HYBRID HYDRODYNAMIC ATTRACTORS, BOTTOM-UP THERMALIZATION AND MORE..

based on arXiv:2006.09383 with, Sukrut Mondkar, Ayan Mukhopadhyay, Anton Rebhan, and Alexander Soloviev

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Overview

Introduction

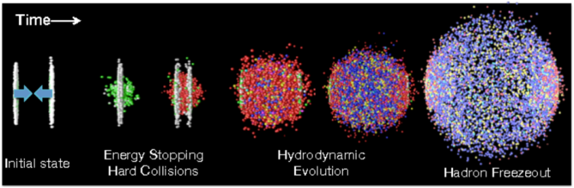


Figure: Time evolution of matter in Heavy Ion Collision, Source: NAYAK T.Heavy ions: Results from the Large Hadron Collider. doi:10.1007/s12043-012-0373-7.

- Different stages of Heavy-ion collision involves different forms of hadronic matter having wide energy scale.
- QGP formed in the intermediate stage behave's as perfect fluid with low ratio of shear viscosity to entropy density ($\eta/s \lesssim 0.2$).
- Experimentally shown, low p_T hadrons has collective behaviour.
- The dynamics of collective motion was successfully studied by the effective theory of hydrodynamics.

Hydrodynamic Attractors

- The degrees of freedom of relativistic hydrodynamics are $\epsilon(t, \vec{x})$ and $u^{\mu}(t, \vec{x})$

- Constitutive relation, $T^{\mu\nu} = T^{\mu\nu}(\epsilon, u^{\mu}, \text{gradients})$

- First-order viscous hydrodynamics: $T^{\mu\nu} = T^{\mu\nu}_{(0)} + \Pi^{\mu\nu}$

- $\nabla_{\mu} T^{\mu\nu} = 0$, not hyperbolic PDEs and thus its not a well defined initial value problem.

- Propogation of linear perturbation about equilibrium,

$$v_g \propto k$$

- Resummation of hydrodynamic expansion to all orders in derivative is needed to have a causal and consistent effective theory.
- One phenomenological approach to it is the Müller-Israel-Stewart Mechanism.
- **MIS theory**, promoting $\Pi^{\mu\nu}$ to an independent dynamical variable,

$$\tau_{\Pi}(u.\partial + 1)\Pi^{\mu\nu} = -\eta\sigma^{\mu\nu} + \ldots \tag{1}$$

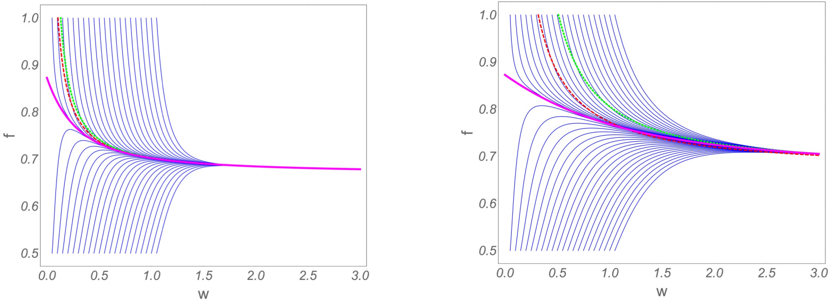
τ_{Π} is the relaxation time and $\Pi^{\mu\nu}$ is the shear-stress tensor

- Propagation of linear perturbation about equilibrium at $k \to \infty$ [arxiv: 1707.02282] ,

$$v_{max} = \frac{1}{\sqrt{3}}\sqrt{1 + 4\frac{\eta/s}{T\tau_{\Pi}}}$$

- This implies, $\tau_{\Pi} T > 2\eta/s$ (valid for all microscopic theory)
- For $\tau_{\Pi} \to 0$, Navier-Stokes equation is recovered
- Need for causal theory incorporates very large momenta accompanied by damping modes(non-hydrodynamic modes).

- The relaxation mechanism is the key to the emergence of hydrodynamic attractor.
- Attractors can be obtained numerically, by resumming hydrodynamic expansion via transseries and other approaches also.
- Attractor is well-behaved at arbitrarily early time.
- In the paper [arxiv: 1503.07514] by Heller and Spalinski, strong evidence of hydrodynamic as a universal attractor has been motivated.



