Flavor in the SM

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General remarks

- I have to make assumptions about what you know
- Please ask questions
- Email: yg73@cornell.edu
- Some reading stuff: Book and TASI lectures
- The plan:
  1. Introduction to model building and the SM
  2. The SM flavor sector: the CKM matrix
  3. Flavor at one loop: FCNC and GIM
  4. CP violation

Some data

$$\text{Br}(D^+ \to \bar{K}^0 e^+ \nu)\ [c \to s e^+ \nu] = 8.82(13) \times 10^{-2}$$

$$\text{Br}(D^+ \to \bar{K}^0 \mu^+ \nu)\ [c \to s \mu^+ \nu] = 8.74(19) \times 10^{-2}$$

$$\text{Br}(B \to X_c e \nu)\ [b \to c e \nu] = 0.1086(16)$$

$$\text{Br}(B \to X_s \gamma)\ [b \to s \gamma] = 3.49(19) \times 10^{-4}$$

$$\text{Br}(B_s \to \mu^+ \mu^-)\ [b \to s \mu^+ \mu^-] = 2.4(8) \times 10^{-9}$$

$$\text{Br}(B^- \to D^0 \mu^- \bar{\nu})\ [b \to c \mu^- \bar{\nu}] = 2.27(11) \times 10^{-2}$$

$$\text{Br}(B^- \to \pi^0 \mu^- \bar{\nu})\ [b \to u \mu^+ \bar{\nu}] = 7.80(27) \times 10^{-5}$$

$$\text{Br}(K_L \to \mu^+ \mu^-)\ [s \to d \mu^+ \mu^-] = 6.84(11) \times 10^{-9}$$

$$\text{Br}(K^- \to \mu^- \bar{\nu})\ [s \to u \mu^- \bar{\nu}] = 0.6356(11)$$

- What patterns do to see?

What we learn from the data

- **Lepton universality**. Swapping one generation of leptons with another does not appear to affect the branching ratios of these transitions.
- **Flavor-changing neutral currents are small**. On the other hand, processes that change flavor are suppressed for charge-neutral transitions compared to transitions between hadrons of different charge.
- **Generation hierarchy**. Decays between third and first generation are suppressed compared to that of third to second generation.

The hope is that by tomorrow you will know why these patterns emerge in the SM

What is HEP?

What is HEP

Find the basic laws of Nature

More formally

$$\boxed{\mathcal{L} = ?}$$

- We have quite a good answer
- It is very elegant, it is based on axioms and symmetries
- The generalized coordinates are fields
- We use particles to answer this question

Building Lagrangians

- Choosing the generalized coordinates (fields)
- Imposing symmetries and how fields transform (input)
- The Lagrangian is the most general one that obeys the symmetries
- We truncate it at some order, usually fourth
- This truncation may result in accidental symmetries

The SM

The SM

Input: Symmetries and fields

- Symmetry: 4d Poincare and

$$SU(3)_C \times SU(2)_L \times U(1)_Y$$

- Fields:
  - 3 copies of QUDLE fermions

$$Q_L(3,2)_{1/6} \quad U_R(3,1)_{2/3} \quad D_R(3,1)_{-1/3}$$
$$L_L(1,2)_{-1/2} \quad E_R(1,1)_{-1}$$

  - One scalar

$$\phi(1,2)_{+1/2}$$

Then Nature is described by

- Output: the most general $\mathcal{L}$ up to dim 4

$$\mathcal{L} = \mathcal{L}_{kin} + \mathcal{L}_{Higgs} + \mathcal{L}_{Yukawa}$$

- This model has a $U(1)_B \times U(1)_e \times U(1)_\mu \times U(1)_\tau$ accidental symmetry
- Initial set of measurements to find the parameters
  - SSB: $SU(2)_L \times U(1)_Y \ \rightarrow \ U(1)_{EM}$
  - Fermion masses, gauge couplings, and mixing angles

The SM pass (almost) all of it tests

A SM vs The SM

- "A SM" is the theory without the values of the parameters
- "The SM" is the one we have with a given set of values for the parameters

It is important to understand what predictions are from "A SM" and what are only in "The SM"

The gauge interactions

The gauge part

$$SU(3)_C \times SU(2)_L \times U(1)_Y \;\to\; SU(3)_C \times U(1)_{EM}$$

Three parts, each look so different...

- QED - photon interaction: Perturbation theory
- QCD - gluon interaction: Confinement and asymptotic freedom
- Electroweak (EW): SSB and massive gauge bosons

In these lectures we focus on the EW part

$\mathcal{L}_{kin}$ and $SU(2) \times U(1)$

- Four gauge bosons DOFs: W_a^μ and B^μ
- The covariant derivative is

$$D^\mu = \partial^\mu + igW_a^\mu T_a + ig'YB^\mu$$

  - Two parameters g and g'
  - Y is the $U(1)$ charge of the field D_μ work on
  - T_a is the $SU(2)$ representation
  - $T_a = 0$ for singlets. $T_a = \sigma_a/2$ for doublets
  - For example

$$D^\mu L = \left(\partial^\mu + \frac{i}{2}gW_a^\mu\sigma_a - \frac{i}{2}g'B^\mu\right)L$$

$$D^\mu E = (\partial^\mu - ig'B^\mu)\,E$$

SSB in the SM

$$-\mathcal{L}_{Higgs} = \lambda\phi^4 - \mu^2\phi^2 = \lambda(\phi^2 - v^2)^2$$

- We measure the fact that $\mu^2 > 0$ by having SSB
- The minimum is at $|\phi| = v$
- ϕ has 4 DOFs. We can choose

$$\langle\phi_1\rangle = \langle\phi_2\rangle = \langle\phi_4\rangle = 0 \qquad \langle\phi_3\rangle = v$$

- It leads to: $SU(2)_L \times U(1)_Y \ \to \ U(1)_{EM}$
- We call the remaining symmetry EM
- Could we “choose” the vev in the neutral direction?
- We left with one real scalar field: the Higgs boson

QED

- Where is QED in all of this?

$$Q = T_3 + Y$$

- We can write explicitly for $L(1,2)_{-1/2}$ and $\phi(1,2)_{1/2}$

$$L_L = \begin{pmatrix} \nu_L \\ e_L \end{pmatrix} \qquad \phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

- We can “tell” the different component because we have SSB

Spectrum

Gauge boson masses

- $W_1,\ W_2,\ W_3,\ B$
- Gauge bosons masses from $|D_\mu\phi|^2$ (HW: do it)
- Diagonalizing the mass matrix the masses are

$$M_{W^+}^2 = M_{W^-}^2 = \frac{1}{4}g^2v^2 \qquad M_Z^2 = \frac{1}{4}(g^2+g'^2)v^2 \qquad M_A^2 = 0$$

- The mass eigenstates

$$W^\pm = \frac{1}{\sqrt{2}}(W_1 \mp iW_2) \qquad \tan\theta_W \equiv \frac{g'}{g}$$

$$Z = \cos\theta_W W_3 - \sin\theta_W B \qquad A = \sin\theta_W W_3 + \cos\theta_W B$$

- We have a θ_W rotation from (W_3, B) to (Z, A)

The $\rho = 1$ relation

We get the following testable relation

$$\rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \qquad \tan\theta_W \equiv \frac{g'}{g}$$

$\boxed{\text{The above is a signal of SSB}}$

$\mathcal{L}_{\rm Yuk}$ and lepton masses

- There is no way to write a mass term (why?)
- The Yukawa part of the leptons

$$\mathcal{L}_{Yuk} = y_{ij}\overline{L_{Li}}E_{Rj}\,\phi \Rightarrow m_{ij}\overline{E_{Li}}E_{Rj} \qquad m_{ij} = v y_{ij}$$

