The Weak Cosmic Censorship Conjecture: Status Report

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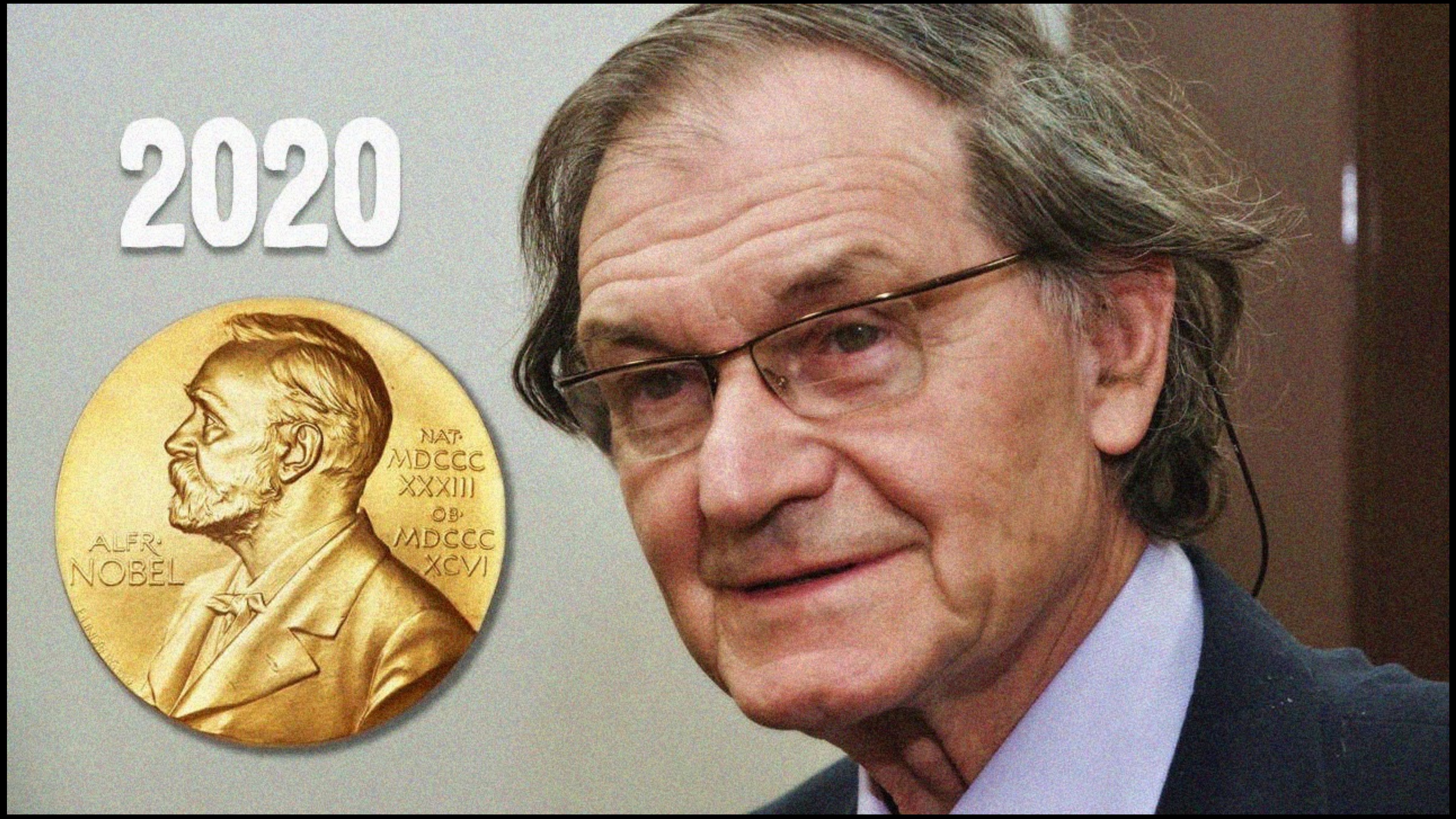
THE ROYAL SOCIETY

Black hole formation in GR



"For the discovery that black hole formation is a robust prediction of the general theory of relativity"

Black hole formation in GR

- Singularity theorems: singularities form generically in GR [Penrose; Penrose and Hawking]

- Some questions not addressed by the singularity theorems:

1. What types of singularities form from generic initial data?

2. Are all singularities hidden inside black holes?

Why are singularities important?

- They are associated to the strong field regime of the theory (GR)

  → Black holes

- GR has a well posed initial value problem [Choquet-Bruhat; Choquet-Bruhat and Geroch]

  → Classically predictive

- If singularities form generically, does GR have any real predictive power?

- If singularities are observable, one could probe Planck scale physics

The Weak Cosmic Censorship Conjecture

"Generic asymptotically flat initial data have a maximal future development possessing a complete future null infinity"

[Penrose; Geroch and Horowitz; Christodoulou]

- Physicists' version:

  Observers can remain forever in low curvature regions of spacetime, where they can conduct classical experiments and predict their outcome ⇔

  we cannot access Planck scale physics in finite time

The Weak Cosmic Censorship Conjecture

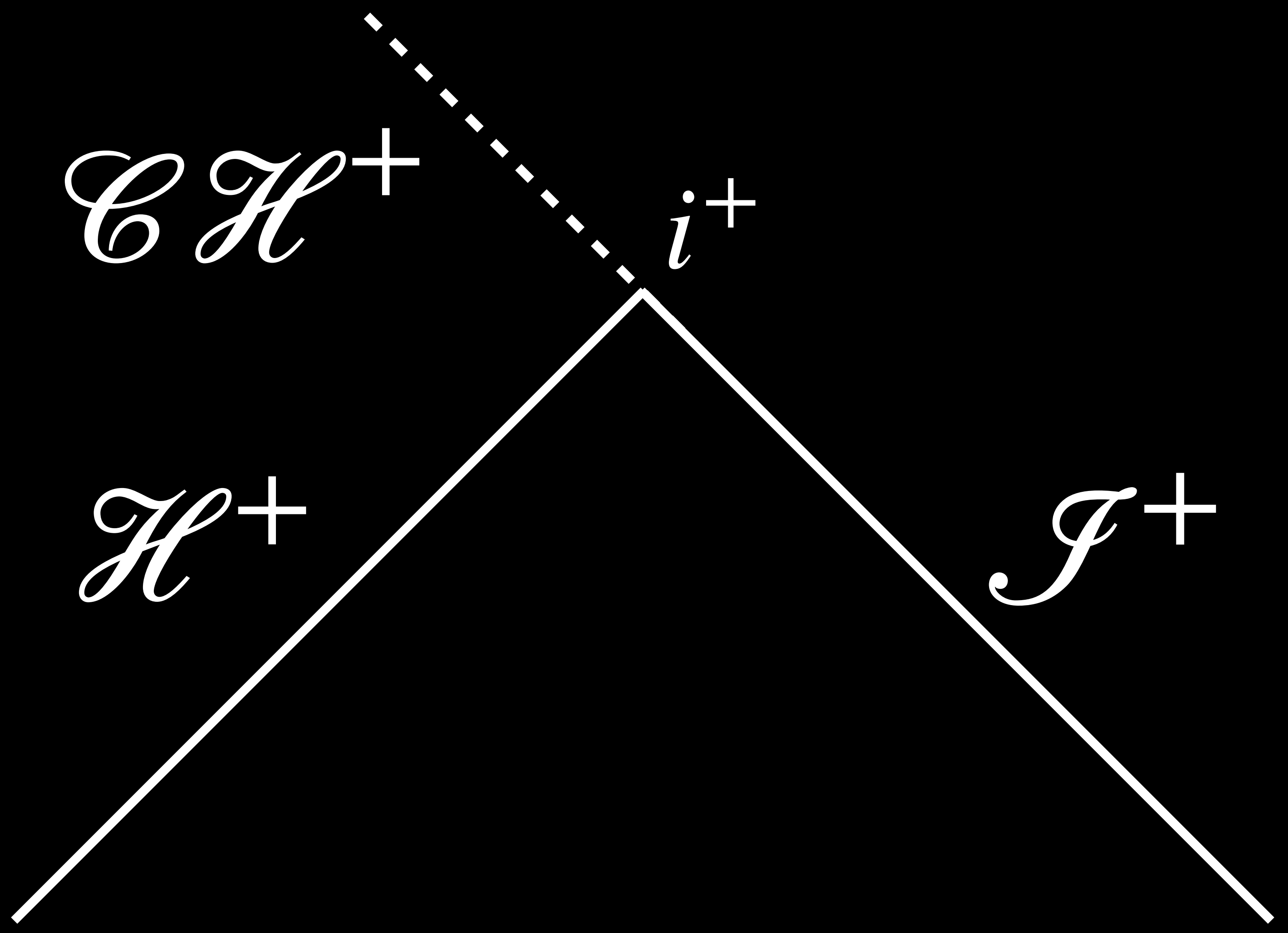
- No known counter-examples in astrophysical settings
- Christodoulou proved it in the Einstein-scalar field model in spherical symmetry
- Finely tuned initial data leads to the formation of a “zero” mass naked singularity [Choptuik]
- It is likely to be false in higher dimensional asymptotically flat spaces
- Some potential counter-examples in asymptotically anti-de Sitter spaces
- General case remains unproven

The Weak Cosmic Censorship Conjecture

- CAUTION: don't confuse with the "Strong Cosmic Censorship Conjecture"

There is no suitably differential continuation across the Cauchy horizon

[Penrose; Christodoulou]



- Also addresses the predictivity of GR as a classical theory
- The two conjectures are unrelated

