Inflation

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Topics Covered

I. Inflation Basics

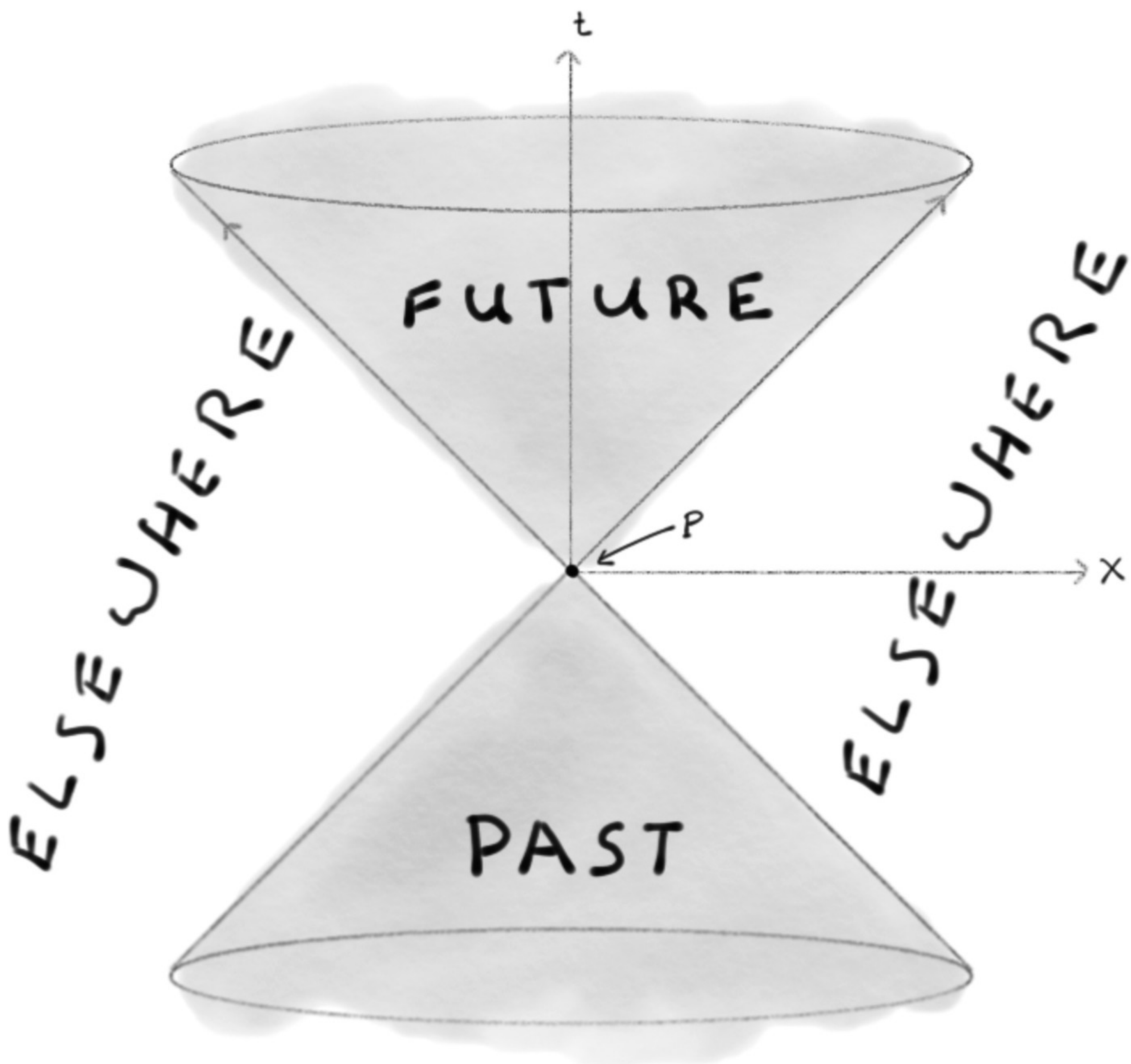
II. Perturbations
- Generation of scalar and tensor pertubations
- The horizon crossing formalism
- Comparison with data

III. Beyond slow roll
- Ultra-slow roll inflation
- Constant roll inflation
- Horizon crossing
- Attractor behavior

IV. Eternal inflation / quantum gravity
- Inflation and the string swampland
- Eternal inflation and the multiverse
- Geodesic completeness

Inflation: basics

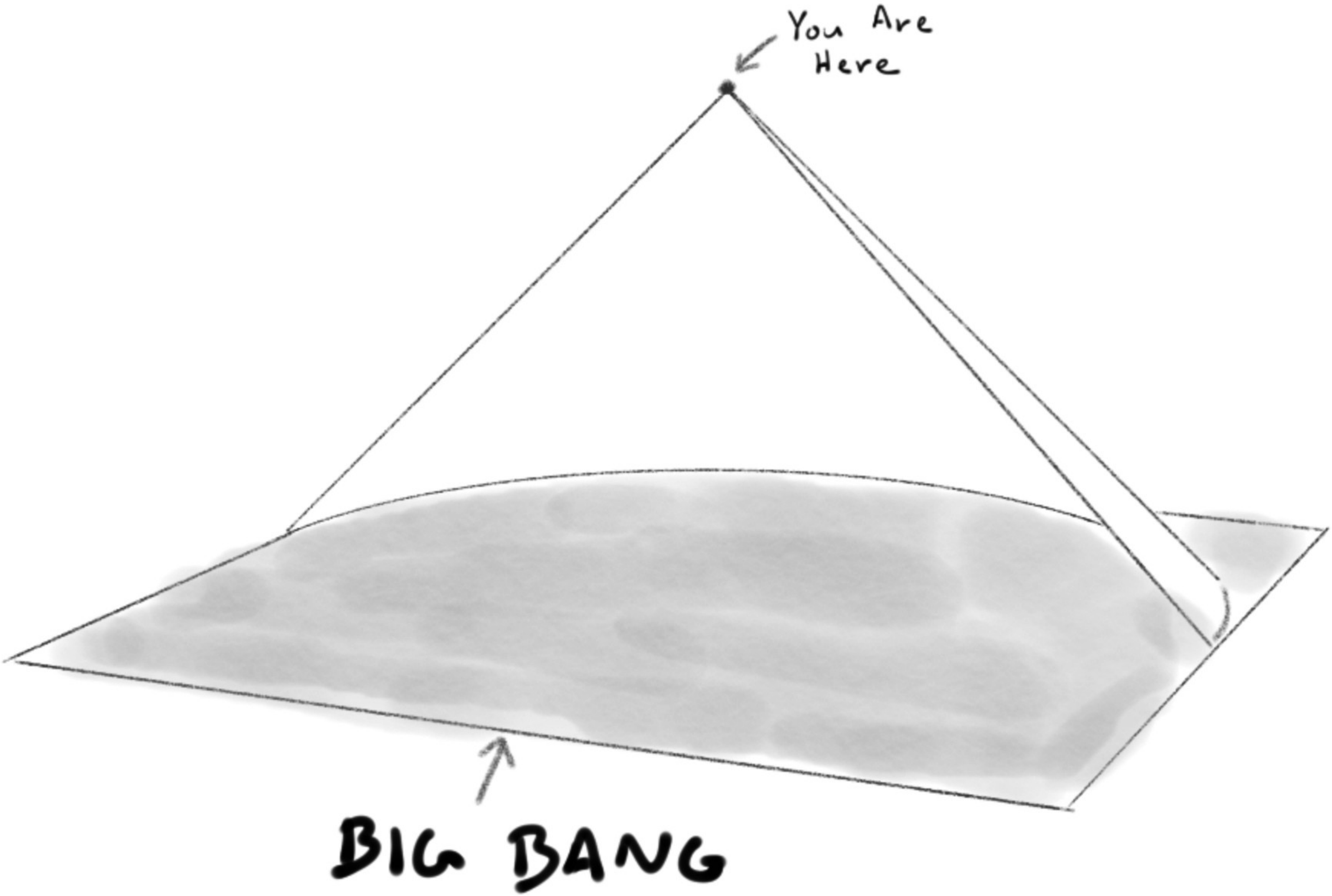
$$ds^2 = dt^2 - d\vec{x}^2$$


t
FUTURE
ELSEWHERE
ELSEWHERE
P
x
PAST

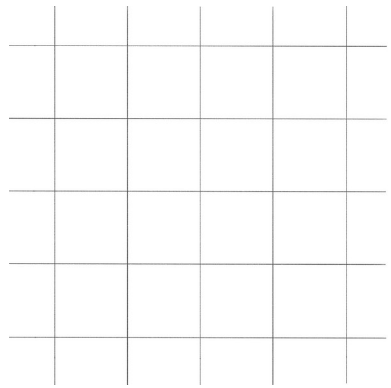
$$ds^2 = dt^2 - a^2(t)d\vec{x}^2$$

$$ds^2 = a^2(\tau)\left(d\tau^2 - d\vec{x}^2\right)$$

The FRW light cone



Dynamics of FRW Space

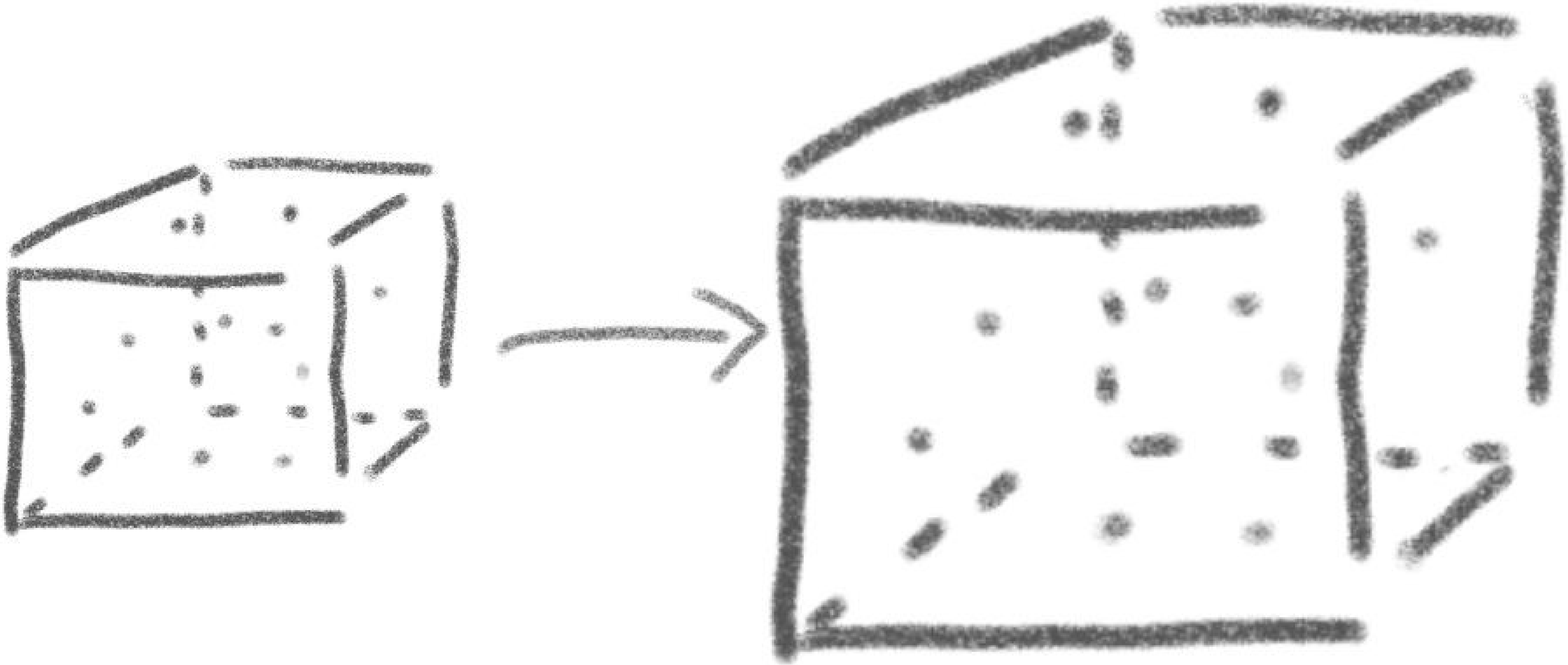


$$ds^2 = dt^2 - a^2(t)\, d\mathbf{x}^2$$

$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\rho$$

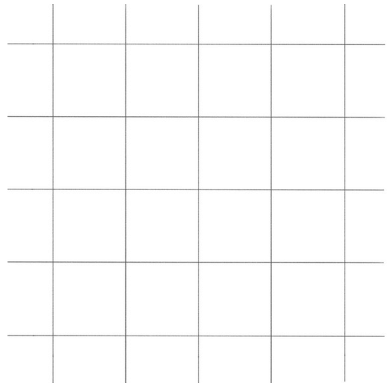
$$\dot{\rho} + 3H(\rho + p) = 0$$

The cosmic cocktail: matter



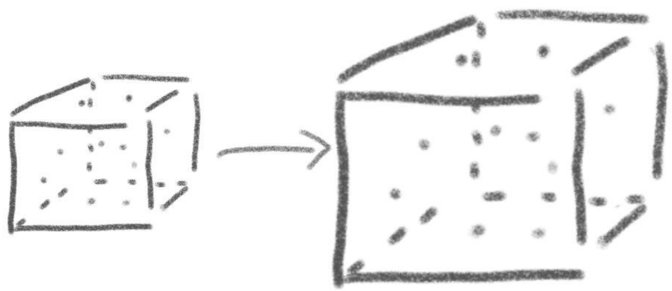
$$\rho \propto 1/a^3$$

Dynamics of FRW Space



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\rho$$

$$\dot{\rho} + 3H\left(\rho + p\right) = 0$$



$$\rho \propto 1/a^3 \Rightarrow \dot{\rho} = -3H\rho$$

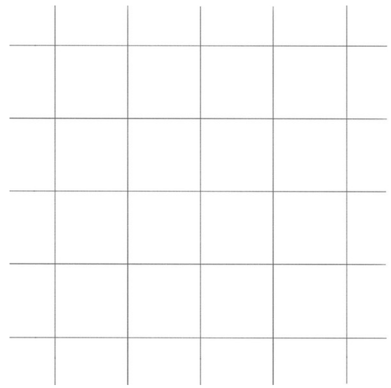
$$p = 0$$

$$a(t) \propto t^{2/3}$$

The cosmic cocktail: radiation

$$\rho \propto 1/a^4$$

Dynamics of FRW Space



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\rho$$

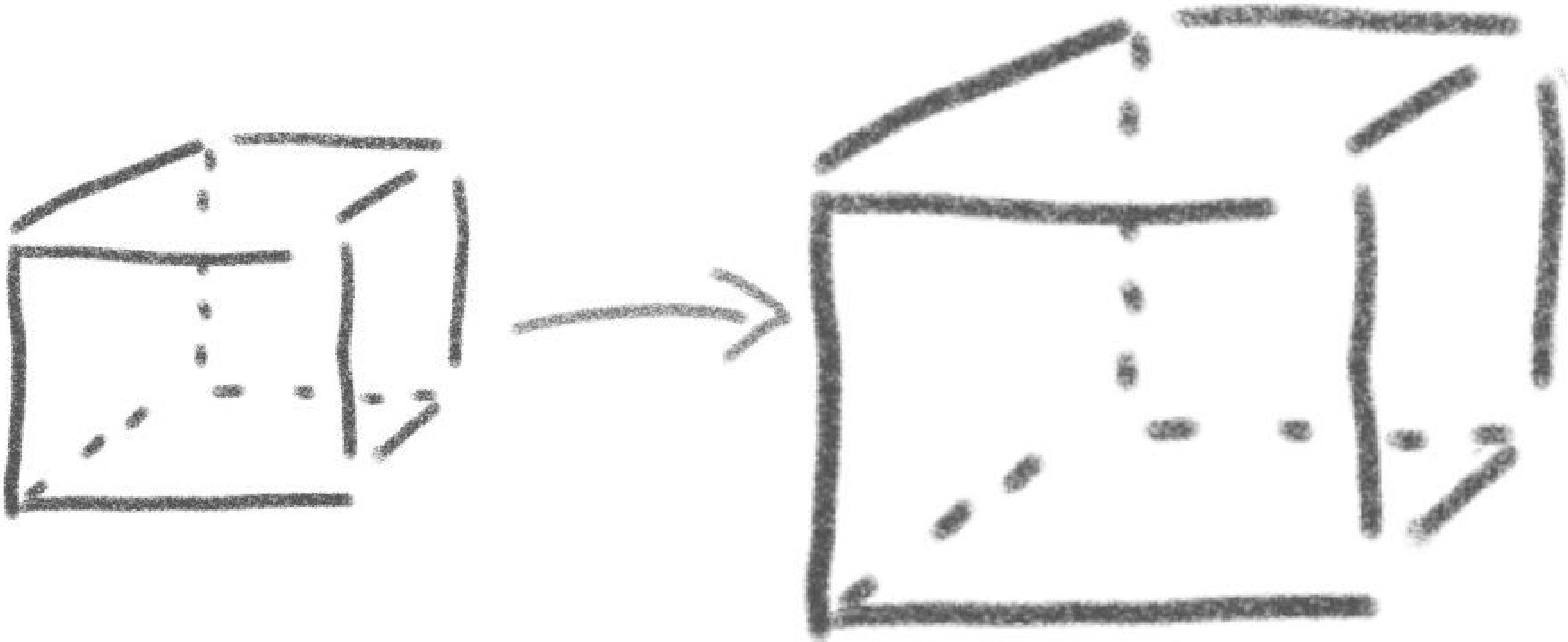
$$\dot{\rho} + 3H\left(\rho + p\right) = 0$$