- $i, j = 1, 2, 3$ are flavor indices
- y is a general complex 3×3 matrix and we can choose a basis where m is diagonal and real

$$m_{ij} = y\, v = \mathrm{diag}(m_e, m_\mu, m_\tau)$$

- Neutrinos are massless
- We will get to the quarks later

Fermion masses in "The SM"

- Lepton masses refer to masses of free particles
- Quark masses are more complicated: quarks are not free
- We will not get into the subtleties here. The best way is to think about it as a parameter
- We see smallness and hierarchies of masses (in GeV)

$$
\begin{array}{ccc}
m_e \sim 10^{-3} & m_\mu \sim 10^{-1} & m_\tau \sim 10^{0} \\
m_d \sim 10^{-2} & m_s \sim 10^{-1} & m_b \sim 10^{1} \\
m_u \sim 10^{-2} & m_c \sim 10^{0} & m_t \sim 10^{2}
\end{array}
$$

  Only the top mass is not small

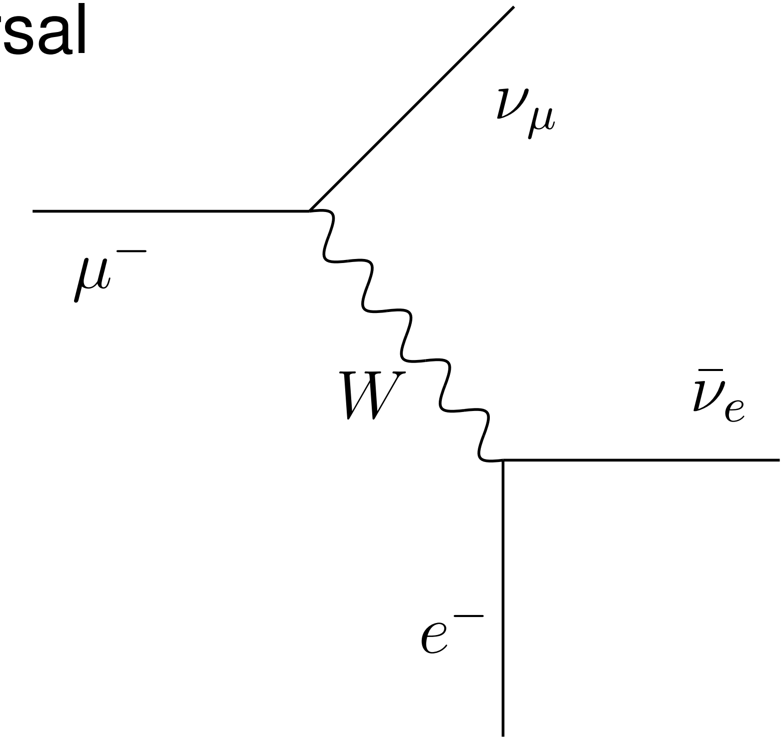
- In the SM these are just the value of the parameters

Interactions

Charged current interactions

$$-\frac{g}{\sqrt{2}}\,\overline{\nu_{eL}}\,W^\mu\gamma_\mu e_L^- + h.c.$$

- Only left-handed fields take part in charged-current interactions. Therefore the W interaction violate parity
- The $W\ell\nu$ interaction is universal
- Can be used to measure g



$$A \sim g^2/m_W^2 \sim G_F$$

Neutral currents

$$\mathcal{L}_{\rm int} = \frac{e}{\sin\theta_W \cos\theta_W}(T_3 - \sin^2\theta_W Q)\; \bar\psi\gamma^\mu\psi\, Z_\mu + e\, Q\bar\psi\gamma^\mu\psi\, A_\mu,$$

- We define

$$Q = T_3 + Y \qquad\qquad e = g\sin\theta_W$$

- Photon coupling is parity invariant
- Z couples to both LH and RH fermions but in a parity violating way
- The coupling to the Z is larger. So why we call it weak interaction?
- Once we know e and g we know θ_W

Higgs interaction with fermions

$$\mathcal{L}_H = y_{ij}(H + v)\bar{\psi}_i \psi_j$$

- The coupling is diagonal in the same basis when the mass is diagonal
- It is proportional to the mass
- The proportionality factor is v

Quarks

Lepton masses

- In a chiral theory fermions are massless
- In the SM they get mass from the interactions with the Higgs
- For leptons only the charged leptons get a mass. We need both LH and RH fields for a mass

$$Y_{ij}\left(\bar{L}_L\right)_i \phi\left(E_R\right)_j \rightarrow Y_{ij}\left(v+H\right)\bar{e}_L^i e_R^j + \ldots$$

- The mass is proportional to the Yukawa coupling and the vev $m_{ij} = Y_{ij}v$
- We can choose a basis where Y is diagonal in flavor space. This basis corresponds to mass eigenstates

Quarks

$$Y_{ij}^{D}\left(\bar{Q}_L\right)_i \phi \left(D_R\right)_j + Y_{ij}^{U}\left(\bar{Q}_L\right)_i \tilde{\phi} \left(U_R\right)_j$$

- The Yukawa matrices, Y_{ij}^F, are general complex matrices
- After the Higgs acquires a vev, the Yukawa terms give masses to the fermions. Also, after the breaking we can talk about U_L and D_L, not about Q_L
- If Y is not diagonal, flavor is not conserved (soon we will go over the subtleties here)
- If Y carries a phase, CP is violated (soon we will understand). C and P is violated to start with

CP violation

A simple "hand wave" argument of why CP violation is given by a phase

- It is all in the $+h.c.$ term

$$Y_{ij}\left(\bar{Q}_L\right)_i \phi\,(D_R)_j + Y_{ji}^*\left(\bar{D}_R\right)_j \phi^\dagger\,(Q_L)_i$$

- Under CP

$$Y_{ij}\left(\bar{D}_R\right)_j \phi^\dagger\,(Q_L)_i + Y_{ji}^*\left(\bar{Q}_L\right)_j \phi\,(D_R)_i$$

- CP is conserved if $Y_{ij} = Y_{ij}^*$
- Not a full proof, since there is still a basis choice...

Parameter counting

How many parameters we have?

How many parameters are physical?

- “Unphysical” parameters are those that can be set to zero by a basis rotation
- General theorem

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken})$$

  - $N(\text{Phys})$, number of physical parameters
  - $N(\text{tot})$, total number of parameters
  - $N(\text{broken})$, number of broken generators

Example: Zeeman effect

A hydrogen atom with weak magnetic field

- The magnetic field add one new physical parameter, B

$$V(r) = \frac{-e^2}{r} \qquad \Rightarrow \qquad V(r) = \frac{-e^2}{r} + B_x\hat{x} + B_y\hat{y} + B_z\hat{z}$$

- But there are 3 total new parameters
- The magnetic field breaks explicitly: $SO(3) \to SO(2)$
- 2 broken generators, can be “used” to define the z axis

$$N(\text{Phys}) = N(\text{tot}) - N(\text{broken}) \qquad \Rightarrow \qquad 1 = 3 - 2$$

Back to the flavor sector

Without the Yukawa interactions, a model with N copies of the same field has a $U(N)$ global symmetry

- It is just the symmetry of the kinetic term

$$\mathcal{L} = \bar{\psi}_i D_\mu \gamma^\mu \psi_i, \qquad i = 1, 2, ..., N$$

- $U(N)$ is the general rotation in N dimensional complex space
- $U(N) = SU(N) \times U(1)$ and it has N^2 generators