Outline of the talk

- Higher dimensions
- Anti-de Sitter
- Summary and Discussion

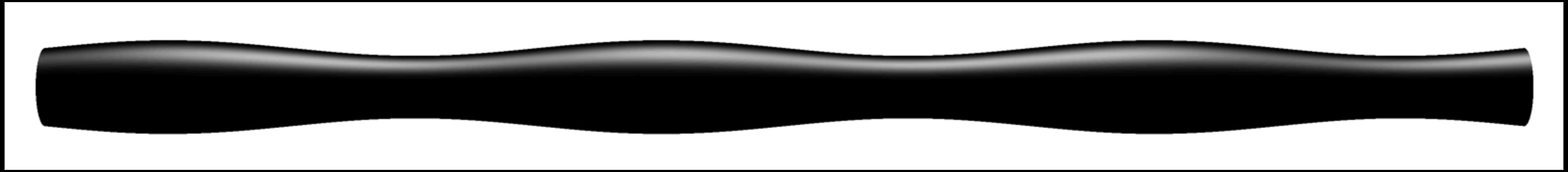
Higher dimensions

Gregory-Laflamme instability of black strings

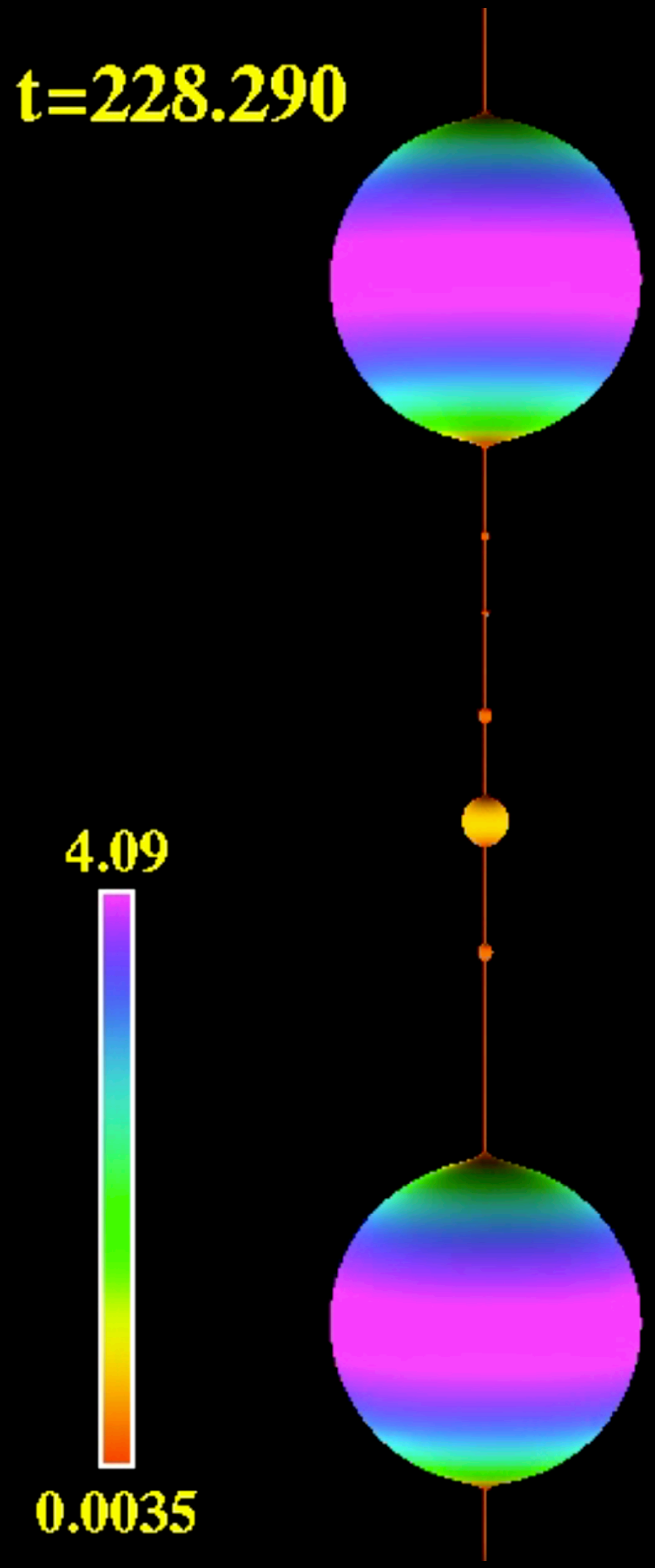
- Black strings: black hole solutions of Einstein's equation in vacuum in $M_4 \mathrm{x} S^1$:

$$ds^2 = -\left(1 - \frac{2M}{r}\right) dt^2 + \frac{dr^2}{1 - \frac{2M}{r}} + r^2 d\Omega^2_{(2)} + dz^2 \quad z \sim z + L$$

- If M/L < O(1), black strings are unstable to develop ripples along the extra dimension [Gregory and Laflamme]