$$\rho \propto 1/a^4 \Rightarrow \dot{\rho} = -4H\rho$$

$$p = (1/3)\rho$$

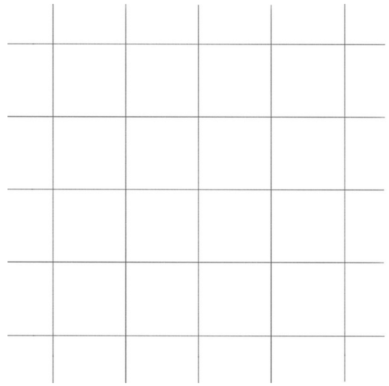
$$a(t) \propto t^{1/2}$$

The cosmic cocktail: vacuum



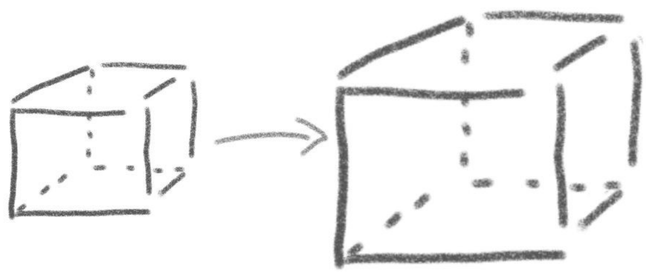
$$\rho = \text{const.}$$

Dynamics of FRW Space



$$H^2 = \left(\frac{\dot{a}}{a}\right)^2 = \frac{1}{3M_P^2}\rho$$

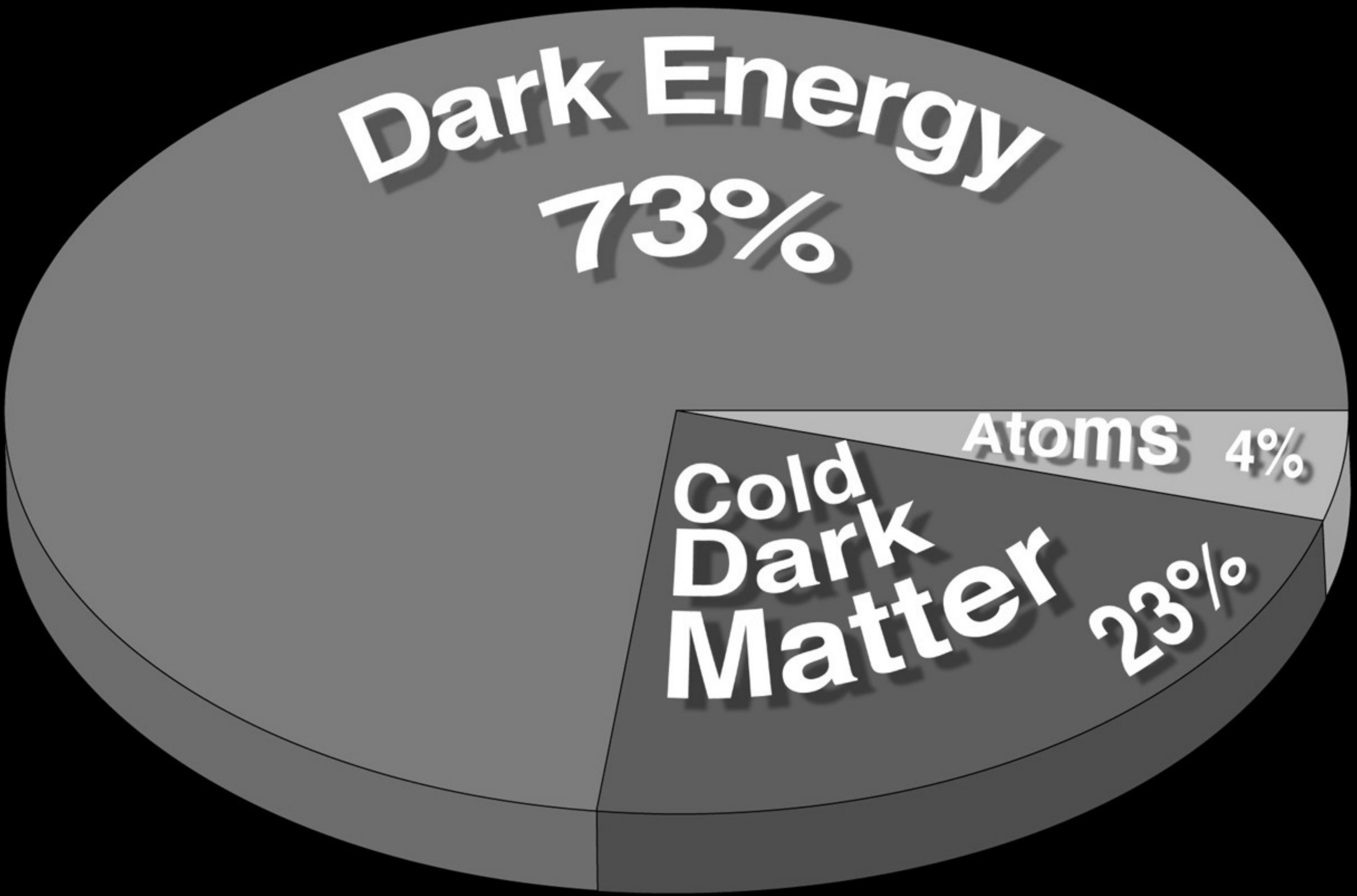
$$\dot{\rho} + 3H\left(\rho + p\right) = 0$$



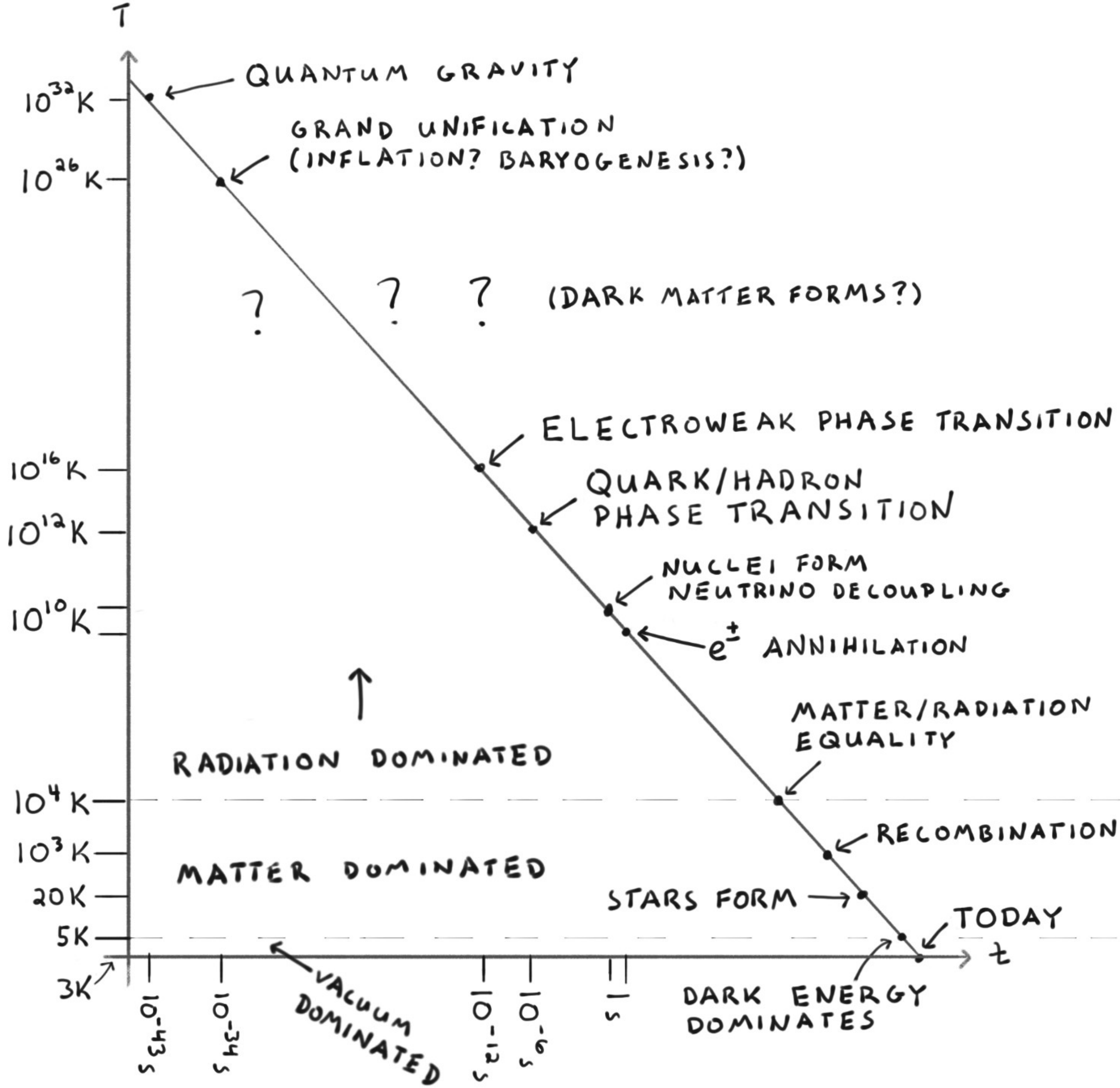
$$\rho = \text{const.} \Rightarrow \dot{\rho} = 0$$

$$p = -\rho$$

$$a(t) \propto e^{Ht} \qquad H = \text{const.}$$


Dark Energy
73%
Atoms 4%
Cold
Dark
Matter
23%

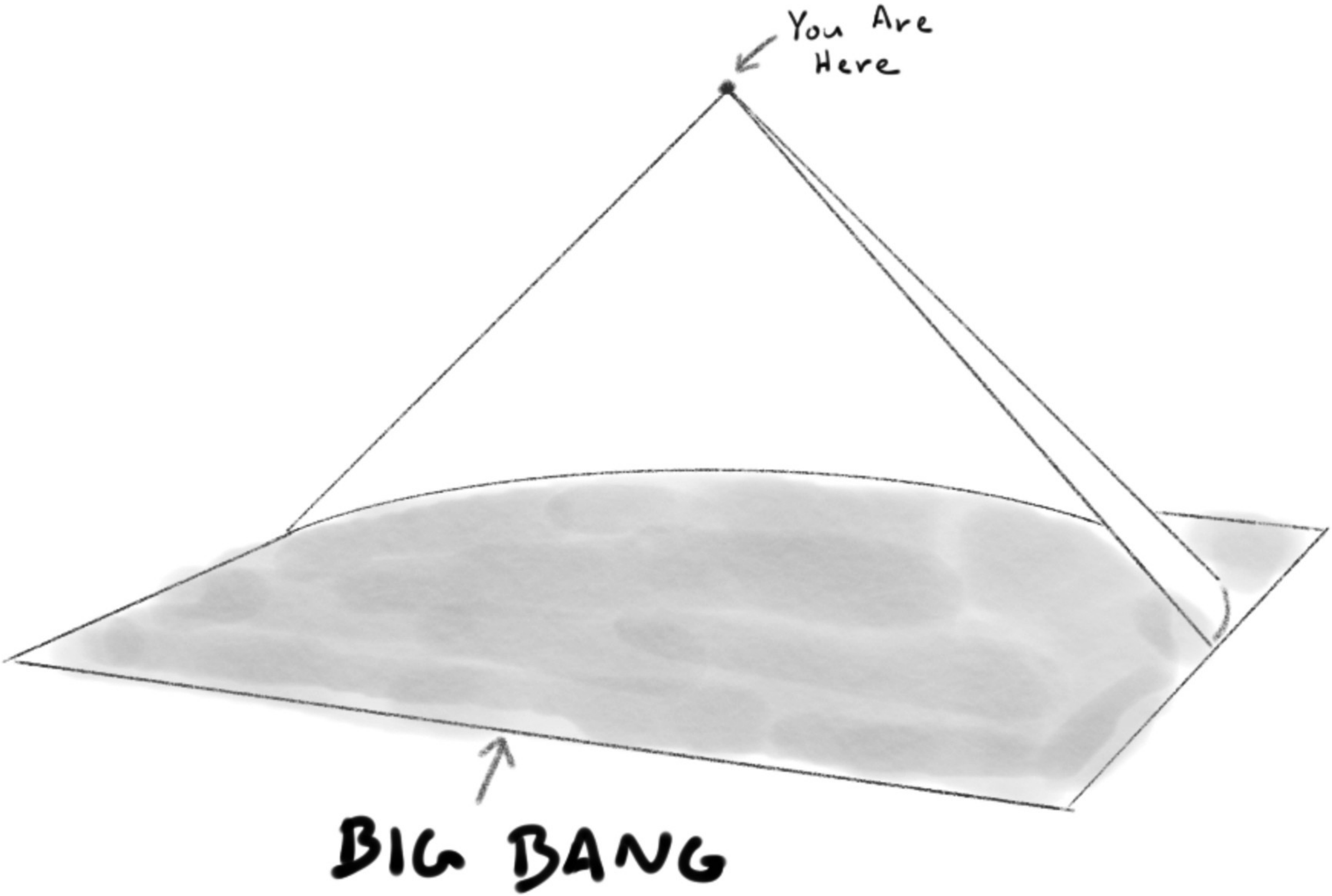
History of the universe: abridged

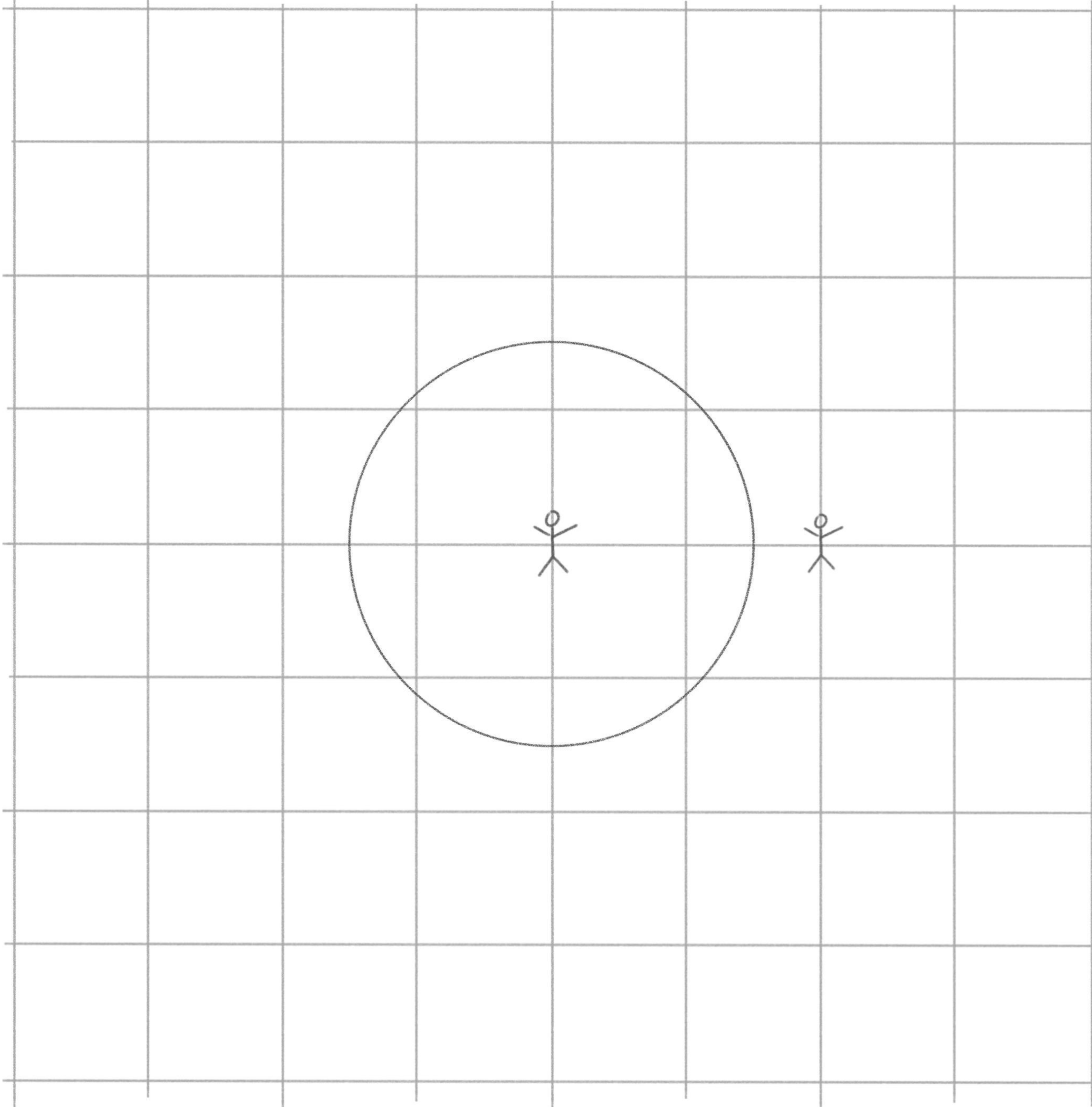


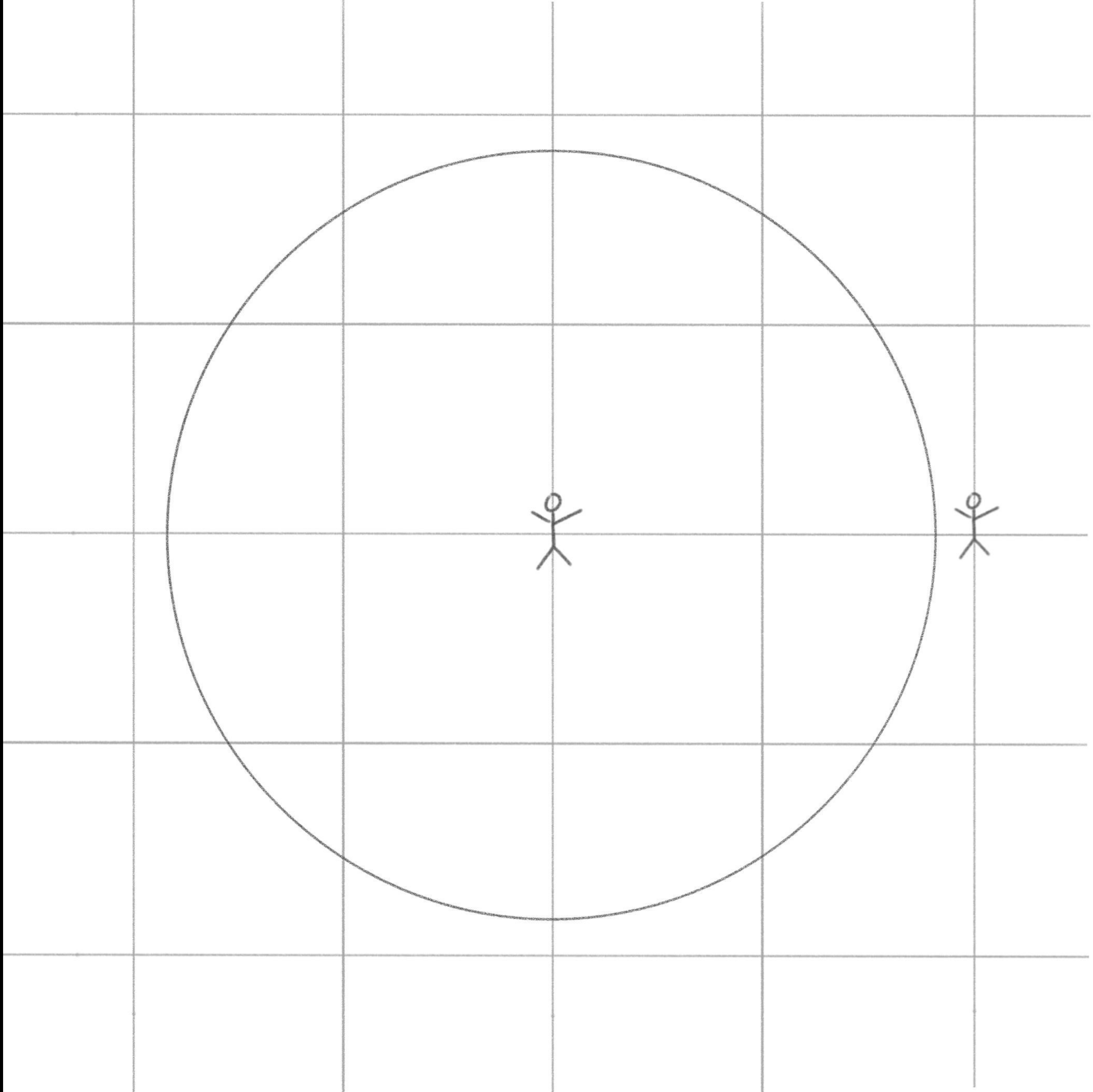
If the Lord Almighty had consulted me before embarking on creation thus, I should have recommended something simpler.