Two generation SM

First example, two generation SM

- Two Yukawa matrices: Y^D, Y^U, $N_{Total} = 16$
- Global symmetries of the kinetic terms: $U(2)_Q \times U(2)_D \times U(2)_U$, 12 generators
- Exact accidental symmetries: $U(1)_B$, 1 generator
- Broken generators due to the Yukawa: $N_{Broken} = 12 - 1 = 11$
- Physical parameters: $N_{Physical} = 16 - 11 = 5$. They are the 4 quarks masses and the Cabibbo angle

The SM flavor sector

Back to the SM with three generations. Do it yourself

- Total parameters (in Yukawas): $N_T =$
- Symmetry generators of kinetic terms: $N_G =$
- Unbroken global generators: $N_U =$
- Broken generators: $N_B =$
- Physical parameters: $N_P =$

The SM flavor sector

Back to the SM with three generations. Do it yourself

- Total parameters (in Yukawas): $N_T = 2 \times 18 = 36$
- Symmetry generators of kinetic terms: $N_G = 3 \times 9 = 27$
- Unbroken global generators: $N_U = 1$
- Broken generators: $N_B = 27 - 1 = 26$
- Physical parameters: $N_P = 36 - 26 = 10$
- 6 quark masses, 3 mixing angles and one CPV phase

Remark: The broken generators are 17 Im and 9 Re. We have 18 real and 18 imaginary to “start with” so the physical ones are $18 - 17 = 1$ and $18 - 9 = 9$

The flavor parameters

- The 6 masses. We kind of know them. There is a lot to discuss, but I will not do it in these lectures
- The CKM matrix has 4 parameters
  - 3 mixing angles (the orthogonal part of the mixing)
  - One phase (CP violating)
- Next we discuss the CKM in some details

The CKM matrix

Basic of basis rotation

- We want to move to another basis
- Usually, we like to rotate fields with the same QN (same representation). Why?

$$\psi' = U\psi \qquad UU^\dagger = 1$$

- What it does to the kinetic term?

$$\bar{\psi}^i \delta_{ij} D_\mu \gamma_\mu \psi^j \to \bar{\psi}^i U^\dagger U \delta_{ij} D_\mu \gamma_\mu U^\dagger U \psi^j \to \bar{\psi}'^i U \delta_{ij} D_\mu \gamma_\mu U^\dagger \psi'^j$$

- What it does to the mass term, $\bar{\psi}^i_L m_{ij} \psi^j_R$?

$$\bar{\psi}^i_L m_{ij} \psi^j_R = \bar{\psi}^i_L U^\dagger U m_{ij} V^\dagger V \psi^i_R = \bar{\psi}'^i_L m'_{ij} \psi'^j_R$$

- How we use it for the leptons?

Neutral currents

How the rotation to the mass basis affect neutral currents?

- Photon, gluon, Z, and Higgs
- All of them are diagonal in flavor space
- The Photon, gluon, and Z couplings are also universal
- The Higgs couplings are proportional to the masses
- We get back to it later in more details

The CKM matrix

$$\mathcal{L}_{Yuk} \sim Y^U \phi \bar{Q} U + Y^D \tilde{\phi} \bar{Q} D$$

- After SSB we define $Q = (U \;\; D)$

$$\mathcal{L} \sim g\, \bar{u}^i_L \delta_{ij} d^j_L\, W + m^D_{ij} \bar{d}^i_R d^j_L + m^U_{ij} \bar{u}^i_R u^j_L$$

- The mass matrices, m^F_{ij}, are general complex matrices
- We can diagonalize them and move to the mass basis

$$m_{diag} = V_L m V_R^\dagger \qquad V_L V_L^\dagger = V_R V_R^\dagger = 1 \qquad V_L \neq V_R$$

$$(d_L)_i \to (V_L^D)_{ij} (d_L)_j \qquad (d_R)_i \to (V_R^D)_{ij} (d_R)_j$$

Finding the CKM

- For the W the rotation to the mass basis is important

$$\mathcal{L}_W \sim \bar{u}_L^i \delta_{ij} d_L^j \;\to\; \bar{u}_i V_L^{U\dagger} V_L^U \delta_{ij} V_L^{D\dagger} V_L^D d_j \sim \bar{u}_i' V d_i'$$

- V is the CKM matrix

$$V = V_L^U V_L^{D\dagger}$$

- The point is that we cannot have m_U, m_D, and the couplings to the W diagonal at the same basis
- In the mass basis the W interaction change flavor, that is, flavor and generation number is not conserved
- We can work in another basis. The point is that at most 2 out of the 3 matrices can be diagonal

The CKM matrix

$$\mathcal{L}_W = \frac{g}{\sqrt{2}} \overline{U_L} V \gamma^\mu D_L W_\mu^+ + \text{h.c.}$$

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- CKM is unitary

$$\sum V_{ij} V_{ik}^* = \delta_{jk}$$

- Experimentally, $V \sim 1$. Off diagonal terms are small
- Many ways to parametrize the matrix

CKM parametrization

- The standard parametrization

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

  where $c_{ij} \equiv \cos\theta_{ij}$ and $s_{ij} \equiv \sin\theta_{ij}$.

- In general there are 5 entries that carry a phase
- Experimentally:

$$|V| \approx \begin{pmatrix} 0.97383 & 0.2272 & 3.96 \times 10^{-3} \\ 0.2271 & 0.97296 & 4.221 \times 10^{-2} \\ 8.14 \times 10^{-3} & 4.161 \times 10^{-2} & 0.99910 \end{pmatrix}$$

The Wolfenstein parametrization

- Since $V \sim 1$ it is useful to expand it

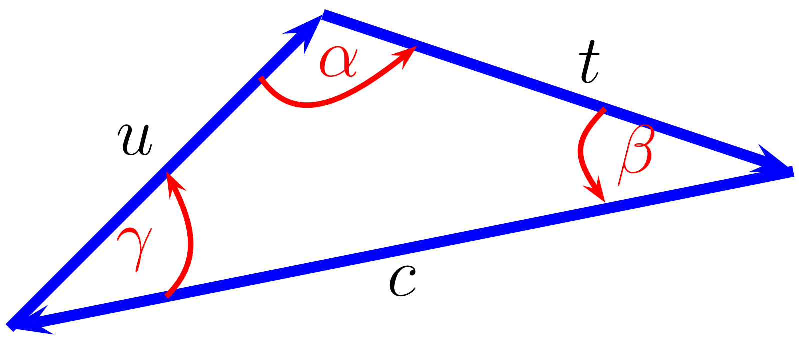
$$V \approx \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix}$$

- One small parameter $\lambda \sim 0.2$, and three (A, ρ, η) that are roughly $O(1)$
- As always, be careful (unitarity...)
- Note that to this order only V_{13} and V_{31} have a phase
- Also we use

$$\bar{\rho} = \rho(1-\lambda^2/2) \qquad \bar{\eta} = \eta(1-\lambda^2/2)$$

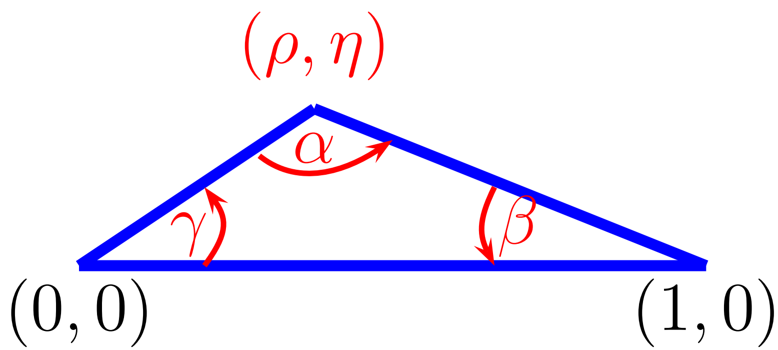
The unitarity triangle

A geometrical presentation of $V_{ub}^* V_{ud} + V_{tb}^* V_{td} + V_{cb}^* V_{cd} = 0$



