Signatures of deconfined quantum criticality in a spin-1 model on the square lattice

Vikas Vijigiri

Collaborators:

Dr. Nisheeta Desai (PDF, TIFR)

Prof. Sumiran Pujari (IIT Bombay)

Department of Physics, IIT Bombay



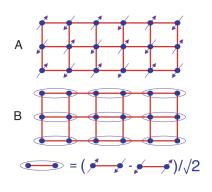
Outline

- Deconfined quantum criticality
- Earlier studies
- 3 Designer model Hamiltonian
- Results





Introduction to DQC



Néel (A): Antiferromagnetic state, breaks rotational symmetry, SU(N).

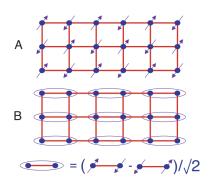
Sensitive to quantum fluctuations.

Low energy excitations: Spin-waves.

Valence bond solid (VBS) (B): A non-magnetic state, and breaks lattice symmetry (e.g. translational for spin-1/2). Product of quantum fluctuations. Localized triplets ("confined spinons").



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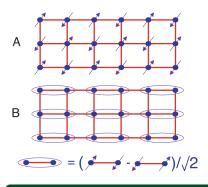
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Introduction to DQC



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$$\mathcal{H} = J \sum_{\langle i,j \rangle} \vec{S}_i \cdot \vec{S}_j + \cdots$$

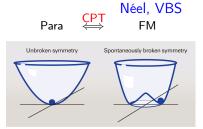
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Deconfined quantum criticality is a...

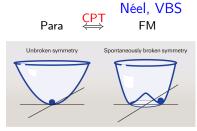
'Critical region of interest' associated with Néel-Valence bond solid (VBS) quantum phase transition in magnetic systems. Can see 'deconfined' nature of spinons.



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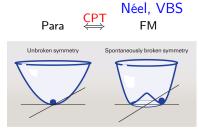






Both the Néel and VBS phase lie to the broken symmetry side.

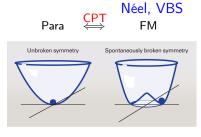




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Order of the phase transition?

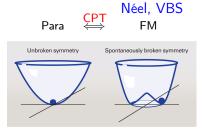




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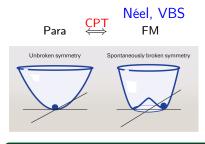


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Order of the phase transition?
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1<sup>st</sup> order? (LGW)
co-existence of two orders? (LGW)
2<sup>nd</sup> order?
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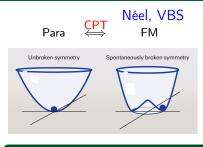
Order of the phase transition?

1st order? (LGW) co-existence of two orders? (LGW) 2nd order?

Continuous phase transitions by Landau-Ginzberg-Wilson (LGW) paradigm...

Ground state to break the continuous symmetry of the Hamiltonian.





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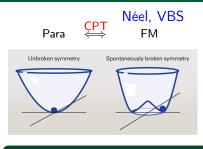
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1. Order parameter description of phases.





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Continuous phase transitions by Landau-Ginzberg-Wilson (LGW) paradigm...

Ground state to break the continuous symmetry of the Hamiltonian.

- 1. Order parameter description of phases.
- 2. An emergent gauge field and "deconfined" degrees of freedom associated with fractionalization of the order parameters. (Beyond LGW paradigm)









Prior to 2015...

Evidence for Deconfined Quantum Criticality in a Two-Dimensional Heisenberg Model with Four-Spin Interactions.
Anders W. Sandvik, PRL 98, 227202 (2007).



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Post 2015...

Emergent SO(5) Symmetry at the Neel to Valence-Bond-Solid Transition.
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- Diagnosing weakly first-order phase transitions by coupling to order parameters. Jonathan D'Emidio, Alexander A. Eberharter, Andreas M. Läuchli, SciPost,





Quantum Spin Nematics, Dimerization, and Deconfined Criticality in Quasi-1D Spin-One Magnets.

