ENTANGLEMENT DYNAMICS AND DISSIPATION

ICTS Workshop “Hydrodynamics and fluctuations”

September 6-10, 2021

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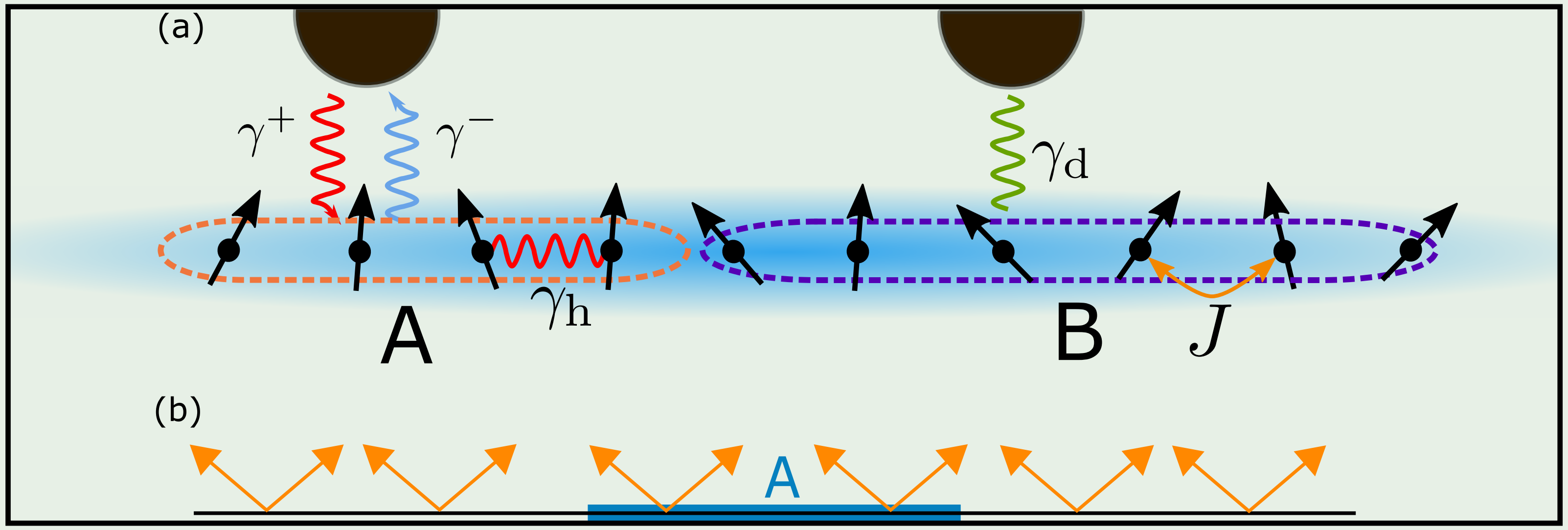


OUTLINE

- Goal: Quantum information spreading in "integrable" systems with dissipation

GLOBAL DISSIPATION

- Lindblad dynamics in free-fermion and free-bosons models
- Hydrodynamic description of entanglement spreading: Quasiparticle picture

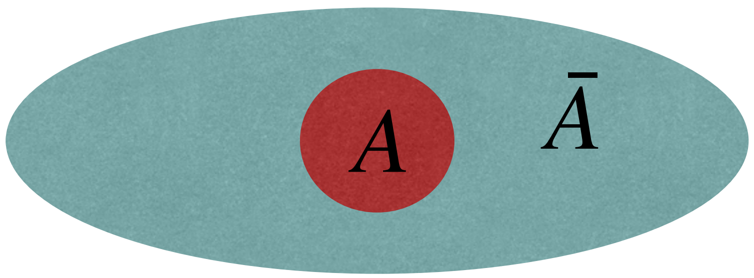
Generic quadratic systems, generic quench, generic linear dissipation

LOCAL DISSIPATION

- Dissipative impurities:
  - Non-equilibrium steady-state: hydrodynamic limit
  - Entanglement production

ENTANGLEMENT MEASURES

- Entanglement measures for global pure states:



$$S_A = -\,\mathrm{Tr}\rho_A \ln(\rho_A)$$ Von Neumann entropy

$$S_A^{(n)} = \frac{1}{1-n}\ln(\mathrm{Tr}\rho_A^n)$$ Renyi entropies

G. Vidal and R. F. Werner, PRA 65, 032314 (2002

- Dissipation ➡ global density matrix is mixed ➡ logarithmic negativity

Partial transpose

$e_i, \bar{e}_j$ bases for $A, \bar{A}$

$$\langle e_i, \bar{e}_j | \rho^T | e_k, \bar{e}_l \rangle = \langle e_i, \bar{e}_l | \rho_A | e_k, \bar{e}_j \rangle,$$

V. Eisler and Z. Zimboras, New J. Phys 17 (2015) 053048

H. Shapourian and S. Ryu, PRA 99, 022310 (2019)

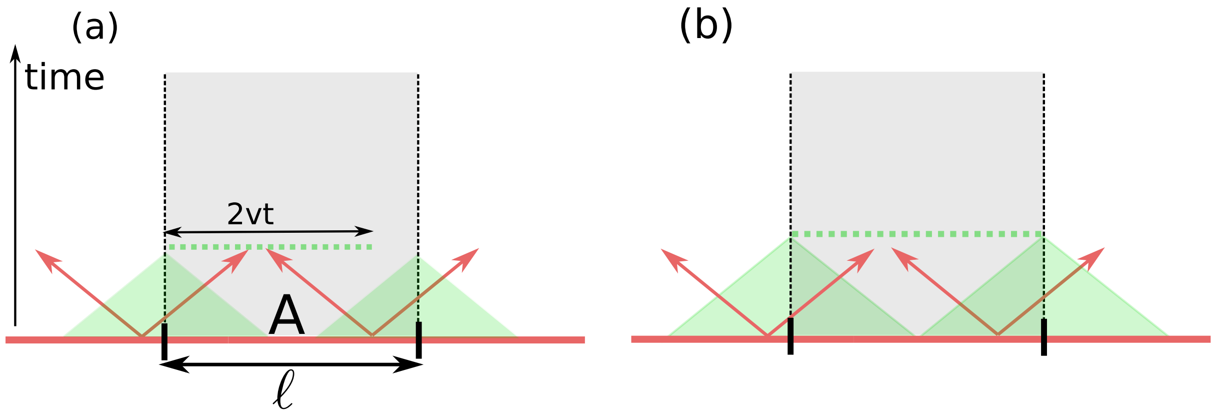
$$\boxed{\mathscr{E} = \ln(\mathrm{Tr}|\rho_A^T|)}$$

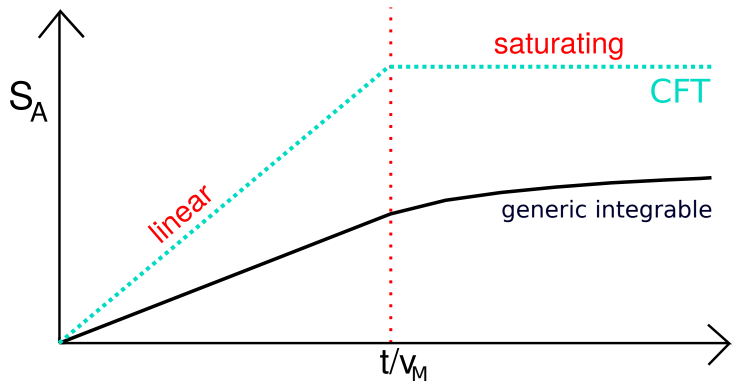
QUASIPARTICLE PICTURE

- Out-of-equilibrium strongly correlated quantum system → entanglement production

- Hydrodynamic description of entanglement dynamics via emergent quasiparticles

P. Calabrese and J. Cardy, JSTAT 2005





QUASIPARTICLE PICTURE

V.A.,P. Calabrese, PNAS,114, 7947 (2017) + a lot more

Universality from thermodynamics

$$S_A = \sum_k \left[t \int_{|v_k|t<\ell} d\lambda v_k(\lambda) s_k(\lambda) + \ell \int_{|v_k|t>\ell} d\lambda s_k(\lambda) \right]$$

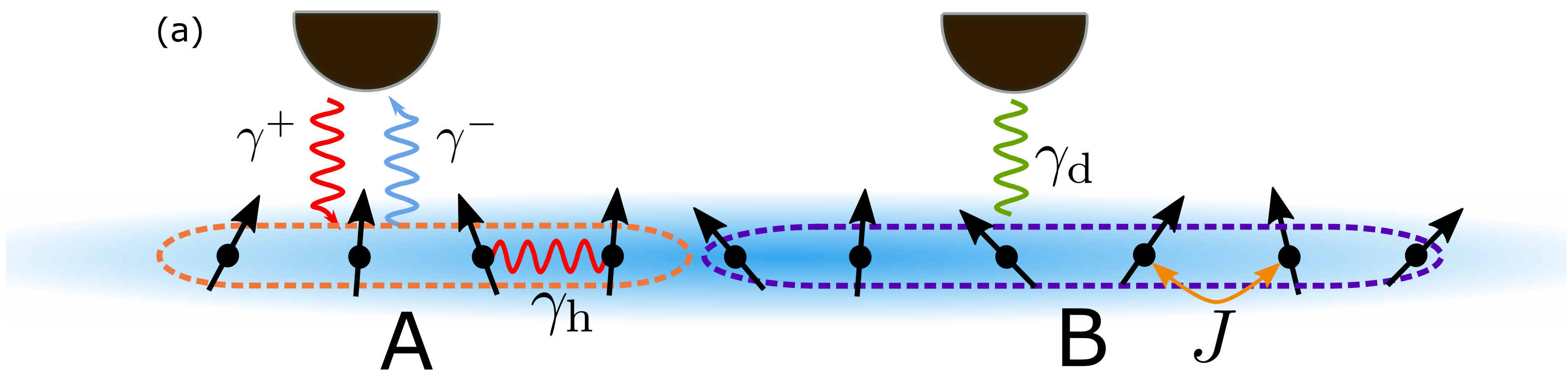
s_k GGE thermodynamic entropy

v_k Entangling quasiparticles are the excitations above GGE

- For free models quasiparticles are the eigenmodes

LINDBLAD EQUATION

- System coupled with bath → Lindblad provides most general linear equation



Breuer, Heinz-Peter; Petruccione, F. (2002). *The Theory of Open Quantum Systems*. Oxford University Press

$$\frac{d\rho}{dt} = -i[H,\rho] + \sum_{\mu}\left(L_\mu \rho L_\mu^\dagger - \frac{1}{2}\{L_\mu^\dagger L_\mu, \rho\}\right)$$

Non unitary dynamics

L_μ encodes system+bath interaction

LINDBLAD EQUATION: INTEGRABLE CASES

- Quadratic Hamiltonian + linear dissipation ➡ exact solvable (third quantisation).

