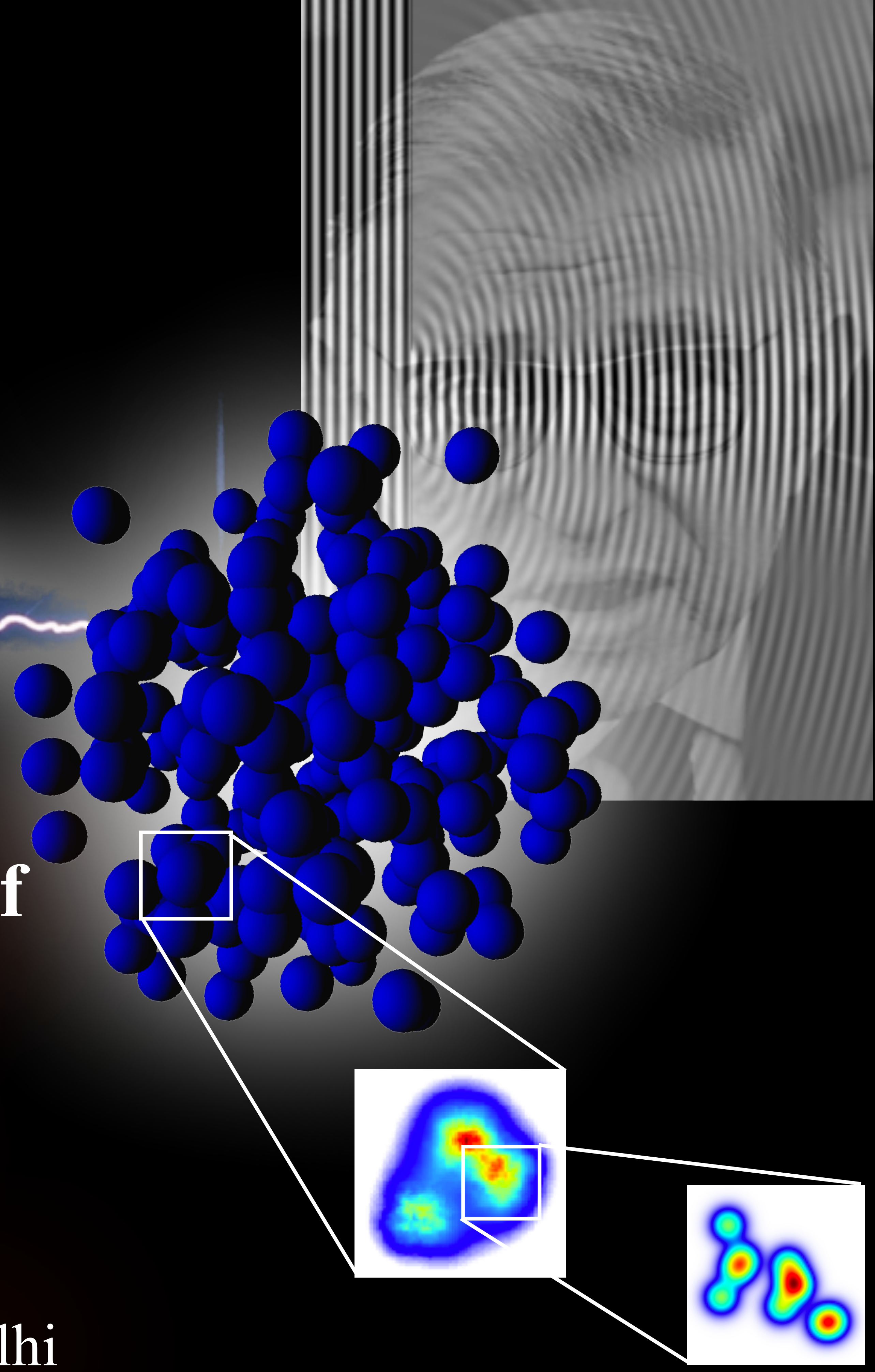
“Small-*x* distributions of hadrons and ions in *Sartre*”

QEIC III, ICTS

February 7, 2024

Tobias Toll

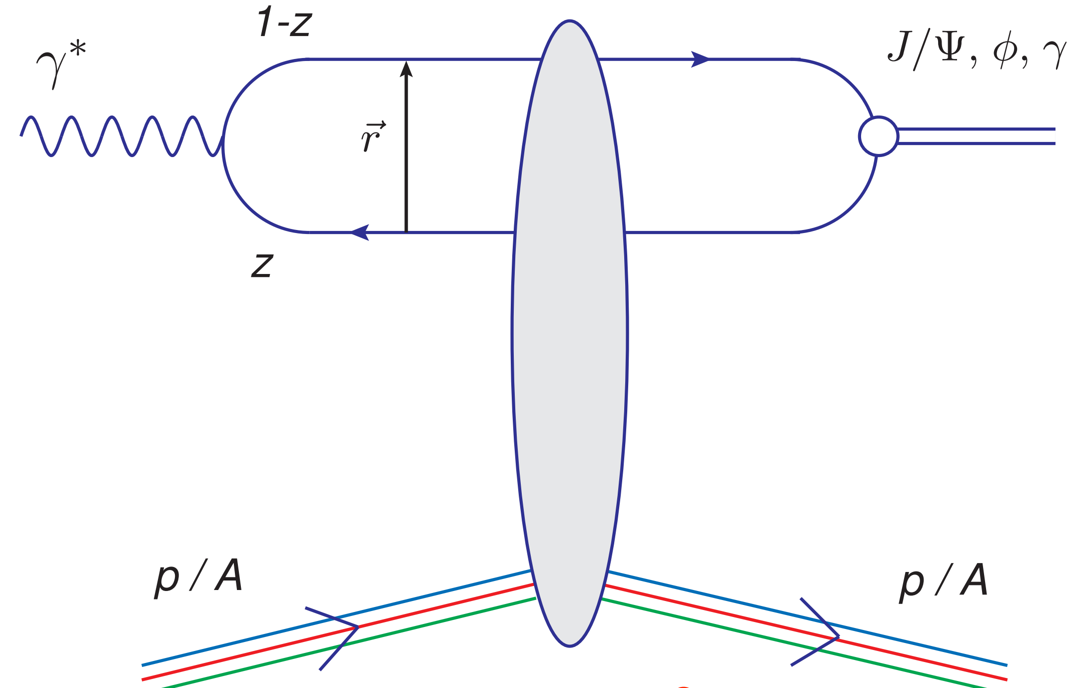
Indian Institute of Technology Delhi



H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

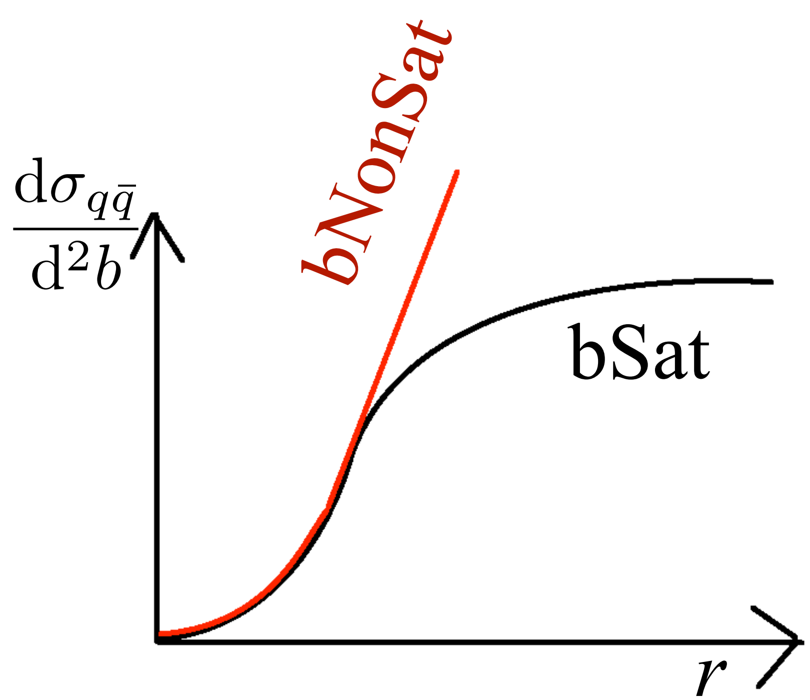
Exclusive diffraction in the Dipole Model

$$\frac{d\sigma^{\gamma^* p \to Vp}}{dt} = \frac{1}{16\pi} \left| \mathscr{A}_{T,L}^{\gamma^* p \to Vp} \right|^2$$



$$t = -\Delta^2$$

$$\mathscr{A}_{T,L}^{\gamma^* p \to Vp}(x_{I\!P}, Q^2, \Delta) = i \int 2\pi r \mathrm{d}r \int \frac{\mathrm{d}z}{4\pi} \int \mathrm{d}^2\vec{b} \left(\Psi_V^* \Psi\right)(r,z) J_0([1-z]r\Delta) e^{-\vec{b}\cdot\vec{\Delta}} \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b})$$



$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)\right)\right]$$

$$\frac{\mathrm{d}\sigma^{\mathrm{nosat}}_{q\bar{q}}}{\mathrm{d}\mathbf{b}} = \frac{\pi^2}{N_C} r^2 \alpha_S(\mu^2) x g(x, \mu^2) T(b)$$

2

H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

Exclusive diffraction in the Dipole Model

$$\mathcal{A}_{T,L}^{\gamma^* p \to Vp}(x_{I\!P}, Q^2, \Delta) = i \int 2\pi r \mathrm{d}r \int \frac{\mathrm{d}z}{4\pi} \int \mathrm{d}^2\vec{b} \left(\Psi_V^* \Psi\right)(r,z) J_0([1-z]r\Delta) e^{-\vec{b}\cdot\vec{\Delta}} \frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b})$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) xg(x,\mu^2) T(b)\right)\right]$$

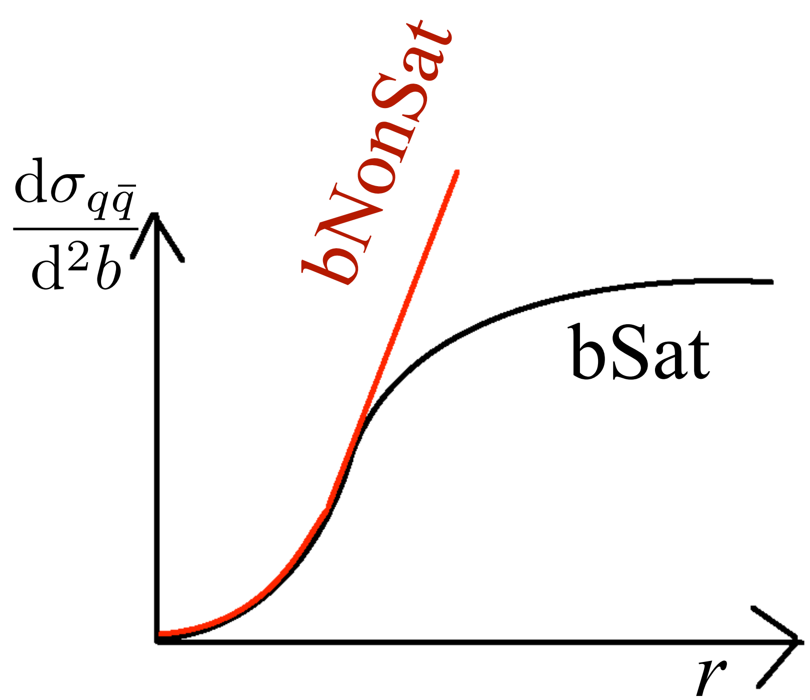
Saturation Scale:

$$Q_S^2 = \frac{2}{r_S^2}$$

$$\frac{1}{2} = \frac{\pi^2}{2N_C} {r_S}^2 \alpha_S(\mu^2(r_S)) xg(x, \mu^2(r_S)) T(b)$$

$$\mu^2(r) = \frac{C}{r^2} + \mu_0^2$$

$$Q_S^2 \simeq T(b)$$



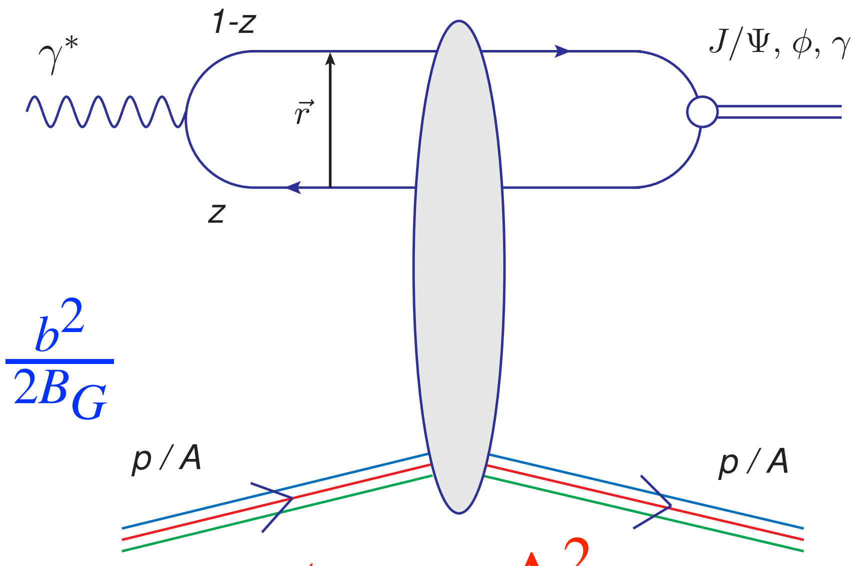
$$\frac{\mathrm{d}\sigma_{\mathrm{q\bar{q}}}^{\mathrm{nosat}}}{\mathrm{d}\mathbf{b}} = \frac{\pi^2}{N_C} r^2 \alpha_S(\mu^2) xg(x,\mu^2) T(b)$$

H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

Exclusive diffraction in the Dipole Model

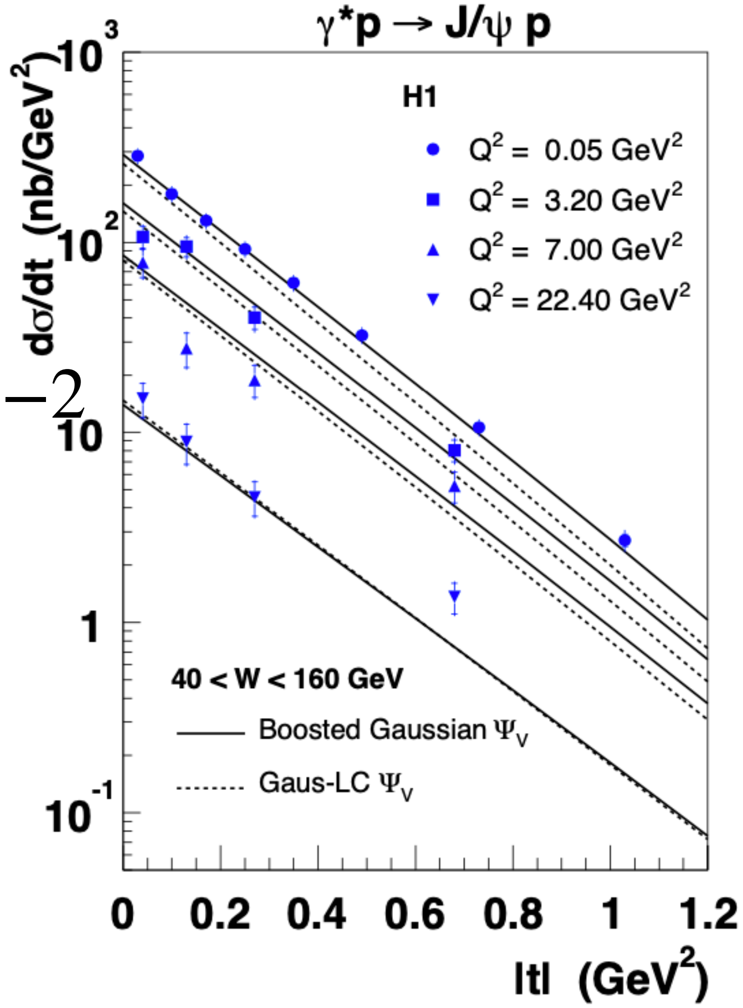
$$\frac{d\sigma_{nosat}}{dt} \propto \left| \mathscr{F}\text{ourier}(T(b)) \right|^2$$

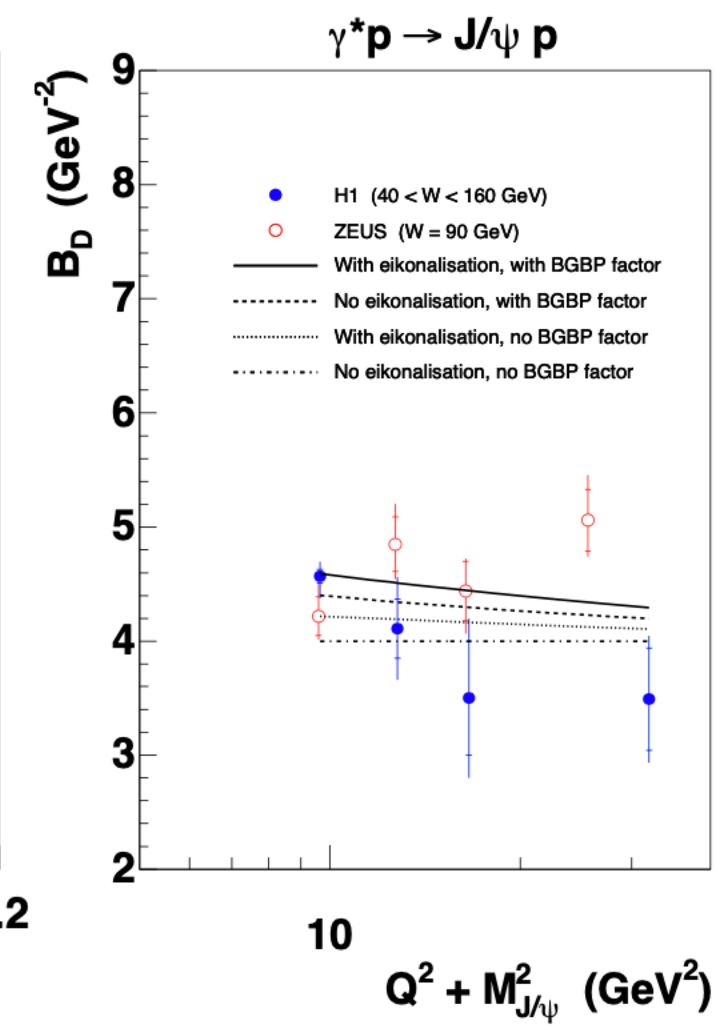
$$\frac{d\sigma}{dt} \propto e^{Bt} = e^{-B\Delta^2} \qquad T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

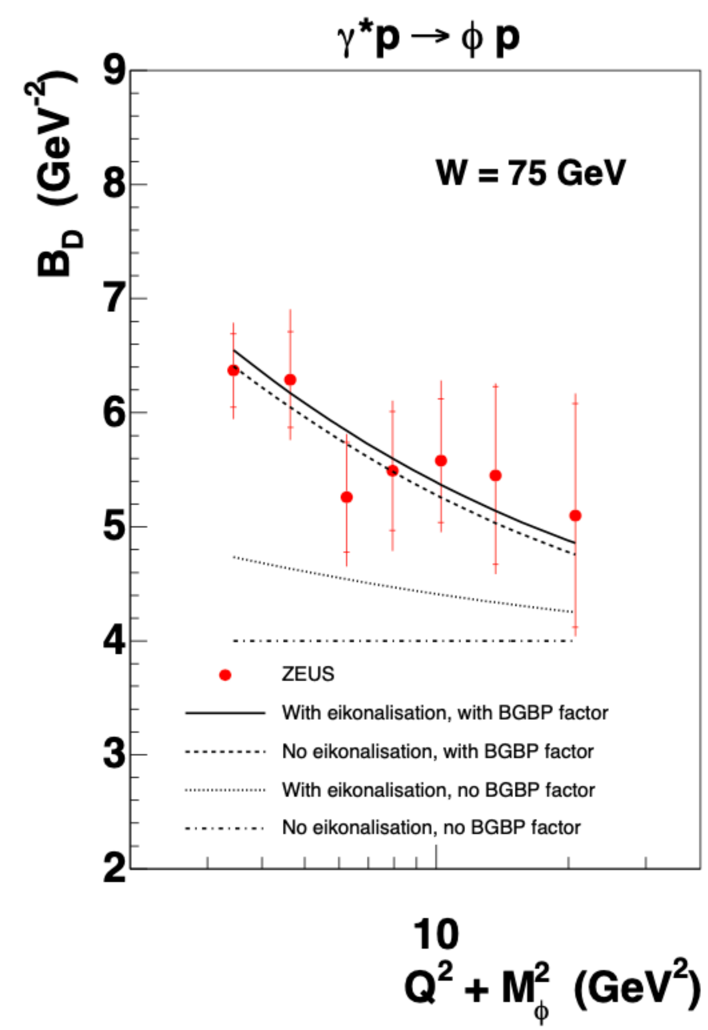


$$t = -\Delta^2$$

$$B_G = 4\ \text{GeV}^{-2}$$







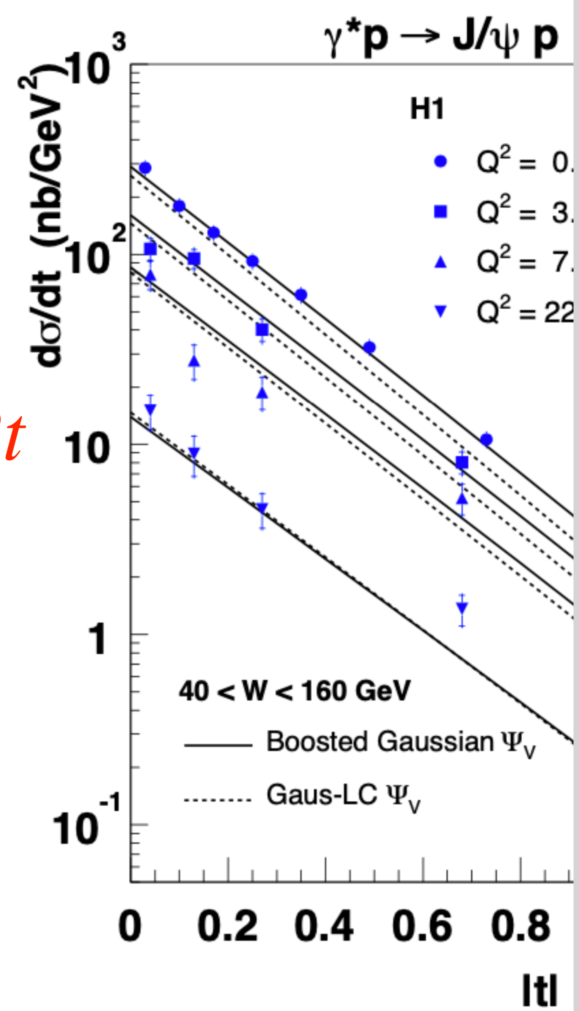
H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016, arXiv: hep-ph/0606272

Exclusive diffraction in the Dipole Model

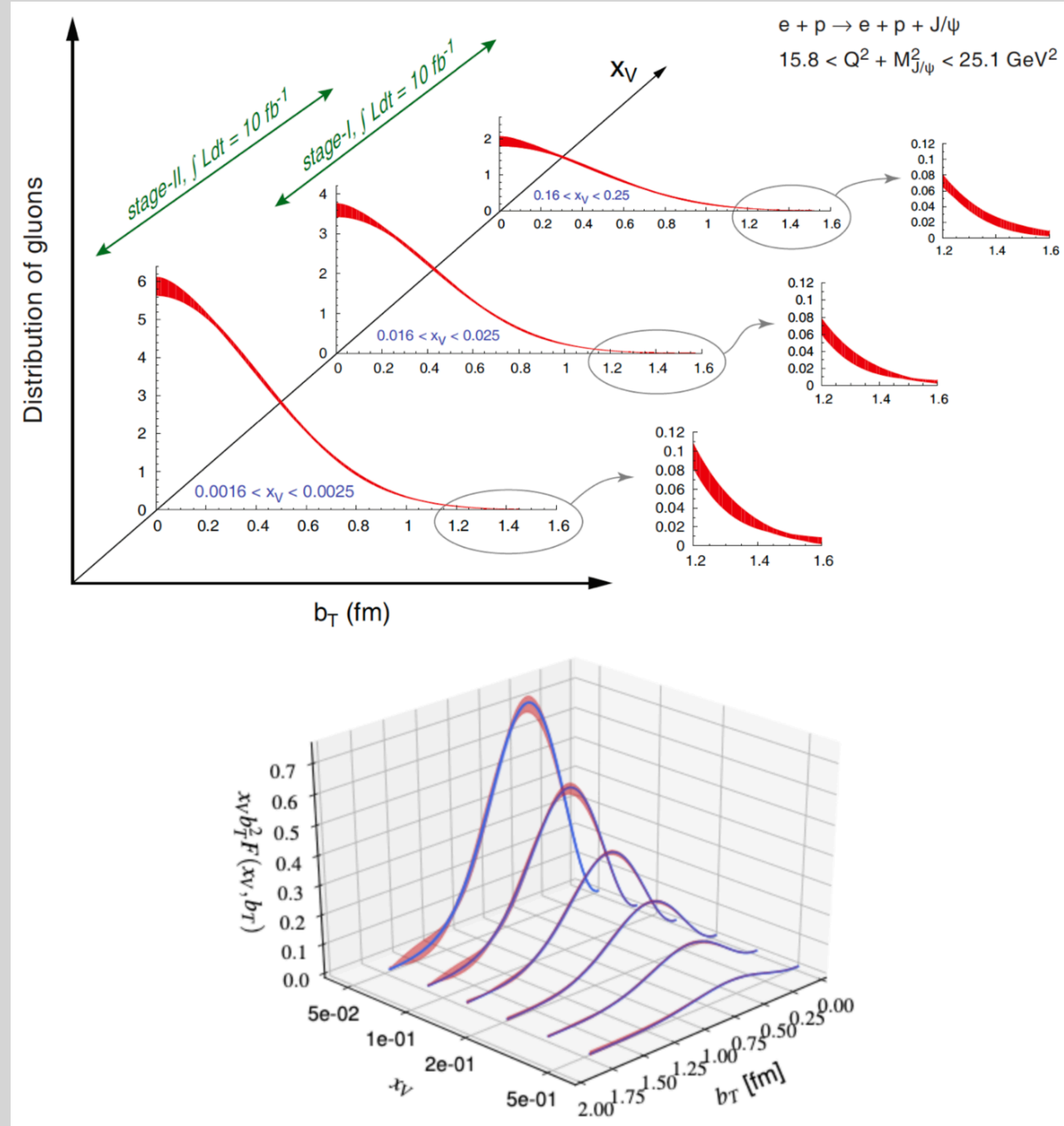
$$\frac{d\sigma}{dt} \propto \left| \mathscr{F}\text{ourier}(T(b)) \right.$$

$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

$$\frac{d\sigma}{dt} \propto e^{-Bt}$$

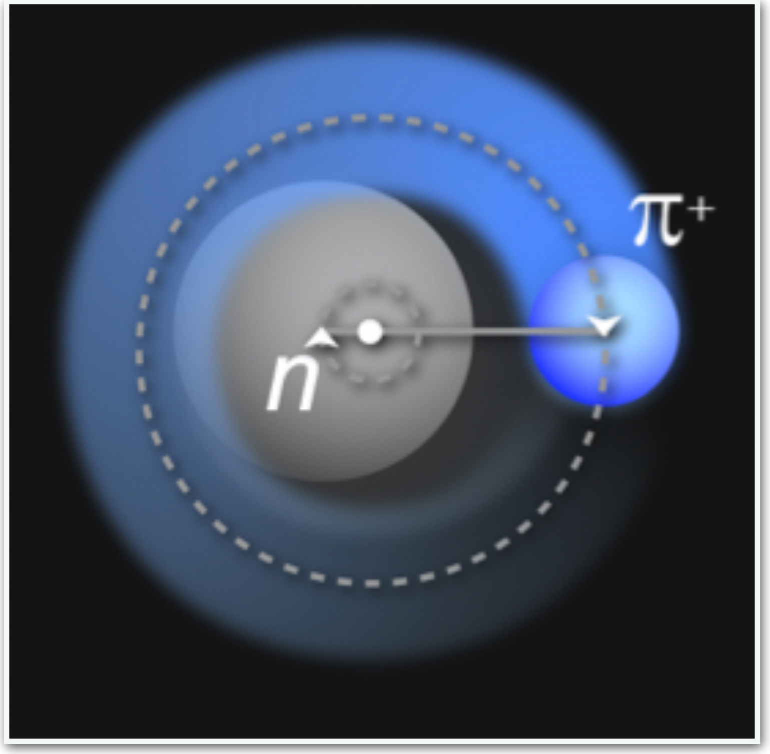


GPDs with EIC:



Nucl. Phys. A 1026 (2022) ; arXiv:2103.05419

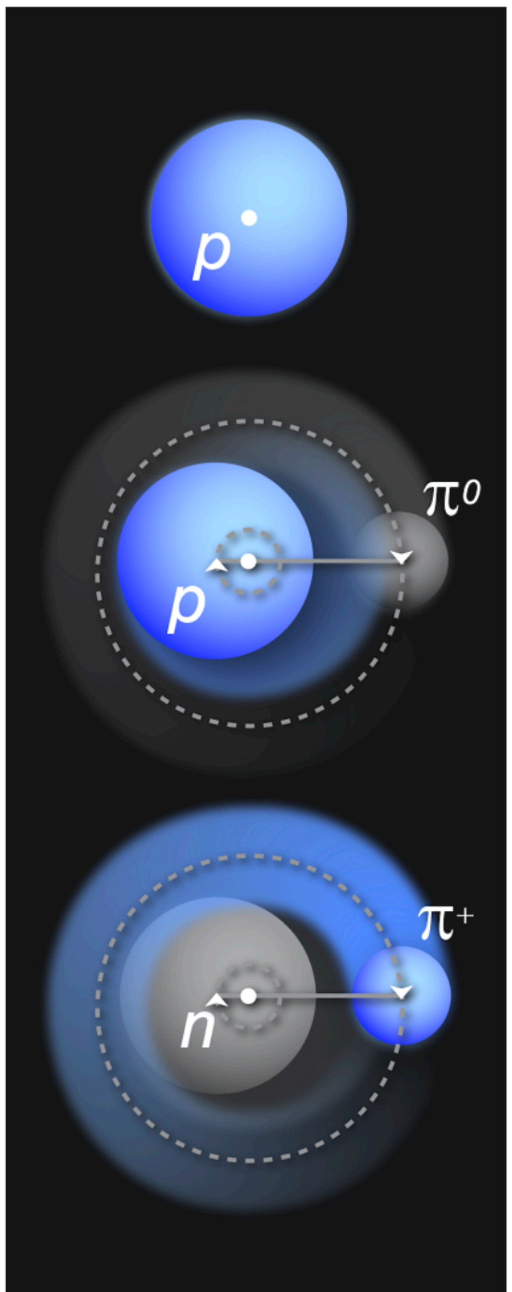
The "Pion Cloud"-and Pion Thickness functions

