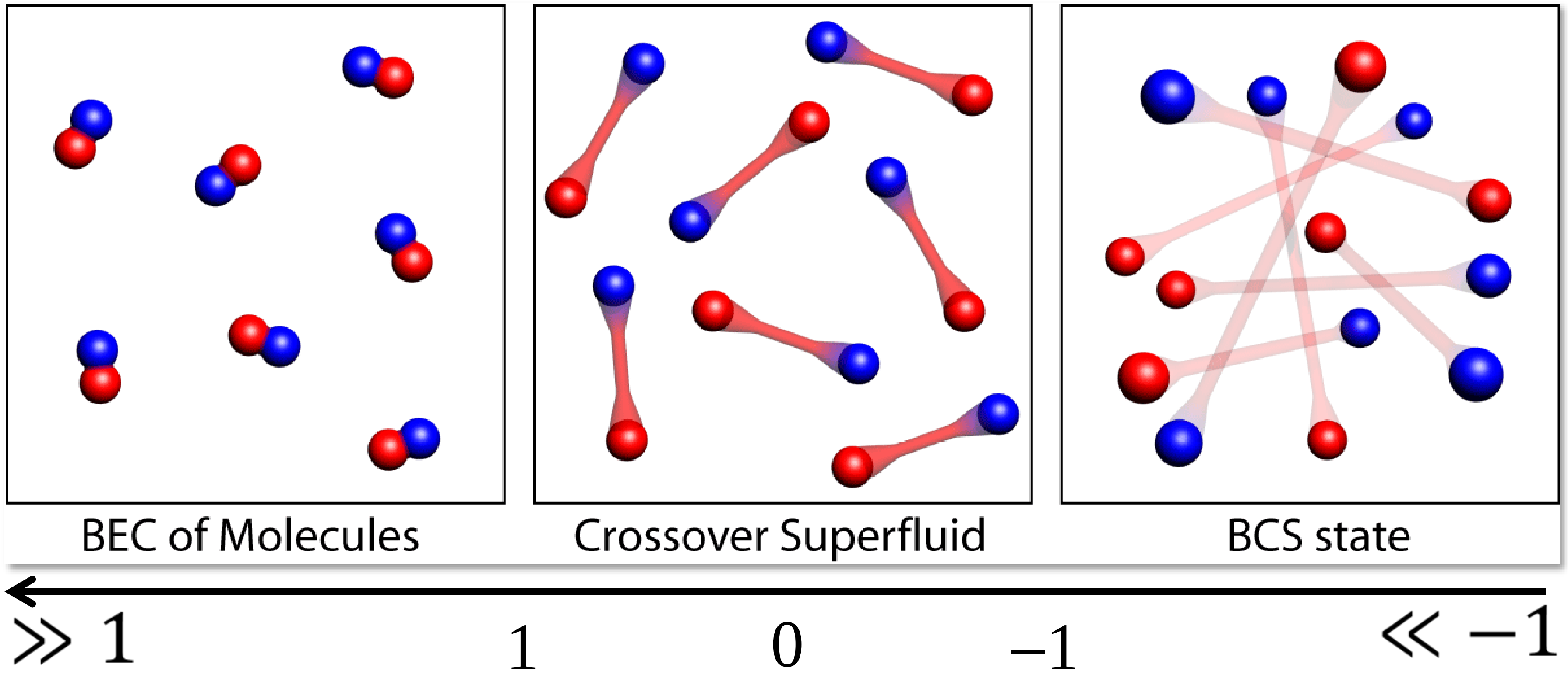
e.g.: e^- + p, ^{1}H, ^{23}Na, ^{6}Li$_2$, ^{6}Li + ^{6}Li

$$T_C \approx T_F$$

e.g.: e^-, ^{3}He, ^{6}Li, ^{40}K

$$T_C \approx T_F e^{-\pi/2k_F|a|}$$

From BEC to BCS



$$(k_F a)^{-1} = \frac{\text{Interparticle Distance}}{\text{Scattering Length}}$$

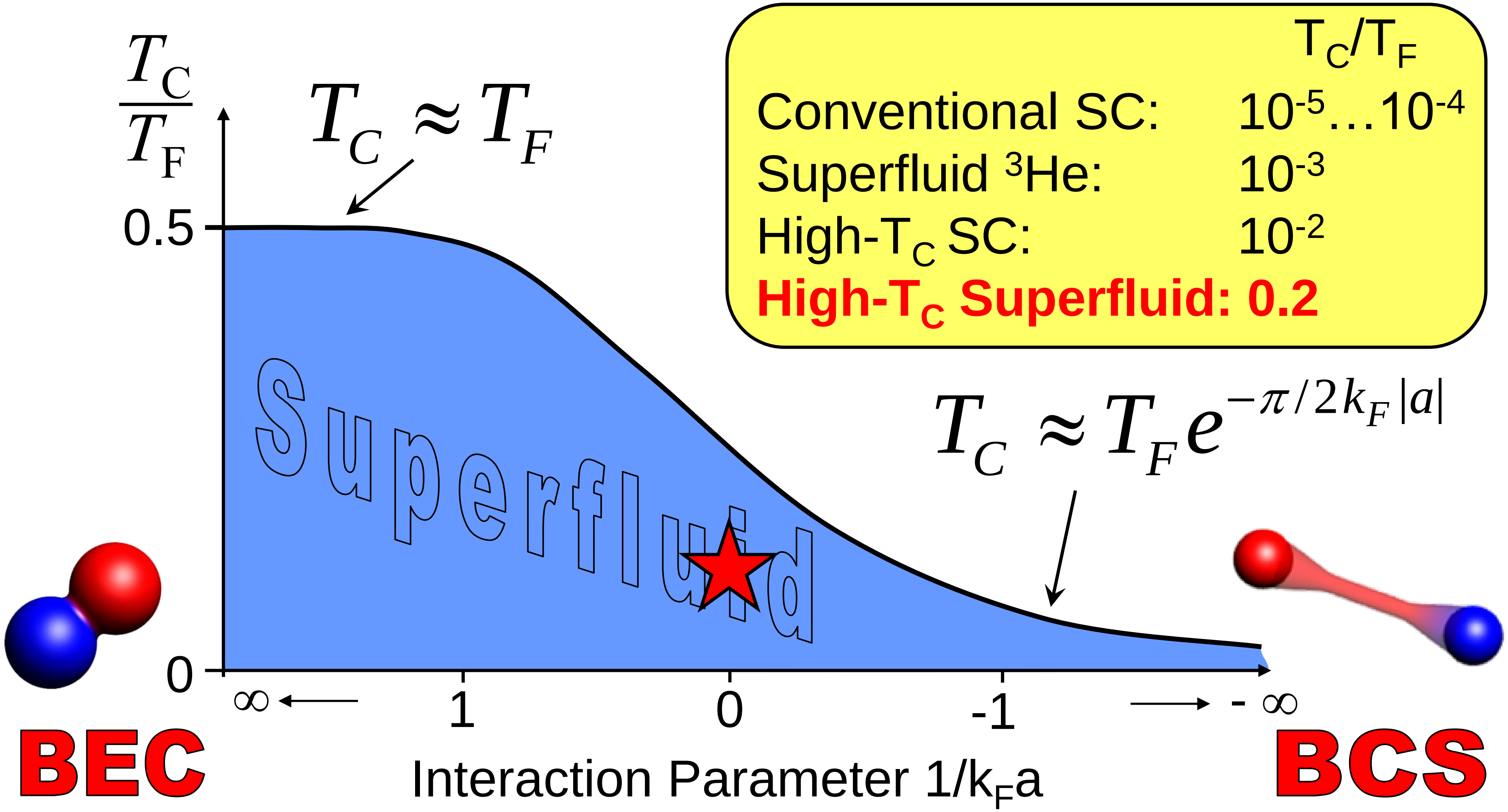
Weakly Interacting Bosons

→ Strongly Interacting Bosons

→ Strongly Interacting Fermions

→ Weakly Interacting Fermions

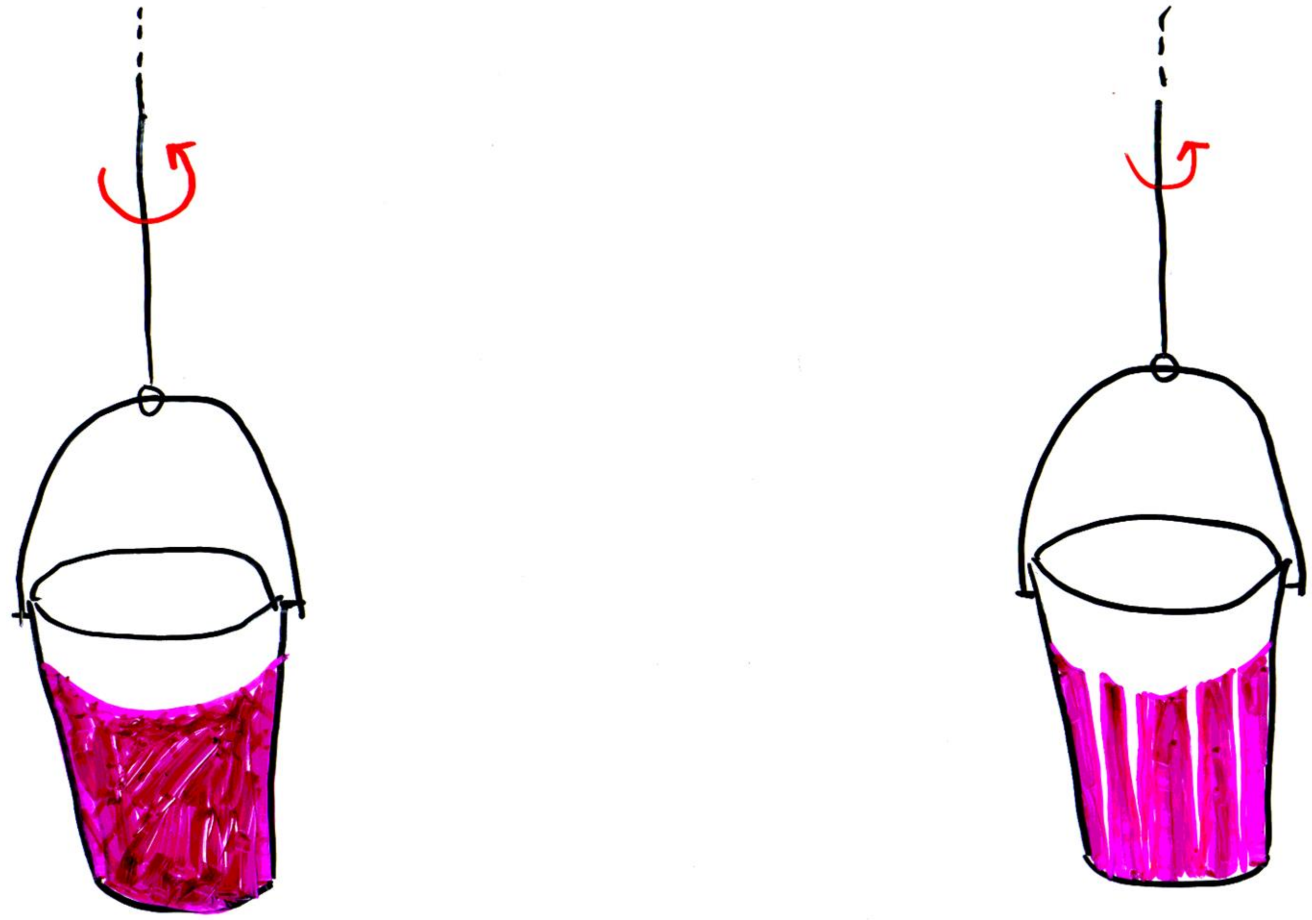
Critical Temperature for Fermionic Superfluidity



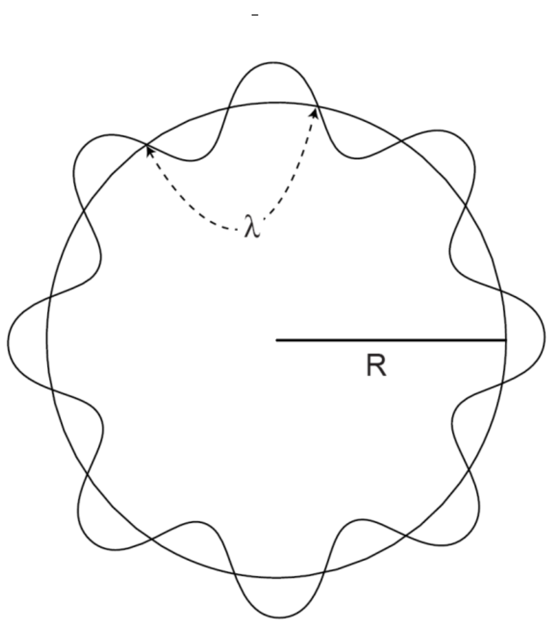
Scaled to the density of electrons in solids:

Superconductivity far above room temperature!

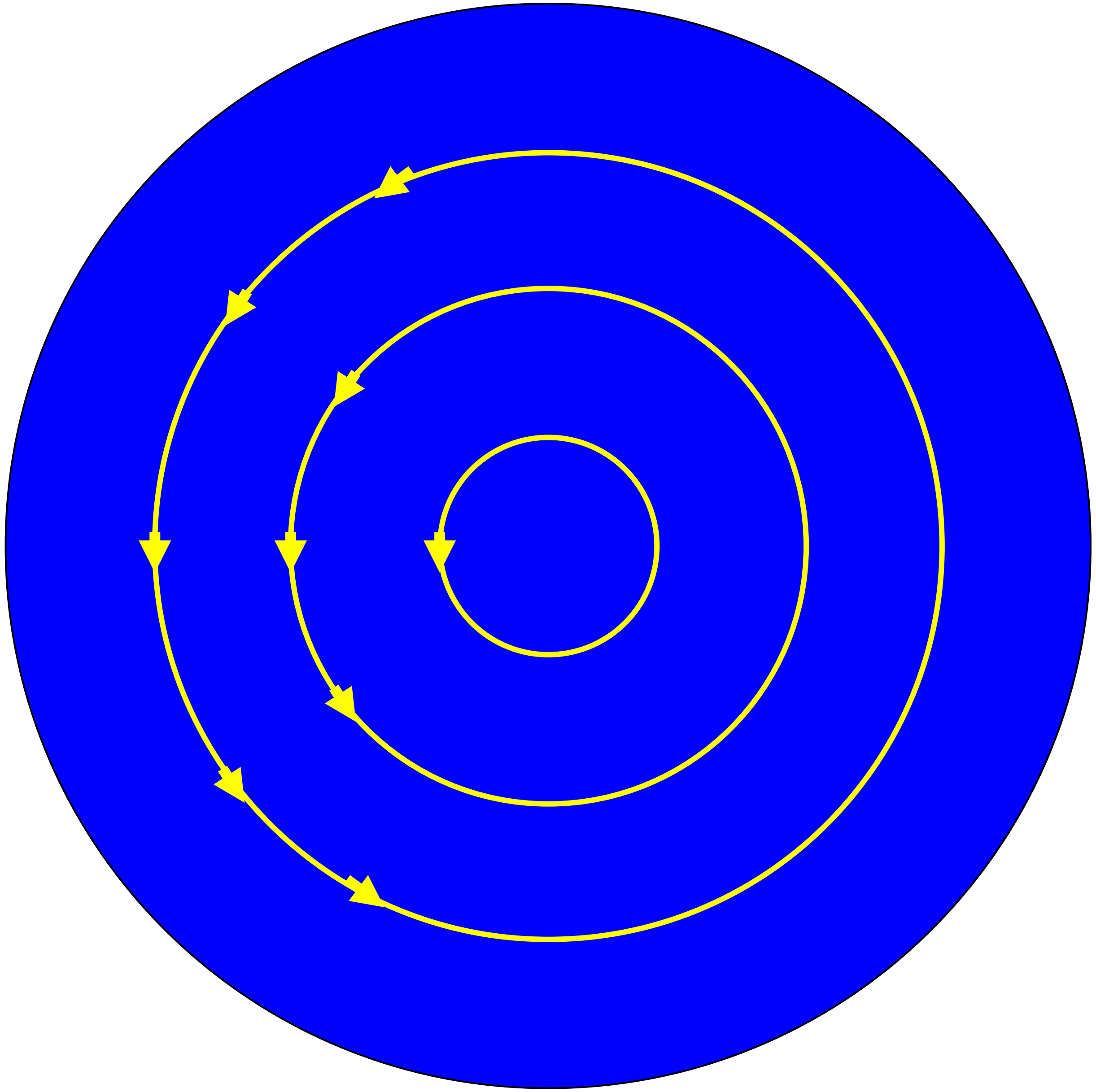
Rotating Fluids



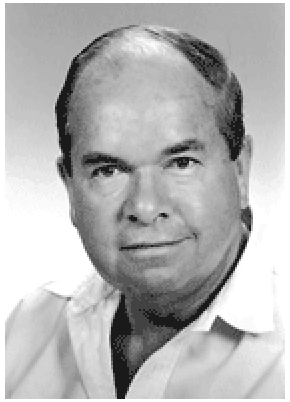
Rotating superfluid



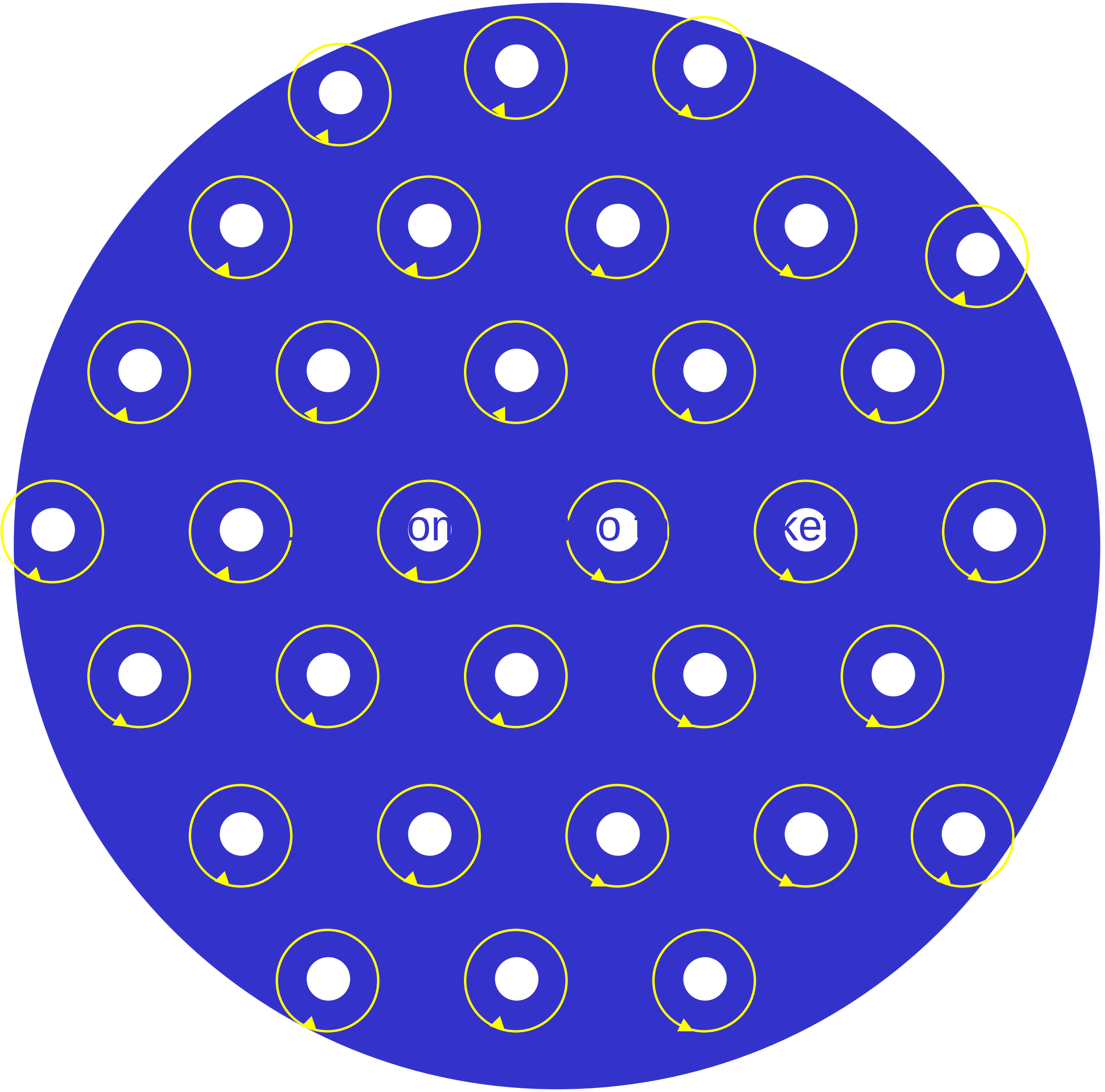
Does it rotate like this?



