On chiral MHD

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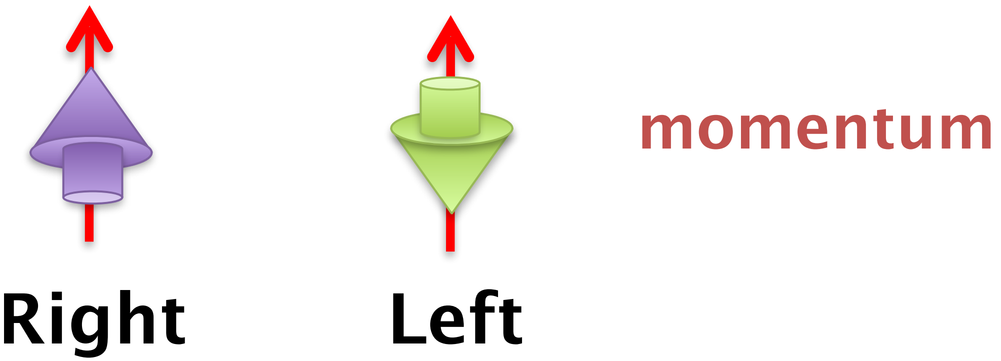
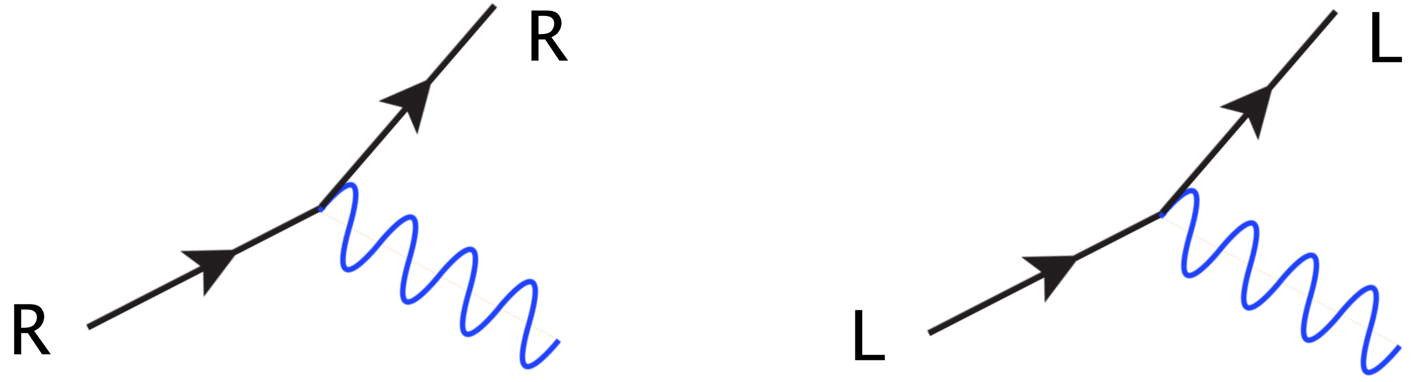
Outline

- Chiral MHD
- Higher-form symmetries
  & Nambu-Goldstone modes

Chiralities of massless fermions



- Chirality does not change through interaction classically



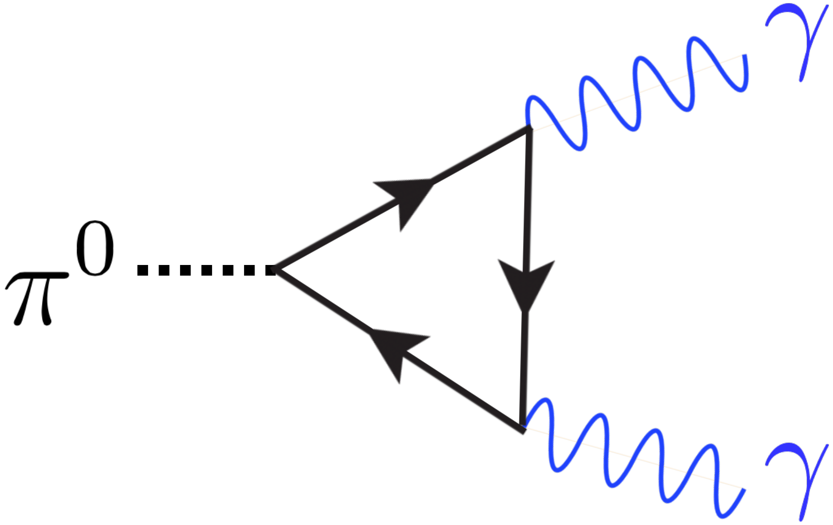
- **Quantum effects** breaks the chirality conservation

Chiral anomaly [Adler, Bell-Jackiw ’69]

Chiral anomaly

$$j^\mu_{\rm A} = j^\mu_{\rm R} - j^\mu_{\rm L}$$ Axial current

$$\partial_\mu j^\mu_{\rm A} = C_{\rm A} \boldsymbol{E} \cdot \boldsymbol{B}$$ $$C_{\rm A} = \frac{e^2}{2\pi^2}$$



Chiral Magnetic Effect (CME)

[Kharzeev-Warringa-Mclerran ’07]

$$j_{\mathrm{CME}} = C_{\mathrm{A}} \mu_{\mathrm{A}} \boldsymbol{B}$$

- Macroscopic transport
- Dissipationless (no heat production)
- Transport coefficient is universal

- Where does it happen?
  - Heavy-ion collisions/Early Universe
Dirac/Weyl semimetals

LETTERS
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nature physics

Chiral magnetic effect in $ZrTe_5$

Qiang Li[1*], Dmitri E. Kharzeev[2,3*], Cheng Zhang[1], Yuan Huang[4], I. Pletikosić[1,5], A. V. Fedorov[6], R. D. Zhong[1], J. A. Schneeloch[1], G. D. Gu[1] and T. Valla[1*]

TOPOLOGICAL MATTER
SCIENCE sciencemag.org

Evidence for the chiral anomaly in the Dirac semimetal Na_3Bi

Jun Xiong,[1] Satya K. Kushwaha,[2] Tian Liang,[1] Jason W. Krizan,[2] Max Hirschberger,[1] Wudi Wang,[1] R. J. Cava,[2] N. P. Ong[1*]

Giant negative magnetoresistance induced by the chiral anomaly in individual Cd_3As_2 nanowires