Rescale by the c size and rotated

$$A\lambda^3 \left[(\rho + i\eta) + (1 - \rho - i\eta) + (-1)\right] = 0$$



The Jarlskog invariant

- There is a freedom to define phases
- There are things that are invariant under phase rotation
- In the SM there is one basis independent invariant, J

$$\mathrm{Im}\left[V_{ij}V_{kl}V_{i\ell}^*V_{kj}^*\right] = J\sum_{mn}\epsilon_{ikm}\epsilon_{j\ell n}\ ,$$

- J corresponds to

$$J = c_{12}c_{23}c_{13}^2 s_{12}s_{23}s_{13}\sin\delta_{\mathsf{KM}} \approx \lambda^6 A^2 \eta$$

- The areas of all the UT is $|J|/2$

CKM determination

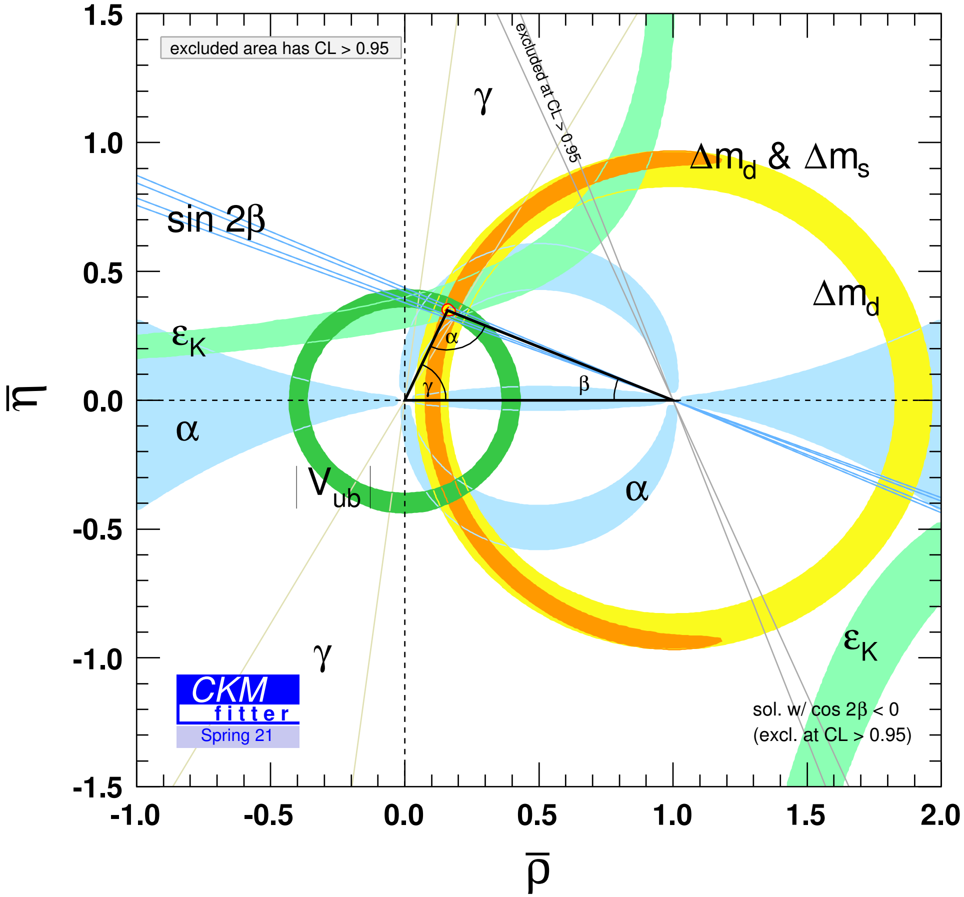
CKM determination

- Basic idea: Measure the 4 parameters in many different ways. Any inconstancy is a signal of NP
- Problems: Experimental errors and theoretical errors
- Have to be smart...
  - Smart theory to reduce the errors
  - Smart experiment to reduce the errors
- There are cases where both errors are very small

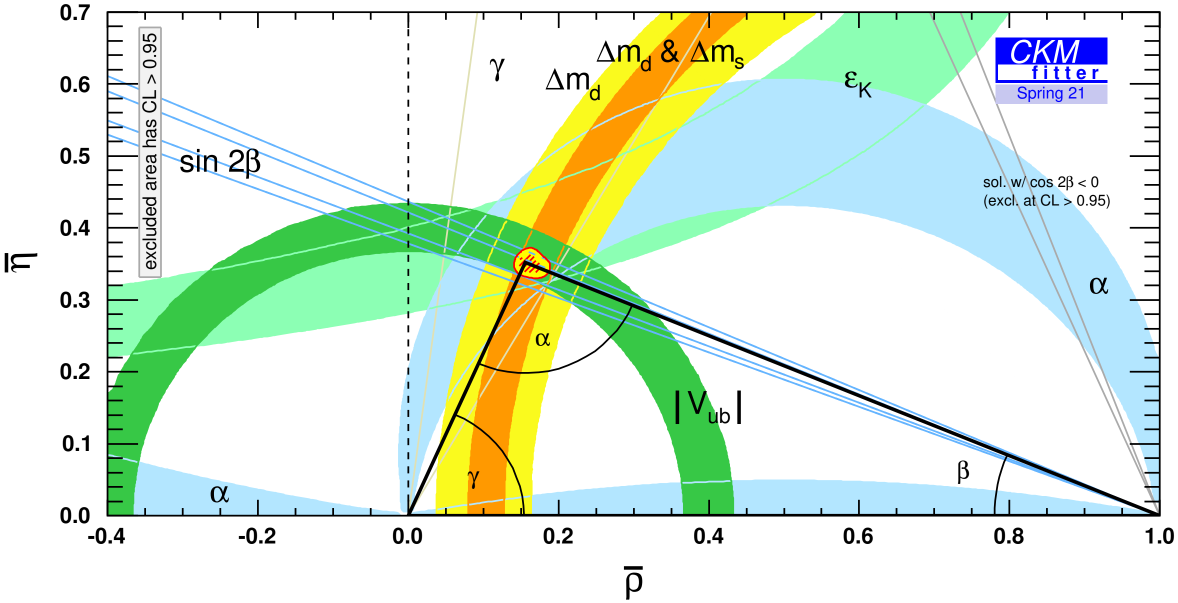
Measuring sides: examples

- β-decay, $d \to ue\bar{\nu} \propto V_{ud}$; Isospin
- K-decay, $s \to ue\bar{\nu} \propto V_{us}$; Isospin and SU(3)
- D-decay, $c \to qe\bar{\nu} \propto V_{cq}\ q = d, s$; HQS
- B-decays $b \to ce\bar{\nu} \propto V_{cb}$; HQS
- Not easy with top. Cannot tag the final flavor, low statistics

All together now



Zoom in



CKM fitter
Spring 21
excluded area has CL > 0.95
sin 2β
γ
Δm_d
Δm_d & Δm_s
ε_K
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
|V_ub|
β
η̄
ρ̄