Like this!

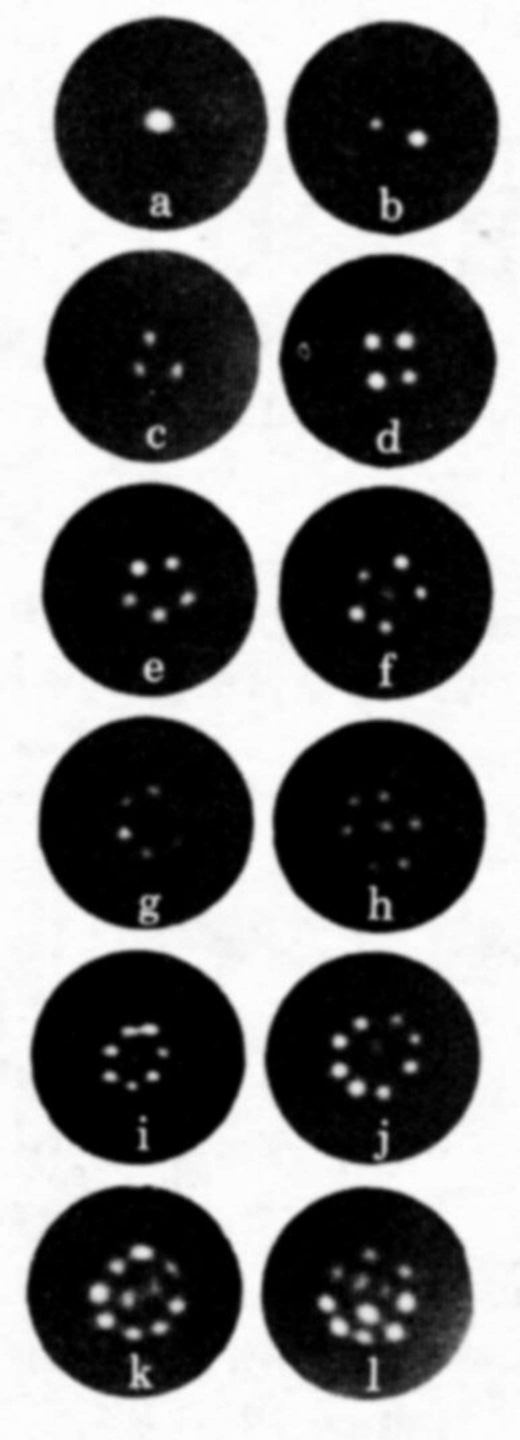


Aleksei A.
Abrikosov

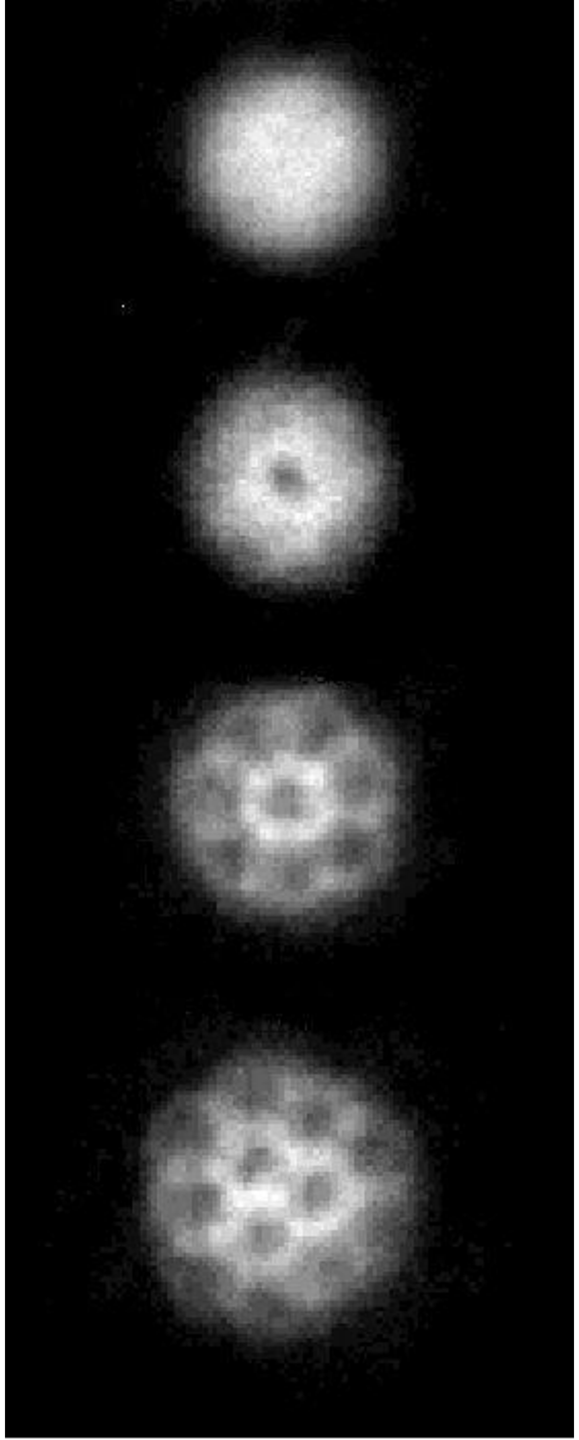


Abrikosov lattice (triangular lattice)

Vortex Arrays
in Bosonic Gases / Fluids



Berkeley
(R.E. Packard, 1979)
Helium-4



ENS
(J. Dalibard, 2000)
Rubidium BEC

Also: Phase engineering of single vortices in BEC: JILA (1999)

THE DIRECT OBSERVATION OF INDIVIDUAL FLUX LINES IN TYPE II SUPERCONDUCTORS

U. ESSMANN and H. TRÄUBLE

Institut für Physik am Max-Planck-Institut für Metallforschung, Stuttgart and Institut für theoretische und angewandte Physik der Technischen Hochschule Stuttgart

Received 4 April 1967

Neutral superfluids under rotation

$$\vec{F} = 2m\vec{v}\times\vec{\omega}$$

Coriolis force in rotating frame

$$\Leftrightarrow$$

Superconductors in magnetic field

$$\vec{F} = q\vec{v}\times\vec{B}$$

Lorentz Force

U. Essmann and H. Träuble,
Physics Letters A, **24**, 526 (1967)

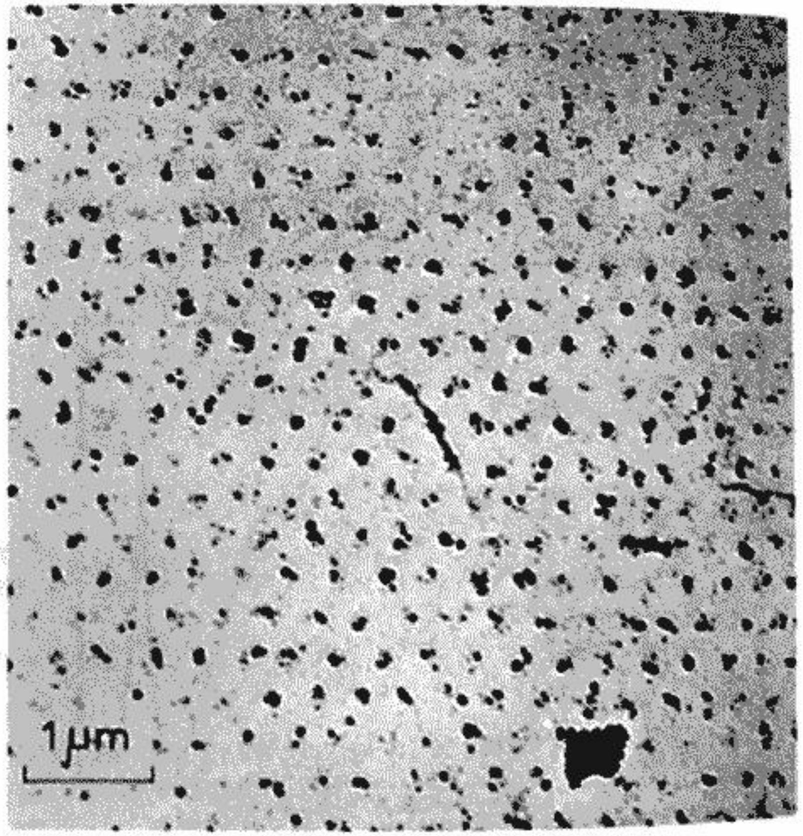
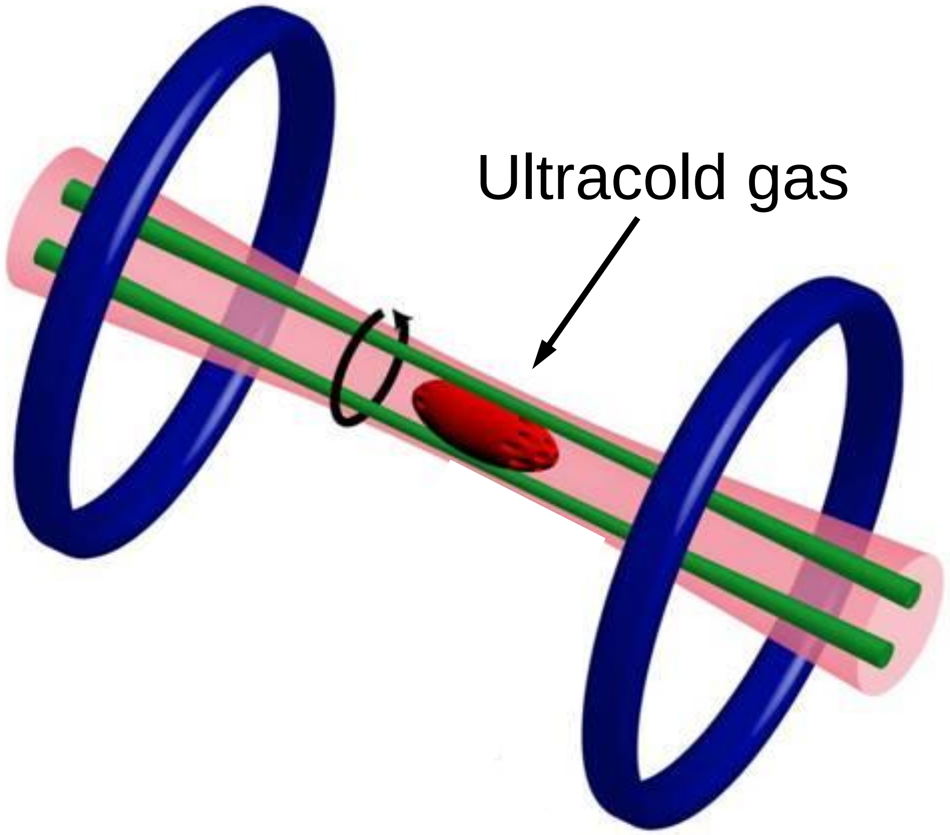
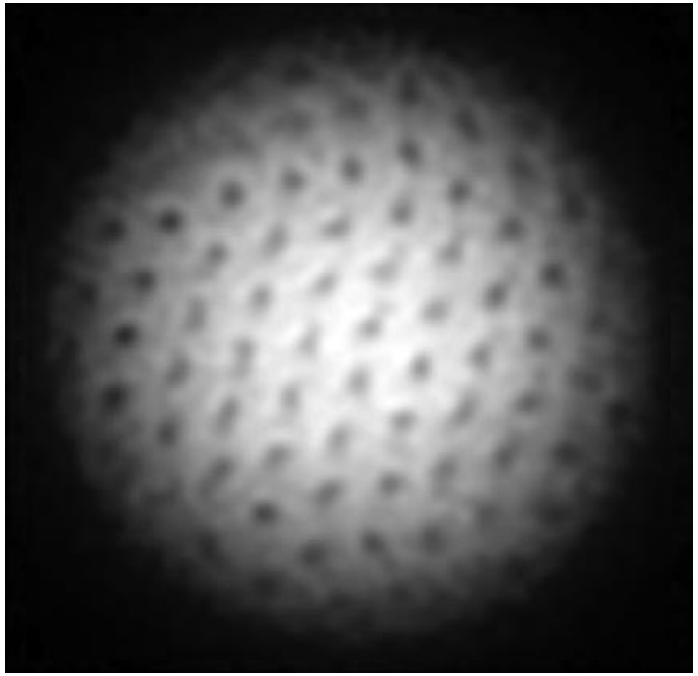


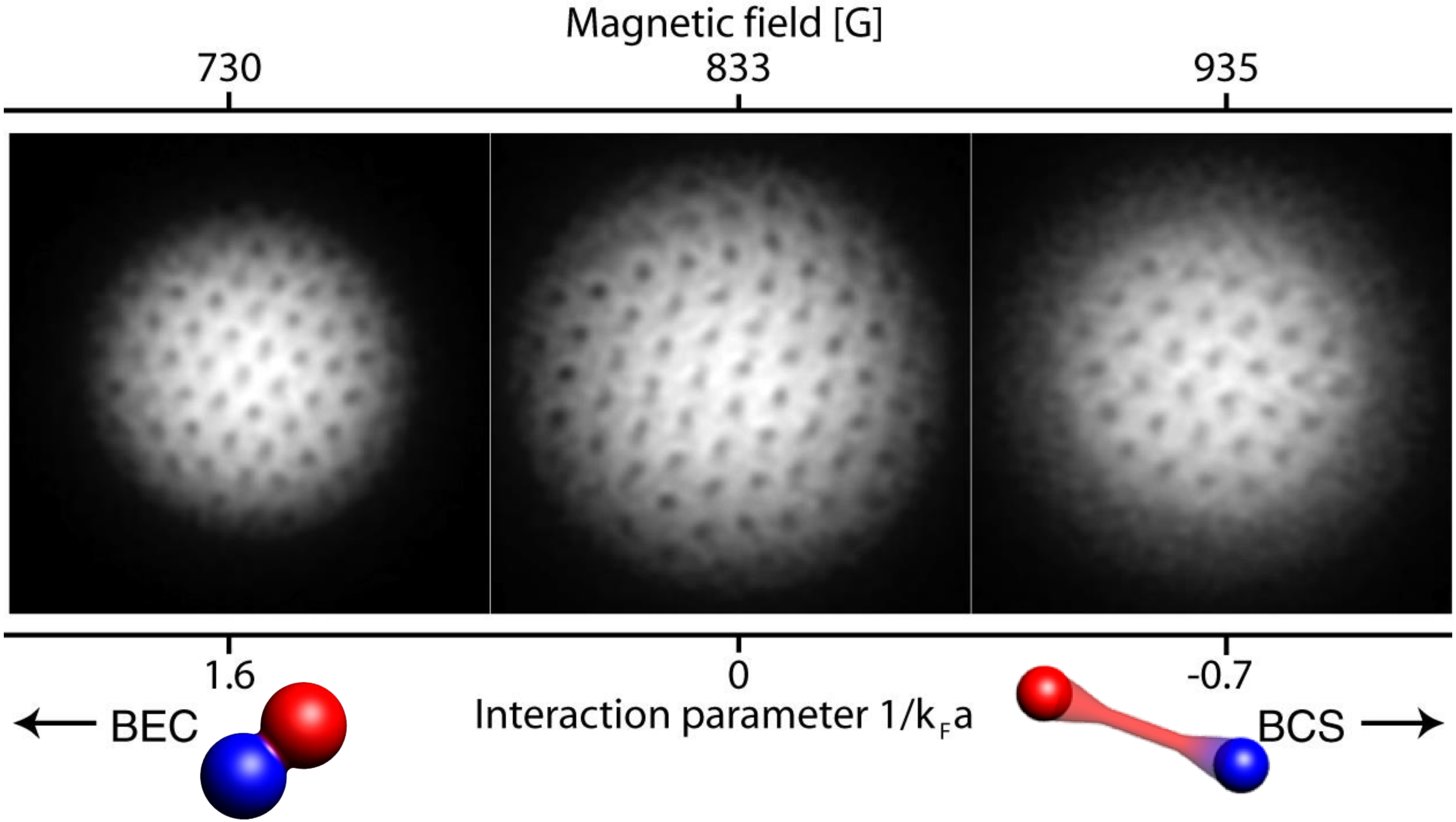
Fig. 1. "Perfect" triangular lattice of flux lines on the surface of a lead-4at% indium rod at 1.1°K. The black dots consist of small cobalt particles which have been stripped from the surface with a carbon replica.

Demonstration of superfluidity in a Fermi gas





Vortex lattices in the BEC-BCS crossover

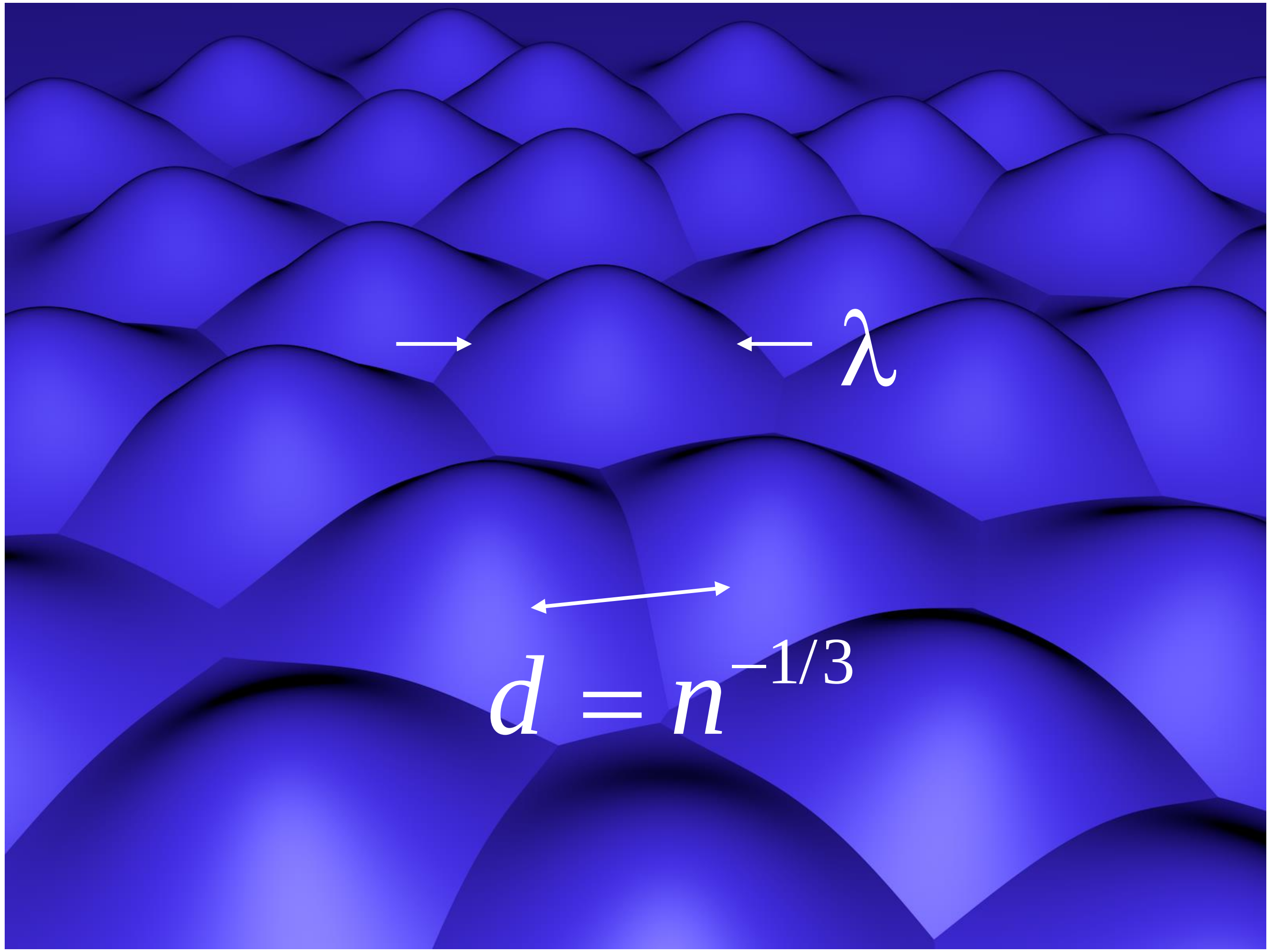


M.W. Zwierlein, J.R. Abo-Shaeer, A. Schirotzek, C.H. Schunck, W. Ketterle,
Nature 435, 1047-1051 (2005)

The Unitary Fermi Gas

Only two length scales:
Interparticle spacing $n^{-1/3}$
De Broglie wavelength λ_{dB}

Only two corresp. energy scales:
Fermi energy E_F
Temperature $k_B T$



The Unitary Fermi Gas

Only two length scales:
Interparticle spacing $n^{-1/3}$
De Broglie wavelength λ_{dB}