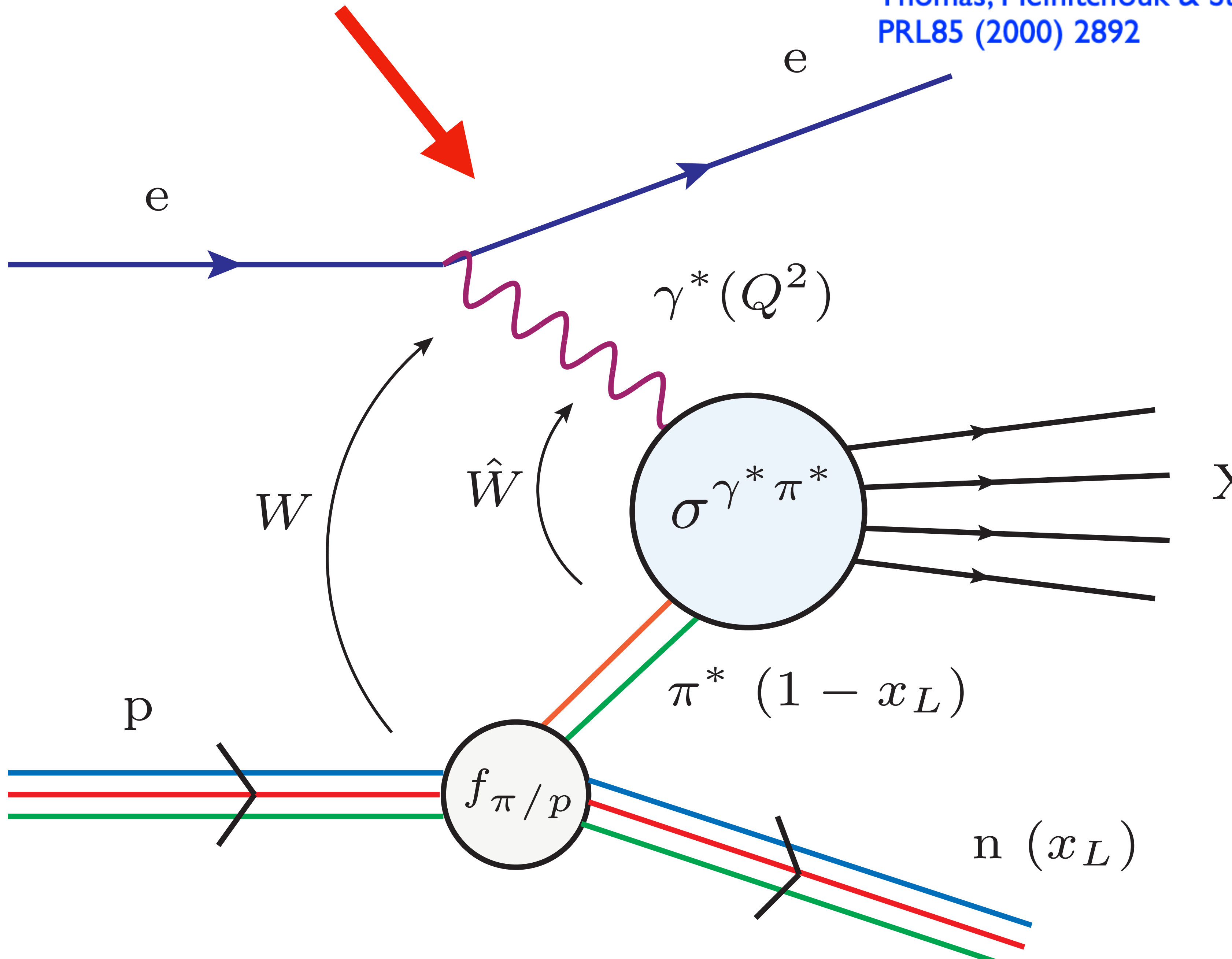


The “Pion Cloud”

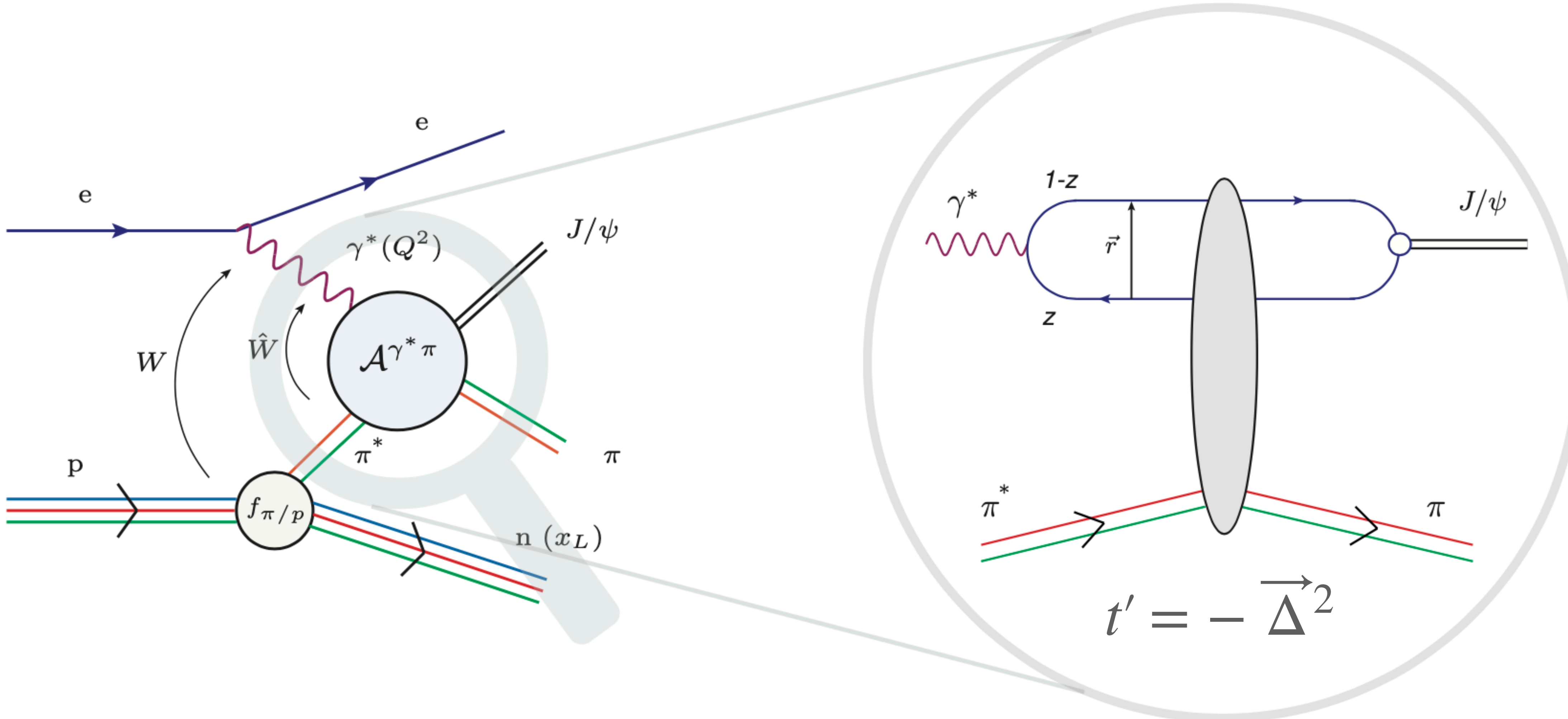
$$|p> \rightarrow \sqrt{1-a-b}\ |p_o> + \sqrt{a}\left(-\sqrt{\frac{1}{3}}\ |p_0\ \pi^0> + \sqrt{\frac{2}{3}}\ |n_0\ \pi^+>\right) + \sqrt{b}\left(-\sqrt{\frac{1}{2}}\ |\Delta_0^{++}\ \pi^-> - \sqrt{\frac{1}{3}}\ |\Delta_0^{+}\ \pi^0> + \sqrt{\frac{1}{6}}\ |\Delta_0^{0}\ \pi^+>\right)$$

Chiral approach: a=0.24, b=0.12
Thomas, Melnitchouk & Steffens, PRL85 (2000) 2892





Exclusive Diffraction



$$\mathscr{A}_{T,L}^{\gamma^*\pi^*\to J/\psi\pi}(\hat{x}, Q^2, \Delta) = i\int \mathrm{d}^2\vec{r}\,\mathrm{d}^2\vec{b}\,\frac{\mathrm{d}z}{4\pi}\left(\Psi^*\Psi_V\right)_{T,L}(Q^2, r, z)e^{-i[\vec{b}-(1-z)\vec{r}]\cdot\vec{\Delta}}\frac{\mathrm{d}\sigma_{q\bar{q}}^{(\pi)}}{\mathrm{d}^2\vec{b}}(\vec{b}, \vec{r}, \hat{x})$$

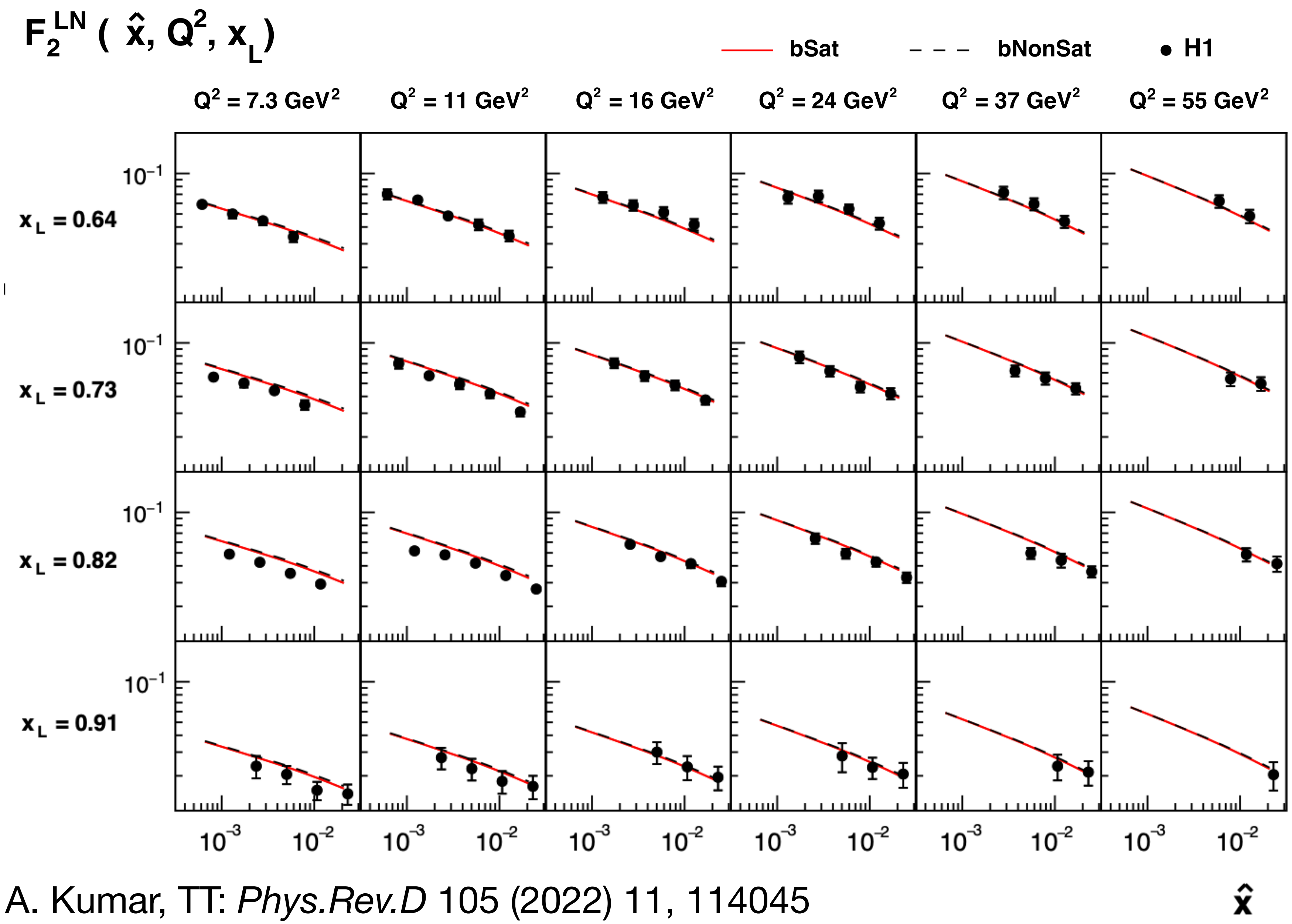
$$\frac{\mathrm{d}\sigma^{\mathrm{sat}}}{\mathrm{d}^2\mathrm{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_C}r^2\alpha_S(\mu^2)xg(x, \mu^2)T(b)\right)\right]$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}^{(\pi)}}{\mathrm{d}^2\vec{b}}(\vec{b}, \vec{r}, \hat{x}) = R_g\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\vec{b}}$$

$$\frac{\mathrm{d}\sigma^{\mathrm{nosat}}}{\mathrm{d}^2\mathrm{b}} = \frac{\pi^2}{N_C}r^2\alpha_S(\mu^2)xg(x, \mu^2)T(b)$$

Pion Longitudinal Structure

$$\frac{\mathrm{d}\sigma_{q\bar{q}}^{(\pi)}}{\mathrm{d}^2\mathrm{b}}(\mathrm{b}, \mathrm{r}, \hat{x}) = R_g \, \frac{\mathrm{d}\sigma_{q\bar{q}}^{(p)}}{\mathrm{d}^2\mathrm{b}}(\mathrm{b}, \mathrm{r}, \hat{x})$$

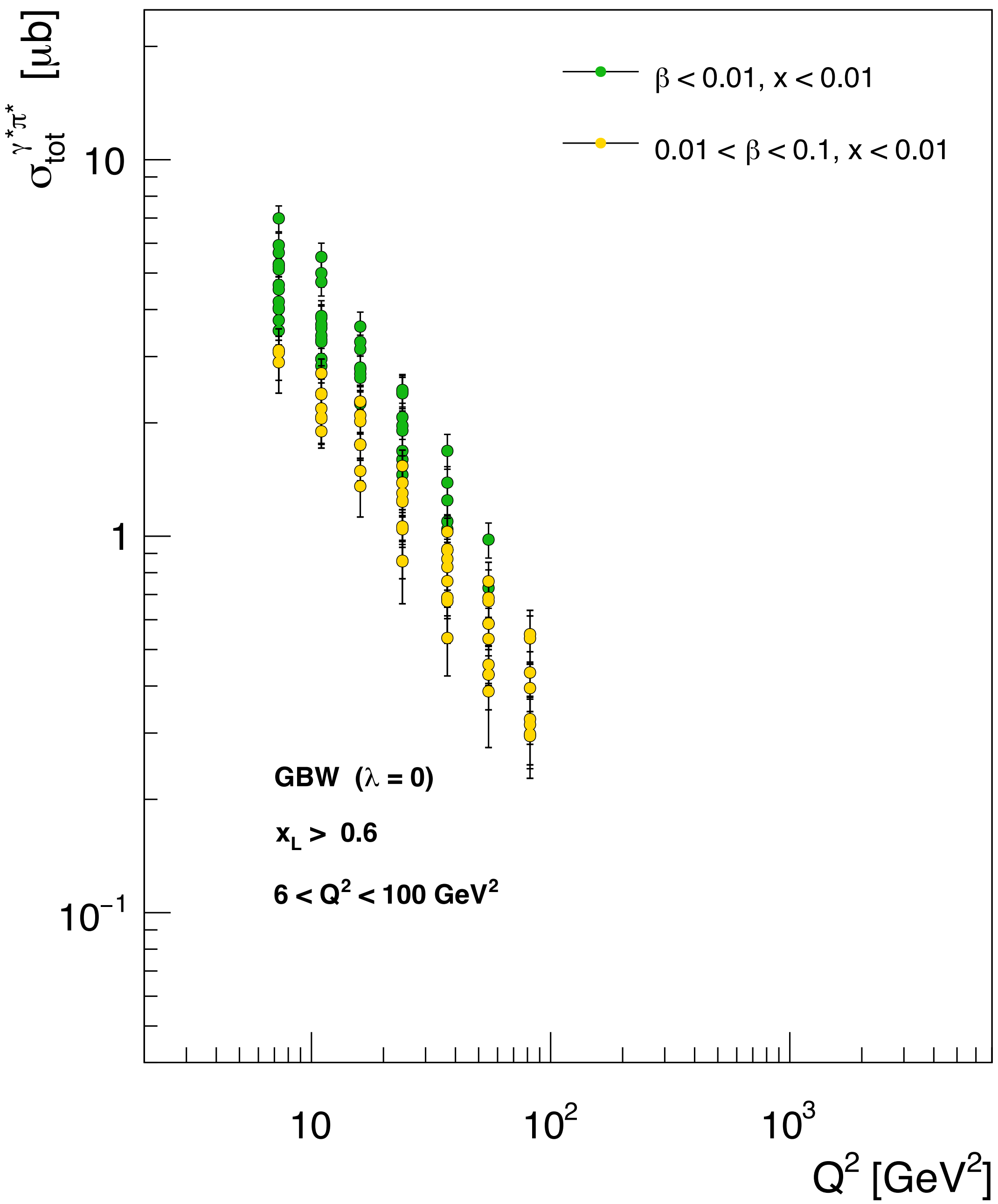


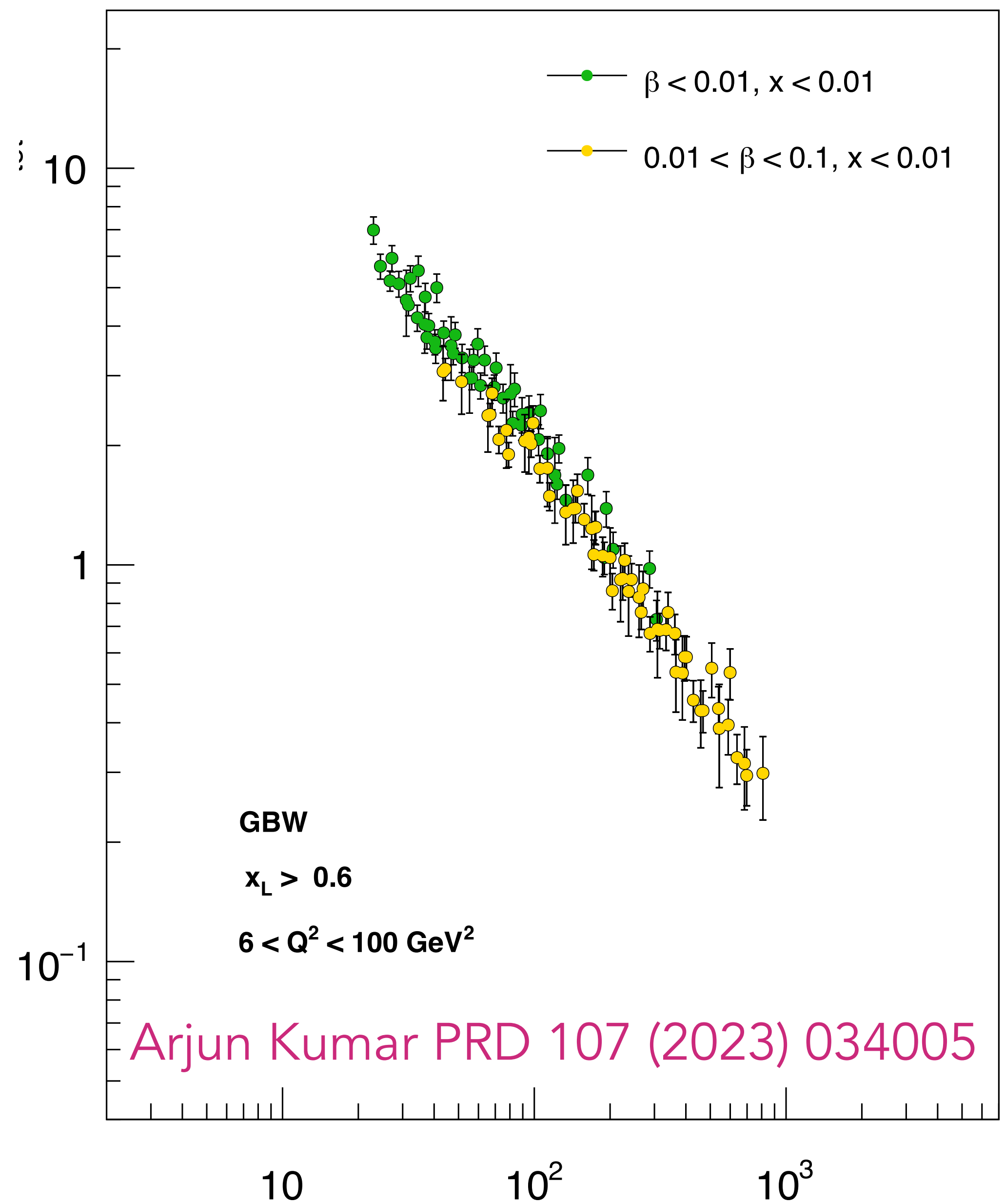
$R_g = 0.5$

A. Kumar, TT: *Phys.Rev.D* 105 (2022) 11, 114045

Pion Longitudinal Structure

$$\frac{\mathrm{d}\sigma_{q\bar{q}}^{(\pi)}}{\mathrm{d}^2\mathrm{b}}(\mathrm{b},\mathrm{r},\hat{x}) = R_g \, \frac{\mathrm{d}\sigma_{q\bar{q}}^{(p)}}{\mathrm{d}^2\mathrm{b}}(\mathrm{b},\mathrm{r},\hat{x})$$

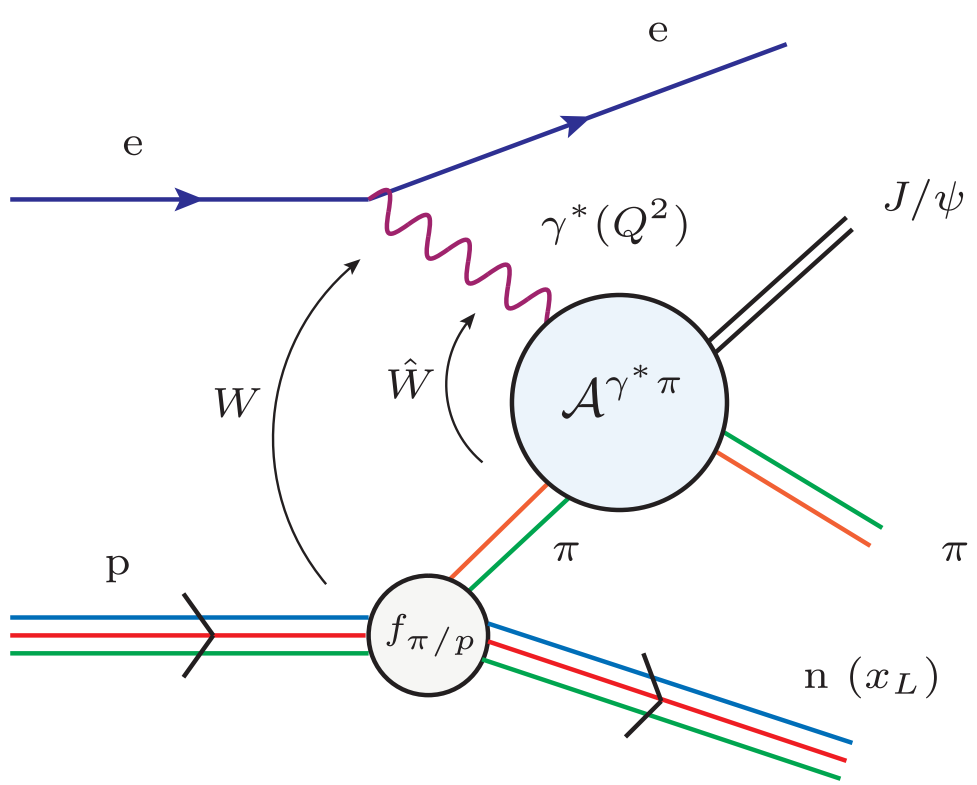




τ

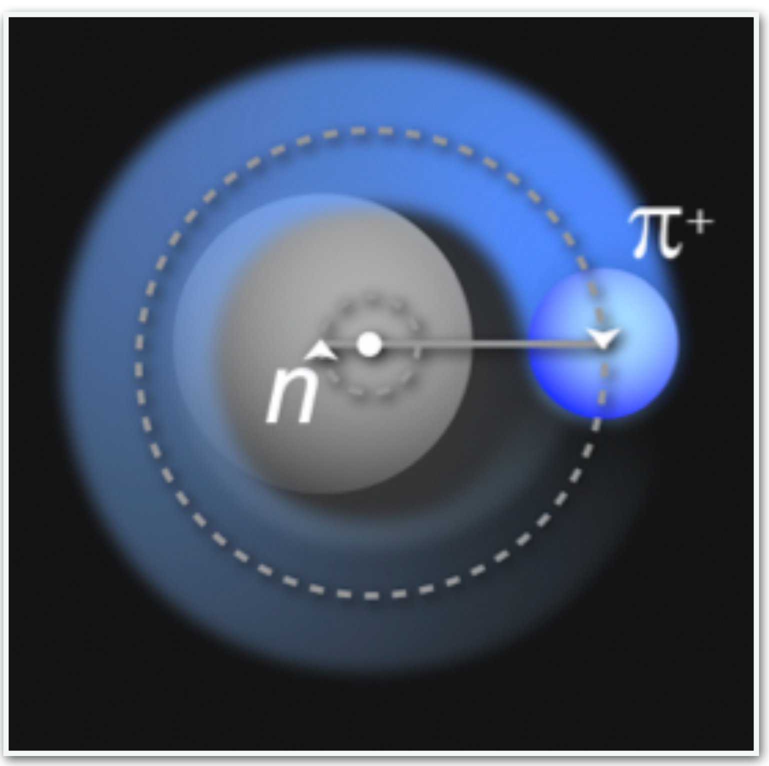
$$\tau = \frac{Q^2_{\hat{x}}}{Q_s^2(\beta)}; \quad Q_s^2(\beta) = Q_0^2\left(\frac{\beta}{x_0}\right)^{-\lambda}$$

The Pion Thickness



$$\frac{d\sigma}{dt} \propto |\mathscr{A}|^2$$

t-spectrum off the proton
slope ∝ (target size)²
Pion cloud is larger than the proton
Expect: Steeper t-spectrum.



$$T_{\pi^*}(b) = \int_{-\infty}^{\infty} dz \rho_{\pi^*}(b, z)$$

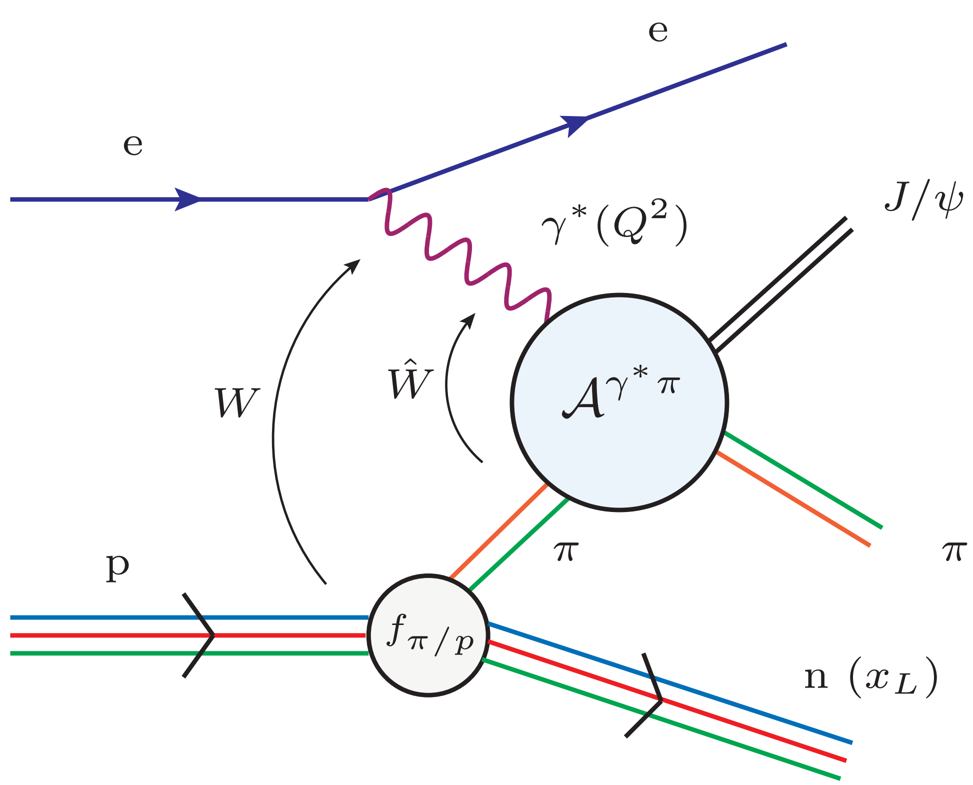
$$\rho_{\pi^*}(b, z) = \frac{m_\pi^2}{4\pi} \frac{e^{-m_\pi\sqrt{b^2+z^2}}}{\sqrt{b^2+z^2}}$$

Yukawa distribution

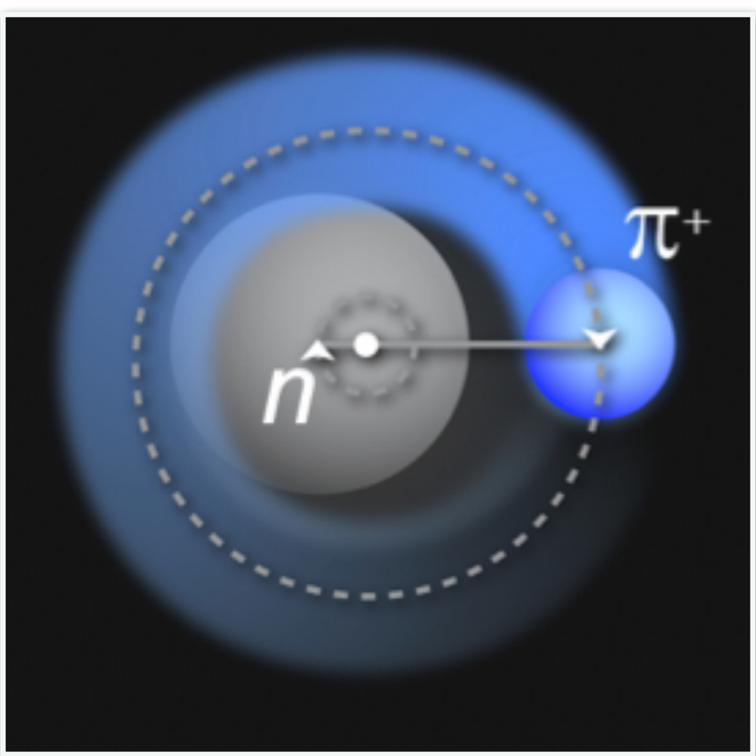
$$T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{-b^2}{2B_\pi}}$$

A. Kumar, TT: *Phys.Rev.D* 105 (2022) 11, 114045

The Pion Thickness



$$\frac{d\sigma}{dt} \propto \left\langle |\mathscr{A}|^2 \right\rangle_{\pi^*-\text{position}}$$



$$T_{\pi^*}(b) = \int_{-\infty}^{\infty} dz \rho_{\pi^*}(b,z)$$

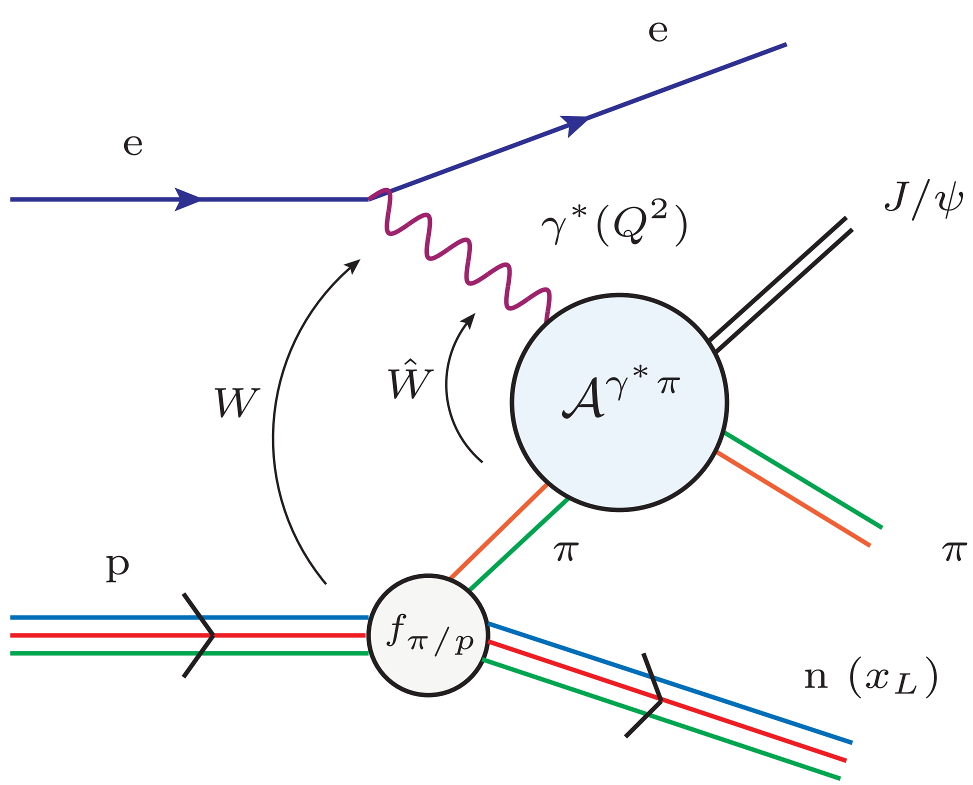
$$T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{-b^2}{2B_\pi}}$$

$$\rho_{\pi^*}(b,z) = \frac{m_\pi^2}{4\pi} \frac{e^{-m_\pi\sqrt{b^2+z^2}}}{\sqrt{b^2+z^2}}$$

