# The vortex ansatz as a fertile testing ground for certain systems of PDE 

Vamsi Pritham Pingali<br>Indian Institute of Science

## The bottom line

## The bottom line

- The vortex ansatz


## The bottom line

- The vortex ansatz produces vector bundles
- The vortex ansatz produces vector bundles with many symmetries.
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to Hermitian-Einstein metrics (GP),
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to Hermitian-Einstein metrics (GP), the KYM equations (GP-GF-AC-P-Y),
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to Hermitian-Einstein metrics (GP), the KYM equations (GP-GF-AC-P-Y), the vector bundle MA equation (P),
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to Hermitian-Einstein metrics (GP), the KYM equations (GP-GF-AC-P-Y), the vector bundle MA equation (P), Gieseker stability (Ghosh),
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to Hermitian-Einstein metrics (GP), the KYM equations (GP-GF-AC-P-Y), the vector bundle MA equation ( P ), Gieseker stability (Ghosh), CYM equations ( P , Ghosh),
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to Hermitian-Einstein metrics (GP), the KYM equations (GP-GF-AC-P-Y), the vector bundle MA equation ( P ), Gieseker stability (Ghosh), CYM equations ( P , Ghosh), the Demailly systems (Mandal), and
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to Hermitian-Einstein metrics (GP), the KYM equations (GP-GF-AC-P-Y), the vector bundle MA equation ( P ), Gieseker stability (Ghosh), CYM equations ( P , Ghosh), the Demailly systems (Mandal), and the vector bundle J-equation (Takahashi)
- The vortex ansatz produces vector bundles with many symmetries.
- It can be used to dimensionally reduce several PDE to simpler PDE.
- Since systems of PDE might be an important future direction the vortex ansatz is a fertile testing ground.
- Has been applied to Hermitian-Einstein metrics (GP), the KYM equations (GP-GF-AC-P-Y), the vector bundle MA equation ( P ), Gieseker stability (Ghosh), CYM equations ( P , Ghosh), the Demailly systems (Mandal), and the vector bundle J-equation (Takahashi) to prove Kobayashi-Hitchin-Donaldson-Uhlenbeck-Yau-type correspondences.