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Our approach lies in the idea that the above problem can be recasted into spin-1 with SU(3) symmetry and see the effect of criticality under reduced symmetry conditions, SU(2).

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Spin-1, SU(3) symmetric

Hamiltonian:

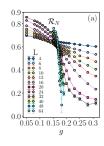
$$SU(3): \mathcal{H} = J_{Bi} \sum_{\langle i,j \rangle} (\vec{S}_i \cdot \vec{S}_j)^2 - Q_n \sum_{ijkl} (\vec{S}_i \cdot \vec{S}_j)^2 (\vec{S}_k \cdot \vec{S}_l)^2$$
 (1)

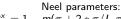
$$SU(2): \mathcal{H}_{p} = \mathcal{H} + J_{H} \sum_{\langle i,j \rangle} \vec{S}_{i} \cdot \vec{S}_{j}$$
 (2)

- **1** It can be shown that the terms in \mathcal{H} are SU(3) symmetric.
- It is of the following reasons, interesting to see the effect of a lower symmetric perturbation, SU(2), upon the deconfined critical point (DCP). There can be three questions now:
 - Does the phases survive?
 - If so, what is the universality class without perturbation?
 - And the effect under reduced symmetry (SU(2)) perturbation?



Scaling of order parameter ratios, in the absence of perturbation





$$R_N^{\times} = 1 - m(\pi + 2 * \pi/L, \pi)/m(\pi, \pi)$$

 $R_N^{y} = 1 - m(\pi, \pi + 2 * \pi/L)/m(\pi, \pi)$
and $m \sim \langle S_i^z S_j^z \rangle$

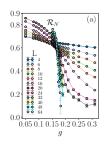
0.4 0.2 0.05 0.1 0.15 0.2 0.25 0.3

VBS order parameter:

$$\begin{split} R_{B}^{\mathsf{x}} &= 1 - \frac{\tilde{\mathsf{c}}^{\mathsf{x}}(\pi, 2\pi/L)}{\tilde{\mathsf{c}}(\pi, 0)}, \\ R_{B}^{\mathsf{y}} &= 1 - \frac{\tilde{\mathsf{c}}^{\mathsf{y}}(2\pi/L, \pi)}{\tilde{\mathsf{c}}(0, \pi)} \\ C^{\alpha} &\sim \langle S_{\vec{r}} \cdot S_{\vec{r}^{\mathsf{y}}+\hat{\alpha}} S_{\vec{r}^{\mathsf{y}}} \cdot S_{\vec{r}^{\mathsf{y}}+\hat{\alpha}} \rangle \end{split}$$

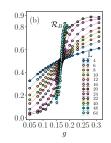


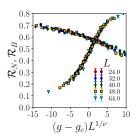
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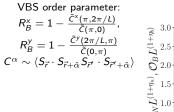


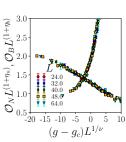
Neel parameters:
$$R_N^{\times} = 1 - m(\pi + 2*\pi/L, \pi)/m(\pi, \pi)$$

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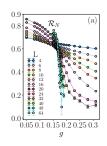






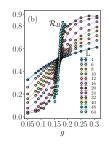


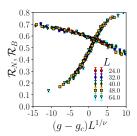
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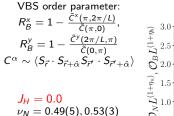


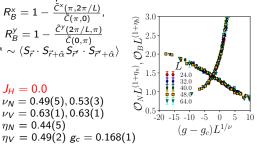
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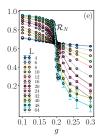




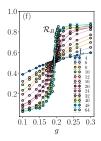
 $\nu_V = 0.63(1), 0.63(1)$

 $\eta_N = 0.44(5)$

In the presence of perturbation, $J_H = 0.05$

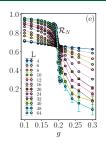


Look at the value of η_N , there is a consistent decrease in the values compared to the case of $J_H=0.0$