T. Prosen, New J. Phys. 10, 043026 (2008)

$$H = \sum_{j=1}^{2L} w_j H_{jk} w_k$$

$$L_\mu = \sum_{j=1}^{2L} l_{\mu,j} w_j$$

w_j Majorana fermions

- Effective equation for two-point correlators (incoherent hopping, dephasing):

V. Eisler, J. Stat. Mech. (2011) P06007

$$G_{x,y} = \mathrm{Tr}(c_x^\dagger c_y \rho)$$

$$L_j^{(h)} = c_j^\dagger c_{j+1}$$

$$\frac{d}{dt} G_{kl} = i(G_{k-1,l} + G_{k+1,l} - G_{k,l-1} - G_{k,l+1}) - 2\gamma_{\mathrm{h}} G_{k,l} + \gamma_{\mathrm{h}} \delta_{k,l}(G_{k-1,k-1} + G_{k+1,k+1})$$

$$L_j^{(d)} = c_j^\dagger c_j$$

$$\frac{d}{dt} G_{kl} = i(G_{k-1,l} + G_{k+1,l} - G_{k,l-1} - G_{k,l+1}) - \gamma_{\mathrm{d}}(G_{kl} - \delta_{kl} G_{kk})$$

- Bethe Ansatz solutions, hydrodynamic treatment:

M. V. Medvedyeva, F. H. L. Essler and T. Prosen, Phys. Rev. Lett. 117, 137202 (2016)

B. Buca, C. Booker, M. Medenjak and D. Jaksch, arXiv:2004.05955

A. Bastianello, J. De Nardis and A. De Luca, Phys. Rev. B 102, 161110 (2020)

A. A. Ziolkowska and F. H. Essler, SciPost Phys. 8, 44 (2020)

M. de Leeuw, C. Paletta and B. Pozsgay, arXiv:2101.08279

I. Bouchoule, B. Doyon and J. Dubail, SciPost Phys. 9, 44 (2020)

FREE FERMIONS WITH GAIN/LOSS DISSIPATION

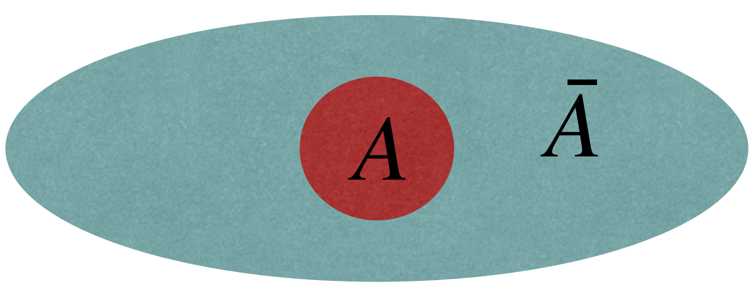
- Simple free fermion model:

$$H = \sum_{x=-\infty}^{\infty} (c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x) \qquad \epsilon_k = 2\cos(k) \qquad v_k = \frac{d\epsilon_k}{dk} = -2\sin(k)$$

- Gain/loss dissipation

$$L_j^+ = \sqrt{\gamma^+} c_j^\dagger \qquad L_j^- = \sqrt{\gamma^-} c_j$$

- Diagonal dissipation ➡ simple relationship for eigenvalues λ_j of G



$$\lambda_j = n_\infty(1 - b(t)) + b(t)\tilde{\lambda}_j$$

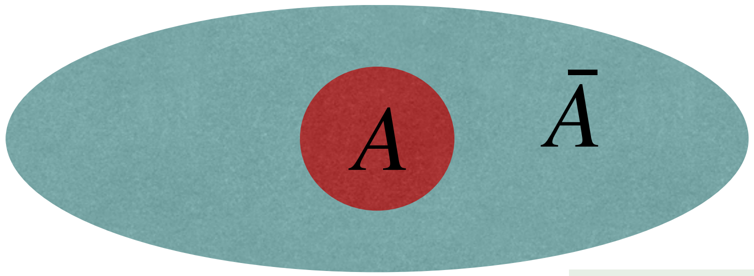
$$b(t) = e^{-(\gamma^- + \gamma^+)t}$$

$$n_\infty = \frac{\gamma^+}{\gamma^- + \gamma^+}$$

- Simple evolution for the density

$$n_{t,n_0} = n_\infty(1 - b(t)) + n_0 b(t)$$

FERMIONIC NEEL QUENCH



$$|0101010101\ldots\rangle$$

$$G_{x,y} = \mathrm{Tr}(c_x^\dagger c_y \rho)$$

G restricted to A

$$S_A = -\,\mathrm{Tr}(G \ln G + (1-G)\ln(1-G))$$

V. Eisler, I. Peschel, J. Phys. A: Math. Theor. 42 504003 (2009)

- $\mathrm{Tr}(G^n)$ in terms of $\mathrm{Tr}(\widetilde{G}^n)$

$\widetilde{G}$ Correlation matrix in the unitary case

$$S_\ell^{(n)}(t) = \int \frac{dq}{2\pi} \left\{ \ell s_q^{(n),\,\mathrm{mix}}(t) + \min(2|v_q|t, \ell)\left[s_q^{(n),\,\mathrm{YY}}(t) - s_q^{(n),\,\mathrm{mix}}(t) \right] \right\}$$

- Hydrodynamic limit $t, \ell \to \infty, \gamma^\pm \to 0, t/\ell, \gamma^\pm \ell$ fixed

$$s^{(\alpha),YY}(n_k) = \frac{1}{1-\alpha} \ln(n_k^\alpha + (1-n_k)^\alpha)$$

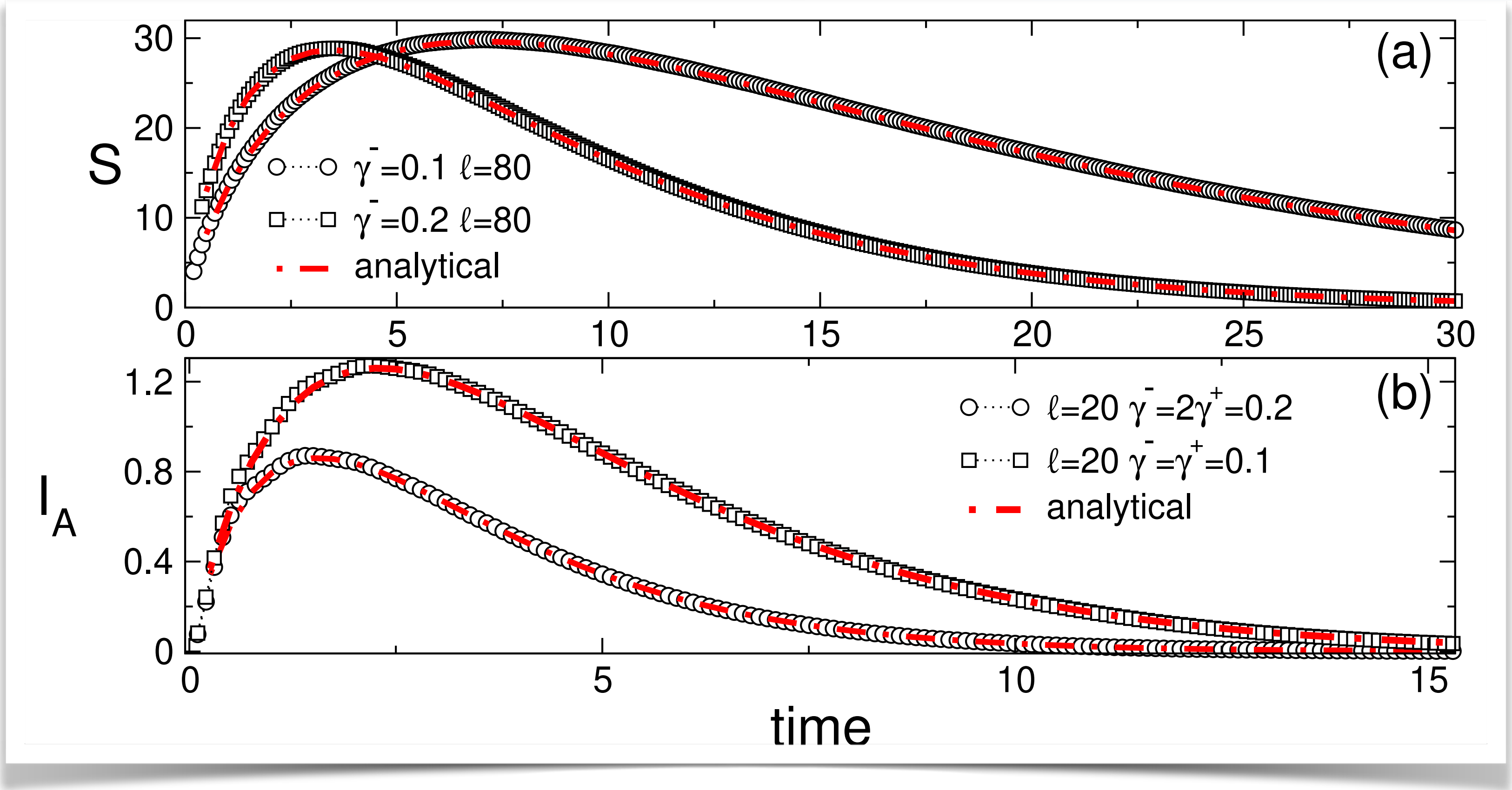
- Simple heuristic interpretations:

$$s^{(\alpha),mix} = \frac{s^{(\alpha),YY}(n_{t,1}) + s^{(\alpha),YY}(n_{t,0})}{2}, \qquad s_q^{(\alpha),YY} = s^{(\alpha),YY}(n_{t,1/2})\,.$$