## The vortex ansatz

## The vortex ansatz

- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$.
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- If $\phi \in H^{0}(\Sigma, L), V$ has second fundamental form $\pi_{1}^{*} \phi \otimes \pi_{2}^{*} \zeta$ where $\zeta=\frac{\sqrt{8 \pi} d z}{\tau\left(1+|z|^{2}\right)} \otimes d \bar{z}$.
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- If $\phi \in H^{0}(\Sigma, L), V$ has second fundamental form $\pi_{1}^{*} \phi \otimes \pi_{2}^{*} \zeta$ where $\zeta=\frac{\sqrt{8 \pi} d z}{\tau\left(1+|z|^{2}\right)} \otimes d \bar{z}$.
- Consider a smooth metric $h$ on $L$ and
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- If $\phi \in H^{0}(\Sigma, L), V$ has second fundamental form $\pi_{1}^{*} \phi \otimes \pi_{2}^{*} \zeta$ where $\zeta=\frac{\sqrt{8 \pi} d z}{\tau\left(1+|z|^{2}\right)} \otimes d \bar{z}$.
- Consider a smooth metric $h$ on $L$ and a smooth function $f_{2}$ on $\Sigma$.
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- If $\phi \in H^{0}(\Sigma, L), V$ has second fundamental form $\pi_{1}^{*} \phi \otimes \pi_{2}^{*} \zeta$ where $\zeta=\frac{\sqrt{8 \pi} d z}{\tau\left(1+|z|^{2}\right)} \otimes d \bar{z}$.
- Consider a smooth metric $h$ on $L$ and a smooth function $f_{2}$ on $\Sigma$. Put $h_{1}=\pi_{1}^{*}\left(h f_{2} \frac{8 \pi}{\tau} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}}\right)$ on $L_{1}$ and
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- If $\phi \in H^{0}(\Sigma, L), V$ has second fundamental form $\pi_{1}^{*} \phi \otimes \pi_{2}^{*} \zeta$ where $\zeta=\frac{\sqrt{8 \pi} d z}{\tau\left(1+|z|^{2}\right)} \otimes d \bar{z}$.
- Consider a smooth metric $h$ on $L$ and a smooth function $f_{2}$ on $\Sigma$. Put $h_{1}=\pi_{1}^{*}\left(h f_{2} \frac{8 \pi}{\tau} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}}\right)$ on $L_{1}$ and $h_{2}=\pi_{1}^{*}\left(f_{2} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}+2}\right)$ on $L_{2}$.
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- If $\phi \in H^{0}(\Sigma, L), V$ has second fundamental form $\pi_{1}^{*} \phi \otimes \pi_{2}^{*} \zeta$ where $\zeta=\frac{\sqrt{8 \pi} d z}{\tau\left(1+|z|^{2}\right)} \otimes d \bar{z}$.
- Consider a smooth metric $h$ on $L$ and a smooth function $f_{2}$ on $\Sigma$. Put $h_{1}=\pi_{1}^{*}\left(h f_{2} \frac{8 \pi}{\tau} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}}\right)$ on $L_{1}$ and $h_{2}=\pi_{1}^{*}\left(f_{2} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}+2}\right)$ on $L_{2}$.
- Can be extended
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- If $\phi \in H^{0}(\Sigma, L), V$ has second fundamental form $\pi_{1}^{*} \phi \otimes \pi_{2}^{*} \zeta$ where $\zeta=\frac{\sqrt{8 \pi} d z}{\tau\left(1+|z|^{2}\right)} \otimes d \bar{z}$.
- Consider a smooth metric $h$ on $L$ and a smooth function $f_{2}$ on $\Sigma$. Put $h_{1}=\pi_{1}^{*}\left(h f_{2} \frac{8 \pi}{\tau} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}}\right)$ on $L_{1}$ and $h_{2}=\pi_{1}^{*}\left(f_{2} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}+2}\right)$ on $L_{2}$.
- Can be extended to higher ranks
- Take $\left(S=\Sigma \times \mathbb{C P}^{1}, \omega=\pi_{1}^{*} \omega_{\Sigma}+\frac{4}{\tau} \pi_{2}^{*} \omega_{F S}\right)$ where $\Sigma$ is a compact Riemann surface and $\omega_{\Sigma}$ is the curvature of an ample bundle $\left(L, h_{0}\right)$. $S U(2)$ acts on $S$.
- Let $V$ be an extension

$$
0 \rightarrow L_{1} \rightarrow V \rightarrow L_{2} \rightarrow 0
$$

- $L_{1}=\pi_{1}^{*}\left(\left(r_{1}+1\right) L\right) \otimes \pi_{2}^{*}\left(r_{2} \mathcal{O}(2)\right)$
- $L_{2}=\pi_{1}^{*}\left(r_{1} L\right) \otimes \pi_{2}^{*}\left(\left(r_{2}+1\right) \mathcal{O}(2)\right)$
- If $\phi \in H^{0}(\Sigma, L), V$ has second fundamental form $\pi_{1}^{*} \phi \otimes \pi_{2}^{*} \zeta$ where $\zeta=\frac{\sqrt{8 \pi} d z}{\tau\left(1+|z|^{2}\right)} \otimes d \bar{z}$.
- Consider a smooth metric $h$ on $L$ and a smooth function $f_{2}$ on $\Sigma$. Put $h_{1}=\pi_{1}^{*}\left(h f_{2} \frac{8 \pi}{\tau} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}}\right)$ on $L_{1}$ and $h_{2}=\pi_{1}^{*}\left(f_{2} h_{0}^{r_{1}}\right) \otimes \pi_{2}^{*}\left(h_{F S}^{2 r_{2}+2}\right)$ on $L_{2}$.
- Can be extended to higher ranks and higher-dimensional $\Sigma$.


## Hermitian-Einstein (HE) metrics

## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.
- This equation can be solved


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.
- This equation can be solved using the Kazdan-Warner theory (Garcia-Prada)


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.
- This equation can be solved using the Kazdan-Warner theory (Garcia-Prada) iff $c_{1}(L)<\frac{\tau(\text { Vol }(\Sigma)}{4 \pi}$, which is


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.
- This equation can be solved using the Kazdan-Warner theory (Garcia-Prada) iff $c_{1}(L)<\frac{\tau(V o l(\Sigma)}{4 \pi}$, which is precisely the Mumford stability condition


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.
- This equation can be solved using the Kazdan-Warner theory (Garcia-Prada) iff $c_{1}(L)<\frac{\tau(\text { Vol }(\Sigma)}{4 \pi}$, which is precisely the Mumford stability condition for invariant subsheaves.