Yukawa distribution

A. Kumar, TT: *Phys.Rev.D* 105 (2022) 11, 114045

The Pion Thickness

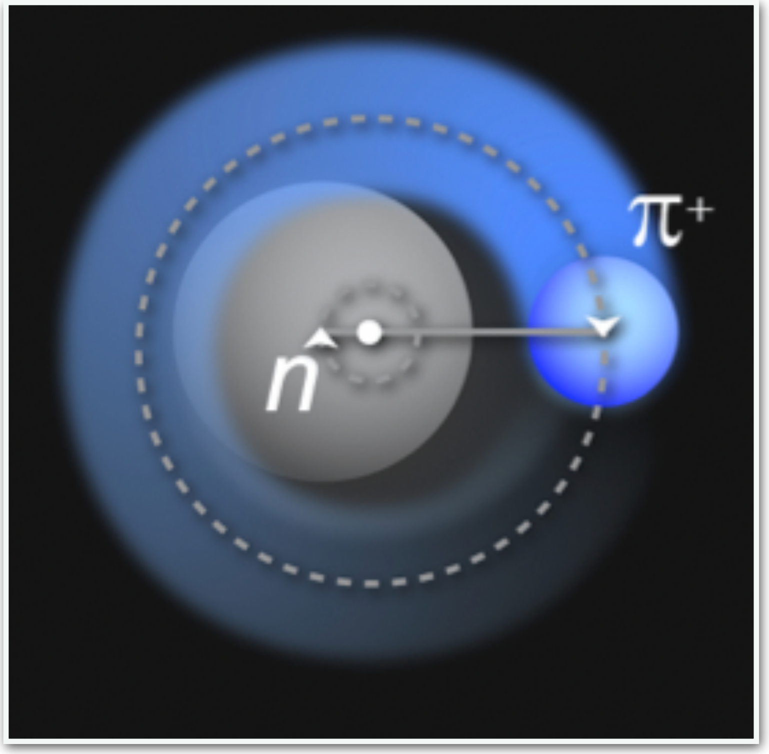


$$\frac{d\sigma}{dt} \propto \left\langle |\mathscr{A}|^2 \right\rangle_{\pi^*-\text{position}}$$

$$= |\langle \mathscr{A} \rangle_{\pi^*}|^2 + \left(\left\langle |\mathscr{A}|^2 \right\rangle_{\pi^*} - |\langle \mathscr{A} \rangle_{\pi^*}|^2 \right)$$

Average pion position

Fluctuations in pion position



$$T_{\pi^*}(b) = \int_{-\infty}^{\infty} dz \rho_{\pi^*}(b,z)$$

$$\rho_{\pi^*}(b,z) = \frac{m_\pi^2}{4\pi} \frac{e^{-m_\pi\sqrt{b^2+z^2}}}{\sqrt{b^2+z^2}}$$

Yukawa distribution

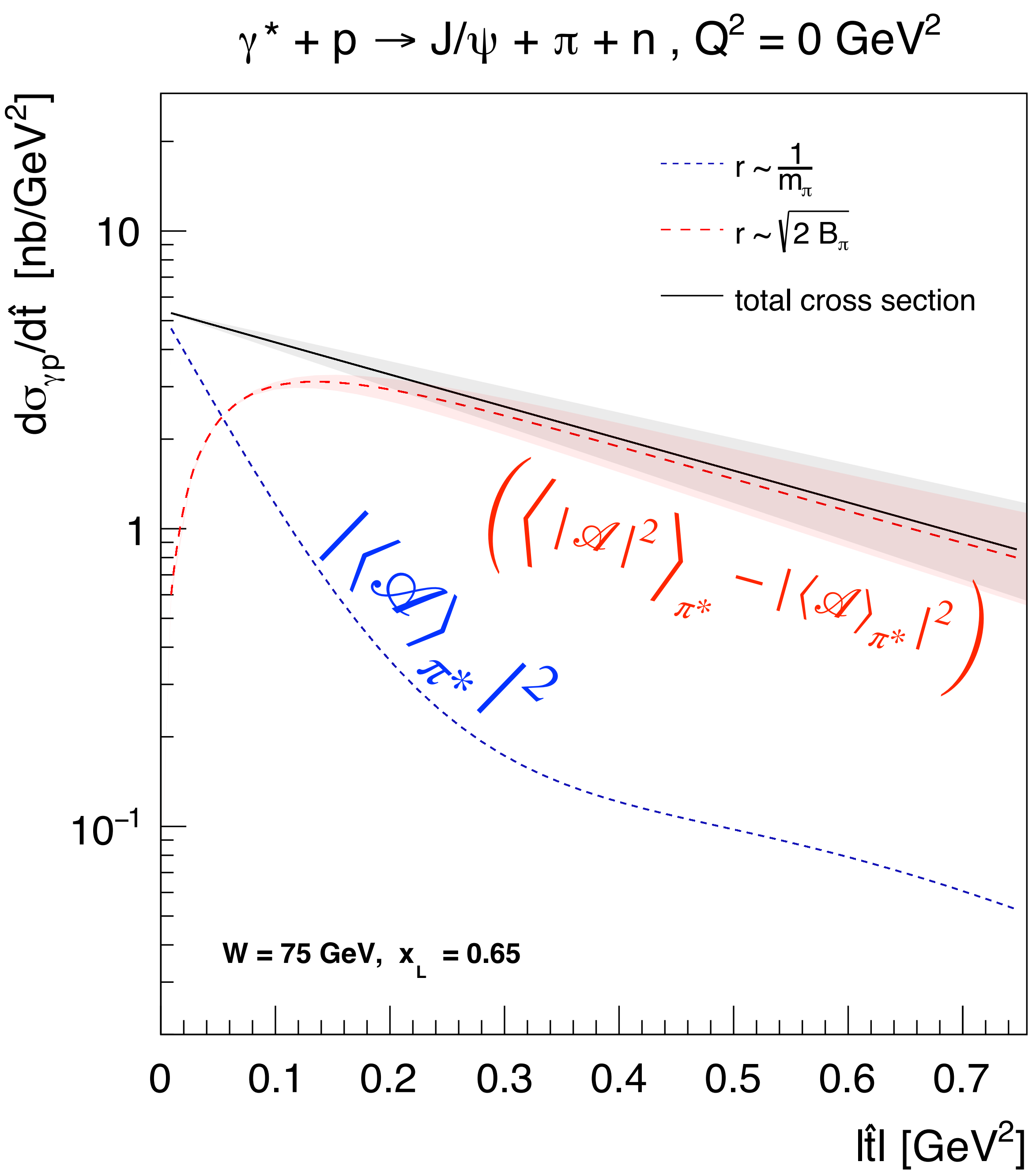
$$T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{-b^2}{2B_\pi}}$$

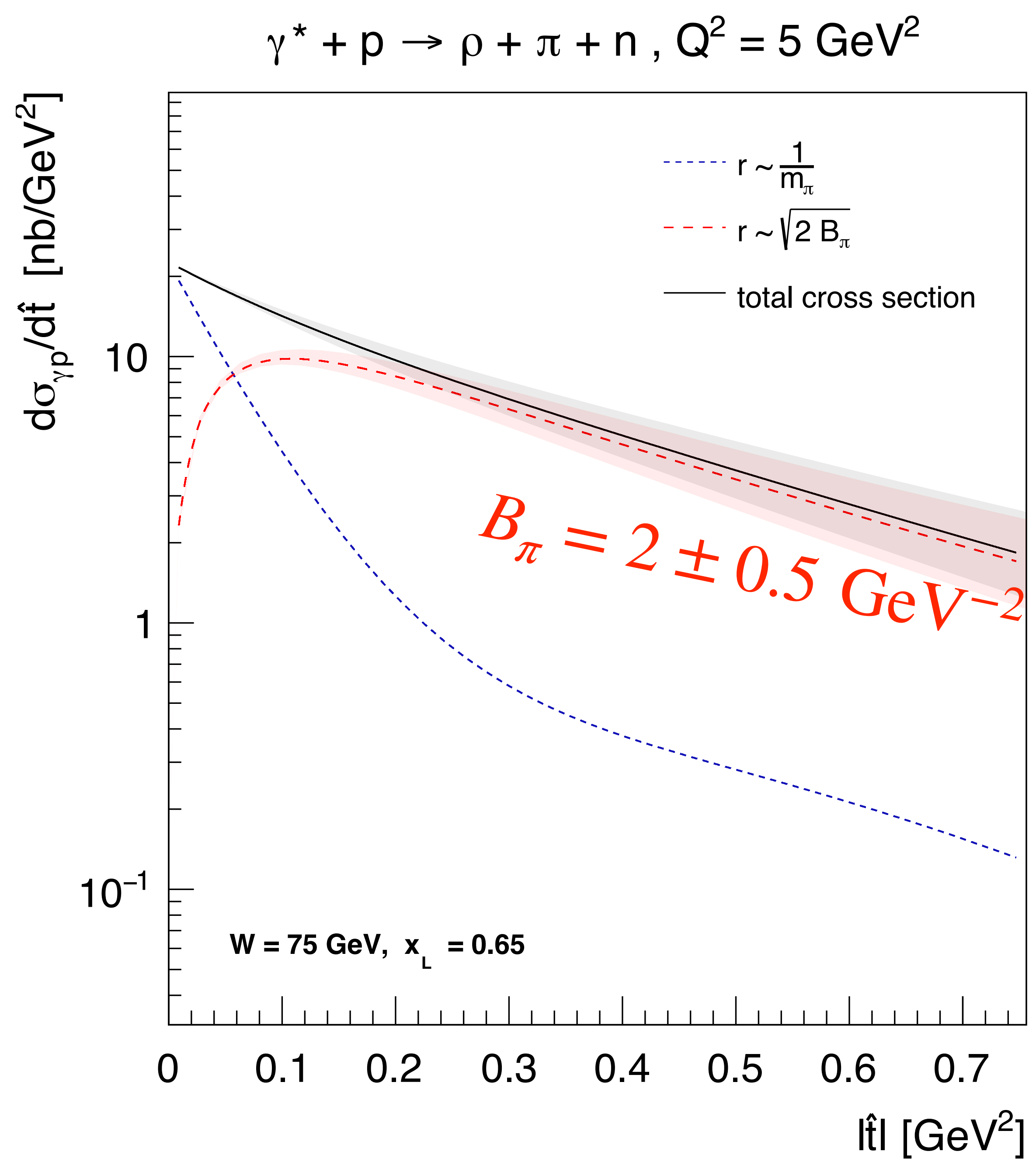
t-spectrum off the proton
slope ∝ (target size)²
Pion cloud is larger than the proton
Expect: Steeper *t*-spectrum.

A. Kumar, TT: *Phys.Rev.D* 105 (2022) 11, 114045

The Pion Thickness

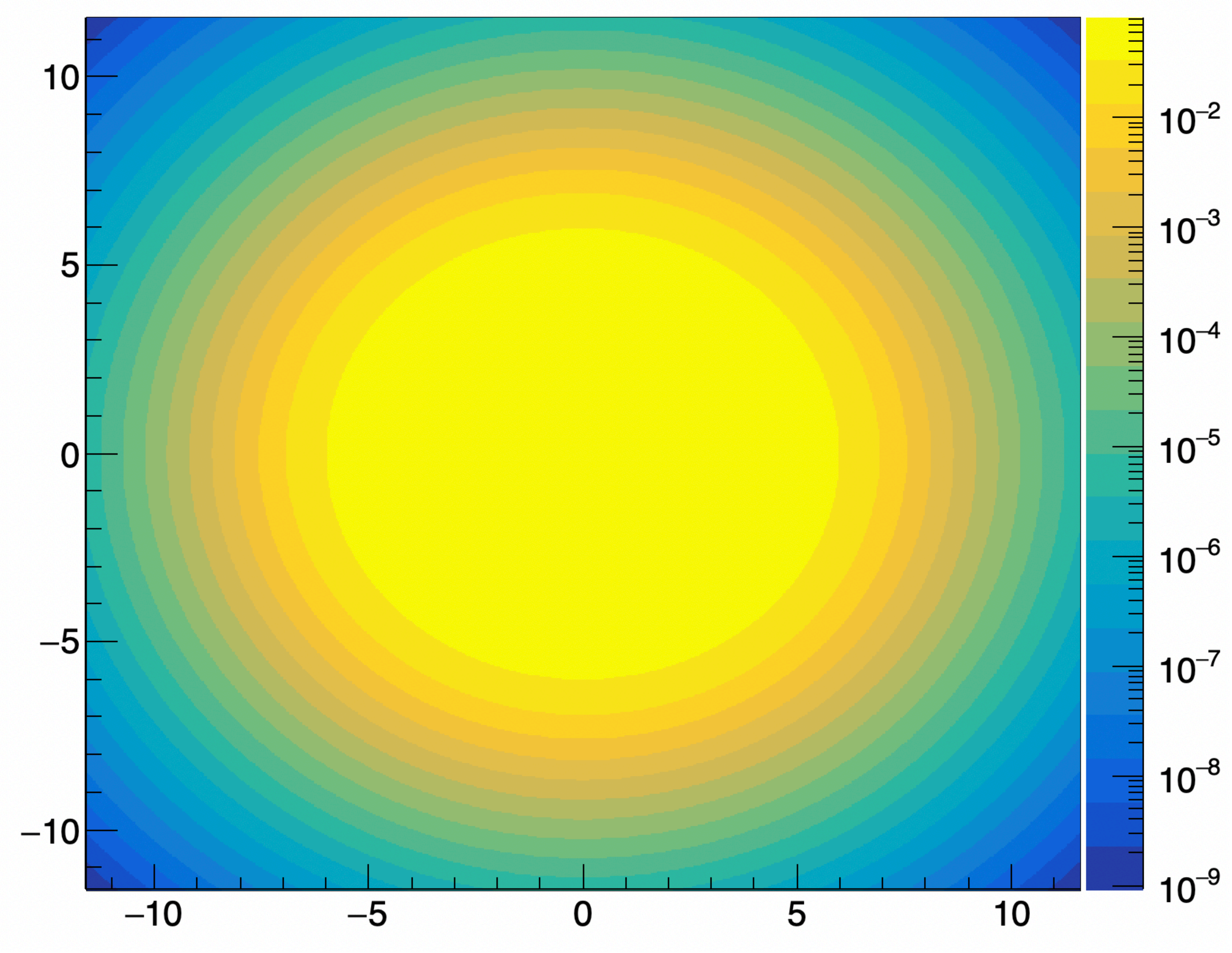
$$\frac{d\sigma}{dt} \propto |\langle \mathscr{A} \rangle_{\pi^*}|^2 + \left(\left\langle |\mathscr{A}|^2 \right\rangle_{\pi^*} - |\langle \mathscr{A} \rangle_{\pi^*}|^2 \right)$$





A. Kumar, TT: *Phys.Rev.D* 105 (2022) 11, 114045

The nucleus thickness



The nucleus thickness

Naively, use a Woods-Saxon distribution:

$$T_A(\vec{b}) = \int \mathrm{d}z \frac{\rho_0}{1 + \exp\left(\frac{\sqrt{\vec{b}^2 + z^2} - R_0}{d}\right)}$$

$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)\right)\right]$$

Nuclear PDF

The nucleus thickness

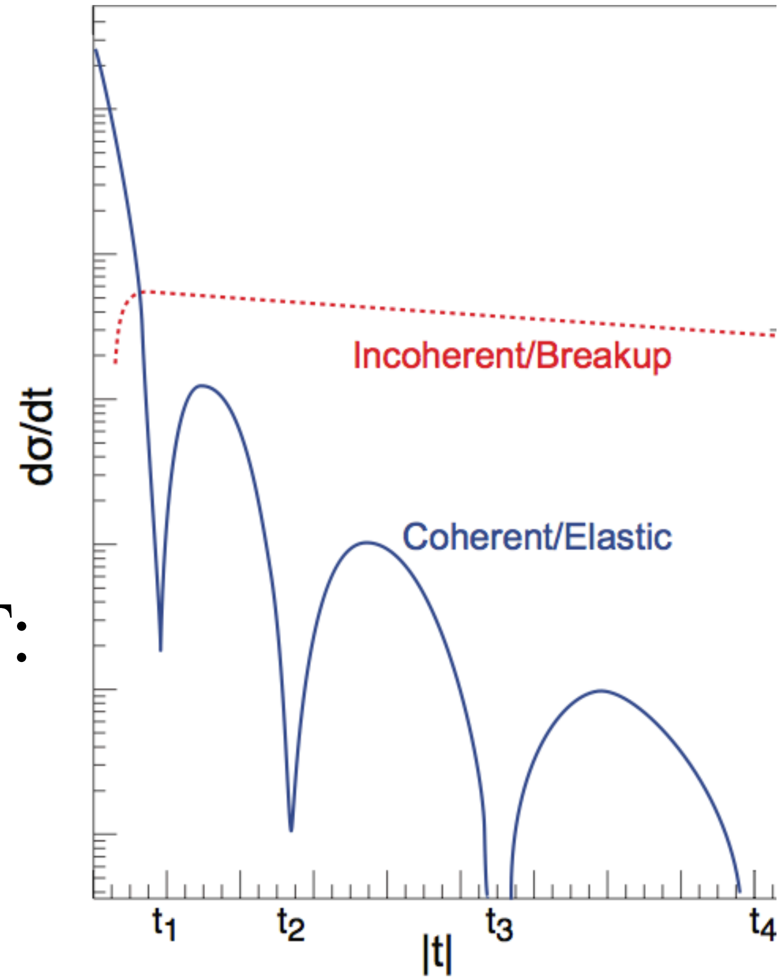
Naively, use a Woods-Saxon distribution:

$$T_A(\vec{b}) = \int dz \frac{\rho_0}{1 + \exp\left(\frac{\sqrt{\vec{b}^2 + z^2} - R_0}{d}\right)}$$

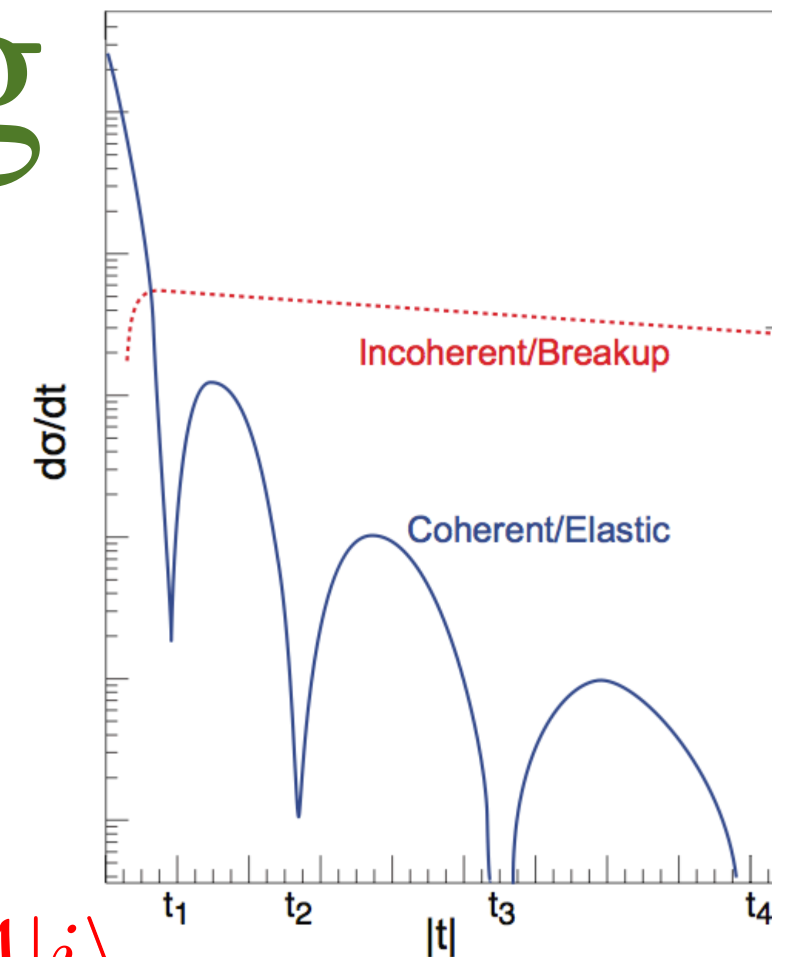
$$\frac{d\sigma_{q\bar{q}}}{d^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x, \mu^2) T(b)\right)\right]$$

Nuclear PDF

BUT:



Incoherent Scattering



Good, Walker:

Nucleus dissociates ($f \neq i$):

$$\sigma_{\text{incoherent}} \propto \sum_{f \neq i} \langle i|\mathcal{A}|f\rangle^\dagger \langle f|\mathcal{A}|i\rangle$$

complete set

$$= \sum_{f} \langle i|\mathcal{A}|f\rangle^\dagger \langle f|\mathcal{A}|i\rangle - \langle i|\mathcal{A}|i\rangle^\dagger \langle i|\mathcal{A}|i\rangle$$

$$= \langle i \left| |\mathcal{A}|^2 \right| i\rangle - |\langle i|\mathcal{A}|i\rangle|^2 = \left\langle |\mathcal{A}|^2 \right\rangle - |\langle \mathcal{A}\rangle|^2$$

The incoherent CS is the variance of the amplitude!

$$\frac{d\sigma_{\text{total}}}{dt} = \frac{1}{16\pi} \left\langle |\mathcal{A}|^2 \right\rangle$$

$$\frac{d\sigma_{\text{coherent}}}{dt} = \frac{1}{16\pi} |\langle \mathcal{A}\rangle|^2$$

The nucleus as a collection of nucleons

TT, Thomas Ullrich
Phys.Rev.C 87 (2013) 2, 024913, arXiv: 1211.3048
Comput.Phys.Commun. 185 (2014) 1835-1853 arXiv:1307.8059

Independent scattering approximations:

$$1-\frac{1}{2}\frac{\mathrm{d}\sigma^{(A)}_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b}) = \prod_{i=1}^{A}\left(1-\frac{1}{2}\frac{\mathrm{d}\sigma^{(p)}_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, |\vec{b}-\vec{b}_i|)\right)$$

Proton PDF

$$\frac{1}{2}\frac{\mathrm{d}\sigma^{(A)}_{q\bar{q}}}{\mathrm{d}^2\vec{b}}(x_{I\!P}, r, \vec{b}) = 1 - \exp\left(\frac{\pi^2}{2N_C}r^2\alpha_S(\mu^2)xg(x,\mu^2)\sum_{i=1}^{A}T_p(|\vec{b}-\vec{b}_i|)\right)$$

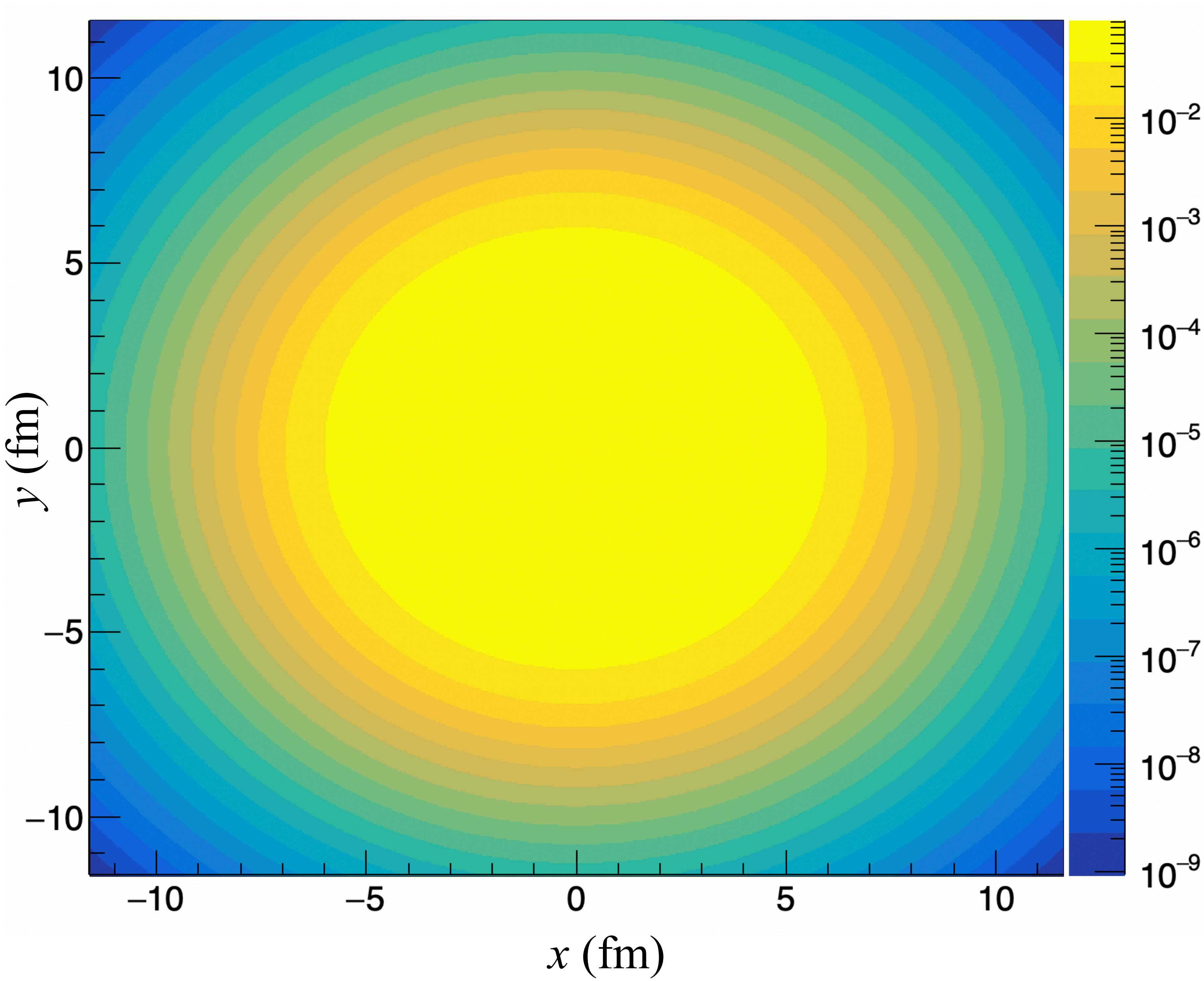
$$T_A(\vec{b}) = \int \mathrm{d}z \frac{\rho_0}{1+\exp\left(\frac{\sqrt{\vec{b}^2+z^2}-R_0}{d}\right)}$$

$$\rightarrow$$

$$T_A(\vec{b}) = \sum_{i=1}^{A} T_p(|\vec{b}-\vec{b}_i|)$$

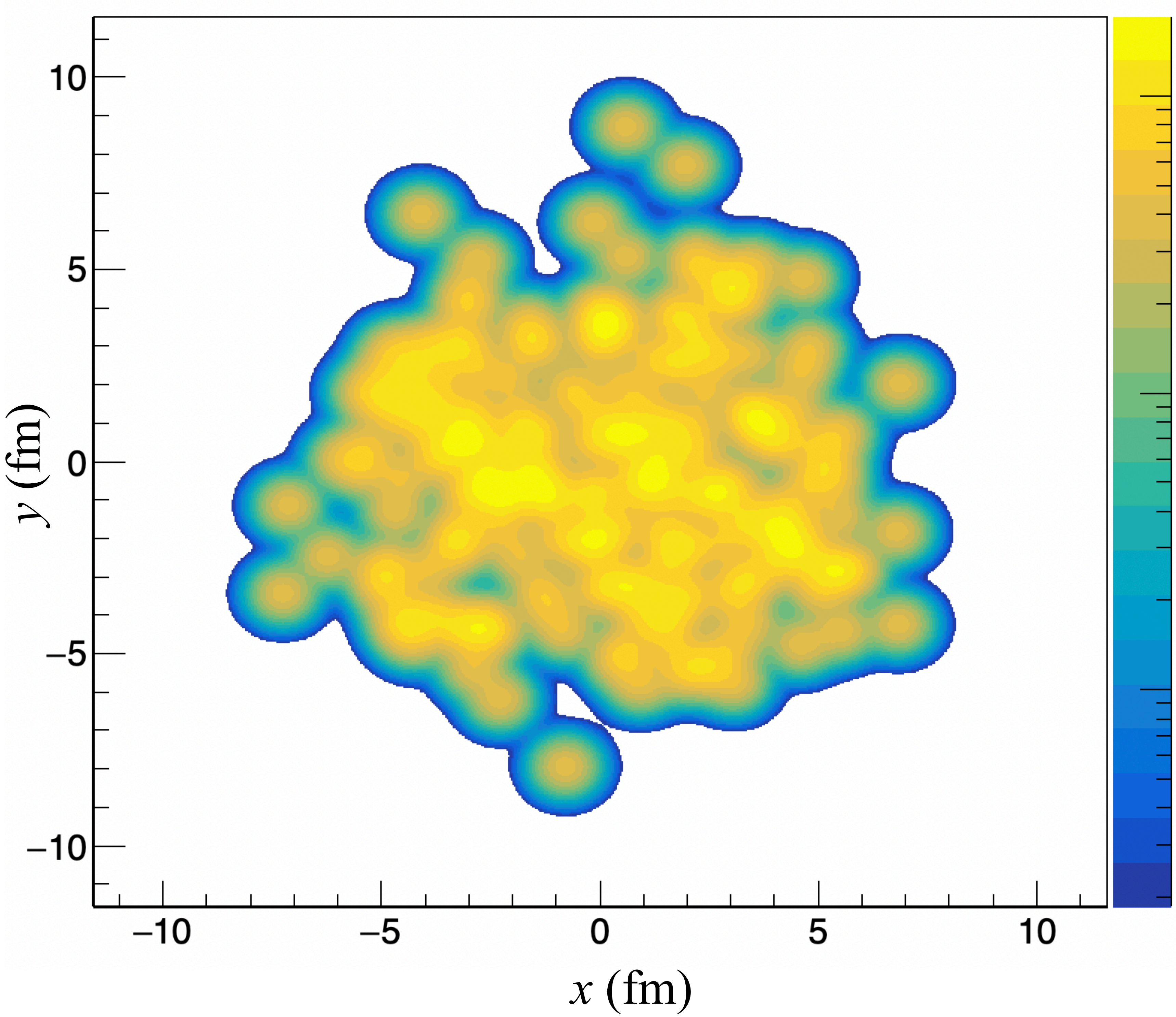
$$T_p(b) = \frac{1}{2\pi B_G}e^{-\frac{b^2}{2B_G}}$$

Into the heavy nucleus

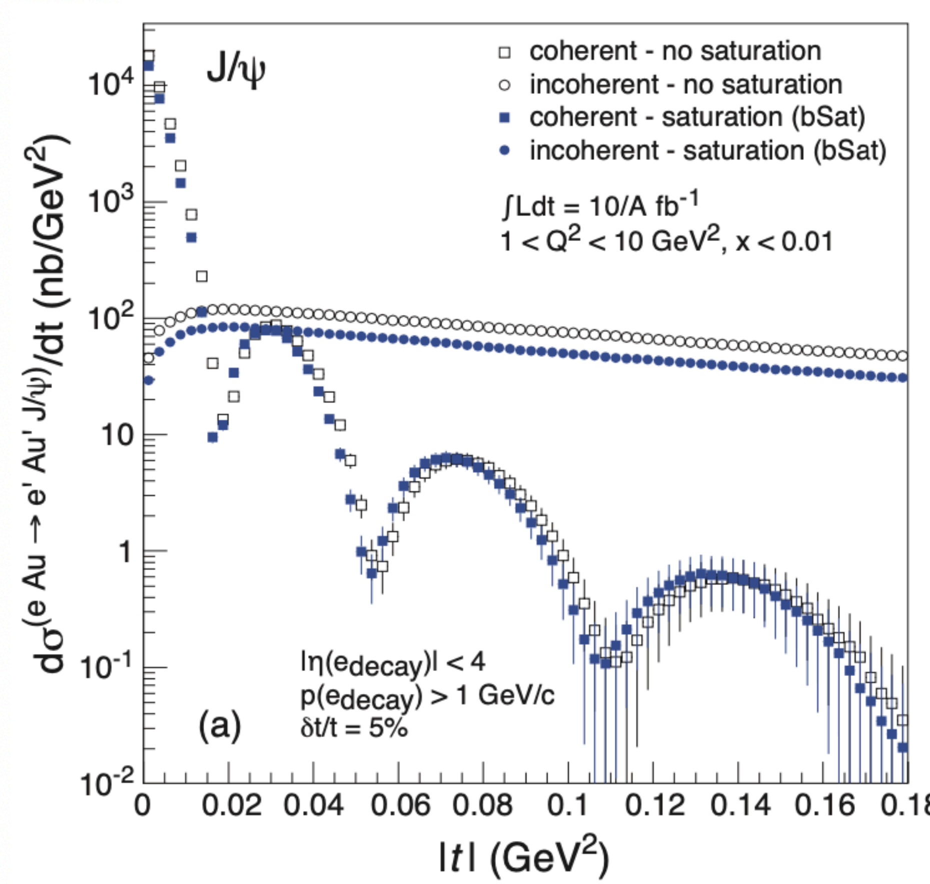


$|t| = 0$

Into the heavy nucleus



$|t| \lesssim 0.2\ \mathrm{GeV}^2$



$|t|$

Hotspot model for incoherent *ep*-scattering

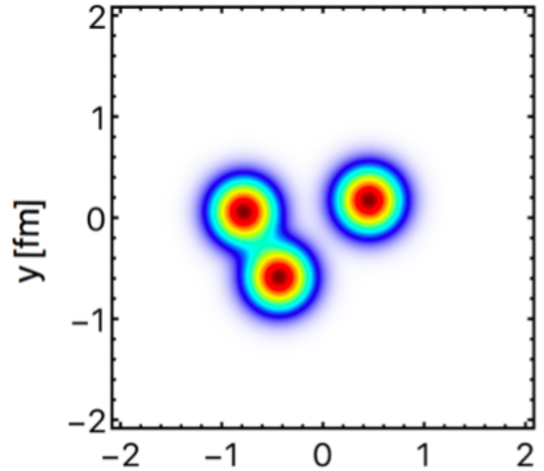
$$\frac{\mathrm{d}\sigma_{q\bar{q}}}{\mathrm{d}^2\mathbf{b}} = 2\left[1 - \exp\left(-\frac{\pi^2}{2N_c} r^2 \alpha_s(\mu^2) x g(x,\mu^2) T(b)\right)\right]$$