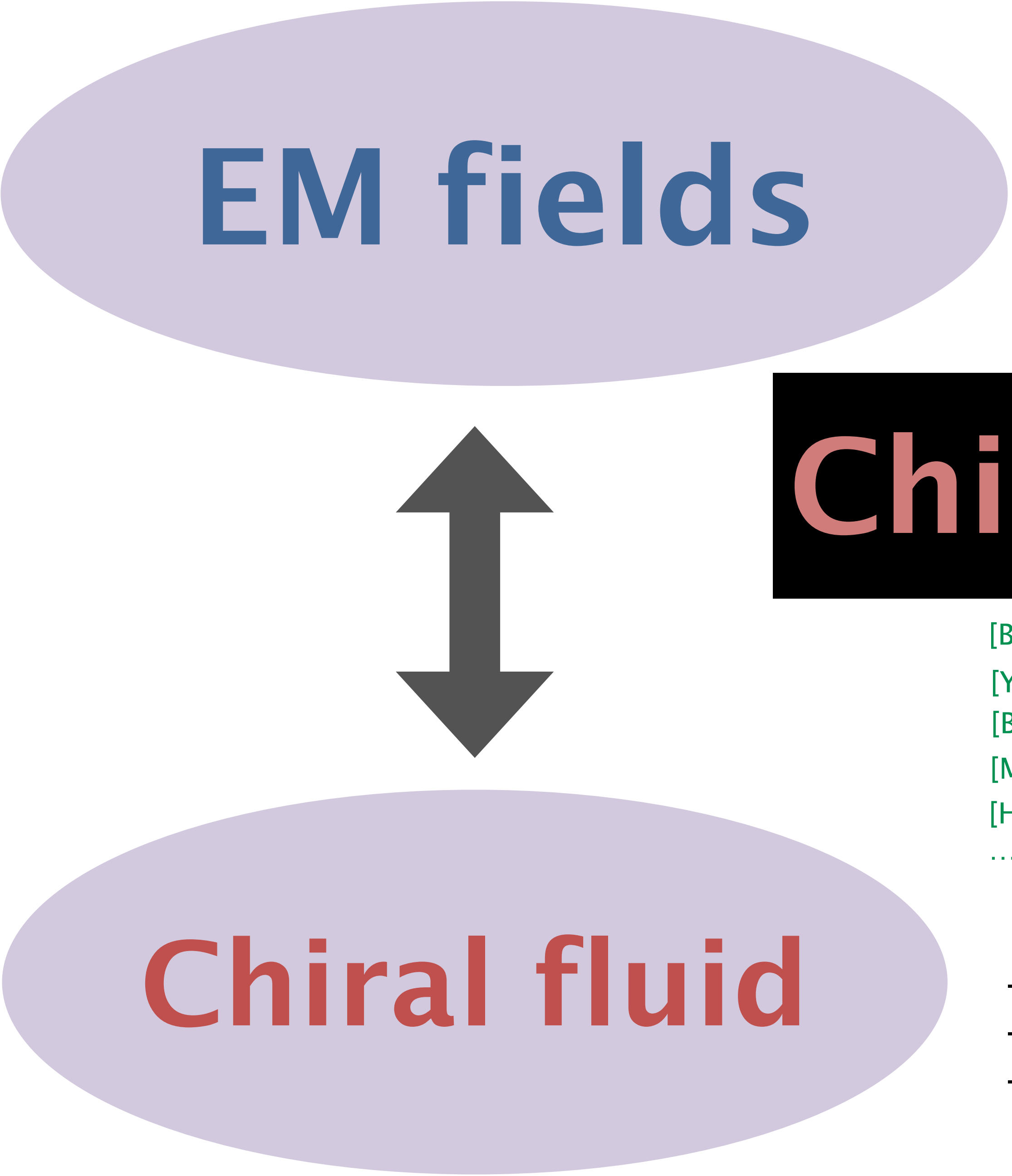
nature COMMUNICATIONS

Cai-Zhen Li[1,*], Li-Xian Wang[1,*], Haiwen Liu[2], Jian Wang[2,3], Zhi-Min Liao[1,3] & Da-Peng Yu[1,3]

Signatures of the Adler–Bell–Jackiw chiral anomaly in a Weyl fermion semimetal

Cheng-Long Zhang[1,*], Su-Yang Xu[2,*], Ilya Belopolski[2,*], Zhujun Yuan[1,*], Ziquan Lin[3], Bingbing Tong[1], Guang Bian[2], Nasser Alidoust[2], Chi-Cheng Lee[4,5], Shin-Ming Huang[4,5], Tay-Rong Chang[2,6], Guoqing Chang[4,5], Chuang-Han Hsu[4,5], Horng-Tay Jeng[6,7], Madhab Neupane[2,8,9], Daniel S. Sanchez[2], Hao Zheng[2], Junfeng Wang[3], Hsin Lin[4,5], Chi Zhang[1,10], Hai-Zhou Lu[11], Shun-Qing Shen[12], Titus Neupert[13], M. Zahid Hasan[2] & Shuang Jia[1,10]

nature COMMUNICATIONS



EM fields

Chiral fluid

Chiral MHD

[Boyarsky-Fröhlich-Ruchayskiy '15]

[Yamamoto ’16]

[Brandenburg et. al. '17]

[Masada-Kotake-Takiwaki-Yamamoto '18]

[Hattori-Hirono-Yee-Yin ’19]

…

Physical systems:

- Heavy-ion collisions
- Early Universe
- Neutrino matter in core-collapse supernovae
- Weyl/Dirac semimetals?