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.
- This equation can be solved using the Kazdan-Warner theory (Garcia-Prada) iff $c_{1}(L)<\frac{\tau(\text { Vol }(\Sigma)}{4 \pi}$, which is precisely the Mumford stability condition for invariant subsheaves.
- Solving using the method of continuity:


## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.
- This equation can be solved using the Kazdan-Warner theory (Garcia-Prada) iff $c_{1}(L)<\frac{\tau(\operatorname{Vol}(\Sigma)}{4 \pi}$, which is precisely the Mumford stability condition for invariant subsheaves.
- Solving using the method of continuity:

$$
\sqrt{-1} \Theta_{0}+\sqrt{-1} \partial \bar{\partial} f_{t}=u^{1-t} \frac{\tau-|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} \omega_{\Sigma} \text { where }
$$

## Hermitian-Einstein (HE) metrics

- The Hermitian-Einstein equation $\sqrt{-1} \Theta \wedge \omega=\lambda \omega^{2}$ on a vortex bundle boils down to the vortex equation $\sqrt{-1} \Theta_{h}=\frac{\tau-|\phi|_{h}^{2}}{2} \omega_{\Sigma}$.
- This equation can be solved using the Kazdan-Warner theory (Garcia-Prada) iff $c_{1}(L)<\frac{\tau(\operatorname{Vol}(\Sigma)}{4 \pi}$, which is precisely the Mumford stability condition for invariant subsheaves.
- Solving using the method of continuity:

$$
\begin{aligned}
& \sqrt{-1} \Theta_{0}+\sqrt{-1} \partial \bar{\partial} f_{t}=u^{1-t} \frac{\tau-|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} \omega_{\Sigma} \text { where } f_{0}=0 \\
& \sqrt{-1} \Theta_{0}=c \omega_{\Sigma}>0,|\phi|_{h_{0}}^{2}<\frac{\tau}{2} \text {, and } u=\frac{2 c}{\tau-|\phi|_{h_{0}}^{2}}
\end{aligned}
$$

## Hermitian-Einstein (HE) metrics

## Hermitian-Einstein (HE) metrics

- Linearisation:


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism.


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT,


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.
- Closedness:


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.
- Closedness: Since

$$
\sqrt{-1} \partial \bar{\partial}|\phi|_{h_{t}}^{2}=-\sqrt{-1} \Theta_{t}|\phi|_{h_{t}}^{2}+\nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}
$$

## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.
- Closedness: Since
$\sqrt{-1} \partial \bar{\partial}|\phi|_{h_{t}}^{2}=-\sqrt{-1} \Theta_{t}|\phi|_{h_{t}}^{2}+\nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}$, by max. princ. $|\phi|_{h_{t}}^{2} \leq \tau$.


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.
- Closedness: Since
$\sqrt{-1} \partial \bar{\partial}|\phi|_{h_{t}}^{2}=-\sqrt{-1} \Theta_{t}|\phi|_{h_{t}}^{2}+\nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}$, by max. princ. $|\phi|_{h_{t}}^{2} \leq \tau$. By max prin,


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.
- Closedness: Since
$\sqrt{-1} \partial \bar{\partial}|\phi|_{h_{t}}^{2}=-\sqrt{-1} \Theta_{t}|\phi|_{h_{t}}^{2}+\nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}$, by max. princ. $|\phi|_{h_{t}}^{2} \leq \tau$. By max prin, $\left\|f_{t}\right\| \leq C$ (when $0<\delta \leq t \leq 1$ ) and hence


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.
- Closedness: Since
$\sqrt{-1} \partial \bar{\partial}|\phi|_{h_{t}}^{2}=-\sqrt{-1} \Theta_{t}|\phi|_{h_{t}}^{2}+\nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}$, by max. princ. $|\phi|_{h_{t}}^{2} \leq \tau$. By max prin, $\left\|f_{t}\right\| \leq C$ (when $0<\delta \leq t \leq 1$ ) and hence by elliptic theory we have a priori estimates.