The SM is very special

- “A SM” is special
- “The SM” is even more special

The fact that the data confirm the SM is far from trivial

FCNC

FCNC

FCNC=Flavor Changing Neutral Current

FCCC=Flavor Changing Charged Current

- $b \to c\ell\nu$ vs $b \to s\ell^+\ell^-$

- Very important concept in flavor physics
- Important: Diagonal couplings vs universal couplings

FCNC

- In Nature FCNC are highly suppressed
  - Historically, $K \to \mu\nu$ vs $K_L \to \mu\mu$
  - The suppression was also seen in charm and B
- In the SM there are no FCNC at tree level. Very nice!
  - In the SM we have four neutral bosons, g, γ, Z, h. Their couplings are diagonal
  - The reasons why they are diagonal, and what it takes to have FCNC, is not always trivial
  - Of course we have FCNC at one loop (two charged current interactions give a neutral one)

Photon and gluon tree level FCNC

- For exact gauge interactions the couplings are always diagonal. It is part of the kinetic term

$$\partial_\mu \delta_{ij} \to (\partial_\mu + iqA_\mu)\delta_{ij}$$

- Symmetries are nice...
- In any extension of the SM the photon couplings are flavor diagonal

Higgs tree level FCNC

- The Higgs is a possible source of FCNC
- With one Higgs doublet, the mass matrix is aligned with the Yukawa. $\phi \sim v + H$

$$\mathcal{L}_m \sim Y_{ij}\, v\, \bar{d}^i_L d^j_R \qquad \mathcal{L}_{int} \sim Y_{ij} H \bar{d}^i_L d^j_R$$

- With two doublets we could have tree level FCNC

$$\mathcal{L}_m \sim \bar{d}_L (Y_1 v_1 + Y_2 v_2) d_R$$

$$\mathcal{L}_{int} \sim H_1 \bar{d}_L Y_1 d_R + H_2 \bar{d}_L Y_2 d_R$$

- There are “ways” to avoid it, by imposing extra symmetries

Z exchange FCNC

- For broken gauge symmetry there is no FCNC when: "All the fields with the same QN in the unbroken symmetry also have the same QN in the broken part"
- In the SM the Z coupling is diagonal since all $q = -1/3$ RH quarks are $(3,1)_{-1/3}$ under $SU(2) \times U(1)$
- What we have in the couplings is

$$\bar{d}_i \, (T_3)_{ij} \, d_j \;\to\; \bar{d} \, V \, (T_3)_{ij} \, V^\dagger d_j \qquad V T_3 V^\dagger \propto I \;\text{ if }\; T_3 \propto I$$

- Adding quarks with different representations can generate tree level FCNC Z couplings, like $\psi_L(3,1)_{-1/3}$
- Same condition for new neutral gauge bosons (usually denoted by Z')

Tree level FCNC in “a SM”

- In a generic model we expect tree-level FCNC
- In a SM there are specific reasons for not having it
  - All fermions with the same charge also have the same hypercharge
  - There is only one Higgs doublet
- We need to be careful when extending the SM

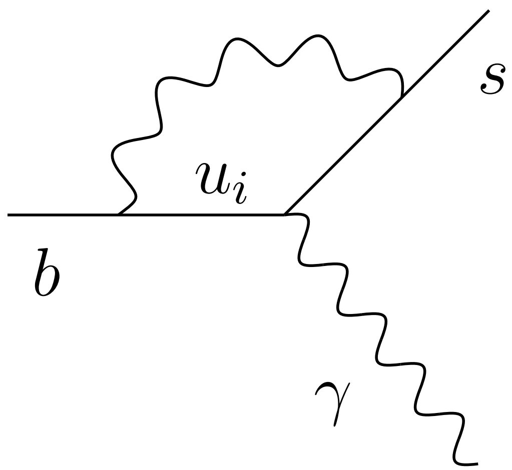
FCNCs at one loop

FCNC at one loop

- We understand why FCNC are suppressed in “a SM”: There is no tree level exchange
- Yet, there are more suppression factors in “The SM”
  - CKM factors
  - Mass factors: GIM mechanism
- The loop factor in “a SM” is universal
- The other factors in “the SM” are not universal

Loop: example

$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^*$



What is $\sum V_{ib} V_{is}^*$?

GIM Mechanism

what we really have is

$$A(b \to s\gamma) \propto \sum V_{ib} V_{is}^* f(m_i)$$

- Because the CKM is unitary, the m_i independent term in f vanishes
- The amplitude must depend on the mass
- For small $x_i \equiv m_i^2/m_W^2$ we have

$$A \sim x_i \quad \text{or} \quad x_i \log x_i$$

  - In s decays this gives m_c^2/m_W^2 extra suppression
  - For charm it gives m_s^2/m_W^2 extra suppression
  - Numerically not important for b decays, $m_t \sim m_W$

Remarks about GIM

- CKM unitarity and tree level Z exchange are related. (Is the diagram divergent?)
- What about the decoupling theorem? Is seems to be violated here

Meson mixing

Two level system

Two level system in QM. $|1\rangle$ and $|2\rangle$ are energy E.S.

$$|f_a\rangle = \frac{|1\rangle + |2\rangle}{\sqrt{2}}, \qquad |f_b\rangle = \frac{|1\rangle - |2\rangle}{\sqrt{2}},$$

The time evolution

$$|f_a\rangle(t) = \exp\left[i\Delta E t/2\right]|1\rangle + \exp\left[-i\Delta E t/2\right]|2\rangle$$

The probability to measure flavor f_i at time t is

$$|\langle f_a|f_a(t)\rangle|^2 = \frac{1+\cos\Delta E t}{2} \qquad |\langle f_b|f_a(t)\rangle|^2 = \frac{1-\cos\Delta E t}{2}$$

- Oscillations with frequency ΔE
- The relevant parameter is $x \equiv \Delta E t$

Meson mixing

For relativistic case

- $E \to m$. Roughly,

$$K_{S,L} = \frac{K \pm \bar{K}}{\sqrt{2}}$$

- "Measurement" is done by the decay

The probability to measure flavor f_i at time t is

$$|\langle f_a | f_a(t) \rangle|^2 = \frac{1 + \cos \Delta m t}{2} \qquad |\langle f_b | f_a(t) \rangle|^2 = \frac{1 - \cos \Delta m t}{2}$$

- Oscillations with frequency Δm
- The relevant time scale is $x \equiv \Delta m / \Gamma$

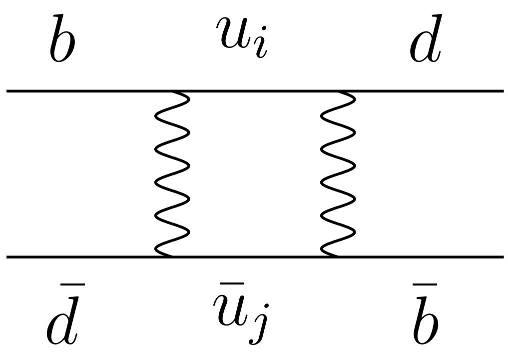
Calculations of Δm

- There are 4 mesons: $K(\bar{s}d)$, $B(\bar{b}d)$, $B_s(\bar{b}s)$, $D(c\bar{u})$
  - Why not charged mesons?
  - Why not the neutral pion?
  - Why not the K^*
  - Why not T mesons?
- The two flavor eigenstate B and $\bar{B}$ mix via the weak interactions. It is an FCNC process $m_{weak} = A(B \to \bar{B})$
- In the SM it is a loop process, and it gives an effect that is much smaller than the mass

$$M = \begin{pmatrix} m_B & m_{weak} \\ m_{weak} & m_B \end{pmatrix} \qquad \begin{aligned} &\Rightarrow \quad M_{H,L} = m_B \pm m_{weak} \\ &\Rightarrow \quad \Delta M = m_{weak} \end{aligned}$$