Hydrodynamics

- Low-energy theory based on conservation law & derivative expansion
- Degrees of freedom:
  - **Conserved densities**: particle number, energy, momentum, etc.

$$\{n, e, \boldsymbol{v} \cdots\}$$

  - **Nambu-Goldstone modes**
- "Hydrodynamic variables"
- Time evolution is given by the conservation law

$$\partial_t n + \nabla \cdot \boldsymbol{j} = 0$$

Magnetohydrodynamics (MHD)

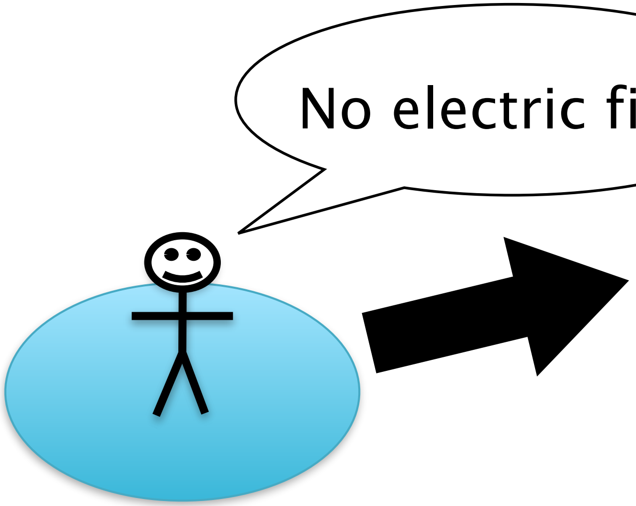
- Hydro variables $\{e(x), u^{\mu}(x), B^{\mu}(x)\}$

$$E^{\mu} := F^{\mu\nu}u_{\nu} \quad B^{\mu} := \tilde{F}^{\mu\nu}u_{\nu} \quad \tilde{F}^{\mu\nu} := \frac{1}{2}\epsilon^{\mu\nu\rho\sigma}F_{\rho\sigma}$$

- EOM

$$\partial_{\mu}T^{\mu\nu}_{\mathrm{tot}} = 0 \qquad \partial_{\mu}\tilde{F}^{\mu\nu} = 0$$

Ideal MHD: no electric field in the fluid frame



$$E^{\mu}_{(0)} = 0$$

In the lab-frame variable,

$$\boldsymbol{E} + \boldsymbol{v} \times \boldsymbol{B} = 0$$

Corresponds to large conductivity

Constitutive relation for ideal MHD

$$T^{\mu\nu}_{\mathrm{fluid}(0)} = (e+p)u^\mu u^\nu - p\eta^{\mu\nu}$$

$$T^{\mu\nu}_{\mathrm{EM}(0)} = \boldsymbol{B}^2 \left[u^\mu u^\nu - b^\mu b^\nu - \frac{1}{2}\eta^{\mu\nu} \right]$$

$$B^\mu = |\boldsymbol{B}| b^\mu \quad b_\mu b^\mu = -1$$

$$F^{\mu\nu}_{(0)} = \epsilon^{\mu\nu\rho\sigma} u_\rho B_\sigma$$

Chiral MHD

- Hydro variables $\{e(x), u^\mu(x), B^\mu(x), n_{\mathrm{A}}(x)\}$

$$E^\mu := F^{\mu\nu} u_\nu \quad B^\mu := \tilde{F}^{\mu\nu} u_\nu \qquad \tilde{F}^{\mu\nu} := \frac{1}{2}\epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}$$

- EOM

$$\partial_\mu T^{\mu\nu}_{\mathrm{tot}} = 0 \qquad \partial_\mu \tilde{F}^{\mu\nu} = 0 \qquad \partial_\mu j^\mu_{\mathrm{A}} = C E_\mu B^\mu$$

- Requirement of the 2nd law of thermodynamics leads to a constitutive relation including CME

[Hattori-Hirono-Yee-Yin ’19]

Constitutive relation for ideal CMHD

$$T^{\mu\nu}_{\mathrm{fluid}(0)} = (e+p)u^\mu u^\nu - p\eta^{\mu\nu}$$

$$T^{\mu\nu}_{\mathrm{EM}(0)} = \boldsymbol{B}^2 \left[u^\mu u^\nu - b^\mu b^\nu - \frac{1}{2}\eta^{\mu\nu} \right]$$

$$B^\mu = |\boldsymbol{B}| b^\mu \quad b_\mu b^\mu = -1$$

$$F^{\mu\nu}_{(0)} = \epsilon^{\mu\nu\rho\sigma} u_\rho B_\sigma$$

$$j^\mu_{\mathrm{A}(0)} = n_{\mathrm{A}} u^\mu$$

CME doesn't play any role

First order in derivative expansion

$$T^{\mu\nu}_{\mathrm{fluid}(1)} = \zeta \Delta^{\mu\nu} \partial \cdot u + 2\eta \nabla^{<\mu} u^{\nu>}$$

$$\sigma E^{\mu}_{(1)} = \epsilon^{\mu\nu\alpha\beta} u_\nu \partial_\alpha B_\beta + \epsilon^{\mu\nu\alpha\beta} u_\nu B_\alpha \partial_\beta \ln T - \underbrace{\sigma_B B^\mu}_{\text{CME}}$$

σ : electric conductivity

$\sigma_B := C_{\mathrm{A}} \mu_{\mathrm{A}}$

First order in derivative expansion

EM tensor and field strength are modified accordingly

$$T^{\mu\nu}_{\mathrm{EM}(1)} = 2\epsilon^{(\mu\alpha\beta}u^{\nu)}E_{\alpha(1)}B_{\beta}$$

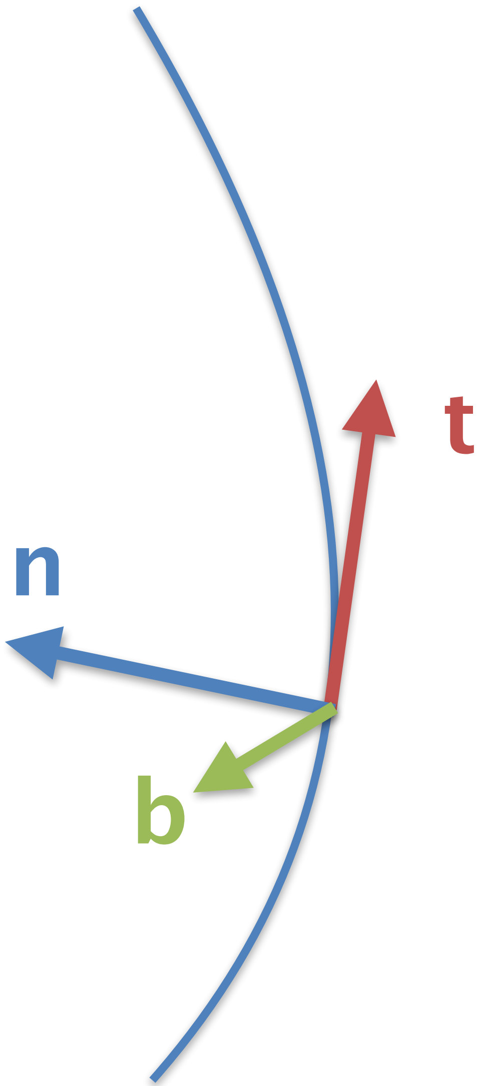
$$\tilde{F}^{\mu\nu}_{(1)} = \epsilon^{\mu\nu\rho\sigma}E_{(1)\rho}u_{\sigma}$$