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.
- Closedness: Since
$\sqrt{-1} \partial \bar{\partial}|\phi|_{h_{t}}^{2}=-\sqrt{-1} \Theta_{t}|\phi|_{h_{t}}^{2}+\nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}$, by max. princ. $|\phi|_{h_{t}}^{2} \leq \tau$. By max prin, $\left\|f_{t}\right\| \leq C$ (when $0<\delta \leq t \leq 1$ ) and hence by elliptic theory we have a priori estimates. By Arzela-Ascoli we are done.


## Hermitian-Einstein (HE) metrics

- Linearisation: $\sqrt{-1} \partial \bar{\partial} v-u^{1-t} \frac{|\phi|_{h_{0}}^{2} e^{-f_{t}}}{2} v \omega_{\Sigma}$ whose kernel is trivial and is hence an isomorphism. By the inf.dim. IFT, the set of $0 \leq t \leq 1$ solving the equation is open.
- Closedness: Since
$\sqrt{-1} \partial \bar{\partial}|\phi|_{h_{t}}^{2}=-\sqrt{-1} \Theta_{t}|\phi|_{h_{t}}^{2}+\nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}$, by max. princ. $|\phi|_{h_{t}}^{2} \leq \tau$. By max prin, $\left\|f_{t}\right\| \leq C$ (when $0<\delta \leq t \leq 1$ ) and hence by elliptic theory we have a priori estimates. By Arzela-Ascoli we are done.
- Uniqueness is by max prin.


## Kähler-Yang-Mills (KYM) and offshoots

## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$,


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied)


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples ( $M, L, E$ ), and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE:


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples ( $M, L, E$ ), and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model:


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples ( $M, L, E$ ), and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$. Ghosh came up with a KE version of the KYM equations:


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples ( $M, L, E$ ), and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:

$$
\begin{aligned}
& \sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} I d \\
& \omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}
\end{aligned}
$$

## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples ( $M, L, E$ ), and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$, $\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples ( $M, L, E$ ), and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$,
$\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.
- The vortex ansatz provides


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$,
$\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.
- The vortex ansatz provides examples of solutions by reducing


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$, $\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.
- The vortex ansatz provides examples of solutions by reducing the PDE to a system on a Riemann surface.


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples ( $M, L, E$ ), and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$, $\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.
- The vortex ansatz provides examples of solutions by reducing the PDE to a system on a Riemann surface. AC,GF, GP, P, and Yao studied the resulting gravitating vortex equations. (


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$, $\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.
- The vortex ansatz provides examples of solutions by reducing the PDE to a system on a Riemann surface. AC,GF, GP, P, and Yao studied the resulting gravitating vortex equations. (A special case is EB equations studied extensively by Yang.)


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$,
$\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.
- The vortex ansatz provides examples of solutions by reducing the PDE to a system on a Riemann surface. AC,GF, GP, P, and Yao studied the resulting gravitating vortex equations. (A special case is EB equations studied extensively by Yang.) Ghosh proved existence for


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$,
$\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.
- The vortex ansatz provides examples of solutions by reducing the PDE to a system on a Riemann surface. AC,GF, GP, P, and Yao studied the resulting gravitating vortex equations. (A special case is EB equations studied extensively by Yang.) Ghosh proved existence for the CYM equations, and


## Kähler-Yang-Mills (KYM) and offshoots

- Motivated by the moduli problem for triples $(M, L, E)$, and physical considerations AC, GF, and GP came up with (and studied) the KYM system of PDE: $\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d$, $S_{\omega} \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Likewise, we came up with a toy model: The CYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} l d, \omega^{n}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=\eta$.
Ghosh came up with a KE version of the KYM equations:
$\sqrt{-1} \Theta \wedge \omega^{n-1}=\lambda \omega^{n} / d$,
$\omega^{n}-C e^{-\phi}+\alpha \operatorname{tr}\left((\sqrt{-1} \Theta)^{2}\right) \omega^{n-2}=c \omega^{n}$.
- Further generalisations due to Schlitzer-Stoppa and Scarpa-Stoppa.
- The vortex ansatz provides examples of solutions by reducing the PDE to a system on a Riemann surface. AC,GF, GP, P, and Yao studied the resulting gravitating vortex equations. (A special case is EB equations studied extensively by Yang.) Ghosh proved existence for the CYM equations, and almost HE equations (Gieseker stability) with this ansatz.


## Vector bundle Monge-Ampére equation

## Vector bundle Monge-Ampére equation

- Mumford stability involves


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics.


