Angular momentum distribution in the proton

Yoshitaka Hatta

BNL/RIKEN BNL

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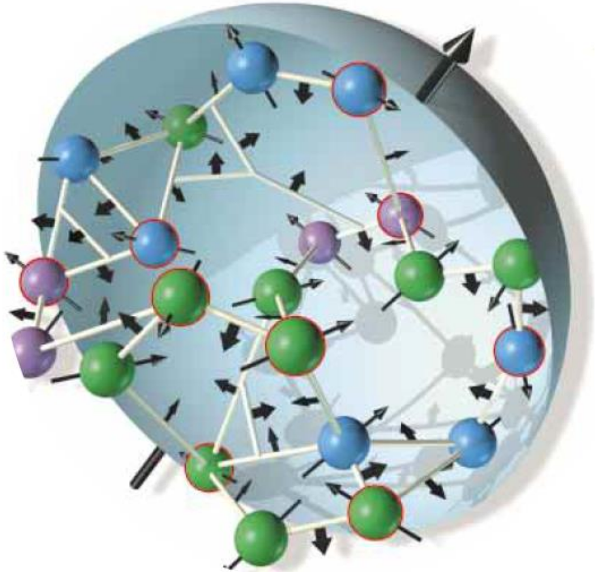
The proton spin problem



The proton has spin ½.

The proton is not an elementary particle.

$$\Rightarrow \quad \frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L^q + L^g$$

Jaffe-Manohar sum rule

$$= \frac{1}{2}\Delta\Sigma + L^q_{kin} + J_g$$

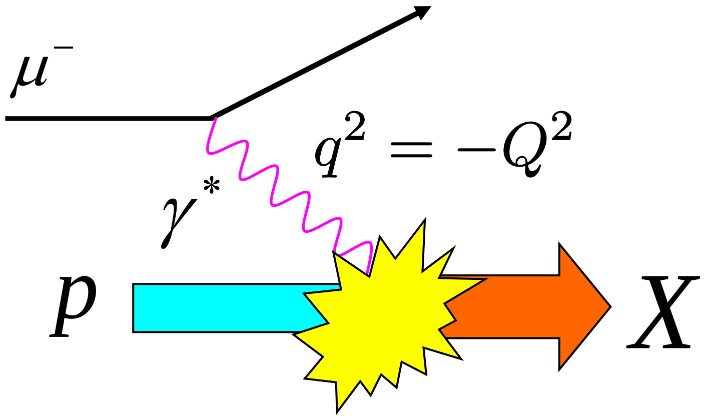
Ji sum rule

$\Delta\Sigma = 1$ in the naïve quark model

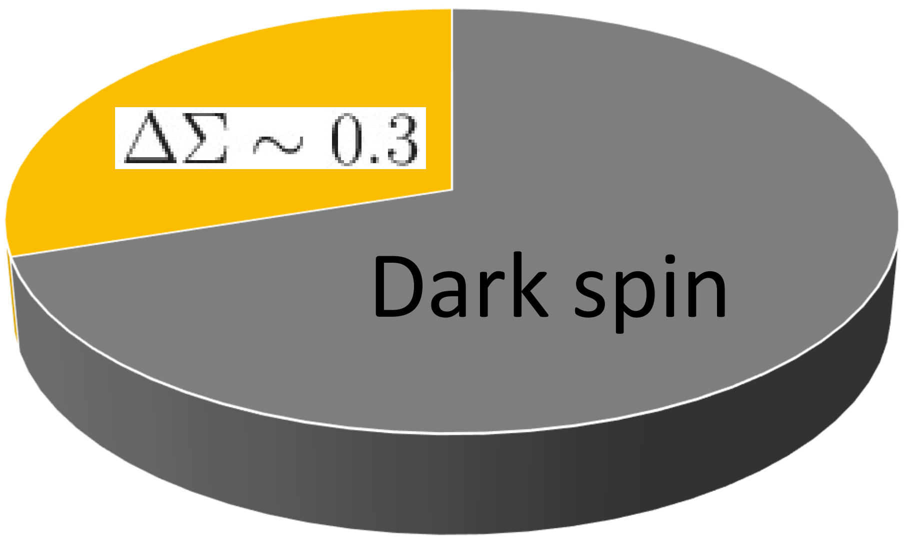
$\Delta\Sigma$ from polarized DIS

Longitudinal double spin asymmetry in polarized DIS

$$A_{LL} = \frac{\mu^\uparrow p^\downarrow - \mu^\uparrow p^\uparrow}{\mu^\uparrow p^\uparrow + \mu^\uparrow p^\downarrow} \sim \left(1 + \frac{\sigma_L}{\sigma_T}\right) \frac{2x g_1}{F_2}$$



$$\int_0^1 dx g_1(x) = \frac{1}{9}(\Delta u + \Delta d + \Delta s) + \frac{1}{12}(\Delta u - \Delta d) + \frac{1}{36}(\Delta u + \Delta d - 2\Delta s) + \mathcal{O}(\alpha_s)$$