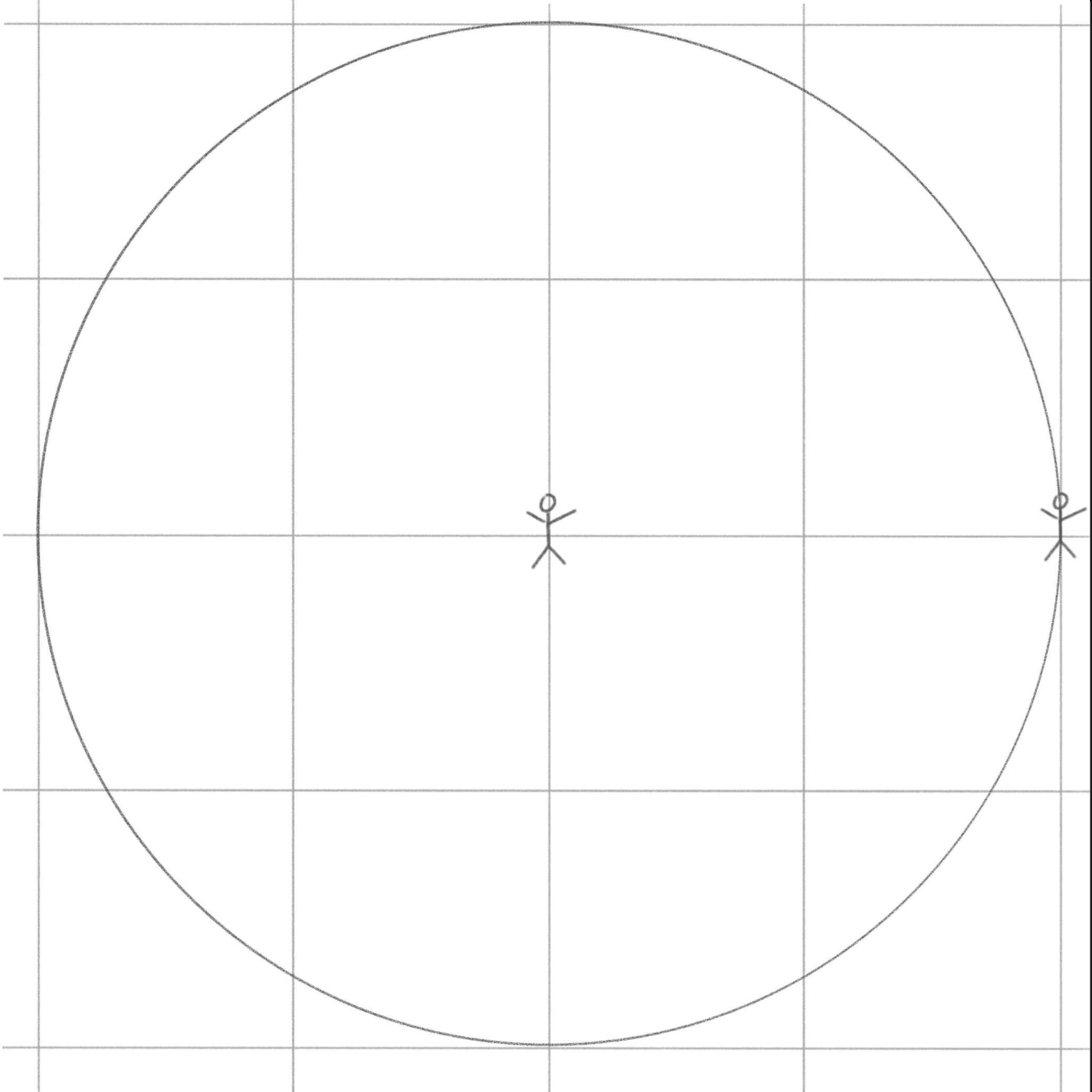
Alfonso X of Castile (r. 1252–84) on the Ptolemaic system

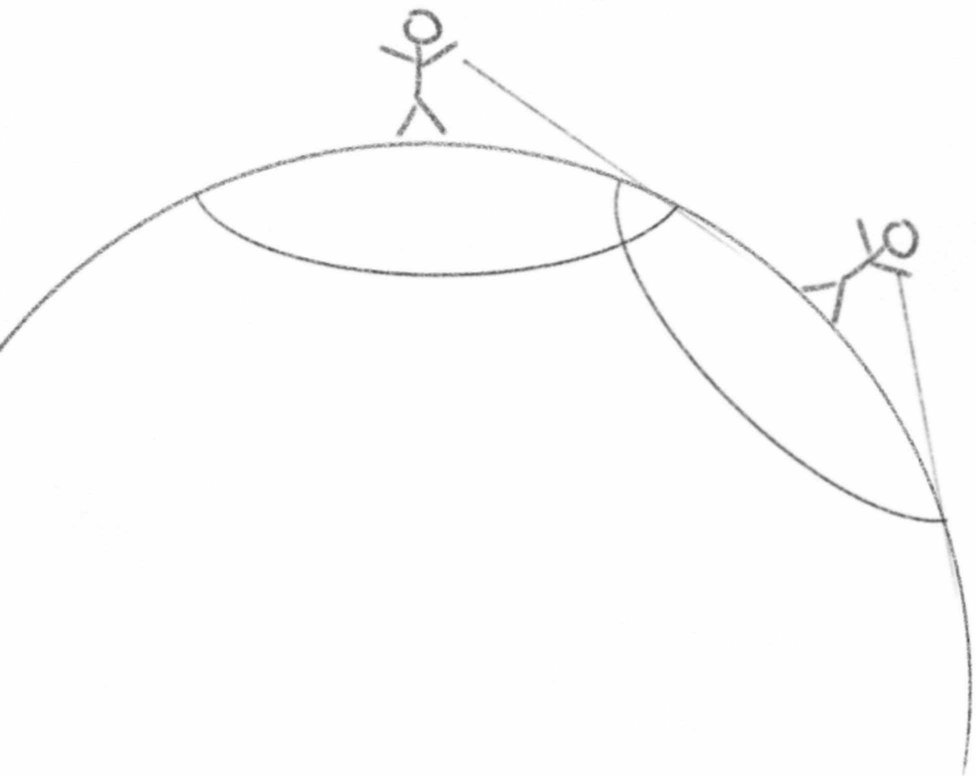
Looking out in space is looking back in time