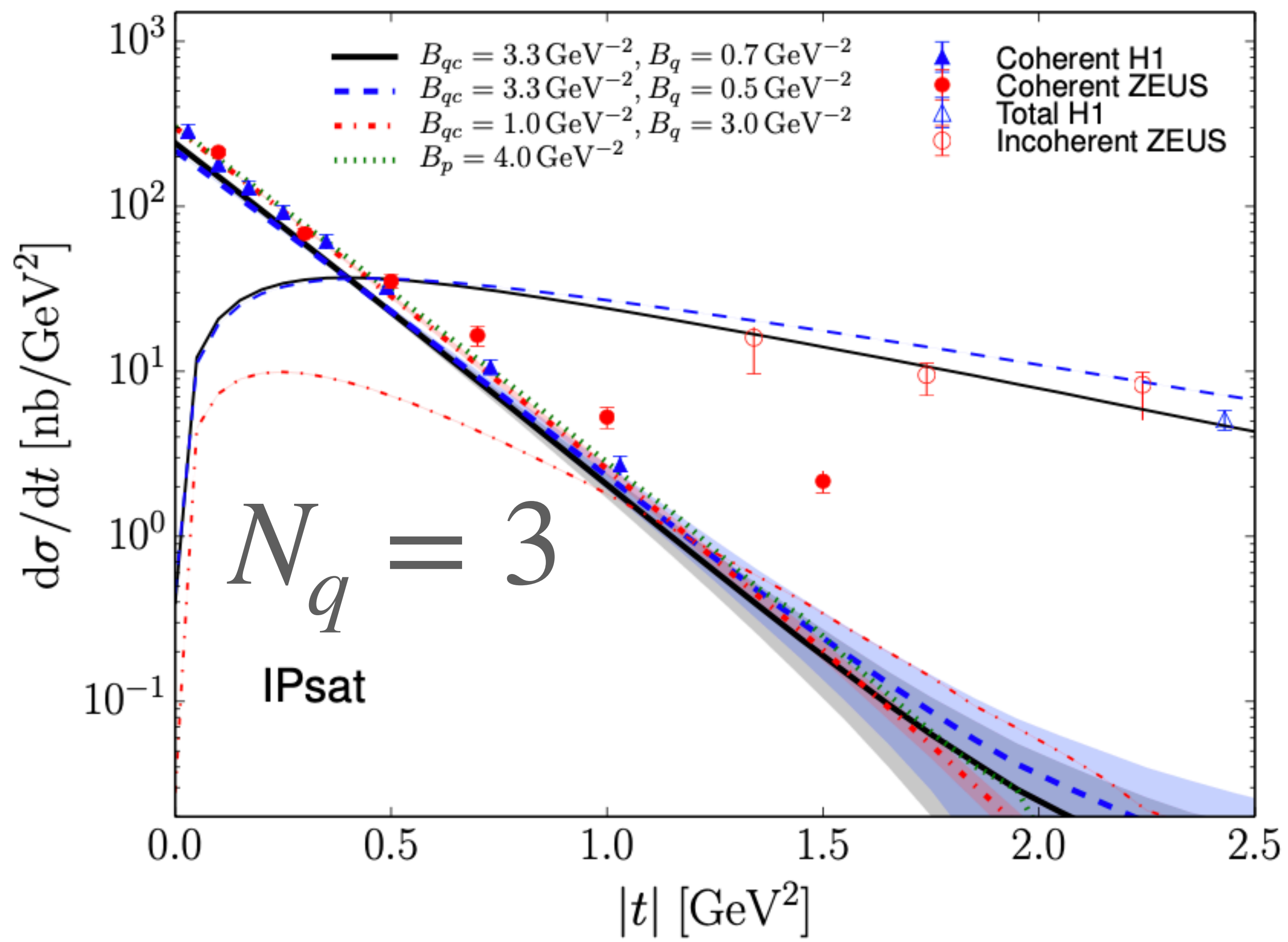
$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

$$\downarrow$$

$$T_p(b) = \frac{1}{2\pi N_q B_q} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b}-\vec{b}_i)^2}{2B_q}}$$

$\vec{b}_i$ with a Gaussian distribution of width B_{qc}

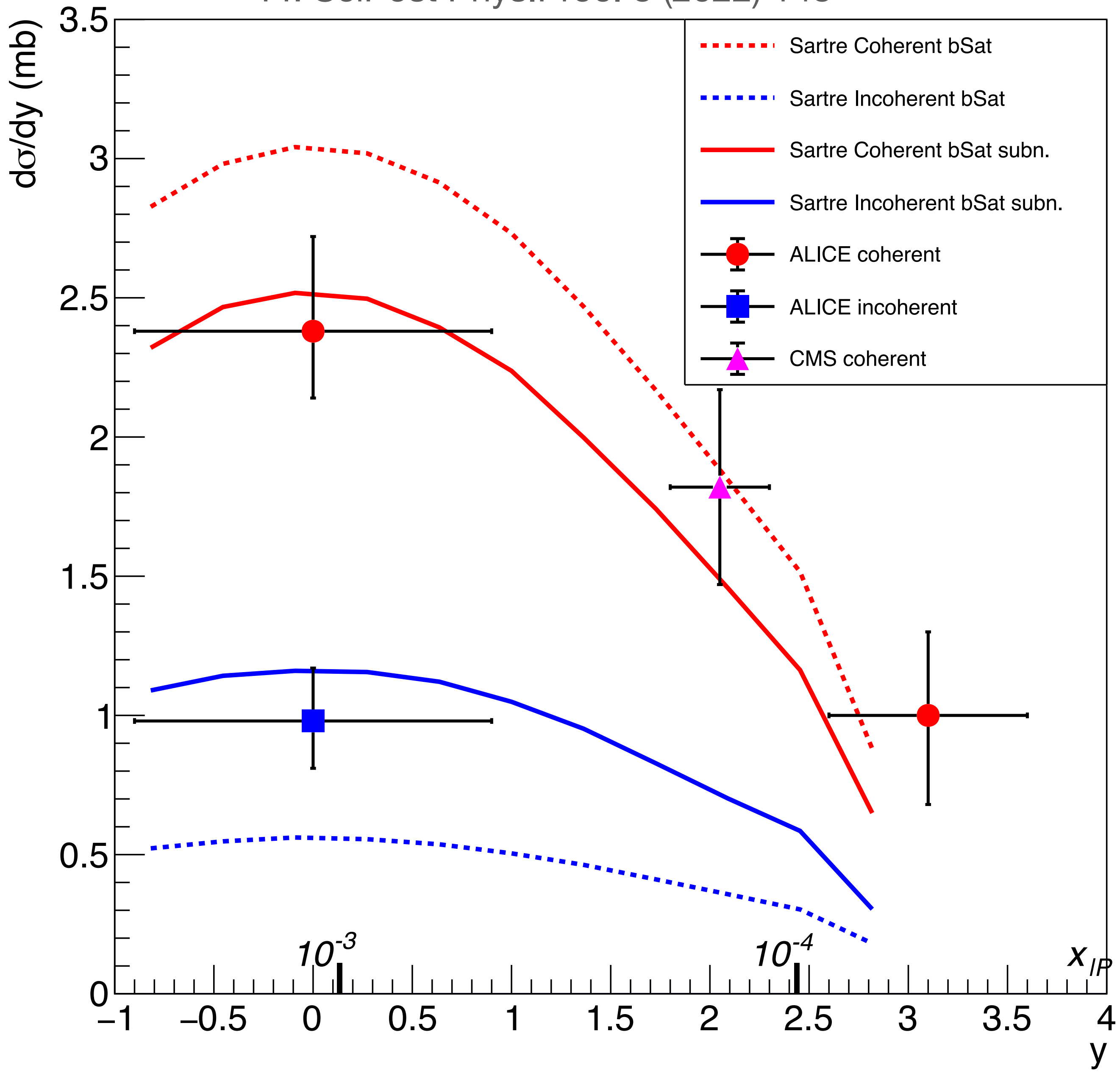




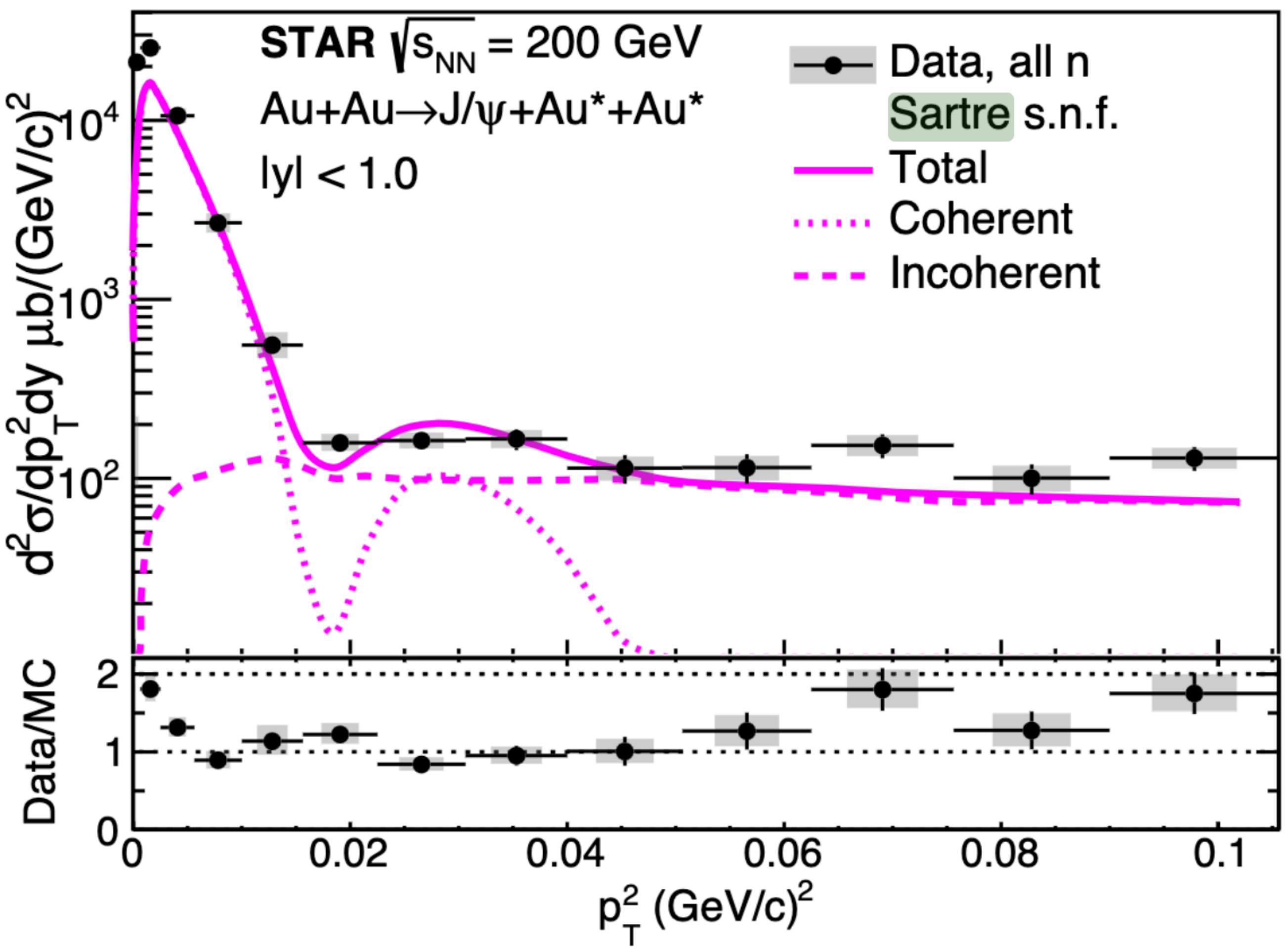
 H. Mäntysaari and B. Schenke Phys. Rev. Lett., 117(5):052301, 2016.

A-A UPC at the LHC & RHIC

TT: SciPost Phys.Proc. 8 (2022) 148



STAR Collaboration, e-Print: 2311.13632 [nucl-ex]



Even though coherent events dominate, the large $|t|$ tails have a significant effect on the cross sections!

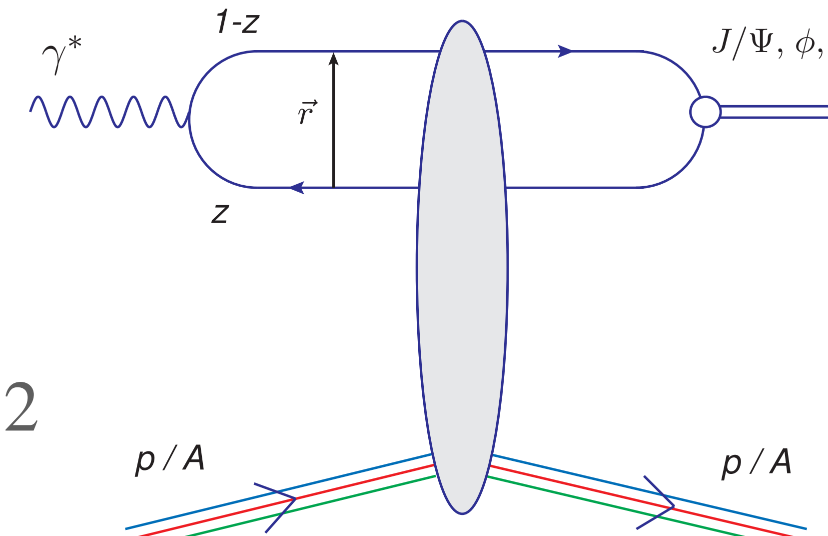
Subnucleon structure becomes important for $|t| > 0.2$ GeV^2

The proton thickness revisited

The proton thickness revisited

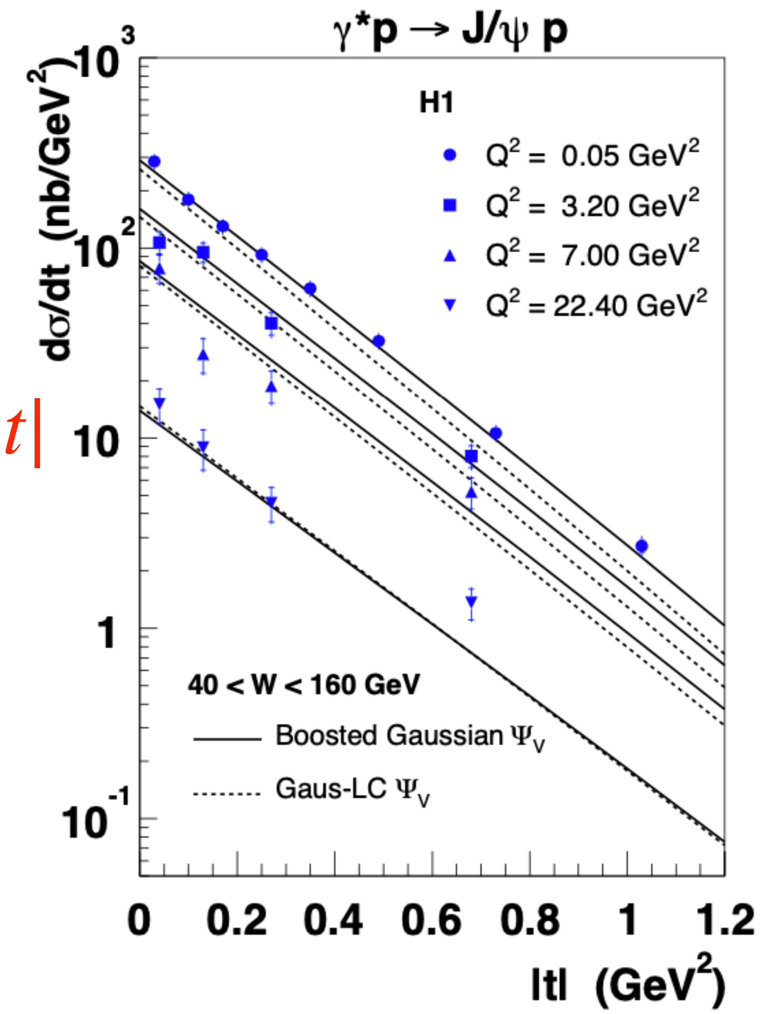
$$\frac{d\sigma}{dt} \propto \left| \mathscr{F}\text{ourier}(T(b)) \right|^2$$

$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}} \quad B_G = 4\ \text{GeV}^{-2}$$



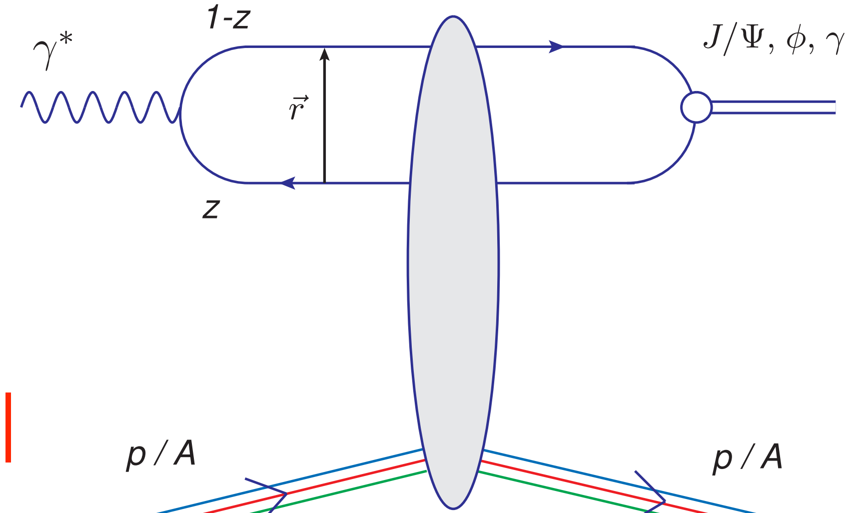
$t = -\Delta^2$

$$\frac{d\sigma}{dt} \propto e^{-B|t|}$$



The proton thickness revisited

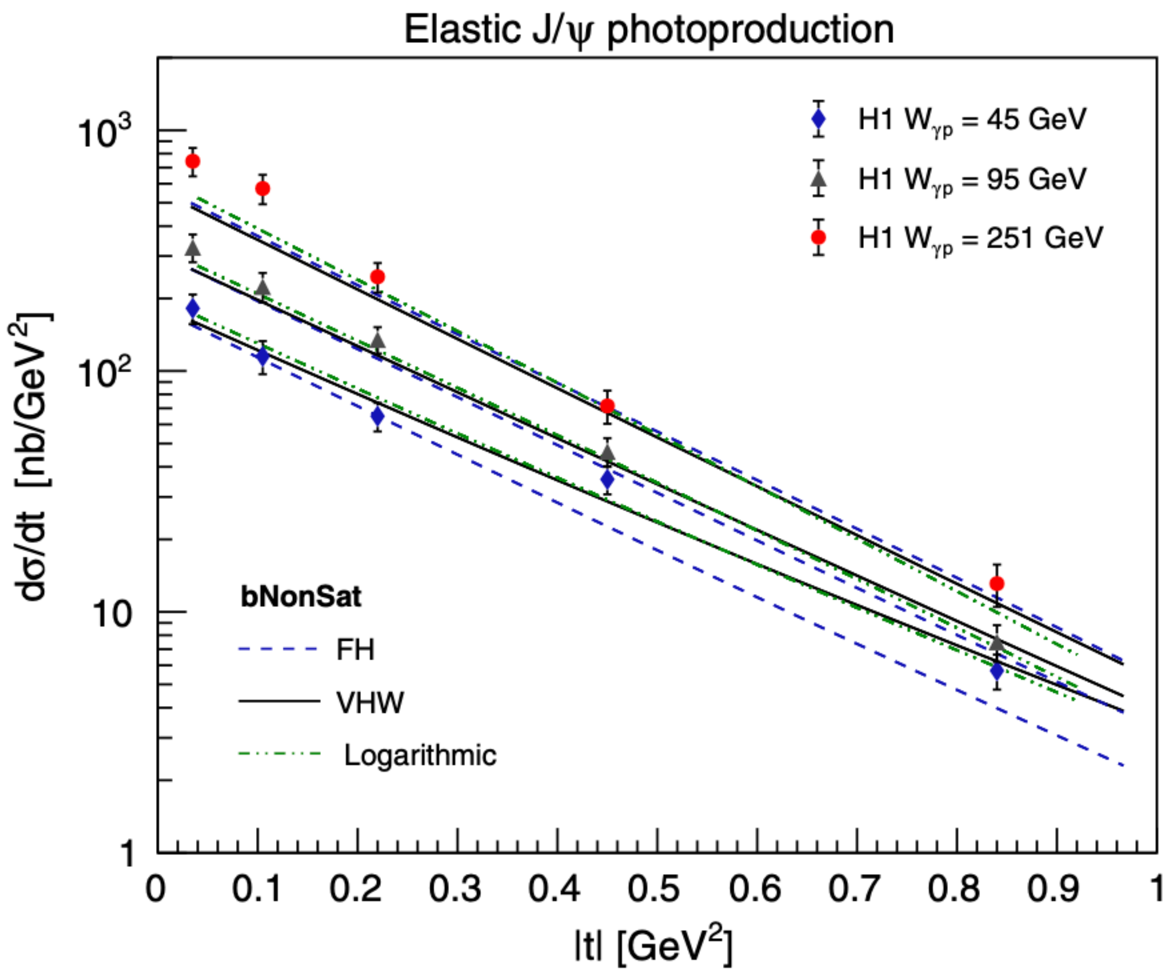
$$\frac{d\sigma}{dt} \propto \left| \mathscr{F}\text{ourier}(T(b)) \right|^2$$

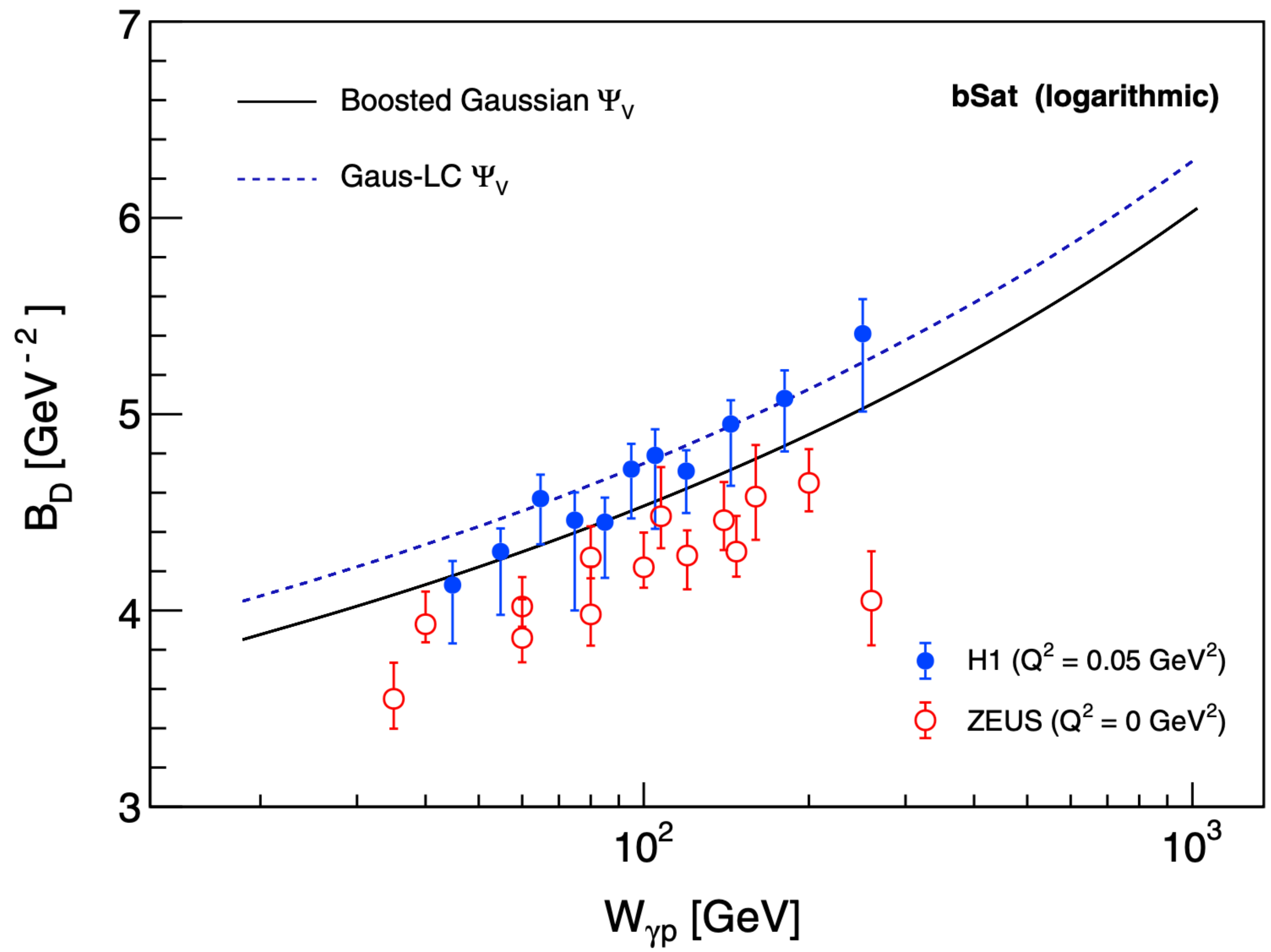


$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$

$$\frac{d\sigma}{dt} \propto e^{-B|t|}$$

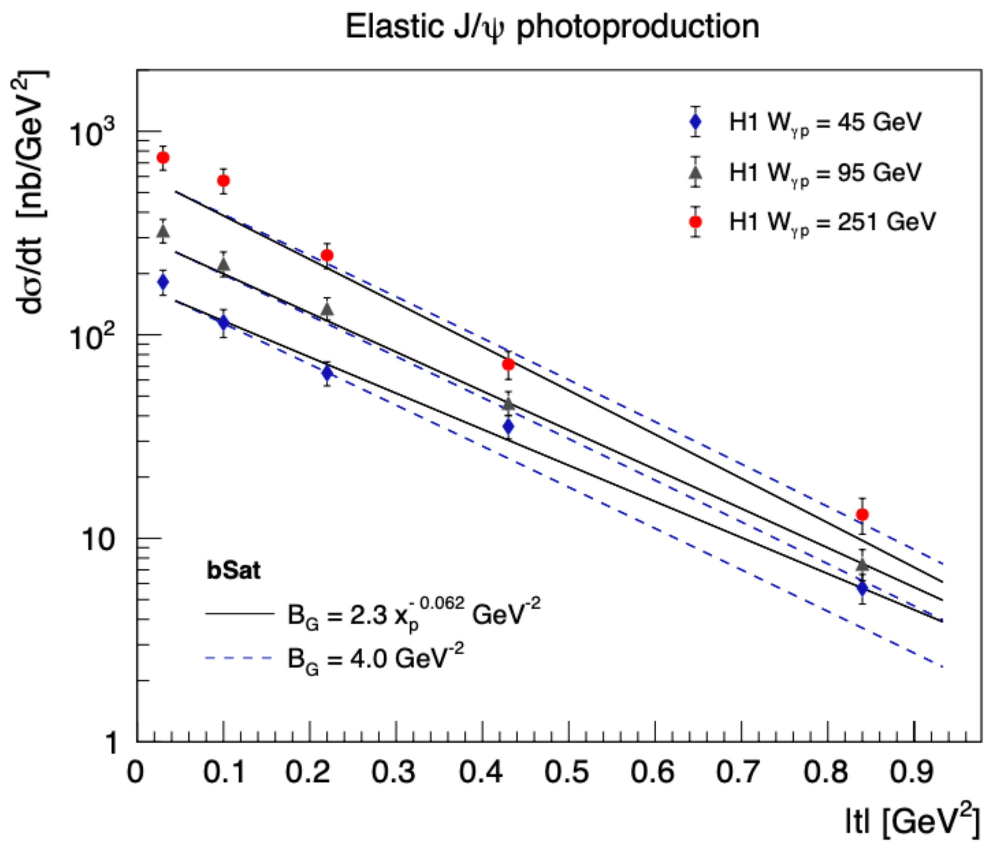
$$t = -\Delta^2$$





The proton thickness revisited

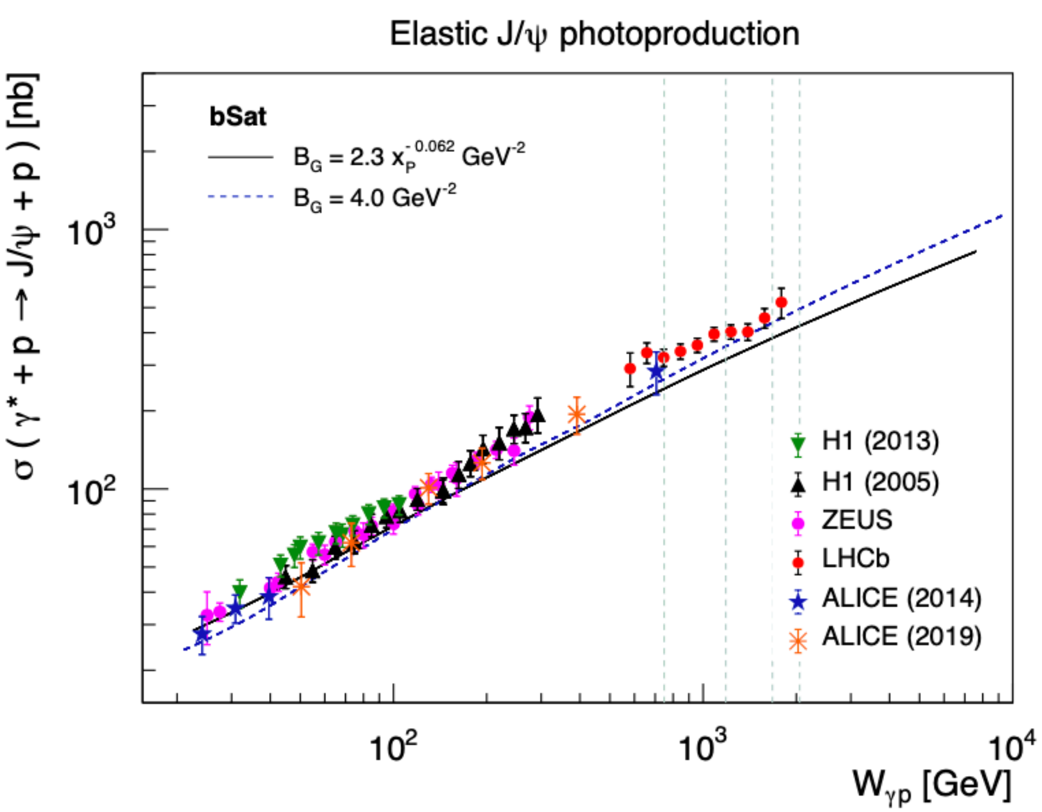
A Kumar, TT, Phys.Rev.D 105 (2022) 11, 114011 arXiv: 2202.06631



$$T_p(b) \to T(b, x_{I\!P}) = \frac{1}{2\pi B_G(x_{I\!P})} e^{-\frac{b^2}{2B_G(x_{I\!P})}}$$