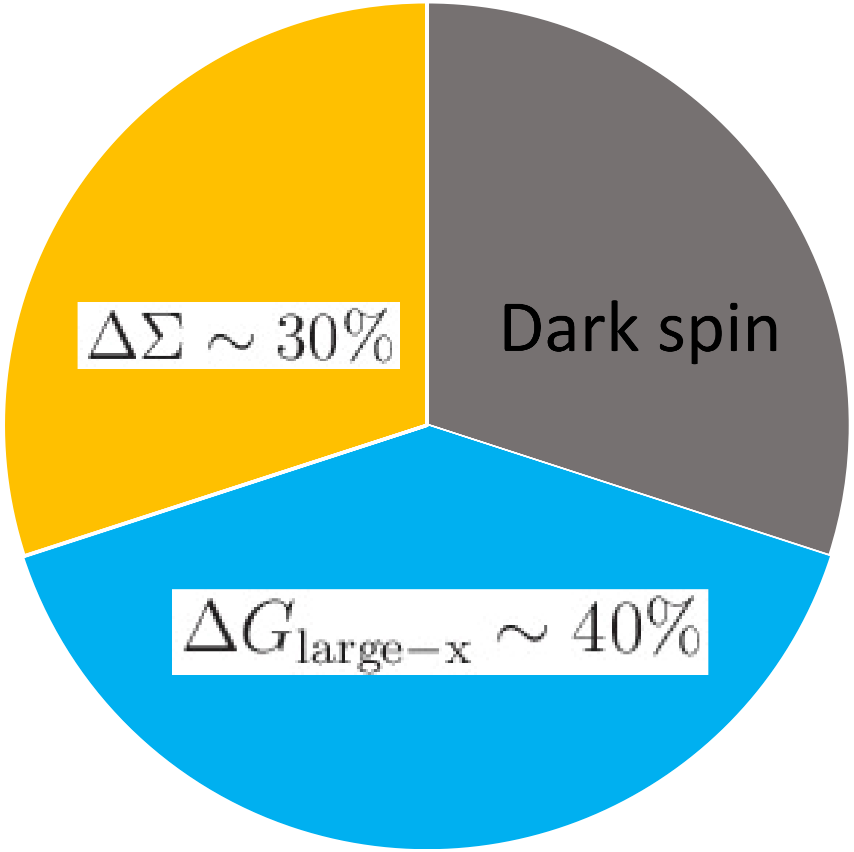
Evidence of nonzero gluon helicity $\Delta G = \int_0^1 dx \Delta G(x)$

A major achievement of the RHIC spin program!

$$\int_{0.05}^{1} dx \Delta G(x, Q^2 = 10\mathrm{GeV}^2) = 0.20^{+.06}_{-.07}$$ DSSV

$$\int_{0.05}^{0.2} dx \Delta G(x, Q^2 = 10\mathrm{GeV}^2) = 0.17 \pm 0.06$$ NNPDF

$$\int_{0.05}^{1} dx \Delta G(x, Q^2 = 10\mathrm{GeV}^2) = 0.23 \pm 0.03$$ JAM



Huge uncertainty from the small-x region

→ Renewed interest in the small-x resummation for helicity PDFs

Regge intercept at small-x, revisited

Bartels, Ermolaev, Ryskin (1996)

$$\Delta\gamma_{GG}^{BER}(\omega) = \frac{4\,\bar{\alpha}_s}{\omega} + \frac{8\,\bar{\alpha}_s^2}{\omega^3} + \frac{56\,\bar{\alpha}_s^3}{\omega^5} + \frac{504\,\bar{\alpha}_s^4}{\omega^7} + \ldots$$

Borden, Kovchegov (2023)

$$\Delta\gamma_{GG}^{us}(\omega) = \frac{4\,\bar{\alpha}_s}{\omega} + \frac{8\,\bar{\alpha}_s^2}{\omega^3} + \frac{56\,\bar{\alpha}_s^3}{\omega^5} + \frac{496\,\bar{\alpha}_s^4}{\omega^7} + \ldots$$

Discrepancy at 4-loops!

$$\Delta q(x), \Delta G(x) \sim \frac{1}{x^\alpha}$$

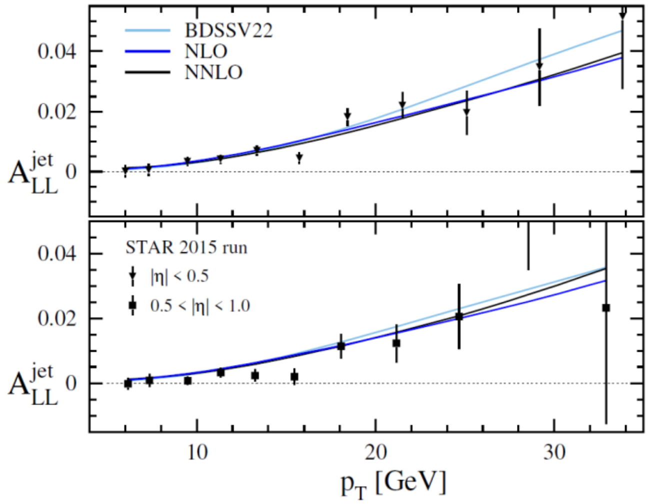
$$\alpha_{BER} \approx 3.664\sqrt{\frac{\alpha_s N_c}{2\pi}} \qquad \alpha_{BK} \approx 3.661\sqrt{\frac{\alpha_s N_c}{2\pi}}$$

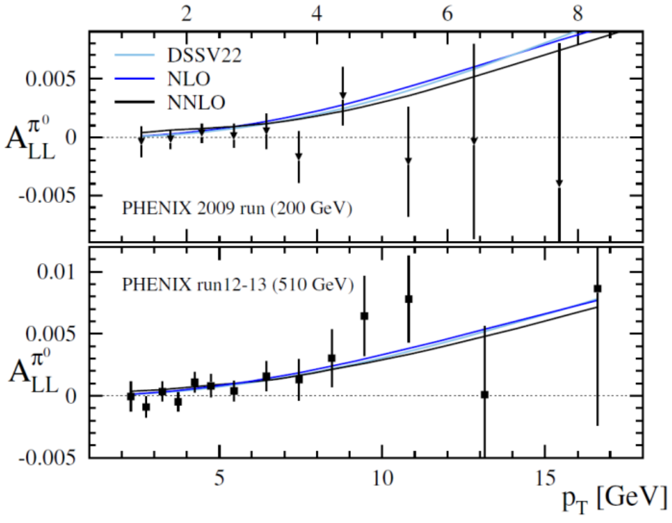
NNLO global analysis for helicity PDF on the way

Vogelsang, talk at SPIN2023

Data:

DIS:	EMC,SMC,E142,E143,E154,E155, HERMES, COMPASS, HALL-A,CLAS (p,n,d,He)	**378**
SIDIS:	HERMES, COMPASS (p-$\pi^{\pm}$,d-$\pi^{\pm}$)	**80**
PP-JETS:	STAR run 5,6,9,12,13,15 $(\sqrt{s} = 200, 510\, GeV)$ (no dijets yet)	**91**
PP-$\pi^0/\pi^{\pm}$:	PHENIX, STAR	**82**
PP $W^{\pm}$:	PHENIX, STAR	**22**
Total:		**653**





An elephant in the room: Orbital angular momentum

It’s an undeniable fact that experimentally we know nothing about OAM

Spin sum rule cannot be complete without understanding OAM

Helicity is not a conserved quantity.