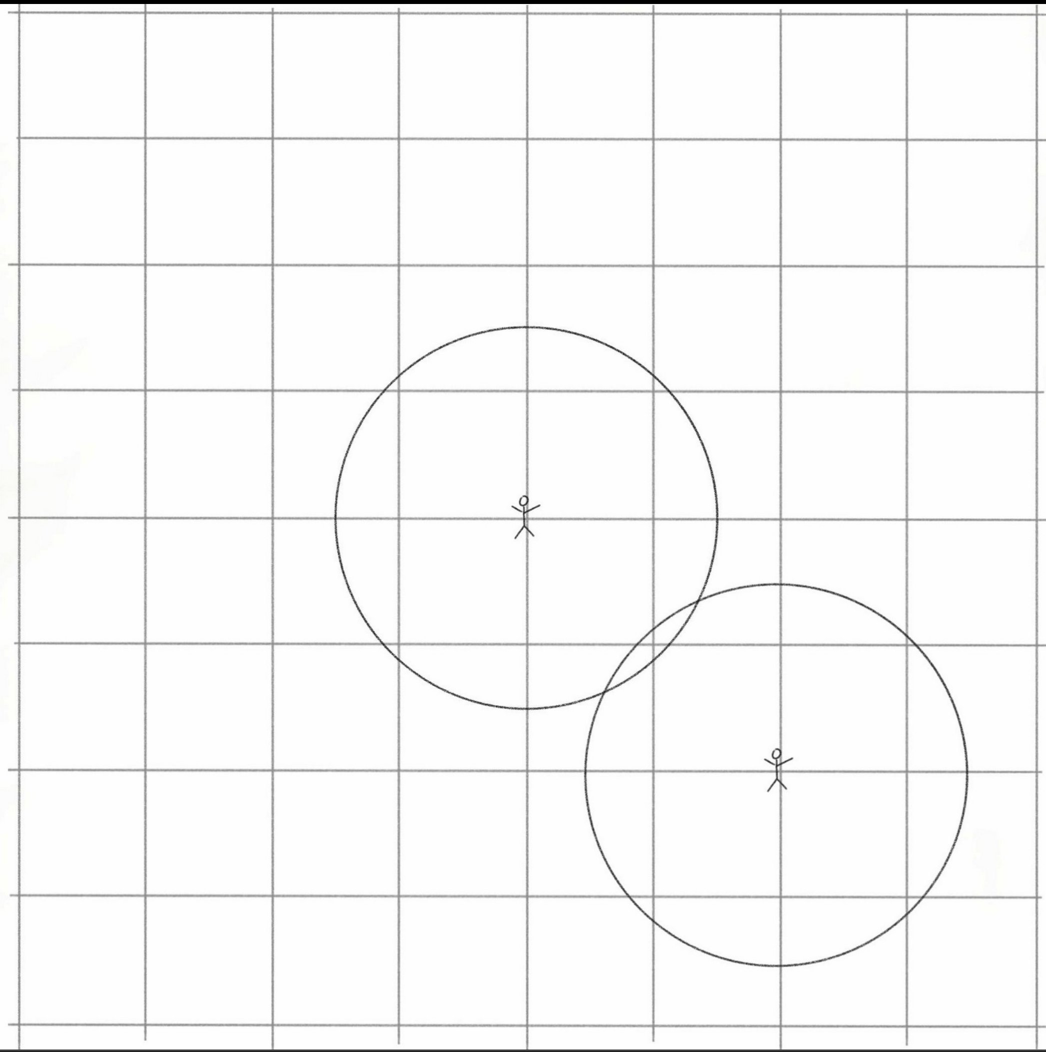






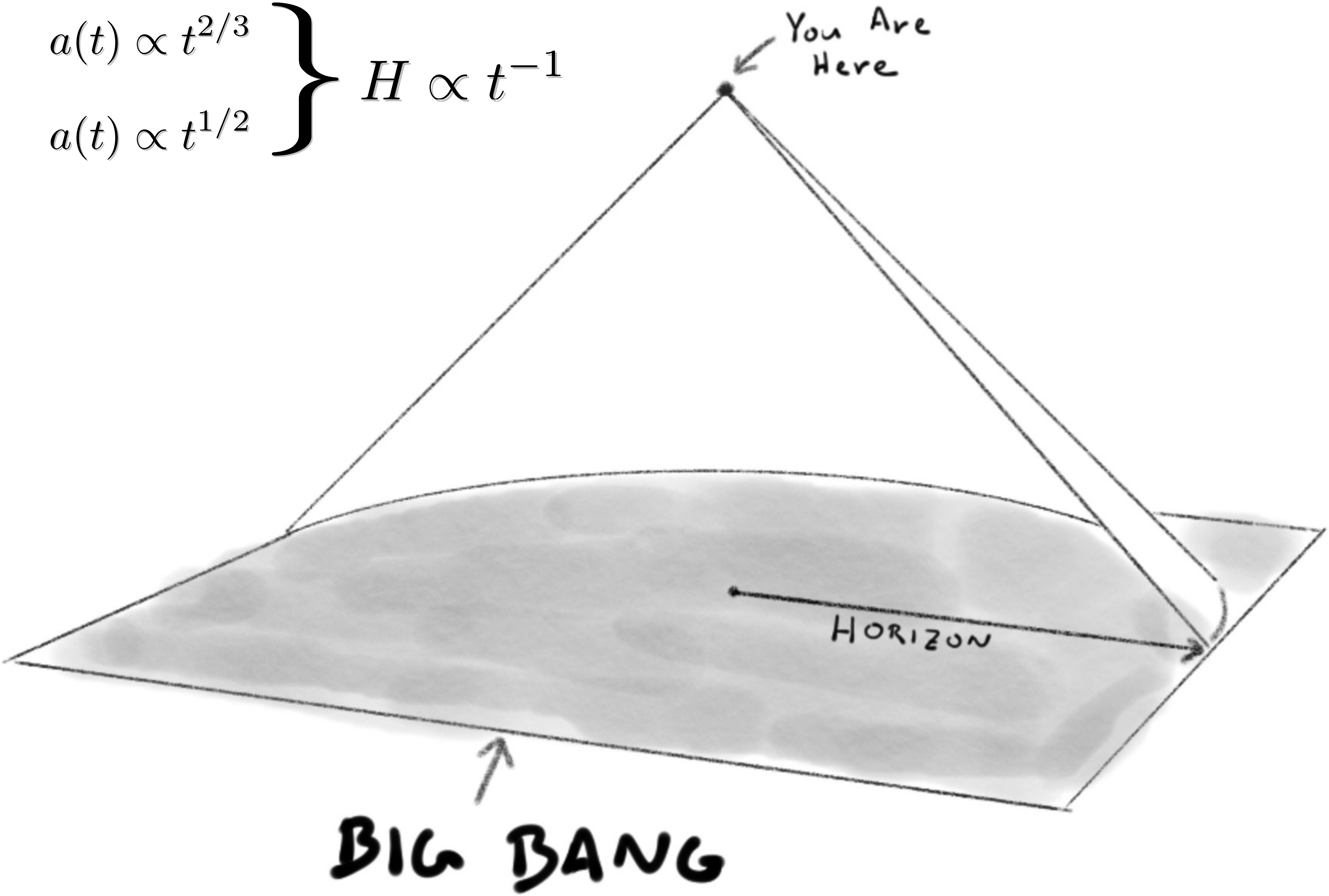


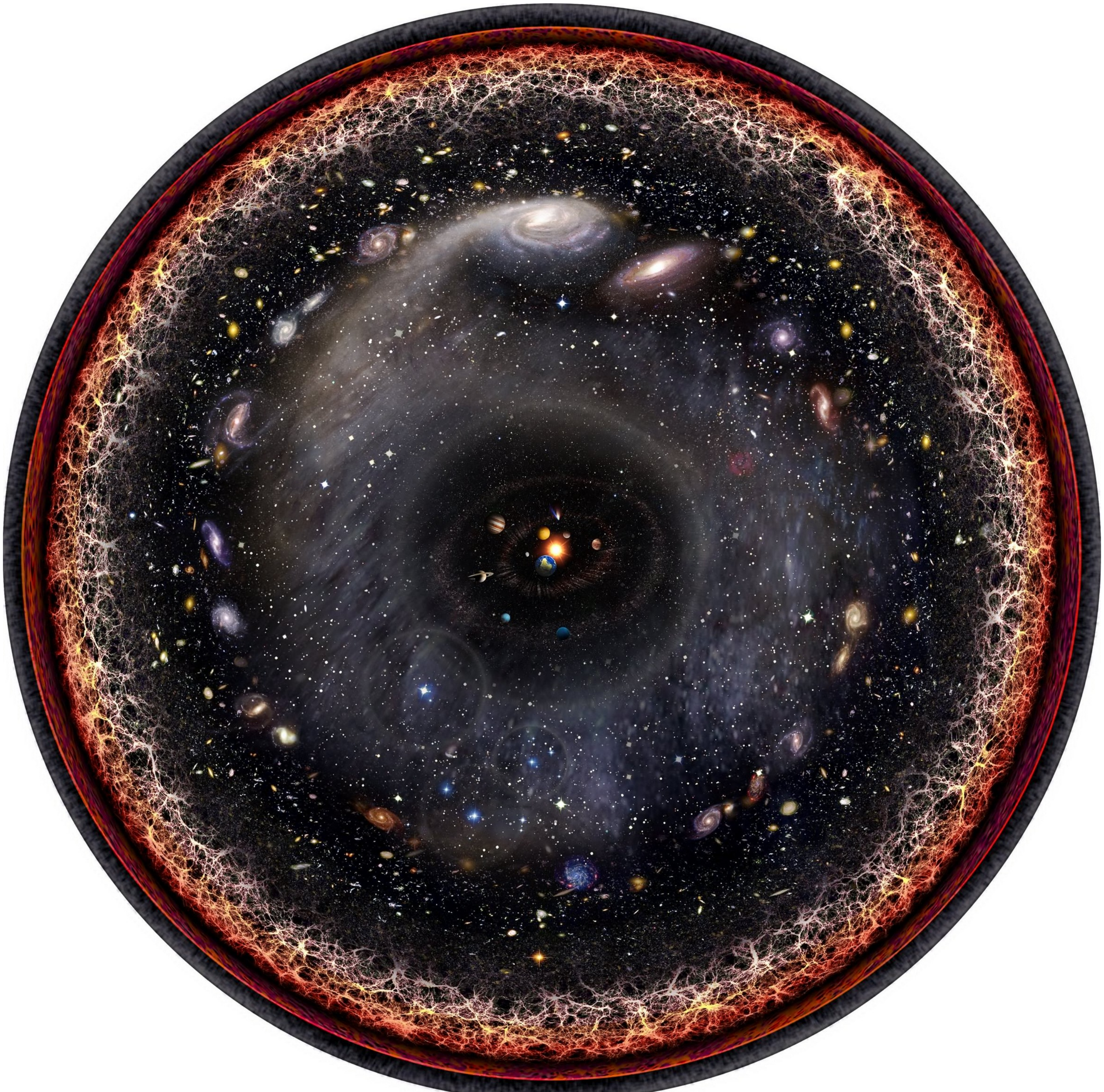




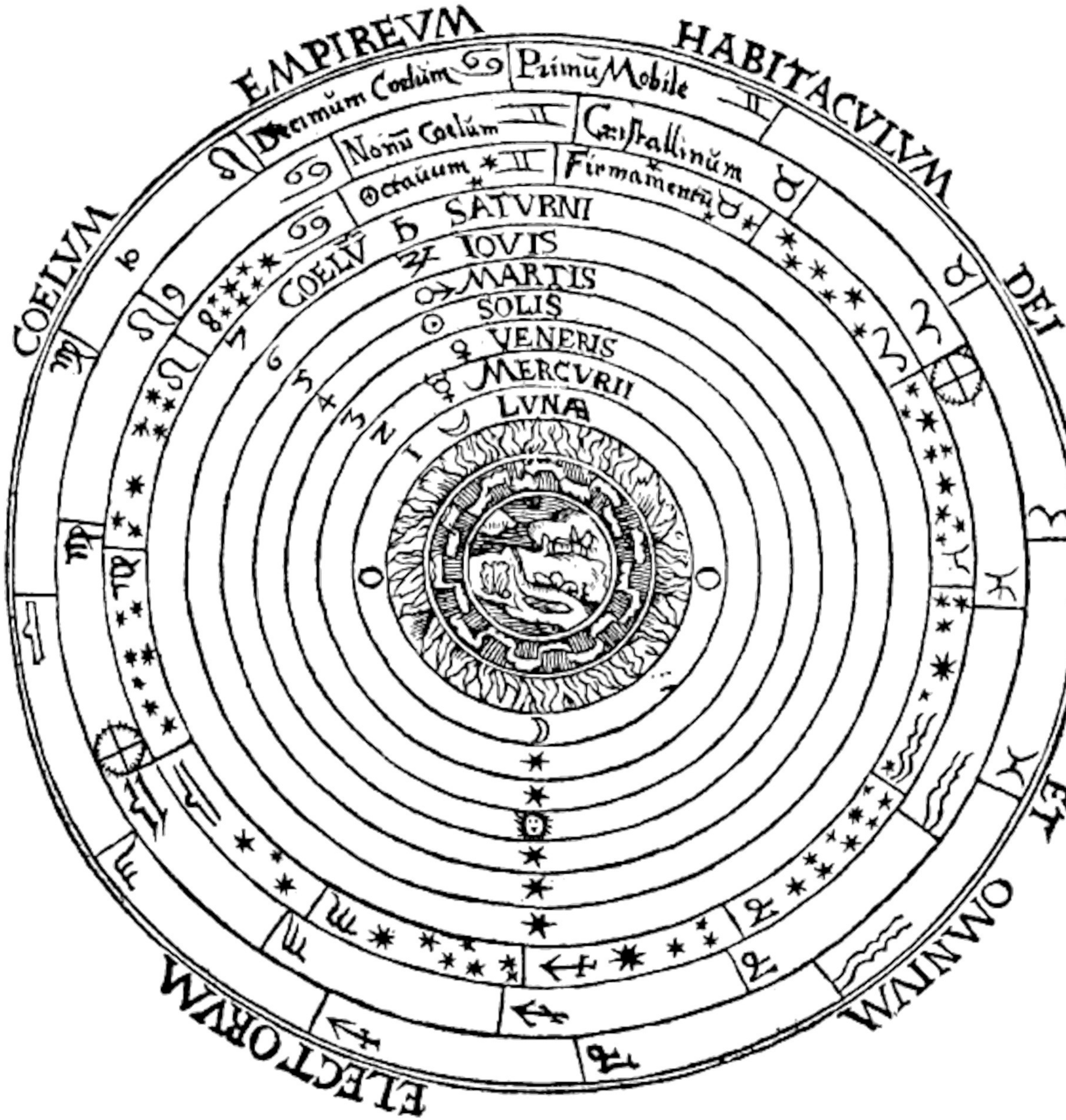
The cosmic horizon

$$\left.\begin{array}{l} a(t) \propto t^{2/3} \\ a(t) \propto t^{1/2} \end{array}\right\} H \propto t^{-1}$$

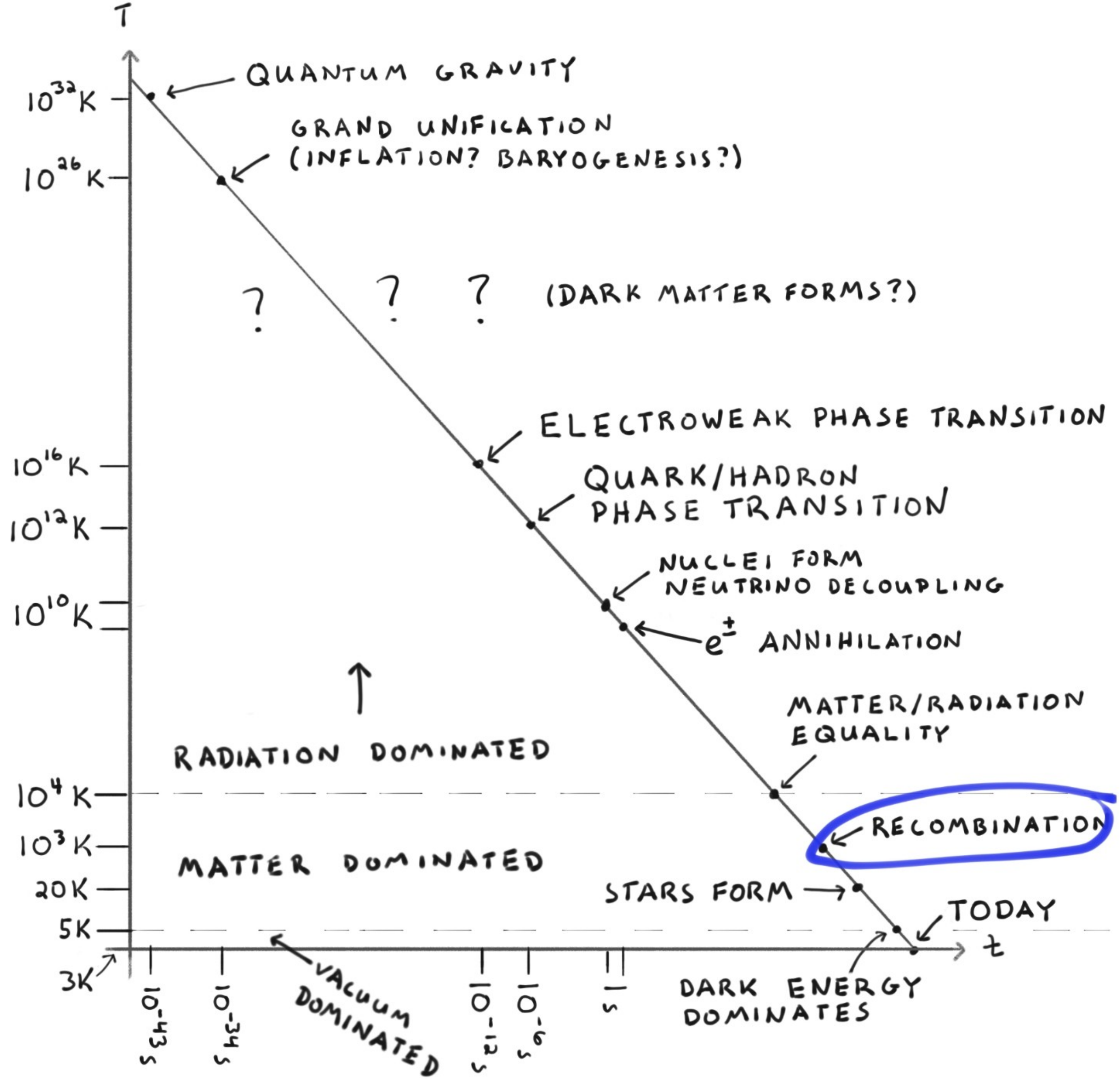




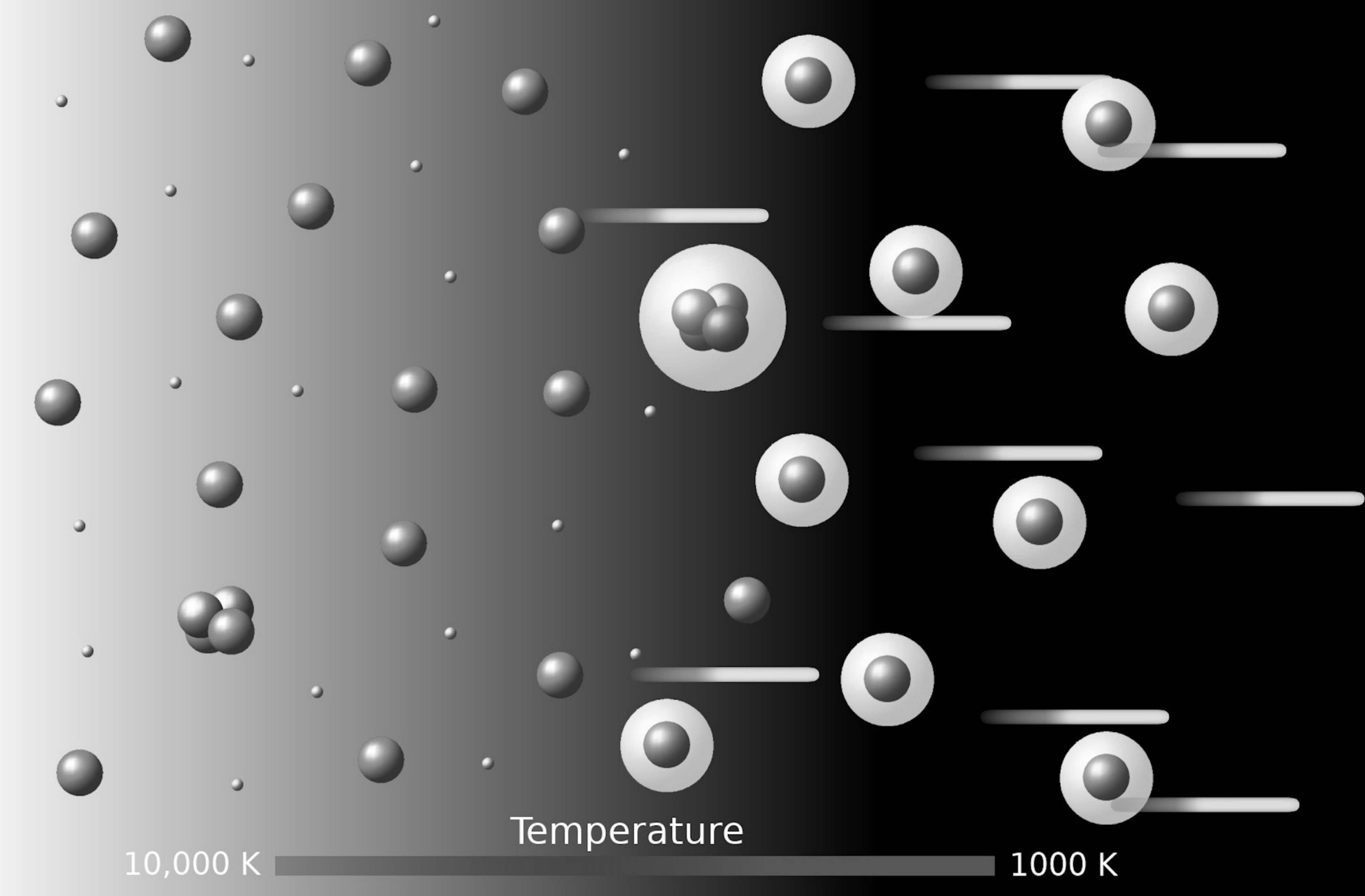
Schema huius præmissæ diuisionis Sphærarum.