The box diagram

- In the SM the mixing is giving by the box diagram



- The result is: $\Delta M \propto \sum_{i,j} V_{is} V_{id}^* V_{js} V_{jd}^* f(m_i, m_j)$
- The constant term vanish due to unitarity (GIM)
- To leading order $f \sim m_i^2/m_W^2$

Meson mixing: remarks

- The different meson have different GIM suppression
  - K mixing: m_c^2/m_W^2
  - D mixing: m_s^2/m_W^2
  - B and B_s mixing: no suppression
- Mixing can be used to determine magnitude of CKM elements. For example B mixing is used to get $|V_{td}|$
- There are still hadronic uncertainties. We calculate at the quark level and we need the meson. Lattice QCD is very useful here
- My treatment was very simplistic, there are more effects
- Each meson has its own set of approximations

Meson mixing

In general we have also width difference between the two eigenstates. They are due to common final states

$$x \equiv \frac{\Delta m}{\Gamma} \qquad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

$$\begin{array}{lll} K & x \sim 1 & y \sim 1 \\ D & x \sim 10^{-2} & y \sim 10^{-2} \\ B_d & x \sim 1 & y \sim 10^{-2} (th) \\ B_s & x \sim 10 & y \sim 10^{-1} \end{array}$$

Mixing measurements

How this is done?

- Need the flavor of the initial state. Usually the mesons are pair produced
  - Same side tagging ($D^* \to D\pi$)
  - Other side tagging (semileptonic B decays)
- The final flavor
  - Use time dependent (easier for highly boosted mesons)
  - Use time integrated signals
  - The final state may not be a flavor eigenstate, but we still can have oscillations as long as it is not a mass eigenstate

Meson mixing

Meson mixing is an FCNC process with

- Loop suppression
- CKM suppression
- GIM suppression

In which meson we get each of them? Which is from “a SM” and which from “the SM”?

CPV

What is CP

- A symmetry between a particle and its anti-particle
- CP is violated if we have

$$\Gamma(A \to B) \neq \Gamma(\bar{A} \to \bar{B})$$

- In the SM it is closely related to flavor
- We do not discuss the strong CP problem that is not directly related to flavor
- We also do not discuss the need for CP for baryogenesis

What to do with CPV

- The basic idea is to find processes where we can measure CPV
- In some cases they are very clean so we get sensitivity to the CKM matrix elements and phase
- Examples: $K_L \to \pi\pi$ and $B(t) \to \psi K_S$
- We can use flavor physics observables to test the CKM picture of the SM

How to find CPV

It is not easy to detect CPV

- Always need interference of two (or more) amplitudes
- CPT implies that the total widths of a particles and it anti-particles are the same, so we need at least two modes with CPV
- To see CPV we need 2 amplitudes with different weak and strong phases

All these phases

- Weak phase (CP-odd phase)
  - Phase in $\mathcal{L}$
  - In the SM they are only in the weak part so they are called weak phases

$$CP(Ae^{i\phi}) = Ae^{-i\phi}$$

Strong phase

- Strong phase (CP-even phase). Do not change under CP

$$CP(Ae^{i\delta}) = Ae^{i\delta}$$

- Due to time evolution

$$\psi(t) = e^{-iHt}\psi(0)$$

- For a free particle this is easy
- Yet, for resonances there are “rescattering” of hadrons
- Such strong phases are very hard to calculate

Why we need the two phases?

Intuitive argument

- If we have only one amplitude $|A|^2 = |\bar{A}|^2$
- If we have two but with a different of only a weak phase

$$\left|A + be^{i\phi}\right|^2 = \left|A + be^{-i\phi}\right|^2$$

- When both are not zero, we can get CP symmetries

$$\left|A + be^{i(\phi+\delta)}\right|^2 \neq \left|A + be^{i(-\phi+\delta)}\right|^2$$

Calculate the difference

CPV remarks

- The basic idea is to find processes where we can measure CPV
- In some cases they are clean so we get sensitivity to the phases of the UT (or of the CKM matrix)
- We can be sensitive to the CP phase without measuring CP violation
- Triple products and EDMs are also probes of CPV. I will not talk about that
- So far CPV was only found in meson decays, K, D, and B, and we will concentrate on that

The three types of CPV

The three classes of CPV

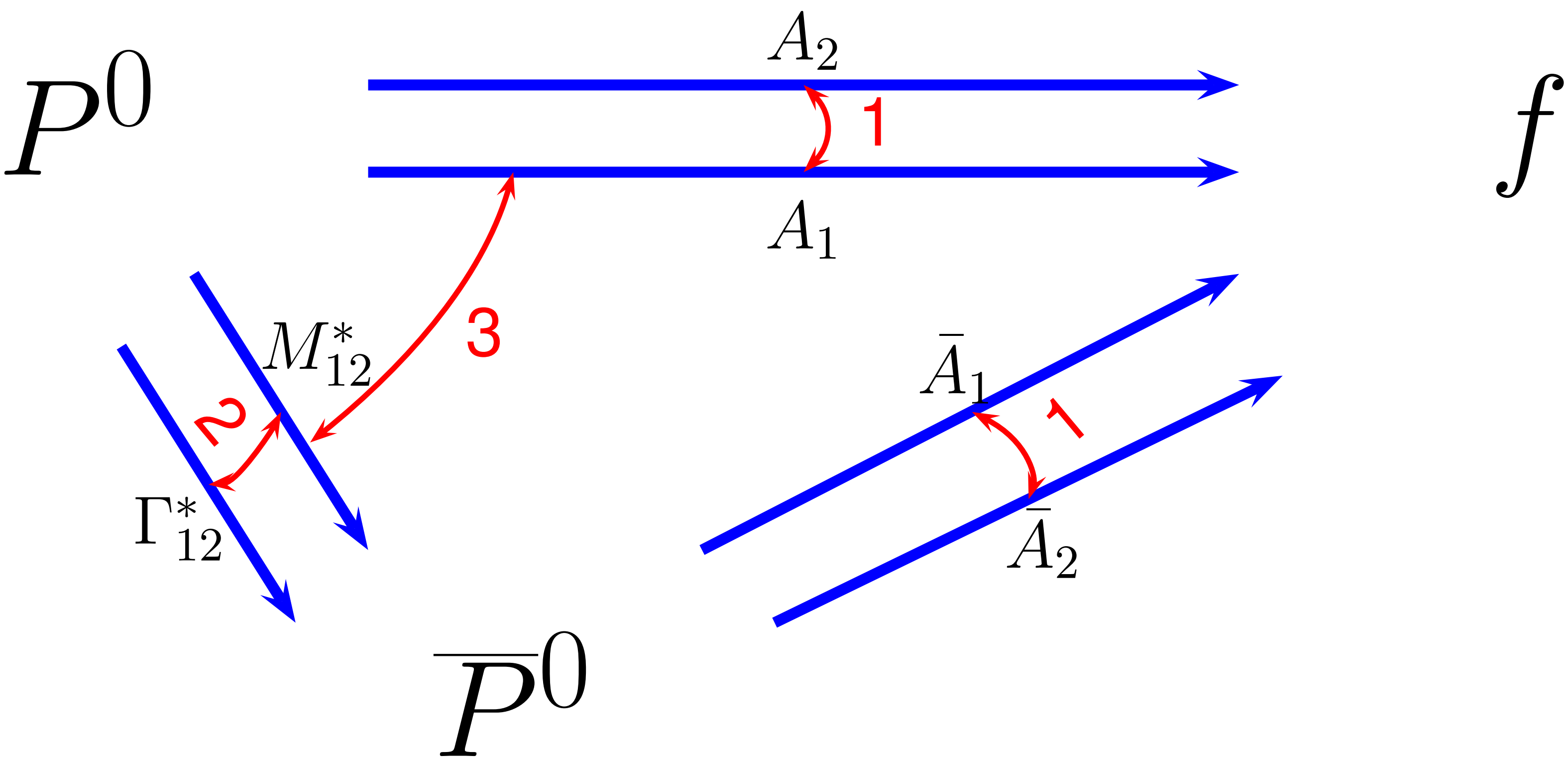
We need to find processes where we have two interfering amplitudes

- Two decay amplitudes
- Two oscillation amplitudes
- One decay and one oscillation amplitudes

Where the phases are coming from?