(a) Strongly coupled system ($\mathcal{N} = 4$SYM)

(b) weakly coupled system (η/s and $C_{\tau\pi}$ increased by 3)

Figure: Numerically obtained attractor solution
Numerical solutions for other initial condition
First order hydrodynamics
Second order hydrodynamics

- The weaker system evolves towards the attractor curve at large value of w,

$$w = \tau T \text{ and } f = \tau \dot{w}/w \text{ [arxiv: 1103.3452]}$$

- Truncated gradient expansion evolves to the attractor.

- The hydrodynamic attractors establishes the applicability of hydrodynamic far from equilibrium.

- Evolution of a system to an attractor adds more precise meaning to hydrodynamization of the system [arxiv: 1703.09681, arxiv: 1103.3452].

Atttractor from ADS/CFT

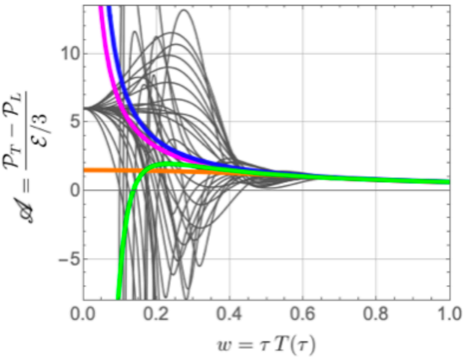


Figure: The plot shows trajectories of 29 different initial condition which evolves to attractor curve.

1st order hydro, 2nd order hydro and 3rd order hydro [arxiv: 2005.12299] .

Why Semi-holographic approach?

- QCD is characterized by asymptotic freedom, running coupling (α_s large in IR and small in UV).
- HIC gives an opportunity to enrich the understanding of QCD.
- To some extent perturbative QCD and kinetic theory describes the behaviour of hard quasi-particle(large p_T momenta).
- Soft gluons (low p_T momenta) exhibit strong coupling, need non-perturbative description.

- Semi-holographic approach incorporates both perturbative and non-perturbative descriptions consistently in one framework [arxiv: 1001.5049, arxiv: 1306.3941].
- It has been applied to study out-of-equilibrium dynamics in HIC by Mukhopadhyay and Iancu [arxiv: 1410.6448].
- Developing an effective description of a full hybrid system from the coarse-grained description of weakly-coupled(perturbative) and strongly coupled(non-perturbative) degrees of freedom.
- Coupling of the individual sectors in a consistent manner is the concern —— **Democratic Coupling**, consistent with the principle of thermodynamics, statistical mechanics and Wilson renormalization group RG flow [arxiv:1805.05213, arxiv: 1701.01229].
  (Talk by Mukhopadhyay)

Essence:

- **Coupling of two fluid in democratic manner.**
- **Emergence of hydrodynamic attractor in a composite system??**
- **Bottom-up thermalization is universal in such a system.**
- **Well-behaved 2-D attractor surface at early time.**
- **Full system as a Single fluid??**
- **Insight into small and large system.**

Caveat

- In the toy model of semi-holographic glasma, democratic coupling of kinetic sector with dynamical black hole exhibits irreversible energy transfer to the soft sector over a larger time [arxiv: 1806.01850].
- In our drastically simplified two fluid model there is no such universal way of energy transfer.
- Model can capture only certain aspect of evolution described by semi-holography.
- Instead of coupling two MIS, coupling of black hole with kinetic theory could have been more specific to semi-holography.

Metric coupling

- Physical background metric of the full system **F**: $g_{\mu\nu}^{(B)}$
- The two subsystems interacts by promoting effective metric of each sector as a functional of the operator of the other sector.
  - System **W**, lives in the effective metric :$g_{\mu\nu}[\tilde{t}^{\gamma\delta}]$
  - System **S**, lives in the effective metric:$\tilde{g}_{\mu\nu}[t^{\alpha\beta}]$
- The subsystems are closed with respect to their individual metrics but can exchange energy and momentum defined with respect to $g_{\mu\nu}^{(B)}$.