➡

$$\partial_\mu\left[T^{\mu\nu}_{\mathrm{tot}(0)} + T^{\mu\nu}_{\mathrm{tot}(1)}\right] = 0$$

$$\partial_\mu\left[\tilde{F}^{\mu\nu}_{(0)} + \tilde{F}^{\mu\nu}_{(1)}\right] = 0$$

Force from $\mathbf{B}$



Stress tensor

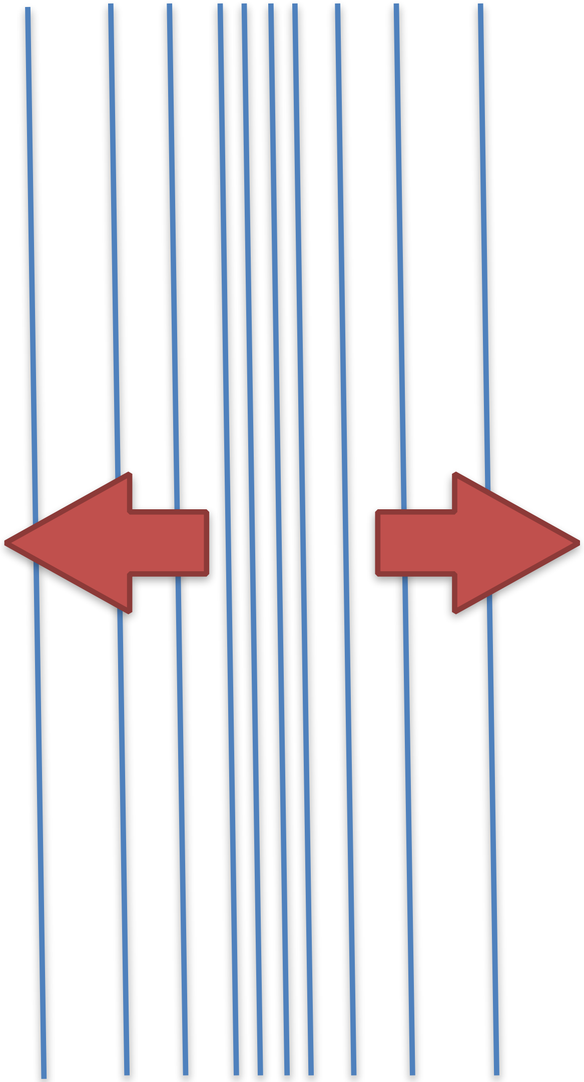
$$T_{ij} = B_i B_j - \frac{1}{2} B^2 \delta_{ij}$$

Force = $\partial_i T_{ij}$

$$\boldsymbol{F} = B^2 \kappa \boldsymbol{n} - \nabla_\perp \frac{1}{2} B^2$$

κ : curvature of $\mathbf{B}$ field line

Force from **B**



Stress tensor

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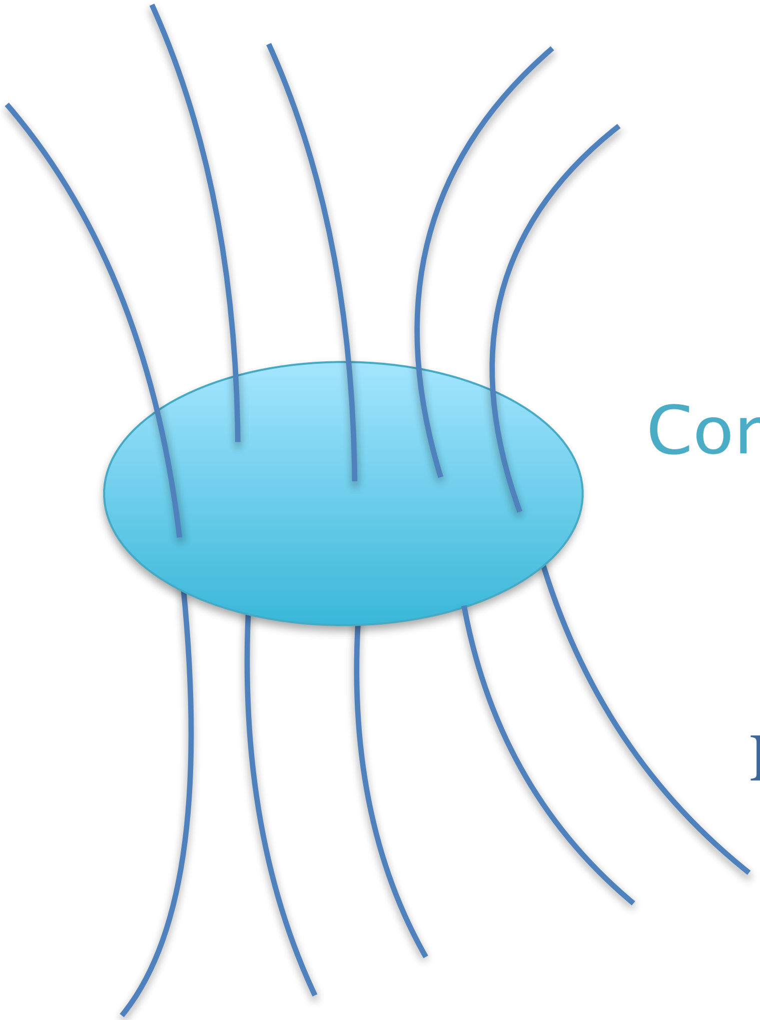
Force = $\partial_i T_{ij}$