$$x_{I\!P} = \frac{M_V^2 + Q^2 + |t|}{W_{\gamma p}^2 + Q^2 - m_p^2}$$

$$B_G(x_{I\!P}) = B_p x_{I\!P}^{\lambda_p}$$

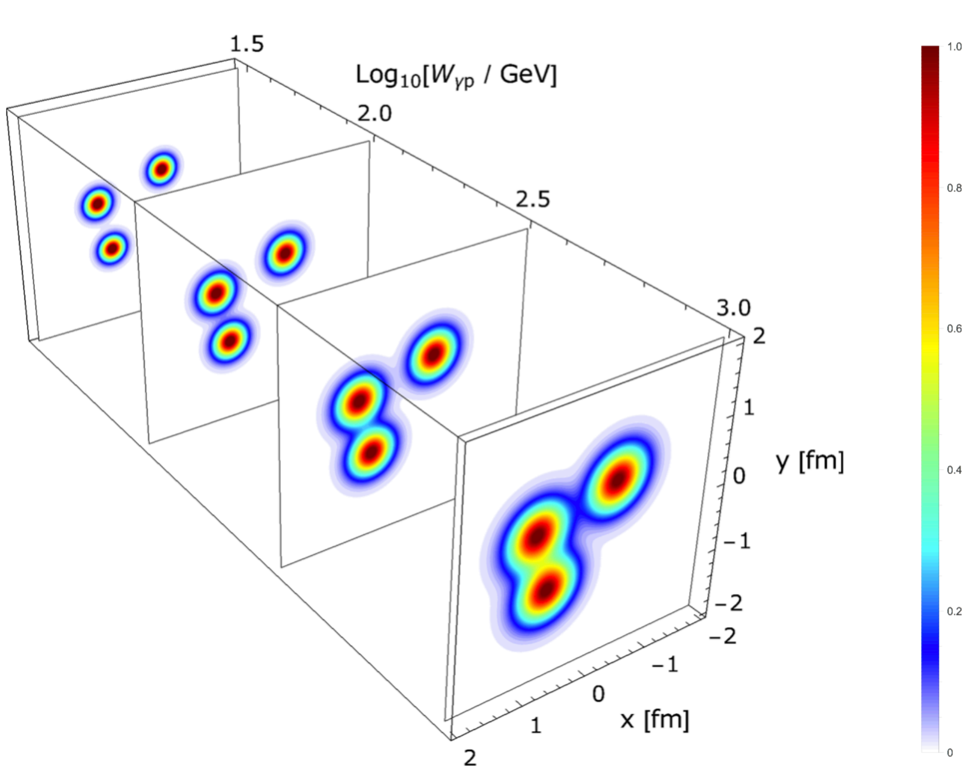


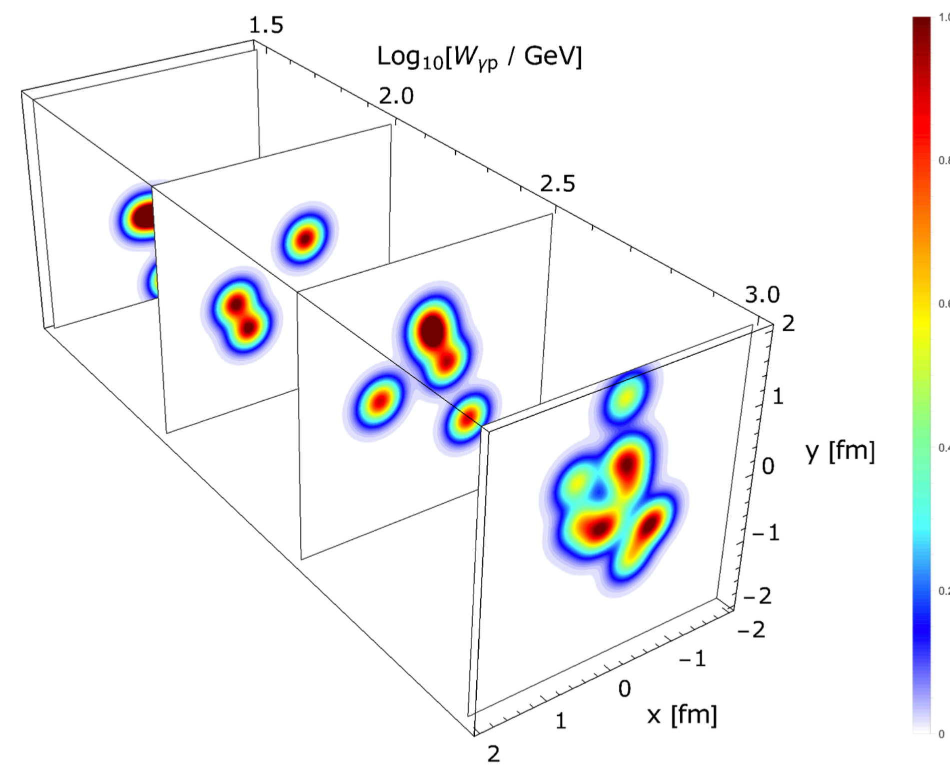
$$r_{\rm rms} = \sqrt{2B_G(x_{I\!P})}$$

$$B_p = 2.3 \text{ GeV}^{-2} \qquad \lambda_p = -0.062$$

The proton thickness revisited

A Kumar, TT, Phys.Rev.D 105 (2022) 11, 114011 arXiv: 2202.06631





$$T_p(b, x_{I\!P}) = \frac{1}{2\pi N_q B_q(x_{I\!P})} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b}-\vec{b}_i)^2}{2B_q(x_{I\!P})}}$$

$$T_p(b, x_{I\!P}) = \frac{1}{2\pi N_q(x_{I\!P}) B_q} \sum_{i=1}^{N_q(x_{I\!P})} e^{-\frac{(\vec{b}-\vec{b}_i)^2}{2B_q(x_{I\!P})}}$$

$$B_q(x_{I\!P}) = b_0 \ln^2 \frac{x_0}{x_{I\!P}}$$

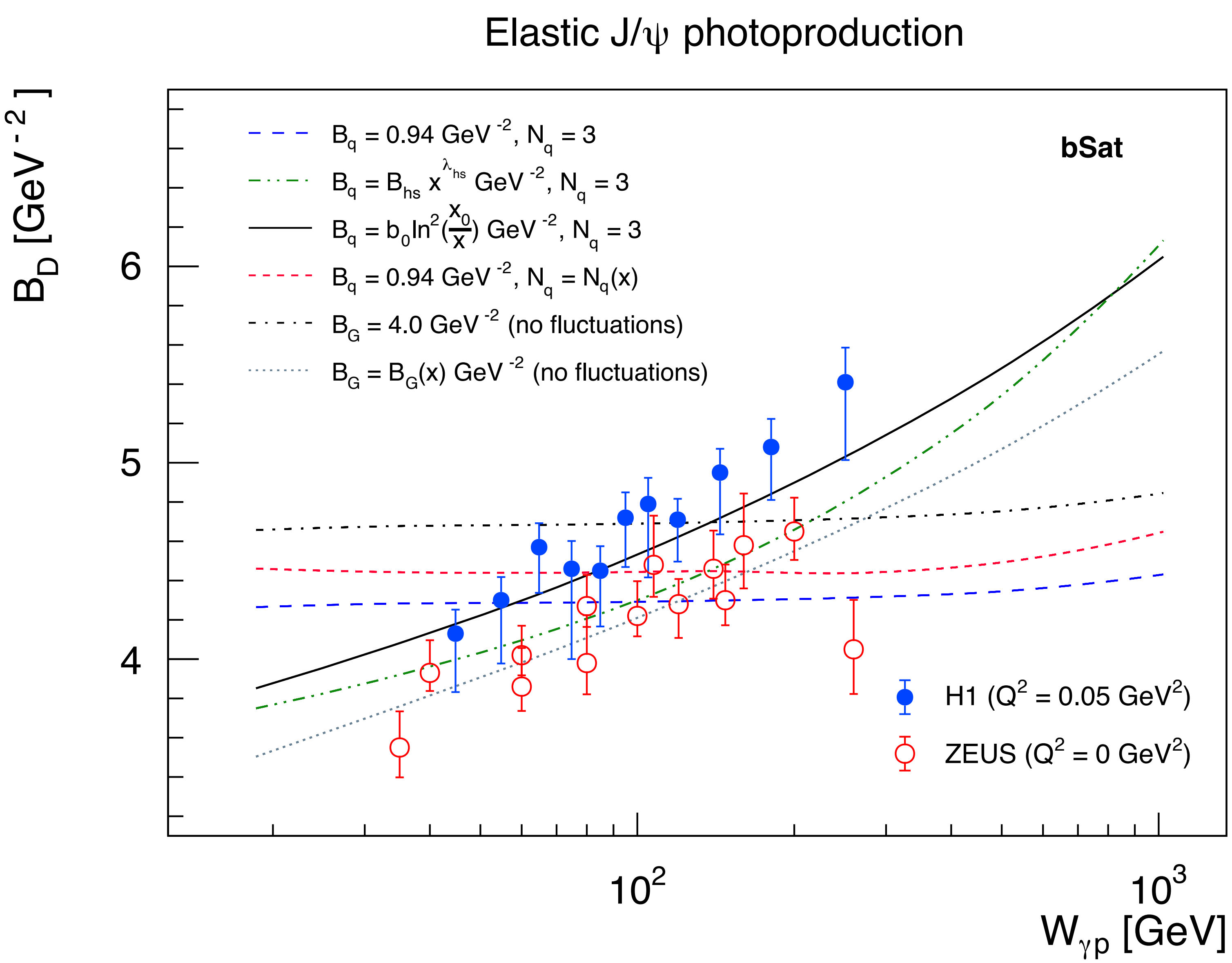
$$r_{\rm rms} = \sqrt{2(B_{qc} + B_q(x_{I\!P}))}$$

$$N_q \rightarrow N_q(x_P) = p_0\ x_{I\!P}^{p_1}(1+p_2\sqrt{x_{I\!P}})$$

$$p_0 = 0.011,\ p_1 = -0.56,\ p_2 = 165$$

J. Cepila, J. G. Contreras, J. D. Tapia Takaki,
Phys. Lett. B 766 (2017) 186–191.

The proton thickness revisited

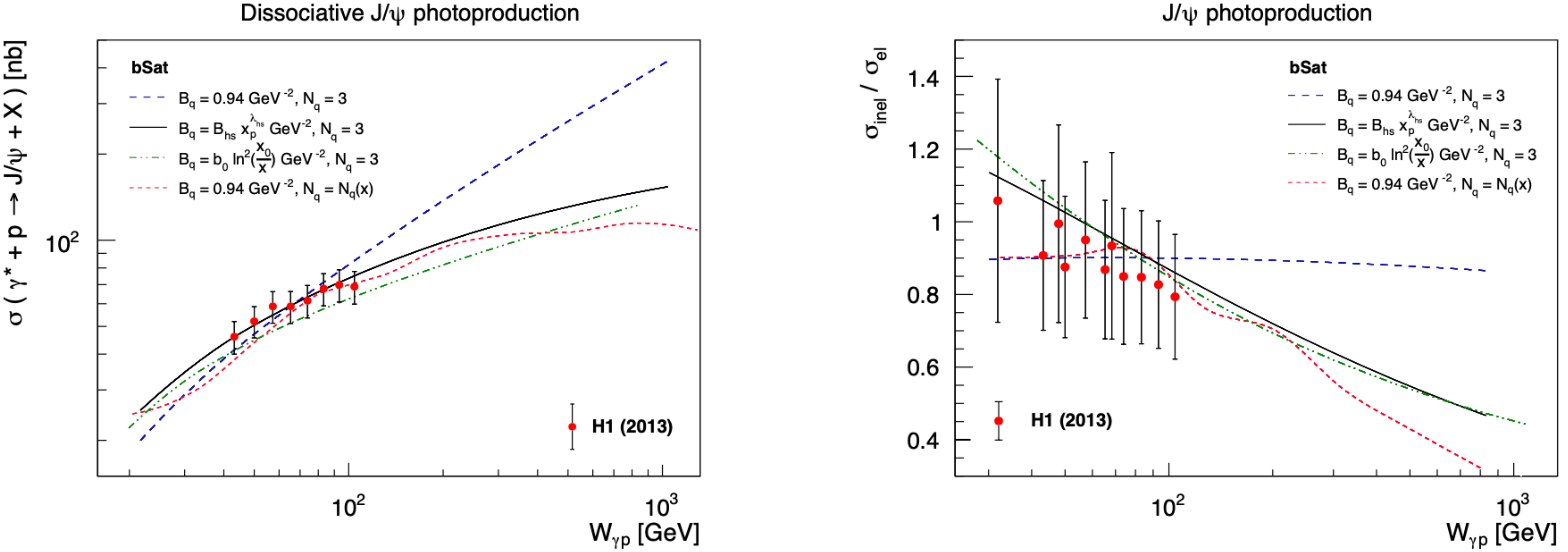


Arjun Kumar, TT, Phys.Rev.D 105 (2022) 11, 114011 arXiv: 2202.06631

The proton thickness revisited

The incoherent cross section gets suppressed as hotspots begin to overlap!

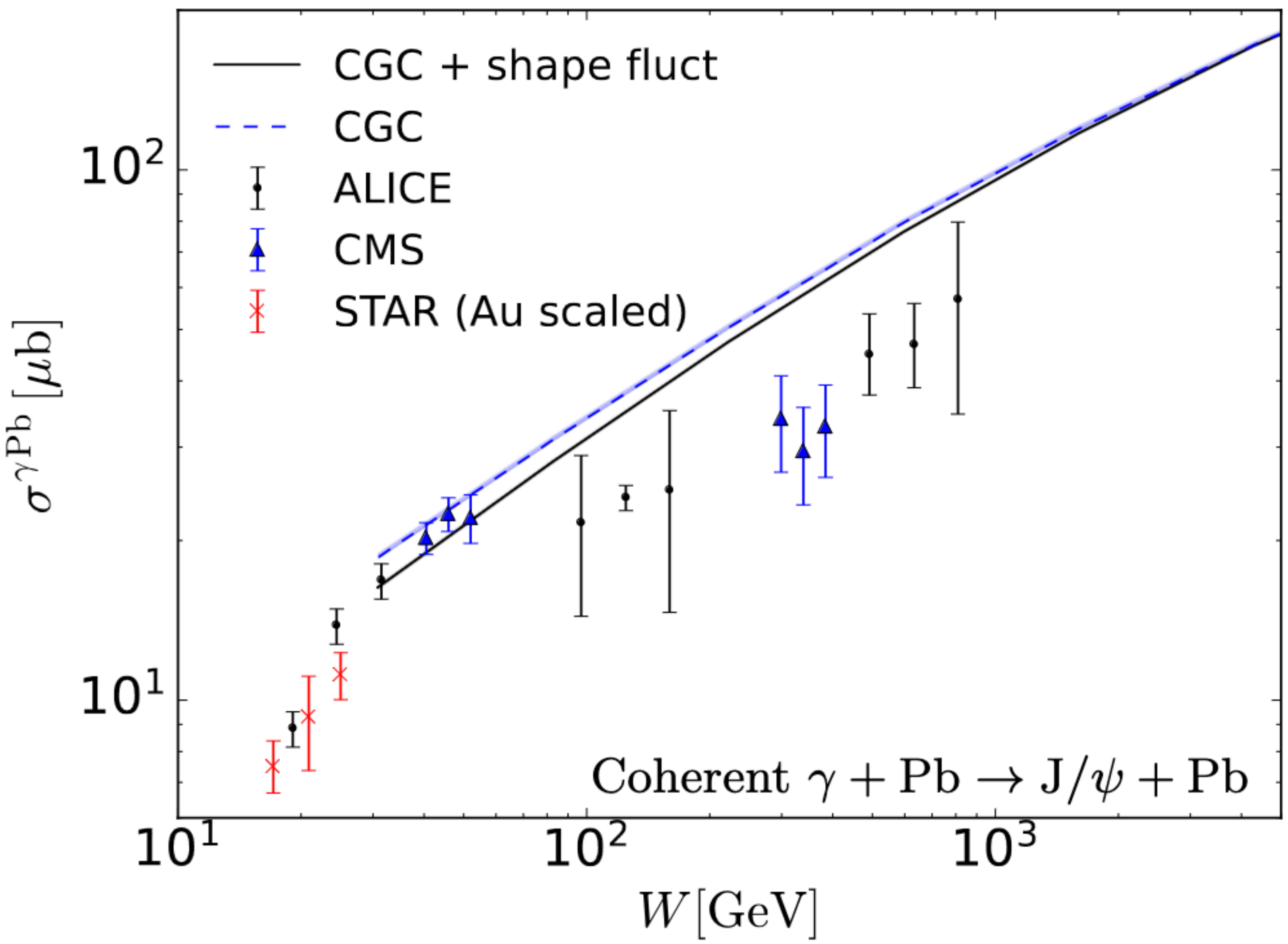
Arjun Kumar, TT, Phys.Rev.D 105 (2022) 11, 114011 arXiv: 2202.06631

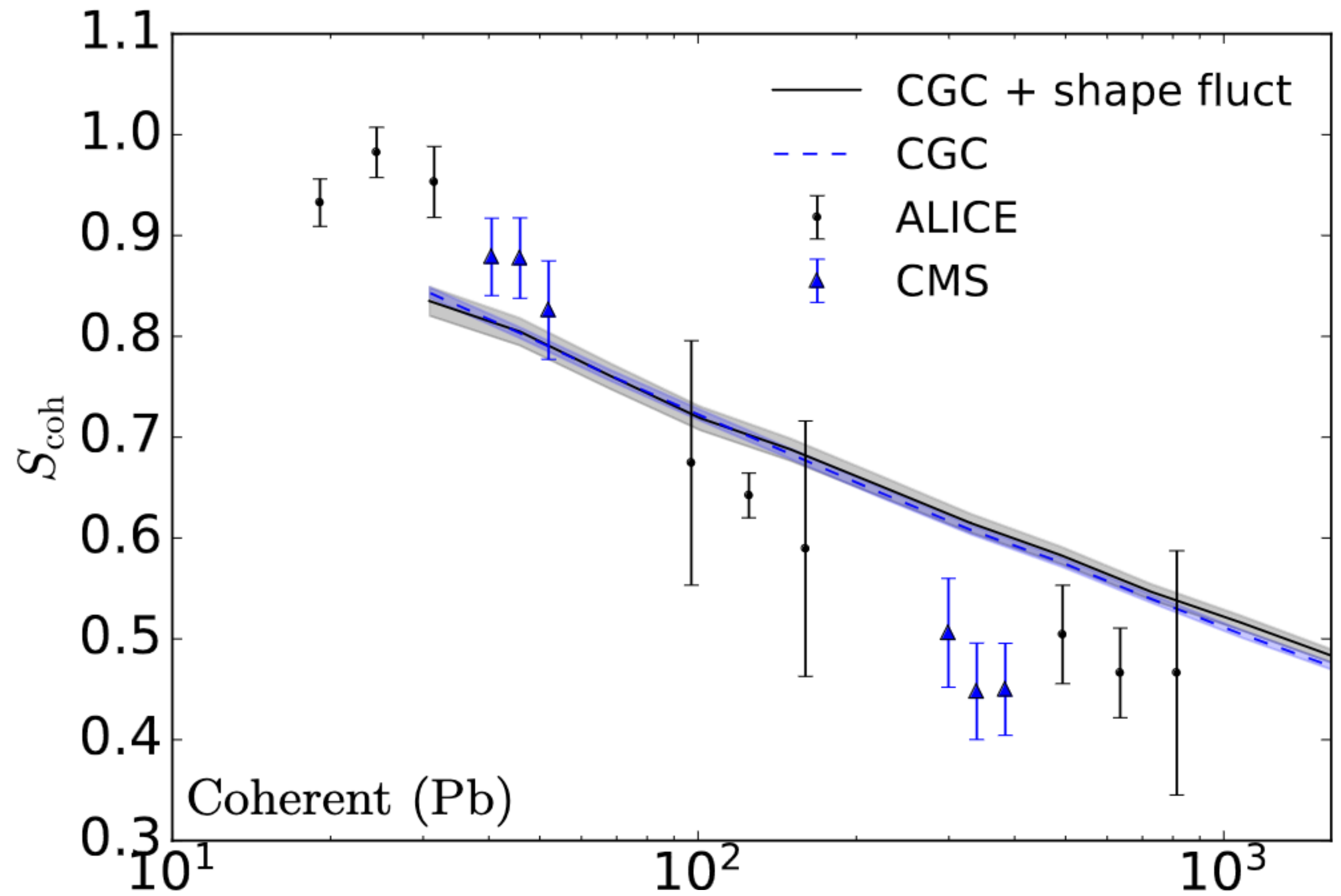


For similar predictions in the IP-Glasma framework, see:
H. Mäntysaari, B. Schenke, Phys.Rev.D 98 (2018) 3, 034013; B. Schenke, Rept. Prog. Phys. 84 (2021) 8, 082301

The proton thickness revisited

Tension in the heavy ion data





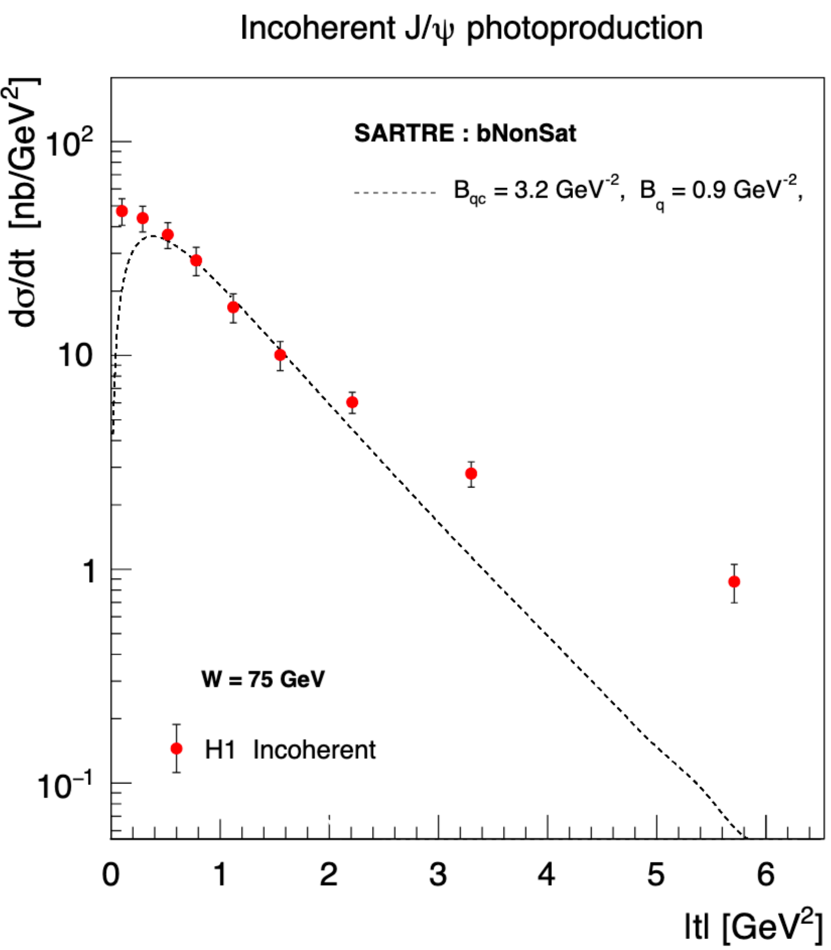
$$S_{\mathrm{coh}} = \sqrt{\frac{\sigma^{\gamma A}}{\sigma^{\mathrm{IA}}}}.$$

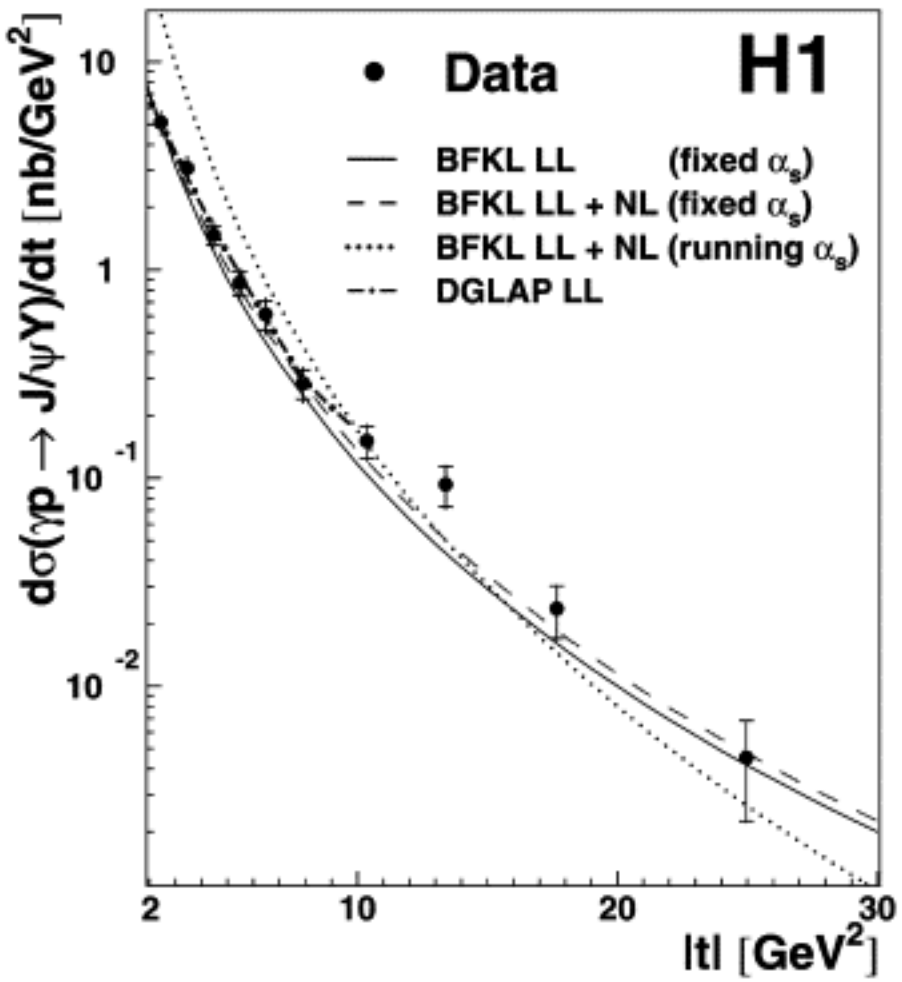
Heikki Mäntysaari, Farid Salazar, Björn Schenke e-Print: 2312.04194 [hep-ph]

$$\sigma^{\mathrm{IA}} = \frac{\mathrm{d}\sigma^{\gamma p}}{\mathrm{d}t}(t=0) \int_{-t_{\min}} \mathrm{d}t\, |F(t)|^2$$

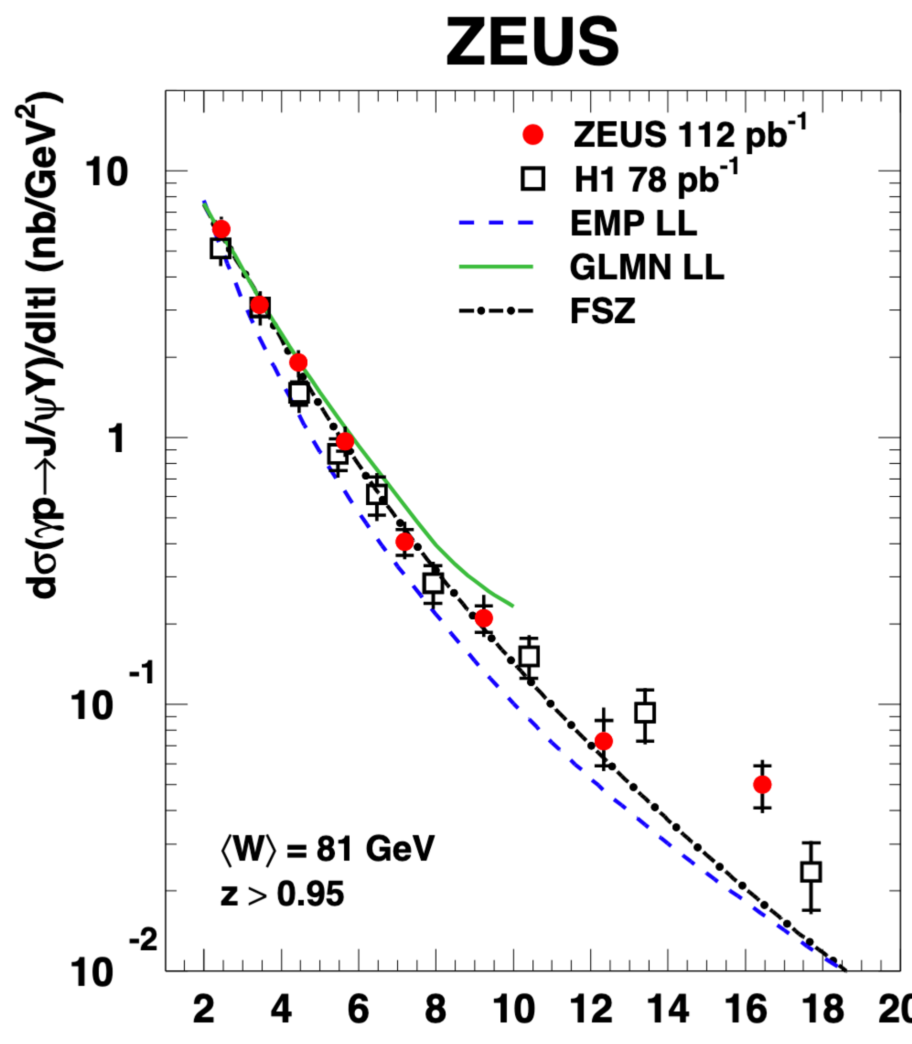
Large $|t|$?

Large $|t|$?





Phys. Lett. B 568 (2003) 205–218

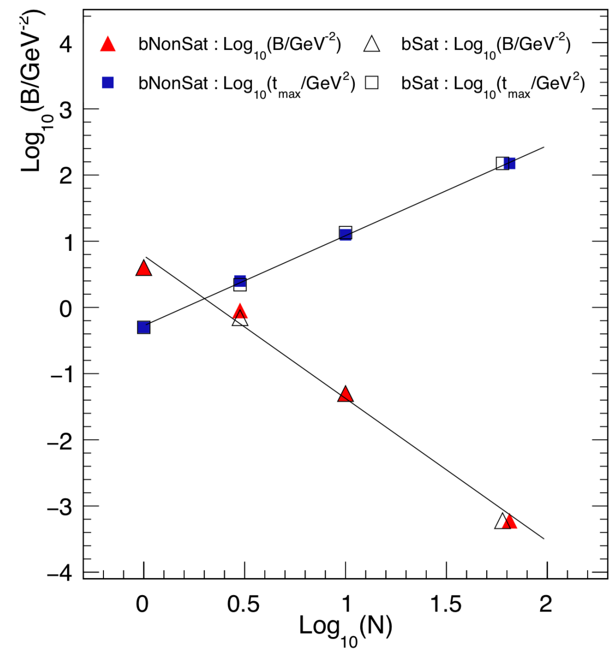


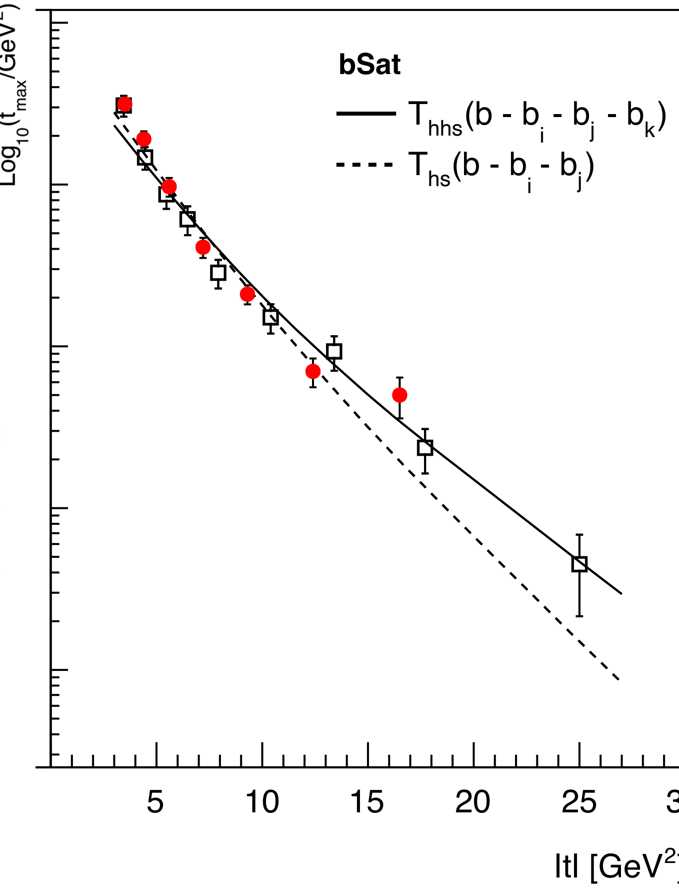
JHEP 05 (2010) 085

Non-perturbative phenomenology. Only valid for $|t| \lesssim 1\ \mathrm{GeV}^2$.
What about larger $|t|$?