Even if OAM is zero at some scale, it is nonzero at different scales.

$$\frac{d}{d\ln Q^2}\left(\frac{1}{2}\Delta\Sigma(Q^2)+\Delta G(Q^2)+L_q(Q^2)+L_g(Q^2)\right)=0$$

Canonical OAM in QCD

Jaffe, Manohar (1990)

Decomposition of the canonical angular momentum tensor operator $M^{\mu\nu\rho}_{can}$

$$\Delta L_q = \frac{1}{2E(2\pi)^3\delta^3(0)} \left\langle p^0_\infty, s^0 \middle| \int \mathrm{d}^3x\, i\psi^\dagger (\boldsymbol{x} \times \boldsymbol{\nabla})^3 \psi \middle| p^0_\infty, s^0 \right\rangle,$$

$$\Delta L_g = \frac{1}{2E(2\pi)^3\delta^3(0)} \left\langle p^0_\infty, s^0 \middle| \int \mathrm{d}^3x\, \mathrm{Tr}\{E^k (\boldsymbol{x} \times \boldsymbol{\nabla})^3 A^k\} \middle| p^0_\infty, s^0 \right\rangle.$$

To be understood in the light-cone gauge $A^+ = 0$

Gauge invariant completion of Jaffe-Manohar

YH (2011)

see also, Bashinsky, Jaffe (1999);

Exact definition of OAM to be used in the Jaffe-Manohar decomposition

$$\lim_{\Delta\to 0}\langle P'S|\bar{\psi}\gamma^+ i \overleftrightarrow{D}^i_{pure}\psi|PS\rangle = iS^+\epsilon^{ij}\Delta_{\perp j}L^q_{can}$$

$$\lim_{\Delta\to 0}\langle P'S|F^{+\alpha}\overleftrightarrow{D}^i_{pure}A^{phys}_\alpha|PS\rangle = -i\epsilon^{ij}\Delta_{\perp j}S^+L^g_{can}$$

$$A^\mu_{phys}(x) = \frac{1}{D^+}F^{+\mu} = \int_{x^-}^{\infty} dz^- W[x^-,z^-]F^{+\mu}(z^-,x_\perp)$$

$$D^\mu_{pure} = D^\mu - iA^\mu_{phys}$$

OAM from the Wigner distribution

Wigner distribution

Phase space distribution of partons in QCD

Belitsky, Ji, Yuan (2004)

$$W(x, \vec{k}_\perp, \vec{b}_\perp)$$
$$= \int \frac{d^2\Delta_\perp}{(2\pi)^2} \frac{dz^- d^2 z_\perp}{16\pi^3} e^{ixP^+ z^- - i\vec{k}_\perp \cdot \vec{z}_\perp} \langle P - \frac{\Delta}{2} | \bar{q}(b - z/2)\gamma^+ q(b + z/2) | P + \frac{\Delta}{2} \rangle$$

Define

$$L^q = \int dx \int d^2 b_\perp d^2 k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

Generalized TMD

Fourier transform $W(x, \vec{k}_\perp, \vec{b}_\perp) \rightarrow W(x, \vec{k}_\perp, \vec{\Delta}_\perp)$

$$\int \frac{d^3 z}{2(2\pi)^3} e^{ixP^+ z^- - i\widetilde{k}_\perp \cdot \widetilde{z}_\perp} \langle p' | \bar{\psi}(-z/2)\gamma^+ \psi(z/2) | p \rangle$$

$$= \frac{1}{2M} \bar{u}(p') \left[F^q_{1,1} + i \frac{\sigma^{j+}}{P^+} (\widetilde{k}^j_\perp F^q_{1,2} + \widetilde{\Delta}^j_\perp F^q_{1,3}) + i \frac{\sigma^{ij} \widetilde{k}^i_\perp \widetilde{\Delta}^j_\perp}{M^2} F^q_{1,4} \right] u(p)$$

$$L_{q,g} = - \int dx \int d^2 k_\perp \frac{k^2_\perp}{M^2} F^{q,g}_{1,4}(x)$$

Lorce, Pasquini (2011)
YH (2011)

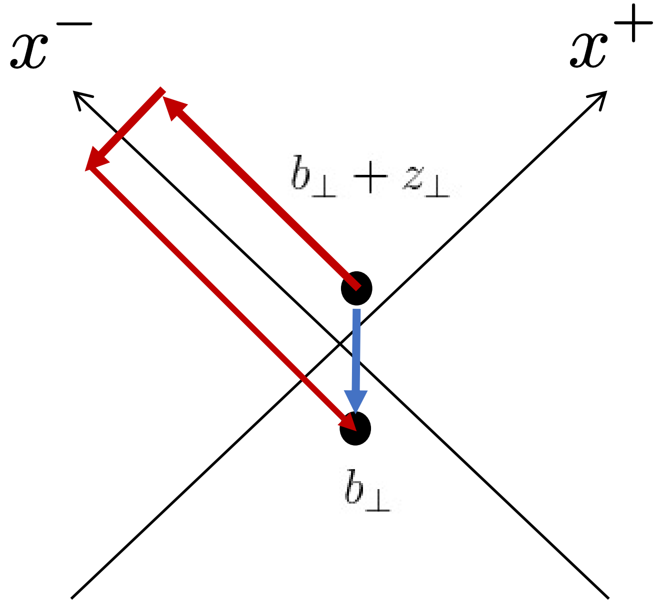
Which OAM is this?