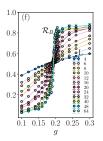


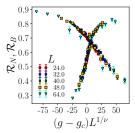


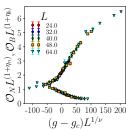
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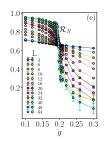




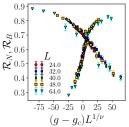


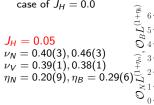


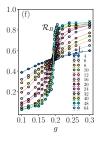
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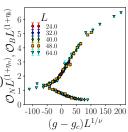


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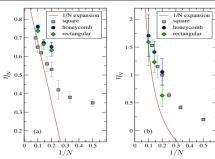
The exponent table

J _H	Parameter	ν_{N}	ν_V	η_{N}	η_V	g _{cN}	g _c V
0.0	Я	0.49(5)	0.63(1)	0	0	0.168(1)	0.167(1)
0.0	0	0.53(3)	0.63(1)	0.44(5)	0.49(2)	0.167	0.167
0.05	${\mathscr R}$	0.40(3)	0.46(3)	0	0	0.196(1)	0.195(1)
0.05	0	0.39(3)	0.38(3)	0.20(9)	0.29(6)	0.195	0.195
0.1	${\mathscr R}$	0.31(1)	0.40(1)	0	0	0.225(1)	0.223(1)
0.1	0	0.33(2)	0.41(2)	0.13(2)	0.28(7)	0.224	0.224
0.15	${\mathscr R}$	0.32(4)	0.44(2)	0	0	0.254(1)	0.252(1)
0.15	0	??` ´	??`	??	??	0.253	0.253



The exponent table

J _H	Parameter	ν_{N}	ν_V	η_{N}	η_V	g _c N	g _c V
0.0	R	0.49(5)	0.63(1)	0	0	0.168(1)	0.167(1)
0.0	O	0.53(3)	0.63(1)	0.44(5)	0.49(2)	0.167	0.167
0.05	${\mathscr R}$	0.40(3)	0.46(3)	0	0	0.196(1)	0.195(1)
0.05	O	0.39(3)	0.38(3)	0.20(9)	0.29(6)	0.195	0.195
0.1	${\mathscr R}$	0.31(1)	0.40(1)	0	0	0.225(1)	0.223(1)
0.1	O	0.33(2)	0.41(2)	0.13(2)	0.28(7)	0.224	0.224
0.15	${\mathscr R}$	0.32(4)	0.44(2)	0	0	0.254(1)	0.252(1)
0.15	Ø	??`	??` ´	??	??	0.253	0.253



Block, Melko, et al, PRL 111, 137202 (2013)





Conclusion

• We have studied and seen a Neel-VBS transition in a spin-1 (SU(3) symmetric) Hamiltonian with Heisenberg (SU(2) symmetric) term as perturbation.



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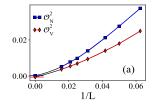
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- ② Our results match with the literature for $J_H = 0.0$ with critical exponents of SU(3) type.
- **1** However, as we turn on perturbation ($J_H = 0.05$) we see a shift in the universality class of SU(2) type.
- ① Our results show some indications of deconfined criticality within the range $J_H \sim 0-0.1$.



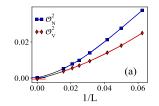


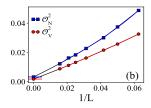
Order parameter extrapolation





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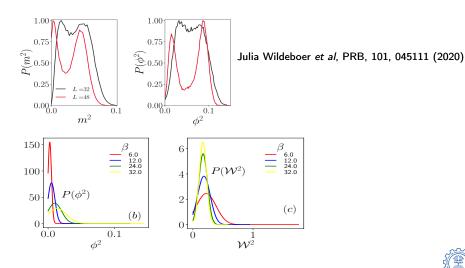




A finite value of order parameter starts appearing as we increase beyond $J_H=0.1$. A signature of first order phase transition (weak).



Histograms







Take-home message

Spin-1 particles can behave like spin-1/2 particles! (Like two independent spins, in constrained environments).



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Thank you ALL