Only two corresp. energy scales:
Fermi energy E_F
Temperature $k_{\mathrm{B}}T$

→ Universal equation of state

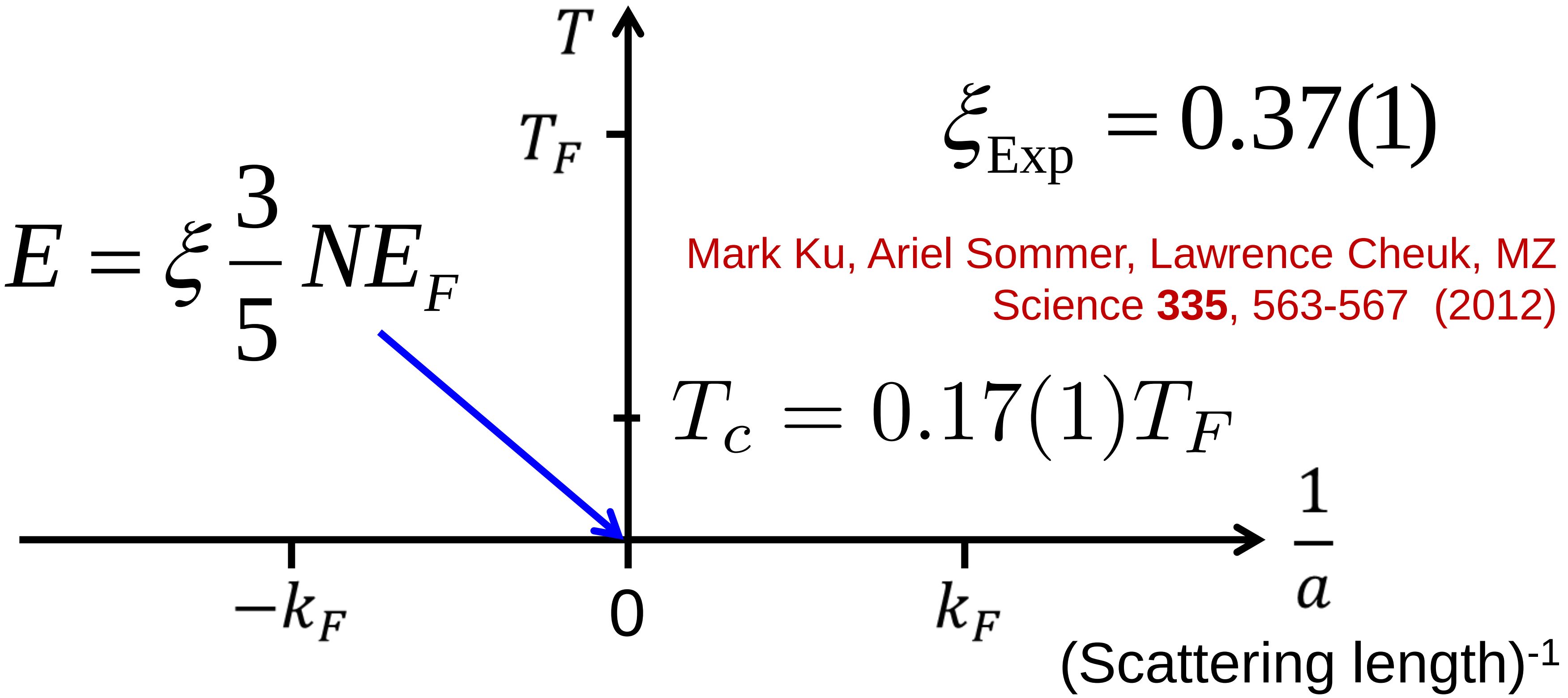
Duke/NC State, ENS, JILA, Innsbruck, Swinburne, MIT

$$\xi_{\mathrm{Exp}} = 0.37(1)$$

$$E = \xi \frac{3}{5} N E_F$$

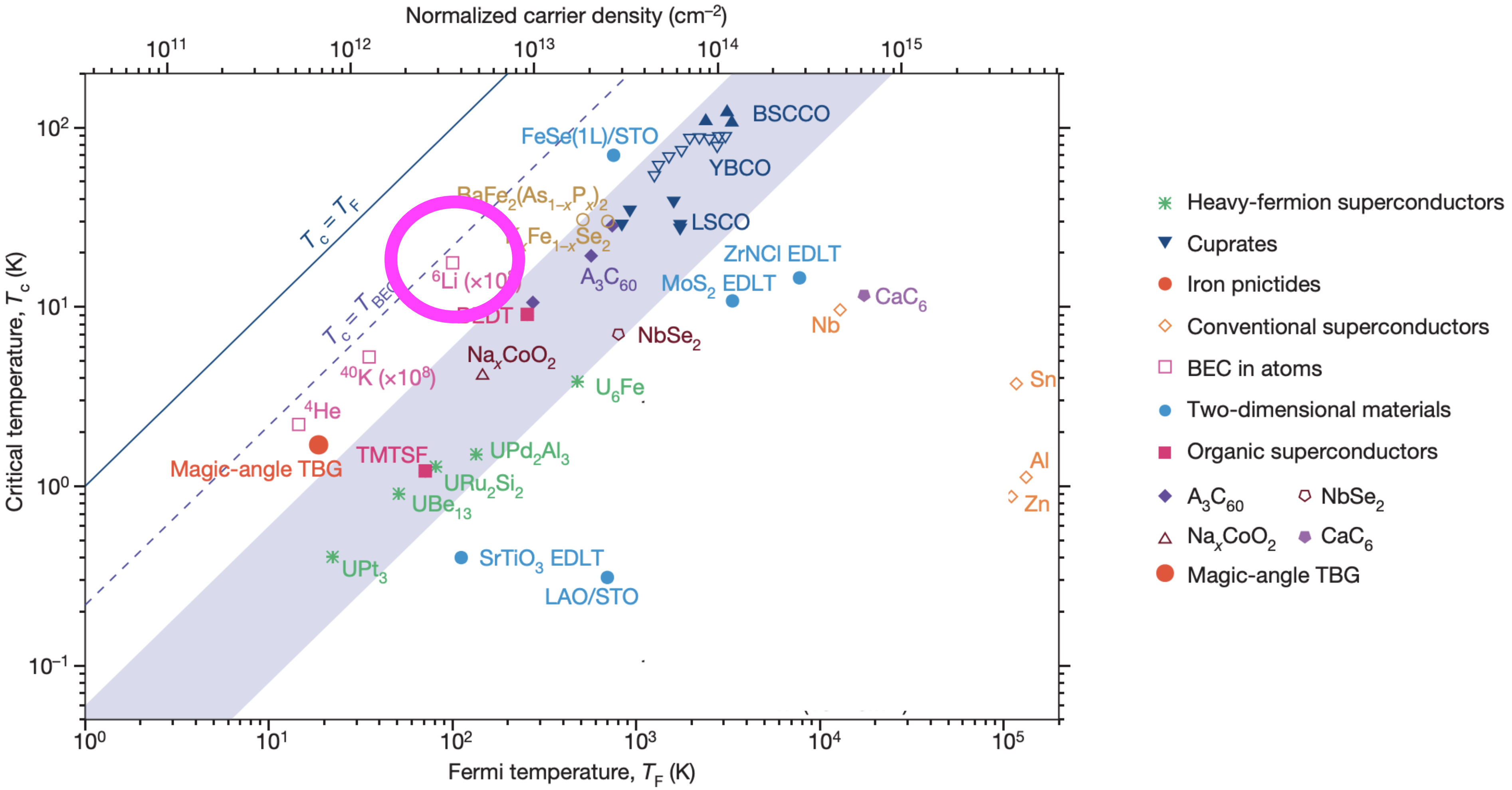
Mark Ku, Ariel Sommer, Lawrence Cheuk, MZ
Science **335**, 563-567 (2012)

$$T_c = 0.17(1) T_F$$



The Unitary Fermi Gas

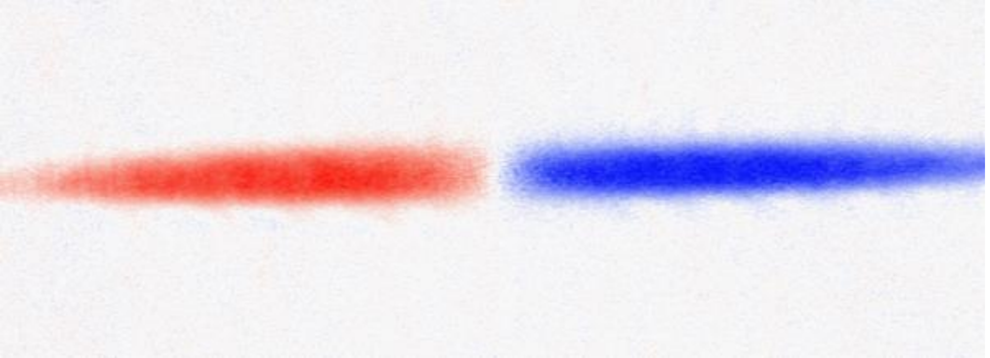
Comparison to other dilute Fermi systems



From Cao et al., Jarillo-Herrero, Nature 2018

Transport in Strongly Interacting Systems

e.g. Unitary Gas @ Feshbach Resonance



Mean free path ~ Interparticle spacing d

Average velocity $v \sim \frac{\hbar}{md}$

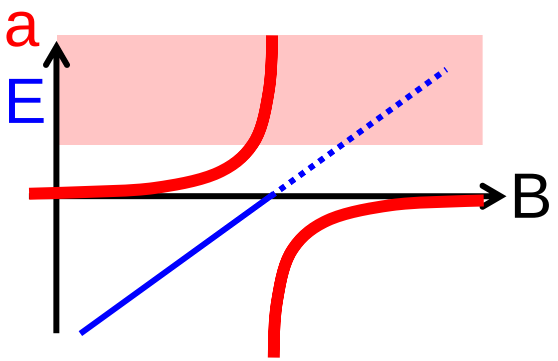
Quantum Limit of Diffusion (charge, spin, momentum, thermal)

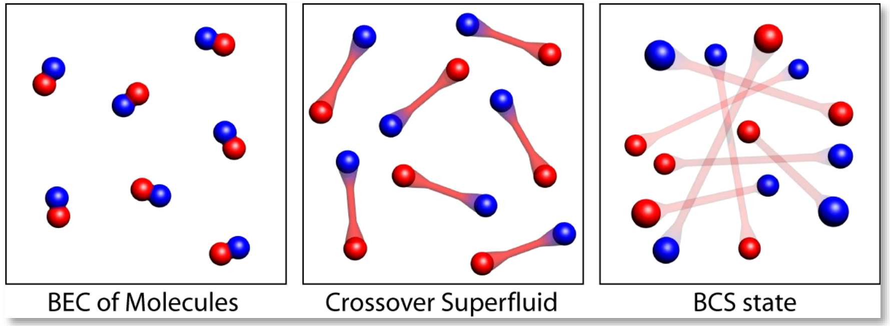
$$D \sim vl \sim \frac{\hbar}{m}$$

A.T. Sommer, M.J.H. Ku, G. Roati, M.W. Zwierlein, Nature 472, 201 (2011)

Sound in strongly interacting Fermi Gases

Feshbach Resonances → e.g. BEC-BCS Crossover





Quantum Gases → Quantum Fluids

Parth Patel

Zhenjie Yan

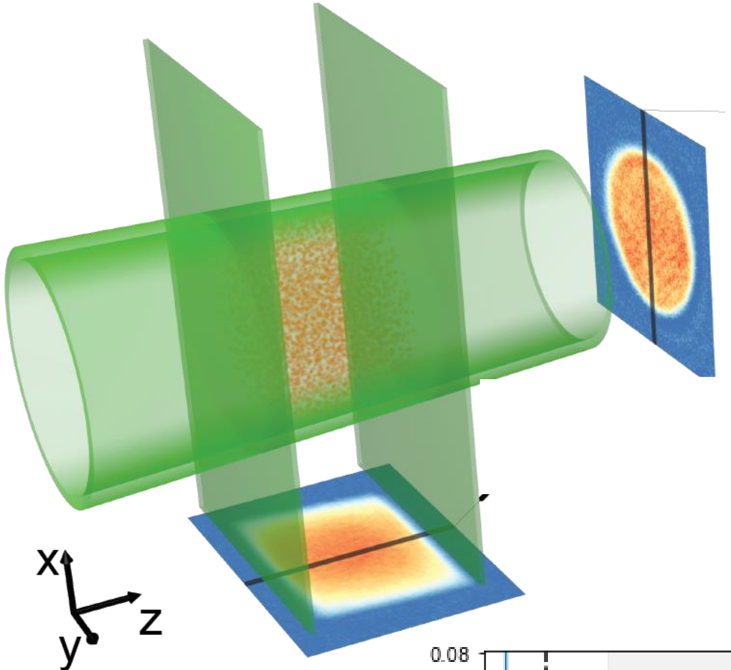
Biswaroop Mukherjee

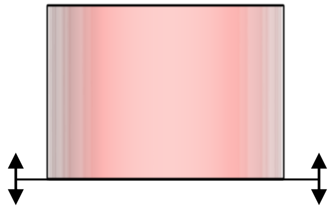
Dr. Richard Fletcher

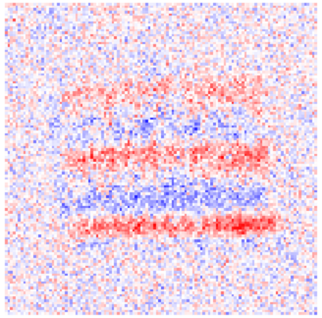
Dr. Julian Struck

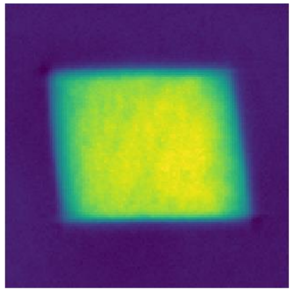
P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

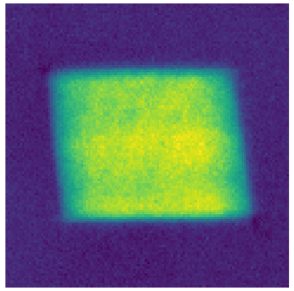
Creating Sound Waves in a Box

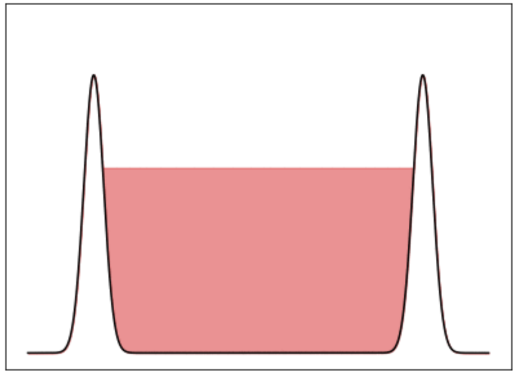


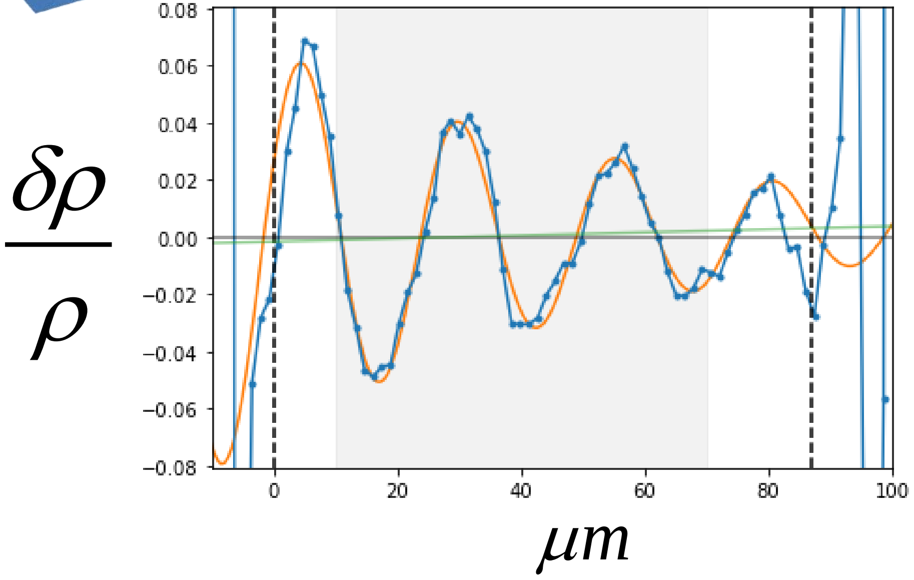






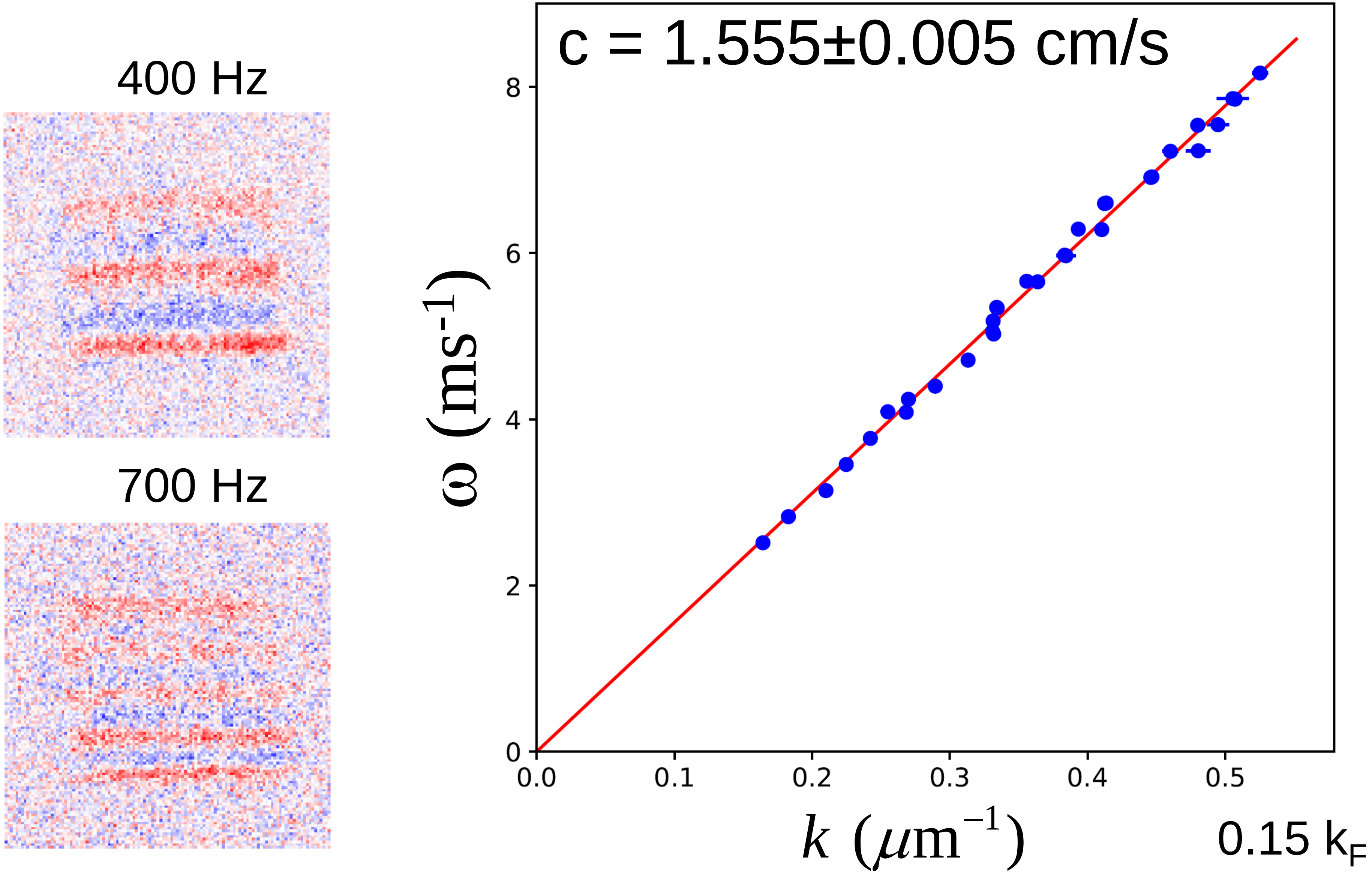






See also: 2D Bose gas, ENS (Dalibard),
3D Bose gas, Cambridge (Hadzibabic), Fermi gas, NCState (Thomas), 2D Fermi Gas, Hamburg (Moritz)