Wilson lines and OAMs

Canonical (JM) OAM from the light-cone staple Wilson line YH (2011)

$$\int d^2k_\perp (b_\perp \times k_\perp) W_{LC}(b_\perp, k_\perp) = \langle \bar{\psi} b_\perp \times i D_\perp^{pure} \psi \rangle$$

$$D_{pure}^\perp = D^\perp - \frac{i}{D^+} F^{+\perp}$$

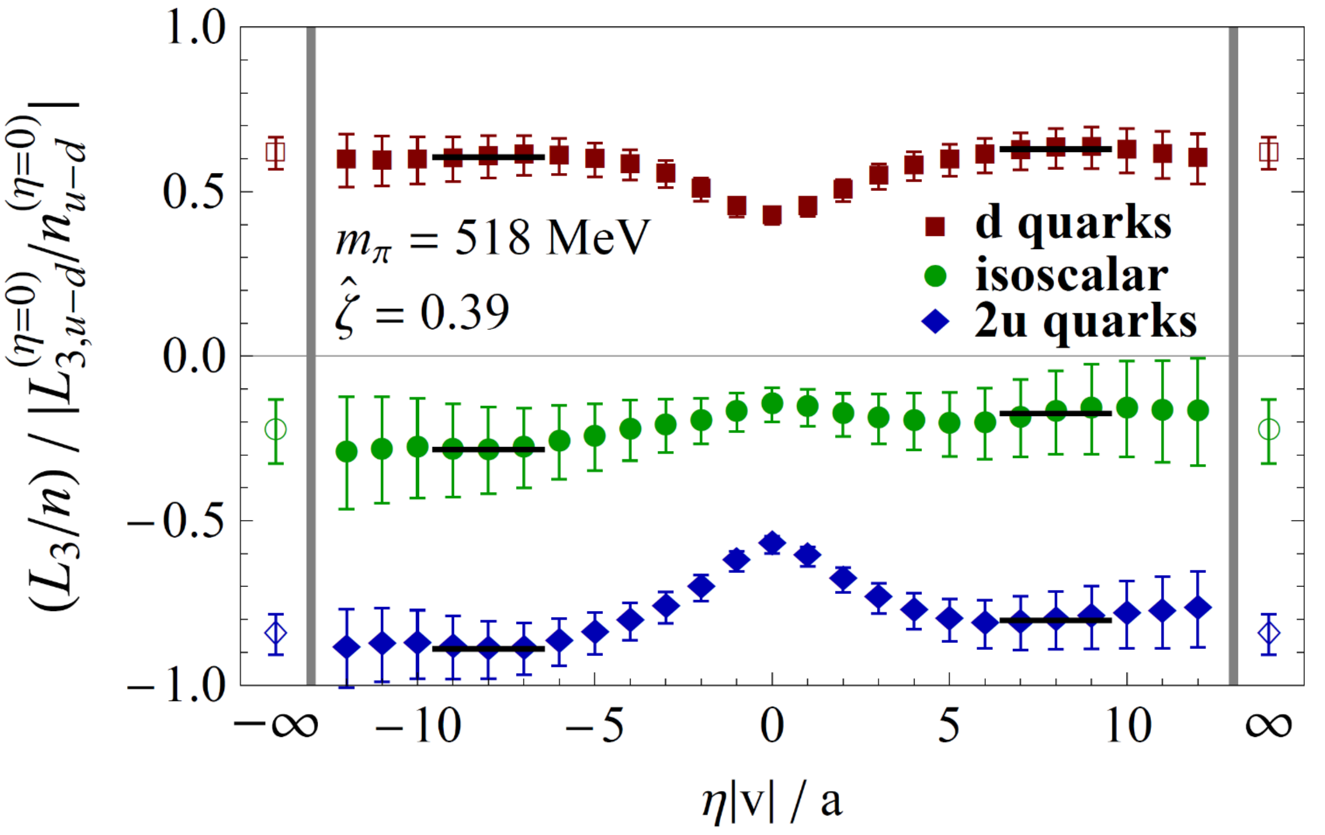


Kinetic (Ji's) OAM from the straight Wilson line

Ji, Xiong, Yuan (2012)

Jaffe-Manohar vs. Ji on a lattice

Engelhardt (2017)
Engelhardt et al. (2020)



Staple length

OAM distributions

In order to determine $\Delta\Sigma, \Delta G$, we first extract the associated PDFs $\Delta q(x), \Delta G(x)$ and integrate over x.

Same with $L_{can}^{q,g}$

OAM from the Wigner distribution

$$L_{can}^{q} = \int dx \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

Define the x-distribution .

$$\Rightarrow \quad L_{can}^{q}(x) = \int d^2b_\perp d^2k_\perp (\vec{b}_\perp \times \vec{k}_\perp)_z W^q(x, \vec{b}_\perp, \vec{k}_\perp)$$

It's a twist-3 PDF, similar to $g_2(x)$.

