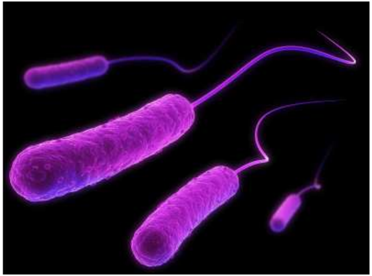
universität
innsbruck

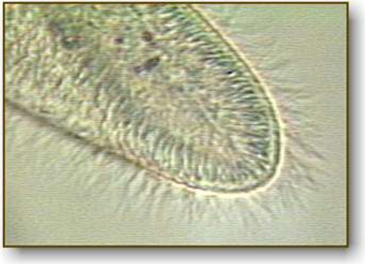
Transport properties of an active Brownian agent in complex environments

Thomas Franosch
Institut für Theoretische Physik
Universität Innsbruck

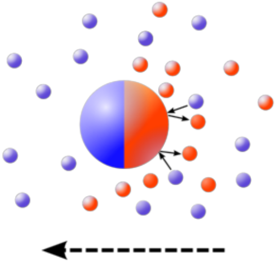
Self-propelled agents



Bacteria using flagella to swim



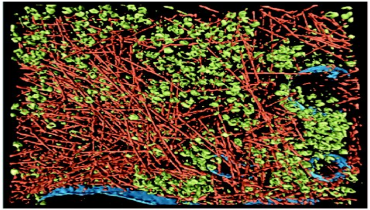
Paramecium uses cilia



self-propelled Janus particle

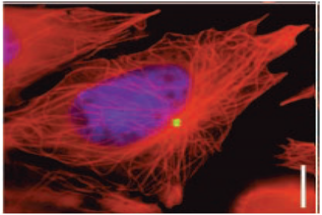
- Swimming mechanism by shape deformations or induced gradients in the fluid
- intrinsically far from equilibrium
- recent experimental progress to build artificial self-propelled particles
- plethora of collective phenomena (flocking, swarms, phase separation, trapping,...)
- mostly simulational studies
- lacking: complete characterization of single particle motion

Motivation- Transport of Stiff Rods



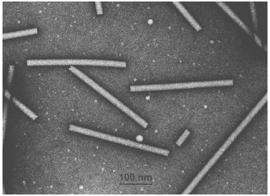
cellular crowding

O. Medalia *et al* (2002) Science



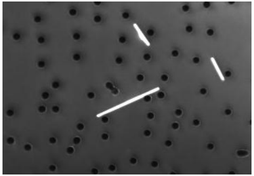
Fibroblast

Rodionov et al., PNAS 96, 1999



tobacco mosaic virus

R.G. Milne



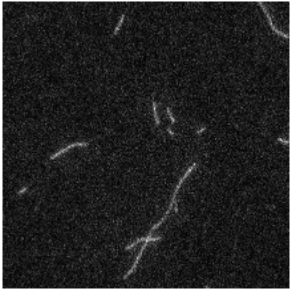
silver rods

Y. Roichman, Tel Aviv

Experimental model systems

reconstituted F-actin, entangled with a network of . . .

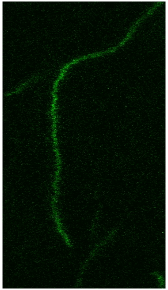
. . . methylcellulose



Miklós Kellermayer

University of Budapest (Hungary)

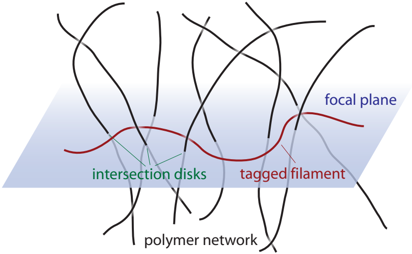
. . . F-actin



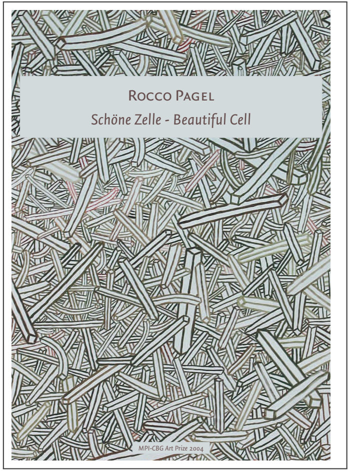
Rudolf Merkel

FZ Jülich (Germany)