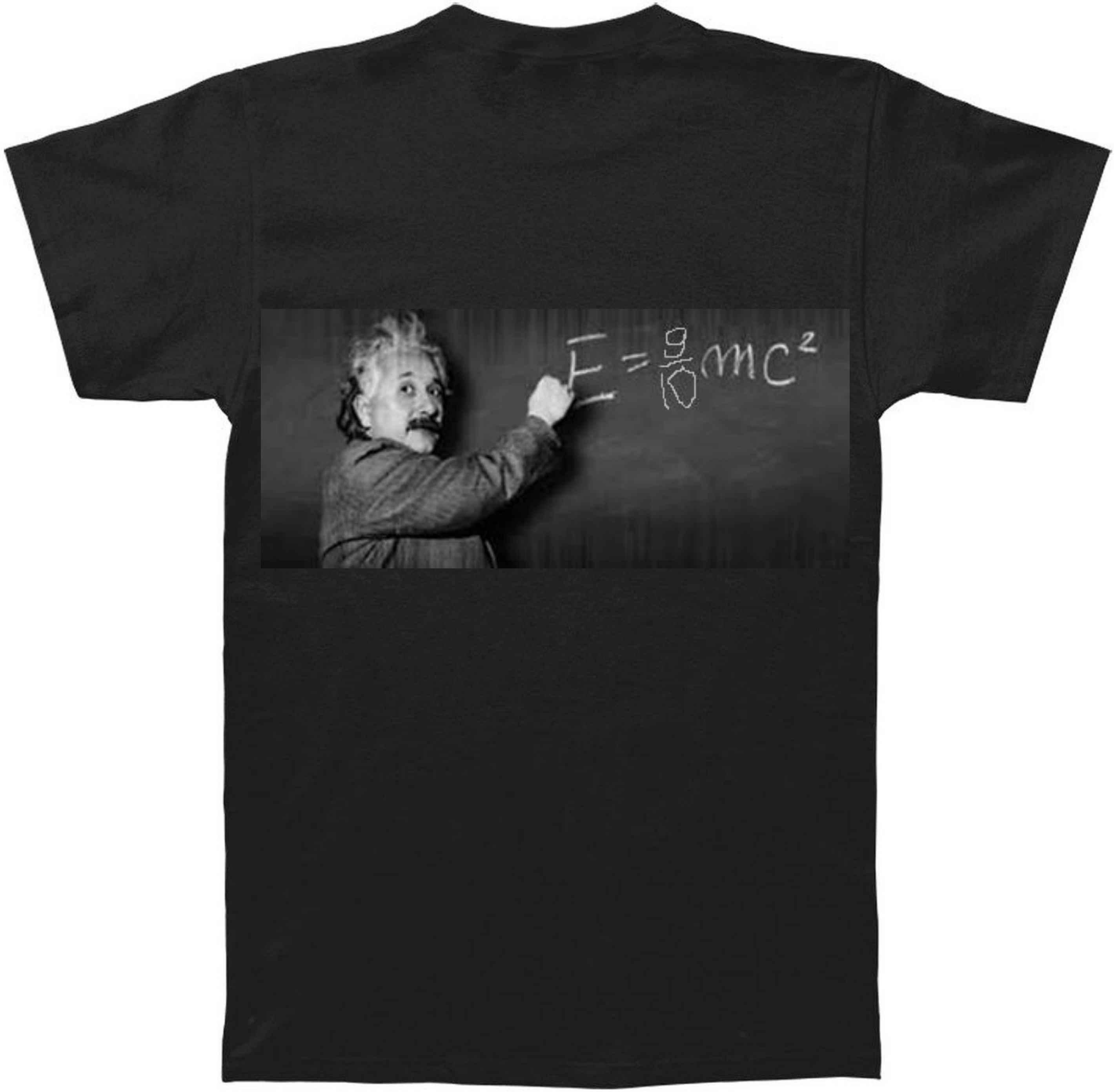
Dispersion relation



P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

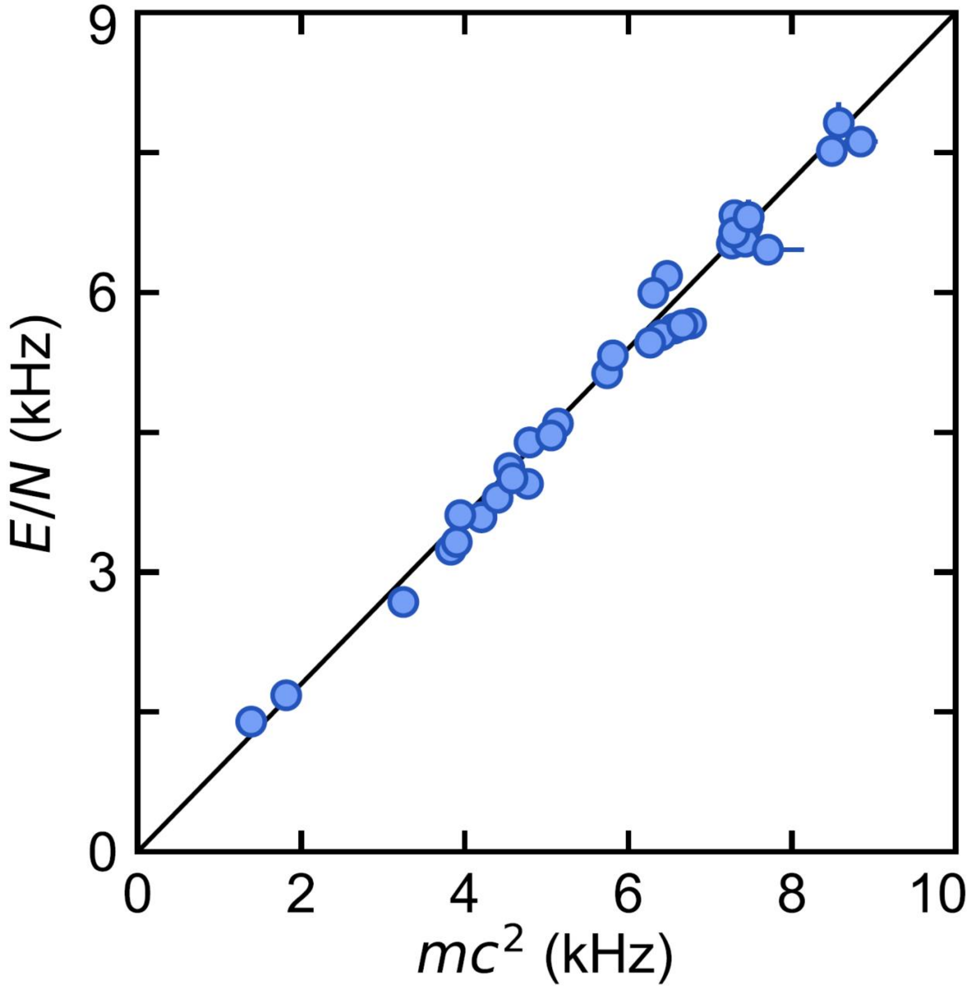
$$E = \frac{9}{10} m c^2$$

?



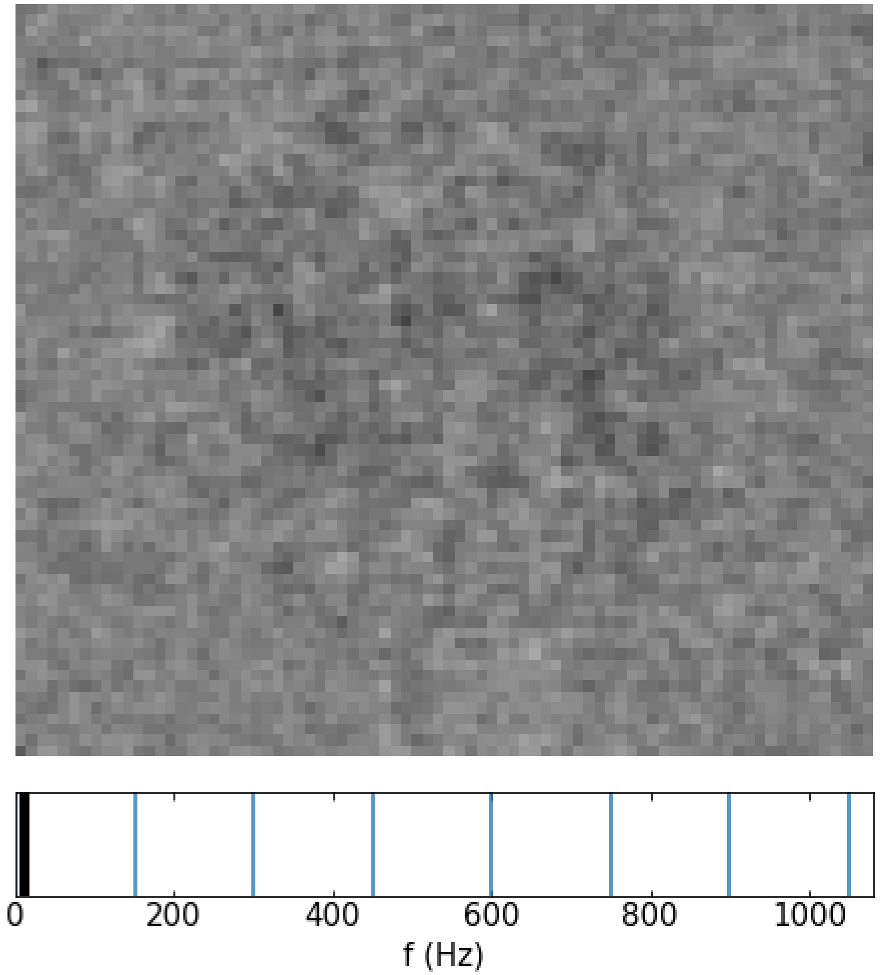
$E = \frac{9}{10} m c^2$

At unitarity: $$mc^2 = \left.\frac{\partial p}{\partial n}\right|_s = f(s) \left.\frac{\partial p_0}{\partial n}\right|_s = \frac{5}{3}\frac{p}{n} = \frac{10}{9}\frac{E}{N}$$



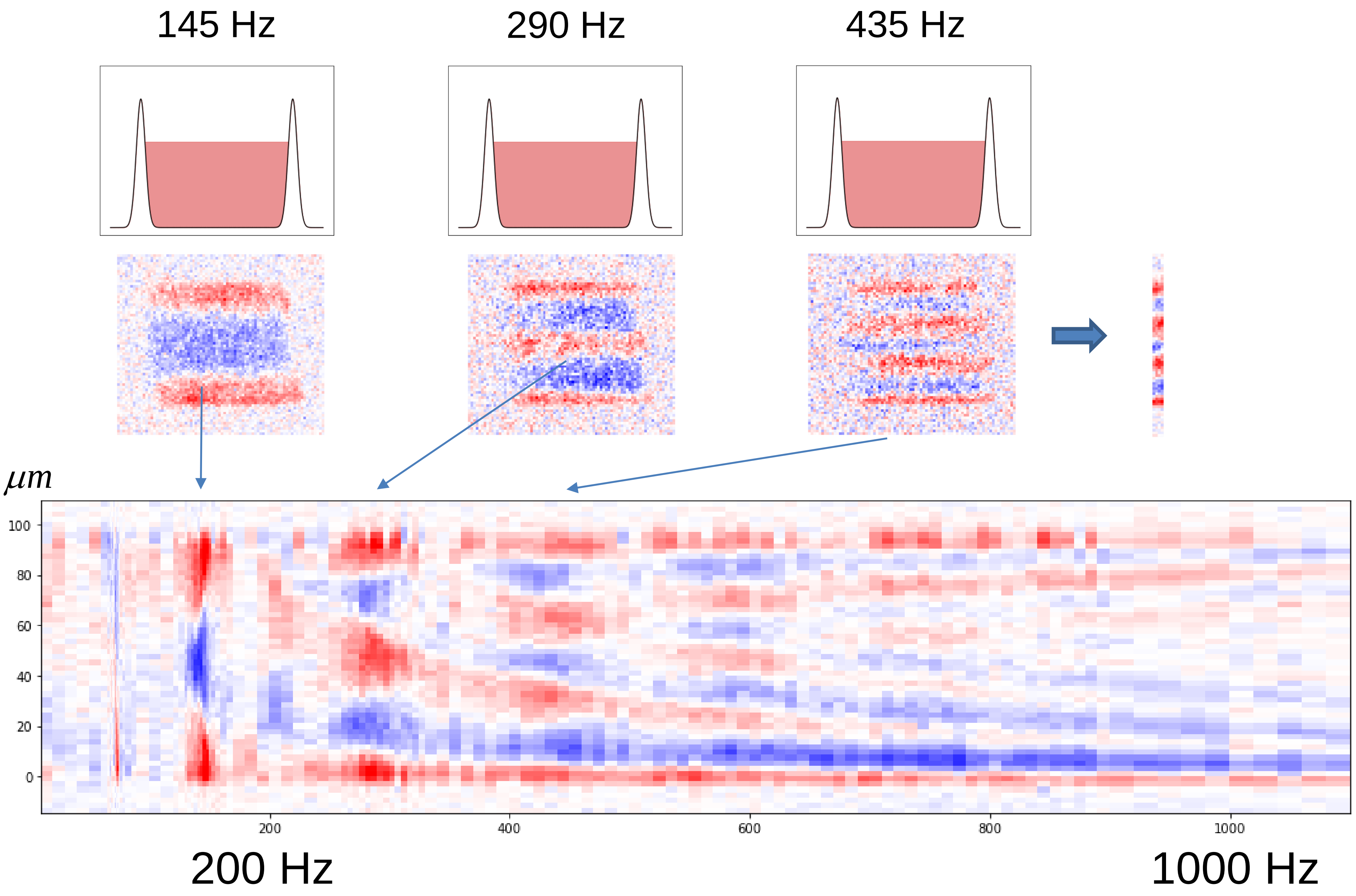
P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

Resonant Modes

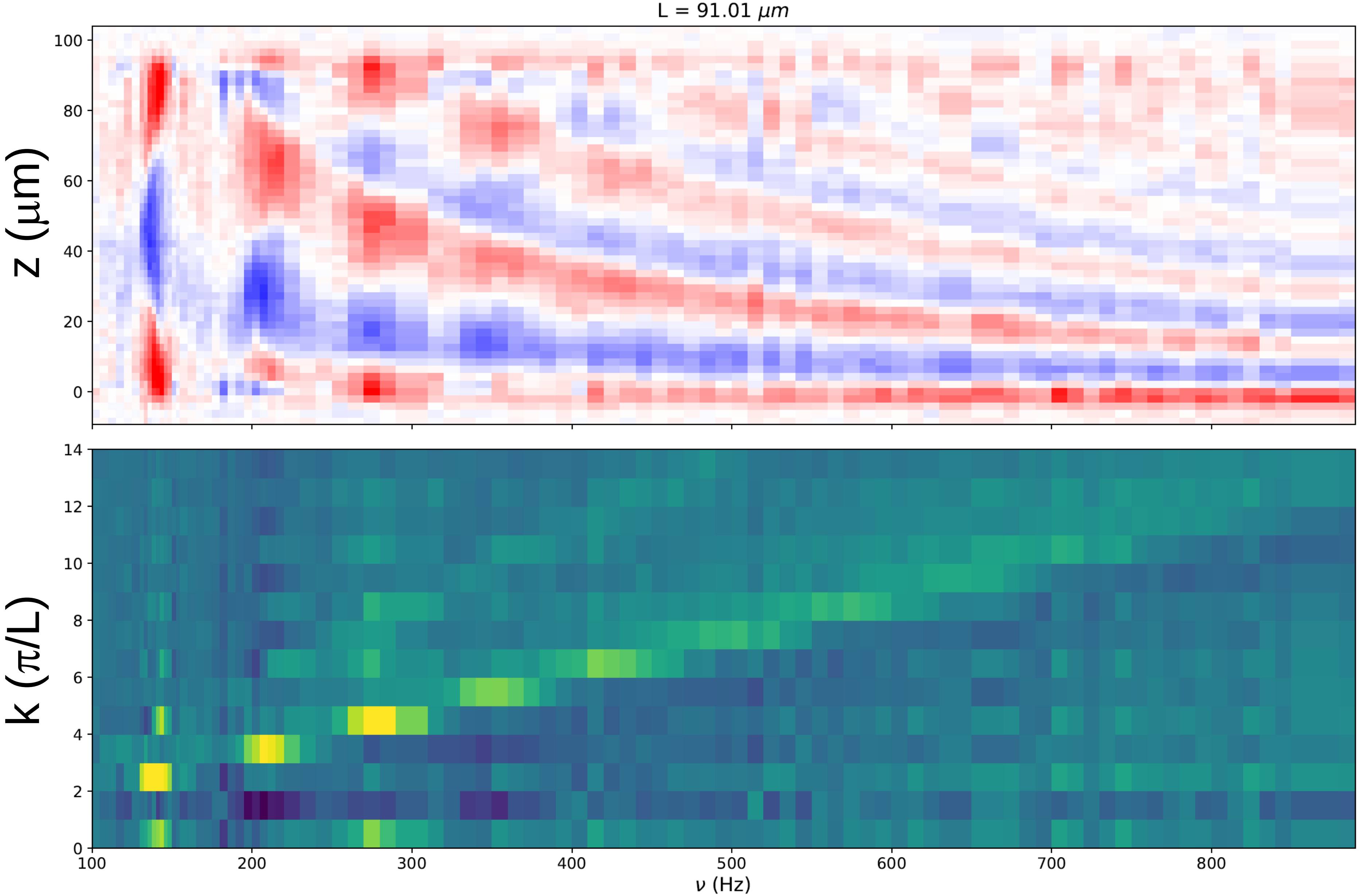


P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

Resonant Modes



One-sided shaking: Even and odd



P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

Unitary Hydrodynamics

Mass $$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial z} = 0$$ Continuity equation

Momentum $$\frac{\partial j}{\partial t} + \frac{\partial p}{\partial z} = \frac{4}{3}\eta \frac{\partial^2 v}{\partial^2 z}$$ Navier-Stokes equation

viscosity damps fluid

No bulk viscosity (ζ) due to scale invariance

Heat $$\frac{\partial s}{\partial t} + \frac{\partial (vs)}{\partial z} = \frac{\kappa}{T} \frac{\partial^2 T}{\partial^2 z}$$ thermal conduction increases entropy

Sound Diffusivity

due to viscosity & thermal conductivity

$$D = \frac{4}{3}\frac{\eta}{\rho} + \frac{4}{15}\frac{\kappa T}{P}$$

Dispersion relation

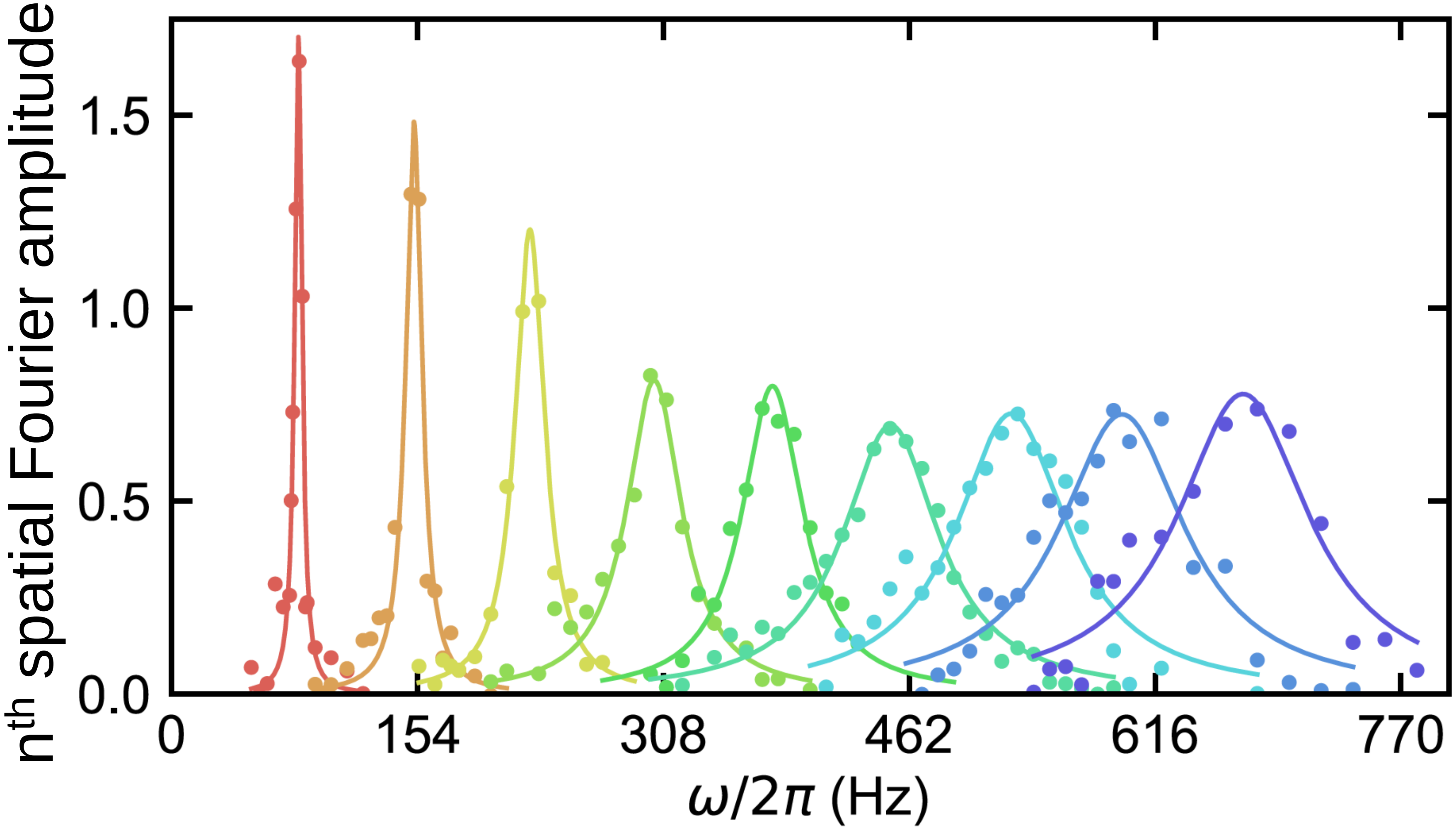
$$\omega^2 = c^2 k^2 + i\omega D k^2$$

Damping rate:

$$\Gamma = D k^2$$

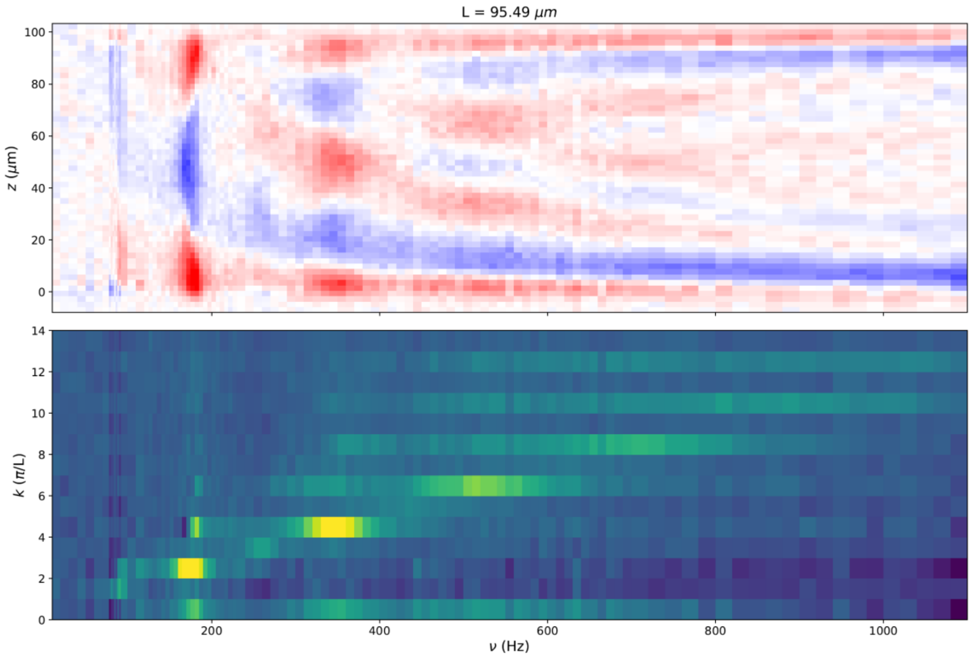
Sound resonances

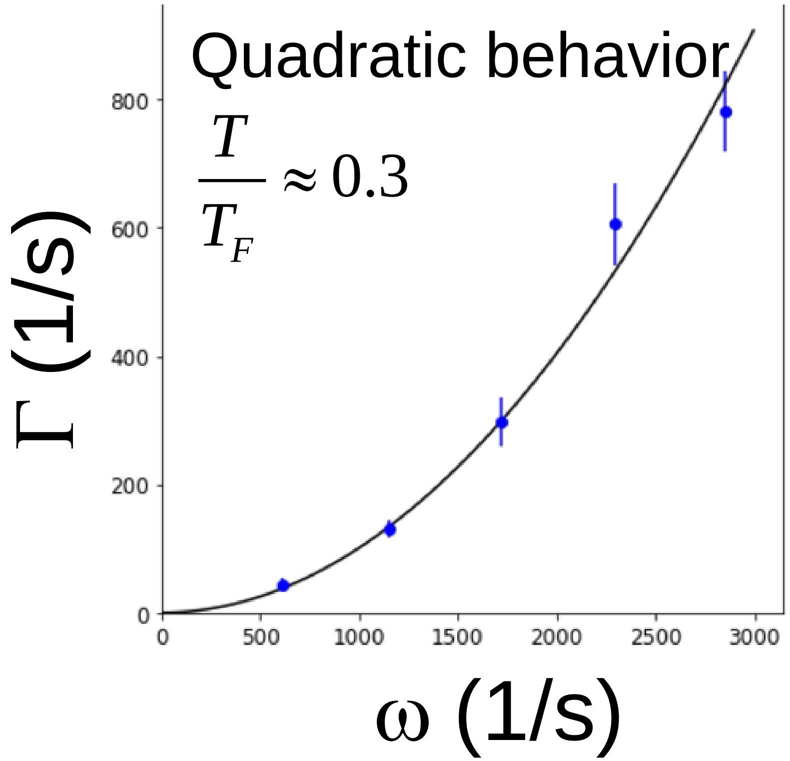
A direct measurement of the density response function



P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

Sound Diffusivity from Sonogram Peaks





$$D = 2.1\frac{\hbar}{m}$$

Quantum limited sound diffusion

Spin diffusion: Sommer et al., MIT 2011

Viscosity: Schaefer, Thomas (e.g. Science 2011)

Theory: Enss, Haussmann, Zwerger, Taylor, Randeria et al.

Viscosity of Superfluid Helium-4

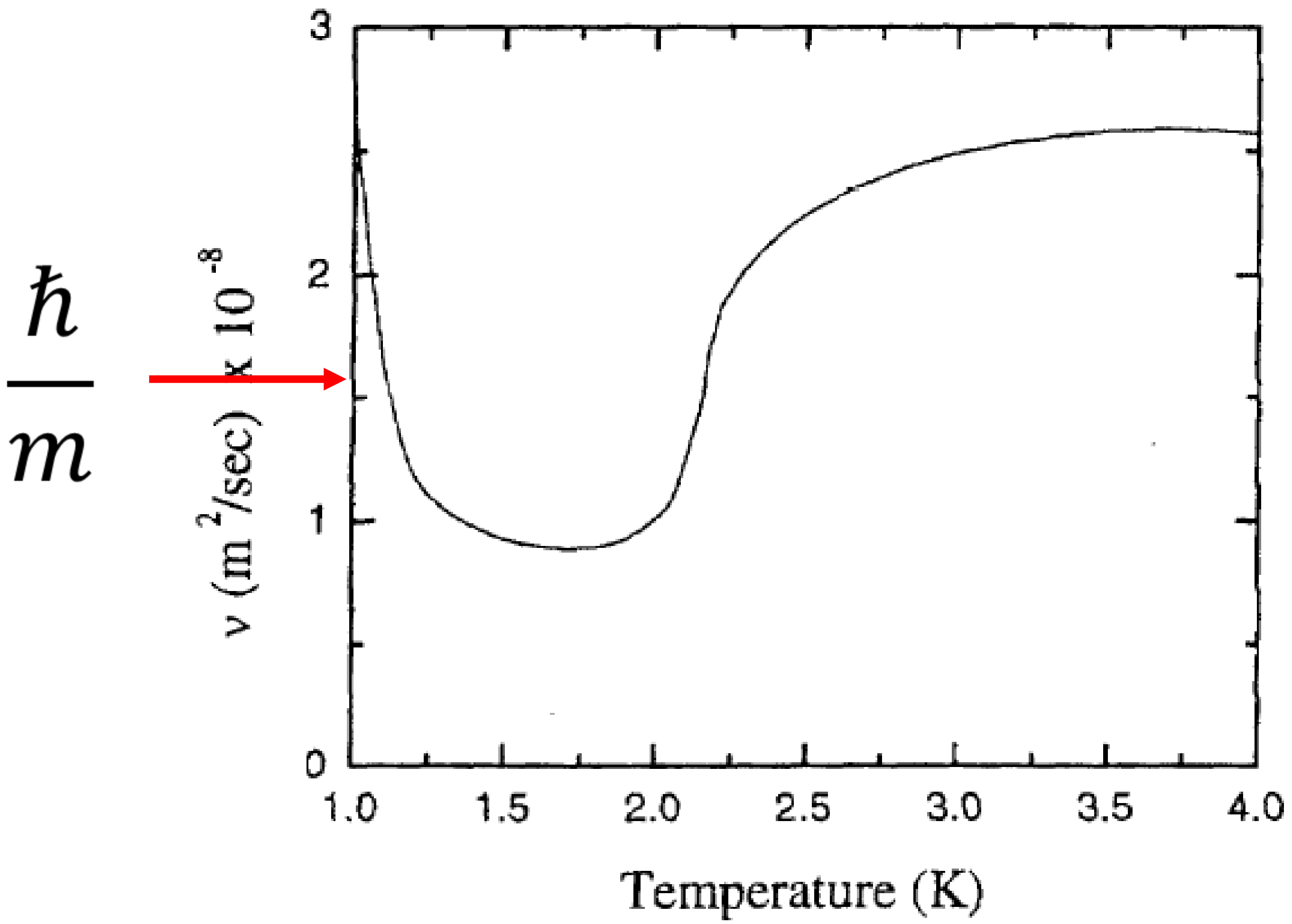
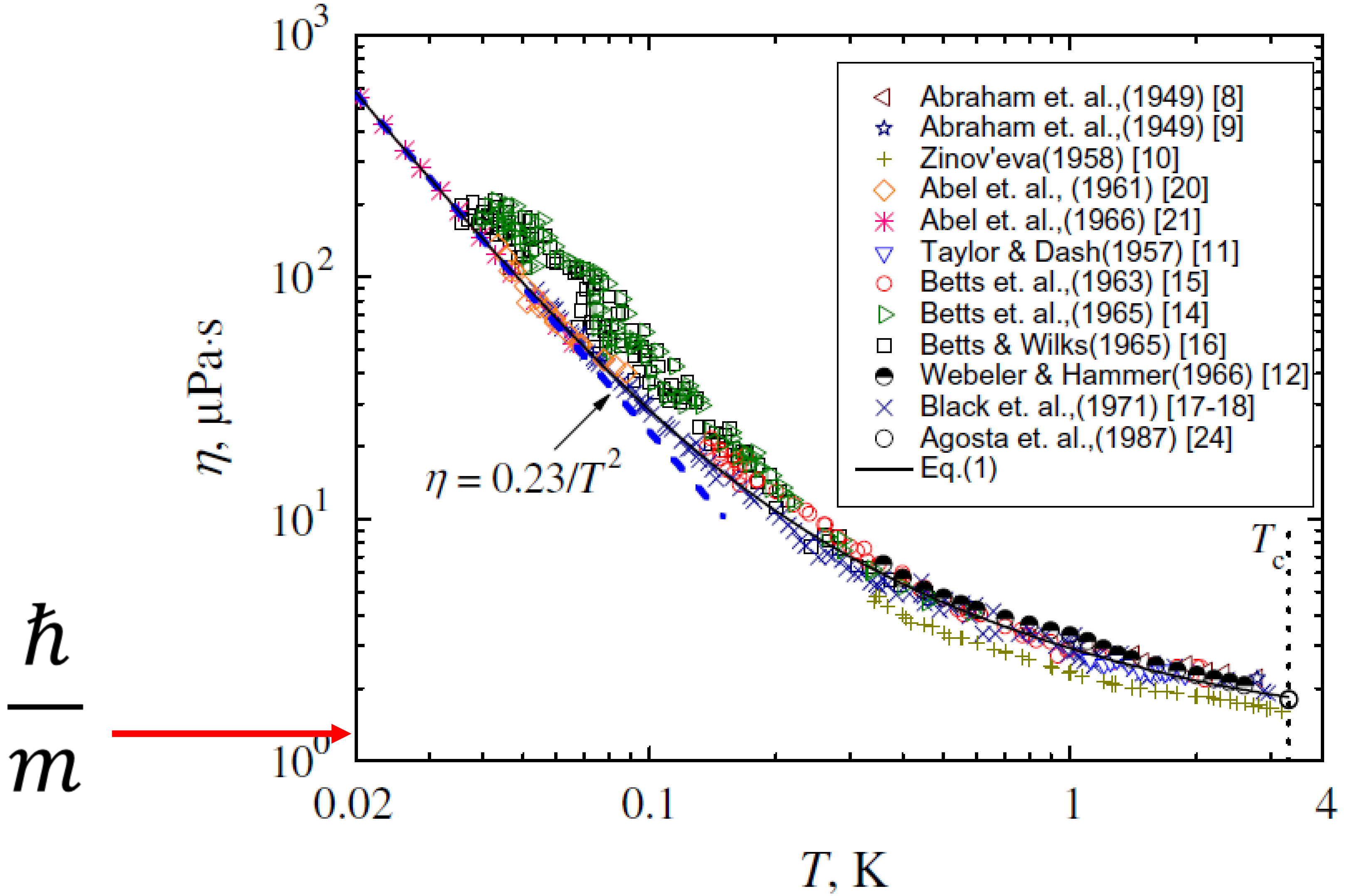


FIG. 12.3. The recommended values of the kinematic viscosity of liquid ^{4}He, $\nu = \eta/\rho$, as a function of temperature at the saturated vapor pressure.

From: Donnelly, Barenghi, JPC Reference Data, 1998

Viscosity of Superfluid Helium-3

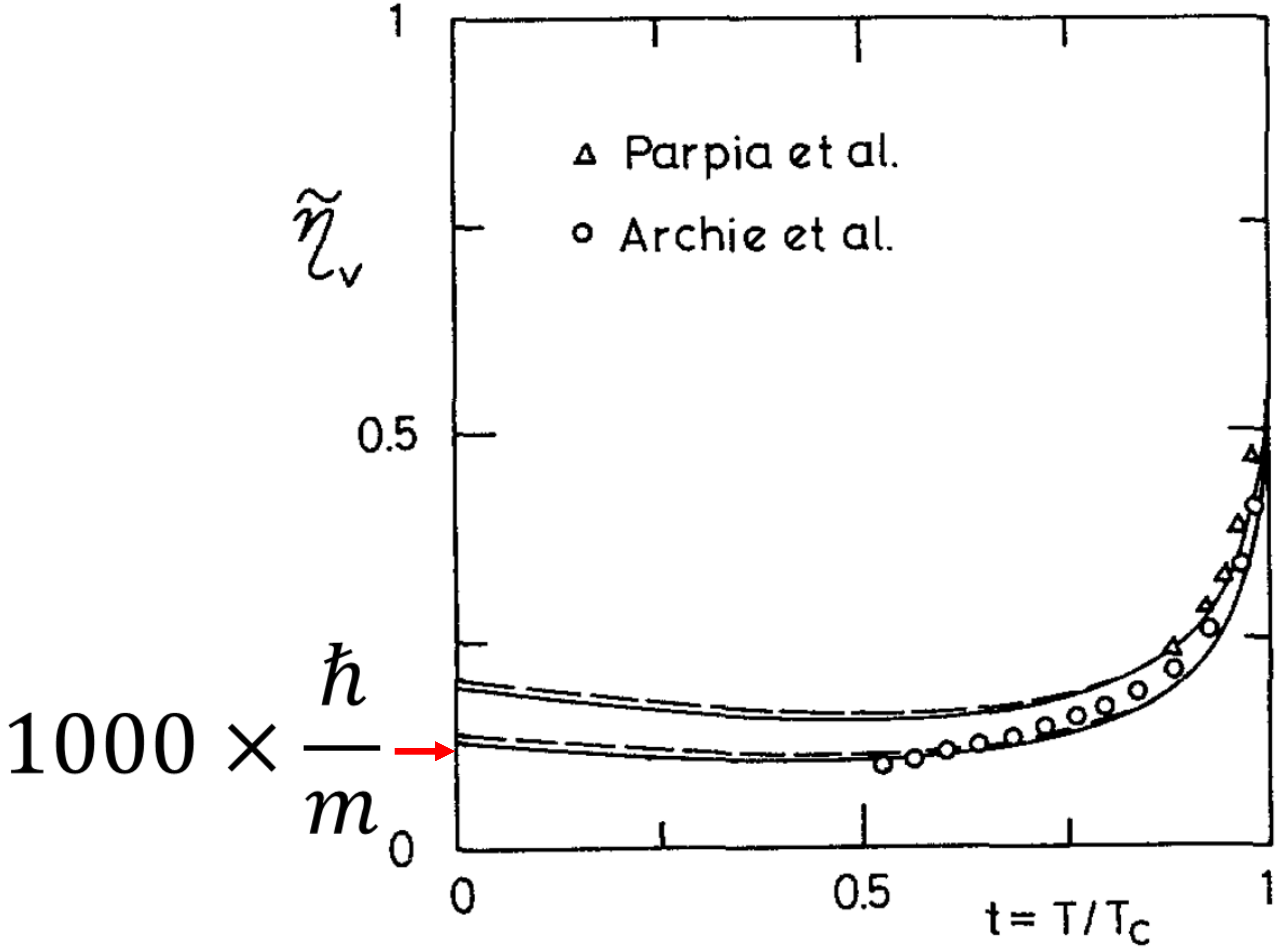
Helium-3 above T_c: Fermi Liquid behavior



Huang et al., Cryogenics 52, 538 (2012)

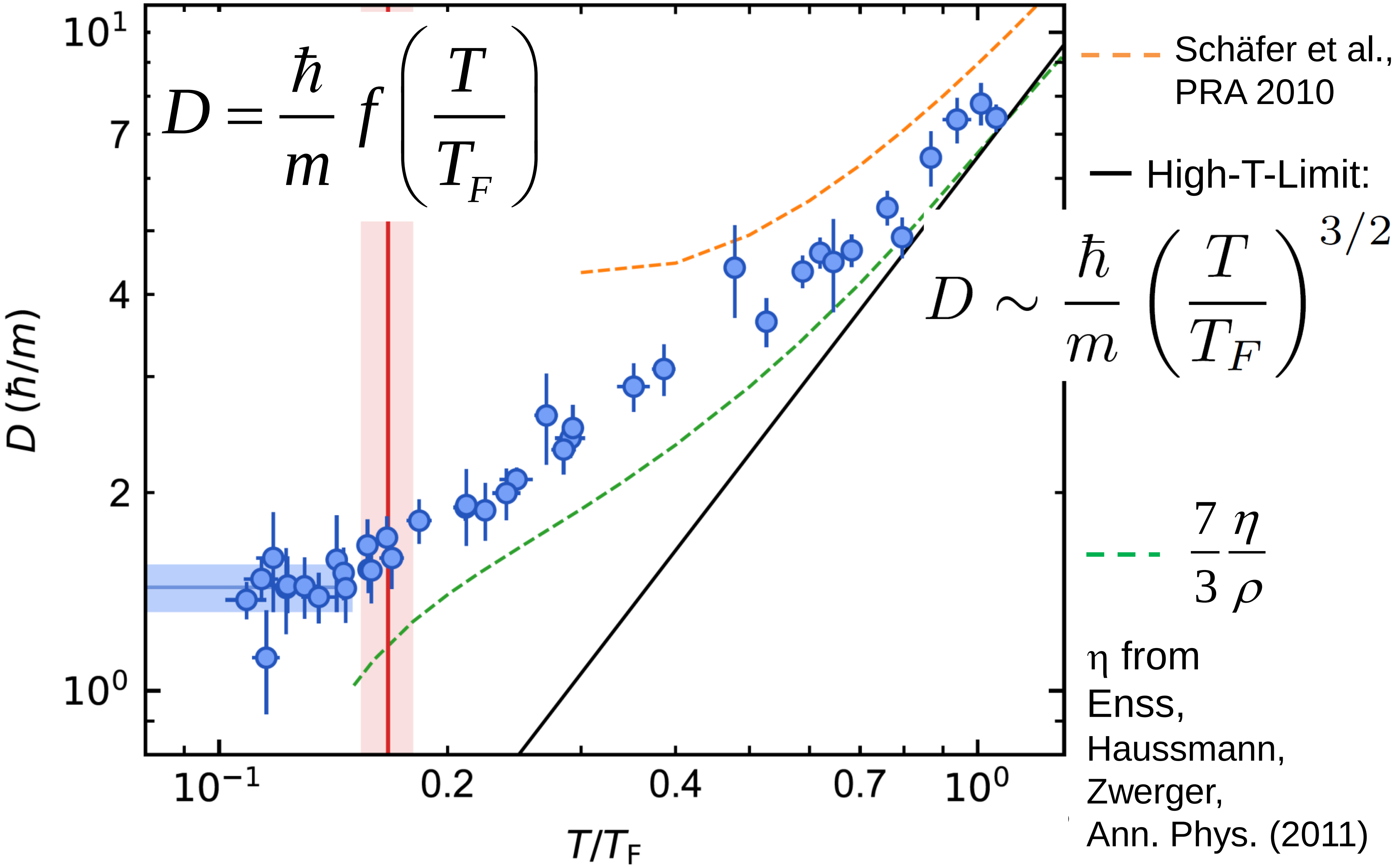
Viscosity of Superfluid Helium-3

Helium-3 below T_c: Sudden decrease, then constant



Ono et al.,
JLTP 48, 167
1982

Quantum Limited Sound Diffusion



P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science, 370, 1222-1226 (2020)

Comparison to state of the art theory

Quantum critical thermal transport in the unitary Fermi gas
Bernhard Frank, Wilhelm Zwerger, Tilman Enss
Phys. Rev. Research 2, 023301 (2020)

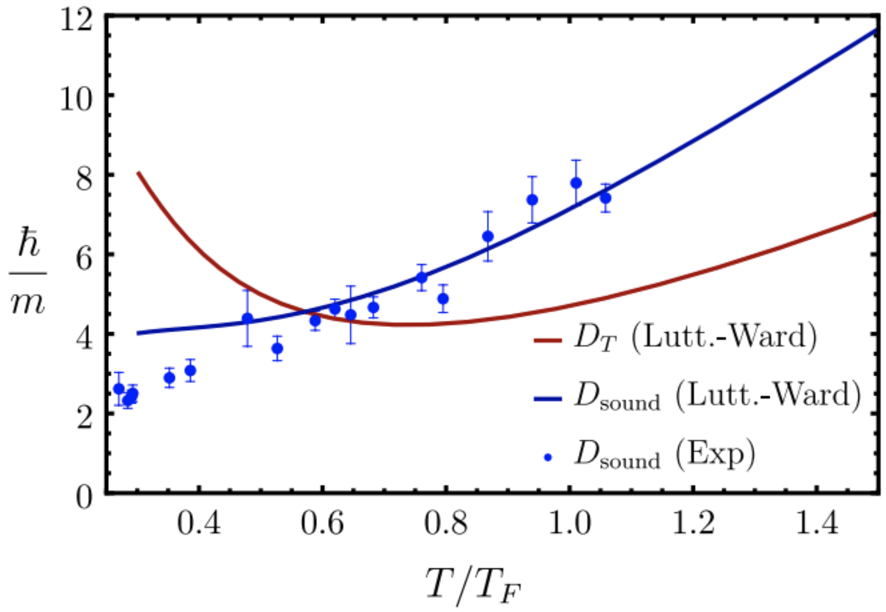


FIG. 1. Thermal diffusivity D_T (red) and sound diffusivity D_{sound} (blue) vs temperature T/T_F in the quantum critical regime of the unitary Fermi gas. Theoretical results from Luttinger-Ward calculations are shown in comparison with sound diffusion measurements [23].