$$\boldsymbol{F} = B^2 \kappa \boldsymbol{n} - \nabla_{\perp} \frac{1}{2} B^2$$

κ : curvature of **B** field line

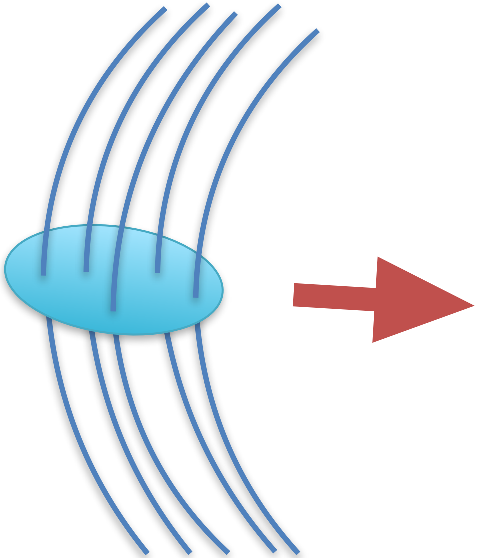
Frozen in of **B** field lines



Conducting fluid

B field lines

Alfven wave



force

B field lines

Momentum along **B**

Dispersion $\omega = \pm v_A \underset{\uparrow}{k_{||}}$

$$v_A^2 = \frac{\boldsymbol{B}^2}{e + p + \boldsymbol{B}^2}$$

Alfven velocity

Alfven wave: with dissipation

Dispersion relation

$$\omega = \pm v_A k_{||} - \underbrace{\frac{i}{2}\left[\bar{\eta} + \lambda\right] k^2}_{\textbf{Damping}}$$

$$\lambda = \frac{1}{\sigma}$$

$$\bar{\eta} \equiv \frac{\eta}{e + p + \boldsymbol{B}^2}$$

σ : electric conductivity

η : shear viscosity

Alfven wave in chiral MHD

Including CME,

$$\omega = \pm v_A k_{||} - \frac{i}{2}(\bar{\eta} + \lambda) k_{||}^2 + \underbrace{\frac{i}{2} s \epsilon_B k_{||}}_{\text{CME}}$$

s indicates the helicity of the mode

$$\epsilon_B := \frac{C_{\mathrm{A}} \mu_{\mathrm{A}}}{\sigma}$$

$$i\boldsymbol{k} \times \underbrace{\boldsymbol{e}^{(s)}}_{\text{helicity eigenstate}} = sk\boldsymbol{e}^{(s)}$$

Alfven wave in chiral MHD

Instability appears
in positive(or negative)-helicity mode

If $\sigma_B > 0$,

- Negative helicity modes decay
- Positive helicity modes with $k < k_c$ exponentially grow

$$k_c = \frac{\sigma_B}{1 + \underset{\cdots\cdots}{\bar{\eta}\sigma}}$$

dissipative effects tame the instability

Bianchi identity

$$\partial_\mu \tilde{F}^{\mu\nu} = 0$$

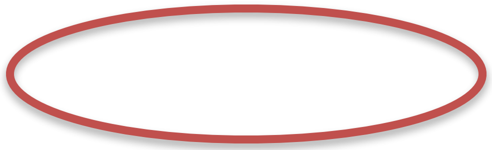
- This can be regarded as a conservation law of charges associated with a $U(1)$ 1-form symmetry

Higher-form symmetries

[Gaiotto-Kapustin-Seiberg-Willett '15]

- Usual symmetry: $\psi_i(x) \mapsto R_i{}^j(g)\,\psi_j(x)$
- Higher-form symmetry
  - Charged objects are **extended**
    - "p-form symmetry" → charged obj. is p-dimensional

0-form symmetry



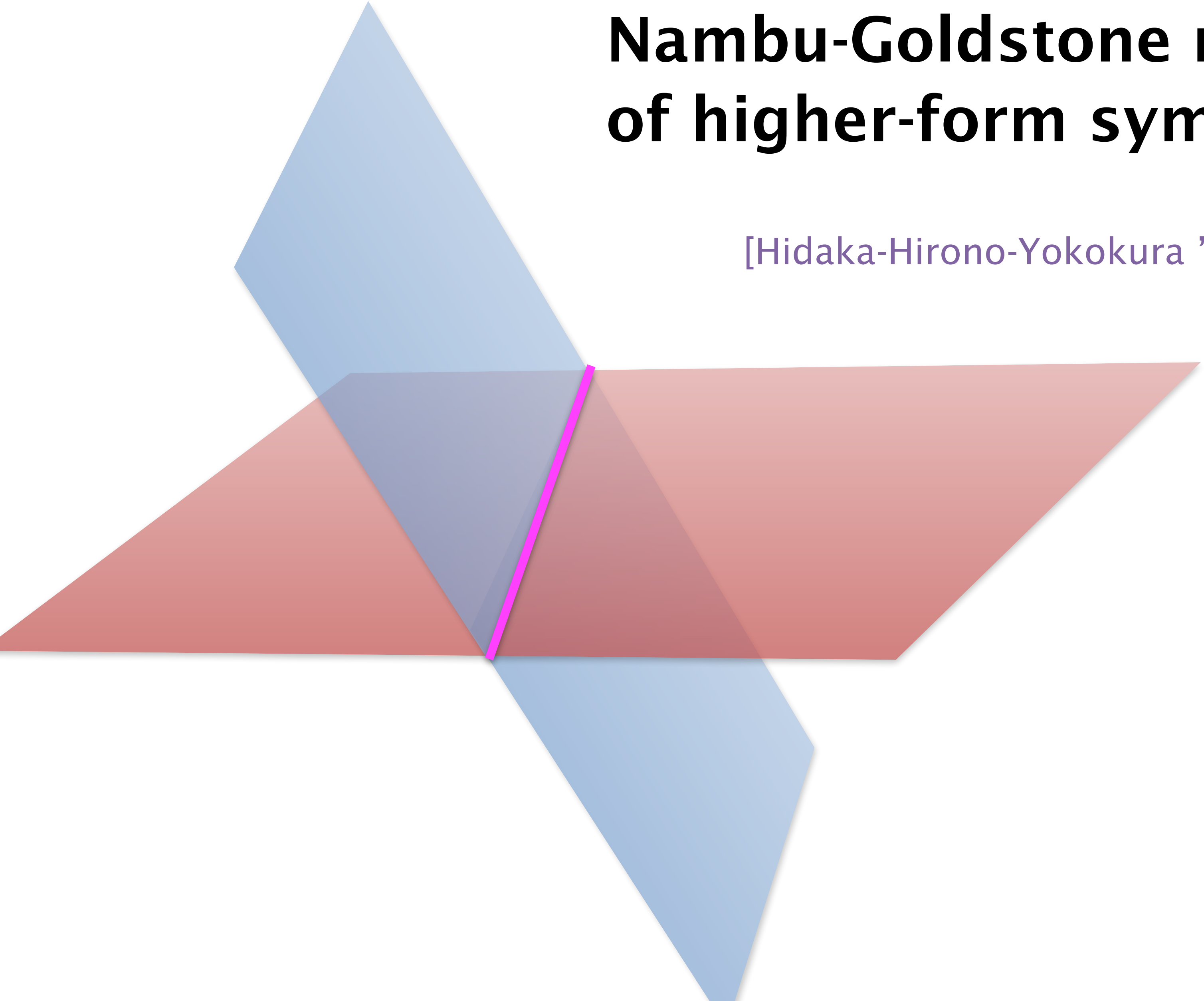
1-form symmetry

- $U(1)^{[p]}$ symmetry: $W(C_p) \to e^{i\alpha}\, W(C_p)$

Higher-form symmetries

[Gaiotto-Kapustin-Seiberg-Willett ’15]