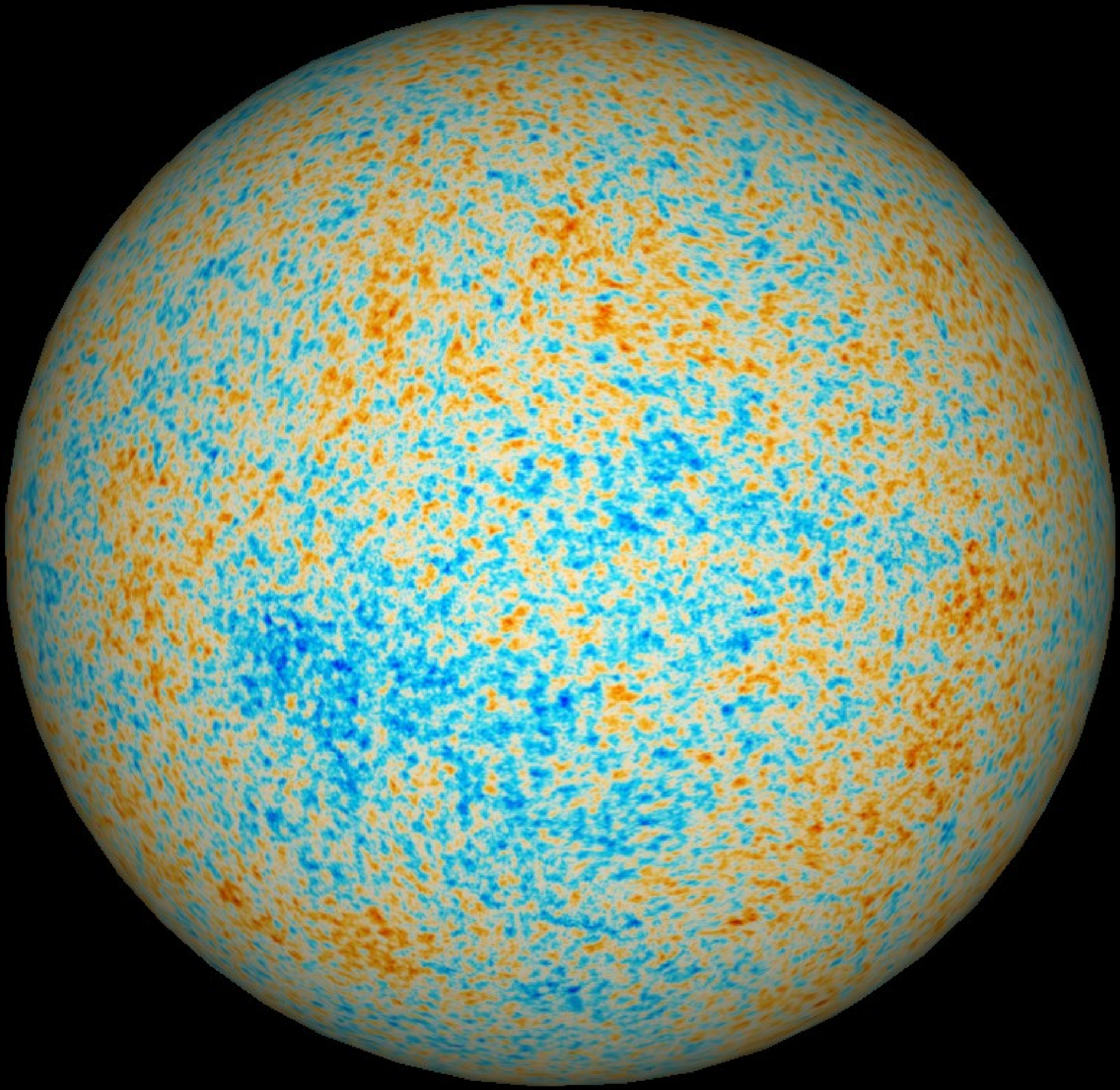
History of the universe: abridged



Recombination

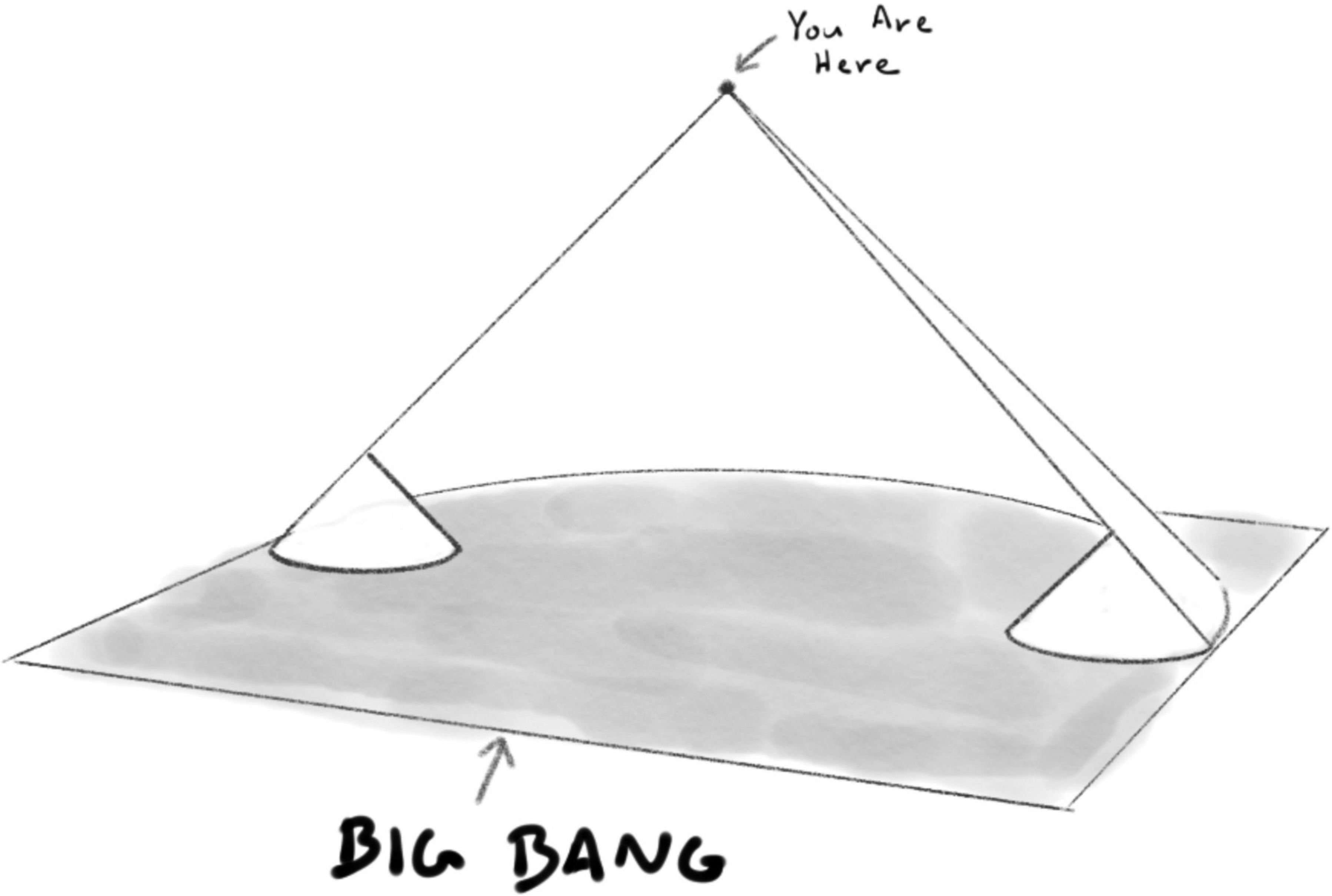


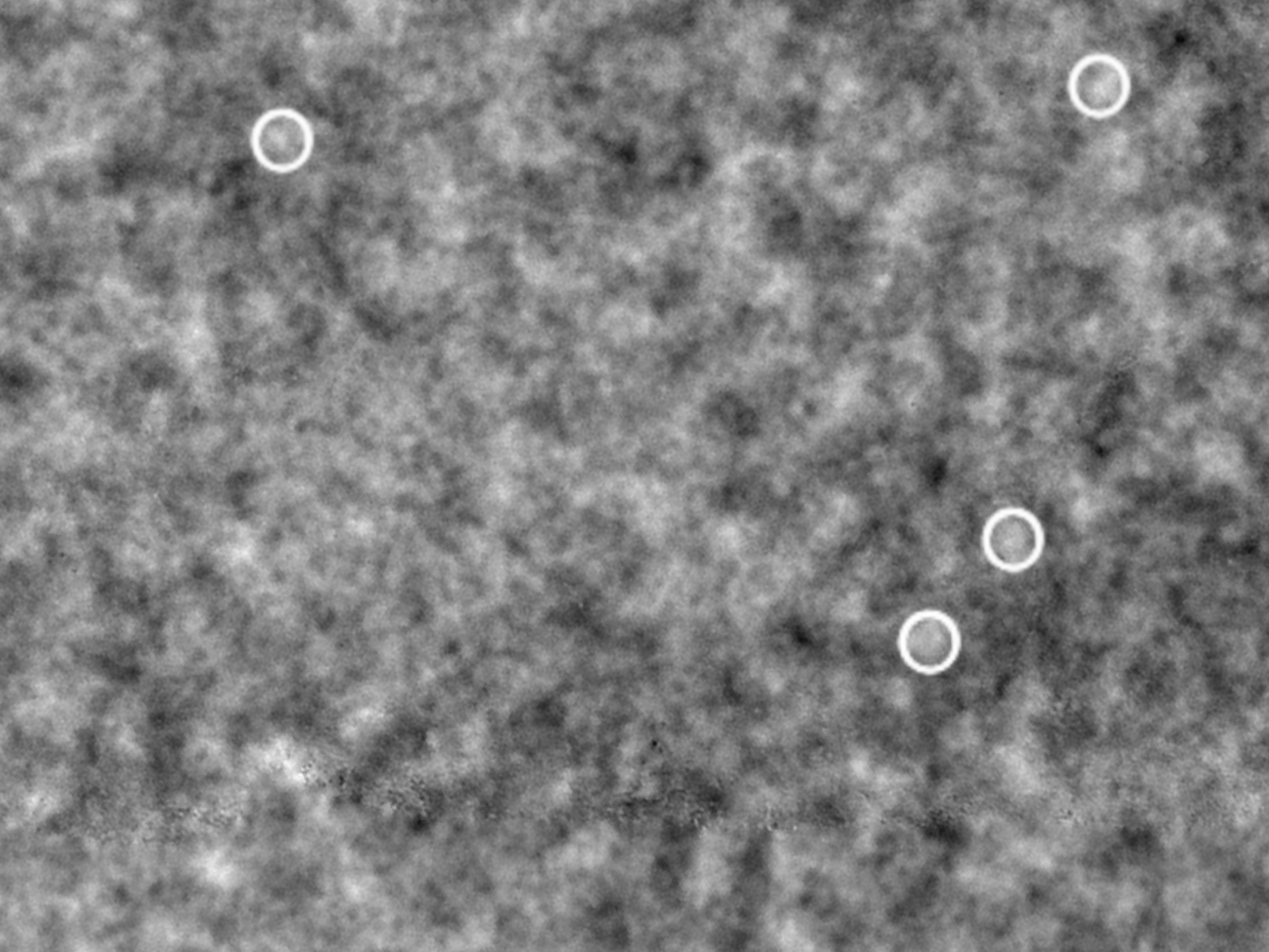
Primordial perturbations



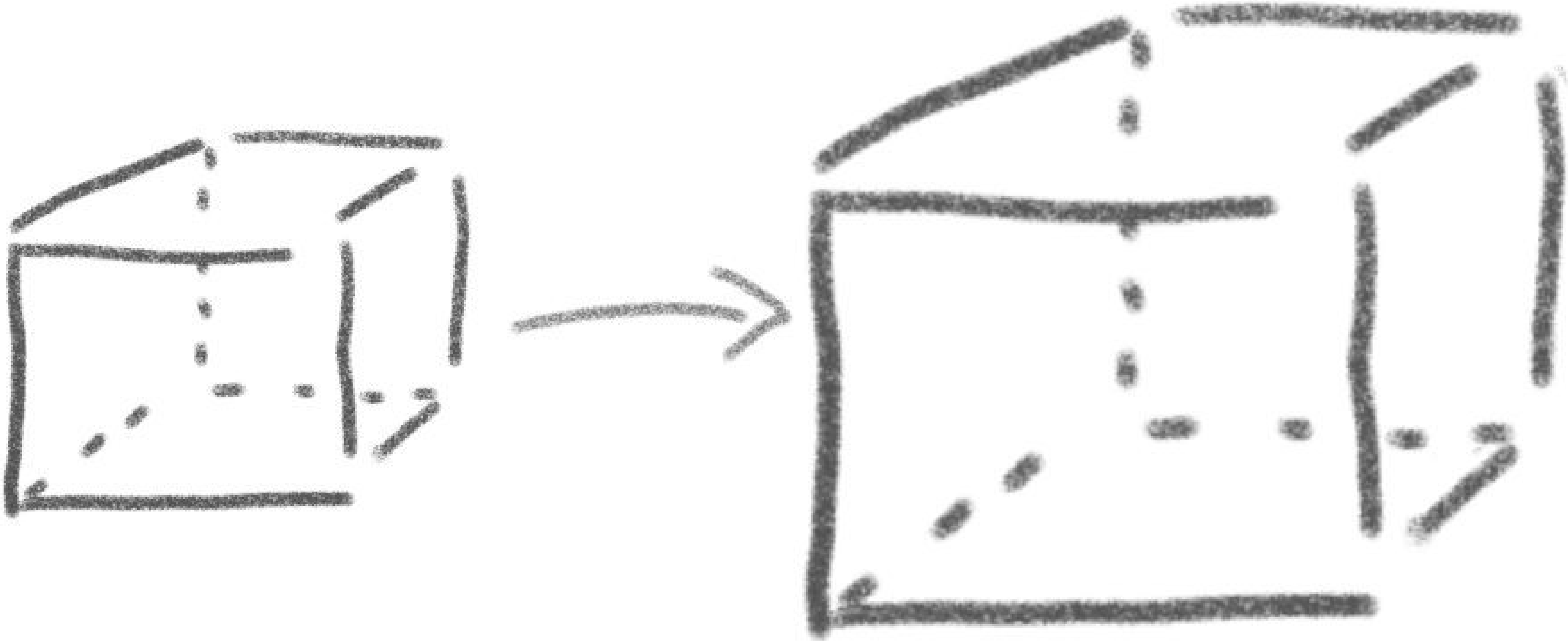
(Image: ESA and the Planck collaboration)