Twist structure of OAM PDF

YH, Yoshida (2012)

$$L^q_{can}(x) = x\int_x^{\epsilon(x)} \frac{dx'}{x'}(H_q(x')+E_q(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\tilde{H}_q(x')$$

Wandzura-Wilczek part

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \Phi_F(x_1,x_2)\mathcal{P}\frac{3x_1-x_2}{x_1^2(x_1-x_2)^2}$$

$$-x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{\Phi}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^2(x_1-x_2)} .$$

genuine twist-3

$$\Phi_F \sim \langle P'|\bar{\psi}\gamma^+ F^{+i}\psi|P\rangle$$

$$M_F \sim \langle P'|F^{+\mu}F^{+i}F^+_\mu|P\rangle$$

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x')+E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$

$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX \Phi_F(X,x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)}$$

$$+2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

Evolution of $L_{q,g}(x)$: WW part

$$\frac{d}{d\ln Q^2}\begin{pmatrix} L_q(x) \\ L_g(x) \end{pmatrix} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\begin{pmatrix} \hat{P}_{qq}(z) & \hat{P}_{qg}(z) & \Delta\hat{P}_{qq}(z) & \Delta\hat{P}_{qg}(z) \\ \hat{P}_{gq}(z) & \hat{P}_{gg}(z) & \Delta\hat{P}_{gq}(z) & \Delta\hat{P}_{gg}(z) \end{pmatrix}\begin{pmatrix} L_q(x/z) \\ L_g(x/z) \\ \Delta q(x/z) \\ \Delta G(x/z) \end{pmatrix},$$

Leading order

Hagler, Schafer (1998)
Harindranath, Kundu (1999)
Hoodbhoy, Ji, Lu (1999)

All orders

Boussarie, YH, Yuan (2019)

$$\frac{\partial}{\partial \ln Q^2}\begin{pmatrix} L_q^\omega \\ L_g^\omega \end{pmatrix} = \begin{pmatrix} \gamma_{qq}^{\omega+1} & \gamma_{qg}^{\omega+1} \\ \gamma_{gq}^{\omega+1} & \gamma_{gg}^{\omega+1} \end{pmatrix}\begin{pmatrix} L_\Sigma^\omega \\ L_g^\omega \end{pmatrix}$$

$$+\frac{1}{\omega+1}\begin{pmatrix} \gamma_{qq}^{\omega+1} - \Delta\gamma_{qq}^\omega & 2\gamma_{qg}^{\omega+1} - \Delta\gamma_{qg}^\omega \\ \gamma_{gq}^{\omega+1} - 2\Delta\gamma_{gq}^\omega & 2\gamma_{gg}^{\omega+1} - 2\Delta\gamma_{gg}^\omega \end{pmatrix}\begin{pmatrix} \Delta\Sigma^\omega \\ \Delta G^\omega \end{pmatrix}$$

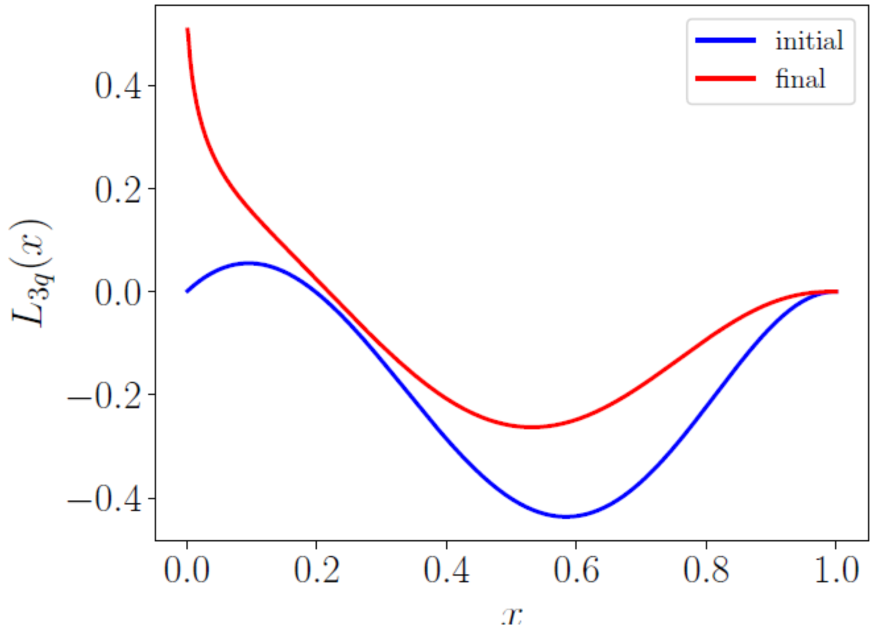
Evolution of $L_{q,g}(x)$: genuine twist-3 part

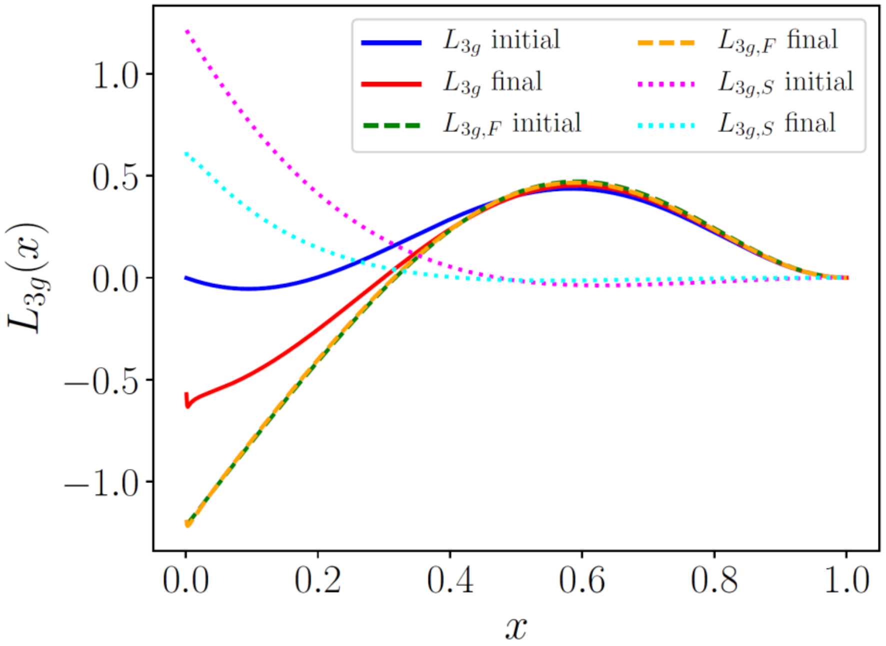
YH, Yao (2019)

Non-forward matrix elements of $\begin{matrix} \bar{\psi}\gamma^+ F^{+i}\psi \\ F^{+\mu}F^{+i}F^+_\mu \end{matrix}$ in the limit $\Delta_\perp \to 0$, zero skewness

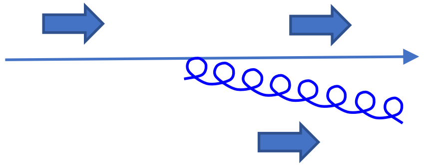
→ The same evolution as for the Efremov-Teryaev-Qiu-Sterman function.

