Future Flavours, ICTS 2022
Lepton Flavour Violation: Exercises

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Problem 1. Find naive dimensional analysis (NDA) predictions for the branching ratios in the Standard Model for the decay modes: $\tau \to \mu\gamma$, $\tau \to e\gamma$, $\tau \to 3\mu$, $\tau \to 3e$.

(a) First find the NDA predictions for the amplitudes for the main SM decay channels of μ and τ. Compare the predicted rates with the measured μ and τ decay times. (Measured mean lifetimes: $\tau_\mu = 2.2 \times 10^{-6}$ s and $\tau_\tau = 2.9 \times 10^{-13}$ s.)

(b) Find the NDA predictions for loop induced $\tau \to \mu\gamma$, $\tau \to e\gamma$, $\tau \to 3\mu$, $\tau \to 3e$ amplitudes and then the corresponding branching ratios.

(c) What happens with these branching ratios in the limit of vanishing neutrino masses?

Problem 2. Imagine that there is a heavy vector boson with flavor off-diagonal couplings, such that it has the interaction Lagrangian in the charge lepton mass basis:

$$\mathcal{L} \supset g_{ij}\, \bar{l}_i \gamma^\mu l_j\, Z'_\mu + \text{h.c.}\,, \tag{1}$$

where $l_i = \{e, \mu, \tau\}$.

(a) Find the branching ratios for $\tau \to 3\mu$, $\tau \to 2\mu + e$, $\tau \to 3e$, $\mu \to 3e$ mediated by Z' exchange. Convert the present experimental bounds on this branching ratios to bounds on $m_{Z'}/g_{ij}$.

(b) Use NDA to make predictions for $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$ decay widths induced by a loop exchange of a Z'. Discuss different cases where the chirality flip comes from the external fermion legs or from internal fermions. Convert the current experimental bounds to bounds on $m_{Z'}/g_{ij}$.

Problem 3. Perform analysis similar to Problem 2 but for an axion like particle (ALP) with flavor violating couplings:

$$\mathcal{L} \supset A_{ij}\, \frac{\partial_\mu a}{f}\, \bar{l}_i \gamma^\mu \gamma^5 l_j + \text{h.c.}\,, \tag{2}$$

where mass of ALP, $m_a \ll m_e$.

Problem 4. Consider that the gauge interactions given in Eq. (1) arise in a $U(1)$ model in which the three generations of SM lepton doublets L_{Li} have charges q_{Li} and the charged lepton singlets l_{Ri} have charges q_{Ri}. With the general mass term for the charged leptons,

(a) show that $g_{ij} \propto \delta_{ij}$ if q_{Li} and q_{Ri} are universal.

(b) for what kind of structures of q_{Li} and q_{Ri} and the mass term, only the off-diagonal couplings g_{ij} can be made non-vanishing?