The horizon problem

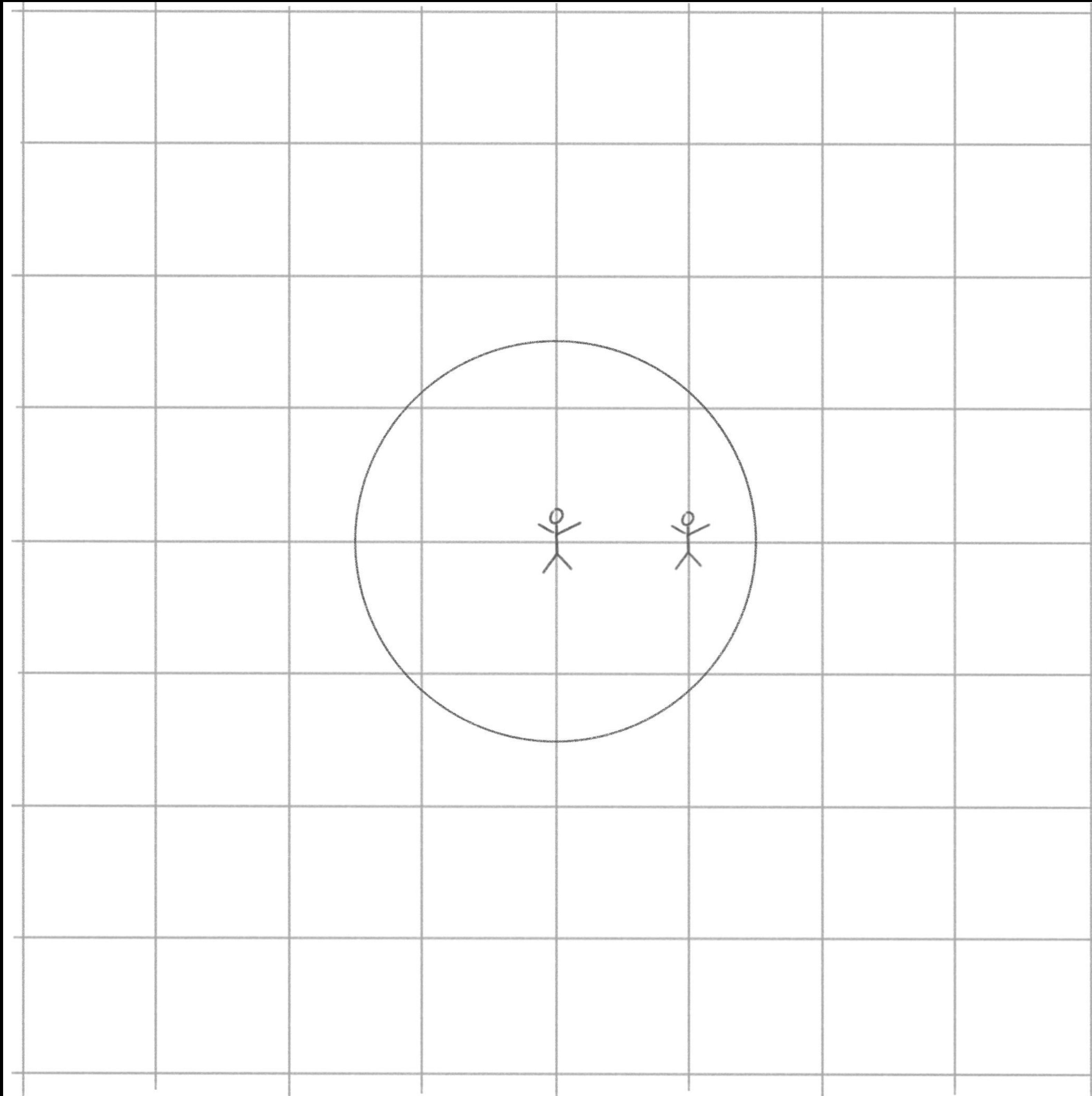


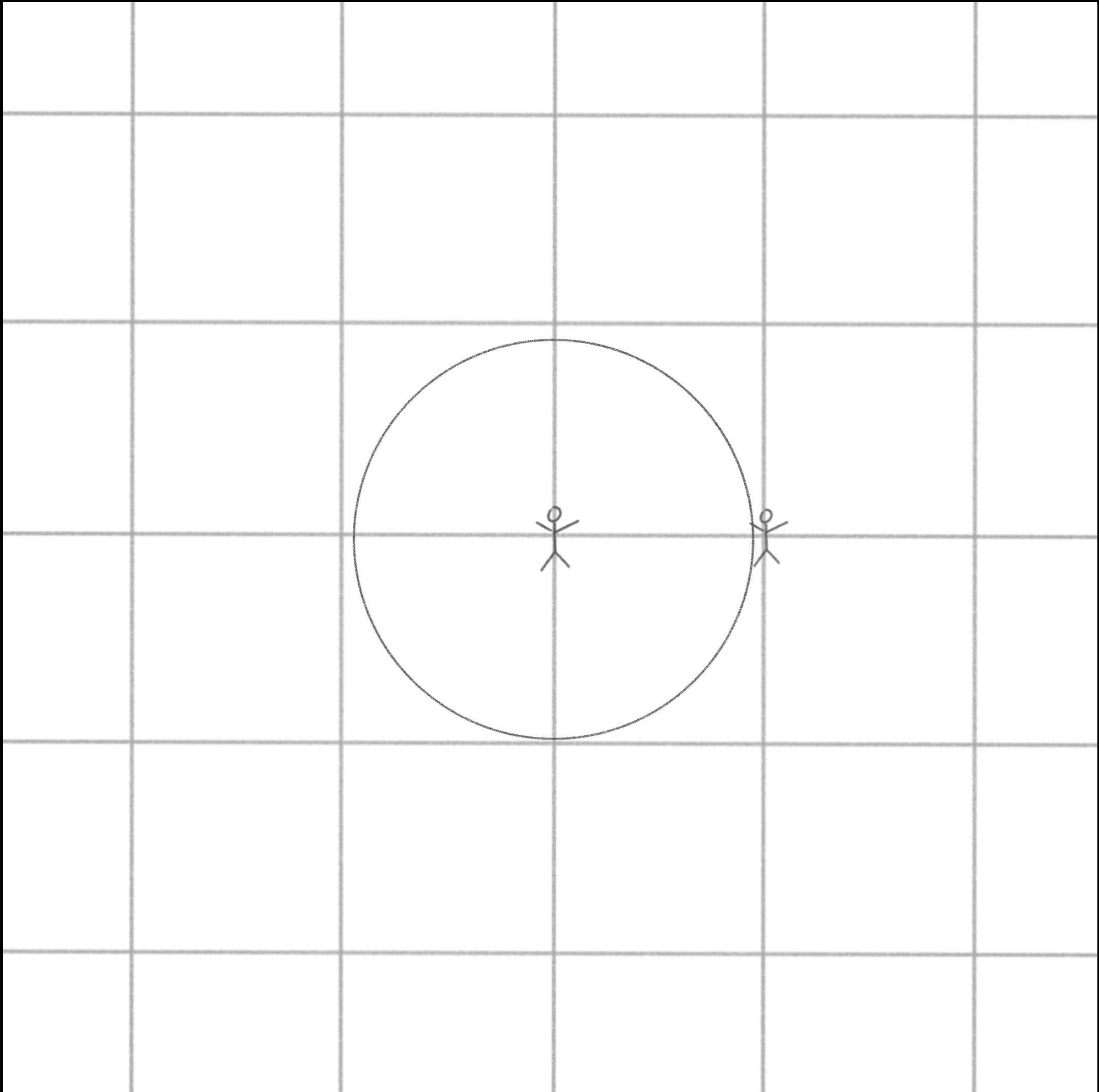


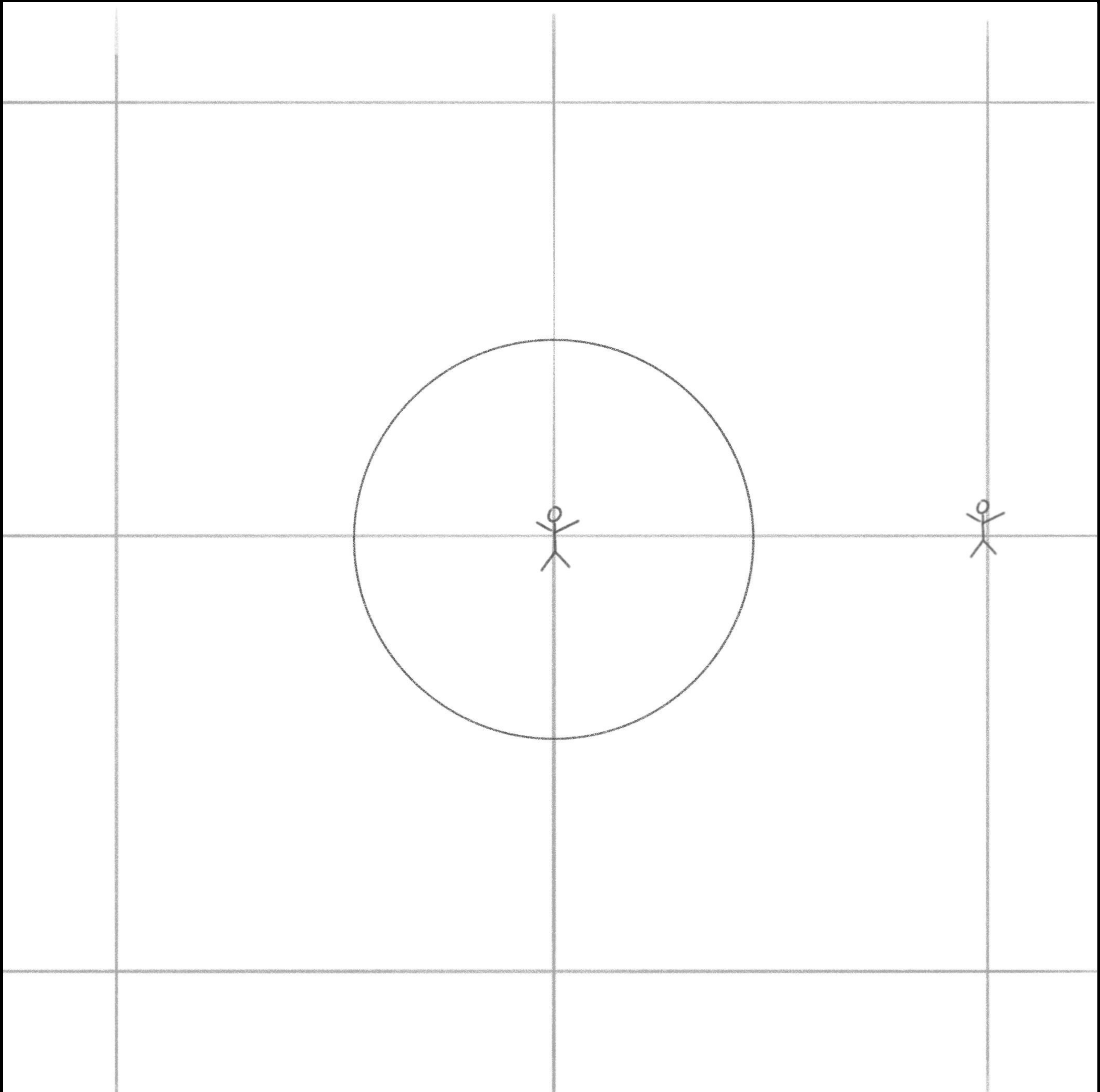
Vacuum energy and inflation



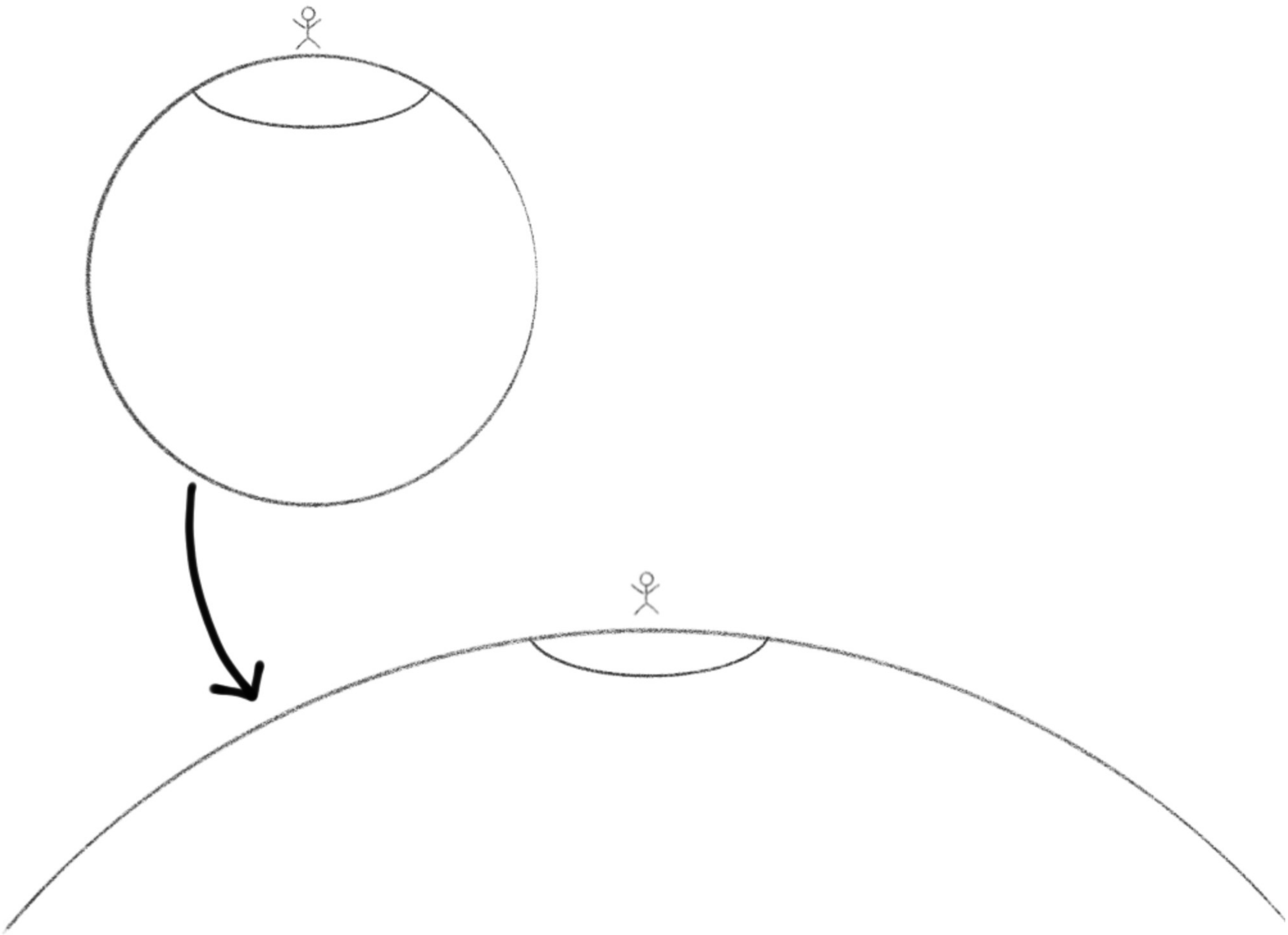
$$\rho = \text{const.} \Rightarrow a \propto e^{Ht}$$







Inflation makes the universe flat



Curvature

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2}\rho$$

$$\frac{\ddot{a}}{a} = -\frac{1}{2M_P^2}(\rho + 3p)$$

$$\Omega \equiv \frac{\rho}{3M_P^2 H^2} = 1 + \frac{k}{(aH)^2}$$

$$\rho_c \equiv 3M_P^2 H^2$$

$$a(t) \propto t^{1/3(1+w)} \qquad p = w\rho$$

$$|\Omega - 1| = \frac{1}{(aH)^2} \longrightarrow$$

- $\Omega = 1$ Unstable: $w > -1/3$ $\quad \ddot{a} < 0$
- $\Omega = 1$ Stable: $w < -1/3$ $\quad \ddot{a} > 0$

Inflation solves the horizon problem

$$H^2 + \frac{k}{a^2} = \frac{1}{3M_P^2}\rho = \text{const.} \qquad a(t) \propto e^{Ht}$$

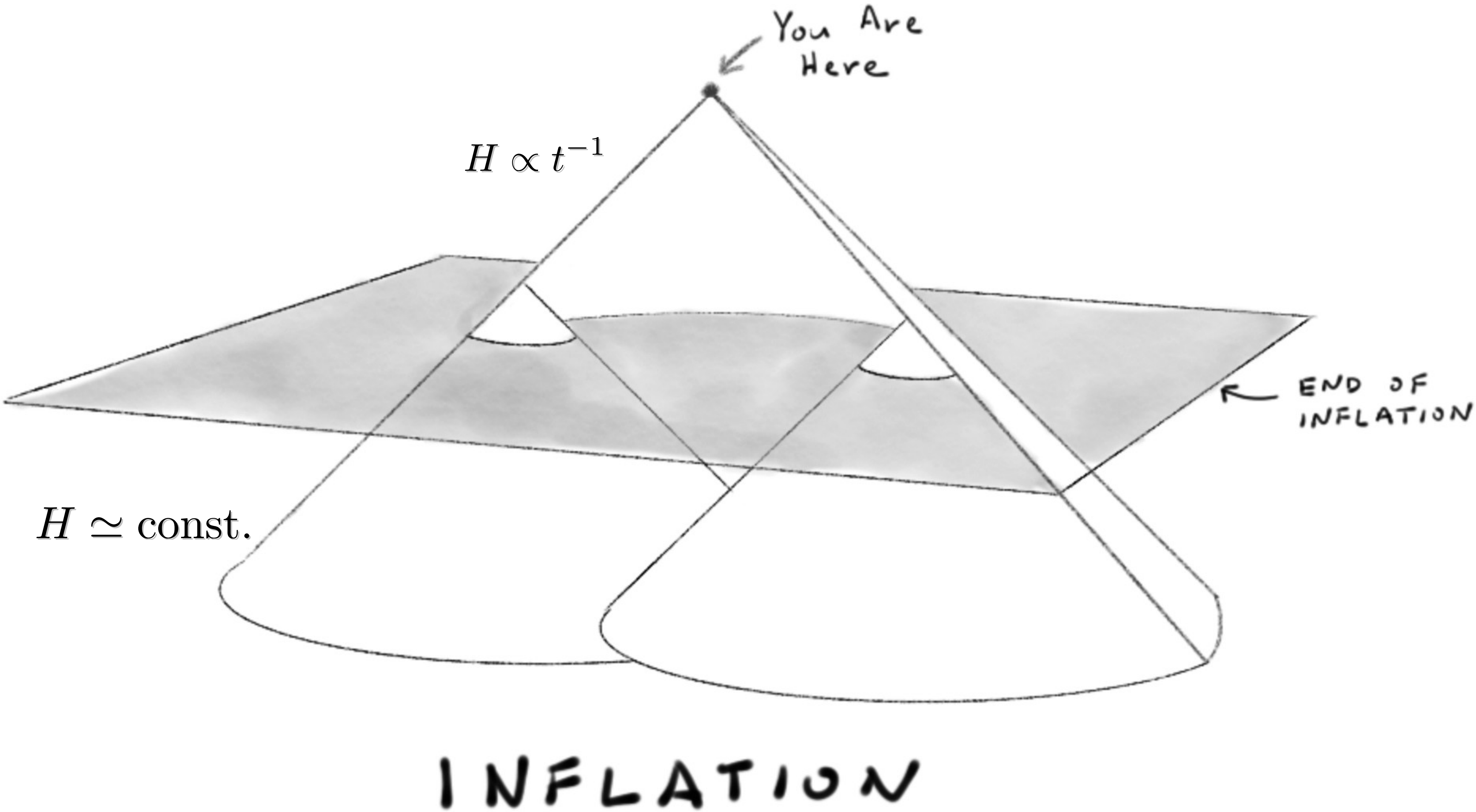
Conformal time: $d\tau = \frac{dt}{a(t)}$

initial singularity (?)

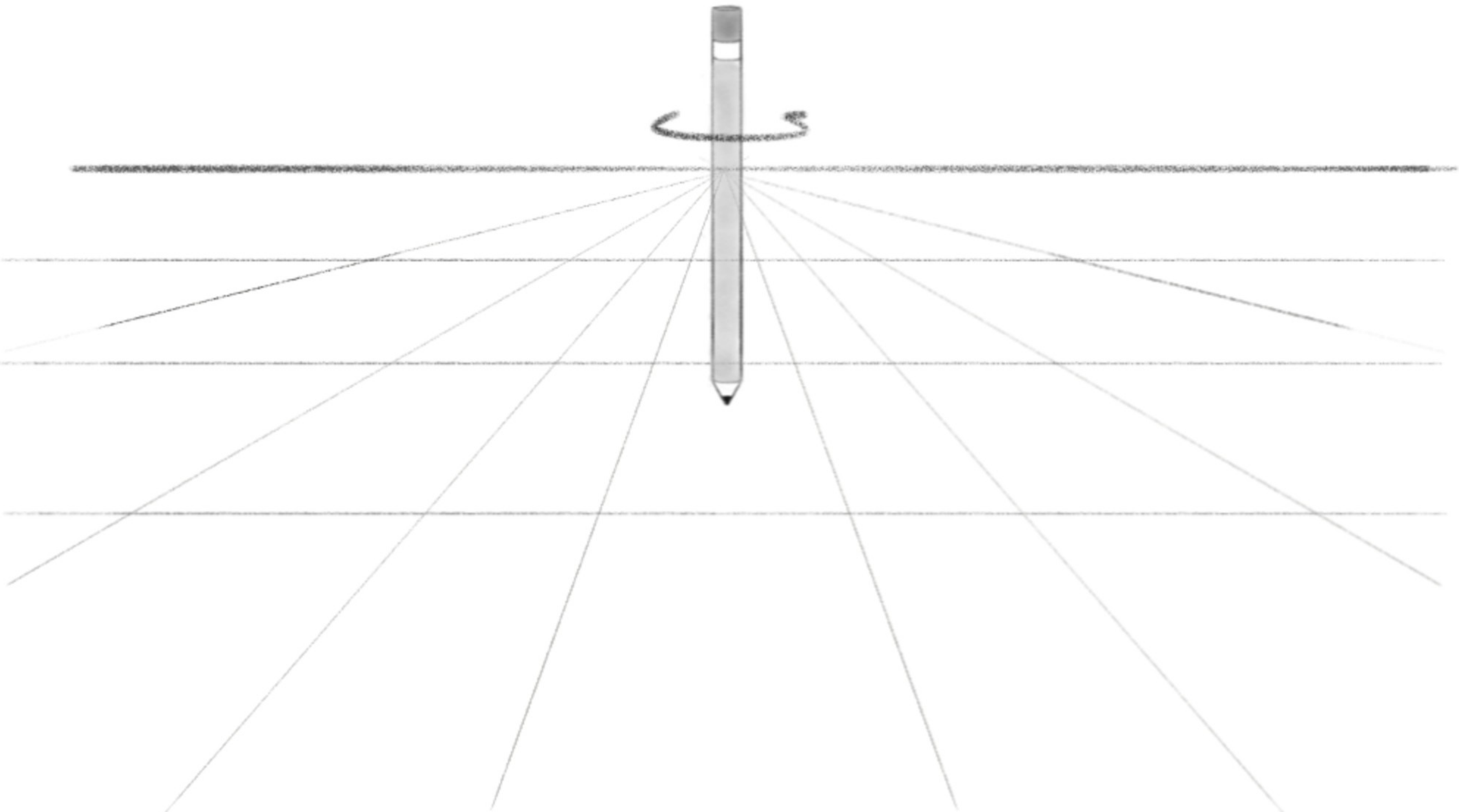
$$a(\tau) = -\frac{1}{H\tau} \quad \begin{cases} \tau \to -\infty \\ \tau \to 0 \end{cases}$$