Lowest order metric coupling:

$$g_{\mu\nu} = g^{(B)}_{\mu\nu} + \left(\gamma\, g^{(B)}_{\mu\alpha} \tilde{t}^{\alpha\beta} g^{(B)}_{\beta\nu} + \tilde{\gamma}\; \tilde{t}^{\alpha\beta} g^{(B)}_{\alpha\beta}\; g^{(B)}_{\mu\nu}\right) \frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(B)}}}$$

$$\tilde{g}_{\mu\nu} = g^{(B)}_{\mu\nu} + \left(\gamma\, g^{(B)}_{\mu\alpha} t^{\alpha\beta} g^{(B)}_{\beta\nu} + \tilde{\gamma}\; t^{\alpha\beta} g^{(B)}_{\alpha\beta}\; g^{(B)}_{\mu\nu}\right) \frac{\sqrt{-g}}{\sqrt{-g^{(B)}}} \qquad (2)$$

γ, $\tilde{\gamma} = -r\,\gamma$ are the coupling constant with mass dimension -4.

For causality and UV completeness of the theory, $\gamma > 0$ and $r > 1$ [arxiv:1805.05213].

- No prior knowledge of microscopic description of the sub-systems is needed.
- The effective metric tensors encode the interactions between the two-subsystems
- Coupling satisfies the Ward identities of the individual sector and demands conservation of the full system energy momentum tensor in the physical background.

$$\nabla_{\mu}^{(B)} T^{\mu}_{\nu} = 0, \quad \nabla_{\mu} t^{\mu}_{\nu} = 0 \text{ and } \nabla_{\mu} \tilde{t}^{\mu}_{\nu} = 0$$

- The full dynamics can be obtained by solving the subsystem in a self-consistent way [arxiv: 1701.01229].

Full-EM Tensor

$$\begin{aligned} T^{\mu}{}_{\nu} &= \frac{1}{2}[(t^{\mu}{}_{\nu} + t_{\nu}{}^{\mu})\frac{\sqrt{-g}}{\sqrt{-g^{(\mathrm{B})}}} + (\tilde{t}^{\mu}{}_{\nu} + \tilde{t}_{\nu}{}^{\mu})\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(\mathrm{B})}}}] + \Delta K \delta^{\mu}_{\nu} \\ &=: T_{1}^{\mu}{}_{\nu}(\mathcal{E}_1, \mathcal{P}_1) + T_{2}^{\mu}{}_{\nu}(\mathcal{E}_2, \mathcal{P}_2) + T^{\mu}{}_{\nu,\mathrm{int}} \end{aligned} \tag{3}$$

where

$$\begin{aligned} \Delta K = & -\frac{\gamma}{2}\left(t^{\rho\alpha}\frac{\sqrt{-g}}{\sqrt{-g^{(\mathrm{B})}}}\right)g^{(\mathrm{B})}_{\alpha\beta}\left(\tilde{t}^{\beta\sigma}\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(\mathrm{B})}}}\right)g^{(\mathrm{B})}_{\sigma\rho} \\ & -\frac{\tilde{\gamma}}{2}\left(t^{\alpha\beta}g^{(\mathrm{B})}_{\alpha\beta}\frac{\sqrt{-g}}{\sqrt{-g^{(\mathrm{B})}}}\right)\left(\tilde{t}^{\sigma\rho}g^{(\mathrm{B})}_{\sigma\rho}\frac{\sqrt{-\tilde{g}}}{\sqrt{-g^{(\mathrm{B})}}}\right). \end{aligned} \tag{4}$$

Hybrid Attractor

- In [arxiv:1805.05213], the coupling of two fluids (hybrid fluid model) analogous to the semi-holographic description has been studied in thermal equilibrium.
- Study of non-equilibrium evolution of viscous hybrid fluid model in a boost-invariant Bjorken-flow.
- Milne coordinate: (τ, x, y, ζ)

  $\tau = \sqrt{t^2 - z^2}$ is the proper time and $\zeta = \mathrm{arctan}(z/t)$ is the rapidity.
- Background metric: $g_{\mu\nu}^{(B)} = \mathrm{diag}(-1, 1, 1, \tau^2)$
  W: $g_{\mu\nu} = \mathrm{diag}(-a^2, b^2, b^2, c^2)$ **S**: $\tilde{g}_{\mu\nu} = \mathrm{diag}(-\tilde{a}^2, \tilde{b}^2, \tilde{b}^2, \tilde{c}^2)$

- The fluid sectors has been characterized by relaxation mechanism (MIS theory).

- Energy-momentum tensor ansatz,

$$t^{\mu\nu} = \text{diag}\left(\frac{\epsilon}{a^2}, \frac{P}{b^2}, \frac{P}{b^2}, \frac{P}{c^2}\right) + \pi^{\mu\nu} \tag{5}$$

where, $\pi^{\mu\nu} = \text{diag}(0, \frac{\phi}{2b^2}, \frac{\phi}{2b^2}, \frac{-\phi}{c^2})$ and $\pi^{\zeta}_{\zeta} = -\phi$

- For simplicity, conformal subsystems, $\epsilon = 3P$

- Full system is not conformal, has a non-vanishing trace of the energy-momentum tensor.