ENTANGLEMENT ENTROPIES

Numerics VS Hydrodynamics

V.A. and F. Carollo, PRB 103 L020302 (2021)



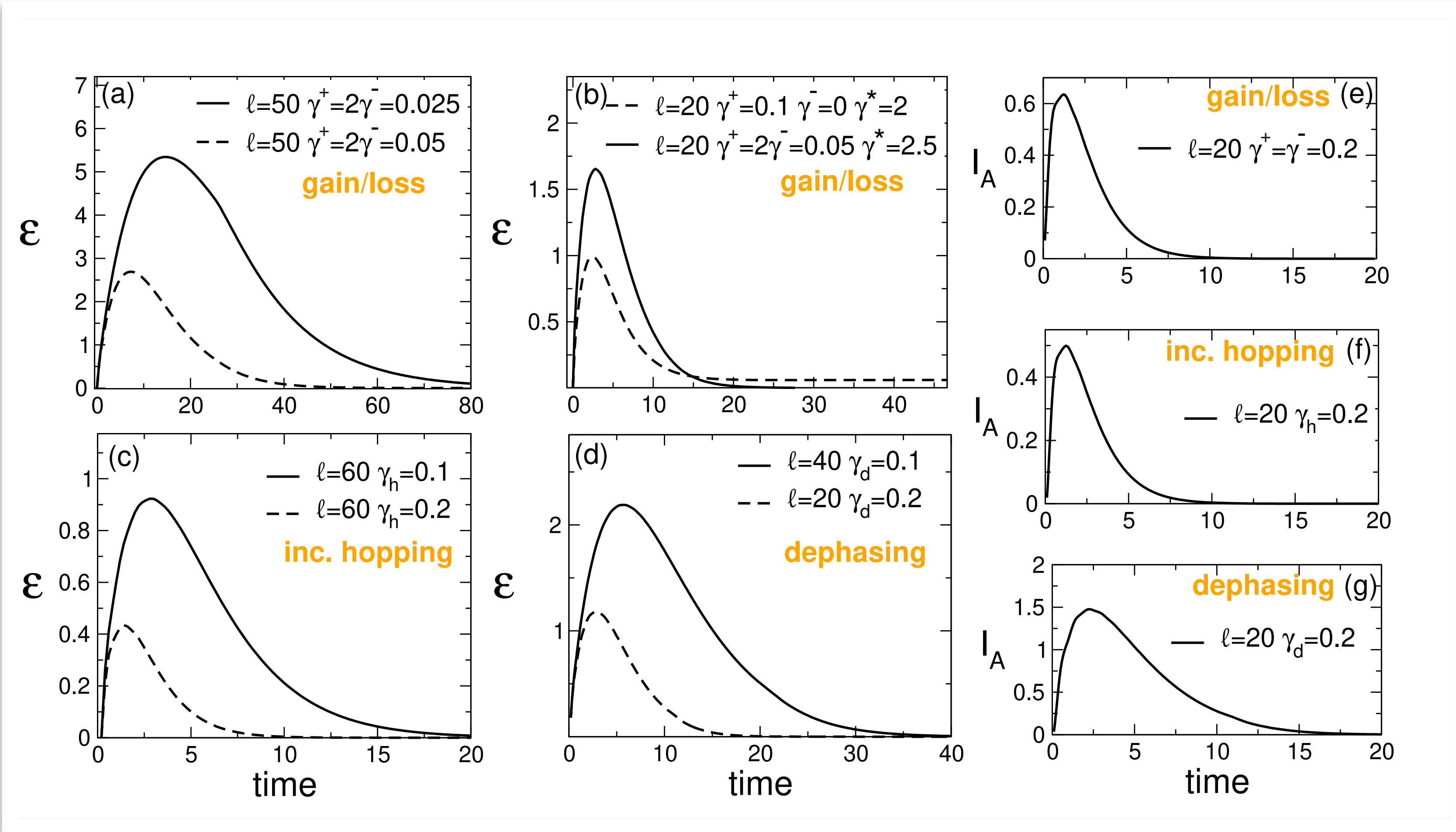
$$I_A = S_A + S_{\bar{A}} - S_{A\cup\bar{A}}$$

$$S_\ell^{(n)}(t) = \int \frac{dq}{2\pi}\Big\{ \ell s_q^{(n),\,\mathrm{mix}}(t) + \min(2|v_q|t,\ell)\Big[s_q^{(n),\,\mathrm{YY}}(t) - s_q^{(n),\,\mathrm{mix}}(t)\Big]\Big\}$$

see also:

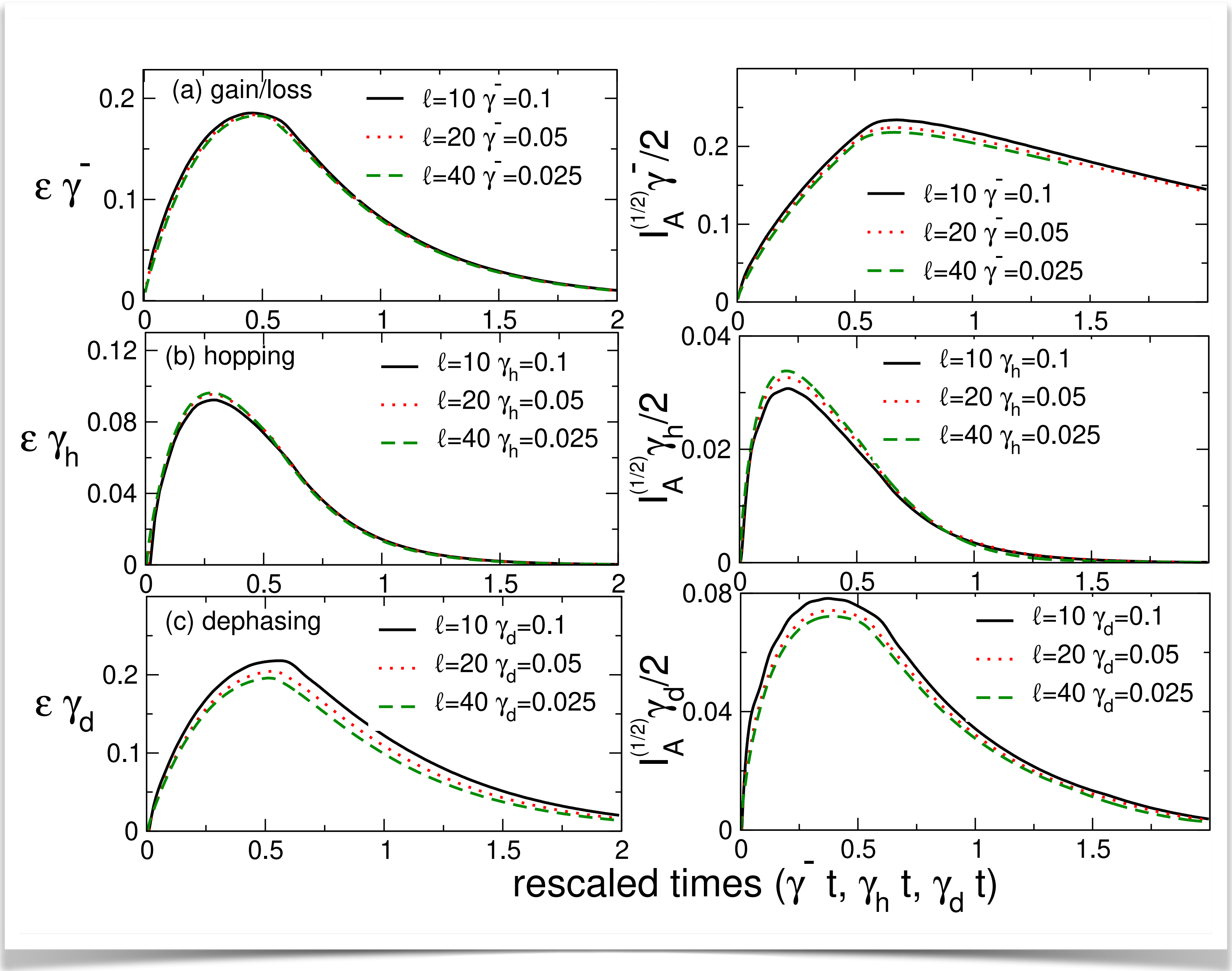
S. Maity, S. Bandyopadhayay, S. Bhattachariee, A. Dutta, PRB 101, 180301(R) (2020)

INCOHERENT HOPPING AND DEPHASING



$$I_A = S_A + S_{\bar{A}} - S_{A\cup\bar{A}}$$

SCALING IN THE WEAK-DISSIPATION LIMIT



$$I_A^{(1/2)} = S_A^{(1/2)} + S_{\bar{A}}^{(1/2)} - S_{A\cup\bar{A}}^{(1/2)}$$

- Nice scaling behaviour for $\gamma \to 0, t, \ell \to \infty, t/\ell$ fixed, $\gamma\ell$ fixed

$$\mathcal{E} \neq I_A^{(1/2)}/2$$

V.A. and P. Calabrese, EPL 126 60001 (2019)