Insights

Arjun Kumar, TT, Eur.Phys.J.C 82 (2022) 9, 837, arXiv: 2106.12855





The transverse gluon structure:

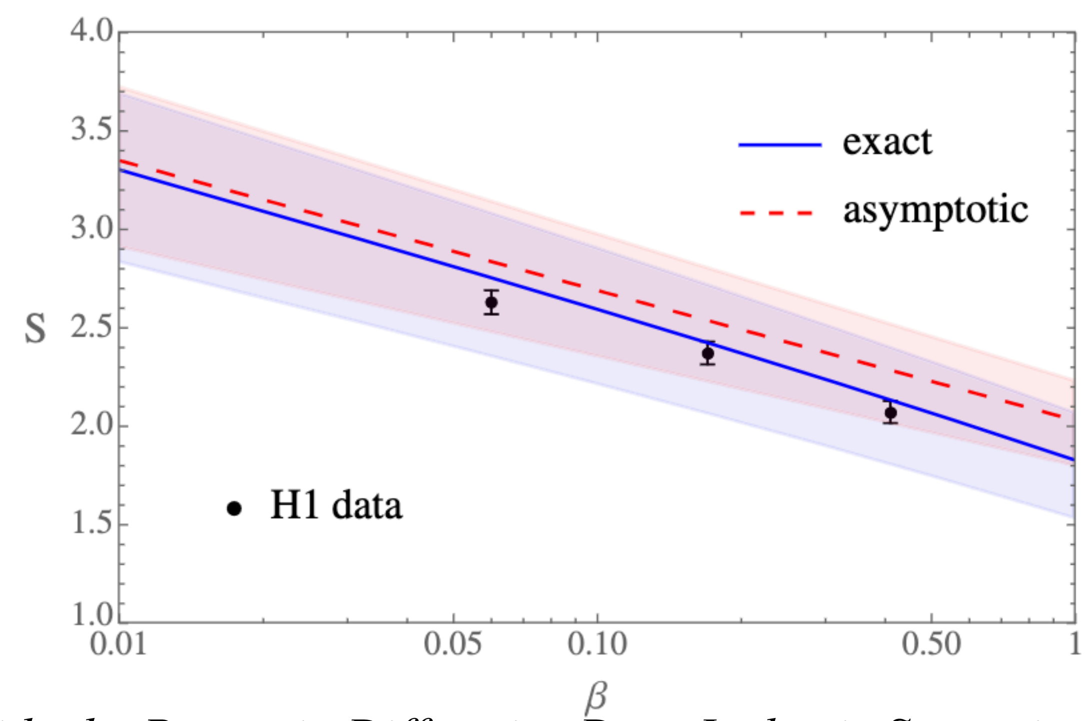
1. t-spectrum can be described by a self-similar structure of hotspots within hotspots

2. Small-x partons are maximally entangled (described by the same wave function)

This suggests that we can describe the hotspot t-spectrum with a linear, scale-independent (in $\log|t|$) evolution

Picture: Transverse part of gluon wavefunction probed with areal resolution $\delta b^2 \sim \frac{1}{|t|}$

Wavefunction collapses into this area.
Increased resolution appears as hotspots splittings.



Probing the Onset of Maximal Entanglement inside the Proton in Diffractive Deep Inelastic Scattering, Hentschinski, Kharzeev, Kutak, Tu: Phys.Rev.Lett. 131 (2023) 24, 241901

Hotspot Evolution

We consider a parton shower-like evolution based on resolution, where a hotspot may split into two as the resolution increases.

Initial State at $t = t_0$:

$$T_p(\vec{b}) = \frac{1}{N_q}\sum_{i=1}^{N_q} T_q(|\vec{b} - \vec{b}_i|)$$

$$T_q(\vec{b}) = \frac{1}{2\pi B_q} e^{-\frac{b^2}{2B_q}}$$

Inital State Parameters:

$B_{qc} = 3.1\ \mathrm{GeV}^{-2}$

$B_q = 1.25\ \mathrm{GeV}^{-2}$

$N_q = 3$

Probability of a hotspot created at t_0 splitting at $|t| > |t_0|$

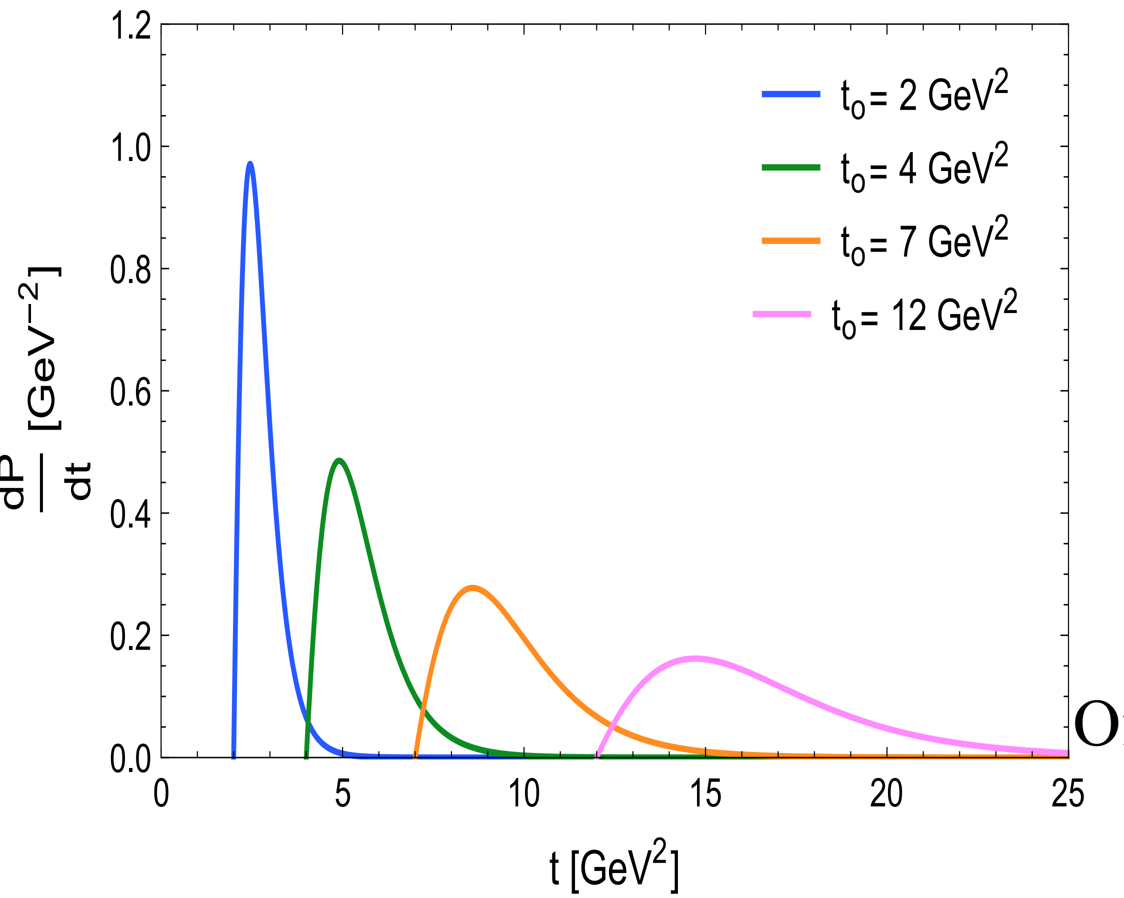
$$\frac{\mathrm{d}P_{\mathrm{split}}}{\mathrm{d}t} = \frac{\alpha}{|t|}\frac{t-t_0}{t}$$

$$\frac{\mathrm{d}P_{\mathrm{nosplit}}}{\mathrm{d}t} = \exp\left(-\int_{t_0}^{t} \mathrm{d}t' \frac{\mathrm{d}P_{\mathrm{split}}}{\mathrm{d}t'}\right)$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\alpha}{|t|}\frac{t-t_0}{t}\exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$

Hotspot Evolution

We consider a parton shower-like evolution based on resolution, where a hotspot may split into two as the resolution increases.



Probability of a hotspot created at t_0 splitting at $|t| > |t_0|$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\alpha}{|t|}\frac{t-t_0}{t}\exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$

Generate offspring $\vec{b}_{i,j}$ from parent $T_{\mathrm{parent}}(\vec{b}_{i,j})$.

Conserve Normalisation in each splitting.

Offspring hotspots i, j created at distance $d_{ij} = |\vec{b}_i - \vec{b}_j|$,

with widths $B_{i,j} = \dfrac{1}{|t|}$

Conditions for resolution:

Probe resolution: $d_{ij} > \dfrac{2}{|\vec{\Delta}|}$ Geometry: $d_{ij} > 2\sqrt{B_{i,j}}$

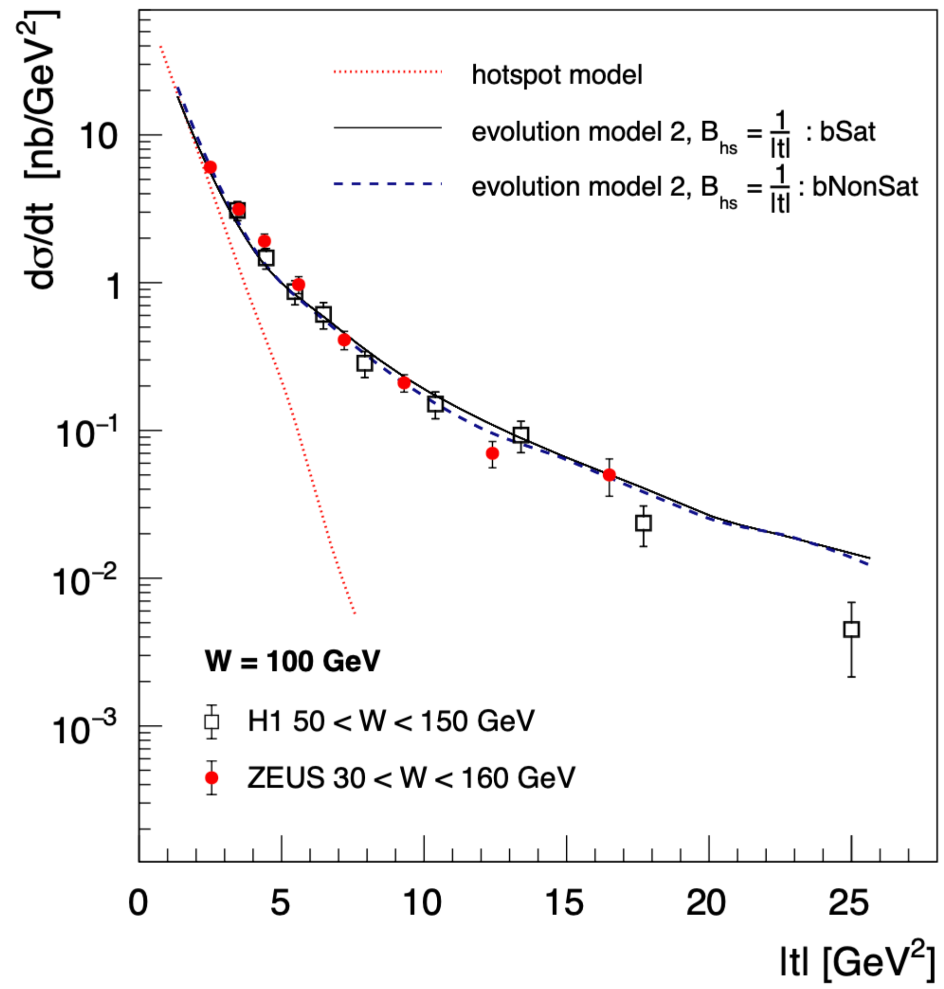
Reject if not resolved.

This becomes an effective hotspot repulsion.

Hotspot Evolution

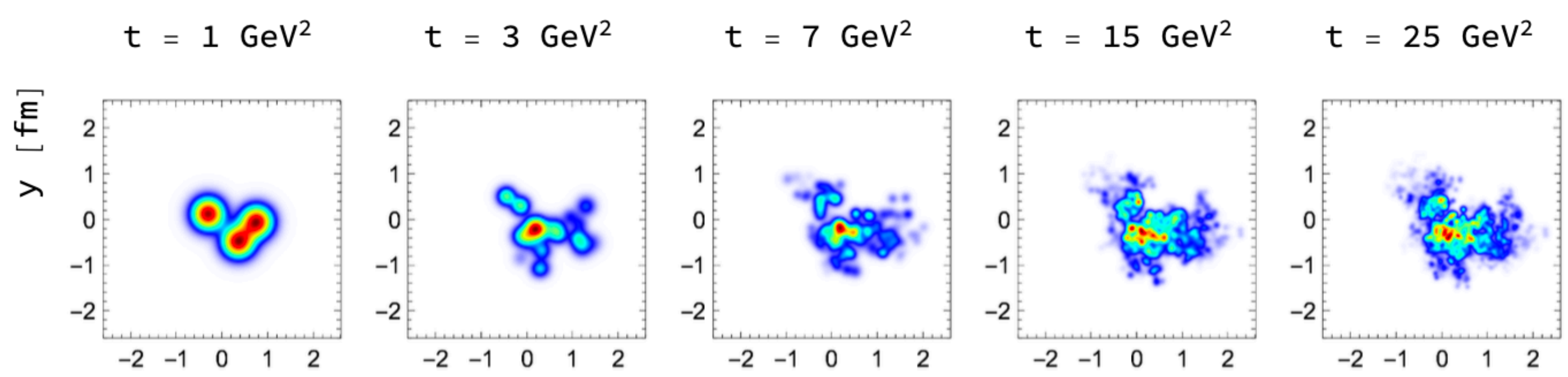
$$\frac{dP}{dt} = \frac{\alpha}{|t|}\frac{t-t_0}{t}\exp\left[-\alpha\left(\frac{t_0}{t} - \ln\frac{t_0}{t} - 1\right)\right]$$

$\alpha = 18.5$
$t_0 = 1.1\ \mathrm{GeV}^2$

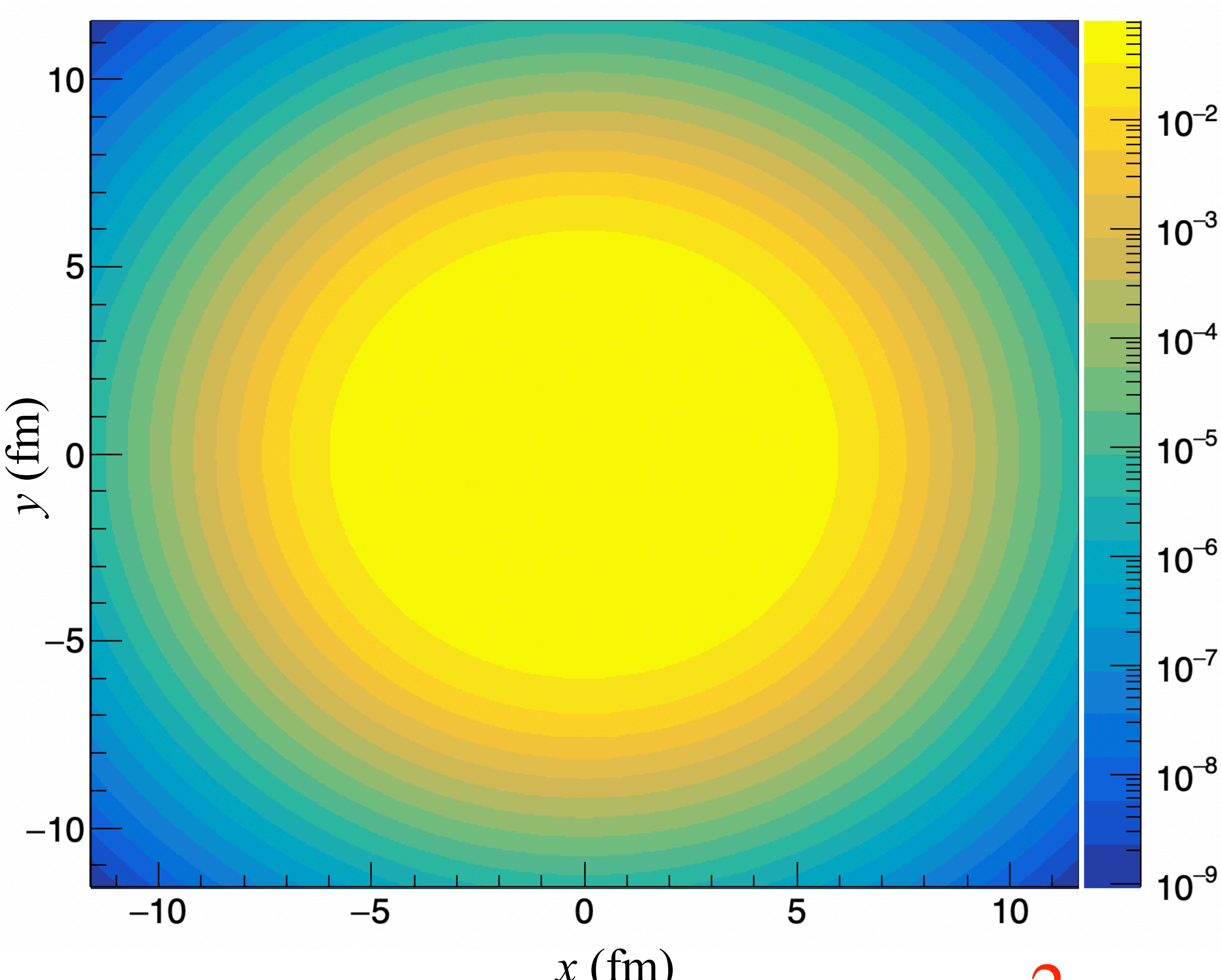


Model can describe all data points for $|t| > |t_0| = 1.1\ \mathrm{GeV}^2$ with only one extra parameter α.

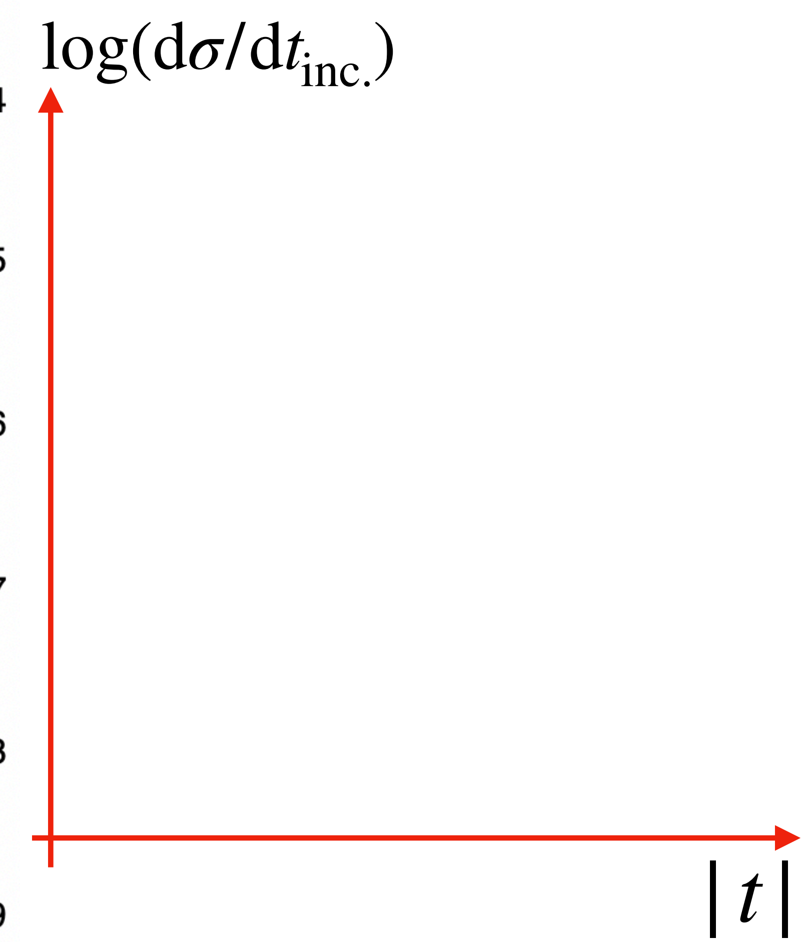
$$T(b) \rightarrow T(b, t)$$



Into the heavy nucleus



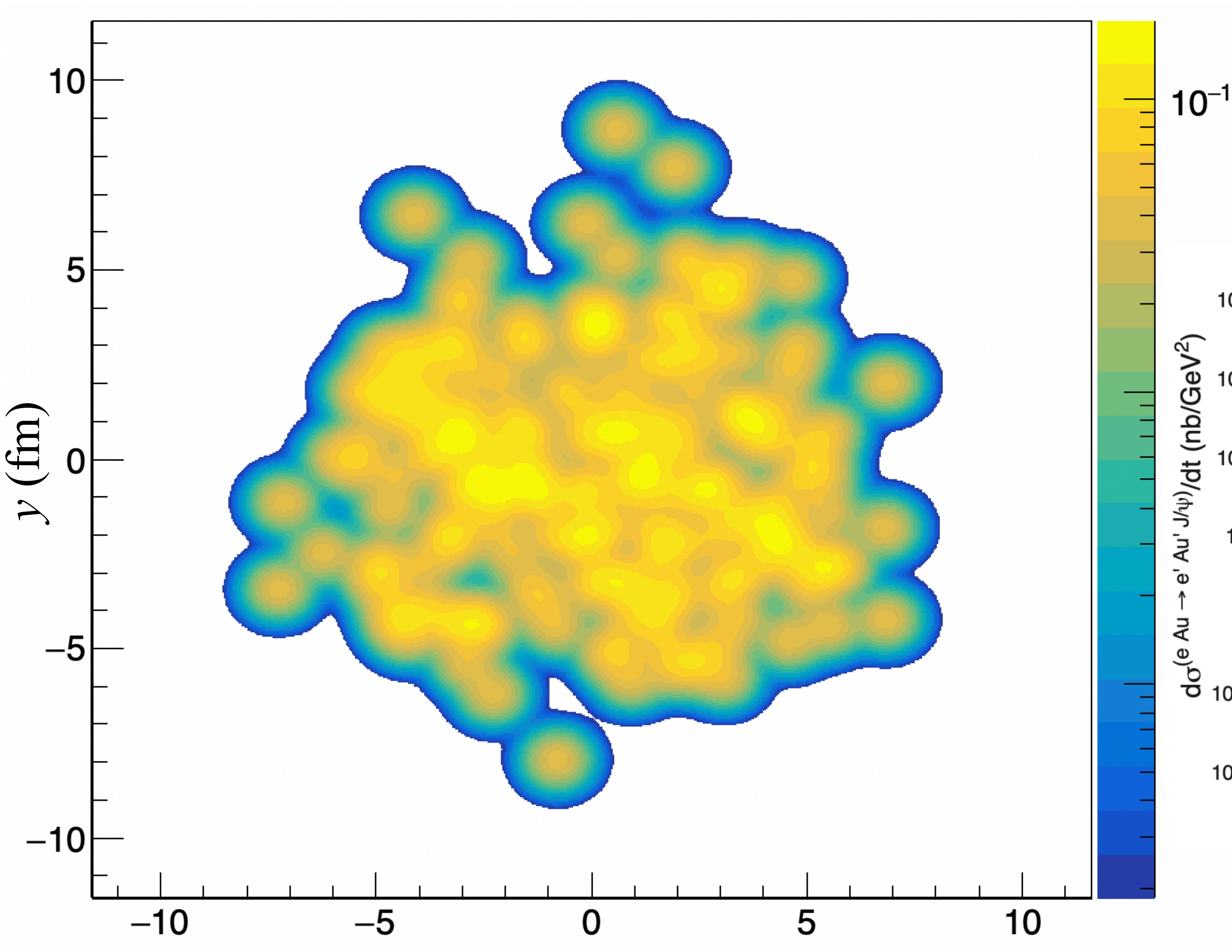
$|t| = 0$



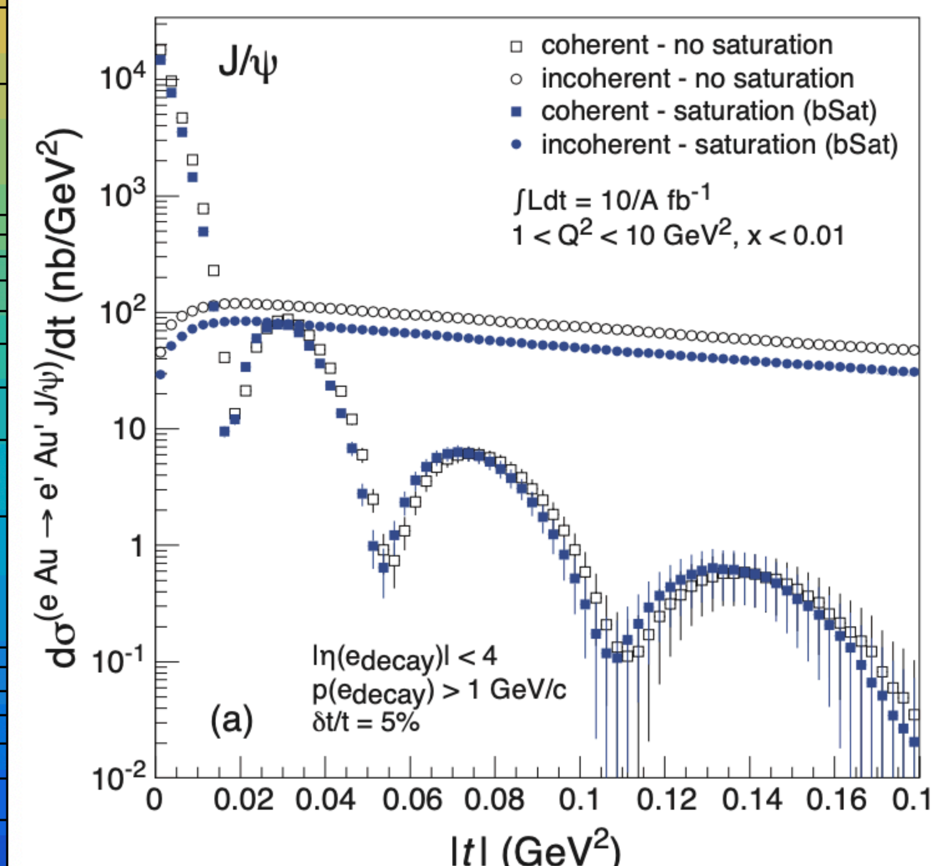
Max thickness: $\sim 1 \cdot 10^{-1}$ GeV^2

$Q_S^2 \simeq T(b)$

Into the heavy nucleus



$|t| \lesssim 0.2\ \text{GeV}^2$

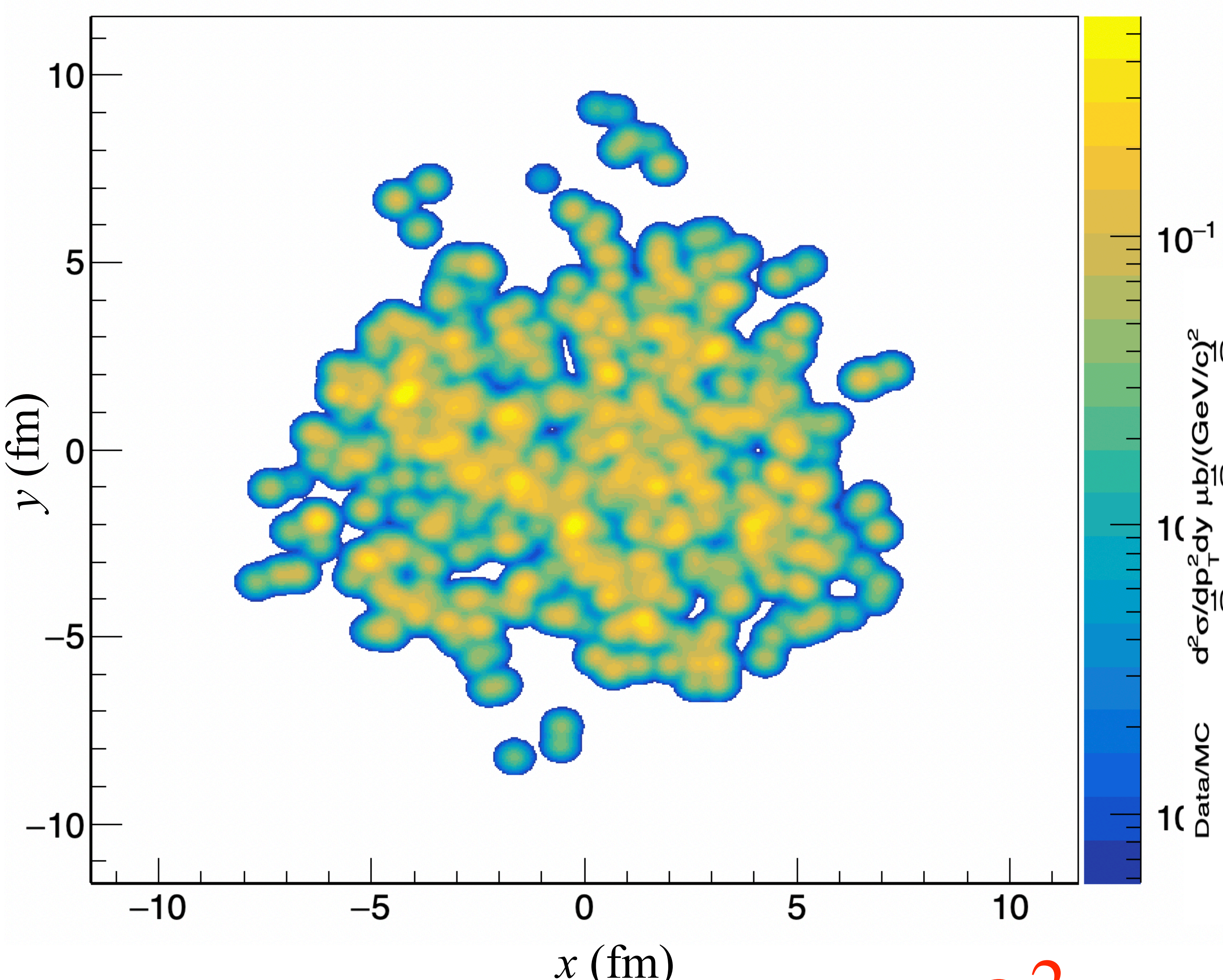


$|t|$

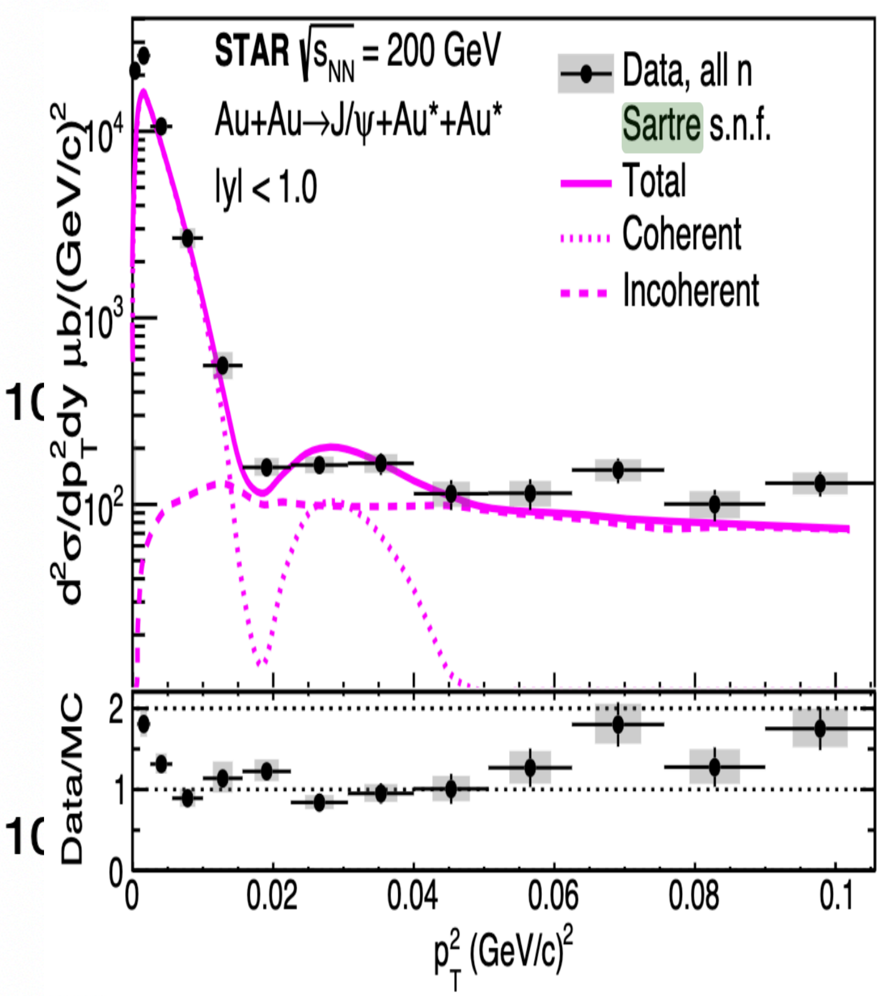
Max thickness: $\sim 2 \cdot 10^{-1}\ \text{GeV}^2$