Toy Model for F-Actin solutions



- filaments are stiff and thin
  → approximate by needles
- single filament dynamics
  → fix surrounding filaments in 3-dim. space
  needle Lorentz model
- restrict motion to a plane
  → surrounding filaments appear as hard disks



An artists view on the cell

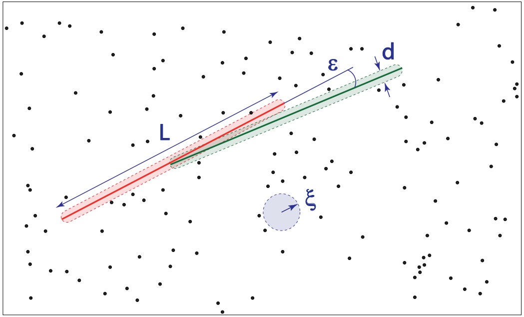
MPI CBG Dresden

Tube Model

- F-actin: very thin filaments
  $L \approx 10 \ldots 100$ μm, $R \approx 0.005$ μm
  → reduce rod to a thin needle and disks to points ($R = 0$)
- isolate entanglement effects (no excluded volume)
  dynamic crowding
- at high filament densities: Tube model
  mesh size $\xi := n^{-1/2}$
  tube diameter $d \sim 1/nL = \xi^2/L$
  tilt angle $\varepsilon = d/L$
  reduced density $n^* = nL^2$
  rotational diffusion constant

$$D_{\text{rot}} \sim \frac{\varepsilon^2}{2\tau_d} \sim \frac{1}{n^2 L^4 \tau_0}$$

Doi & Edwards (1978)



Persistence of the Orientation

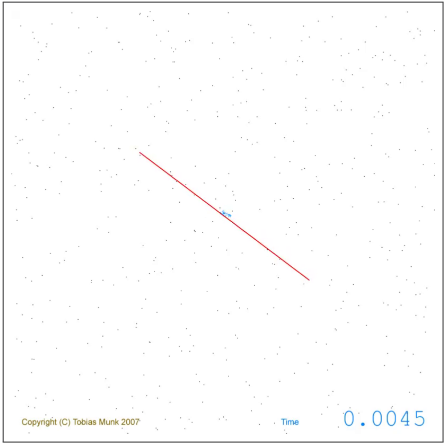
Brownian rod in 2D

- sliding motion of the rod
- orientation changes only gradually
- experiment Yael Roichman (TAU) S3_pillar

Persistence of the Orientation

Brownian rod in 2D

- sliding motion of the rod
- orientation changes only gradually
- experiment Yael Roichman (TAU) S3_pillar



Brownian dynamics simulation

Brownian dynamics in 3D

- infinitely thin needles of number density n, length L
  dimensionless density $n^* = nL^3$
- free motion:
  rotational diffusion D^0_{rot}, anisotropic translational diffusion $D^0_\parallel, D^0_\perp$
- unit vector of orientation $\mathbf{u}$, $|\mathbf{u}| = 1$, position of the needle $\mathbf{r}$
  Stochastic dynamics (Itō)

$$\mathrm{d}\mathbf{u} = -2D^0_{\text{rot}}\mathbf{u}\mathrm{d}t - \sqrt{2D^0_{\text{rot}}}\mathbf{u} \times \boldsymbol{\xi}\mathrm{d}t$$

$$\mathrm{d}\mathbf{r} = \left[\sqrt{2D^0_\parallel}\mathbf{uu} + \sqrt{2D^0_\perp}(\mathbb{I} - \mathbf{uu})\right]\boldsymbol{\eta}\mathrm{d}t$$

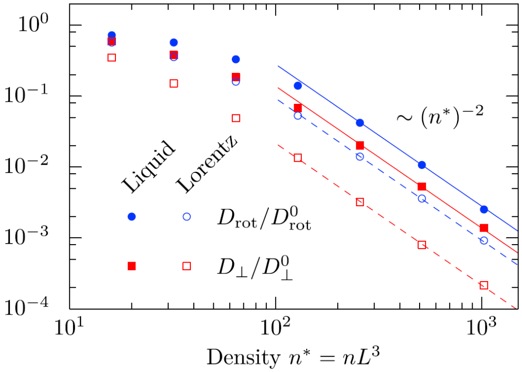
- independent white noise $\boldsymbol{\xi}, \boldsymbol{\eta}$: $\langle \xi_i(t)\xi_j(t')\rangle = \langle \eta_i(t)\eta_j(t')\rangle = \delta_{ij}\delta(t - t')$
- implementation by orthogonal integrator, pseudo-scheme: interrupted free propagation using pseudo-velocities, pseudo-angular velocities drawn from Maxwell distributions
- elastic collisions with other needles symmetry-adapted neighbor list