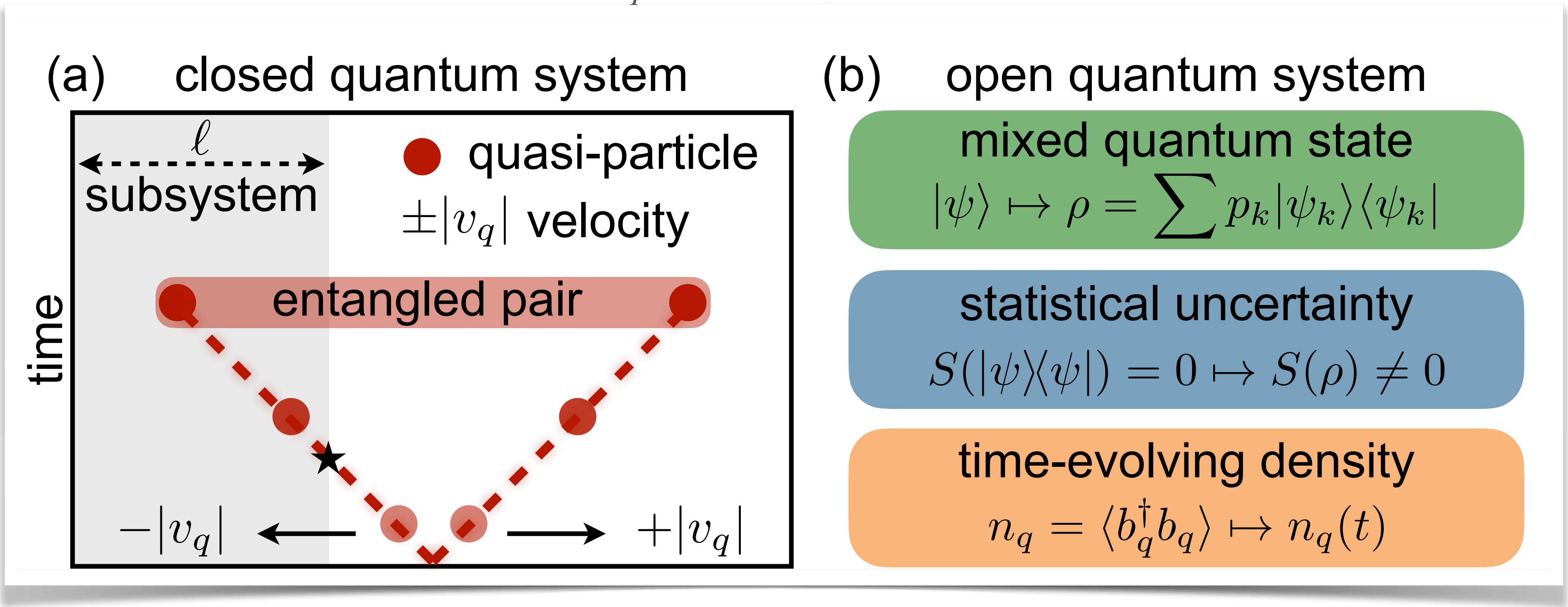
HOW GENERAL IS THIS RESULT?

QUASIPARTICLE PICTURE: WEAK-DISSIPATION LIMIT

- Generic free-fermion or free-boson model:

$$H = \sum_q \varepsilon_q b_q^\dagger b_q + h.c$$

b_q Bogoliubov modes



V.A. and F. Carollo, arXiv:2106.11997)

$$S_\ell^{(n)}(t) = \int \frac{dq}{2\pi}\left\{\ell s_q^{(n),\,\mathrm{mix}}(t) + \min(2|v_q|t, \ell)\left[s_q^{(n),\,\mathrm{YY}}(t) - s_q^{(n),\,\mathrm{mix}}(t)\right]\right\}$$

QUASIPARTICLE PICTURE: THE RECIPE

$$H = \sum_k \varepsilon_k b_k^\dagger b_k + h.c$$

- Determine Liouvillian eigenmodes β_k

$$\mathscr{L}\left[\beta_k\right] = -\left(\frac{\gamma_k}{2} + \mathrm{i}\,\varepsilon_k\right)\beta_k$$

$\beta_k \neq b_k$

$\rho_k = \langle \beta_k^\dagger \beta_k \rangle$

- $s^{(n),YY}(\rho_k)$ is the Yang-Yang entropy of ρ_k

$$s^{(n),YY}(\rho_q) = \frac{1}{1-n}\ln(\rho_q^n + (1-\rho_q)^n)$$

- $s_q^{(n),\mathrm{mix}}$ obtained from the covariance matrix in momentum space.

$$\Gamma_{j,j'} = \frac{1}{2}\begin{pmatrix} \langle w_{1,j}w_{1,j'} - \delta_{j,j'}\rangle & \langle w_{1,j}w_{2,j'}\rangle \\ \langle w_{2,j}w_{1,j}\rangle & \langle w_{2,j}w_{2,j'}\rangle - \delta_{j,j'} \end{pmatrix} \longrightarrow \hat{\Gamma}_k$$

$$s_k^{(n),\mathrm{mix}} = \frac{1}{2}\frac{1}{1-n}\mathrm{Tr}\ln\left[\left(\frac{1_2 + \hat{\Gamma}_k}{2}\right)^n + \left(\frac{1_2 - \hat{\Gamma}_k}{2}\right)^n\right],$$

THE CASE OF THE HARMONIC CHAIN

$$\frac{dO}{dt} = i[H,O] + \sum_{i,j=1}^{2L} C_{ij}\left(r_i O r_j - \frac{1}{2}\left\{O, r_i r_j\right\}\right)$$

$$r = (x_1, p_1, x_2, p_2, \ldots, x_L, p_L)$$

Dissipation

$$H = \frac{J}{2}\sum_{i=1}^{L}\left(p_i^2 + m^2 x_i^2 + (x_i - x_{i+1})^2\right)$$

$$S^{vN}_{\ell,pairs} = 2I^{vN}_A$$

Non local dissipative processes

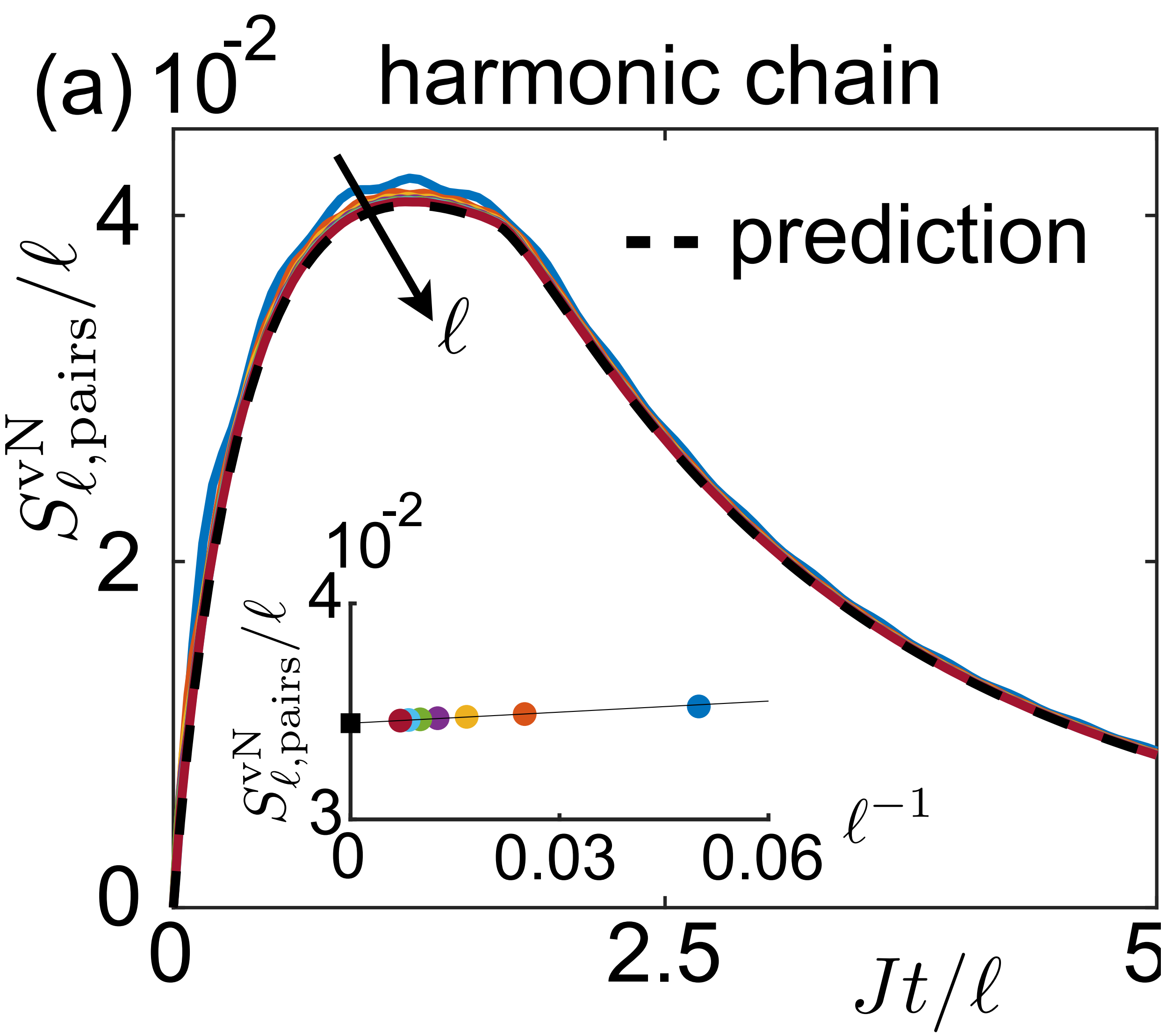
$$C_{ij} = \sum_\alpha C^\alpha_{ij} \qquad C^\alpha_{ij} = \gamma^\alpha f^\alpha_{ij} c^\alpha$$

$$f^\alpha_{ij} = e^{-|i-j|/\xi_\alpha}$$

$$c^\pm = \frac{1}{2}\begin{pmatrix} 1 & \mp i \\ \pm i & 1 \end{pmatrix}, \quad c^x = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad c^p = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