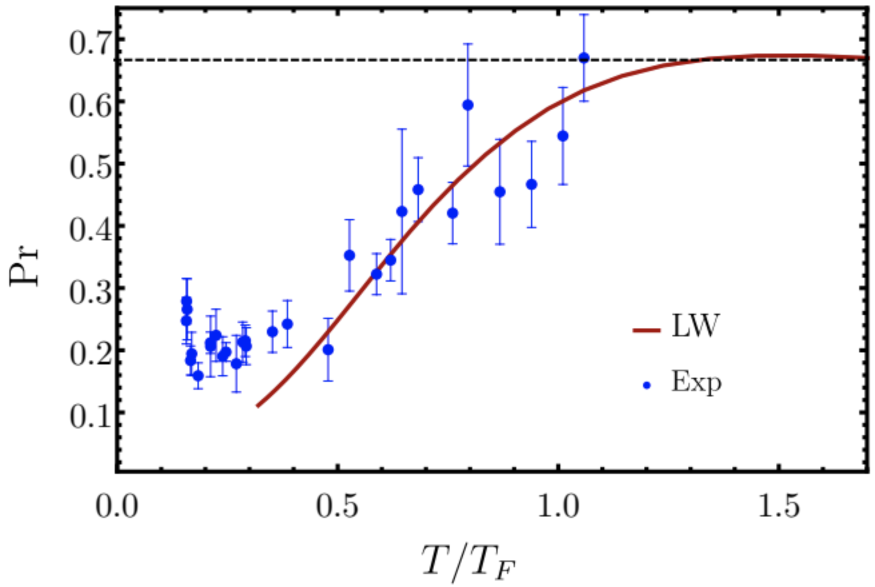


FIG. 8. Prandtl number $\mathrm{Pr} = D_\eta / D_T$ vs temperature T/T_F from Luttinger-Ward calculations (red line) and sound attenuation measurements [23]; the dashed line marks the high-temperature limit $\mathrm{Pr} = 2/3$.

Unitary Two-Fluid Hydrodynamics

For 1D flow, linearized:

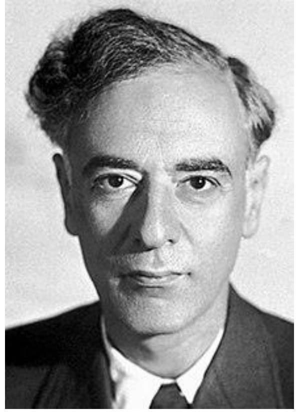
Laszlo Tisza

$$\frac{\partial \rho}{\partial t} + \frac{\partial j}{\partial z} = 0$$

Continuity equation

$$\frac{\partial j}{\partial t} + \frac{\partial p}{\partial z} = \frac{4}{3}\eta \frac{\partial^2 v_n}{\partial^2 z}$$

Navier-Stokes eq.
viscosity
damps normal fluid



Lev Landau

$$\frac{\partial s}{\partial t} + \frac{\partial (v_n s)}{\partial z} = \frac{\kappa}{T}\frac{\partial^2 T}{\partial^2 z}$$

Entropy moving
with normal fluid,
thermal conduction
increases entropy

Superfluid flow:

$$\frac{\partial v_s}{\partial t} + \frac{\partial \mu / m}{\partial z} = \rho_s \zeta_3 \frac{\partial^2 (v_s - v_n)}{\partial^2 z}$$

bulk viscosity
normal-superfluid
interconversion

+Thermodynamics relating p, ρ, T, s, μ

No other transport coefficients ($\zeta_1, \zeta_2, \zeta_4$)
due to scale invariance (Son, Stringari&Pitaevskii)

First and Second Sound

Two fluids → Two sound modes

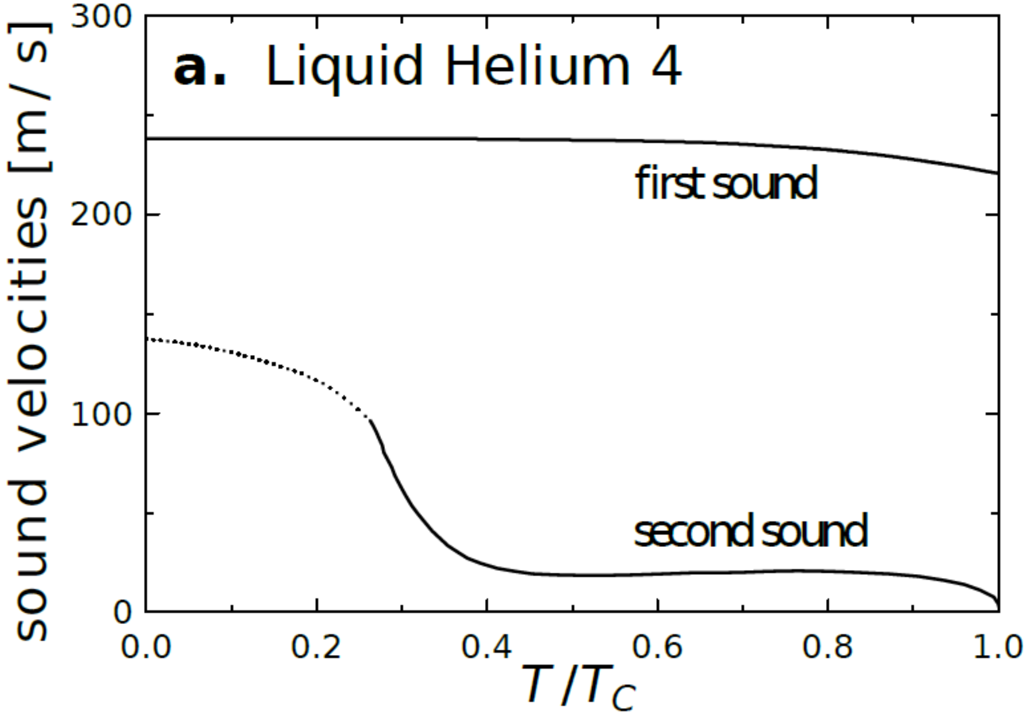
Review: Stringari&Pitaevskii
arXiv:1510.01306

For small expansivity: Density and Heat

$$c_{10}^2 = \left.\frac{\partial p}{\partial \rho}\right|_{S,N} = \frac{1}{\rho \kappa_s}$$

First sound, a density wave
normal and superfluid
oscillate in phase

$$c_{20}^2 = \frac{\rho_s}{\rho_n} \left.\frac{\partial T}{\partial \sigma}\right|_{\rho} \frac{\sigma^2}{\gamma} = \frac{\rho_s}{\rho_n} \frac{\sigma^2 T}{c_p}$$



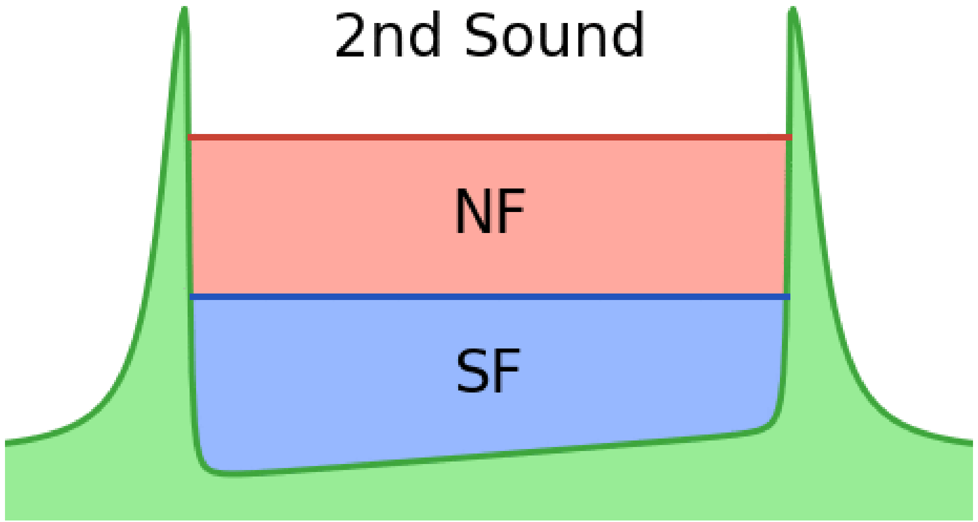
Second sound, a heat wave,
normal and superfluid oscillate out of phase

Second sound in a box

(Isentropic) expansivity at unitarity $-\frac{1}{\rho}\frac{\partial\rho}{\partial T}\Big|_S = -\frac{3}{2}\frac{1}{T}$ always non-zero

→ Can drive temperature wave using potential acting on density

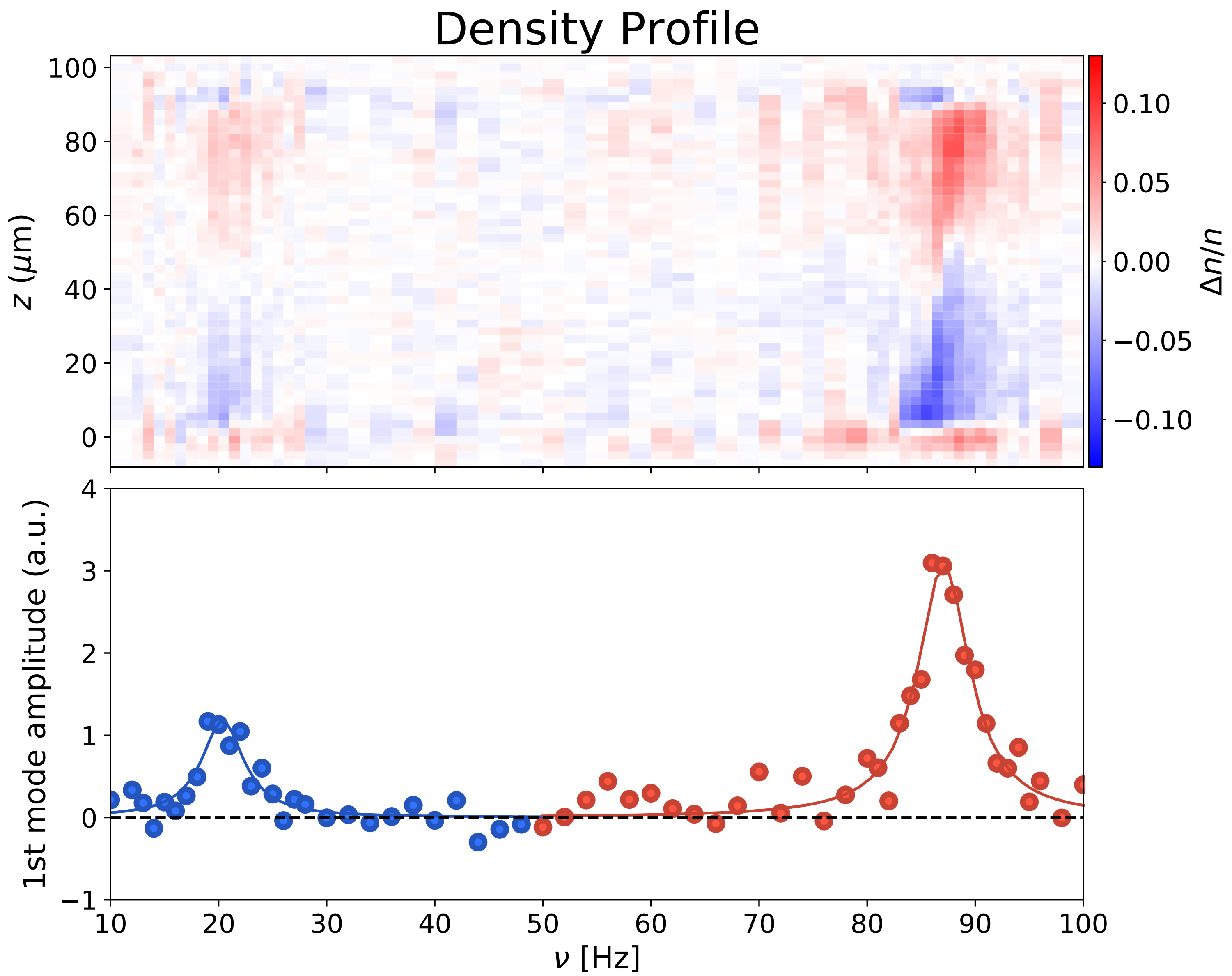
Apply oscillating gradient to the box



Observation of Second Sound in Quasi-1D geometry:
Sidorenkov et al., Grimm, Stringari&Piatevskii, Nature 2013

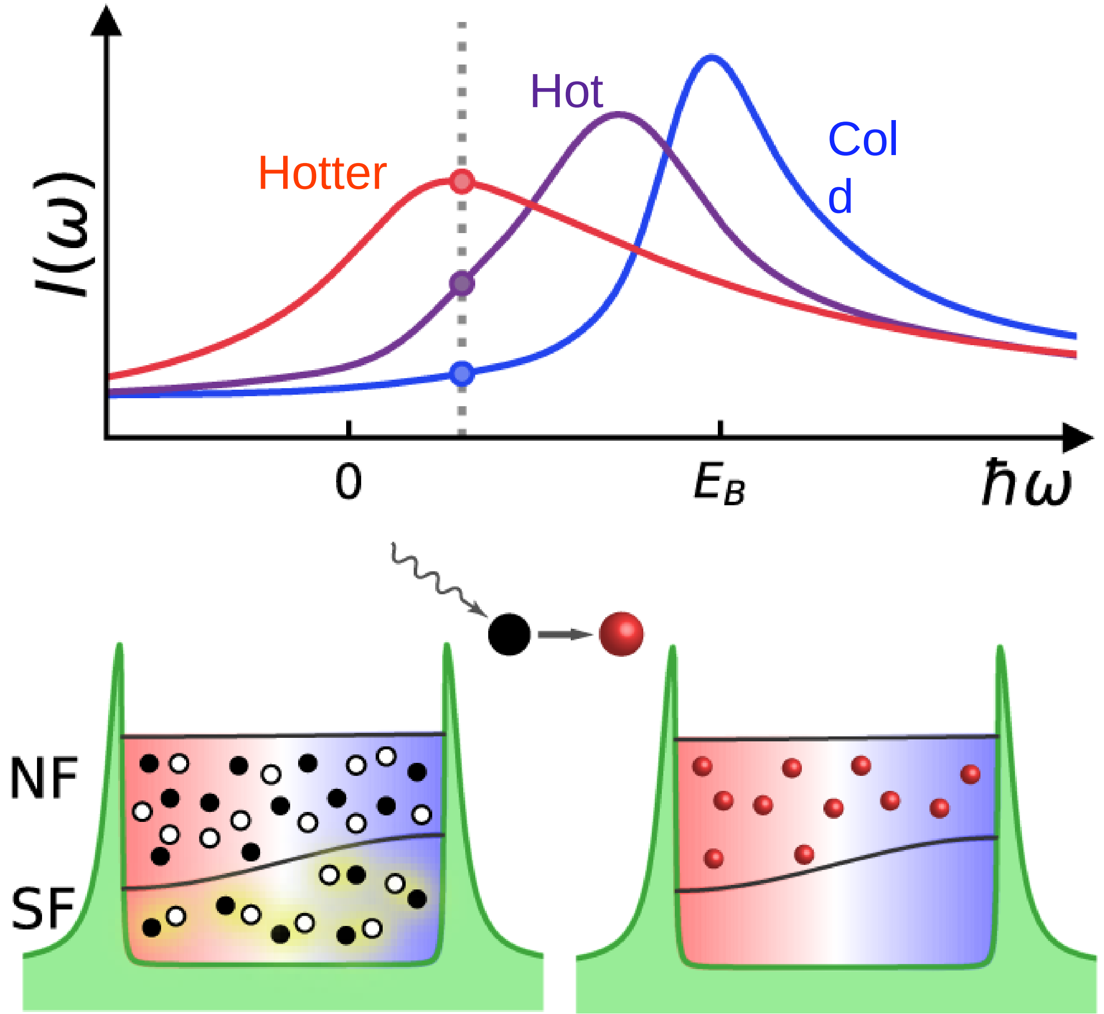
On bosons: weakly interacting situation: JILA (Debbie Jin 1996), MIT
Hydrodynamic gas: van der Straten