Diffusion coefficients

Brownian dynamics simulations

- needle liquids
- needle Lorentz system
  tracer needle explores frozen array of needles
  needle in a haystack
- data to highly entangled regime
- Doi-Edwards scaling $D_{\text{rot}} \sim n^{-2}$ ✓
- Needle Lorentz and Liquids behave similarly
- Prediction for $D_\perp \sim n^{-2}$ by entering new tubes

Tube is collectively build by many surrounding needles



Leitmann, Höfling, Franosch, PRL (2016)

Ramifications of the Tube

Phantom needle

- Tube confines needle
  → motion essentially along the tube, orientational relaxation by tube renewal
  Phantom needle performs anisotropic diffusion with **effective** transport coefficients $D_{\text{rot}}, D_{\perp}, D_{\|}$
  forgotten!(?) prediction of Doi & Edwards (1978)
- conditional probability density $\mathbb{P}(\mathbf{r}, \mathbf{u}, t|\mathbf{u}_0)$ (Green function)
  Perrin equation (Markov process)

$$\partial_t \mathbb{P} = D_{\text{rot}} \Delta_{\mathbf{u}} \mathbb{P} + \partial_{\mathbf{r}} \cdot \left[D_{\|} (\partial_{\mathbf{r}} \mathbb{P}) - \Delta D (\mathbb{I} - \mathbf{u}\mathbf{u}) \cdot (\partial_{\mathbf{r}} \mathbb{P}) \right]$$

orientational diffusion

anisotropic translational diffusion $\Delta D = D_{\|} - D_{\perp}$

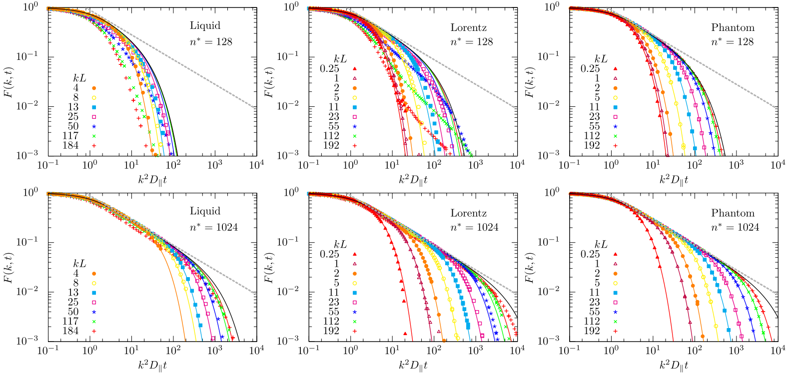
- marginalize for ISF

$$F(k,t) = \langle \exp\left(-i\mathbf{k} \cdot \Delta\mathbf{r}(t)\right) \rangle = \int_{S^2} \mathrm{d}\mathbf{u} \int_{S^2} \frac{\mathrm{d}\mathbf{u}_0}{4\pi} \int_{\mathbb{R}^3} \mathrm{d}^3 r \, \exp(-i\mathbf{k} \cdot \mathbf{r}) \mathbb{P}(\mathbf{r}, \mathbf{u}, t|\mathbf{u}_0)$$

Solve or simulate phantom needle

Spatio-temporal transport

Intermediate scattering function $F(k,t) = \langle \exp[-i\mathbf{k}\cdot\Delta\mathbf{r}(t)]\rangle$



- Phantom needle describes spatio-temporal dynamics for dynamically crowded systems $n^* = nL^3 \gtrsim 100$
- First data in highly entangled regime, first test of Doi-Edwards prediction ✓
- Characteristic tail $t^{-1/2}$ sliding motion in the tube

Leitmann, Höfling, Franosch, PRL (2016)