Mass quench

$$m_0 \to m$$



THE CASE OF THE ISING CHAIN

$$H = -J\sum_{j=1}^{L}(c_j^\dagger c_{j+1} + c_{j+1}^\dagger c_j - \delta c_j c_{j+1} - \delta c_{j+1}^\dagger c_j^\dagger - 2hc_j^\dagger c_j) - JhL\,.$$

$$\frac{dO}{dt} = i[H,O] + \sum_{m,j=1}^{2L} C_{mj}\left(w_m O w_j - \frac{1}{2}\left\{O, w_m w_j\right\}\right)$$

$$w_{+,j} = c_j^\dagger + c_j$$
$$w_{-,j} = c_j^\dagger - c_j$$

$$w = (w_{+,1}, w_{-,1}, w_{+,2}, w_{-,2}, \ldots, w_{+,L}, w_{-,L})$$

$$S^{vN}_{\ell,pairs} = 2I^{vN}_A$$

Non local dissipative processes

$$C_{ij} = \sum_\alpha C^\alpha_{ij} \qquad C^\alpha_{ij} = \gamma^\alpha f^\alpha_{ij} c^\alpha$$

$$f^\alpha_{ij} = e^{-|i-j|/\xi_\alpha}$$

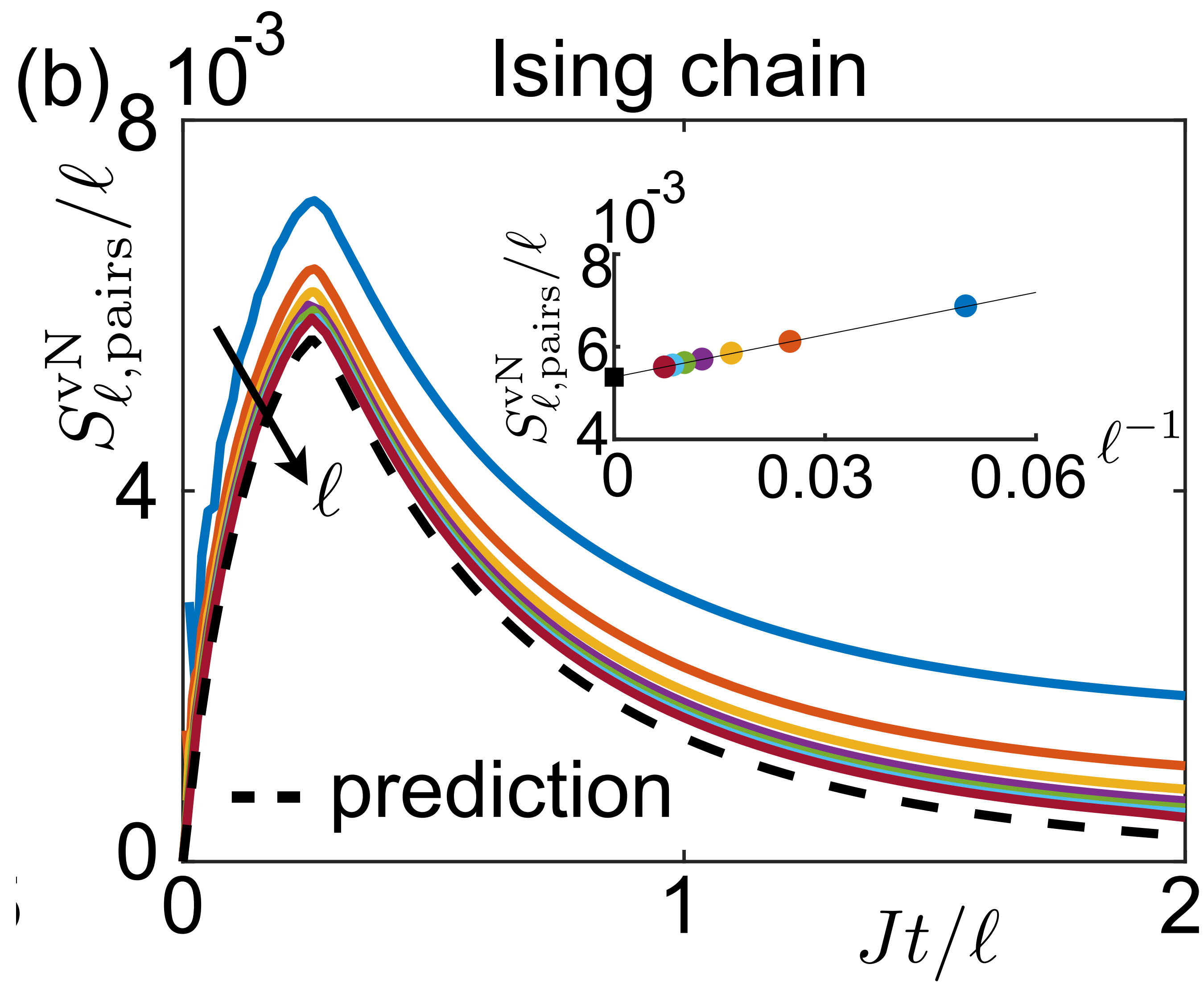
$$\xi_- = 2, \gamma^- = 2/\ell$$

$$c^\pm = \frac{1}{2}\begin{pmatrix}1 & \mp i\\ \pm i & 1\end{pmatrix}$$

Magnetic field quench

$$h_0 \to h$$

$$(h_0 = 3, h = 5)$$



QUANTUM ISING CHAIN: GIST OF THE PROOF

V.A., F. Carollo, arXiv:2109.01836

- Full information contained in the covariance matrix

$$\Gamma_{j,j'} = \frac{1}{2}\begin{pmatrix} \langle w_{1,j}w_{1,j'} - \delta_{j,j'}\rangle & \langle w_{1,j}w_{2,j'}\rangle \\ \langle w_{2,j}w_{1,j}\rangle & \langle w_{2,j}w_{2,j'}\rangle - \delta_{j,j'} \end{pmatrix}$$

- Formal solution of Lindblad equation

$$\frac{dO}{dt} = i[H,O] + \sum_{m,j=1}^{2L} K_{mj}\left(w_m O w_j - \frac{1}{2}\left\{O, w_m w_j\right\}\right) \rightarrow \Gamma(t) = \widetilde{U}(t)\Gamma(0)\widetilde{U}^T(t) + 4i\int_0^t du\, \widetilde{U}(t-u)K^{\mathrm{Im}}\widetilde{U}^T(t-u)$$

$$U(t) = e^{-4iht - 2K^{\mathrm{Re}}t}$$

- In Fourier space

$$\hat{\Gamma}_k(t) = \widetilde{U}_k\hat{\Gamma}_k(0)\widetilde{U}^T_{-k}(t) + 4i\int_0^t du\, \widetilde{U}_k(t-u)\hat{K}_k^{\mathrm{Im}}\widetilde{U}^T_{-k}(t-u)$$

$\hat{\Gamma}_k$ 2x2 matrix

- Generic linear translation invariant dissipation:

$$\hat{K}_k = \gamma\begin{pmatrix} a(k) & c(k) \\ c^*(k) & b(k) \end{pmatrix}$$

a, b, c arbitrary functions

QUANTUM ISING CHAIN: GIST OF THE PROOF

- Weakly-dissipative hydrodynamic limit $t \to \infty, \gamma \to 0, \quad \gamma t \text{ fixed}$

$$\varepsilon_h(k) = 2\sqrt{1 + h^2 - 2h\cos(k)}$$

$$\hat{\Gamma}_k = C_k 1_2 + A_k \sigma_y^{(k)} + B_k \sigma_x^{(k)} e^{2i\varepsilon_h(k)t\sigma_y^{(k)}}$$

Rotated Pauli matrices

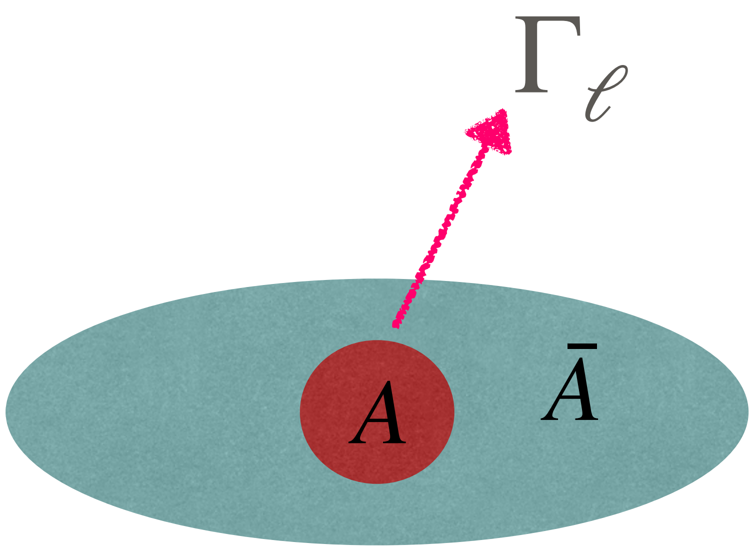
$$\sigma_\alpha^{(k)} := e^{i\vec{v}(k)\cdot\vec{\sigma}}\sigma_\alpha e^{-i\vec{v}(k)\cdot\vec{\sigma}}, \quad \alpha = x, y, z$$

- A_k, B_k, C_k encode dissipation and unitary dynamics

- How do we calculate the entropies?

$$\Gamma_{m,j}(t) = \int_{-\pi}^{\pi} \frac{dk}{2\pi} e^{-ik(m-j)} \hat{\Gamma}_k(t)$$

$$S_A^{(n)} = \frac{1}{2}\frac{1}{1-n}\mathrm{Tr}\ln\left[\left(\frac{1+\Gamma_\ell}{2}\right)^n + \left(\frac{1-\Gamma_\ell}{2}\right)^n\right]$$



- Moments Γ_ℓ^α + analytic continuation

$$S_A^{(n)} = \mathrm{Tr}\,\mathscr{F}(\Gamma_\ell^2) = \sum_\alpha c_\alpha \mathrm{Tr}\,\Gamma_\ell^{2\alpha}$$

QUANTUM ISING CHAIN: GIST OF THE PROOF

- $C_k = 0 \Rightarrow$ Same situation as in the unitary case

P. Calabrese, F. H. L. Essler, M. Fagotti J. Stat. Mech. (2012) P07016

$$\hat{\Gamma}_k = C_k 1_2 + A_k \sigma_y^{(k)} + B_k \sigma_x^{(k)} e^{2i\varepsilon_h(k)t\sigma_y^{(k)}}$$

- Generalization to $C_k \neq 0$ (multidimensional stationary phase approximation)

$t, \ell \to \infty, \gamma \to 0, t/\ell, \gamma\ell$ fixed

V.A., F. Carollo, arXiv:2109.01836

$$S_A^{(n)} = \int_{-\pi}^{\pi} \frac{dk}{2\pi} \left[s_k^{(n),YY} - s_k^{(n),\mathrm{mix}} \right] \min(\ell, |2\varepsilon_h'(k)|t) + \ell \int_{-\pi}^{\pi} \frac{dk}{2\pi} s_k^{(n),\mathrm{mix}}$$

$$s^{(n),YY}(\rho_k) = \frac{1}{1-n} \ln(\rho_k^n + (1-\rho_k)^n)$$

$$\rho_k := \frac{1 - C_k + A_k}{2}$$

$$s_k^{(n),\mathrm{mix}} = \frac{1}{2} \frac{1}{1-n} \mathrm{Tr} \ln \left[\left(\frac{1_2 + \hat{\Gamma}_k}{2} \right)^n + \left(\frac{1_2 - \hat{\Gamma}_k}{2} \right)^n \right]$$