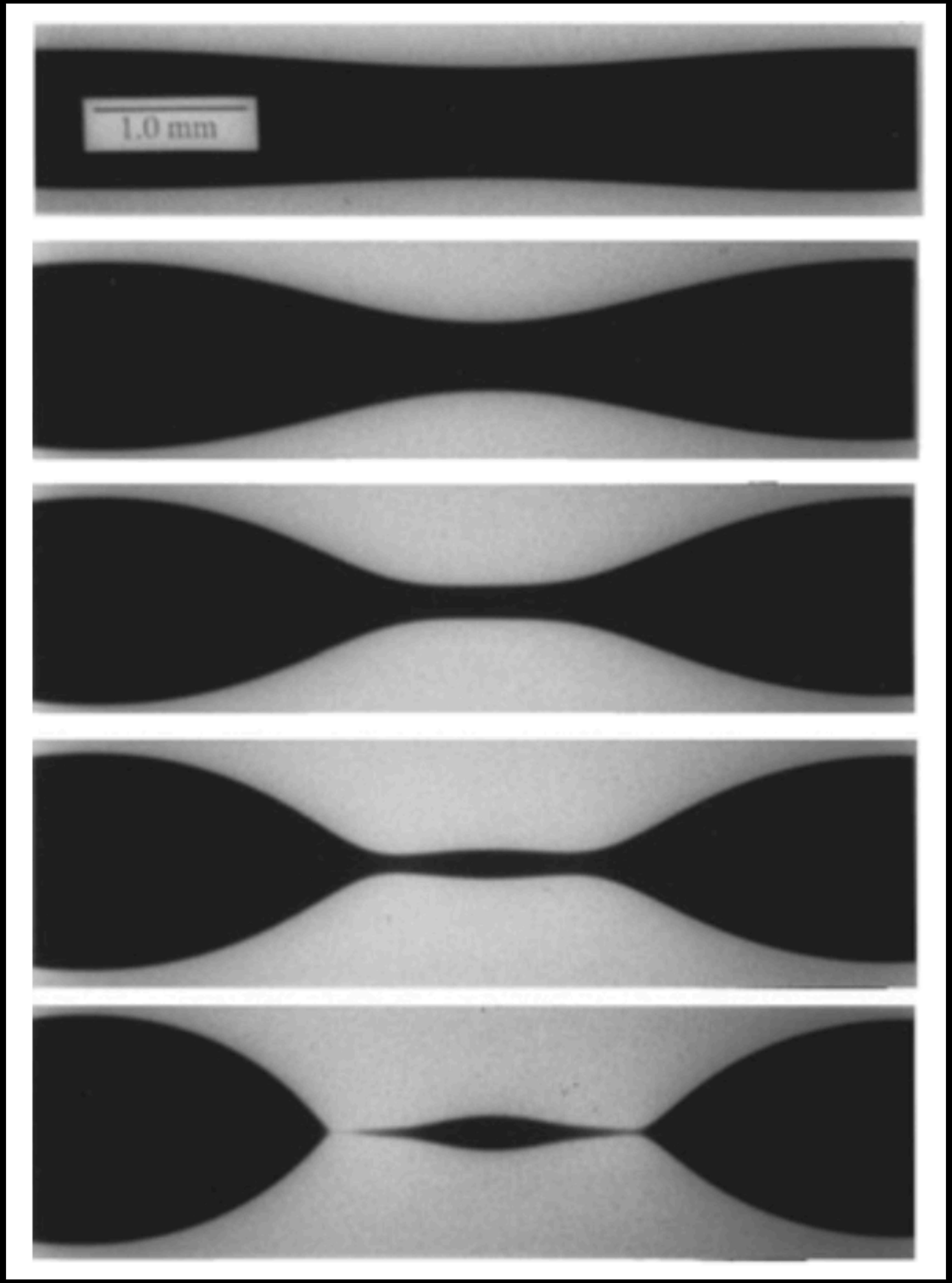


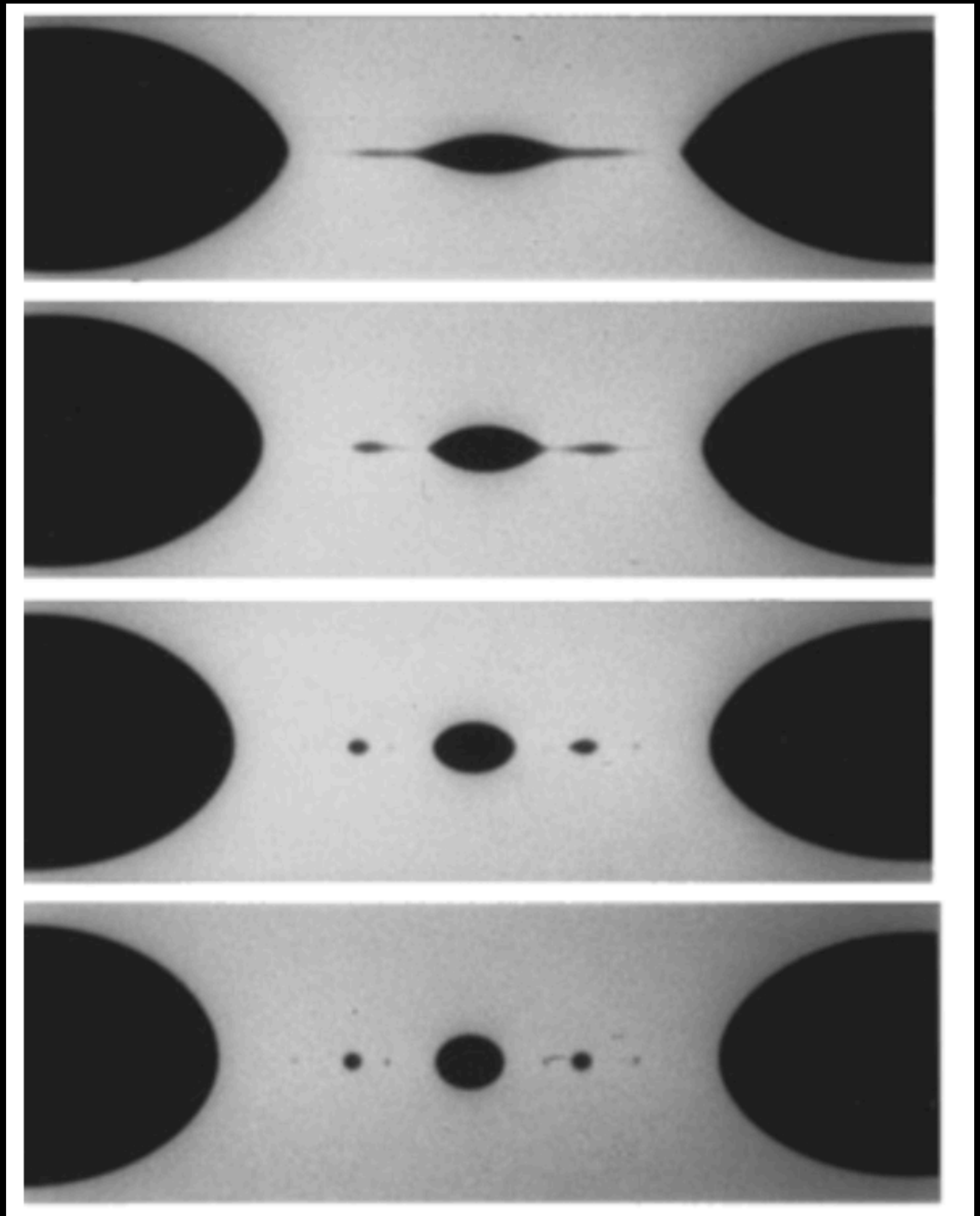
- Endpoint of the instability: [Lehner and Pretorius]