Second Sound seen in Density

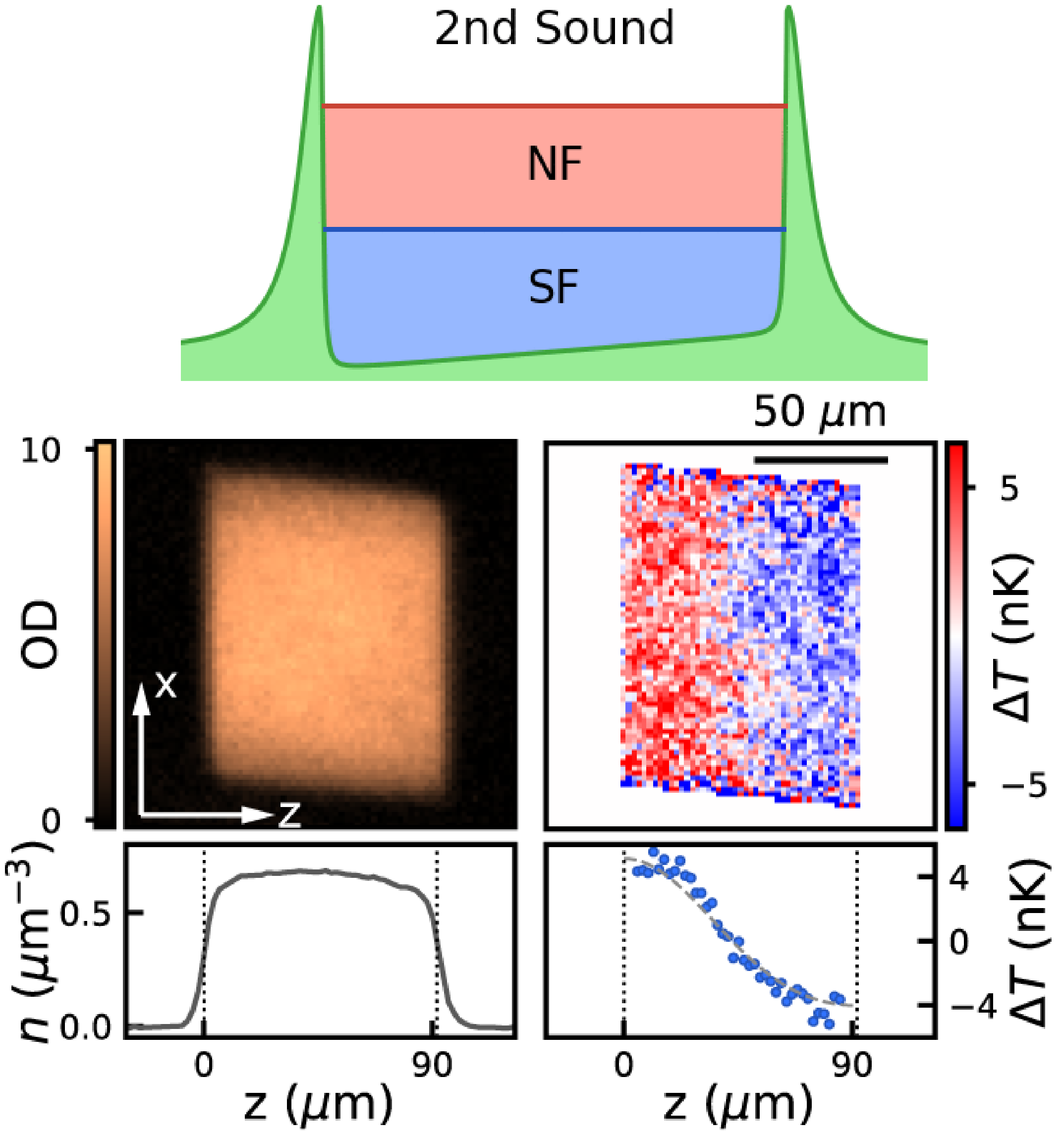


A local thermometer for heat transport

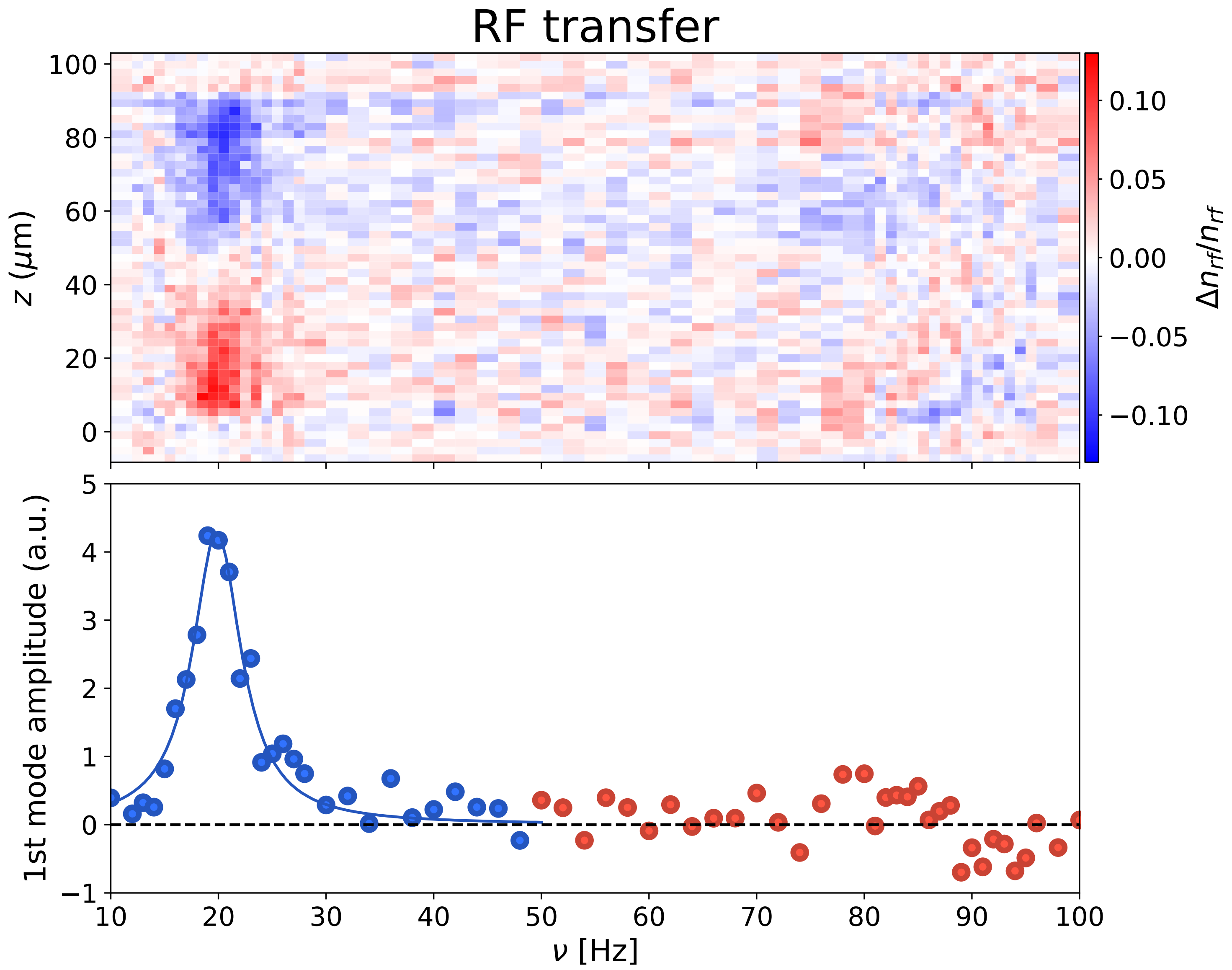
RF absorption spectra



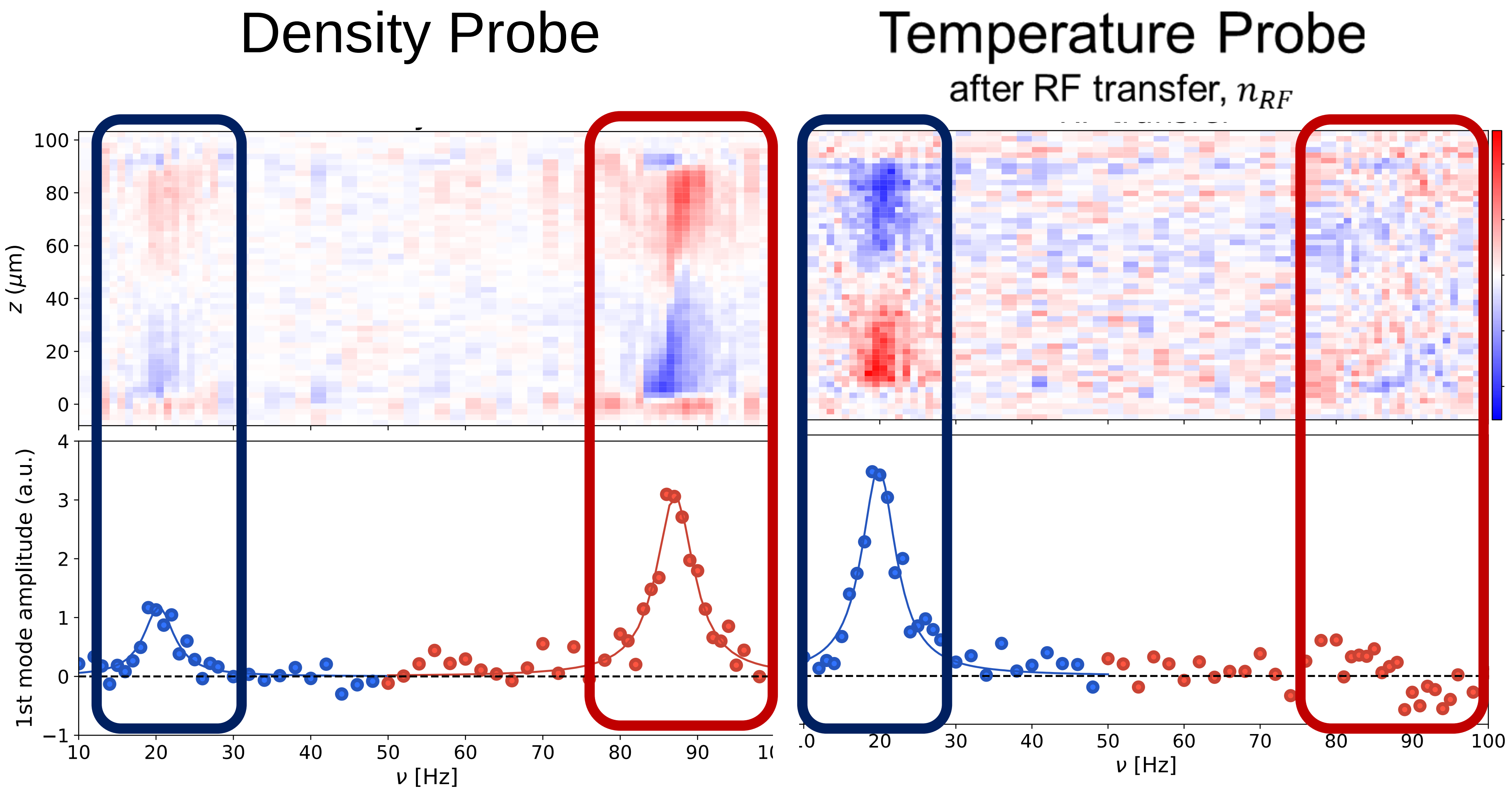
2nd Sound Creation



Second Sound with local thermometer

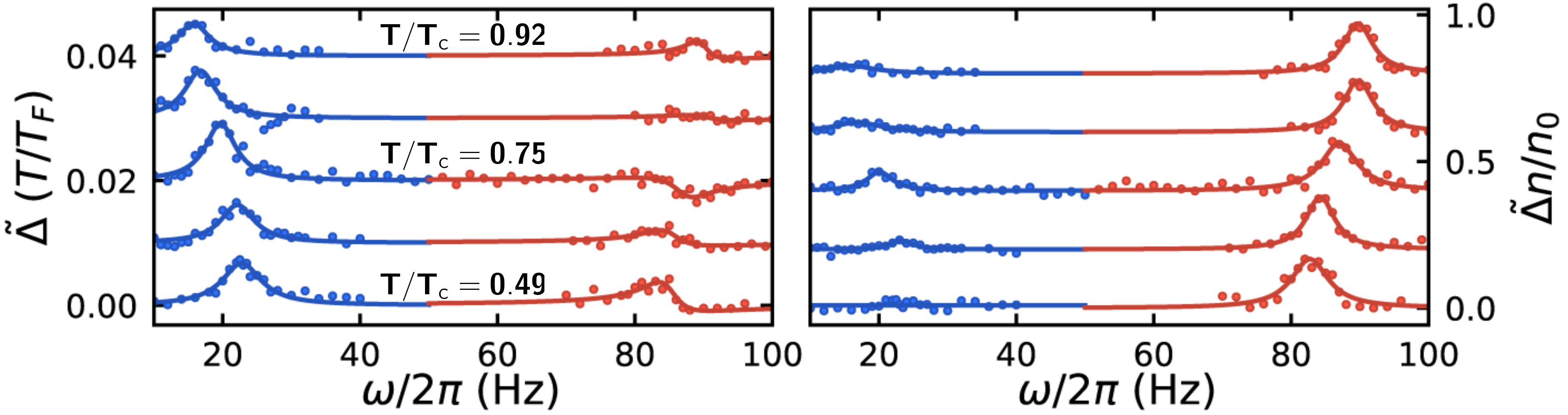


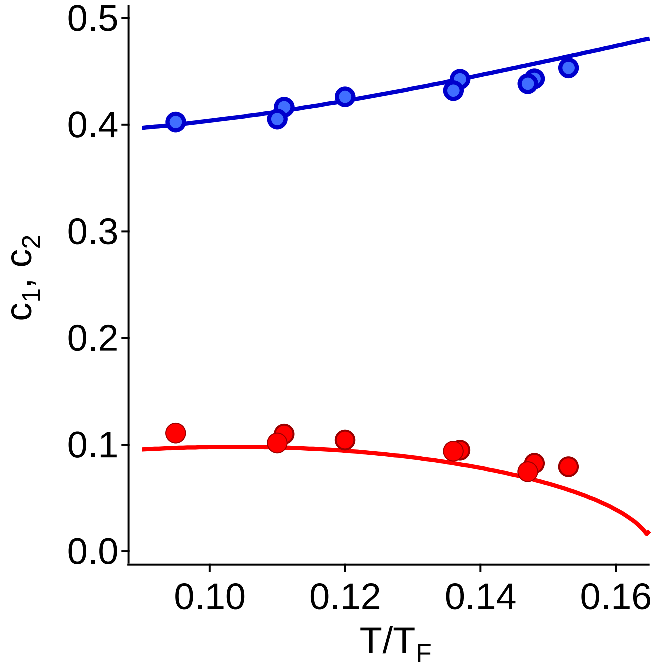
First and Second Sound



→ Speeds and Decay rates of First and Second sound

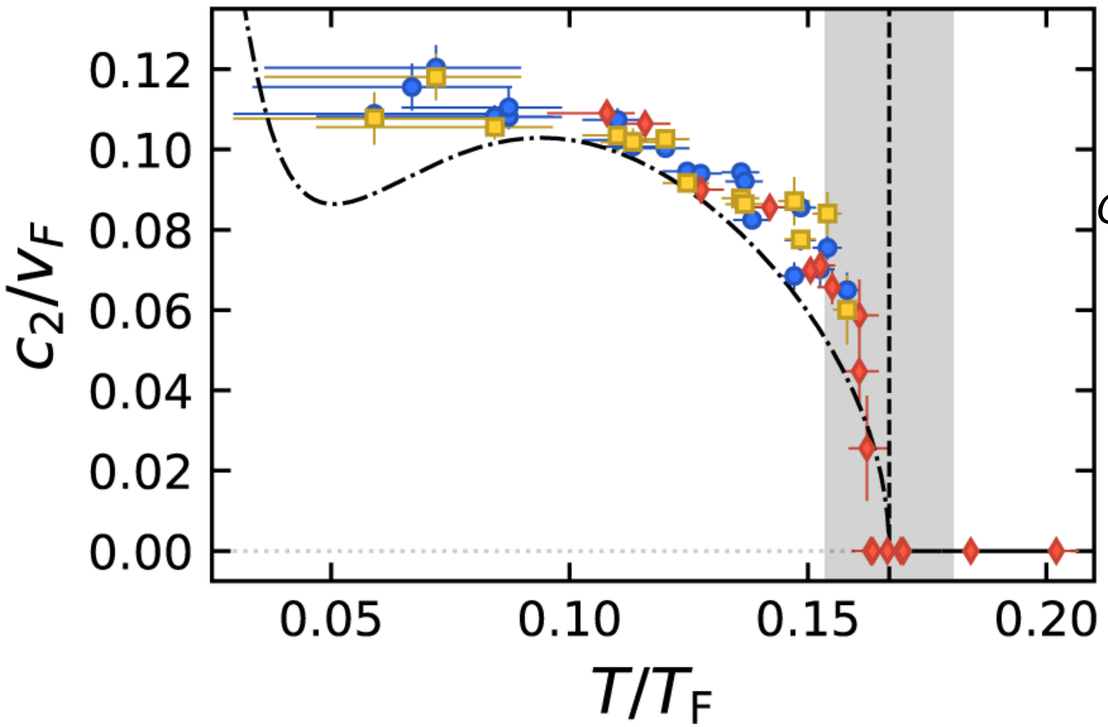
Speed of first and second sound



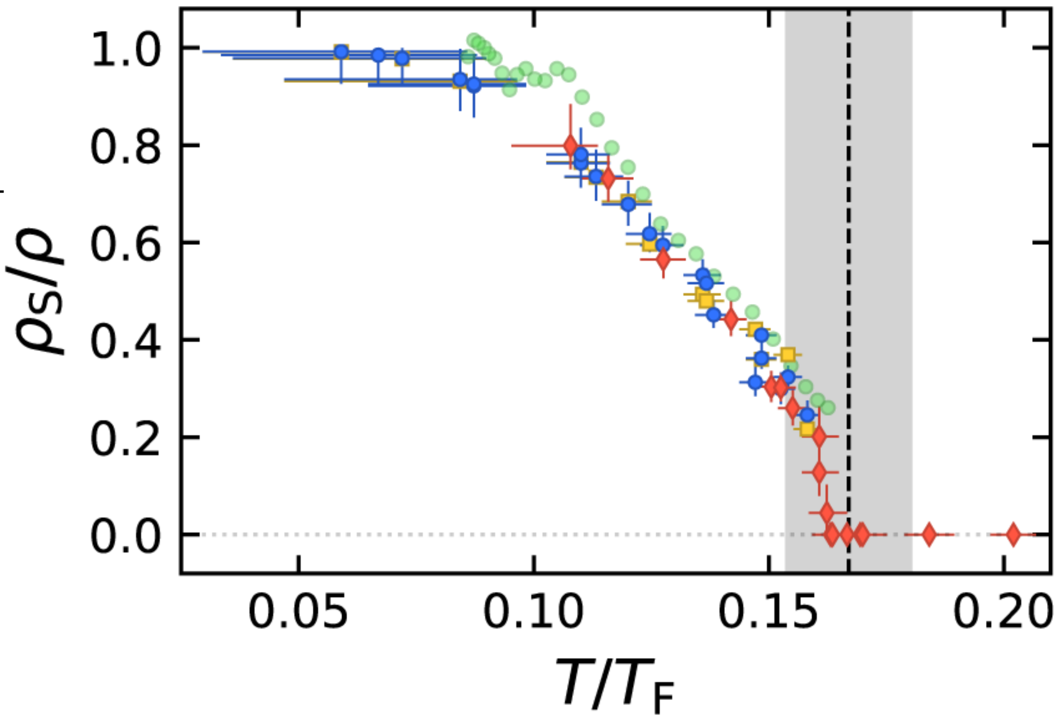


Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, MZ, in prep. (2021)

Superfluid fraction



$$c_2^2 = \frac{\rho_S}{\rho_N}\frac{TS^2}{C_p}$$

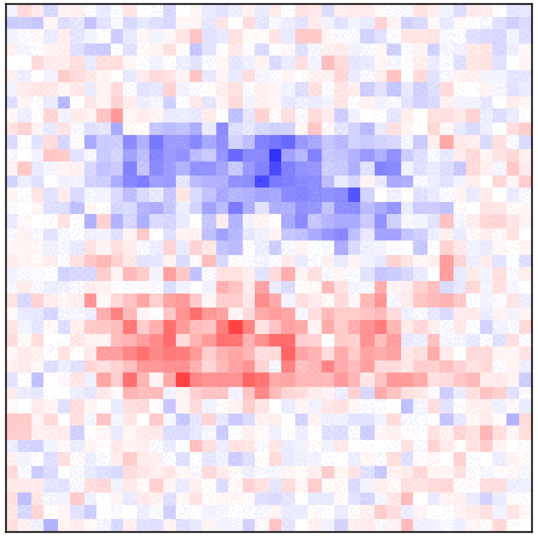


- Steady state response
- Free evolution, resonant shaking
- Free evolution, local heater

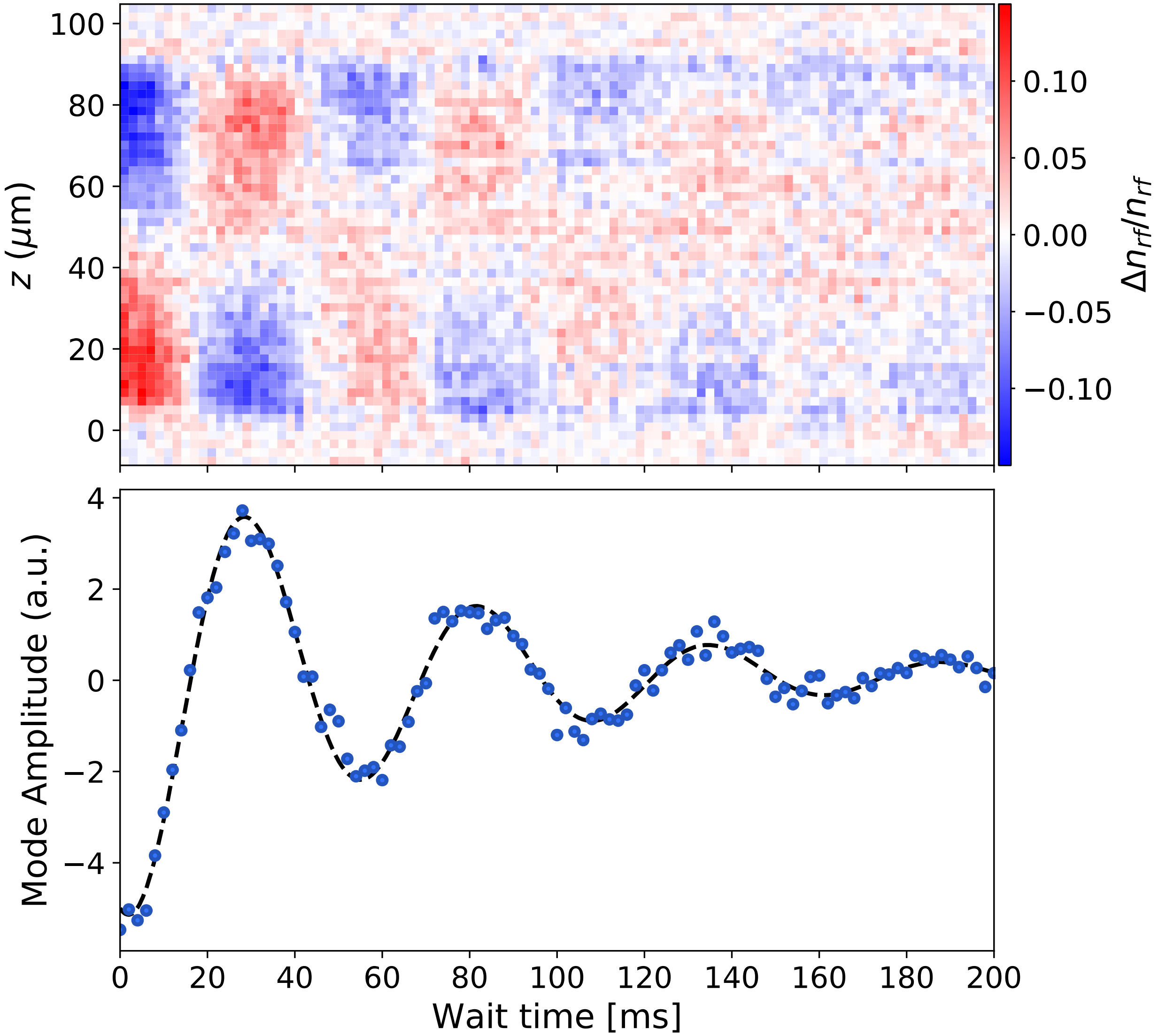
Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, MZ, in prep. (2021)