- Simple dynamics for ρ_k

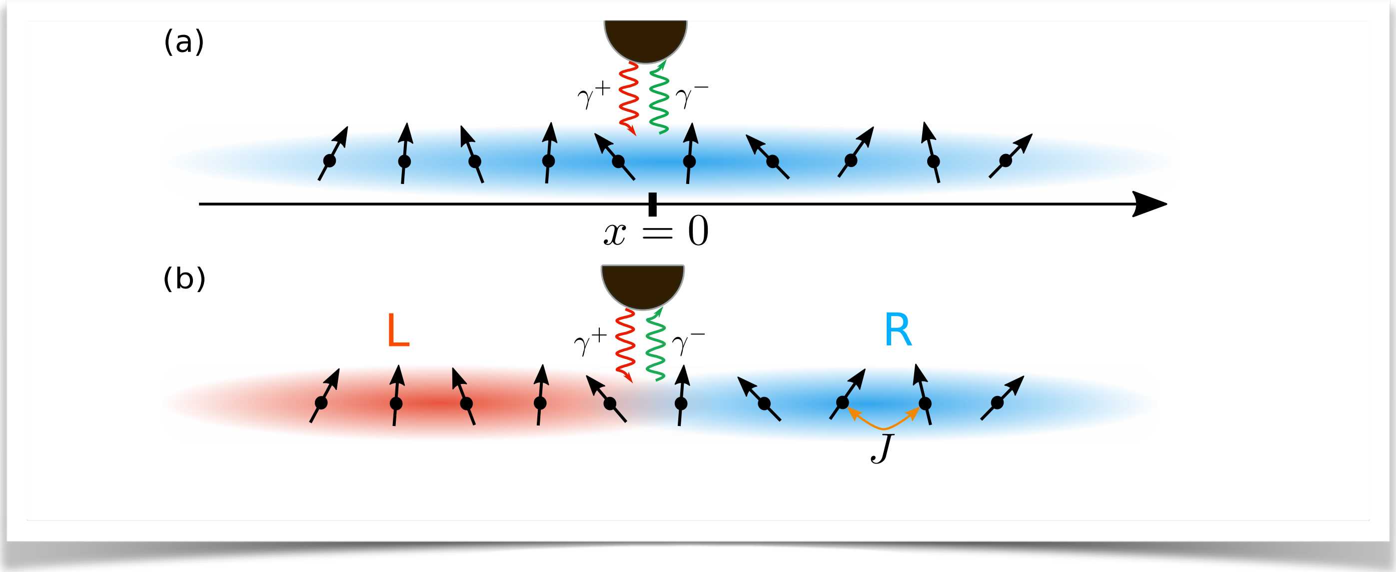
ρ_k not the density of Bogoliubov modes

$$\frac{d\rho_k}{dt} = -4\gamma(s_e + z_o)\rho_k + 2\gamma(s_e - s_o + z_o + z_e)$$

DISSIPATIVE IMPURITIES

V.A. and F. Carollo, arXiv:2103.05671)

V.A., arXiv:2104.10921v2)



CORRELATION FUNCTIONS

$$\frac{dG_{x,y}}{dt} = i(G_{x+1,y} + G_{x-1,y} - G_{x,y+1} - G_{x,y-1}) - \frac{\gamma^+ + \gamma^-}{2}(\delta_{x,0}G_{x,y} + \delta_{y,0}G_{x,y}) + \gamma^+\delta_{x,0}\delta_{y,0}$$

With initial condition $G_{x,y}(0)$

sufficient to restrict to losses

$$\frac{dG_{x,y}}{dt} = i(G_{x+1,y} + G_{x-1,y} - G_{x,y+1} - G_{x,y-1}) - \frac{\gamma^-}{2}(\delta_{x,0}G_{x,y} + \delta_{y,0}G_{x,y})$$

factorized ansatz $$G_{x,y} = \int_{-\pi}^{\pi} \frac{dk}{2\pi} S_{k,x}\bar{S}_{k,y}$$

$$\frac{dS_{k,x}}{dt} = i[S_{k,x+1} + S_{k,x-1}] - \frac{\gamma^-}{2}\delta_{x,0}S_{k,x}$$

Fourier+Laplace transform $$\widehat{S}_{k,x}(s) = \int_0^\infty dt e^{-st} S_{k,x}(t) \qquad \widehat{S}_{k,q} = \sum_{x=-\infty}^{\infty} \widehat{S}_{k,x} e^{-iqx}$$

CORRELATION FUNCTIONS: HYDRODYNAMIC LIMIT

- Almost full analytic solution:

$$S_{k,x}(t) = i^{|x-k|}J_{|x-k|}(2t) - \frac{\gamma^-}{2} i^{|x|+|k|}K_{|x|+|k|}(t)$$

$$K_{|x|+|k|} = \mathcal{L}^{-1}\Big(\frac{1}{(\frac{\gamma^-}{2}+\sqrt{s^2+4})\sqrt{s^2+4}}\Big(\frac{2i}{s+\sqrt{s^2+4}}\Big)^{|k|+|x|}\Big)$$

Dissipation

- Hydrodynamic limit:

$$\frac{x}{2t}, \frac{y}{2t} \quad \text{fixed}$$ doable in some cases

$$\frac{|x-y|}{2t} \to 0$$ easier "universal"

$$S_{k,x} = e^{ikx} + r(k)\Theta(|v_k|t - |x|)e^{i|k||x|}$$

$$r(k) = -\frac{\gamma^-}{2}\frac{1}{\frac{\gamma^-}{2}+|v_k|}$$

SCATTERING WITH IMAGINARY DELTA POTENTIAL

- Scattering by delta potential

$$-\frac{\hbar^2}{2m}\frac{d^2\psi(x)}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$V(x) = -i\frac{\gamma^-}{2}\delta(x)$$

$$r(k) = -\frac{\gamma^-}{2}\frac{1}{\frac{\gamma^-}{2}+k}$$

$$k \to |v_k|$$

- Reflection and transmission amplitudes:

$$\tau(k) = \frac{|v_k|}{\frac{\gamma^-}{2}+|v_k|}$$

$$1-\tau^2-r^2 = a^2 > 0$$

Non unitary dynamics

INHOMOGENEOUS QUENCHES

- Generalizable to other other homogeneous and inhomogeneous quenches

$$H = \sum_{x=-\infty}^{\infty} (c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x)$$

Fermionic Neel quench	$\|0101010101\dots\rangle$
Domain-wall quench	$\|\dots 11111110000000\dots\rangle$
Two Fermi seas	$\|k_F^l\rangle \otimes \|k_F^r\rangle$

T. Antal, Z. Racz, A. Racos, G. M. Schutz, PRE 59, 4912 (1999)

INHOMOGENEOUS QUENCHES

$$H = \sum_{x=-\infty}^{\infty} (c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x)$$

- Two Fermi seas

$$|k_F^l\rangle \otimes |k_F^r\rangle$$

- Factorized ansatz

$$G_{x,y} = \int_{-k_F^l}^{k_F^l} \frac{dk}{2\pi} S_{k,x}^l \bar{S}_{k,y}^l + \int_{-k_F^r}^{k_F^r} \frac{dk}{2\pi} S_{k,x}^r \bar{S}_{k,y}^r$$

$$\chi_x = \Theta(|v_k|t - |x|)$$

- Hydrodynamic limit

$$G_{x,y}(t) = \int_{-k_F^l}^{k_F^l} \frac{dK}{2\pi} \Big\{ e^{iK(x-y)} \Theta\Big(2t\sin(K) - \frac{x+y}{2}\Big) + e^{iK(|x|-|y|)}\Theta(K)(\Theta(x)+\Theta(y))\chi_x\chi_y r$$
$$+\Theta(K)e^{iK(|x|-|y|)}\chi_x\chi_y r^2 \Big\} + l \leftrightarrow r,$$

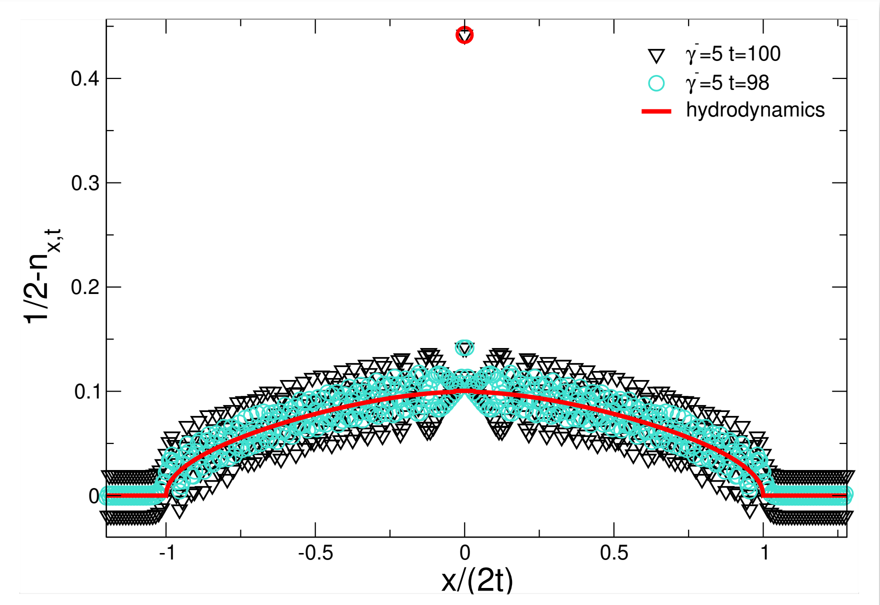
J. Viti, J.-M. Stephan, J. Dubail, M. Haque, EPL 115 (2016) 40011

INHOMOGENEOUS QUENCHES

$$H = \sum_{x=-\infty}^{\infty} (c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x)$$

V.A. and F. Carollo, arXiv:2103.05671)