- The (apparent) horizon develops a fractal structure
- The dynamics after the formation of the first generation is universal and self-similar
- The black string pinches off in finite asymptotic time
- No fine-tuning is required

- Dynamics qualitatively similar to fluids:





Experiment

[Tjahjadi, Stone, Ottino, J. Fluid Mech. **243** 297 (1992)]

- In fluids:
  - Hydrodynamics eventually breaks down → loss of classical predictivity
  - Molecular dynamics takes care of the break up
  - Only a handful of molecules are involved
  - The break up is governed by an attractor [Eggers]

➡ Minimal loss of classical predictivity

Does the same happen in GR?

- Issues with the current state-of-the-art in GR:
  - There has been only one simulation
  - Current simulations show a breaking of the Z_2 symmetry around the big blob
  - At late times numerical noise triggers the formation of new satellites
  - The evolution cannot possibly be self-similar beyond the 3rd generation

- Work in progress with more modern techniques:

[Andrade, PF, Gu]

- Self-similarity as described in [Lehner and Pretorius] seems unlikely

- Evidence for an attractor controlling the pinch off?

Black hole instabilities in asymptotically flat spaces

- In the previous example the space-time is not asymptotically flat and the black hole wraps a topological circle

- Can we find other examples in asymptotically flat spaces?

  YES

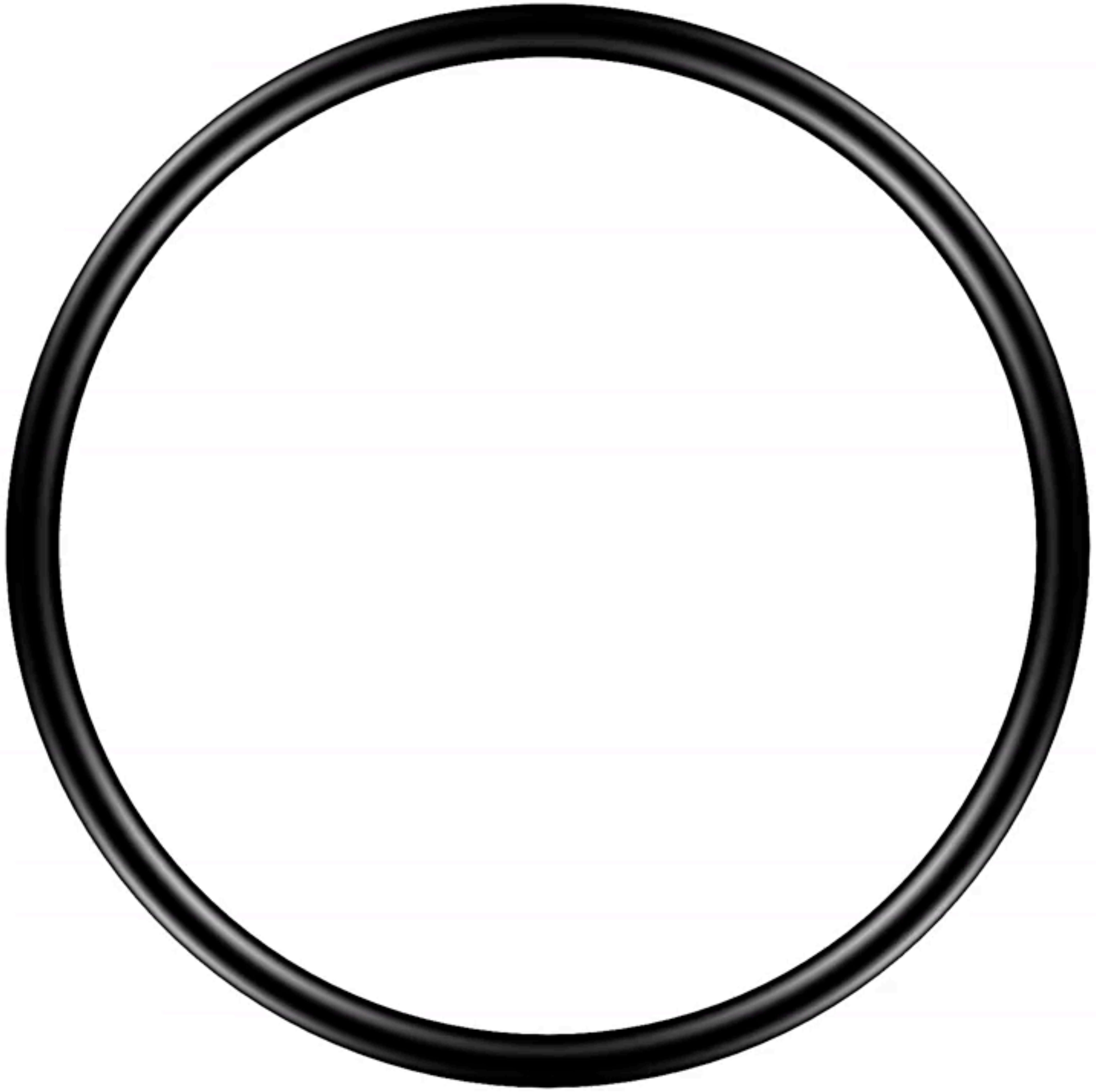
Black hole instabilities in asymptotically flat spaces