Numerical code developed by Pirnay (2013)
Straightforward to adapt to the OAM problem





OAM at small-x



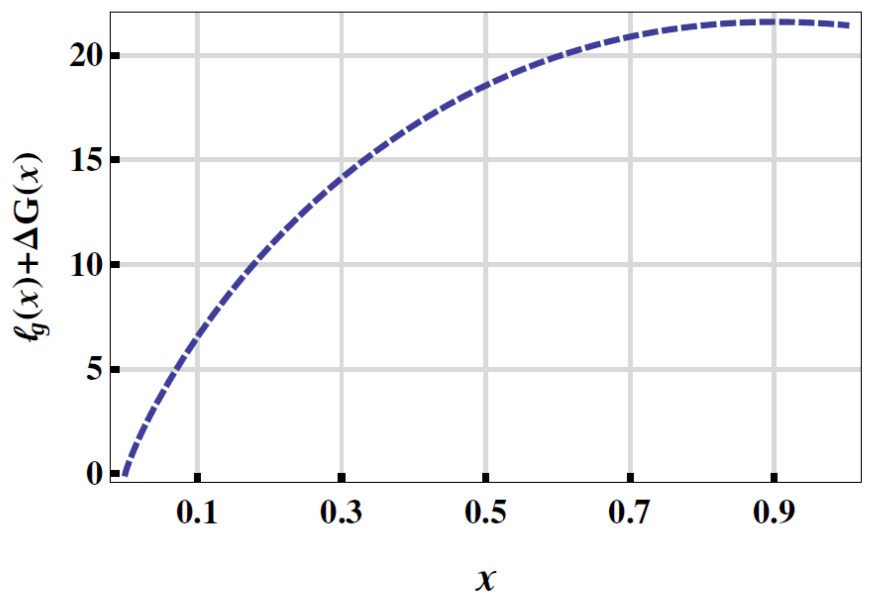
Suppose a quark emits a very soft gluon.
Nothing happens to the quark.
From angular momentum conservation, gluon helicity and OAM must cancel.

$$\frac{d}{d\ln Q^2} L_g(x) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}(-2C_F + \cdots)\Delta q(x/z)$$

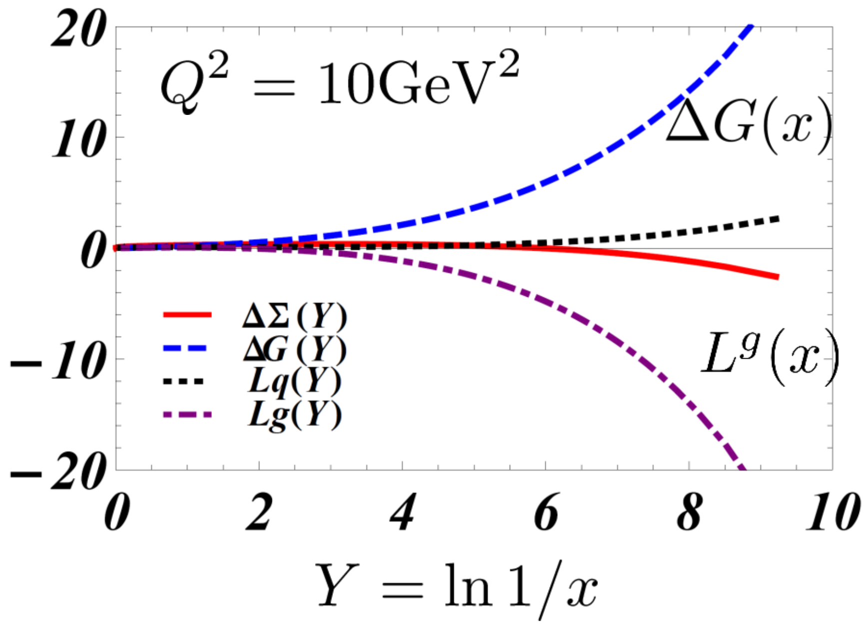
$$\frac{d}{d\ln Q^2} \Delta G(x) = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}(+2C_F + \cdots)\Delta q(x/z)$$

Helicity-OAM cancellation at small-x

More Mukherjee, Nair (2017)



YH, Yang (2018)



All-order argument

Start from the exact formula YH, Yoshida (2013)

$$L^g_{can}(x) = \frac{x}{2}\int_x^{\epsilon(x)} \frac{dx'}{x'^2}(H_g(x')+E_g(x')) - x\int_x^{\epsilon(x)} \frac{dx'}{x'^2}\Delta G(x')$$
$$+2x\int_x^{\epsilon(x)} \frac{dx'}{x'^3}\int dX\,\Phi_F(X,x') + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 \tilde{M}_F(x_1,x_2)\mathcal{P}\frac{1}{x_1^3(x_1-x_2)} + 2x\int_x^{\epsilon(x)} dx_1 \int_{-1}^{1} dx_2 M_F(x_1,x_2)\mathcal{P}\frac{2x_1-x_2}{x_1^3(x_1-x_2)^2}$$

Assume that the helicity term dominates on the rhs

If $\Delta G(x) \sim \frac{1}{x^\alpha}$, then $L_g(x) \approx -\frac{2}{1+\alpha}\Delta G(x)$ Boussarie, YH, Yuan (2019)

Consistent with double log resummation Boussarie, YH, Yuan (2019)

Robust under inclusion of genuine twist-3 effects Kovchegov, Manley (2023); Manley (2024)