- Provides us with a unifying viewpoint
  - Classification of phases
    - Certain topological orders are understood as a consequence of broken discrete HFS
  - Nambu-Goldstone theorem
  - ’t Hooft anomalies
    - Constraints on the low-energy physics
  - Formulation of MHD [Grozdanov-Hofman-Iqbal ’17], …



Counting the number of Nambu-Goldstone modes of higher-form symmetries

[Hidaka-Hirono-Yokokura ’20]

Spontaneous symmetry breaking → low-energy d.o.f

- Examples of Nambu-Goldstone (NG) modes
    - pions in QCD
    - magnons
    - superfluid phonons (He^4, He^3, atomic BEC)
    - lattice phonons
    - Kelvon (vibration of a vortex string)
    - photons
    - ...

Number of NG modes

- (Internal) symmetry breaking $G \to H$
- Lorentz invariance

$$N_{\mathrm{NG}} = N_{\mathrm{BG}}$$

\# of massless NG modes

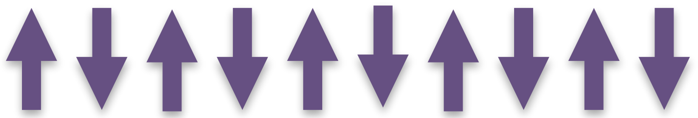
\# of broken generators

- Dispersion relation: $\omega = ck \qquad k = |\mathbf{k}|$

Magnets:Heisenberg model $H = J \sum_{\langle i,j \rangle} \vec{s}_i \cdot \vec{s}_j$

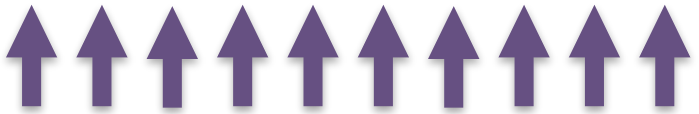
- $J > 0$: Anti-ferromagnetism

$N_{\mathrm{NG}} = 2 \qquad \omega \propto k$



- $J < 0$: Ferromagnetism

$N_{\mathrm{NG}} = 1 \qquad \omega \propto k^2$



Symmetry breaking pattern is common $SO(3) \to SO(2)$

$N_{\mathrm{BG}} = 2$

Thermodynamic properties: heat capacity

- When $\omega \sim k^n$

heat capacity behaves as

$$C \sim T^{\frac{d}{n}}$$

d : spatial dimension

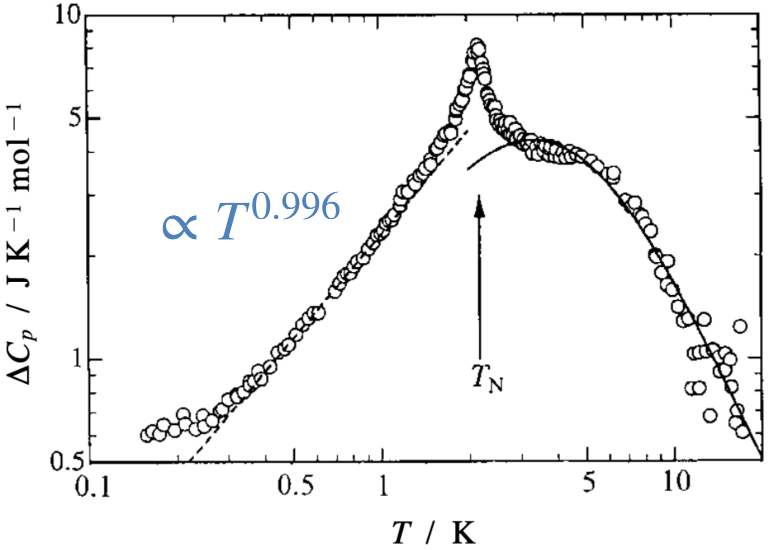


Fig.9 Excess heat capacities of MnCu(obbz)・5H_2O around the antiferromagnetic phase transition temperature T_N. Solid line indicates the theoretical heat capacity curve estimated by the high-temperature series expansion for $S=2$ one-dimensional ferromagnetic Heisenberg model with $J/k_B=0.75$ K. The broken straight line shows the heat capacity due to the spin-wave excitation.[38)]

[K. Asano, et. al., Bull. Chem. Soc. Jpn. 72, 1749 (1999).]

What’s the difference?

- Anti-ferro: $\langle \hat{s}_z \rangle = 0$
- Ferro: $\langle \hat{s}_z \rangle = \langle [\hat{s}_x, \hat{s}_y] \rangle \neq 0$

- $\langle [Q_\alpha, Q_\beta] \rangle = 0$ for $^\forall \alpha, \beta \;\to\; N_{\mathrm{NG}} = N_{\mathrm{BG}}$

[Schafer-Son-Stephanov-Toublan-Verbaarschot '01]

Number of NG modes

- Non-relativistic systems

$$N_{\rm NG} = N_{\rm BG} - \frac{1}{2}{\rm rank}\,\langle [Q_\alpha, Q_\beta] \rangle$$

# of massless NG modes

# of broken generators

[Watanabe-Brauner '11]

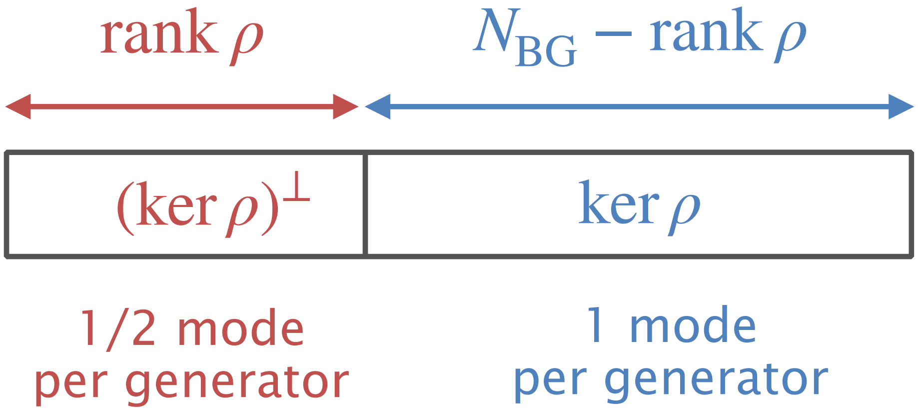
[Watanabe-Murayama’12] [Hidaka’12]