- Some relevant facts about higher dimensional asymptotically flat black holes:
  - Black holes can have non-spherical topologies, e.g., black rings $S^1 x S^2$.
  - In $D \geq 5$, black holes can have arbitrarily large angular momentum ⇒ dynamical instabilities (GL and others)

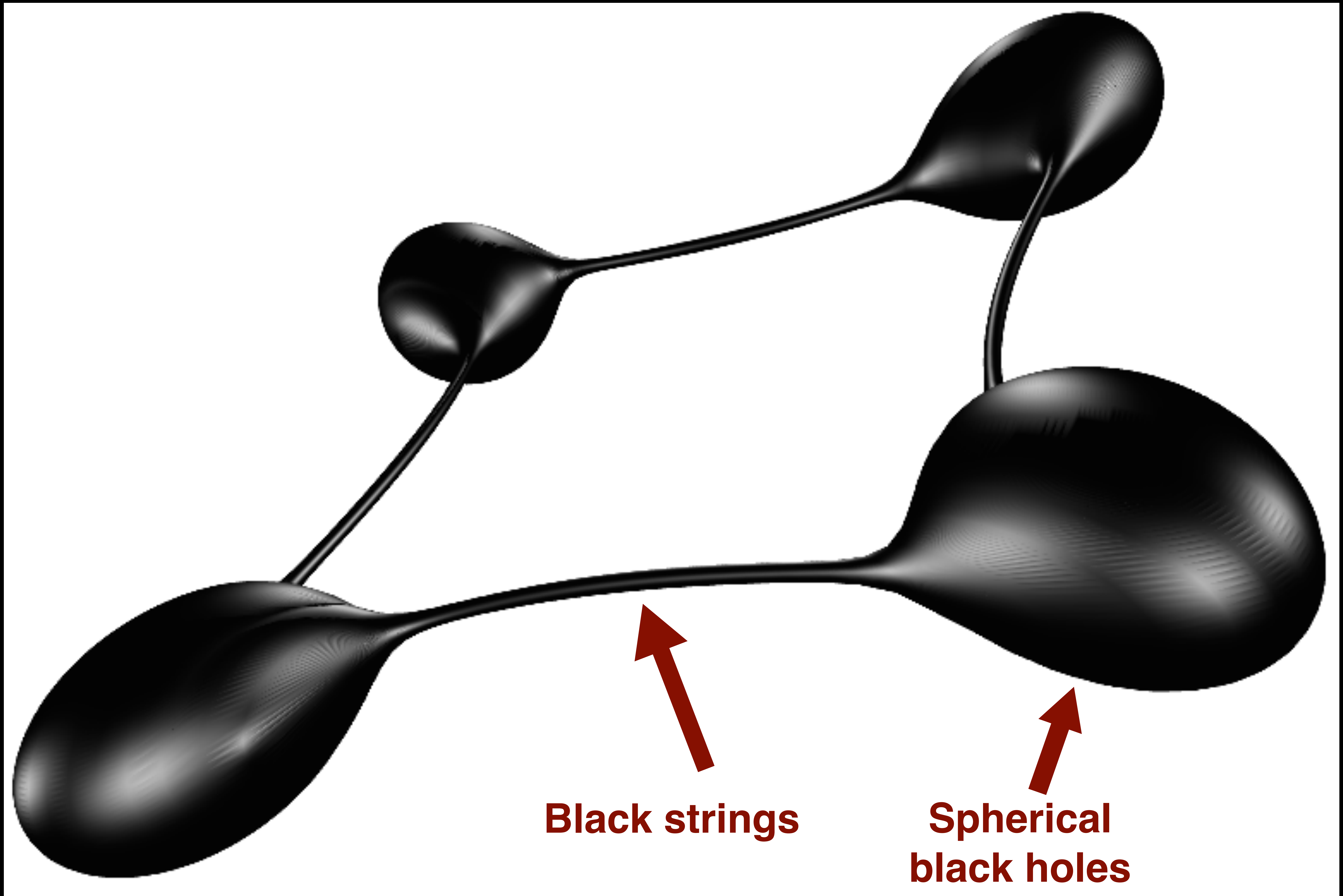
Black Ring Instabilities

- Rapidly spinning black rings are unstable under a GL-type of instability
  [Santos and Way]

- What's the endpoint of this instability?



[PF, Kunesch,
Tunyasuvunakool]


Black strings
Spherical
black holes

Rotating spherical black hole instabilities

- The analogues of the Kerr solution can have arbitrarily large spin for $D \geq 6$