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability,


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions:


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE?


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question,


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes)


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes) is the deformed Hermitian-Yang-Mills (dHYM) equation (Jacob, Yau, Collins, Xie, Han, Jin, G. Chen, Chu, Lee, Takahashi, Ballal, P, etc).


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes) is the deformed Hermitian-Yang-Mills (dHYM) equation (Jacob, Yau, Collins, Xie, Han, Jin, G. Chen, Chu, Lee, Takahashi, Ballal, P, etc).
- The dHYM equation


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes) is the deformed Hermitian-Yang-Mills (dHYM) equation (Jacob, Yau, Collins, Xie, Han, Jin, G. Chen, Chu, Lee, Takahashi, Ballal, P, etc).
- The dHYM equation in the case of surfaces boils down to


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes) is the deformed Hermitian-Yang-Mills (dHYM) equation (Jacob, Yau, Collins, Xie, Han, Jin, G. Chen, Chu, Lee, Takahashi, Ballal, P, etc).
- The dHYM equation in the case of surfaces boils down to the usual Monge-Ampère equation


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes) is the deformed Hermitian-Yang-Mills (dHYM) equation (Jacob, Yau, Collins, Xie, Han, Jin, G. Chen, Chu, Lee, Takahashi, Ballal, P, etc).
- The dHYM equation in the case of surfaces boils down to the usual Monge-Ampère equation $\omega_{\phi}^{n}=\eta$.


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes) is the deformed Hermitian-Yang-Mills (dHYM) equation (Jacob, Yau, Collins, Xie, Han, Jin, G. Chen, Chu, Lee, Takahashi, Ballal, P, etc).
- The dHYM equation in the case of surfaces boils down to the usual Monge-Ampère equation $\omega_{\phi}^{n}=\eta$.
- Motivated by this observation,


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes) is the deformed Hermitian-Yang-Mills (dHYM) equation (Jacob, Yau, Collins, Xie, Han, Jin, G. Chen, Chu, Lee, Takahashi, Ballal, P, etc).
- The dHYM equation in the case of surfaces boils down to the usual Monge-Ampère equation $\omega_{\phi}^{n}=\eta$.
- Motivated by this observation, we came up with


## Vector bundle Monge-Ampére equation

- Mumford stability involves the first Chern class and corresponds to HE metrics. As a consequence of solvability, the Kobayashi-Lübke-Bogomolov-Miyaoka-Yau inequality is met.
- Natural questions: Is there a (non-asymptotic) stability condition involving higher Chern classes that corresponds to a PDE? If so, does its solvability imply inequalities between classes other than $c_{1}, c_{2}$ ?
- For the second question, a line bundle version of it (higher Chern character classes) is the deformed Hermitian-Yang-Mills (dHYM) equation (Jacob, Yau, Collins, Xie, Han, Jin, G. Chen, Chu, Lee, Takahashi, Ballal, P, etc).
- The dHYM equation in the case of surfaces boils down to the usual Monge-Ampère equation $\omega_{\phi}^{n}=\eta$.
- Motivated by this observation, we came up with the vector bundle Monge-Ampère equation:

$$
\left(\sqrt{-1} \Theta_{h}\right)^{n}=\eta / d .
$$

## Vector bundle Monge-Ampére equation

## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation.


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation.


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability:


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf,


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$.


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$,


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive,


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity:


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity: $\int_{M} \sum_{k=0}^{n-1} \operatorname{tr}\left(a\left(\frac{i \Theta_{A}}{2 \pi}\right)^{k} a^{\dagger}\left(\frac{i \Theta_{A}}{2 \pi}\right)^{n-1-k}\right)>0$.


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta I d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity: $\int_{M} \sum_{k=0}^{n-1} \operatorname{tr}\left(a\left(\frac{i \Theta_{A}}{2 \pi}\right)^{k} a^{\dagger}\left(\frac{i \Theta_{A}}{2 \pi}\right)^{n-1-k}\right)>0$.
- We came up with a


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity: $\int_{M} \sum_{k=0}^{n-1} \operatorname{tr}\left(a\left(\frac{i \Theta_{A}}{2 \pi}\right)^{k} a^{\dagger}\left(\frac{i \Theta_{A}}{2 \pi}\right)^{n-1-k}\right)>0$.
- We came up with a moment map interpretation (and a


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity: $\int_{M} \sum_{k=0}^{n-1} \operatorname{tr}\left(a\left(\frac{i \Theta_{A}}{2 \pi}\right)^{k} a^{\dagger}\left(\frac{i \Theta_{A}}{2 \pi}\right)^{n-1-k}\right)>0$.
- We came up with a moment map interpretation (and a pre-quantum line bundle).