- Dispersion relation
  - Type A: $\omega \sim k$
  - Type B: $\omega \sim k^2$

Derivation by effective field theories

- Up to 2nd order in fields & derivatives,

$$\mathscr{L} = \rho_{AB}\,\pi^A \dot{\pi}^B + \frac{1}{2} G_{AB}\,\dot{\pi}^A \dot{\pi}^B - \frac{1}{2} \bar{G}_{AB}\,\partial_i \pi^A \partial^i \pi^B$$



$\mathrm{rank}\,\rho$

$N_{\mathrm{BG}} - \mathrm{rank}\,\rho$

$(\ker\rho)^{\perp}$

$\ker\rho$

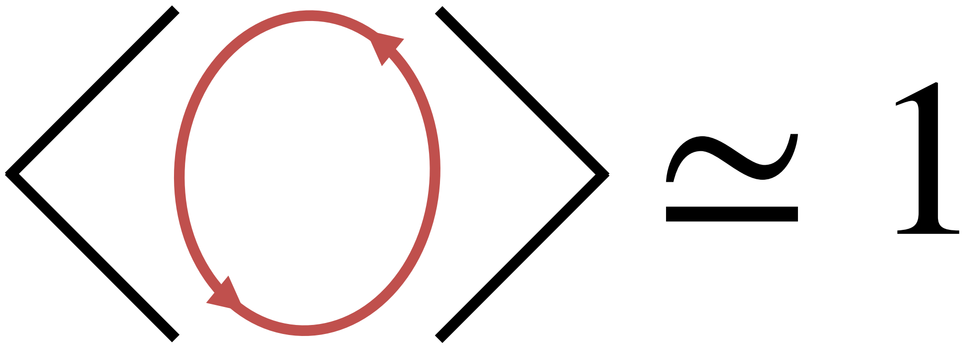
1/2 mode
per generator

1 mode
per generator

$$N_{\mathrm{NG}} = \frac{1}{2} \cdot \mathrm{rank}\,\rho \; + \; 1 \cdot (N_{\mathrm{BG}} - \mathrm{rank}\,\rho) \;\; = N_{\mathrm{BG}} - \frac{1}{2} \mathrm{rank}\,\rho$$

NG modes for higher-form symmetries

- Spontaneous breaking of a continuous HFS → massless particle



Perimeter law
or Coulomb law

- Breaking of $U(1)_e^{[1]}$ 1-form symmetry → photons

- Questions
  - How many NG modes?
  - Dose $\langle [Q_\alpha, Q_\beta] \rangle$ affect the number of NG modes?
    - cf. Non-commuting 0-form and 1-form charges

[Sogabe-Yamamoto'19]

Generalized formula to count NG modes

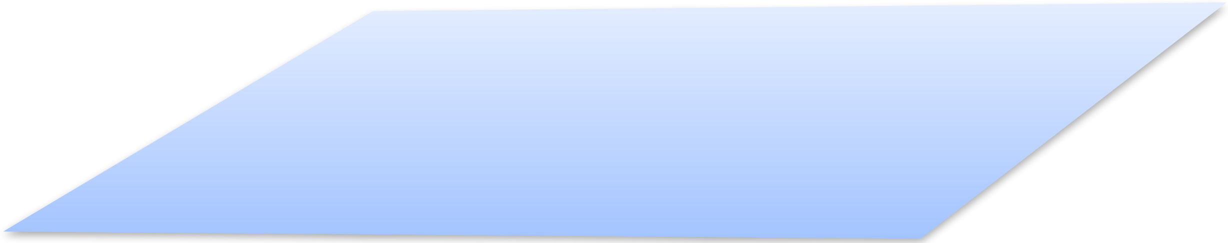
$$N_{\mathrm{NG}} = \sum_A {}_{D-2}C_{p_A} - \frac{1}{2}\mathrm{rank}\,\langle [Q_\alpha, Q_\beta] \rangle$$

[Hidaka-Hirono-Yokokura ‘20]

- SSB of internal symmetries (that can include higher-form)
- D-dim Minkowski space $\mathbb{R}^{1,D-1}$
  - Lorentz symmetry doesn’t have to be there
  - Translational symmetry is intact
- “A” is a label for a p_A-form symmetry

Generalized formula to count NG modes

- Symmetry generators Q_α should be enumerated taking into account how to place them
  - For 0-form symmetries, $Q(V)$ is $(D-1)$-dimensional
  - For a p-form symmetry, the generator is $D-p-1$ -dimensional
  - Ex) 1-form-symmetry in (3+1)-dim. → gen. is 2-dim.



$$U(e^{i\alpha}, S) = e^{i\alpha Q^{[1]}(S)}$$

Generalized formula to count NG modes

$$N_{\mathrm{NG}} = \sum_{A} {}_{D-2}C_{p_A} - \frac{1}{2}\mathrm{rank}\,\langle [Q_\alpha, Q_\beta] \rangle$$

⇓ When all the symmetries are 0-form

$$N_{\mathrm{NG}} = N_{\mathrm{BG}} - \frac{1}{2}\mathrm{rank}\,\langle [Q_\alpha, Q_\beta] \rangle$$

Reproduce the previous result

[Watanabe-Brauner '11]

[Watanabe-Murayama’12] [Hidaka’12]

Generalized formula to count NG modes

$$N_{\mathrm{NG}} = \sum_{A} {}_{D-2}C_{p_A} - \frac{1}{2}\mathrm{rank}\,\langle [Q_\alpha, Q_\beta] \rangle$$

- Photons in $(3+1)$-dimensions
  - Broken symmetry: $U(1)_e^{[1]}$

$$N_{\mathrm{NG}} = {}_{D-2}C_p = 2 \qquad D = 4,\ \ p = 1$$

Ex) Photons under a θ gradient

[Yamamoto '16]