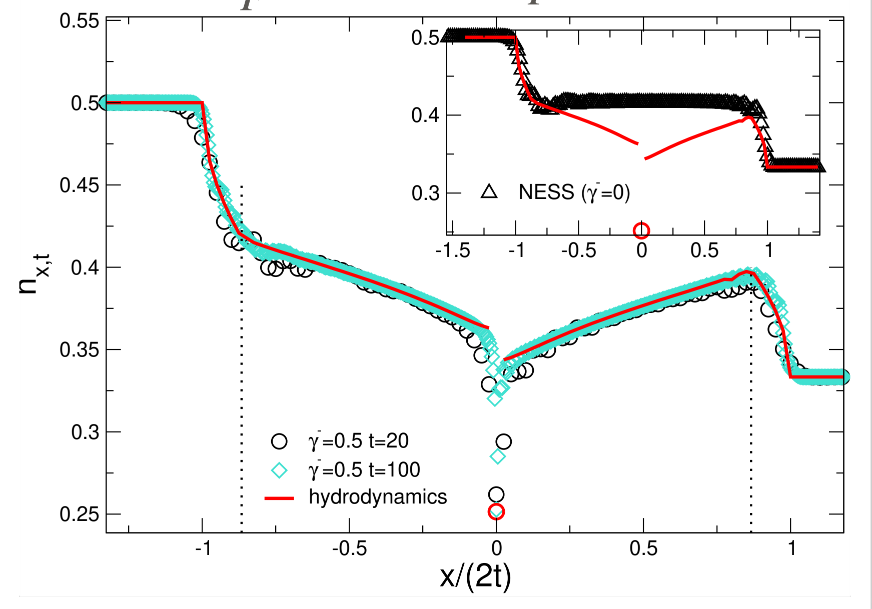
Fermionic Neel quench



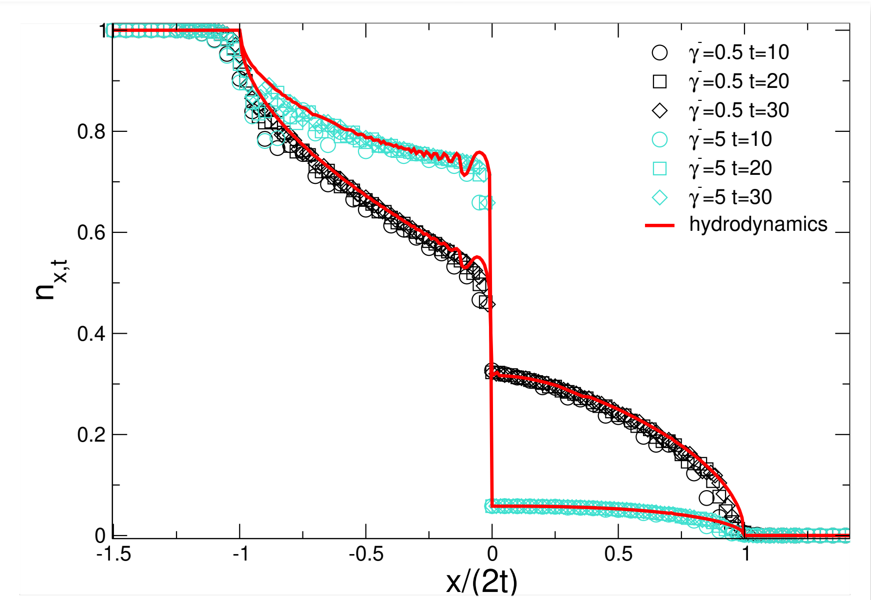
$$n_{x,t} = \langle c_x^\dagger c_x \rangle$$

Two Fermi seas

$k_F^l = \pi/2 \quad k_F^r = \pi/3$



Domain wall quench



- ☑ Singularity at x=0
- ☑ Scaling behaviour
- ☑ Match with analytics

$\gamma^- \to \infty$

Quantum Zeno effect

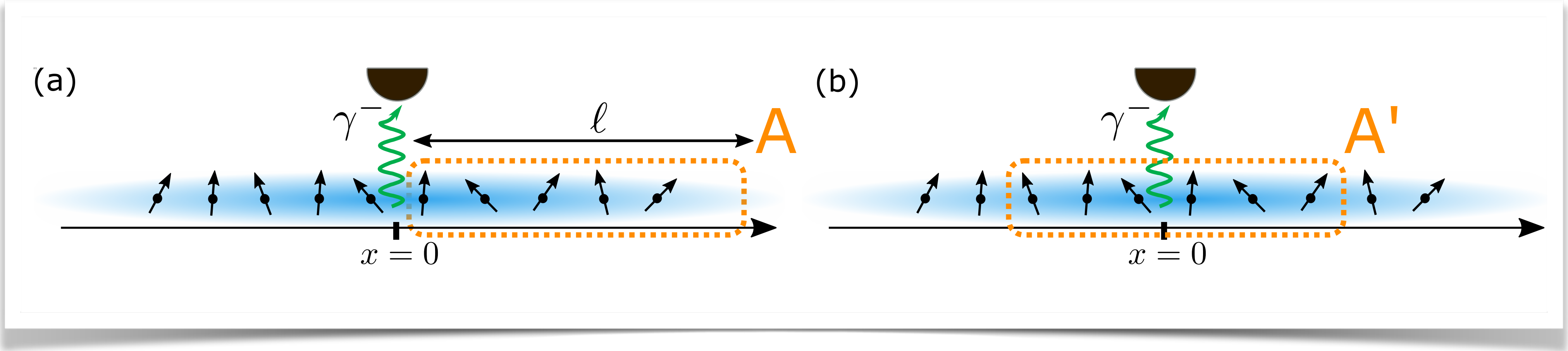
P. Krapyvsky, K. Mallick, D. Sels, J. Stat. Mech. (2019) 113108

HOW ABOUT ENTANGLEMENT?

- Let us consider the uniform Fermi sea with localised losses

V.A., to appear

$$H = \sum_{x=-\infty}^{\infty} (c_x^\dagger c_{x+1} + c_{x+1}^\dagger c_x)$$

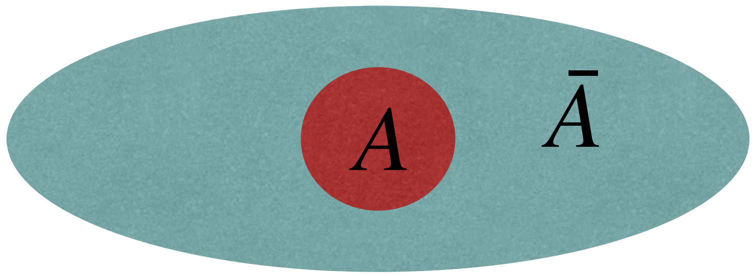


- From previous slides:

$$G_{x,y} = \int_{-k_F}^{k_F} \frac{dk}{2\pi} (e^{ikx} + r(k)\chi_x e^{i|k||x|})(e^{-iky} + r(k)\chi_y e^{-i|k||y|})$$

HOW ABOUT ENTANGLEMENT?

- Entanglement entropy obtained from covariance matrix

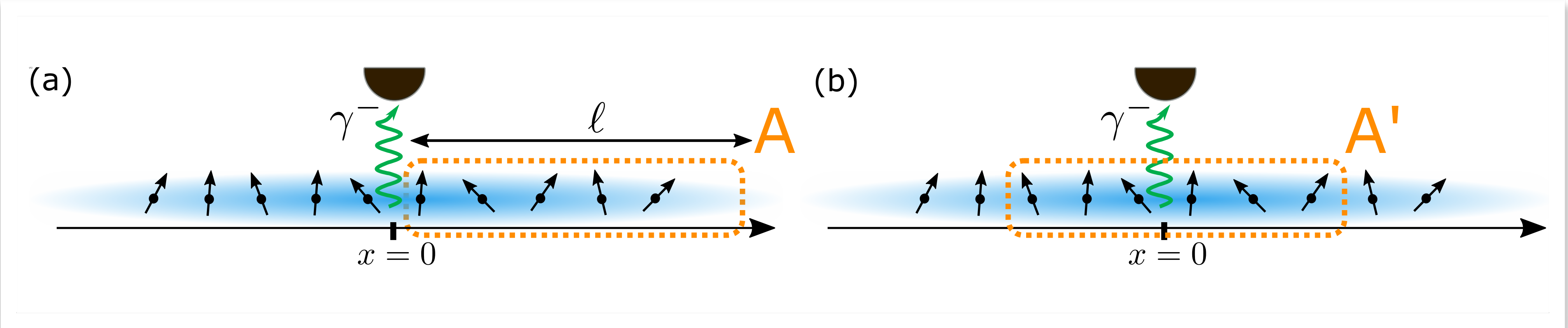


G restricted to A

$$S_A = -\,\mathrm{Tr}(G\ln G + (1-G)\ln(1-G))$$

V. Eisler, I. Peschel, J. Phys. A: Math. Theor. 42 504003 (2009)

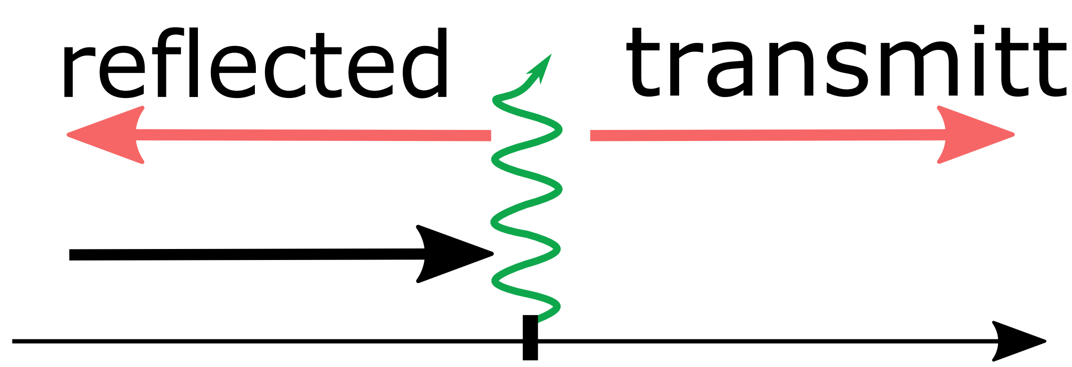
- Requires hydrodynamic limit of $\mathrm{Tr}(G^n)$



$$\mathrm{Tr}G^n = \ell\int_{-k_F}^{k_F}\frac{dk}{2\pi}\Big[\Big(1-\frac{1}{2z_X}\min(z_X|v_k|t/\ell,1)\Big)+\frac{1}{2z_X}(1-z_X|a(k)|^2)^n\min(z_X|v_k|t/\ell,1)\Big],\quad z_{A(A')}=1(2)$$

