The vortex ansatz as a fertile testing ground for certain systems of PDE

Vamsi Pritham Pingali

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$$\sqrt{-1}\Theta_0 + \sqrt{-1}\partial\bar{\partial}f_t = u^{1-t}\frac{\tau - |\phi|_{h_0}^2 e^{-f_t}}{2}\omega_{\Sigma} \text{ where } f_0 = 0,$$

$$\sqrt{-1}\Theta_0 = c\omega_{\Sigma} > 0, \ |\phi|_{h_0}^2 < \frac{\tau}{2}, \text{ and } u = \frac{2c}{\tau - |\phi|_{h_0}^2}.$$

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Vamsi Pritham Pingali The vortex ansatz

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Vamsi Pritham Pingali The vort

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Vamsi Pritham Pingali The vorte

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- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.
- Rewards exist! Demailly proposed several PDE-based approaches towards the Griffiths conjecture in its full generality. All are fully nonlinear systems.
- The vortex ansatz is a fertile testing ground. For instance, Mandal recently proved the feasibility of some of Demailly's methods for the vortex bundle.



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