$Q_S^2 \simeq T(b)$

Into the heavy nucleus



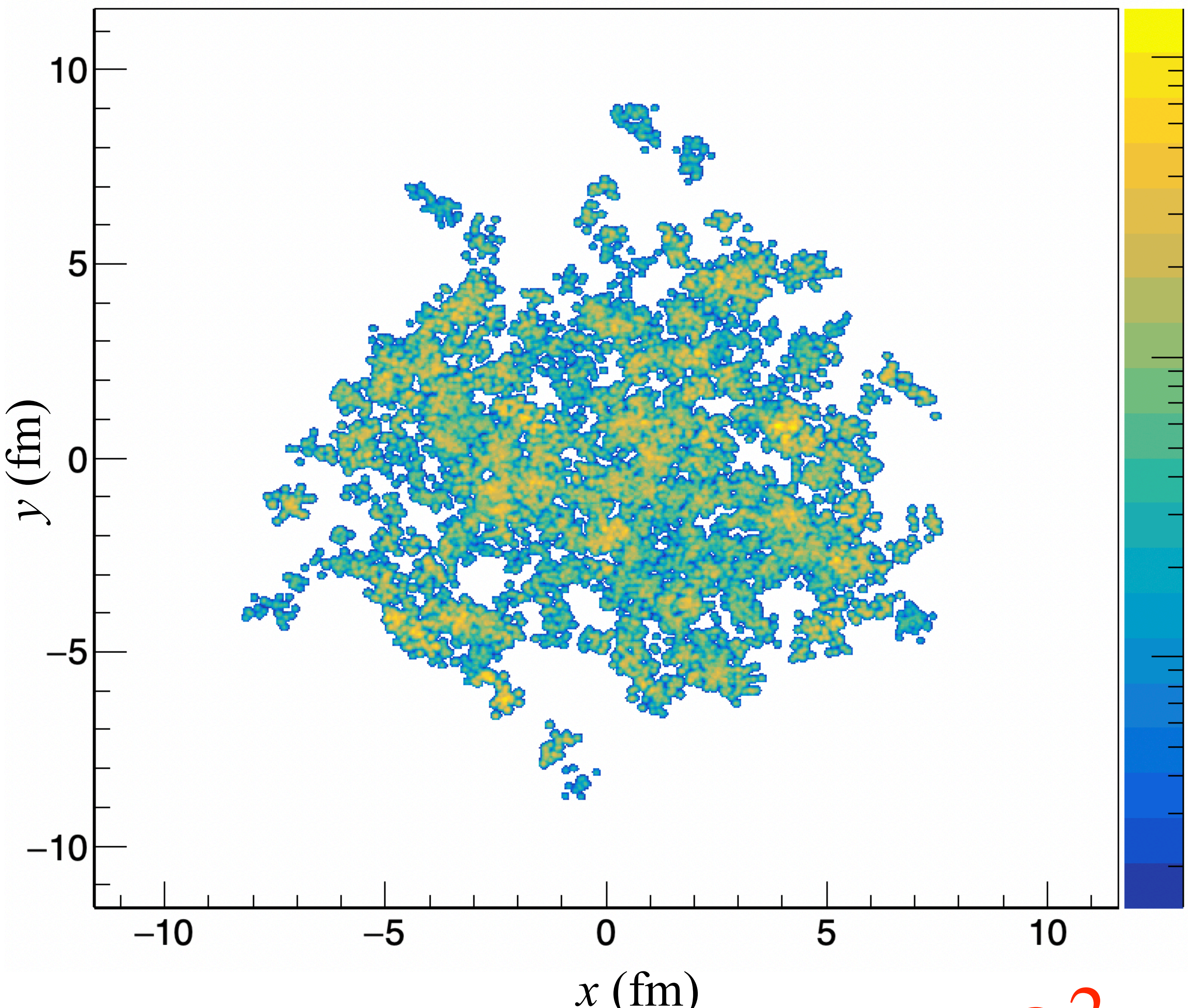
$0.2 \lesssim |t| \lesssim 2\ \mathrm{GeV}^2$



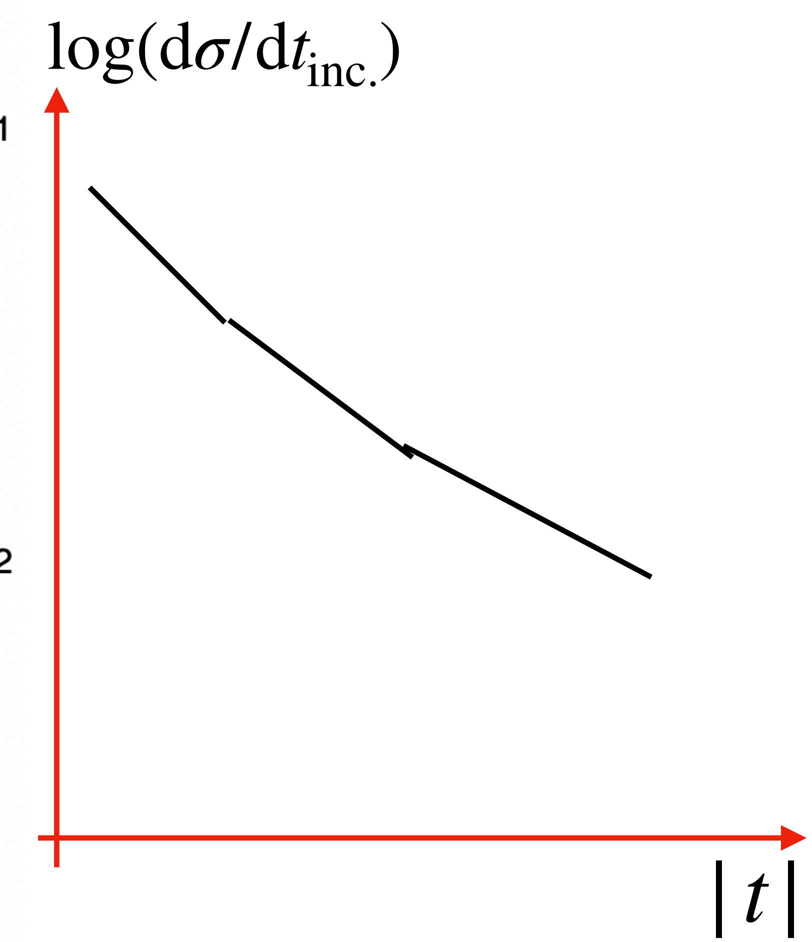
Max thickness: $\sim 6 \cdot 10^{-1}\ \mathrm{GeV}^2$

$Q_S^2 \simeq T(b)$

Into the heavy nucleus



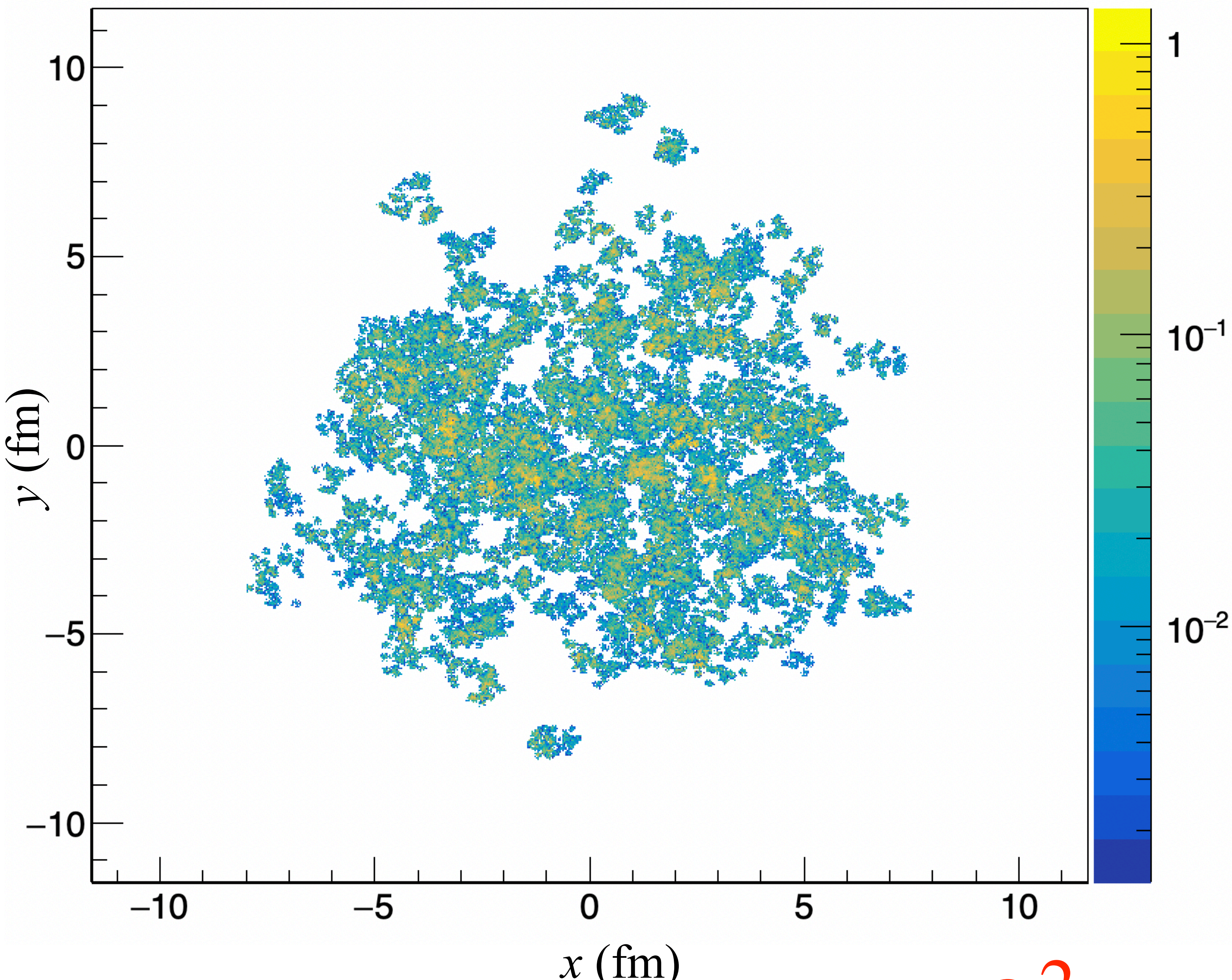
$2 \lesssim |t| \lesssim 12 \text{ GeV}^2$



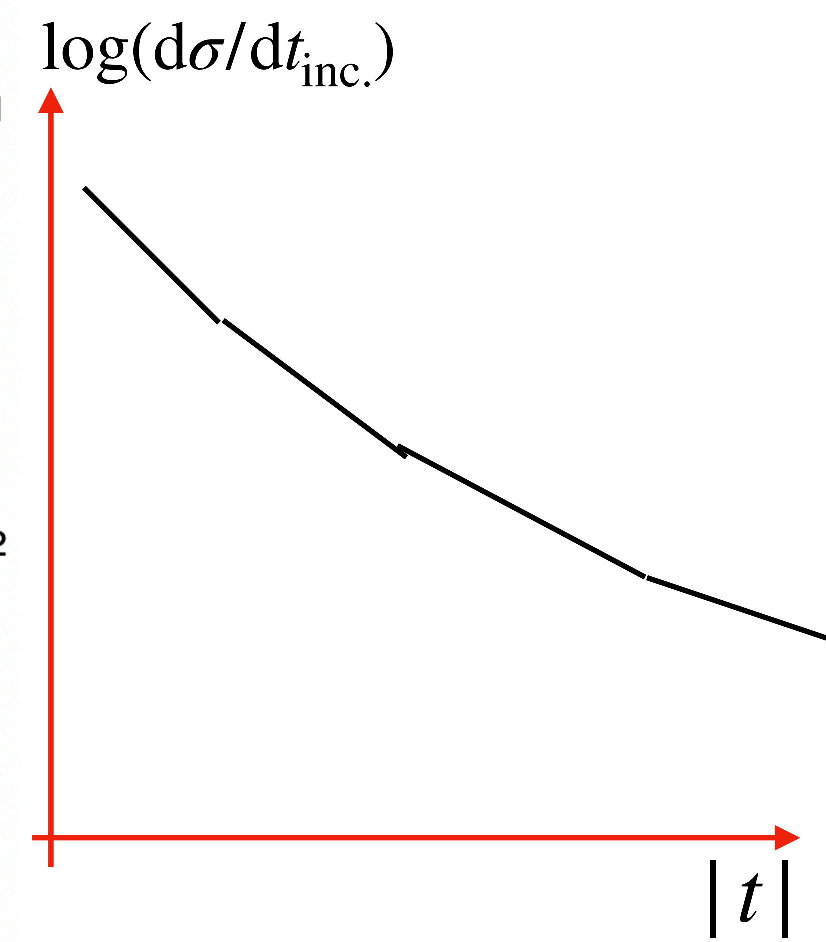
Max thickness: $\sim 1 \cdot 10^0 \text{ GeV}^2$

$$Q_S^2 \simeq T(b)$$

Into the heavy nucleus



$12 \lesssim |t|$ GeV^2

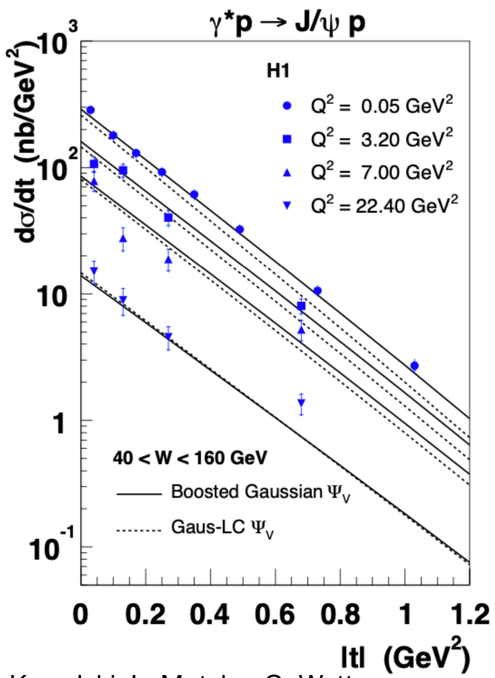


Max thickness: $\sim 1 \cdot 10^0$ GeV^2

$$Q_S^2 \simeq T(b)$$

Summary

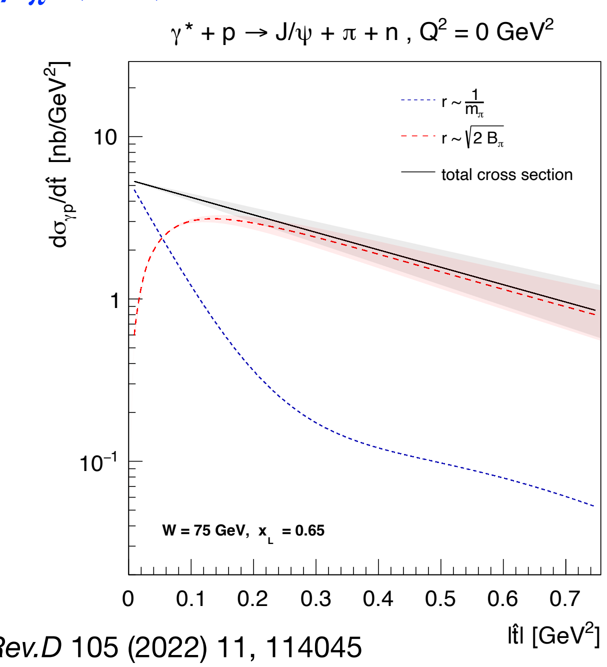
$$T_p(b) = \frac{1}{2\pi B_G} e^{-\frac{b^2}{2B_G}}$$



H. Kowalski, L. Motyka, G. Watt, Phys.Rev.D 74 (2006) 074016

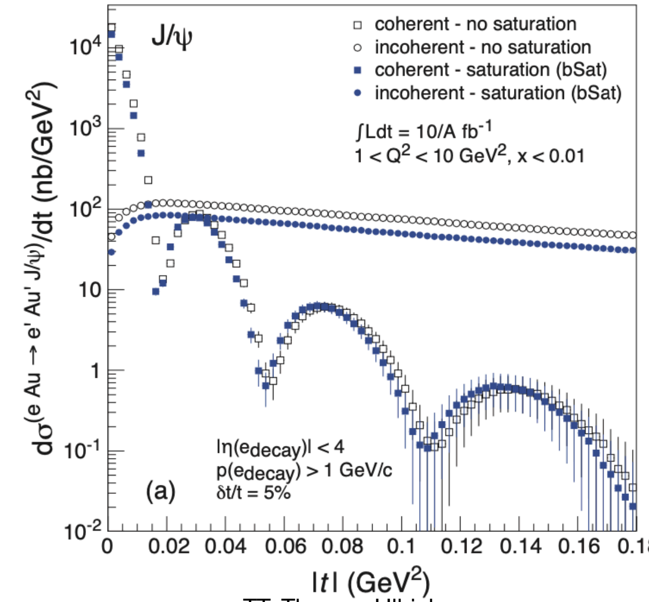
$$\rho_{\pi^*}(b,z) = \frac{m_\pi^2}{4\pi} \frac{e^{-m_\pi\sqrt{(b^2+z^2)}}}{\sqrt{b^2+z^2}} \qquad T_\pi(b) = \frac{1}{2\pi B_\pi} e^{-\frac{-b^2}{2B_\pi}}$$

$$T_{\pi^*}(b) = \int_{-\infty}^{\infty} \mathrm{d}z \rho_{\pi^*}(b,z)$$



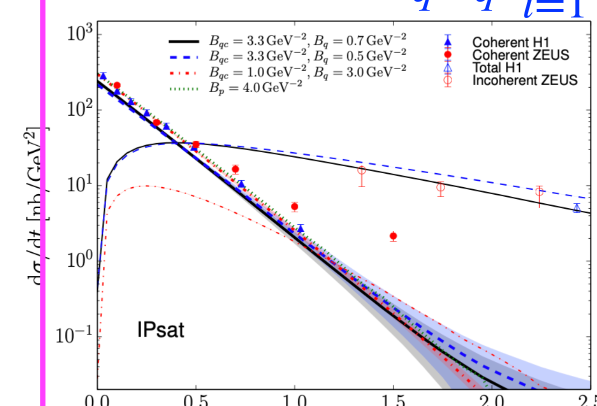
A. Kumar, TT: *Phys.Rev.D* 105 (2022) 11, 114045

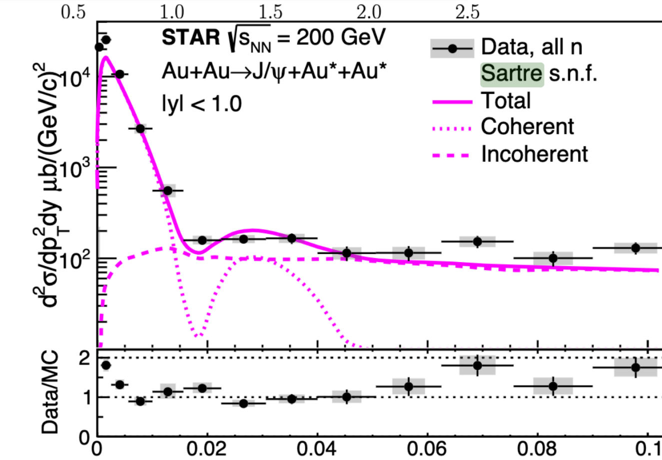
$$T_A(\vec{b}) = \sum_{i=1}^{A} T_p(|\vec{b} - \vec{b}_i|)$$



TT, Thomas Ullrich
Phys.Rev.C 87 (2013) 2, 024913, arXiv: 1211.3048
Comput.Phys.Commun. 185 (2014) 1835-1853 arXiv:1307.8059

$$T_p(b) = \frac{1}{2\pi N_q B_q} \sum_{i=1}^{N_q} e^{-\frac{(\vec{b}-\vec{b}_i)^2}{2B_q}}$$

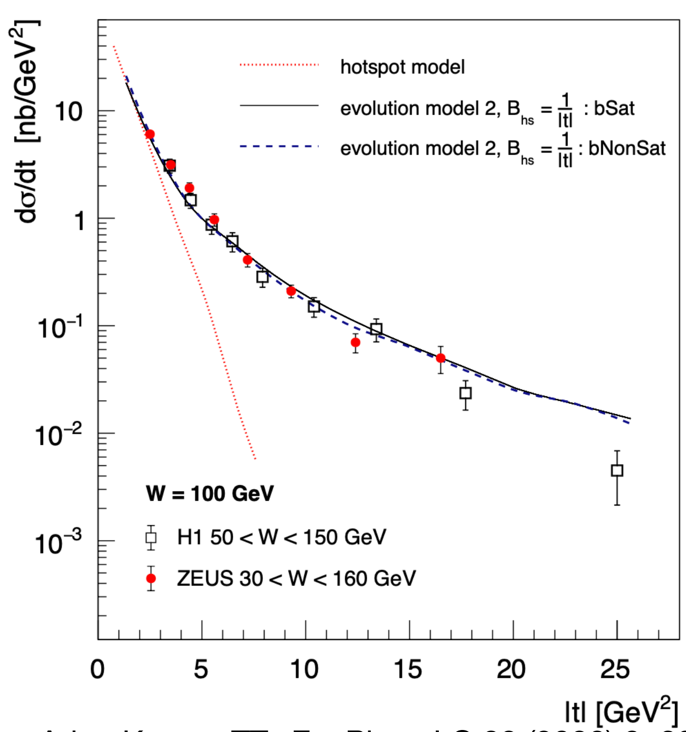




H. Mäntysaari, B. Schenke Phys. Rev. Lett. 117(5)

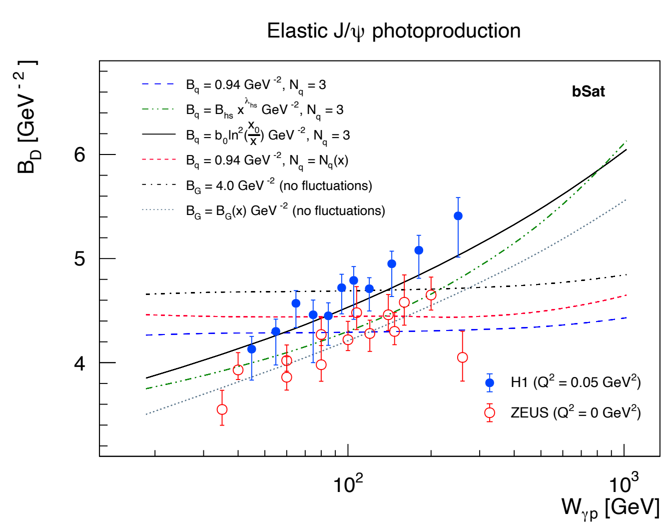
STAR Collaboration, e-Print: 2311.13632 [nucl-ex]

$$T(b) \rightarrow T(b,t)$$



Arjun Kumar, TT, Eur.Phys.J.C 82 (2022) 9, 837

$$T(b) \rightarrow T(b, x_{I\!P})$$



A Kumar, TT, Phys.Rev.D 105 (2022) 11, 114011

Outlook

Still to do:

Extend all these studies to eA

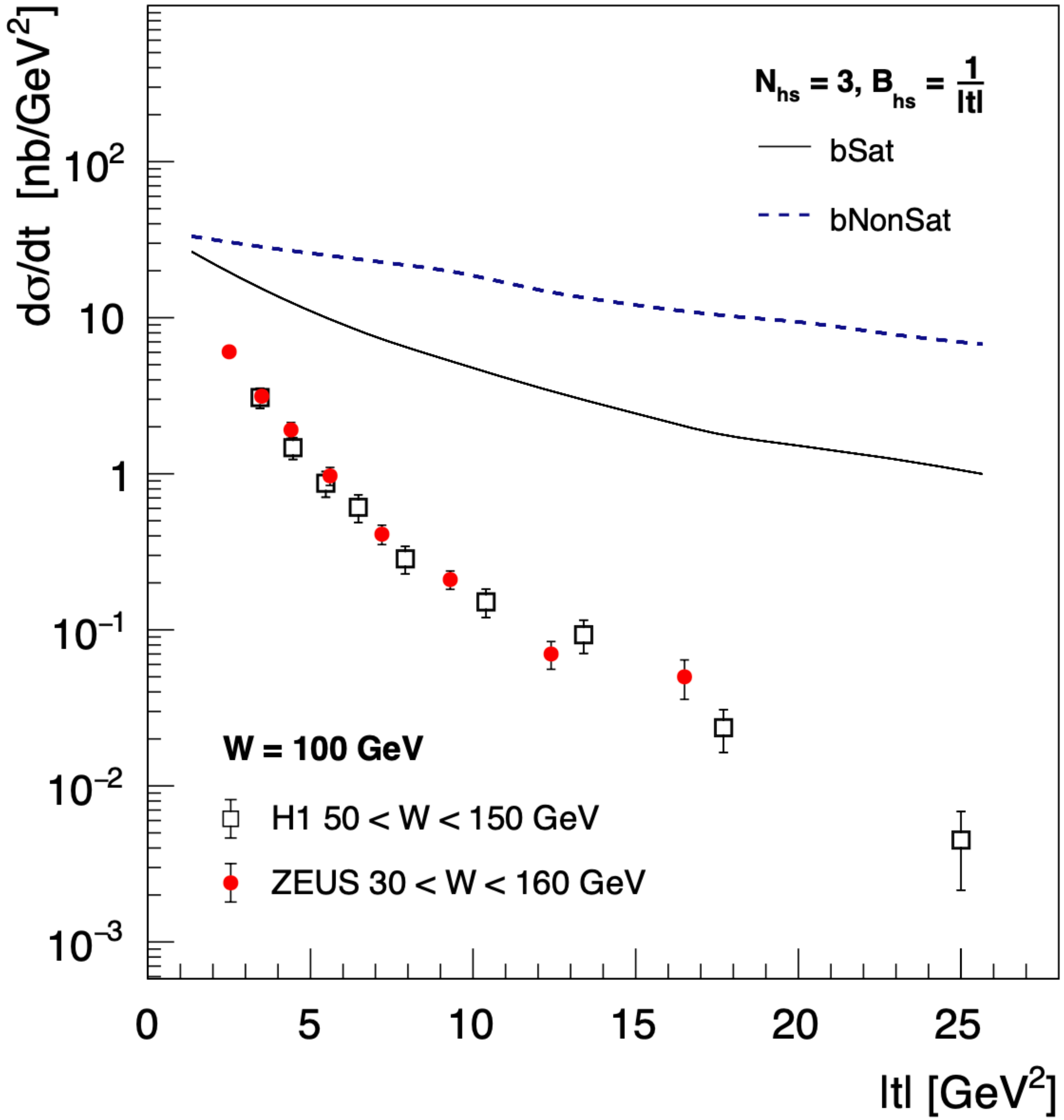
Investigate $T(b) \rightarrow T(b, x_{I\!P}, t)$

Implement in Sar*t*re utilising the thickness functions.

…and much more.

All these compelling processes can be measured at the EIC

Back Up

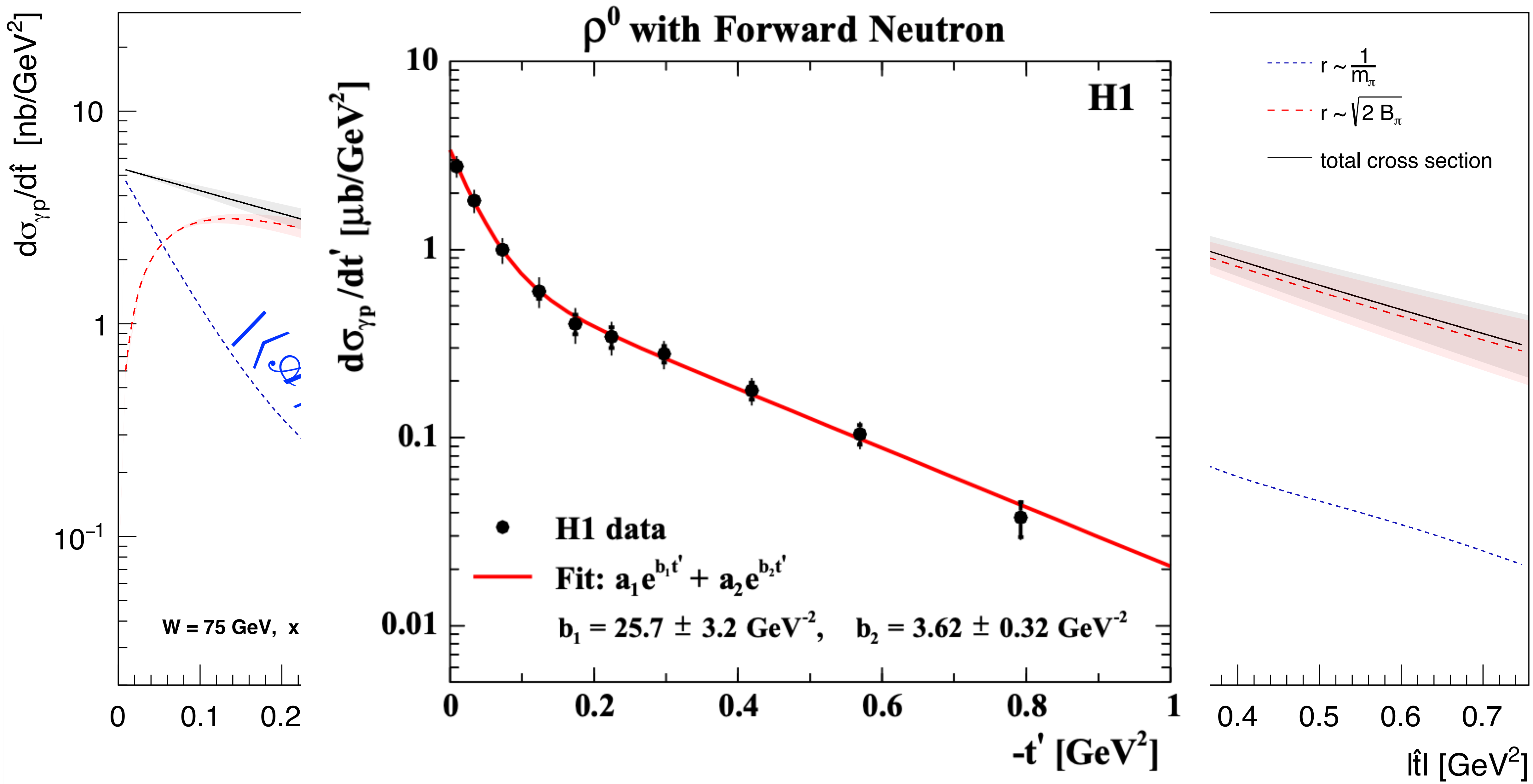


$N_{hs} = 3, B_{hs} = \frac{1}{|t|}$
bSat
bNonSat
W = 100 GeV
H1 50 < W < 150 GeV
ZEUS 30 < W < 160 GeV
dσ/dt [nb/GeV²]
|t| [GeV²]

The Pion Thickness

$$\frac{d\sigma}{dt} \propto \left|\langle \mathscr{A} \rangle_{\pi^*}\right|^2 + \left(\left\langle |\mathscr{A}|^2 \right\rangle_{\pi^*} - \left|\langle \mathscr{A} \rangle_{\pi^*}\right|^2\right)$$