- Weak phases from the decay or mixing amplitudes
- Strong phase is the time evolution (mixing) or the rescattering (decay)

The 3 classes



- 1: Decay   2: Mixing   3: Mixing and decay

Type 1: CPV in decay

Two decay amplitudes

$$|A(B \to f)| \neq |A(\bar{B} \to \bar{f})|$$

- The way to measure it is via

$$a_{CP} \equiv \frac{\Gamma(\bar{B} \to \bar{f}) - \Gamma(B \to f)}{\Gamma(\bar{B} \to \bar{f}) + \Gamma(B \to f)} = \frac{|\bar{A}/A|^2 - 1}{|\bar{A}/A|^2 + 1}$$

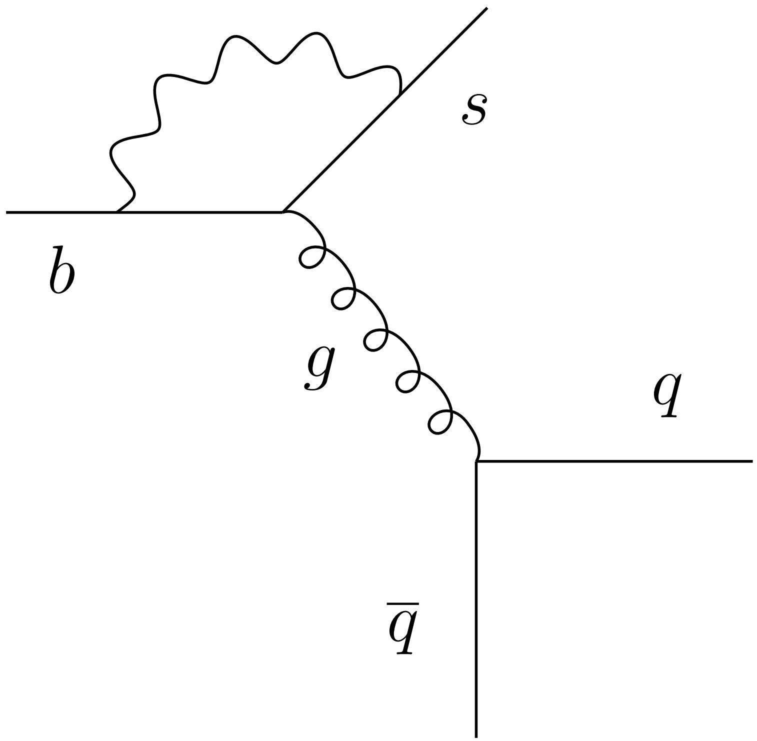
- When write the amplitude as $A\,(1 + r\exp[i(\phi + \delta)])$ then

$$a_{CP} = r \sin\phi \sin\delta + O(r^2)$$

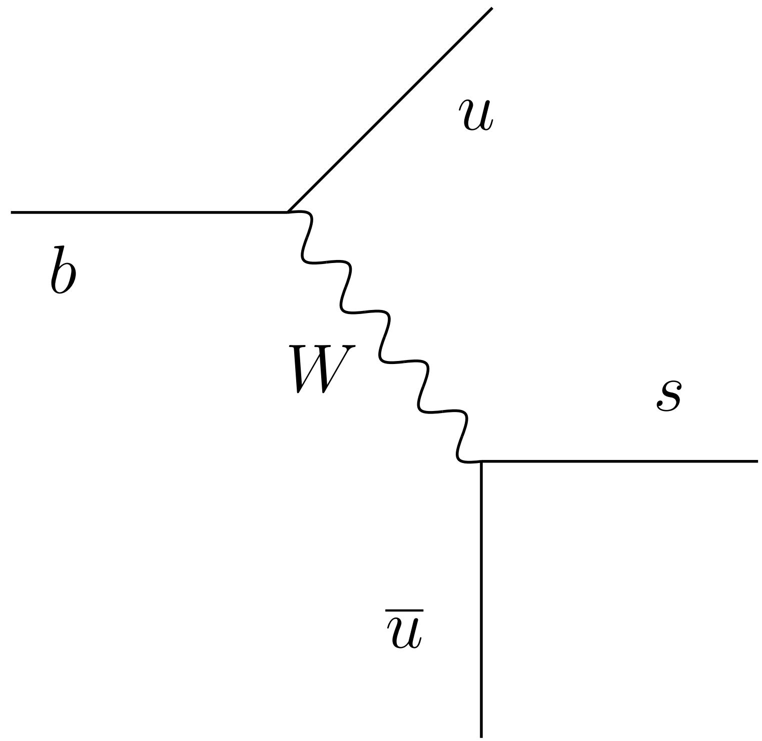
- If we know r and δ we can extract ϕ, the weak phase
- Work for decays of both charged and neutral hadrons

CPV in decay, example: $B \to K\pi$

(P) + (P_{EW})



(T)



P is a loop amplitude, but due to CKM factors $P/T \sim 3$. We also have a strong phase difference

One more example: $B \to DK$

- A bit more “sophisticated” example of CPV in decay
- Theoretically by far the cleanest measurement of any CKM parameter

Mixing formalism with CPV

When there is CPV the mixing formalism is more complicated. Diagonalizing the Hamiltonian we get

$$B_{H,L} = p|B\rangle \pm q|\bar{B}\rangle$$

- In general B_H and B_L are not orthogonal
- This is because they are “resonances” not asymptotic states. Open system
- The condition for the non orthogonality is CPV

2: CPV in mixing

The second kind of CPV is when it is pure in the mixing

$$|q| \neq |p| \qquad (B_{H,L} = p|B\rangle \pm q|\bar{B}\rangle)$$

We measure it by semileptonic asymmetries

- It was measured in

$$\frac{\Gamma(K_L \to \pi\ell^+\nu) - \Gamma(K_L \to \pi\ell^-\bar{\nu})}{\Gamma(K_L \to \pi\ell^+\nu) + \Gamma(K_L \to \pi\ell^-\bar{\nu})} = (3.32 \pm 0.06) \times 10^{-3}$$

- This is so far the only way we can define the electron microscopically!