- The individual sectors are covariantly conserved with respect to effective metrics, $\nabla_\mu t^{\mu\nu} = 0$

$$\partial_\tau \epsilon + \epsilon \partial_\tau \log(b^{8/3} c^{4/3}) + \phi \partial_\tau \log(b/c) = 0 \tag{6}$$

- MIS equation:

$$\tau_\pi \partial_\tau \phi + \frac{4}{3} \eta \partial_\tau \log(b/c) + [a + \frac{4}{3} \tau_\pi \partial_\tau \log(b^2 c)] \phi = 0 \tag{7}$$

- Four PDEs, there exist a four dimensional phase space spanned by the dynamical variable ϵ , ϕ, $\tilde{\epsilon}$ and $\tilde{\phi}$

- Introducing dimensionless variable (Measure of anisotropy) $\chi := \frac{\phi}{\epsilon + P}$

- The sub-systems are parametrized with different shear viscosity η and relaxation time τ_π to behaves as a strongly coupled and less strongly coupled fluid .

$$\eta = \frac{C_\eta(\epsilon + P)}{T}, \quad \tau_\pi = C_\tau / T$$

- Choice of parameter: $0 < \sigma < \tilde{\sigma} < 1/\sqrt{2}$.

- At $\tau \to 0$, Attractor initial condition

$$\chi = \sqrt{\frac{C_\eta}{C_\tau}} := \sigma, \qquad \tilde{\chi} = \sqrt{\frac{\tilde{C}_\eta}{\tilde{C}_\tau}} := \tilde{\sigma}$$

- Metric coupling of the two fluids give rise to a common attractor.

Attractor curve

- $\epsilon_0 = \epsilon(\tau_0)$ and $\tilde{\epsilon_0} = \tilde{\epsilon}(\tau_0)$, τ_0 is some non-zero reference time.
- Fine tune χ and $\tilde{\chi}$, run the solution backward and forward in time.

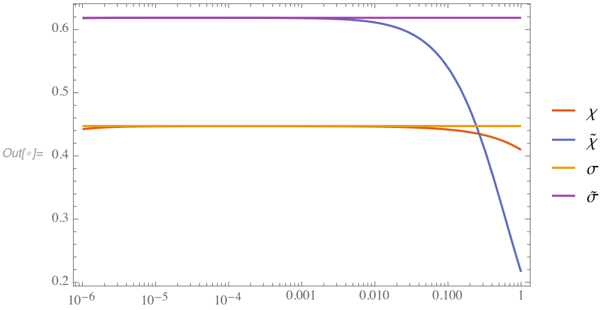


Figure: Numerically determined attractor solution at early time

Attractor curve

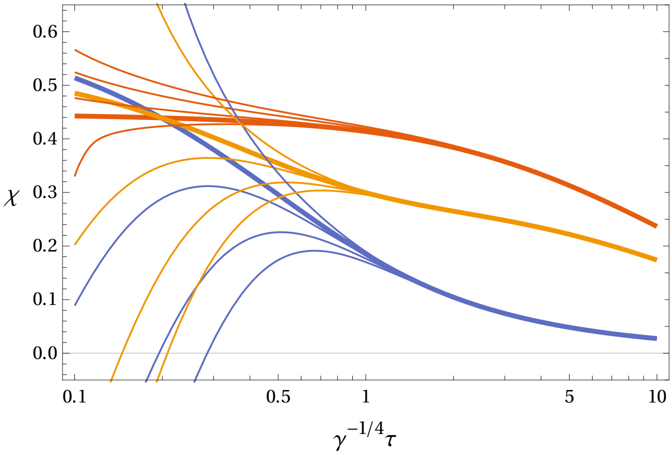


Figure: **Attractor solutions** , thin lines are neighbouring trajectories. *Less viscous system*, *More viscous system* *and* *Full system*

- Solutions with different initial conditions evolves to the attractor.
- Trajectories above the attractor diverges in the early time while below the attractor goes to the negative values of χ.
- The set of attractor solution is a 2-D manifold, parametrized by dimensionless energy densities of the subsystem at some non-zero reference time.
- The attractor surface depends on r, in this case $r = 2$, for cross-over behaviour.
- r closer to 1, the fluid system exhibit first-order transition [arxiv:1805.05213].