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity: $\int_{M} \sum_{k=0}^{n-1} \operatorname{tr}\left(a\left(\frac{i \Theta_{A}}{2 \pi}\right)^{k} a^{\dagger}\left(\frac{i \Theta_{A}}{2 \pi}\right)^{n-1-k}\right)>0$.
- We came up with a moment map interpretation (and a pre-quantum line bundle).
- Chern class inequality for


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity: $\int_{M} \sum_{k=0}^{n-1} \operatorname{tr}\left(a\left(\frac{i \Theta_{A}}{2 \pi}\right)^{k} a^{\dagger}\left(\frac{i \Theta_{A}}{2 \pi}\right)^{n-1-k}\right)>0$.
- We came up with a moment map interpretation (and a pre-quantum line bundle).
- Chern class inequality for rank-2 bundles on surfaces


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity: $\int_{M} \sum_{k=0}^{n-1} \operatorname{tr}\left(a\left(\frac{i \Theta_{A}}{2 \pi}\right)^{k} a^{\dagger}\left(\frac{i \Theta_{A}}{2 \pi}\right)^{n-1-k}\right)>0$.
- We came up with a moment map interpretation (and a pre-quantum line bundle).
- Chern class inequality for rank-2 bundles on surfaces with MA-positively curved solutionsof vbMA:


## Vector bundle Monge-Ampére equation

- For $n=1$ : HE equation. For $r=1$ : MA equation. Thus we expect stability and positivity to be necessary.
- MA-stability: If $S$ is saturated coherent subsheaf, then $\frac{c h_{n}(S)}{r k(S)}<\frac{c h_{n}(E)}{r k(E)}$. P: If $(E, h)$ is an indecomposable Hermitian holomorphic rank-2 bundle on a smooth projective surface $M$, such that $\left(i \Theta_{h}\right)^{2}=\eta l d$ where $\eta>0$ and $\operatorname{tr}\left(i \Theta_{h}\right)$ is positive, then $E$ is MA-stable.
- MA-positivity: $\int_{M} \sum_{k=0}^{n-1} \operatorname{tr}\left(a\left(\frac{i \Theta_{A}}{2 \pi}\right)^{k} a^{\dagger}\left(\frac{i \Theta_{A}}{2 \pi}\right)^{n-1-k}\right)>0$.
- We came up with a moment map interpretation (and a pre-quantum line bundle).
- Chern class inequality for rank-2 bundles on surfaces with MA-positively curved solutionsof vbMA: $c_{1}^{2}(E)-4 c_{2}(E) \leq 0$.


## Vector bundle Monge-Ampére equation

## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces:


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture:


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$,


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to the vbMA for


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to the vbMA for a right-hand-side.


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to the vbMA for a right-hand-side.
- Provides an approach to


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to the vbMA for a right-hand-side.
- Provides an approach to the Griffiths conjecture for


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to the vbMA for a right-hand-side.
- Provides an approach to the Griffiths conjecture for rank-2 stable bundles on surfaces:


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to the vbMA for a right-hand-side.
- Provides an approach to the Griffiths conjecture for rank-2 stable bundles on surfaces: Ballal proved that


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to the vbMA for a right-hand-side.
- Provides an approach to the Griffiths conjecture for rank-2 stable bundles on surfaces: Ballal proved that MA-positivity is preserved along this continuity path.


## Vector bundle Monge-Ampére equation

- Rank-2 on surfaces: Nakano and dual-Nakano positivity imply MA-positivity which implies Griffiths positivity.
- Griffiths conjecture: An ample bundle admits a Griffiths-positively curved metric. (Still open beyond Riemann surfaces.)
- P: If $E$ is Mumford stable w.r.t $L$, then $E \otimes L^{k}$ admits an MA-positive solution to the vbMA for a right-hand-side.
- Provides an approach to the Griffiths conjecture for rank-2 stable bundles on surfaces: Ballal proved that MA-positivity is preserved along this continuity path. So "only" a priori estimates are left.