Damping of Second Sound

2nd sound
Rf transfer

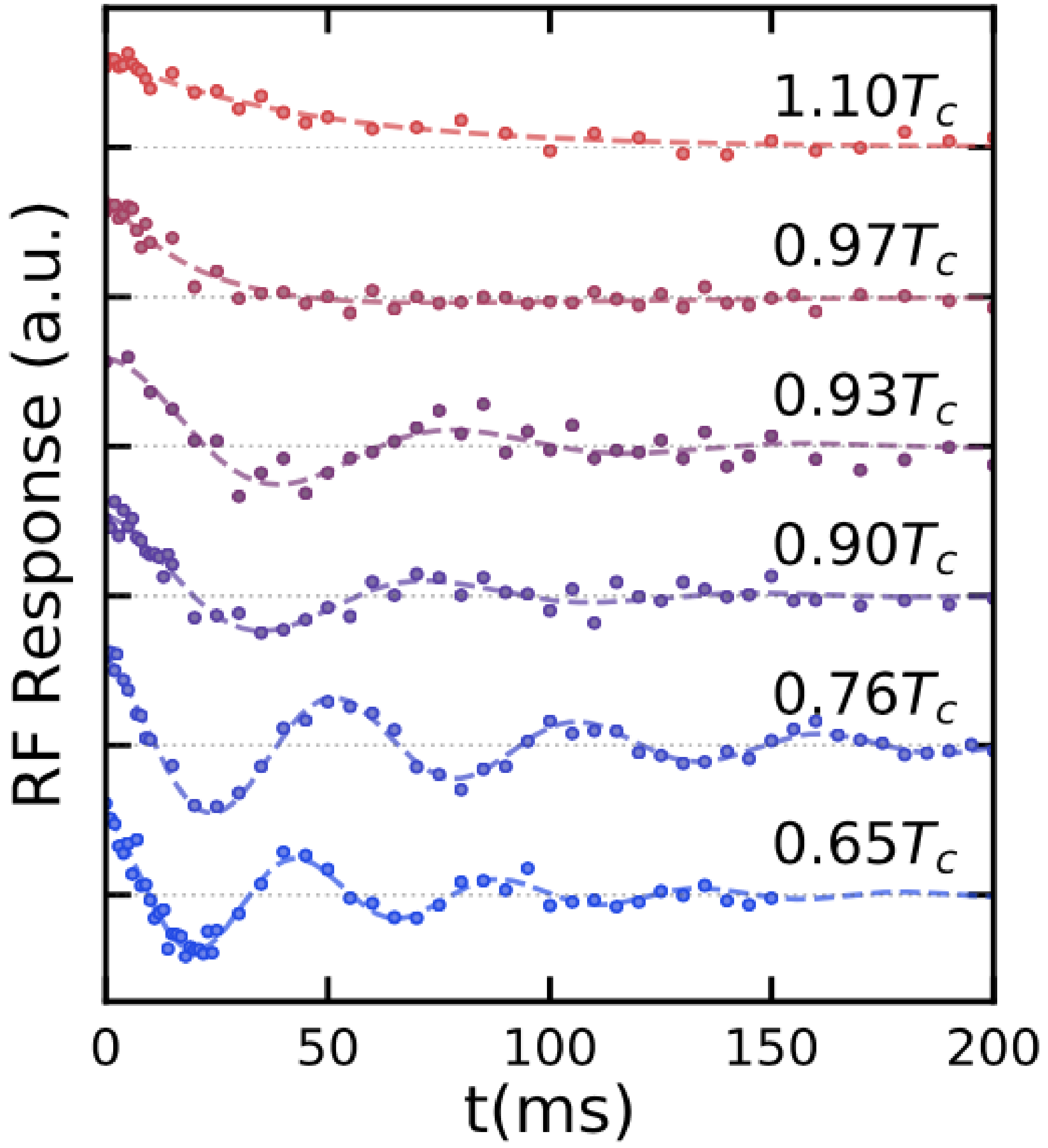


Temperature Probe



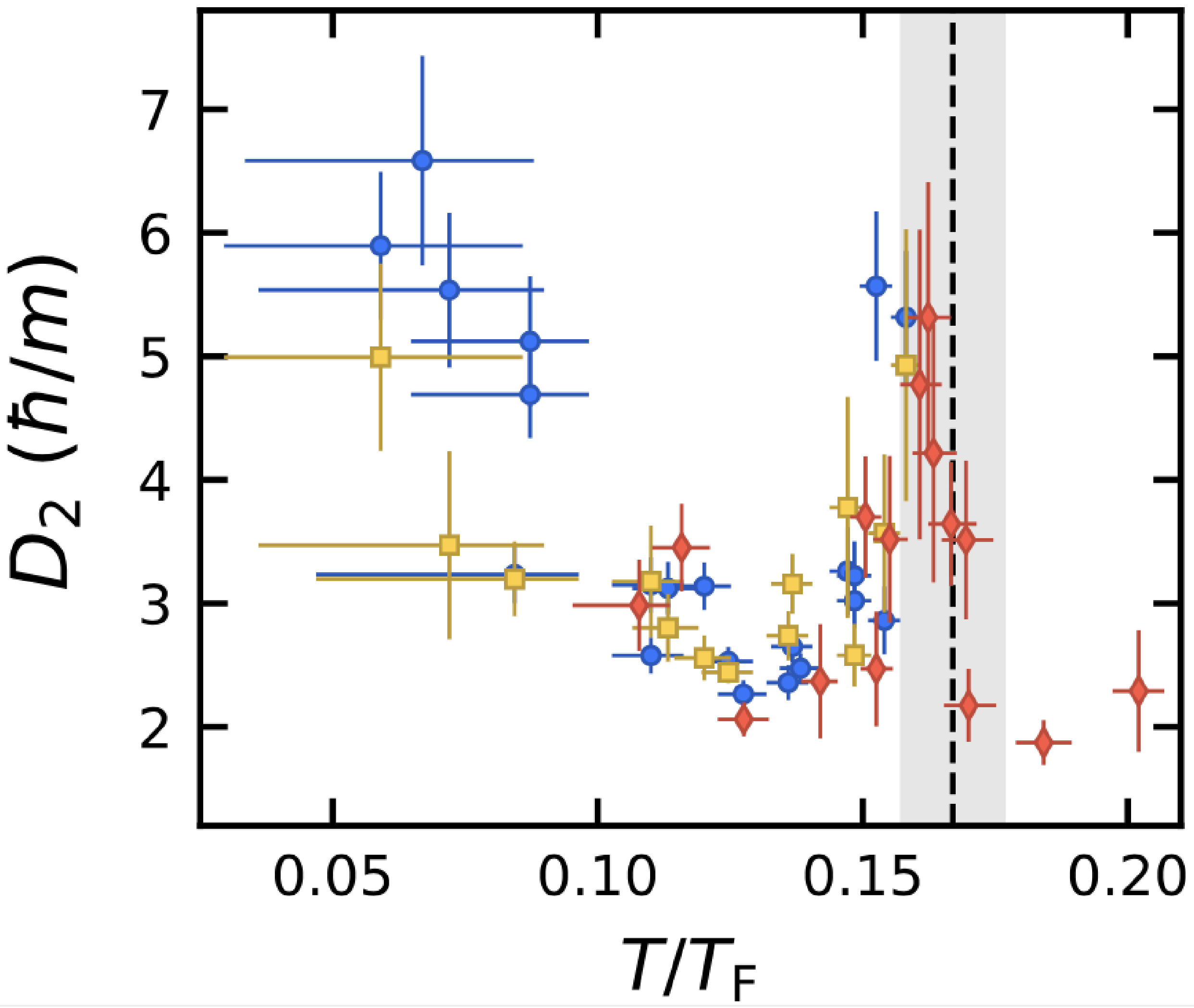
Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, MZ, in prep. (2021)

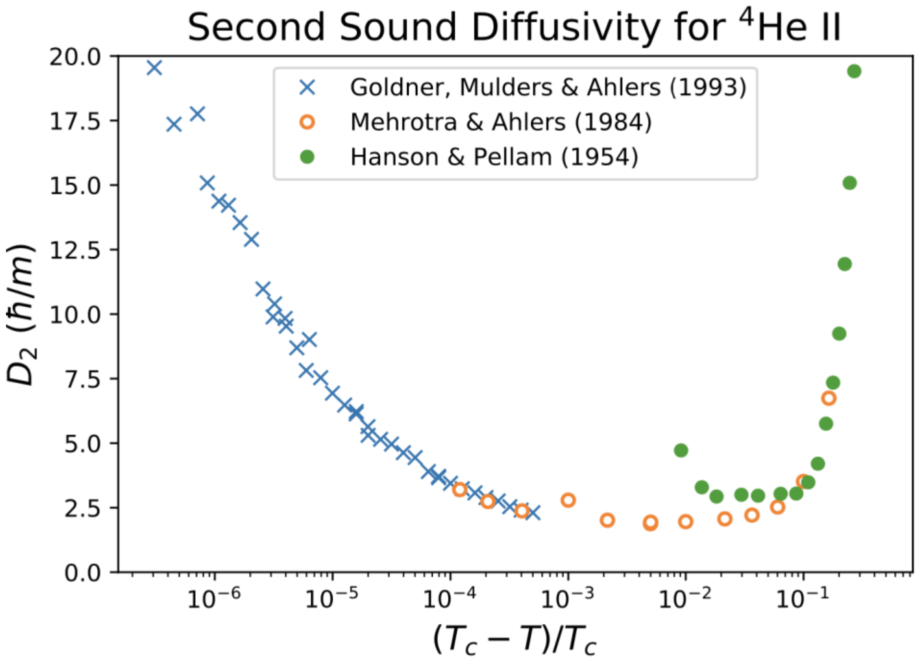
Transition from normal to superfluid



Diffusivity of Second Sound

$$D_2 = \Gamma_2/k^2$$



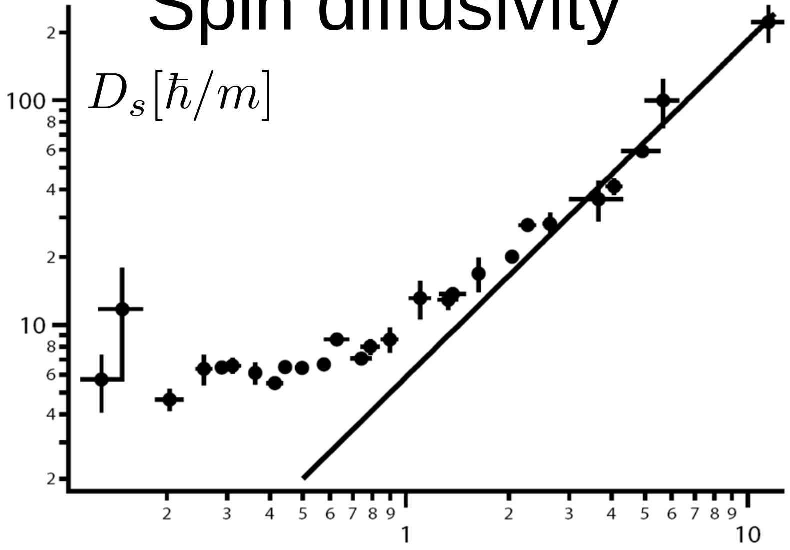


$$D_2 = \frac{\kappa}{c_p} + \frac{\rho_s}{\rho_n}\left(\xi_3\rho + \frac{4}{3}\frac{\eta}{\rho}\right)$$

Z. Yan, P. Patel, B. Mukherjee, R. Fletcher, MZ, in prep. (2021)

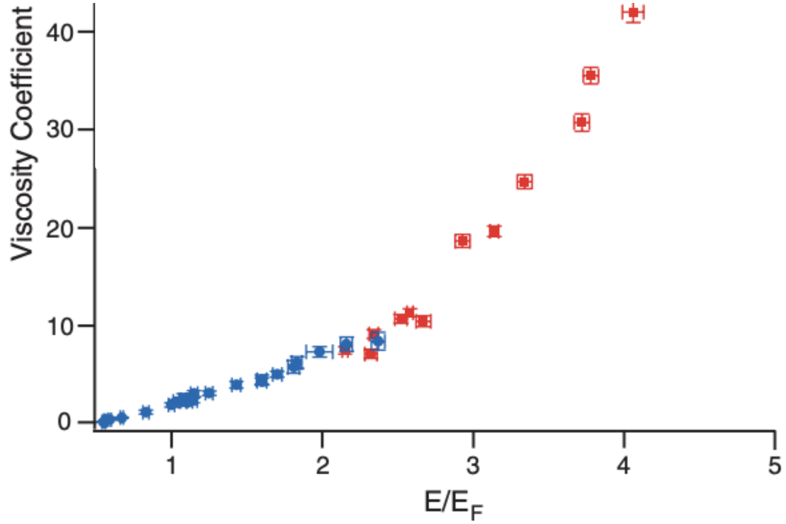
Diffusivities in the Unitary Gas

Spin diffusivity



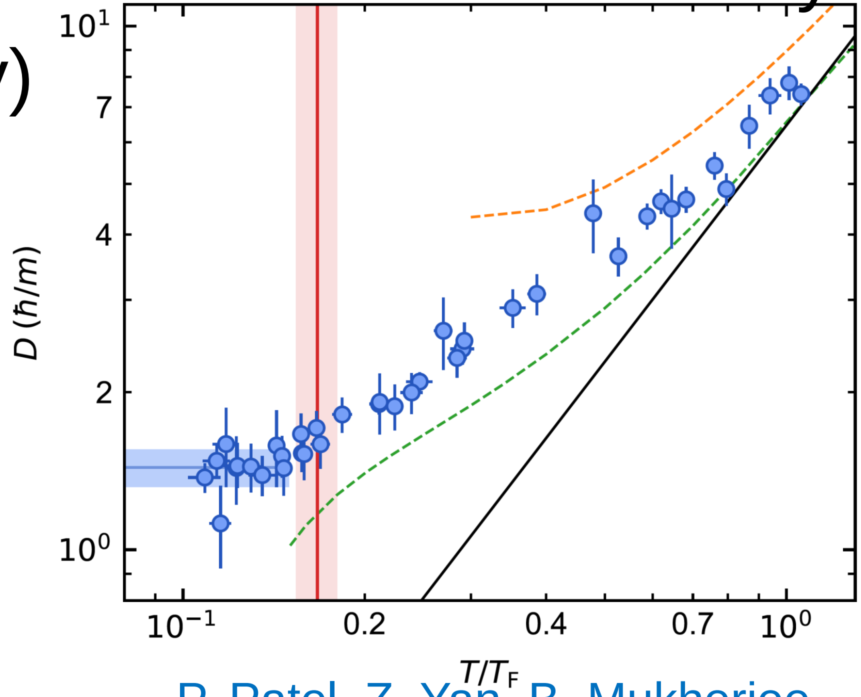
A.T. Sommer, M.J.H. Ku, G. Roati, MZ, Nature 472, 201 (2011)

Kinematic viscosity (= momentum diffusivity)



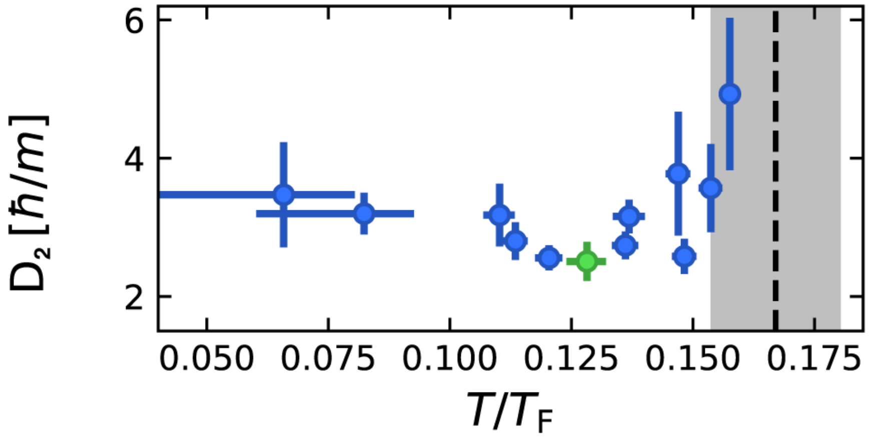
Cao et al., John Thomas group, Science 331, 58 (2011)

1st Sound diffusivity



P. Patel, Z. Yan, B. Mukherjee, R. Fletcher, J. Struck, MZ, Science 370, 1222 (2020)

2nd Sound diffusivity



Diffusivity (charge, spin, momentum, thermal)

$$D \sim vl \sim \frac{\hbar}{m}$$

Transport in Strongly Correlated Quantum Gases

Unifying themes of strongly interacting Fermi systems:

Loss of quasi-particle description: $\tau^{-1} \approx E_F / \hbar$

Quantum-Limited diffusivities: $$D \approx \frac{\hbar}{m}$$

Quantum gas simulators are poised to further elucidate Interplay between charge, spin, heat and Hall transport

- Universal Sound Diffusion in a Strongly Interacting Fermi Gas
  Parth B. Patel, Zhenjie Yan, Biswaroop Mukherjee, Richard J. Fletcher, Julian Struck, MZ
  Science, 370, 1222 (2020)
- Spin Transport in a Mott Insulator of Ultracold Fermions
  Matthew Nichols, Lawrence Cheuk, Melih Okan, Thomas Hartke, Enrique Mendez, T. Senthil, Ehsan Khatami, Hao Zhang, MZ, Science 363, 383 (2019)
- Doublon-Hole Correlations and Fluctuation Thermometry in a Fermi-Hubbard Gas
  T. Hartke, B. Oreg, N. Jia, MZ, PRL 125, 113601 (2020)
- Bose polarons near quantum criticality
  Zoe Z. Yan, Yiqi Ni, Carsten Robens, MZ, Science, 368, 190-194 (2020)
- Geometric squeezing into the lowest Landau level
  Richard J. Fletcher, Airlia Shaffer, Cedric C. Wilson, Parth B. Patel, Zhenjie Yan, Valentin Crépel, Biswaroop Mukherjee, MZ, arXiv:1911.12347 (Science 2021, to be published)

Bosons and Fermions

Bosons and Fermions

BEC 1/Fermi3

Fermionic Superfluids

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Fermi 1

NaK Dipolar Molecules

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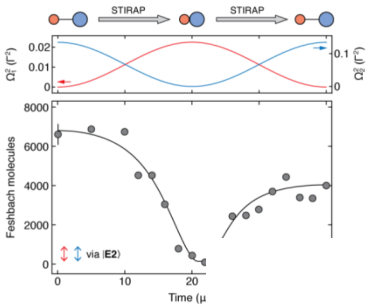
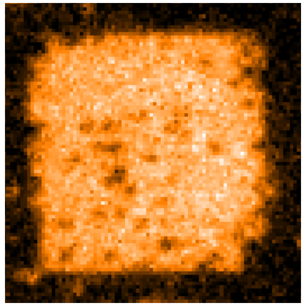
(→Prof., Columbia U.)
Cheng-Hsun Wu (→ Google X)
Dr. Huanqian Loh (→ Singap.)





Fermi 2

Fermi Gas Microscope

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Fermi3

Rotating Superfluids

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