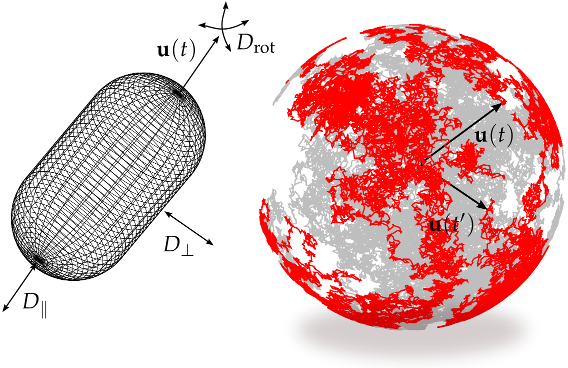
Mini-Résumé

Dynamically crowded needles

- Minimal model for solutions of F-actin
- First simulations of needles deep in the semidilute regime $nL^3 \gg 1$
  symmetry adapted neighbor list
- Tube concept reduces many-body problem to single particle motion
  → non-perturbative approach
- Needle liquids and needle Lorentz systems behave asymptotically identically
  justifies 2D toy model ✓
- Strong suppression of orienatational diffusion $D_{\text{rot}} \sim n^{-2}$ ✓
- Full spatio-temporal information encoded in phantom needle
- algebraic decay is fingerprint of sliding motion
- full analytic solution of the phantom needle
  beyond Doi & Edwards simplified harmonic oscillator analysis
- Form factor can be included easily in simulation

Model set-up

Active Brownian Particle



- Active propulsion with constant velocity v along the long axis $\mathbf{u}$, $|\mathbf{u}| = 1$
- Rotational diffusion D_{rot}
- Anisotropic translational diffusion $D_\parallel, D_\perp$
- ignores microscopic origin of propulsion, effective description
- simplistic model encoding persistent random walk
  persistence length $\ell = v/D_{\text{rot}}$,
  persistence time $\tau = 1/D_{\text{rot}}$

Stochastic equations (3D)

Active Brownian particle

$$\mathrm{d}\mathbf{u} = -2D_{\text{rot}}\mathbf{u}\mathrm{d}t - \sqrt{2D_{\text{rot}}}\mathbf{u} \times \boldsymbol{\xi}\mathrm{d}t$$

$$\mathrm{d}\mathbf{r} = v\,\mathbf{u}\,\mathrm{d}t + \left[\sqrt{2D_{\parallel}}\mathbf{u}\mathbf{u} + \sqrt{2D_{\perp}}(\mathbb{I} - \mathbf{u}\mathbf{u})\right]\boldsymbol{\eta}\mathrm{d}t$$

position — fixed velocity — orientation

$$\langle \xi_i(t)\xi_j(t')\rangle = \langle \eta_i(t)\eta_j(t')\rangle = \delta_{ij}\delta(t-t')$$

independent Gaussian white noise

- multiplicative noise (Itō)
- translational anisotropy $\Delta D = D_{\parallel} - D_{\perp}$
  mean diffusion coefficient $\bar{D} = (D_{\parallel} + 2D_{\perp})/3$
- For long rods $D_{\parallel} = 2D_{\perp}, D_{\text{rot}} = 12D_{\perp}/L^2$, length of the needle L
- Dimensionless parameters

reduced number density $n^* = nL^3$ — anisotropy $\Delta D/\bar{D}$ — Péclet number $\text{Pe} = vL/\bar{D}$

Active Brownian particle in 3D

Separation ansatz

- choose coordinates $\mathbf{k}$ in z-direction, parametrize $\mathbf{u} = (\sin\vartheta\cos\varphi, \sin\vartheta\sin\varphi, \cos\vartheta)$, write $\eta = \cos\vartheta$
- separation ansatz yields superposition of eigenfunctions

$$\tilde{\mathbb{P}}(\mathbf{k}, \mathbf{u}, t|\mathbf{u}_0) = \frac{1}{2\pi} e^{-D_\perp k^2 t} \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} e^{im(\varphi-\varphi_0)} \mathrm{Ps}_\ell^m(c, R, \eta) \mathrm{Ps}_\ell^m(c, R, \eta_0)^* \exp(-A_\ell^m D_{\mathrm{rot}} t)$$