end of inflation

Inflation solves the horizon problem



Symmetry breaking: an analogy



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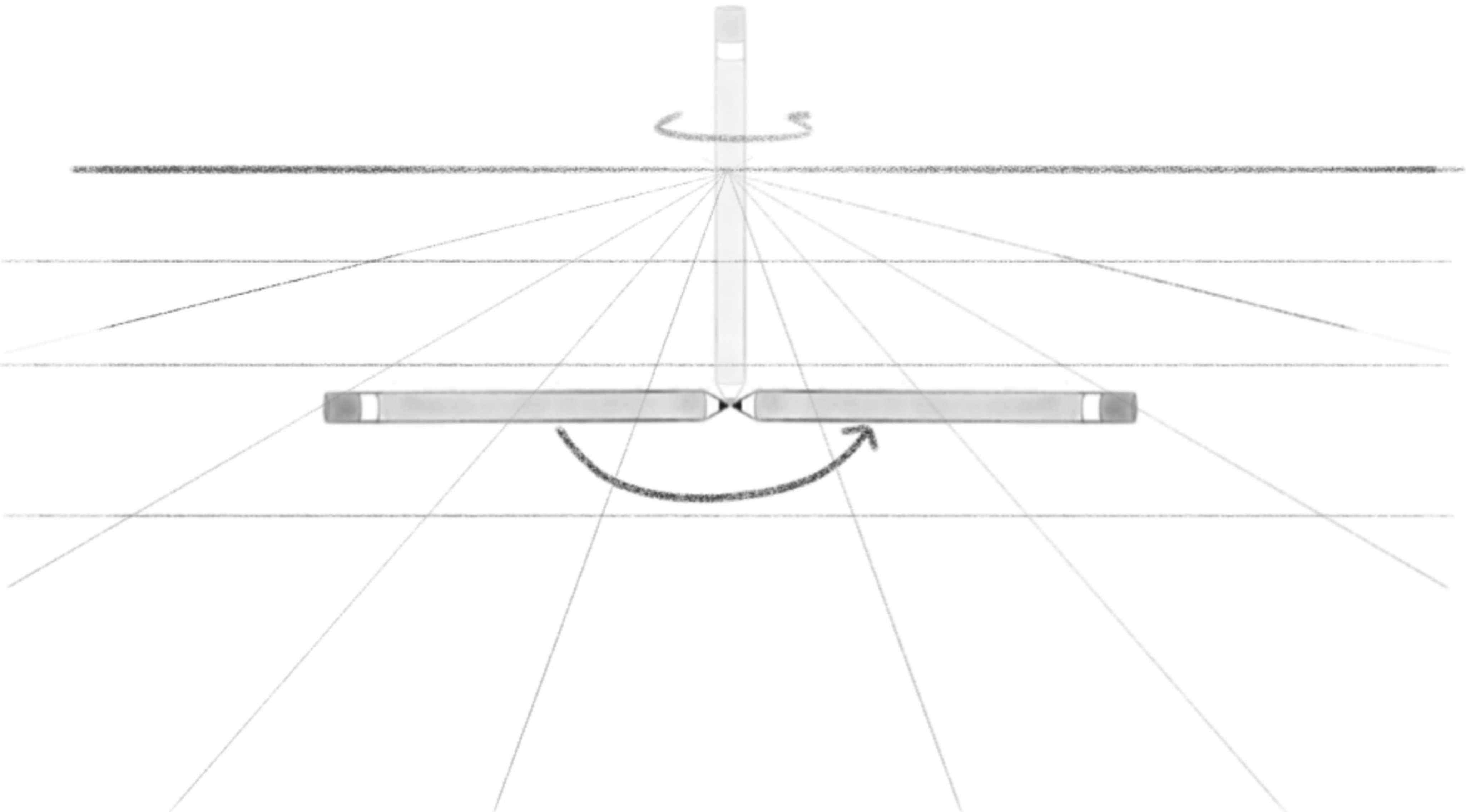
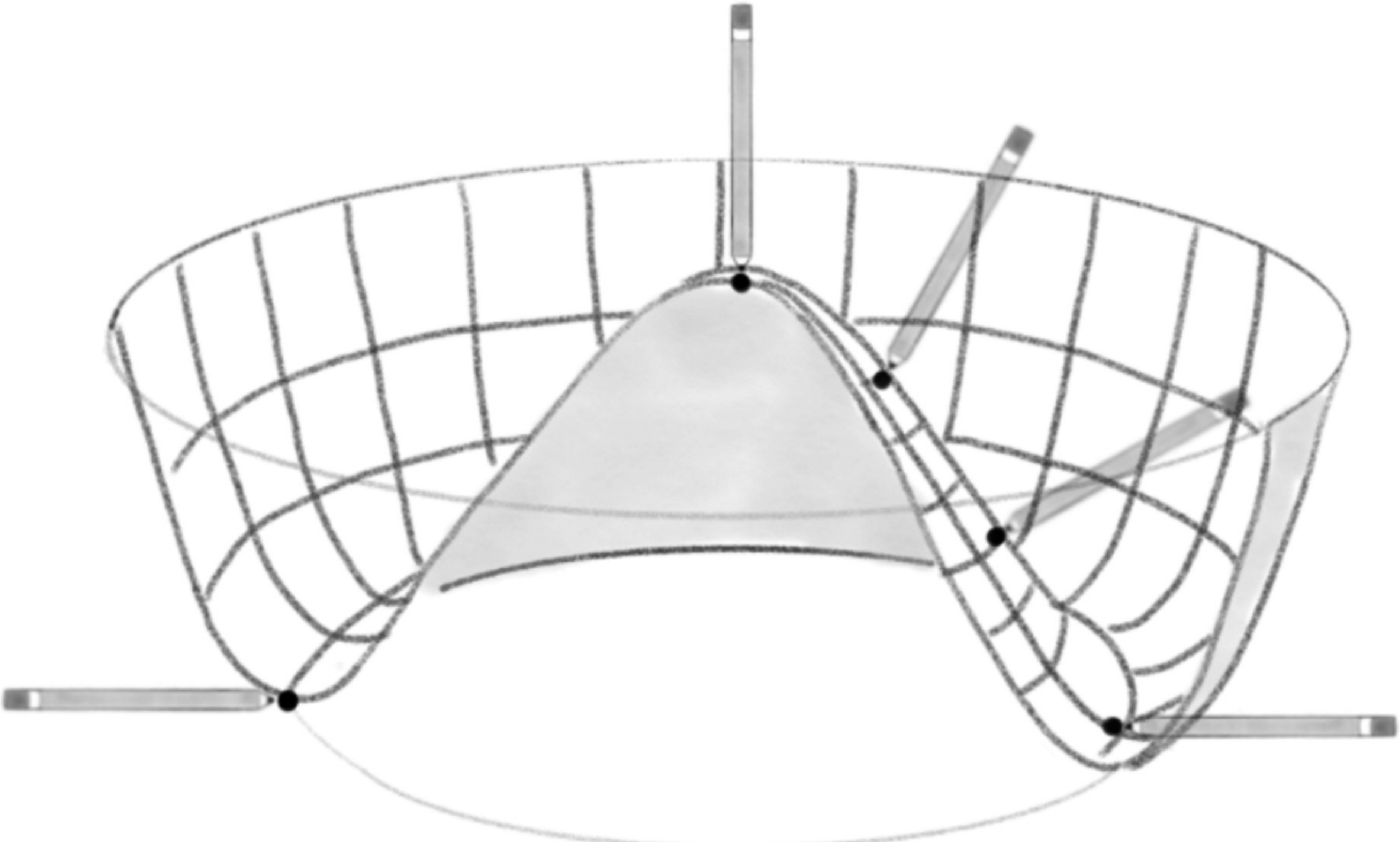


The potential surface



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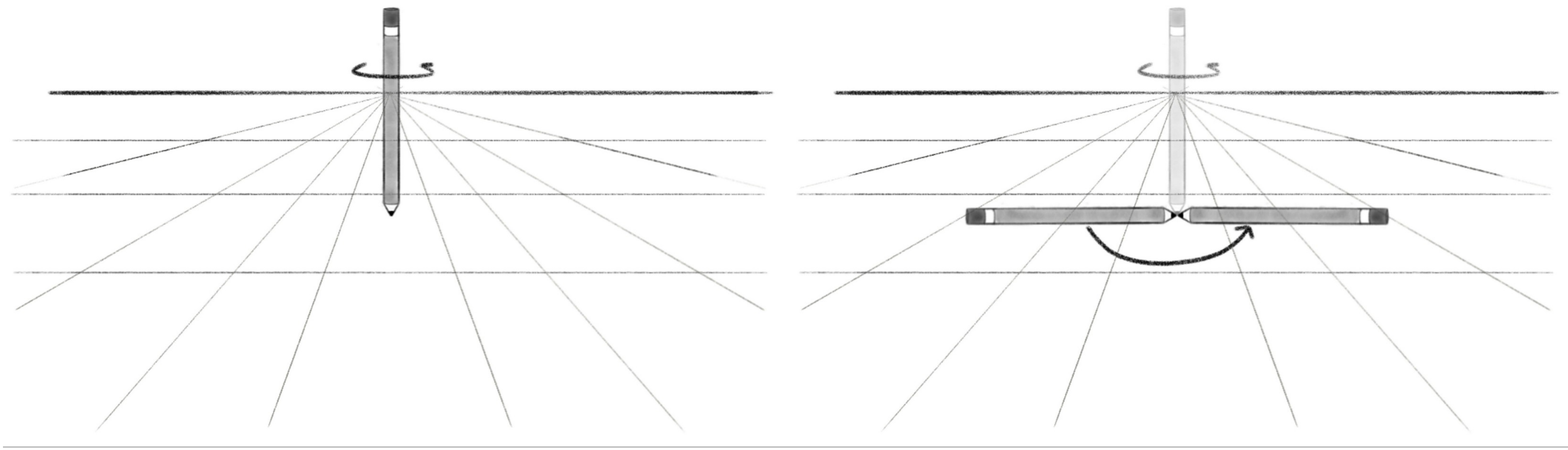
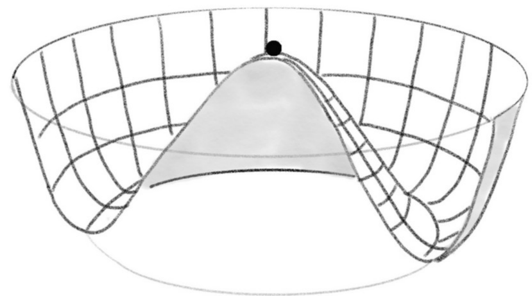
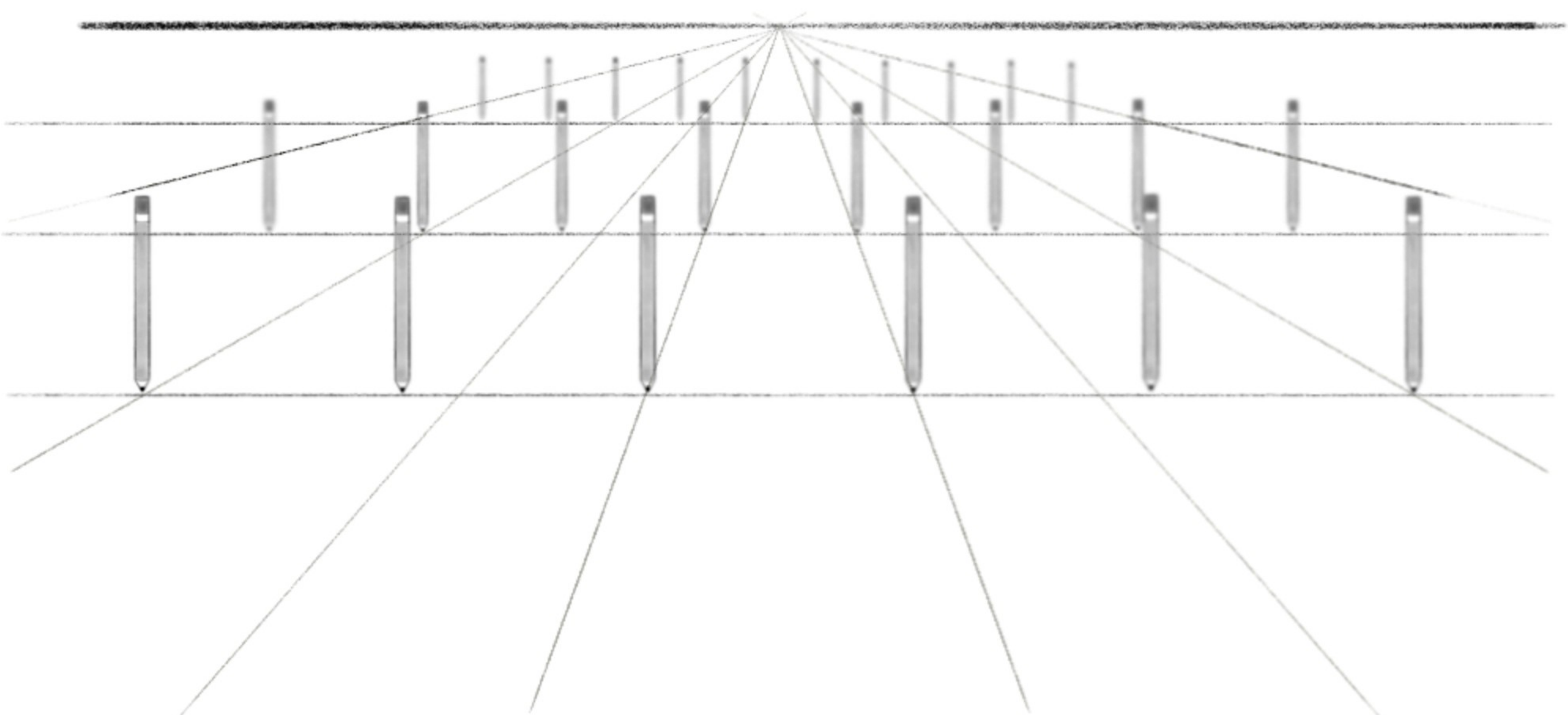
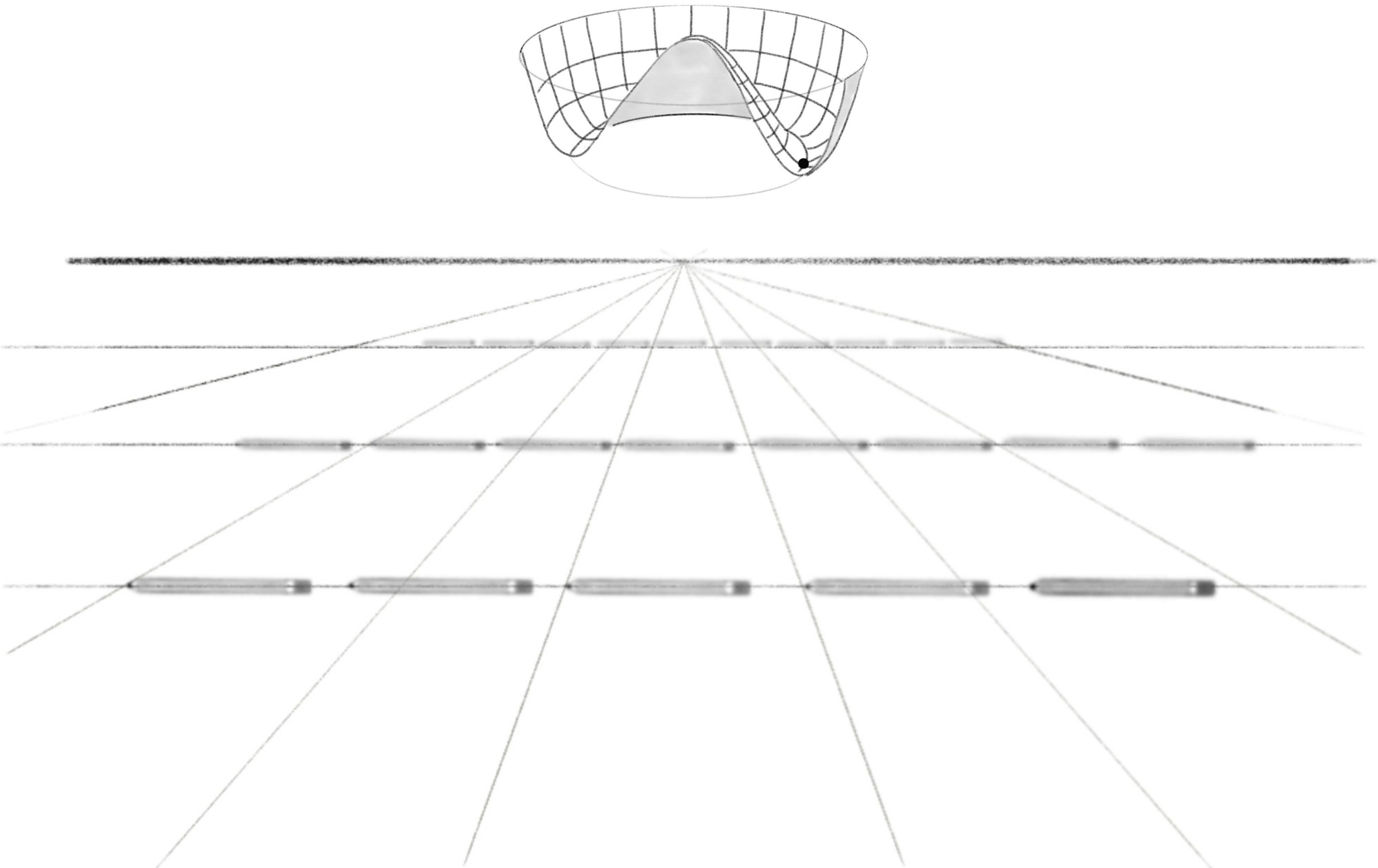


Symmetry breaking and vacuum energy





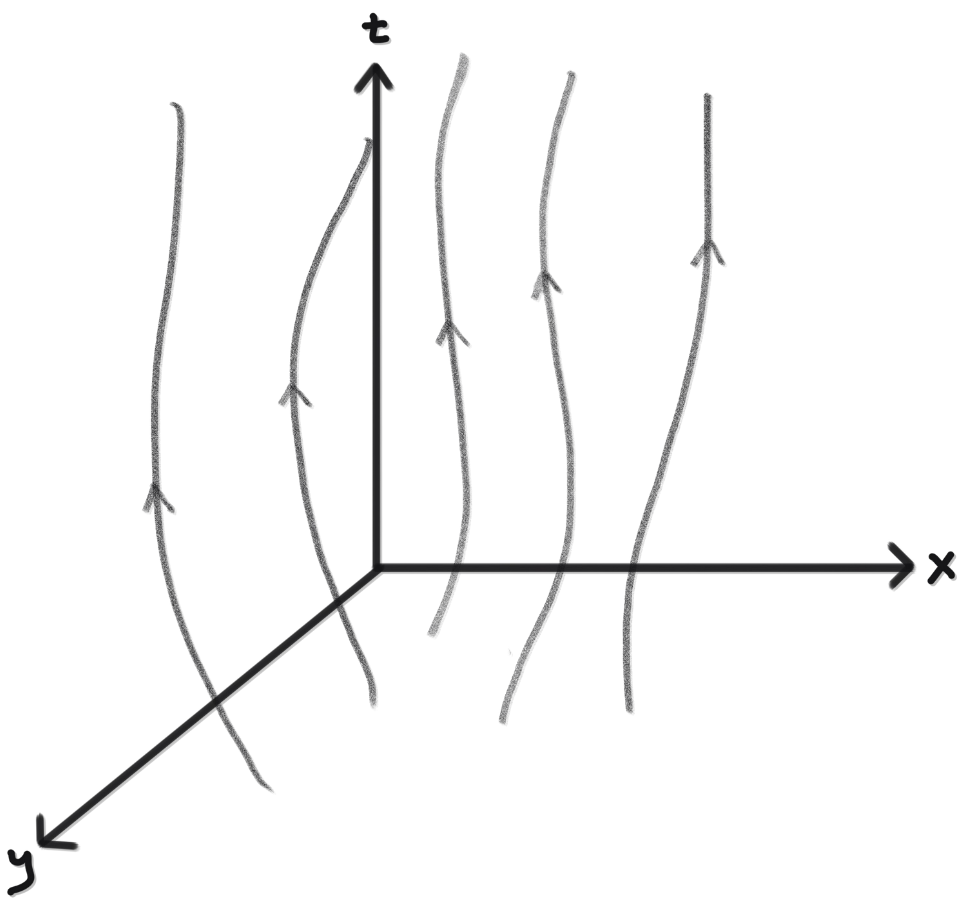
Symmetry breaking and vacuum energy



Scalar fields and stress energy

Stress energy (perfect fluid): $T^{\mu\nu} = pg^{\mu\nu} + (\rho + p)u^{\mu}u^{\nu}$