- The two components of the full-system do not equilibrate locally with each other even at late time due to lack of sufficient time for interaction.
- The full system can be described by standard hydrodynamic expansion with a shear viscosity,

$$C_\eta^{eff} = \lim_{\tau\to\infty} \mathbf{H}(\tau) = \lim_{\tau\to\infty} \frac{C_\eta \epsilon(\tau)^{4/3} + \tilde{C}_\eta \tilde{\epsilon}(\tau)^{4/3}}{[\epsilon(\tau) + \tilde{\epsilon}(\tau)]^{4/3}} \tag{8}$$

- Hydrodynamic expansion of the energy density $\mathcal{E}$ of the full system :

$$\mathcal{E}^{1/4}\tau = \omega - \frac{2}{3} C_\eta^{eff} + O(\omega^{-1}) \tag{9}$$

where, $\omega = \mathcal{E}_{pf}^{1/4}\tau = (\epsilon_{pf} + \tilde{\epsilon}_{pf})^{1/4}\tau$

Interaction energy and H

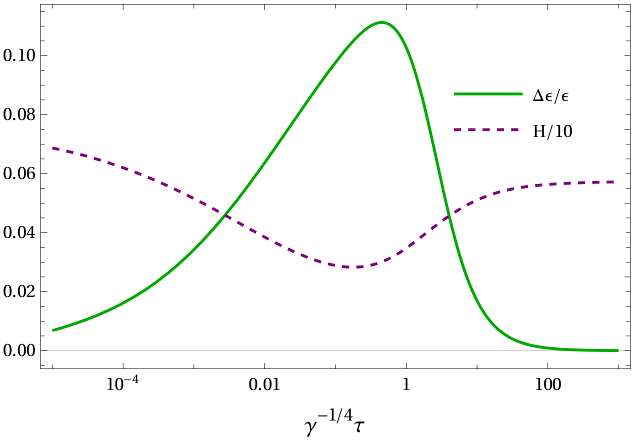


Figure: *Interaction energy of the subsystems over total energy*
Averaged effective shear viscosity

- The interaction energy increases towards the cross-over region and then decays rapidly.
- The full-system along with the subsystem approaches perfect fluid behaviour.
- Effective shear viscosity near the cross-over region lowers and shows a Hadron gas like transition.

- Physical background perspective, as $\tau \to 0$ the contribution to the total energy density,
  - $\mathcal{E}_1 \sim \tau^{4(\sigma-1)/3}$, diverges.
  - For $\tilde{\sigma} > \sigma$, $\mathcal{E}_2/\mathcal{E}_1 \sim \tau^{8(\tilde{\sigma}-\sigma)/3}$ is suppressed.
- **Bottom-Up thermalization!!**: Evolution starts with total energy density concentrated to the weakly coupled sector, redistributed to strongly coupled sector of soft degrees of freedom [0009237,BMSS].
- Away from $\tau \to 0$, the differential equations and the non-linear algebraic equations (coupling equations) can be solved numerically.

Energy density for one attractor curve

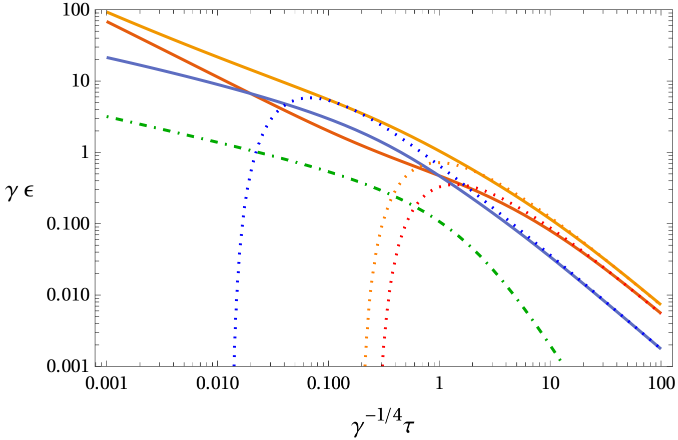


Figure: *Less viscous system*, *More viscous system* *and* *Full system*

· · · *First-order hydrodynamic*

· — · *Interaction energy of the subsystem*

- At initial time total energy is concentrated to hard sector which gets distributed to the soft sector over a stretch of time (around the cross-over region).
- At larger time re-dominance of weak sector can be viewed as hadron gas like transition.
- Anisotropy of sub-system decreases monotonically while the full-system shows a complicated behaviour.
- Weakly coupled sector is strongly anisotropic compared to the soft sector.