- generalized spheroidal wave functions $\mathrm{Ps}_\ell^m(R, c, \eta)$

$$\left[\frac{\mathrm{d}}{\mathrm{d}\eta}\left((1-\eta^2)\frac{\mathrm{d}}{\mathrm{d}\eta}\right) + R\eta - c^2\eta^2 + \frac{m^2}{1-\eta^2} + A_\ell^m\right] \mathrm{Ps}_\ell^m(c, R, \eta) = 0$$

active propulsion $R = -ikv/D_{\mathrm{rot}}$

anisotropy $c^2 = \Delta D k^2/D_{\mathrm{rot}}$

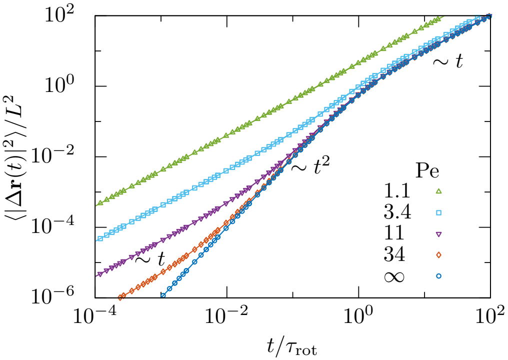
eigenvalue

- deformation of (associated) Legendre polynomials $\mathrm{P}_\ell^m(\eta)$
- Intermediate scattering function

$$F(k, t) = \frac{1}{2\pi} e^{-D_\perp k^2 t} \sum_{\ell=0}^{\infty} \exp(-A_\ell^0 D_{\mathrm{rot}} t) \left| \int_{-1}^{1} \mathrm{d}\eta \, \mathrm{Ps}_\ell^0(c, R, \eta) \right|^2$$

Low-order moments

Mean-square displacement (3D)



- Expansion of ISF for **isotropic system**

$$F(k,t) = \langle \exp(-i\mathbf{k}\cdot\Delta\mathbf{r}(t))\rangle = \left\langle \frac{\sin(k|\Delta\mathbf{r}(t)|)}{k|\Delta\mathbf{r}(t)|}\right\rangle$$

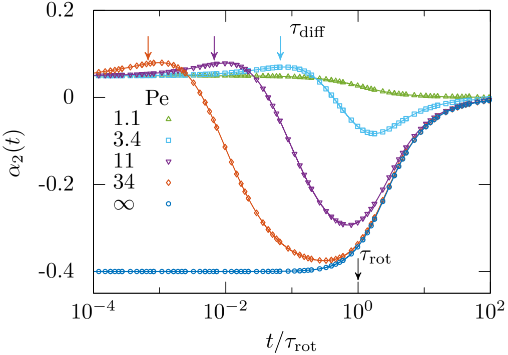
$$F(k,t) = 1 - \frac{k^2}{3!}\langle|\Delta\mathbf{r}(t)|^2\rangle + \frac{k^4}{5!}\langle|\Delta\mathbf{r}(t)|^4\rangle + \mathcal{O}(k^6)$$

- MSD initially translational diffusion dominates
  persistent swimming
  effective diffusion

$$\langle|\Delta\mathbf{r}(t)|^2\rangle = 6\bar{D}t + \frac{v^2}{2D_{\mathrm{rot}}^2}\left(e^{-2D_{\mathrm{rot}}t} + 2D_{\mathrm{rot}}t - 1\right)$$

Low-order moments

Non-Gaussian parameter (3D)

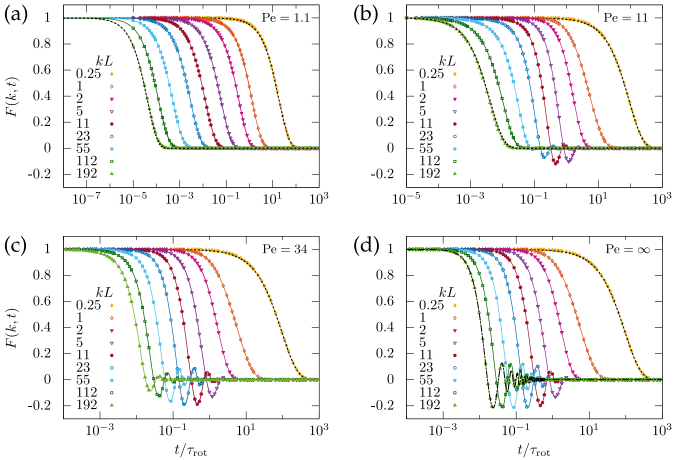


- Non-Gaussian parameter

$$\alpha_2(t) = \frac{3\langle|\mathbf{\Delta r}(t)|^4\rangle}{5\langle|\mathbf{\Delta r}(t)|^2\rangle^2} - 1$$