$u^{\mu}u_{\mu} = -1$



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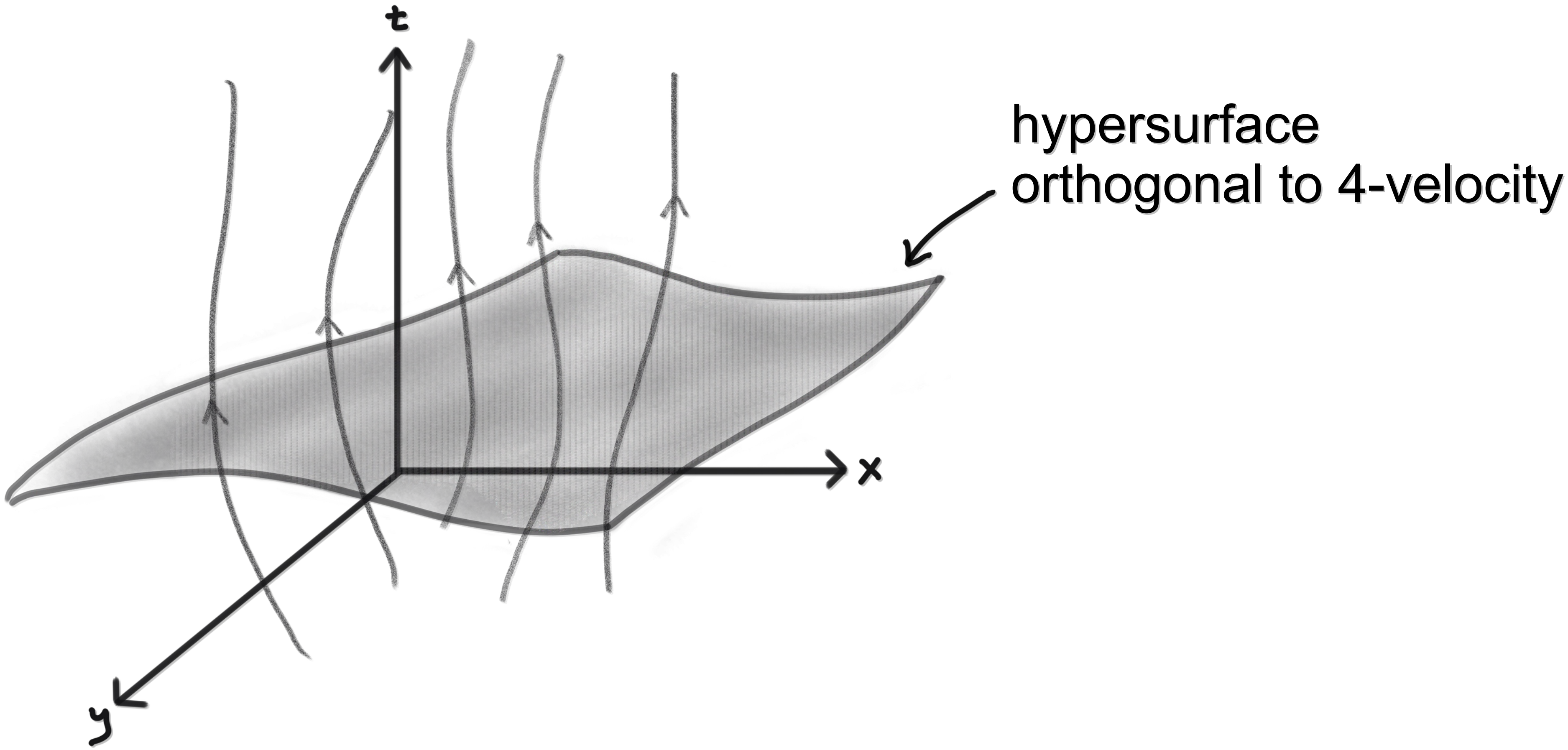
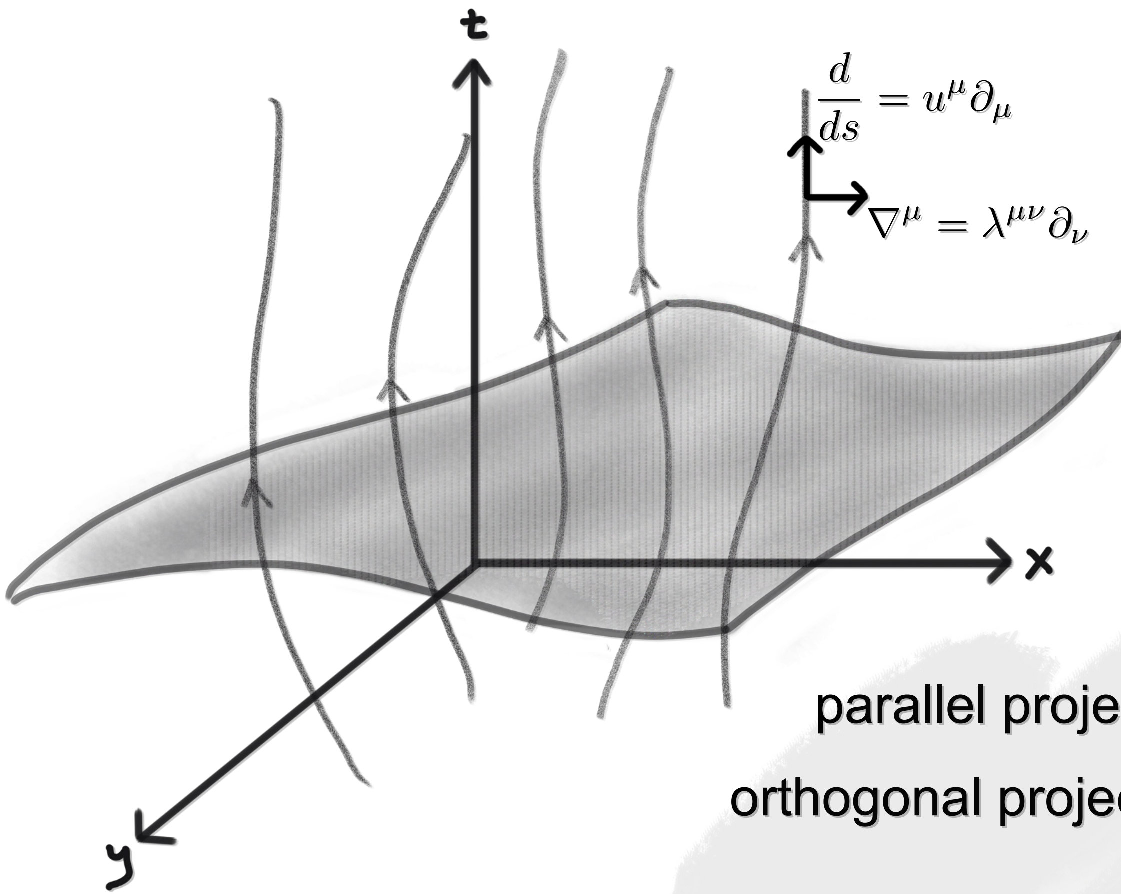


Scalar fields and stress energy

Stress energy (perfect fluid): $T^{\mu\nu} = \rho u^\mu u^\nu + p\lambda^{\mu\nu}$



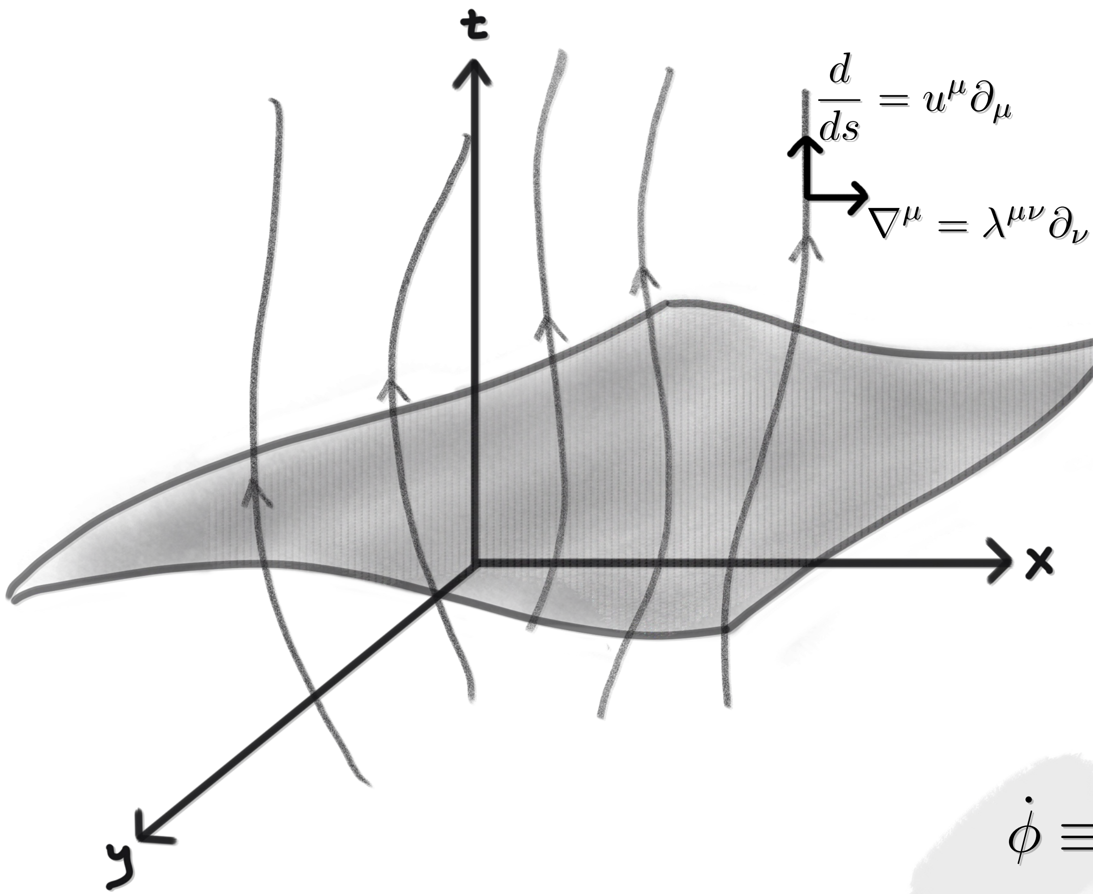
parallel projection: u^μ

orthogonal projection: $\lambda^{\mu\nu} \equiv g^{\mu\nu} + u^\mu u^\nu$

$$\lambda^{\mu\nu} u_\nu = (g^{\mu\nu} + u^\mu u^\nu)\, u_\nu = u^\mu - u^\mu = 0$$

Scalar fields and stress energy

Stress energy (perfect fluid): $T^{\mu\nu} = \rho u^\mu u^\nu + p\lambda^{\mu\nu}$



$$u^\mu = \frac{\partial^\mu \phi}{\sqrt{|g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}|}}$$

$$\uparrow$$

$$u^\mu u_\mu = -1$$

$$\dot{\phi} \equiv \frac{d\phi}{ds} = u^\mu \partial_\mu \phi$$

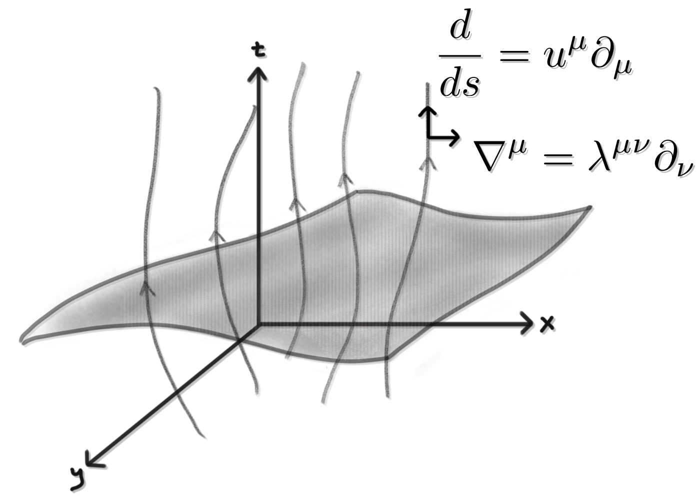
Exercise

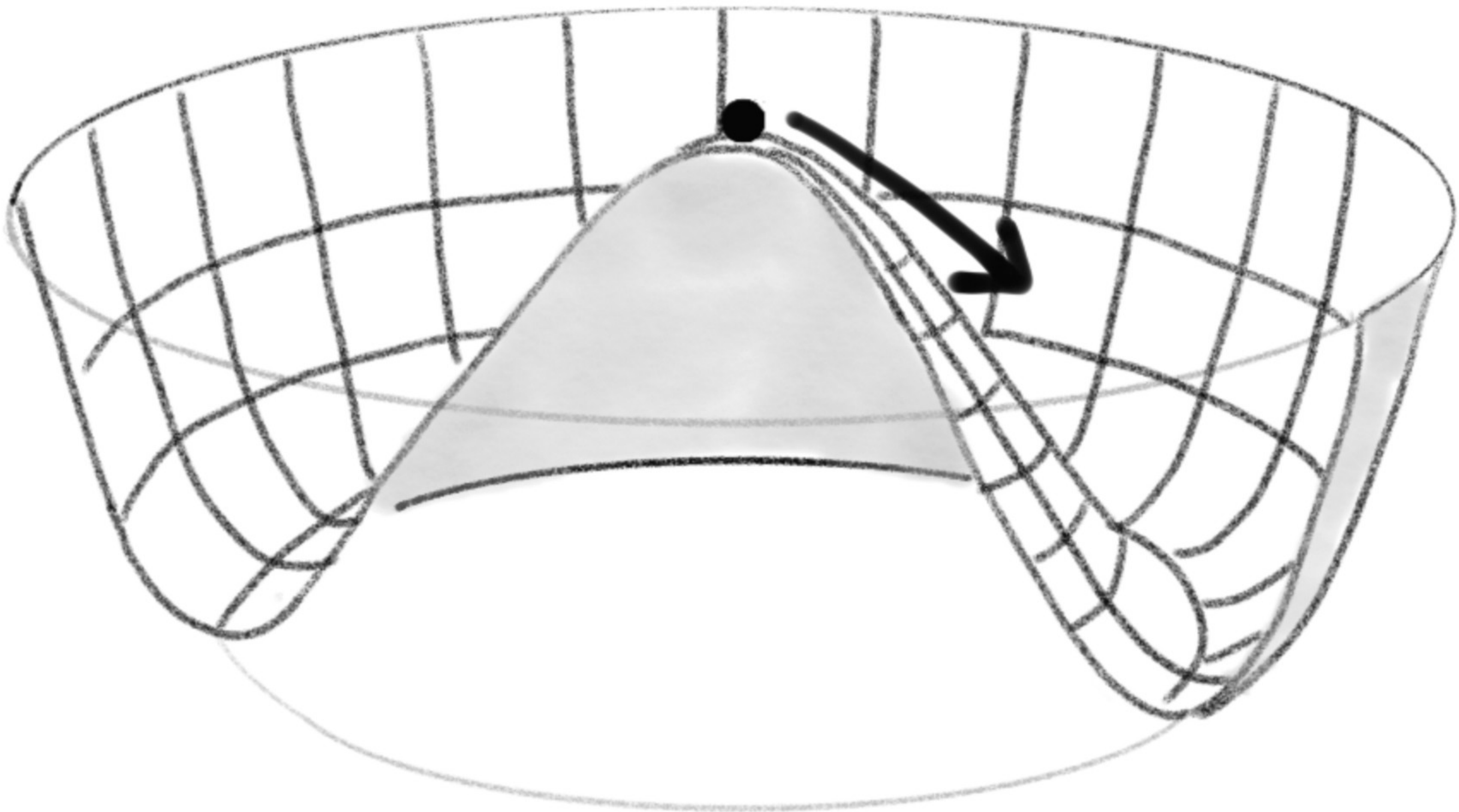
For: $u^\mu = \dfrac{\partial^\mu \phi}{\sqrt{|g^{\mu\nu}\phi_{,\mu}\phi_{,\nu}|}}$ $\quad \dot{\phi} \equiv \dfrac{d\phi}{ds} = u^\mu \partial_\mu \phi$

Show:

(a) $u^\mu u_\mu = -1$

(b) $\dot{\phi}^2 = g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$





Scalar fields and stress energy

$$\mathcal{L} = \frac{1}{2}\dot{\phi}^2 - V(\phi) = p$$

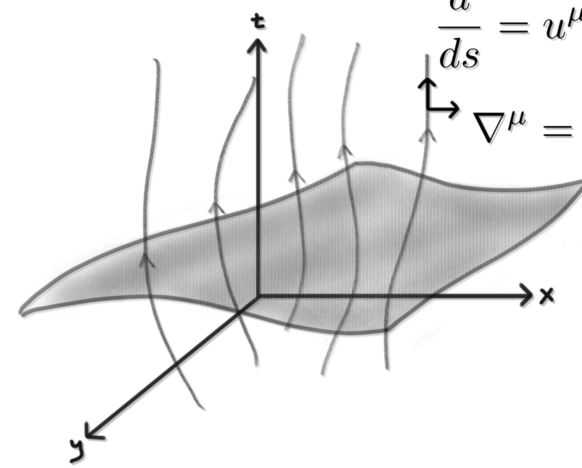
pressure

$$\mathcal{H} = \frac{1}{2}\dot{\phi}^2 + V(\phi) = \rho$$

energy density

$$\frac{d}{ds} = u^\mu \partial_\mu$$

$$\nabla^\mu = \lambda^{\mu\nu}\partial_\nu$$



Exercise:

$$\nabla^\mu \phi = 0$$

$$u^\mu = (1, 0, 0, 0)$$

comoving gauge

$$T^\mu{}_\nu = \begin{pmatrix} \rho & & & \\ & -p & & \\ & & -p & \\ & & & -p \end{pmatrix}$$

Equation of motion

$$T^{\mu\nu} = \rho u^\mu u^\nu + p\lambda^{\mu\nu}$$

$$T^{\mu\nu}{}_{;\nu} = 0 \Rightarrow \begin{cases} \dot{\rho} - (\rho + p)\, u^\mu{}_{;\mu} = 0 & \text{(1)} \\ (\nabla p)^\mu - (\rho + p)\, \dot{u}^\mu = 0 & \text{(2)} \end{cases}$$

$$g^{\mu\nu} = \text{diag.}\left(-1, a^2, a^2, a^2\right) \Rightarrow u^\mu{}_{;\mu} = -3\left(\frac{\dot{a}}{a}\right) = -3H$$

$$p = \frac{1}{2}\dot{\phi}^2 - V(\phi)$$

$$\rho = \frac{1}{2}\dot{\phi}^2 + V(\phi)$$

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\delta V}{\delta\phi} = 0 \qquad \text{(1)}$$

Slow roll

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0$$

$$H^2 = \frac{1}{3M_P^2}\left[\frac{1}{2}\dot{\phi}^2 + V(\phi)\right]$$

$$\left.\begin{aligned} \rho &= \frac{1}{2}\dot{\phi}^2 + V(\phi) \\ p &= \frac{1}{2}\dot{\phi}^2 - V(\phi) \end{aligned}\right\} \quad p \simeq -\rho$$

Attractor solution:

$$\dot{\phi} \simeq -\frac{V'(\phi)}{3H}$$

$$H^2 \simeq \frac{1}{3M_P^2} V(\phi)$$