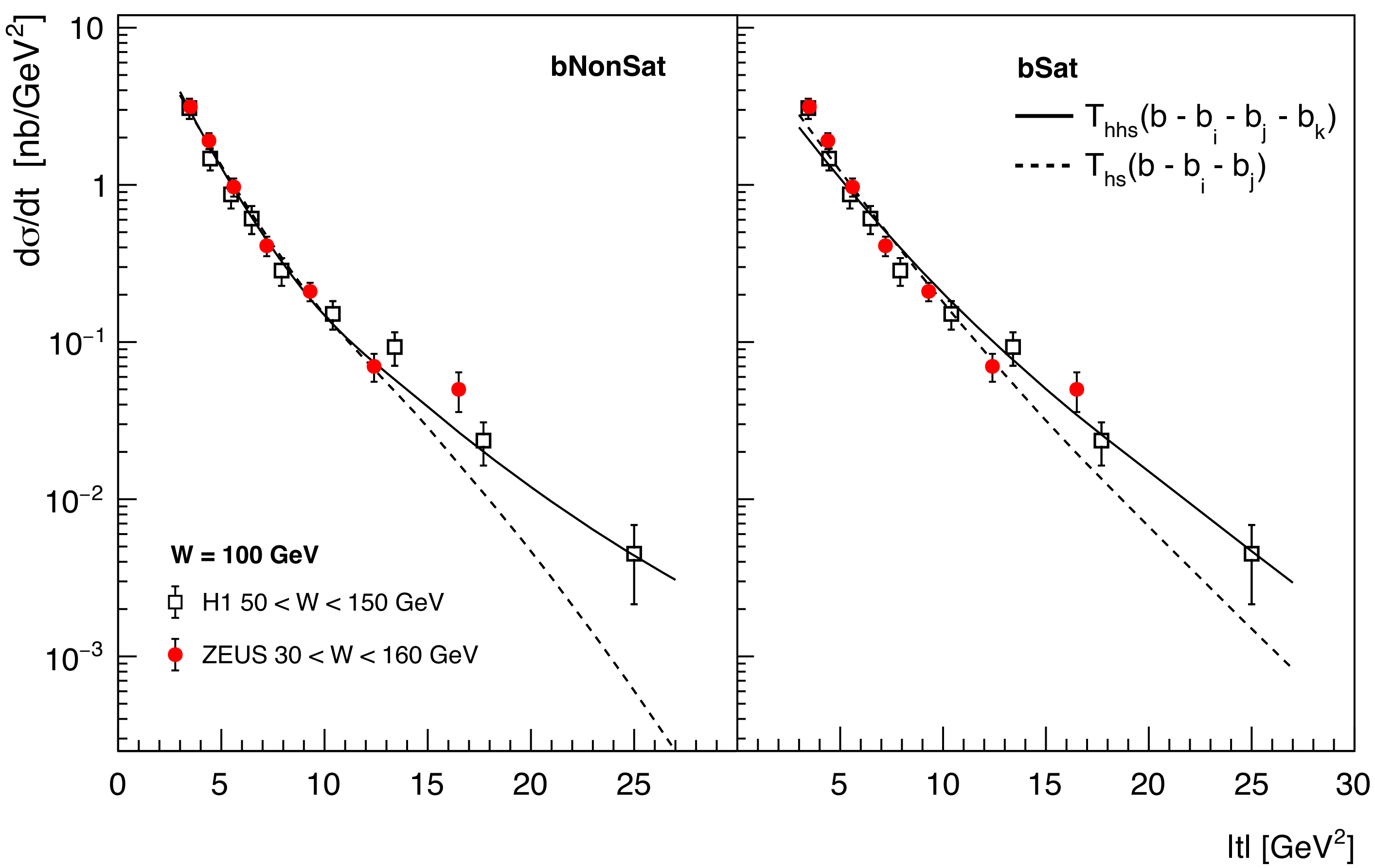
$\gamma^* + p \to J/\psi + \pi + n$, $Q^2 = 0$ GeV^2 $\qquad$ $\gamma^* + p \to \rho + \pi + n$, $Q^2 = 5$ GeV^2

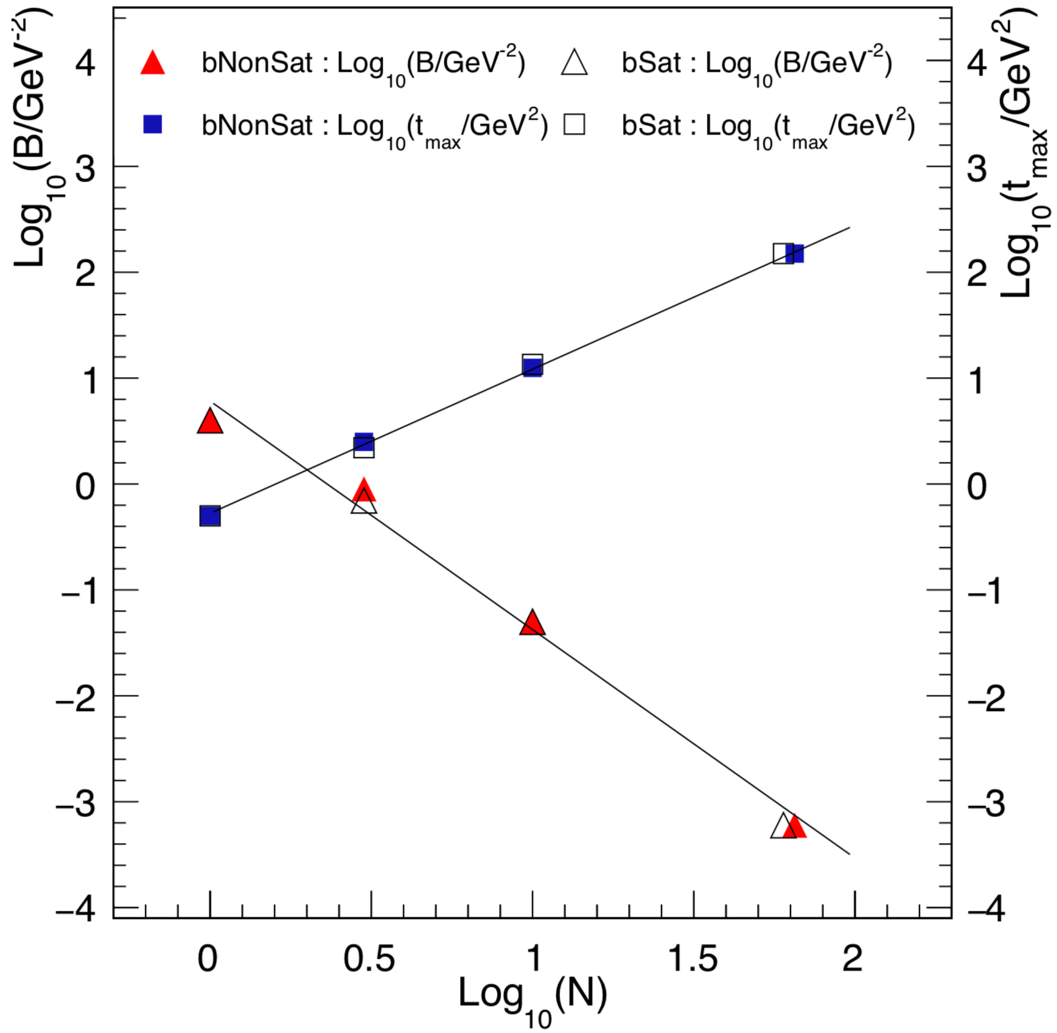


A. Kumar, TT: *Phys.Rev.D* 105 (2022) 11, 114045

Large $|t|$? Self-similar hotspots within hotspots

Model	B_{qc}	B_q	N_q	B_{hs}	N_{hs}	B_{hhs}	N_{hhs}	S_g	σ
bNonSat further refined hotspot	3.2	1.15	3	0.05	10	0.0006	65	–	0.4
bSat further refined hotspot	3.3	1.08	3	0.09	10	0.0006	60	0.4	0.5





$$T_P(\vec{b}) = \frac{1}{2\pi N_q N_{hs} N_{hhs} B_{hhs}} \sum_i^{N_q} \sum_j^{N_{hs}} \sum_k^{N_{hhs}} e^{-\frac{(\vec{b}-\vec{b}_i-\vec{b}_j-\vec{b}_k)^2}{2B_{hhs}}}$$

Arjun Kumar, TT, Eur.Phys.J.C 82 (2022) 9, 837, arXiv: 2106.12855

Outlook

Sartre can utilise the dipole model with a thickness function to describe a plethora of exclusive diffractive processes: Coherent *ep*, Leading neutron pion clouds, Coherent and Incoherent *e*A

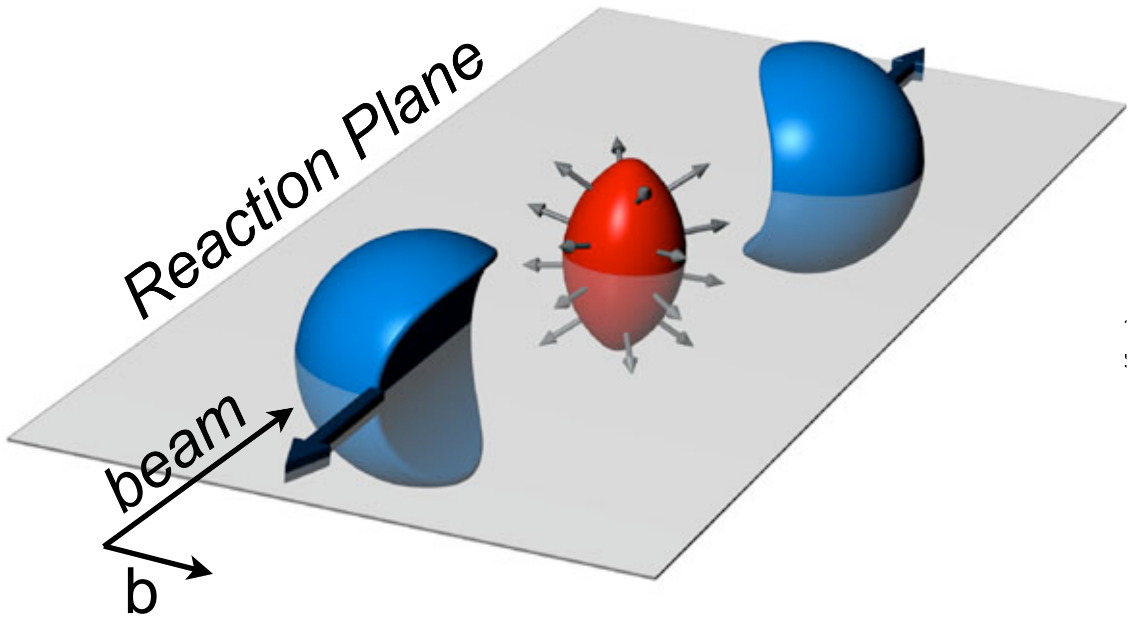
HERA measurements show that the *t*-slope has an energy dependence. This can be taken into account by modelling the thickness with an *x*-dependence:

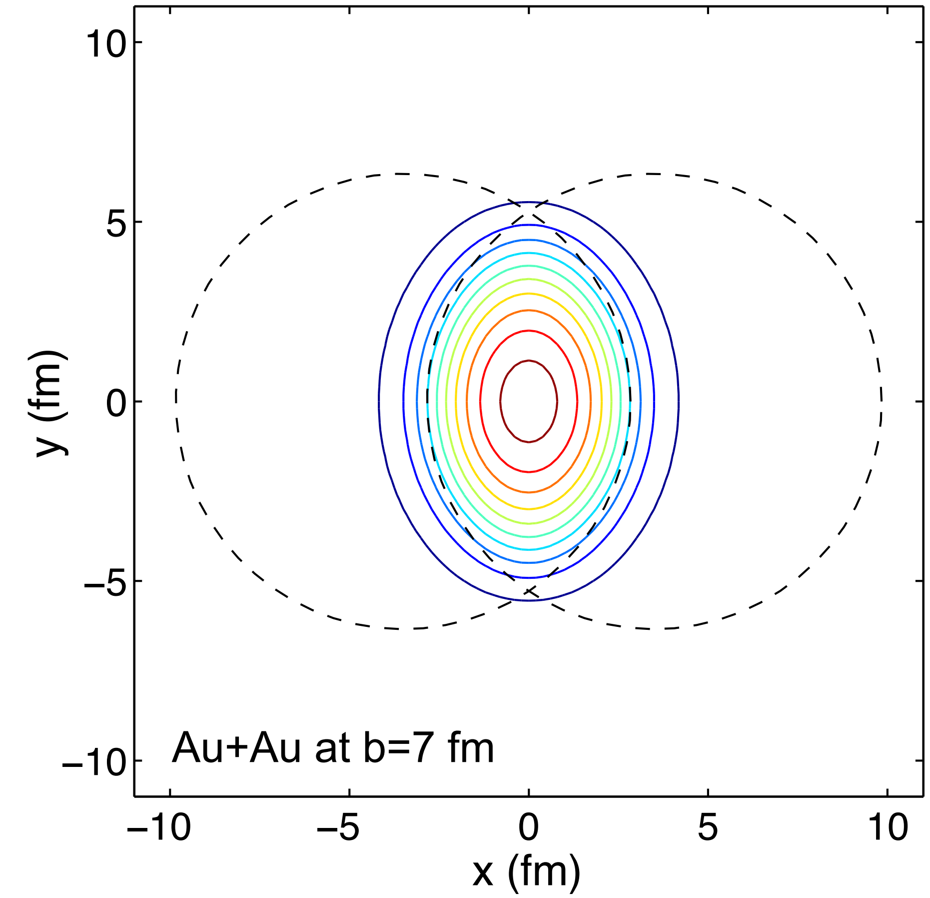
$$T(b) \rightarrow T(b, x_{\mathbb{P}})$$

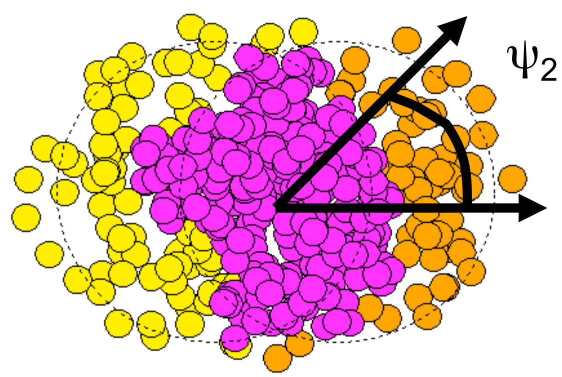
We can also describe the total *ep* *t*-spectrum by introducing a t-dependent hotspot evolution to the thickness function:
$T(b) \rightarrow T(b, t)$

Still to do:
Extend all these to *e*A studies
Investigate $T(b) \rightarrow T(b, x_{\mathbb{P}}, t)$
Implement in Sartre utilising the thickness functions.

All these compelling processes can be measured at the EIC







$$\frac{dN}{d\varphi} \propto 1 + 2v_2 \cos[2(\varphi - \psi_R)] + \ldots$$

$$v_2 = \langle \cos[2(\varphi - \psi_R)] \rangle$$

Sensitive to early interactions and pressure gradients

In ideal hydrodynamics $v_2 \propto$ spatial eccentricity ε_2: $\epsilon_2 = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$

v_2/ε versus particle density is sensitive test of ideal hydrodynamic:

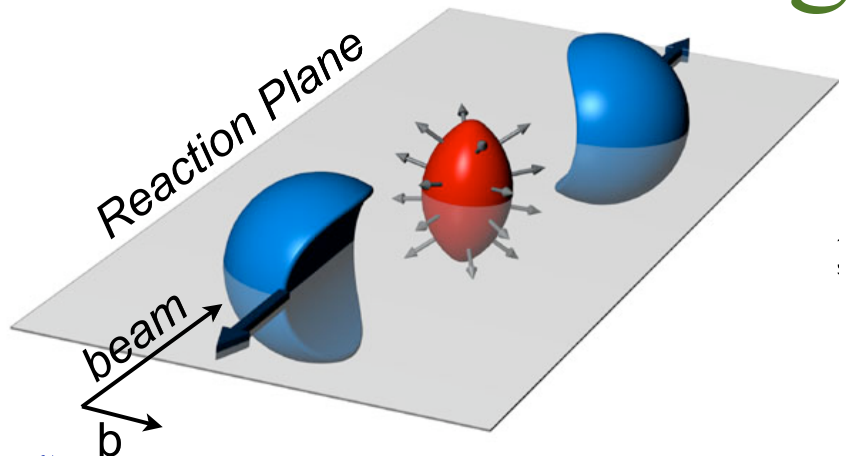
$$\frac{v_2}{\epsilon_2} = \frac{h}{1 + B/\left(\frac{1}{S}\frac{\mathrm{d}N}{\mathrm{d}y}\right)}$$

S= transverse area,

h = hydro limit of v_2/ε and B $\propto$ η/s

Different initial distributions gives different flows!

$$\epsilon_2 = \frac{\langle y^2 - x^2 \rangle}{\langle y^2 + x^2 \rangle}$$



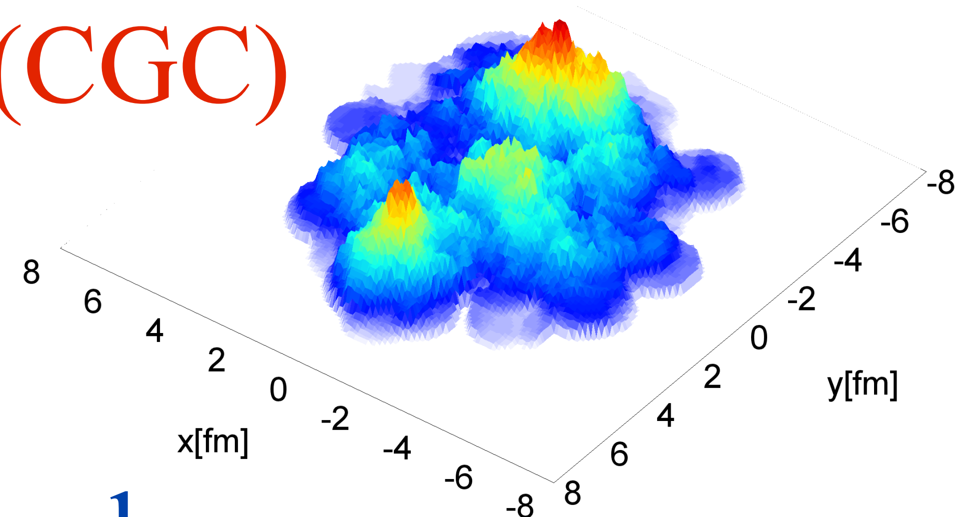
The question is what is ε?

RHIC & LHC: low-p_T realm driven almost entirely by glue ⇒ spatial distribution of glue in nuclei?

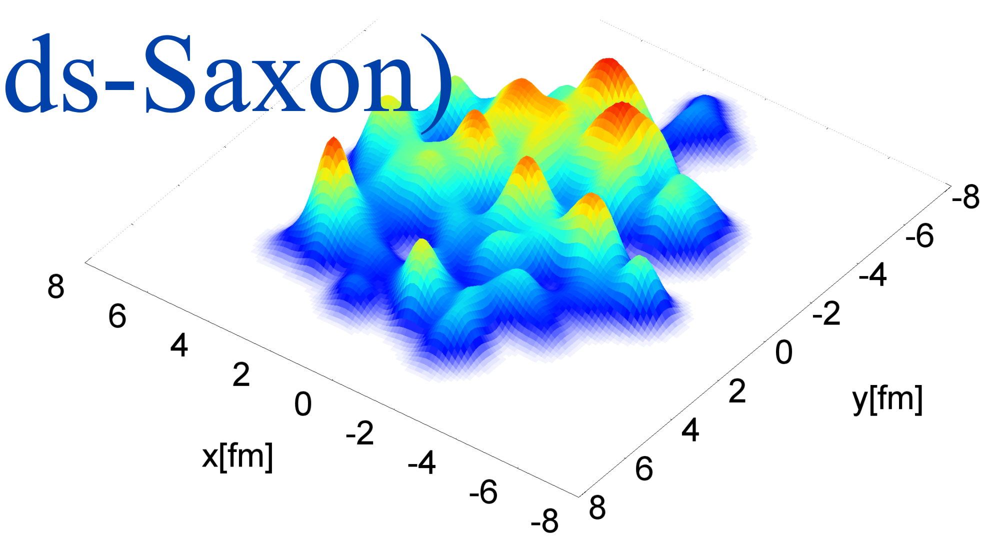
Two methods for ε:

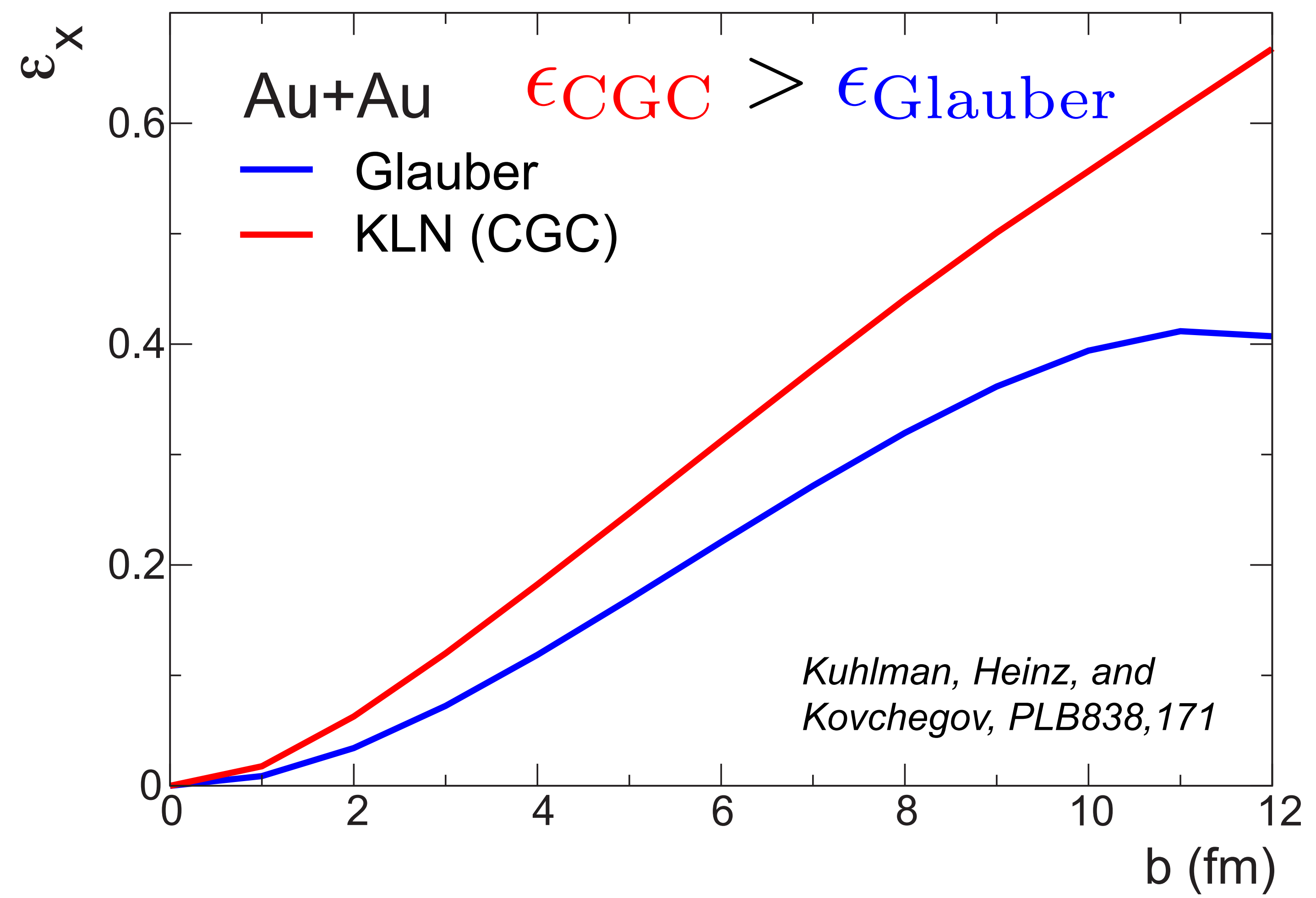
- Glauber (non-saturated)?
- CGC (saturated)?

KLN(CGC)



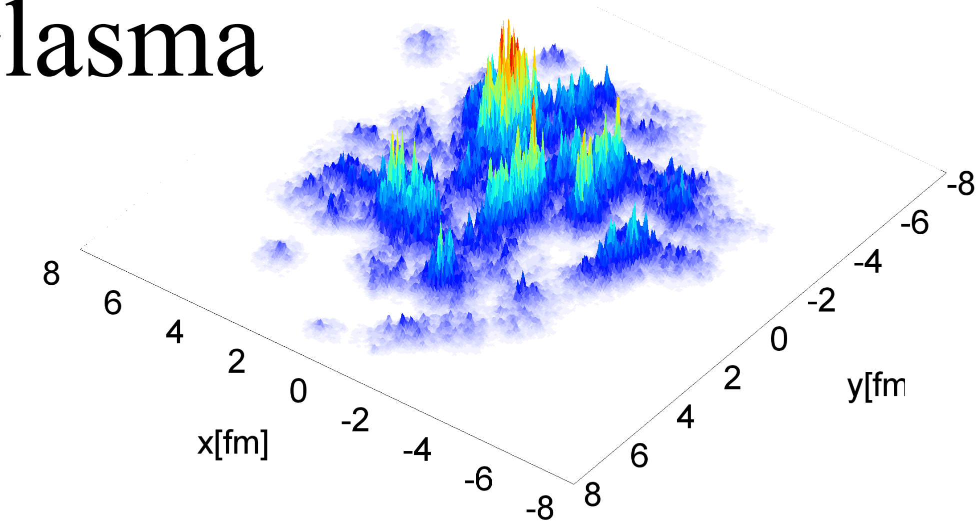
Glauber (Woods-Saxon)



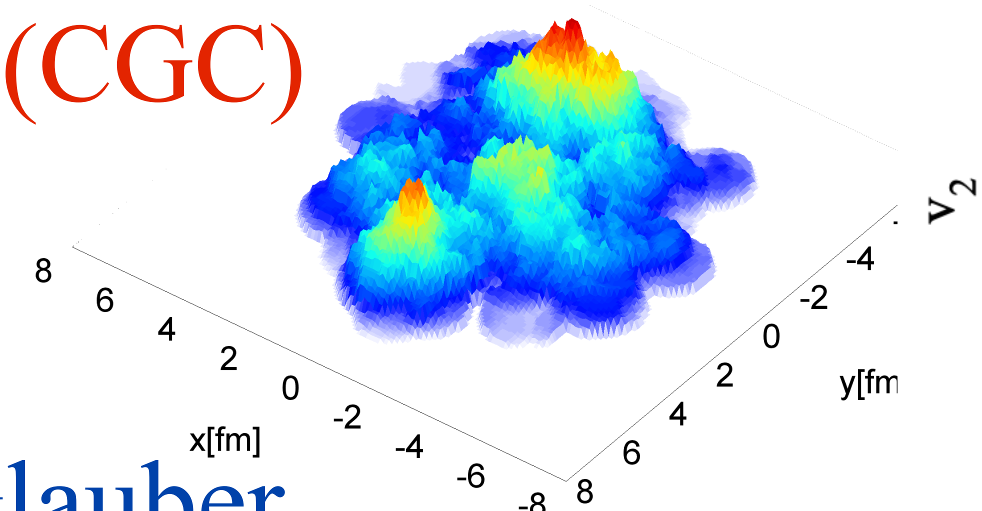


What is η/s?

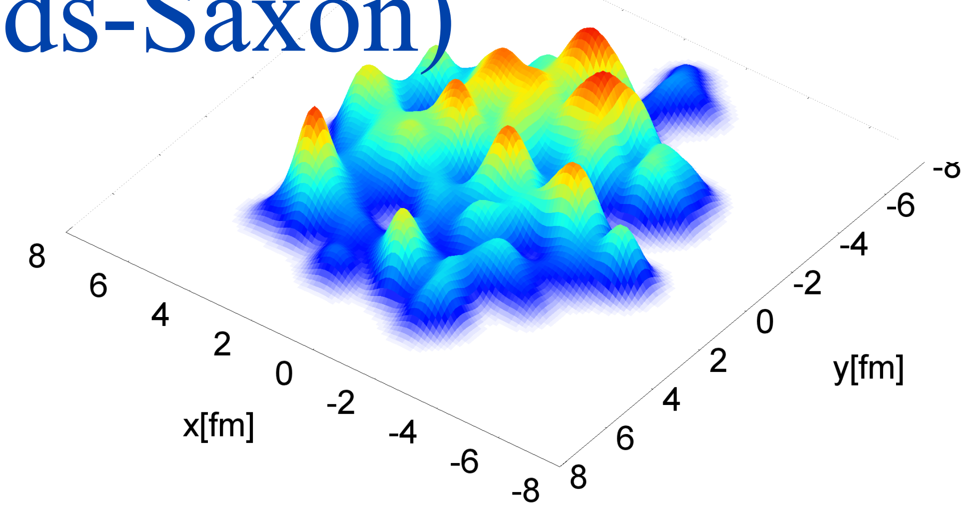
IP-Glasma



KLN(CGC)

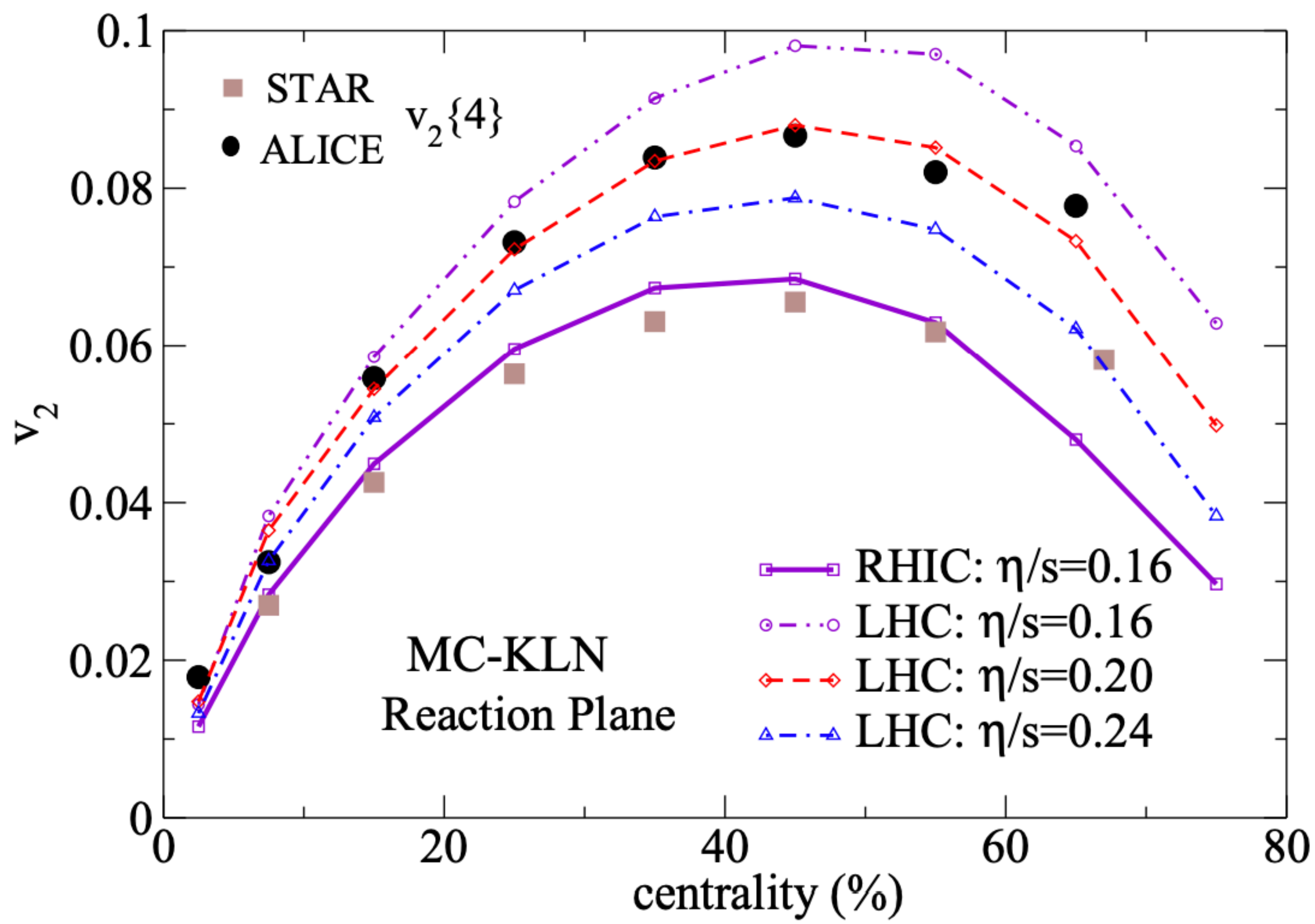


Glauber
(Woods-Saxon)



$1/(4\pi) \sim 0.08$

U. Heinz, C. Shen, H. Song, AIP Conf.Proc. 1441 (2012) no.1, 766-770



Different initial states=
different fluctuation scales

The longitudinal inital state