- initially non-gaussian by translational anisotropy characteristic minimum due to persistent swimming eventually again Gaussian

Intermediate scattering function



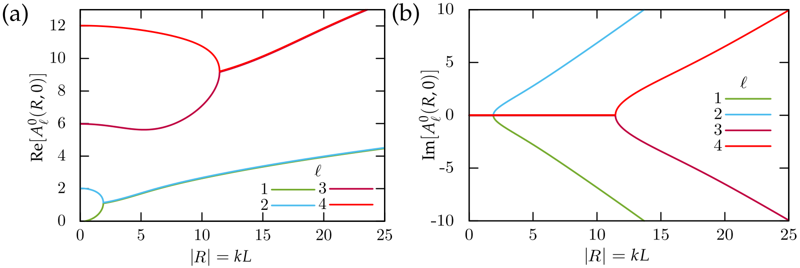
- characteristic oscillations emerge at intermediate wavenumbers
  fingerprint of persistent swimming

$$F(k,t) = \left\langle \frac{\sin(k|\Delta\mathbf{r}(t)|)}{k|\Delta\mathbf{r}(t)|} \right\rangle$$

- large wavenumbers
  anisotropic translational diffusion
- small wavenumbers
  effective diffusion

$$D_{\mathrm{eff}} = \bar{D} + v^2/6D_{\mathrm{rot}}$$

How can oscillations emerge?



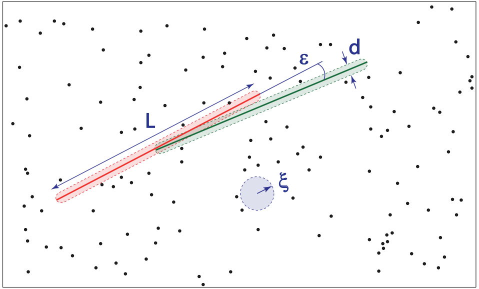
- Intermediate scattering function sum of relaxing exponentials?

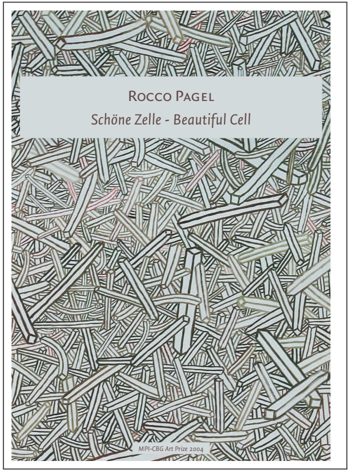
$$F(k,t) = \frac{1}{2\pi} e^{-D_\perp k^2 t} \sum_{\ell=0}^{\infty} \exp(-A_\ell^0 D_{\text{rot}} t) \left| \int_{-1}^{1} \mathrm{d}\eta \, \mathrm{Ps}_\ell^0(c,R,\eta) \right|^2$$

- Eigenvalue problem is non-Hermitian, eigenvalues become complex
  branching in the eigenvalues → no perturbation theory
  fingerprint of active motion

Christina Kurzthaler *et al*, Sci. Rep. (2016)

Crowded suspension of active needles

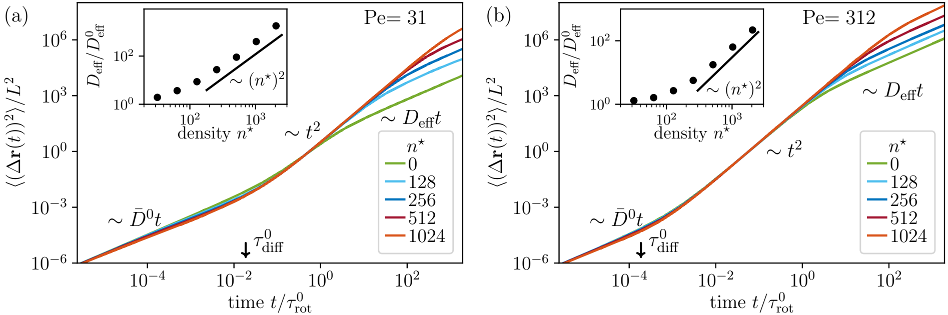




Does the tube model apply to active needles?