## Vortex Monge-Ampére equation

## Vortex Monge-Ampére equation

- With the vortex ansatz,


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation:


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$.

## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough,

## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

- Ghosh generalised the estimates


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

- Ghosh generalised the estimates to general equations that also include


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

- Ghosh generalised the estimates to general equations that also include the CYM equations with the vortex ansatz.


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

- Ghosh generalised the estimates to general equations that also include the CYM equations with the vortex ansatz.
- Currently we (with Ballal) are


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

- Ghosh generalised the estimates to general equations that also include the CYM equations with the vortex ansatz.
- Currently we (with Ballal) are studying a higher-dimensional vortex MA equation.


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

- Ghosh generalised the estimates to general equations that also include the CYM equations with the vortex ansatz.
- Currently we (with Ballal) are studying a higher-dimensional vortex MA equation. Leads to a fully nonlinear system.


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

- Ghosh generalised the estimates to general equations that also include the CYM equations with the vortex ansatz.
- Currently we (with Ballal) are studying a higher-dimensional vortex MA equation. Leads to a fully nonlinear system. Openness is done.


## Vortex Monge-Ampére equation

- With the vortex ansatz, vbMA reduces to the following equation: $\sqrt{-1} \Theta_{h}=\left(1-|\phi|_{h}^{2}\right) \frac{\mu e^{f} \omega_{\Sigma}+\sqrt{-1} \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{h}^{2}\right)\left(2+2 r_{2}-|\phi|_{h}^{2}\right)}$ where $f$ is given and $\mu$ is a constant.
- We proved existence and uniqueness using the method of continuity

$$
\sqrt{-1} \Theta_{h_{t}}=\left(1-|\phi|_{t}^{2}\right) \frac{\mu u^{1-t} \omega_{\Sigma}+\sqrt{-1} t \nabla^{1,0} \phi \wedge \nabla^{0,1} \phi^{\dagger}}{\left(2 r_{2}+|\phi|_{t}^{2}\right)\left(2+2 r_{2}-|\phi|_{t}^{2}\right)}
$$

where $u=\frac{2 r_{2}\left(2 r_{2}+2\right)}{\mu\left(1-|\phi|_{0}^{2}\right)}$. Surprisingly enough, uniqueness and openness proved to be hardest.

- Ghosh generalised the estimates to general equations that also include the CYM equations with the vortex ansatz.
- Currently we (with Ballal) are studying a higher-dimensional vortex MA equation. Leads to a fully nonlinear system. Openness is done. Closedness appears nastily difficult!


## An advertisement for systems of PDE and the vortex ansatz

## An advertisement for systems of PDE and the vortex ansatz

- KE is dead!


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al),


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle,


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc.


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.
- Rewards exist!


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.
- Rewards exist! Demailly proposed several PDE-based approaches towards


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.
- Rewards exist! Demailly proposed several PDE-based approaches towards the Griffiths conjecture in its full generality.


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.
- Rewards exist! Demailly proposed several PDE-based approaches towards the Griffiths conjecture in its full generality. All are fully nonlinear systems.


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.
- Rewards exist! Demailly proposed several PDE-based approaches towards the Griffiths conjecture in its full generality. All are fully nonlinear systems.
- The vortex ansatz is a fertile testing ground.


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.
- Rewards exist! Demailly proposed several PDE-based approaches towards the Griffiths conjecture in its full generality. All are fully nonlinear systems.
- The vortex ansatz is a fertile testing ground. For instance, Mandal recently proved


## An advertisement for systems of PDE and the vortex ansatz

- KE is dead! Long live cscK!
- The subject is branching out into CY manifolds (Tosatti et al), the symplectic side (Minimal surfaces (Székelyhidi), Lagrangian submanifolds and mirror symmetry (Rubinstein, Collins, Yau, etc), and into systems of PDE (Dervan, Stoppa, GF, etc).
- Systems of fully nonlinear PDE have not been studied much and are resistant to techniques like the maximum principle, Evans-Krylov theory, etc. A wide gap in technology available.
- Rewards exist! Demailly proposed several PDE-based approaches towards the Griffiths conjecture in its full generality. All are fully nonlinear systems.
- The vortex ansatz is a fertile testing ground. For instance, Mandal recently proved the feasibility of some of Demailly's methods for the vortex bundle.


## Thank you