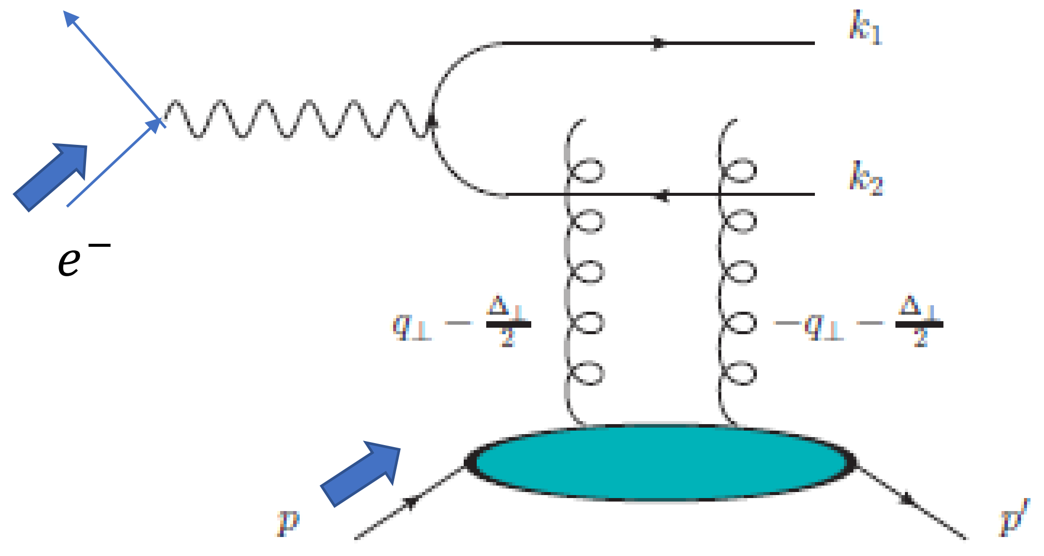
Observables for OAM

- Nonexistent…until recently

- Still at the level of identifying processes that are sensitive to OAM at leading order.

- GTMD factorization hard to establish
  (see however, Echevarria, Gutierrez-Garcia, Scimemi (2023))

Longitudinal double spin asymmetry in diffractive dijets



Bhattacharya, Boussarie, YH, (2022) + paper in preparation

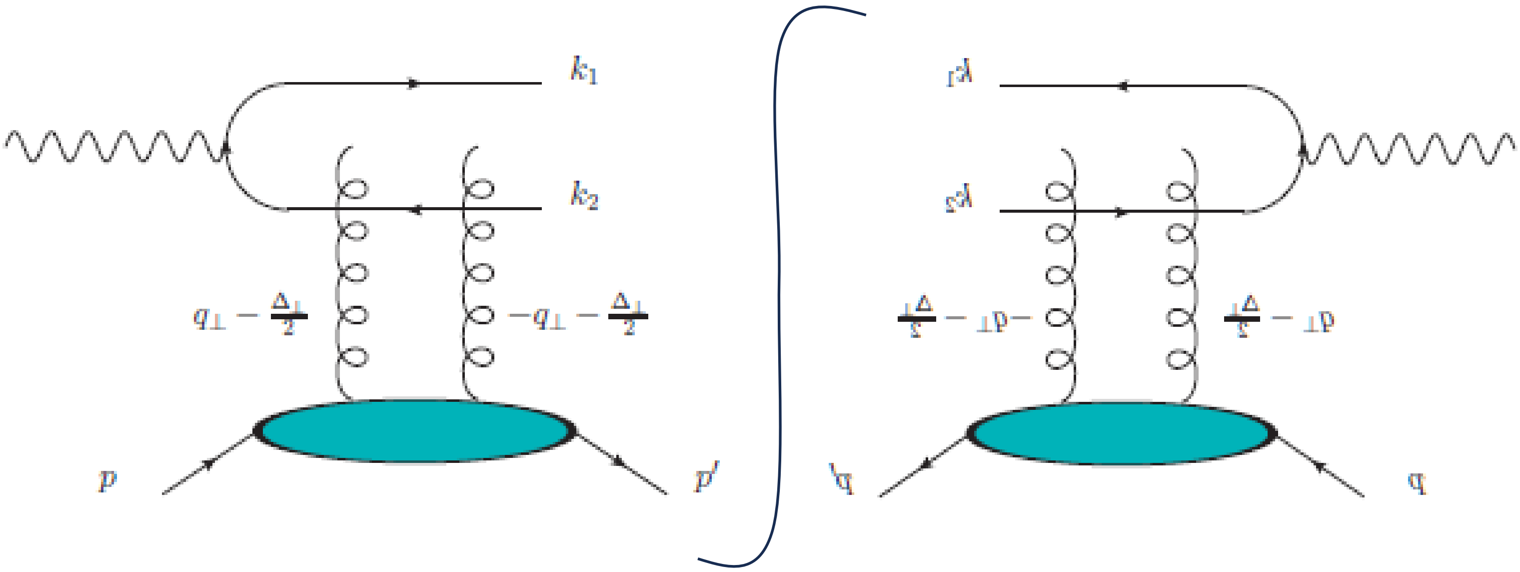
$$L^z \sim b_\perp \times k_\perp$$

conjugate to $\Delta_\perp$ proton recoil momentum

correlated with jet transverse momentum

$$d\sigma^{h_p h_l} \sim h_p h_l \cos(\phi_{l_\perp} - \phi_{\Delta_\perp}) \mathrm{Re}(iA_L^{2*} A_T^{3i} - iA_T^{2i*} A_L^3)$$

Spin, orbit, and spin-orbit



3 contributions:

$\mathcal{H}_g$	$\mathcal{L}_g$	OAM
$\mathcal{H}_g$	$\tilde{\mathcal{H}}_g$	helicity
$\tilde{\mathcal{H}}_g$	$\mathcal{C}_g$	spin-orbit coupling ← New