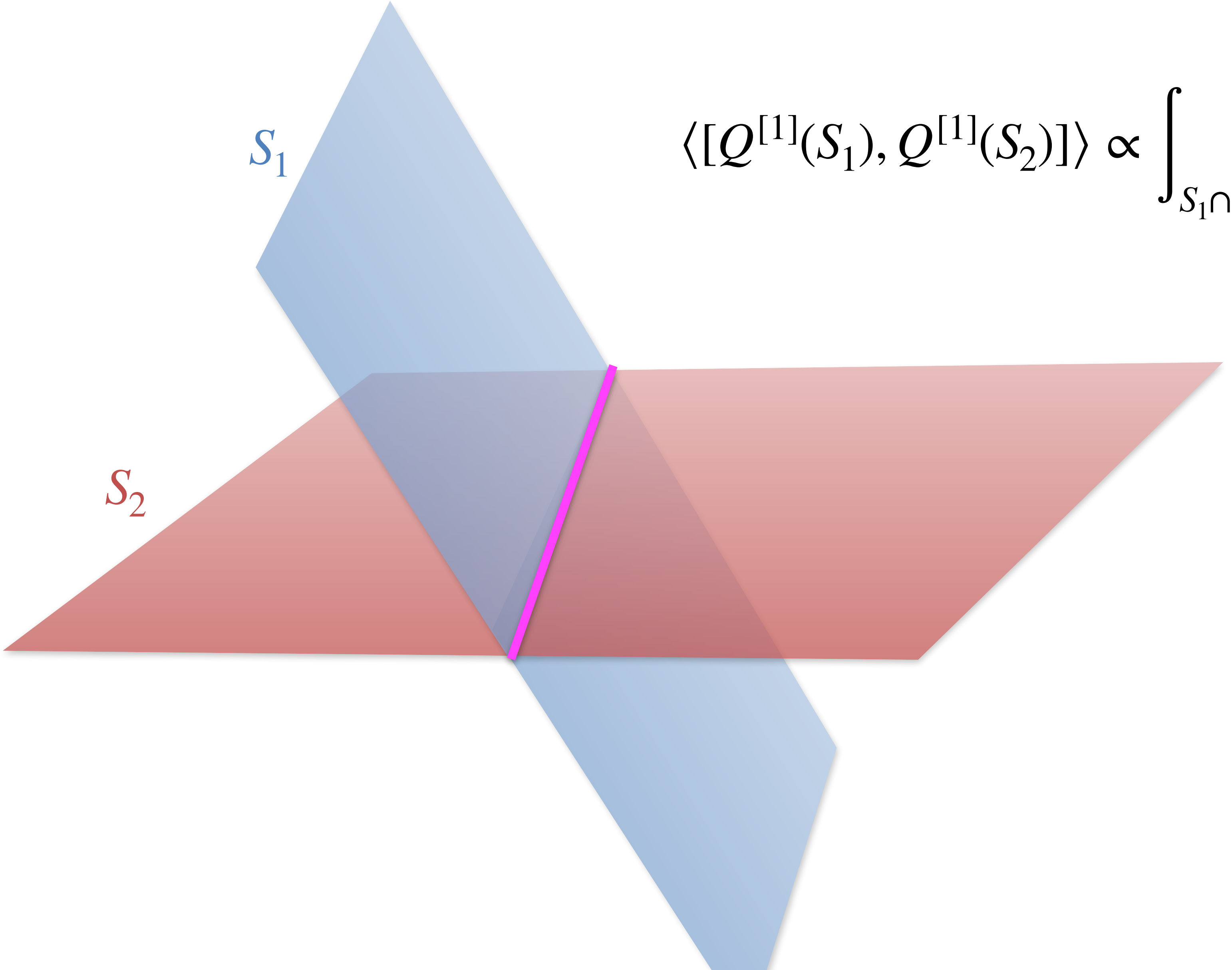
- Lagrangian

$$L = -\frac{1}{2e^2} f \wedge \star f - \frac{\theta}{2\pi} f \wedge f$$

- $d\theta = C = C_i \, dx^i \neq 0$
- Broken symmetry: $U(1)_e^{[1]}$
  - Conserved charge: $Q^{[1]}(S)$
- Charge commutators $\langle [Q^{[1]}(S_1), Q^{[1]}(S_2)] \rangle \propto \int_{S_1 \cap S_2} d\theta \neq 0$

Ex) Photons under a θ gradient

$$\langle [Q^{[1]}(S_1), Q^{[1]}(S_2)] \rangle \propto \int_{S_1 \cap S_2} d\theta \neq 0$$



Ex) Photons under a θ gradient

- Independent generators: $\{Q^{[1]}(S_x), Q^{[1]}(S_y), Q^{[1]}(S_z)\}$

  S_i is a 2-dim surface perpendicular to i-axis

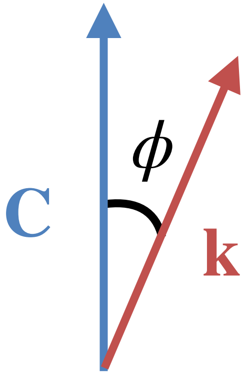
- Charge commutators

$$M_{ij} = \langle\, [Q^{[1]}(S_i), Q^{[1]}(S_j)]\,\rangle \propto \epsilon_{ijk} C_k$$

- Number of NG modes

$$N_{\mathrm{NG}} = {}_{D-2}C_1 - \frac{1}{2}\mathrm{rank}\, M_{ij} = 2 - \frac{1}{2}\cdot 2 = 1$$

Ex) Photons under a θ gradient



Dispersion relation

$$\omega^2(k) = \begin{cases} C^2 + (2 - \sin^2\phi)k^2 + O(k^4) & \text{massive} \\ \sin^2\phi\, k^2 + \dfrac{\cos^4\phi}{C^2}k^4 + O(k^6) & \text{massless} \end{cases}$$

$$C := |\mathbf{C}|$$

- $\omega \sim k^2$ when $\mathbf{C} \,||\, \mathbf{k}$
- $\omega \sim k$ for other angles
- No generic properties about dispersion relations for higher-form symmetries?

Derivation: effective field theories

- EFT of NG modes
  - Write down terms consistent with the symmetries
  - Derivative expansion
- EFT for broken higher-form symmetries
  - Extension of the coset construction

Summary

- Chiral MHD
  - Low-energy theory of chiral matter & EM fields
  - Formation of helical **B** configurations

- Higher-form symmetries & Nambu-Goldstone modes
  - Formula to count the number
  - EFT of NG modes for higher-form symmetries
    - Extension of the coset construction