3: CPV in interference mixing & decay

Interference between decay and mixing amplitudes

$$A(B \to f_{CP}) \qquad A(B \to \bar{B} \to f_{CP})$$

- Best with one decay amplitude
- Very useful when f is a CP eigenstate
- In that case $|A(B \to f_{CP})| = |A(\bar{B} \to f_{CP})|$

Some definitions

$$\lambda_f \equiv \frac{q}{p}\frac{\bar{A}_f}{A_f}$$

In the case of a CP final state

- $\lambda \neq \pm 1 \;\Rightarrow\;$ CPV
  - $|\lambda| \neq 1$ because $|A| \neq |\bar{A}|$. CPV in decay
  - $|\lambda| \neq 1$ because $|q| \neq |p|$. CPV in mixing
  - The $|\lambda| \approx 1$ and $\mathrm{Im}(\lambda) \neq 0$ vase is interference between mixing and decay
- We can have several types in any system
- In the clean cases we have one dominant source

Formalism

B at $t = 0$ compared to a $\bar{B}$ and let them evolve

$$a_{CP}(t) \equiv \frac{\Gamma(B(t) \to f) - \Gamma(\bar{B}(t) \to f)}{\Gamma(B(t) \to f) + \Gamma(\bar{B}(t) \to f)}$$

Consider the case where $|\lambda| = 1$

$$A_{CP}(t) = -\mathrm{Im}\lambda \sin \Delta m t$$

- We know Δm so we can measure $\mathrm{Im}\lambda$
- $\mathrm{Im}\lambda$: the phase between mixing and decay amplitudes
- When we have only one dominant decay amplitude all the hadronic physics cancel (YES!!!)
- In some cases this phase is $O(1)$

Example: $B \to \psi K_S$

Reminder ψ is a $\bar{c}c$, K_S is s and d

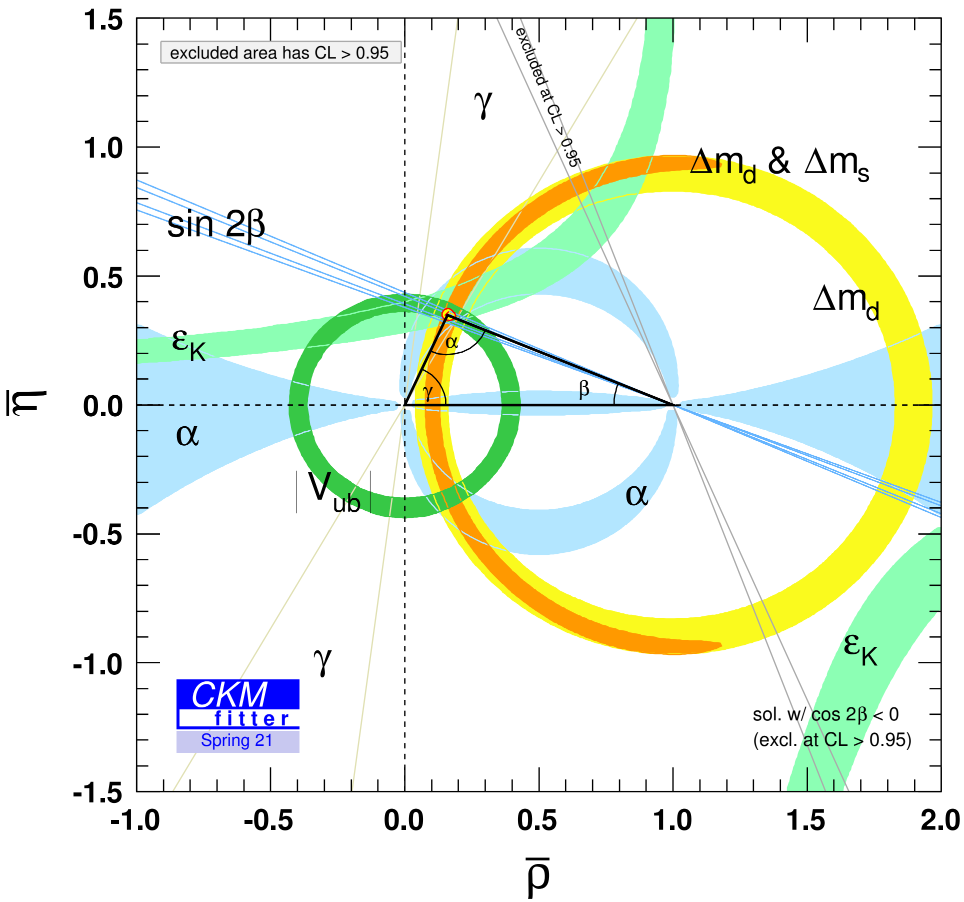
- One decay amplitude, tree level $A \propto V_{cb}V_{cs}^*$. In the standard parametrization it is real
- Very important: $|A| = |\bar{A}|$ to a very good approximation.
- In the standard parametrization $q/p = \exp(2i\beta)$ to a very good approximation
- We then get

$$\mathrm{Im}\lambda = \mathrm{Im}\left[\frac{q}{p}\frac{\bar{A}}{A}\right] = \sin 2\beta$$

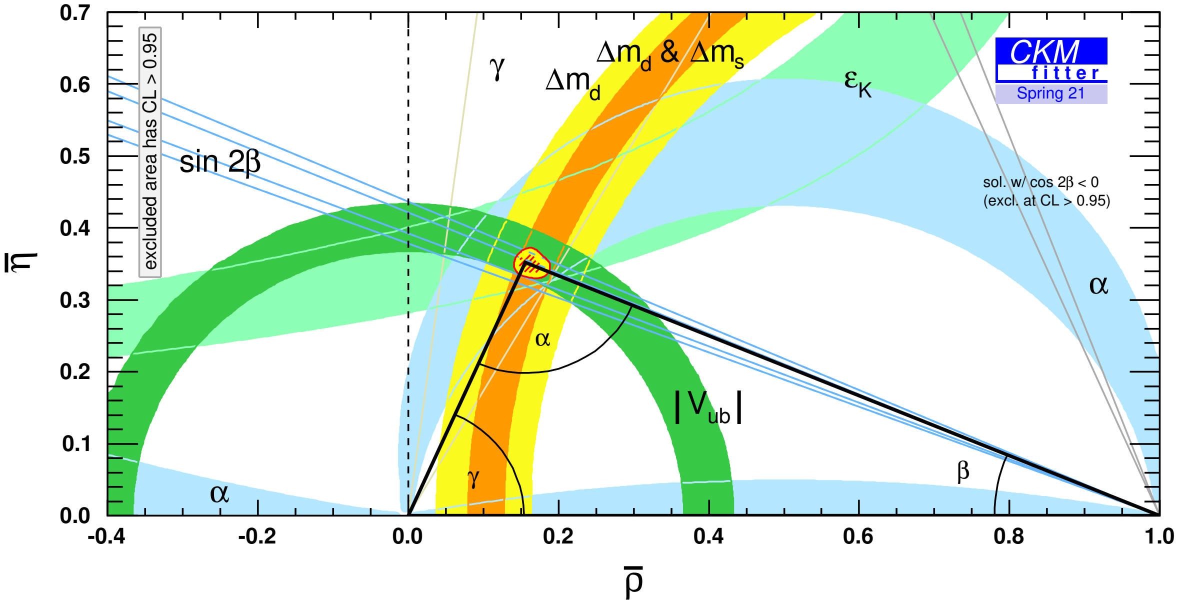
- For HW do some other decays: D^+D^-, $\pi^+\pi^-$, ϕK_S and $B_s \to \psi\phi$ (Ignore the subtleties)

Instead of summary

All together now



Zoom in



excluded area has CL > 0.95
sin 2β
γ
Δm_d
Δm_d & Δm_s
ε_K
CKM fitter
Spring 21
sol. w/ cos 2β < 0
(excl. at CL > 0.95)
α
|V_ub|
β
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0.0
-0.4
-0.2
0.0
0.2
0.4
0.6
0.8
1.0
η̄
ρ̄