ENTANGLEMENT PRODUCTION MECHANISM

- Entanglement between reflected and transmitted wave

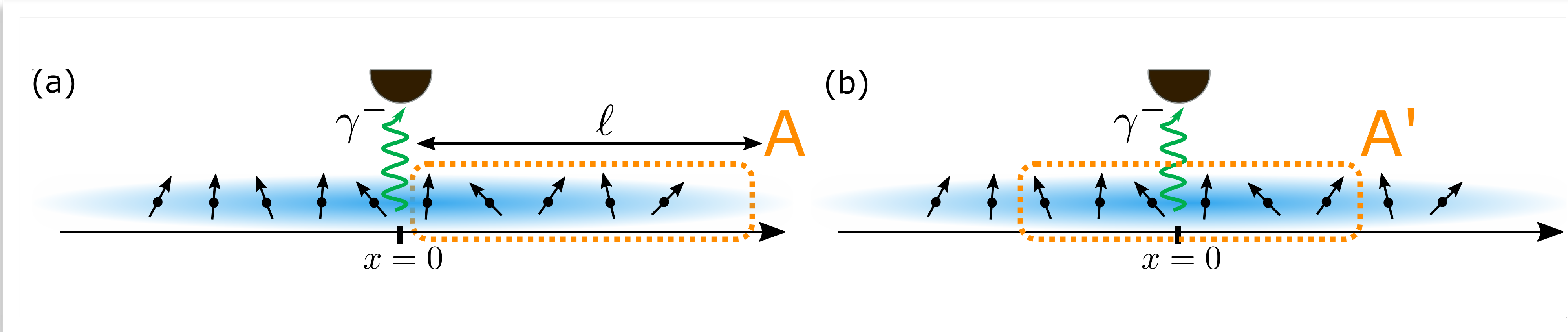


- Similar to unitary case (defect)

V. Eisler and I. Peschel, EPL 99, 20001 (2012)

- Due to non unitarity genuine entanglement mixed with thermodynamic entropy

ENTANGLEMENT ENTROPY



- Main result:

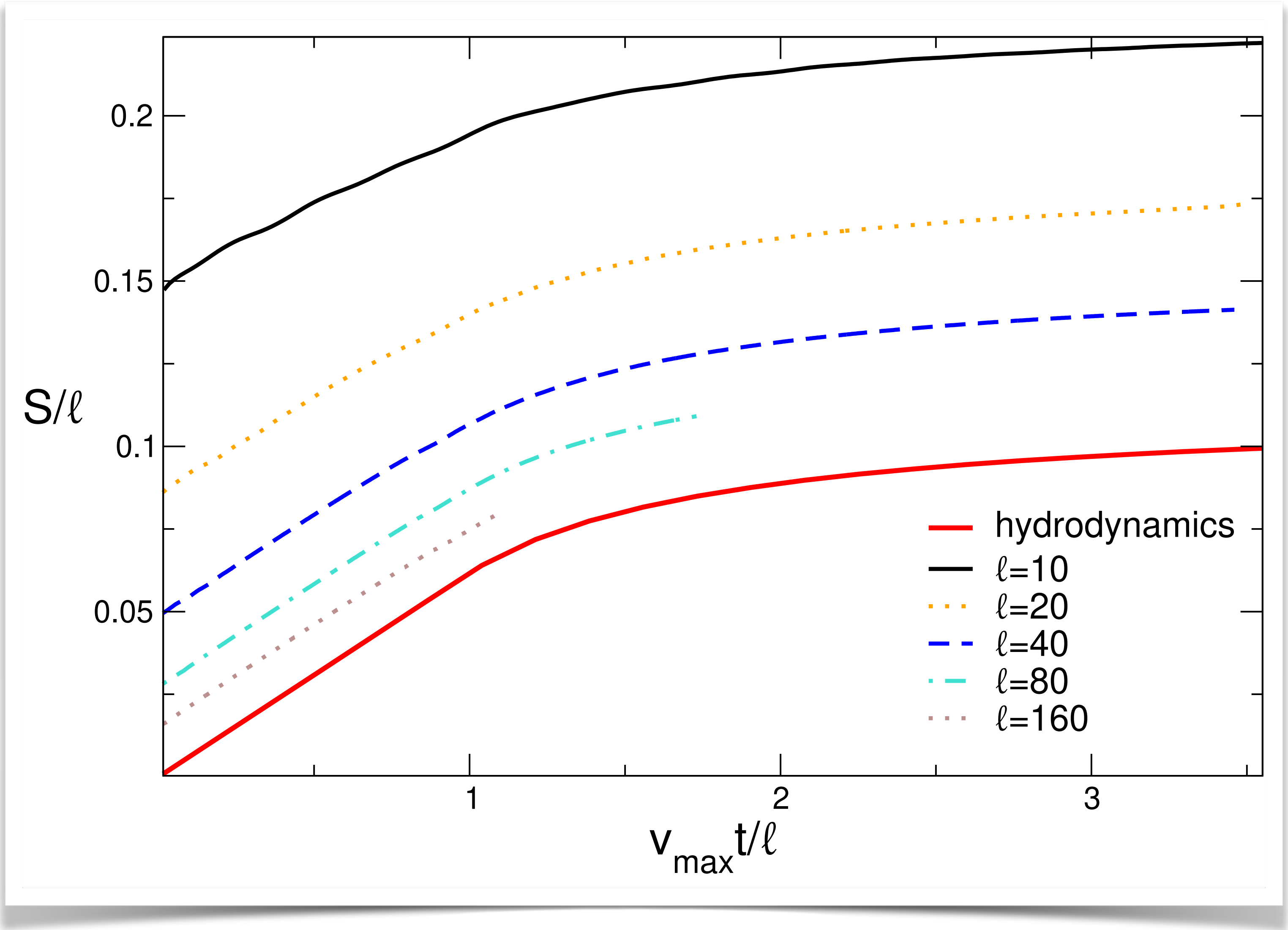
$$S_X^{(n)} = \frac{1}{2z_X}\frac{\ell}{1-n}\int_{-k_F}^{k_F}\frac{dk}{2\pi}H_n(1 - z_X|a|^2)\min(z_X|v_k|t/\ell, 1)$$

$$H_n(z) = ln(z^n + (1-z)^n)$$

- Observations

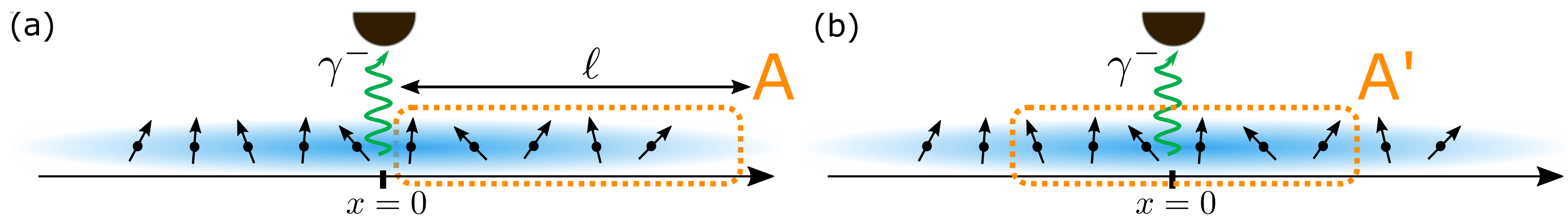
  - Dependence on "surviving" probability of fermions
  - Trivial dependence on geometry
  - Same qualitative behaviour for both bipartitions

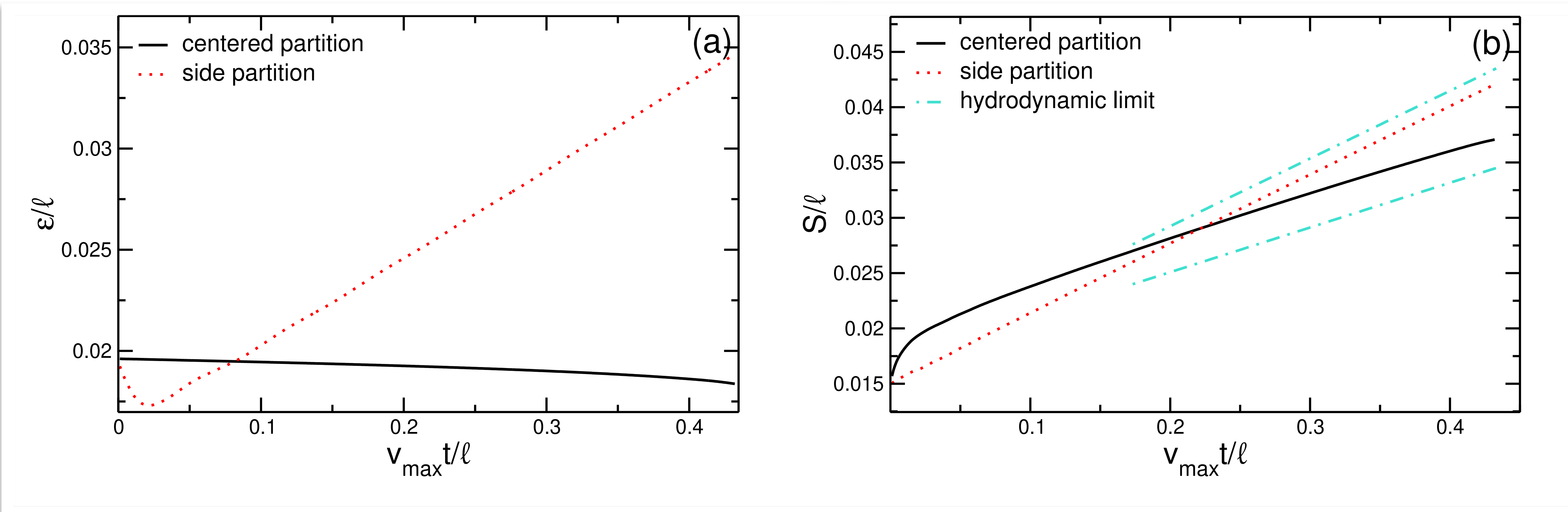
NUMERICAL BENCHMARKS



- ☑ Linear increase + saturation
- ☑ Agreement with hydrodynamics in the scaling limit

ENTANGLEMENT VS THERMODYNAMICS





☑ Genuine entanglement correctly diagnosed by negativity

CONCLUSIONS

A LOT REMAINS TO BE DONE !!

THANK YOU