Gluon spin-orbit coupling

helicity operator

Polarized gluon GTMD

$$-i\epsilon_{ij}\int\frac{d^3z}{(2\pi)^3P^+}e^{ixP^+z^- - i\tilde{k}_\perp\cdot\tilde{z}_\perp}\langle p'|F_a^{+i}(-z/2)F_a^{+j}(z/2)|p\rangle$$

$$= \frac{-i}{2M}\bar{u}(p')\left[\frac{\epsilon_{ij}\tilde{k}_\perp^i\tilde{\Delta}_\perp^j}{M^2}G_{1,1}^g + \frac{\sigma^{i+}\gamma_5}{P^+}(\tilde{k}_\perp^i G_{1,2}^g + \tilde{\Delta}_\perp^i G_{1,3}^g) + \sigma^{+-}\gamma_5 G_{1,4}^g\right]u(p),$$

$$C_g(x) = \int\frac{dk_\perp^2}{M^2}k_\perp^2 G_{1,1}(x,k_\perp)$$

$$= x^2\int_x^{\epsilon(x)}\frac{dx'}{x'^2}(\tilde{H}_g(x') + \tilde{E}_g(x')) - 2x^2\int_x^{\epsilon(x)}\frac{dx'}{x'^2}G(x') + \cdots$$

$$\approx -xG(x) < 0$$

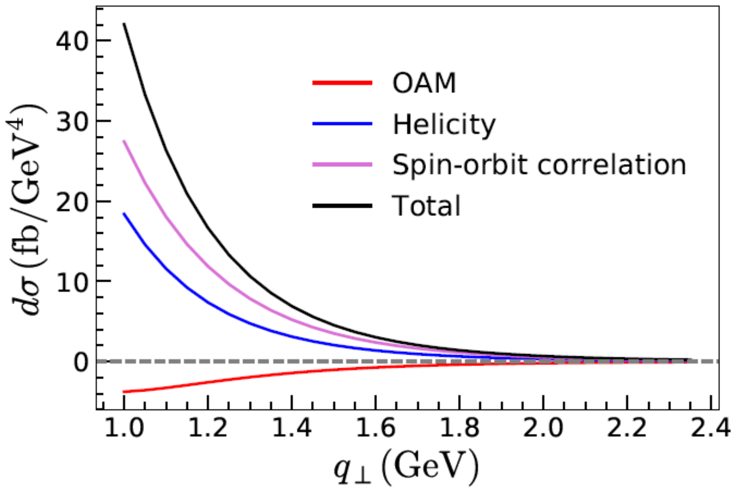
$C > 0$ spin and OAM aligned

$C < 0$ spin and OAM anti-aligned

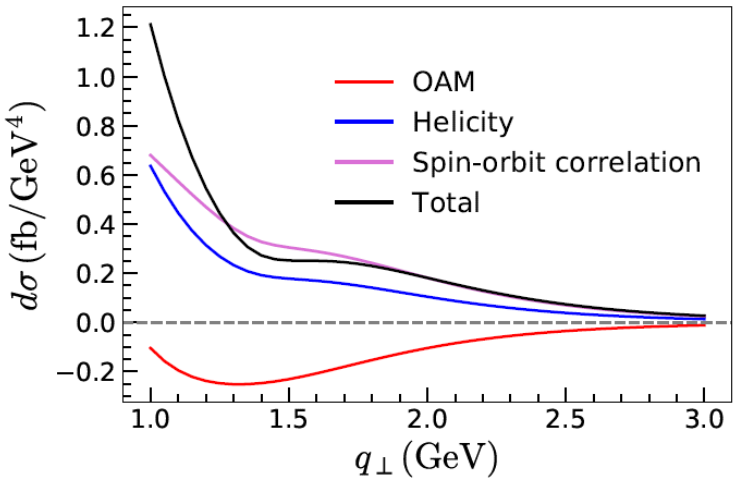
Lorce, Pasquini (2011)

Numerical result (updated)

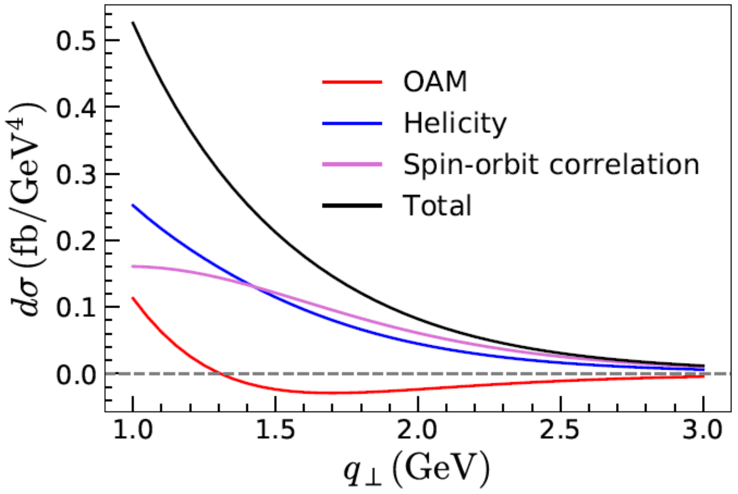
Bhattacharya, Boussarie, YH, in preparation



$$Q^2 = 2.7\,\mathrm{GeV}^2$$



$$Q^2 = 4.8\,\mathrm{GeV}^2$$



$$Q^2 = 10\,\mathrm{GeV}^2$$

In practice, reconstructing jets at low-Pt is challenging.

Re-interpret the process as semi-inclusive diffractive DIS (SIDDIS) YH, Xiao, Yuan (2022)

Conclusions

- OAM is an essential component of the spin sum rule.
- Helicity is not RG invariant. OAM is always there.
- Unraveling the proton spin structure is a key mission of EIC. More attention/discussion needed in the community.
- Our proposal: DSA in diffractive dijet. Other proposals on the market e.g., Bhattacharya, Zheng, Zhou 2312.01309