Evolution of anisotropy

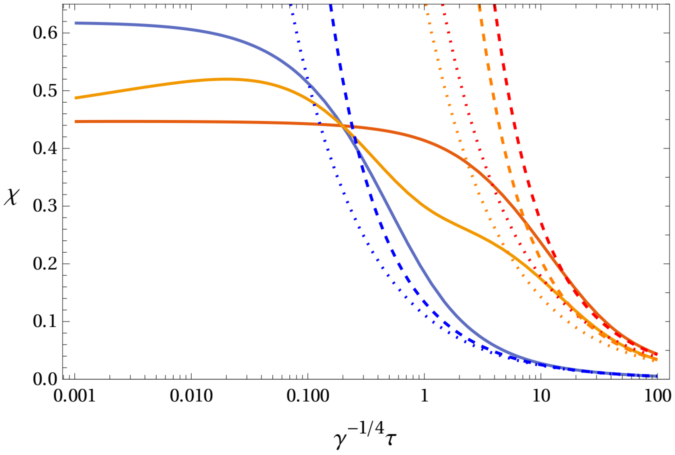


Figure: *Less viscous system*, *More viscous system* *and* *Full system*

$\cdots$ *first-order hydrodynamic*

– – *Second-order hydrodynamics*

- Results have been compared with first order hydrodynamics and second order hydrodynamics.
- Hydrodynamization time: [arxiv: 1609.04803 , arxiv: 1703.09681]

$$\frac{|\Delta P_L|}{P} := \frac{|\phi - \phi_{1st}|}{P} < 0.1, \quad \tau > \tau_{hd}$$

- Above Plots, $\tau_{hd}/\tilde{\tau}_{hd} \approx 5.76$, hydrodynamization time is longer for hard sector than for soft.
- Ratio of hydrodynamization time depends on the initial condition which can give some insight in the small and large system hydrodynamization scenario.

Table

Table: Subsystem hydrodynamization times and the concurrent values of the dimensionless quantities $w = \mathcal{E}_1^{1/4}\tau$, $\tilde{w} = \mathcal{E}_2^{1/4}\tau$, χ, and $\tilde{\chi}$ for three scenarios with different values of $\mathcal{E}_1(1) = \mathcal{E}_2(1) =: \mathcal{E}(1)$, where all dimensionful quantities are given in units of γ. The last column gives the ratio $R_{\rm hd} := \tau_{\rm hd}/\tilde{\tau}_{\rm hd}$.

$\mathcal{E}(1)$	$\tau_{\rm hd}$	$w_{\rm hd}/10$	$\chi_{\rm hd}$	$\tilde{\tau}_{\rm hd}$	$\tilde{w}_{\rm hd}$	$\tilde{\chi}_{\rm hd}$	$R_{\rm hd}$
0.26	12.0	0.609	0.215	2.08	1.42	0.101	5.76
0.32	10.2	0.705	0.203	3.90	2.82	0.0525	2.62
0.052	25.5	0.608	0.210	1.39	0.613	0.211	18.4

Anisotropy, $A = \frac{P_\perp - P_L}{P} \sim 6\chi$

w scales as $\frac{4\pi\eta}{s}$

Summary

- Hydrodynamic expansion resummed to all orders in gradient expansion generates causal evolution for any arbitrary initial condition.
- Democratic coupling allows to work without information of the action principles of the subsystems.
- The hybrid fluid model [arxiv:1805.05213], with MIS equations, is an interesting model for the non-equilibrium dynamics of a two-component system.
- Emergence of a 2-D attractor hypersurface in a four dimensional phase space.

- Boost-invariant expansion provides a model with universal feature of bottom-up themalization provided $\tilde{\sigma} > \sigma$.
- The full system can be described hydrodynamically at late times and the transport coefficient of the full hydro is determined by the attractor curve followed by the system.
- The dependence of hydrodynamization ratio on an extra parameter may account for different hydrodynamization scenario in small and large system produced in HIC.
- Our model is successful in capturing certain aspects of heavy-ion collisions and is open to further generalization like study of first order transition.

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Thank You