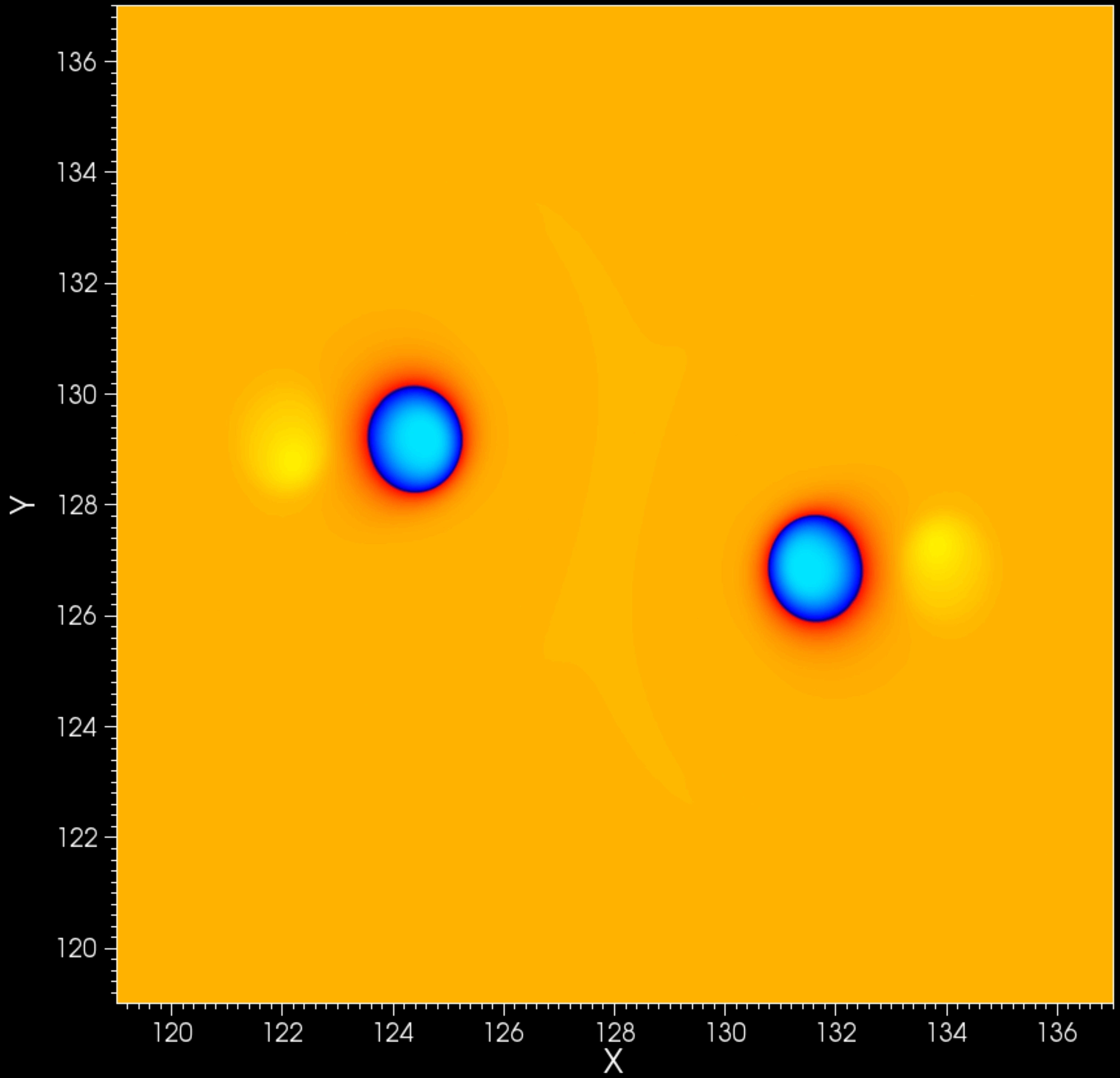
➡ Dynamical instabilities of the GL-type [Emparan and Myers; Dias, PF, Monteiro, Santos and Emparan]

$t/\mu^{\frac{1}{3}} = 29.9121$

[PF, Kunesch, Lehner, Tunyasuvunakool]

- In all the previous examples, the initial black hole is unstable. Is this generic?

  YES



[Andrade, PF and Sperhake]

Open issues

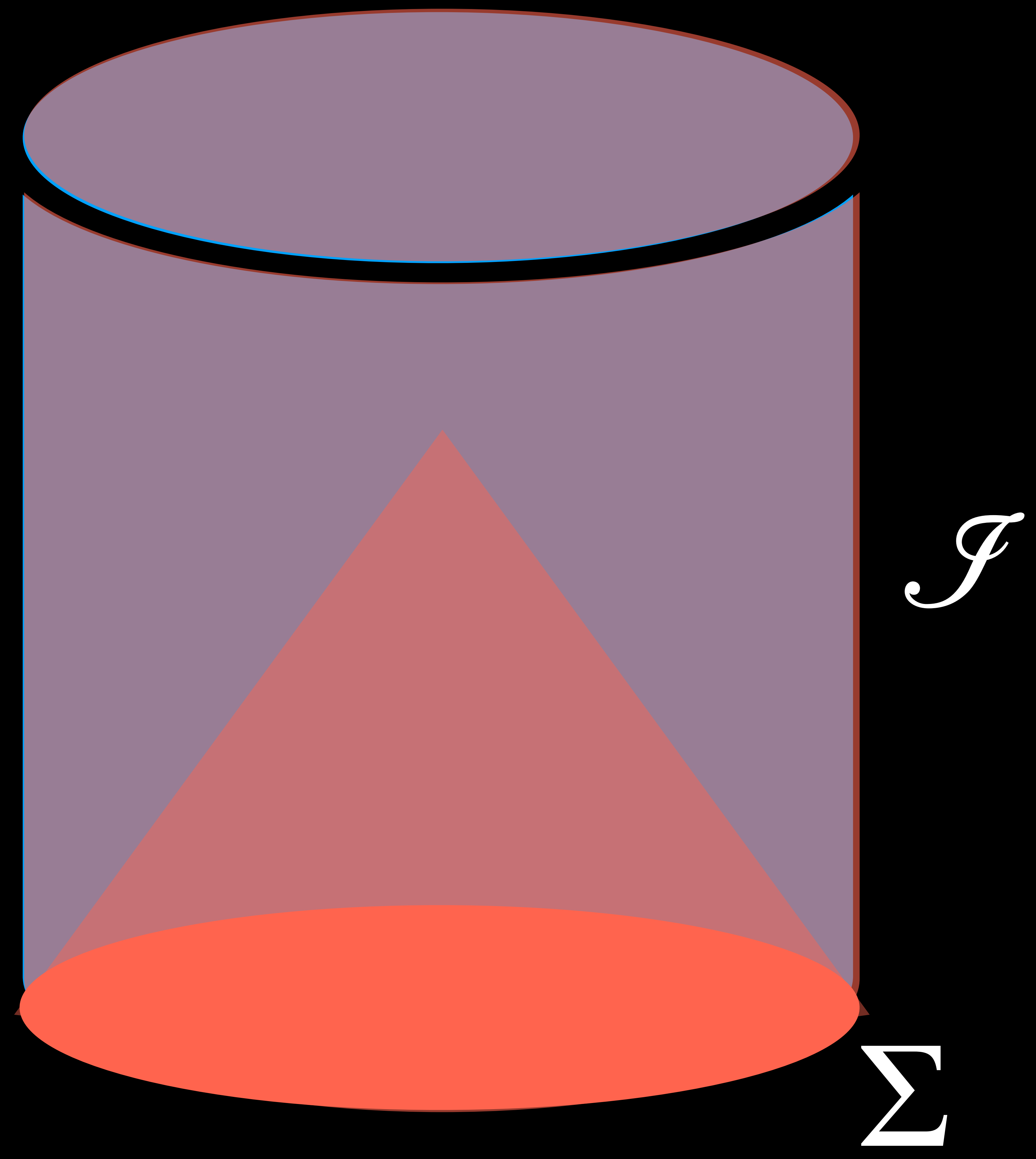
- In all the previous works, one tracks the Apparent Horizon and not the Event Horizon. Does the latter pinch off? What happens to null infinity?
- Is there an attractor controlling the pinch off?

Anti-de Sitter spacetimes

Anti-de Sitter space

- Well motivated from the point of view of GR and also from string theory and the AdS/CFT correspondence

- Initial boundary value problem:



NOTE: all the previous examples in AF should have their counterparts in AdS for small enough black holes

First class of examples: time-dependent boundary conditions

[Crisford,Horowitz, Santos]

- Consider the theory: $$I = \frac{1}{16\pi G}\int d^4x\left(R + \frac{6}{L^2} - F_{\mu\nu}F^{\mu\nu}\right)$$

- Boundary conditions: $$ds^2\Big|_{\partial M} = -\,dt^2 + dR^2 + R^2 d\phi^2$$ $$F\Big|_{\partial M} = \frac{a(t)\,R\,\gamma}{\sigma^2\left(1+\frac{R^2}{\sigma^2}\right)^{\frac{\gamma}{2}+1}}\,dt\wedge dR\,, \qquad \gamma \geq 1$$

- Initial conditions: empty AdS

- No charged matter in the bulk and T=0

- Under these conditions, regular extremal planar black holes exist for $a<a_{max}$

- Strategy: evolve $a(t)$ at the boundary from 0 to $a>a_{max}$

- Result:

  - The curvature on the horizon grows without bound *but* no singularity forms in finite asymptotic time

- Question: is it reasonable to impose boundary conditions for which no regular black hole can exist?

Connection to the weak gravity conjecture

[Crisford,Horowitz, Santos]

- Weak gravity conjecture: any consistent theory of quantum gravity must have a stable particle with $q/m \geq 1$ [Arkani-Hamed, Motl, Nicolis, Vafa]

- Observation:

  - Adding charged matter saves WCC

  - If WGC holds, one cannot violate WCC with this model

  - The minimum charge require to save WCC agrees with the WGC bound

Second class of "examples": turbulent gravitational dynamics in AdS

- AdS is non-linearly unstable to black hole formation [Bizon and Rostworowski,…]
- Black holes in AdS have been conjectured to be non-linearly stable for high ℓ modes [Holzegel and Smulevici]
- Superradiant instability of Kerr-AdS [Chesler and Lowe]

➡ In all these cases there seems to be a strong energy cascade to the UV

Second class of "examples": turbulent gravitational dynamics in AdS

- With reflective boundary conditions, can a naked singularity form in finite time?
- <u>CAUTION:</u> in AdS_3, the evidence suggests that the singularity forms in infinite time [Bizon and Jalmuzna]

Summary and Discussion

Summary and Discussion

- We have seen various scenarios where the WCCC may be violated
- Goal: Sharpen the conjecture so that it can be proven
- Known mechanism for violations in KK/AF spaces: Gregory-Laflamme instability
- Superradiant instability in AF spaces? [Eperon, Ganchev, Santos]
- Role of boundary conditions in AdS and connections between WCCC and WGC

Summary and Discussion

- What if WCCC is violated?

  GL instability leads to the formation of a naked singularity of Planck mass: if an attractor controls the pinch off, the loss of predictivity is minimal

- Are naked singularities bad?

  Maybe for mathematicians but not necessarily for physicists: Choptuik's critical solution requires polynomial fine-tuning of the initial data → OK for experiments

  Quantum gravity effects?

Thanks for your attention!