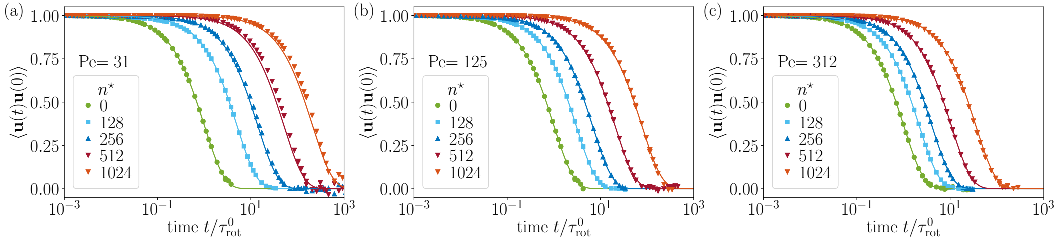
How do the dynamics change?

Crowded is faster



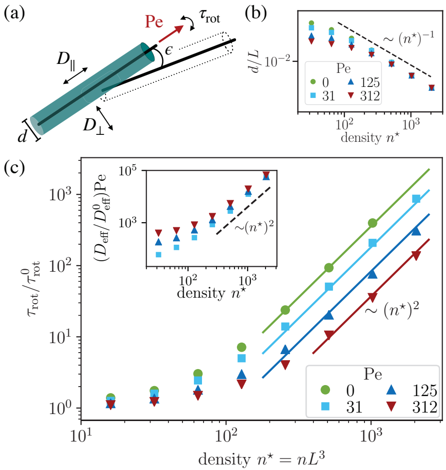
- mean-square displacement $\langle[\Delta\mathbf{r}(t)]^2\rangle$ displays three regimes
  short-time diffusion – directed motion – effective diffusion
- free needle $D^0_{\rm eff} = \bar{D}^0 + v^2\tau^0_{\rm rot}/3$
- Entanglement speeds up the effective diffusion $D_{\rm eff}/D^0_{\rm eff} \sim (n^*)^2$

Orientational dynamics



- Orientation correlation function $\langle \mathbf{u}(t) \cdot \mathbf{u}(0) \rangle$ approaches perfect exponential
- relaxation time τ_{rot} increases by orders of magnitude

Relaxation time



- tube picture:

  disengagement time $\tau_0 \hat{=}$ time to move length L

  interpolation formula $\tau_0^{-1} = D_\parallel^0/L^2 + v/L$

  rotation angle $\varepsilon \sim d/L \sim (n^*)^{-1}$

$$\tau_{\rm rot} \sim \frac{1}{\varepsilon^2 \tau_0^{-1}} \sim \frac{(n^*)^2}{D_\parallel^0/\bar{D}^0 + \text{Pe}} \tau_{\rm rot}^0$$

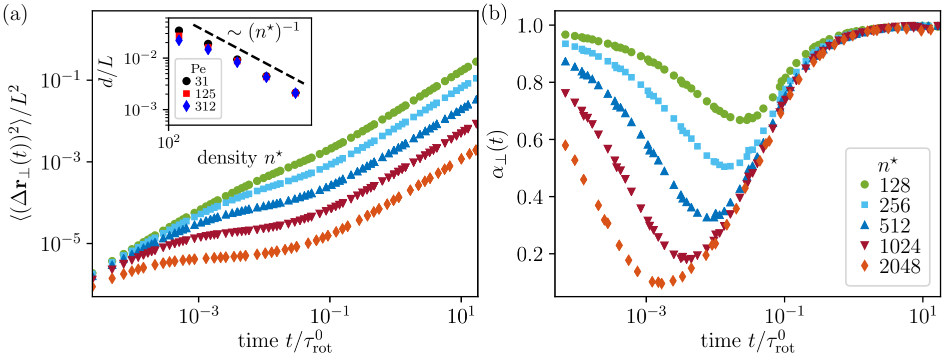
  reduces to Doi-Edwards for Pe $= 0$

  additional decrease by activity

- master plot for effective diffusion

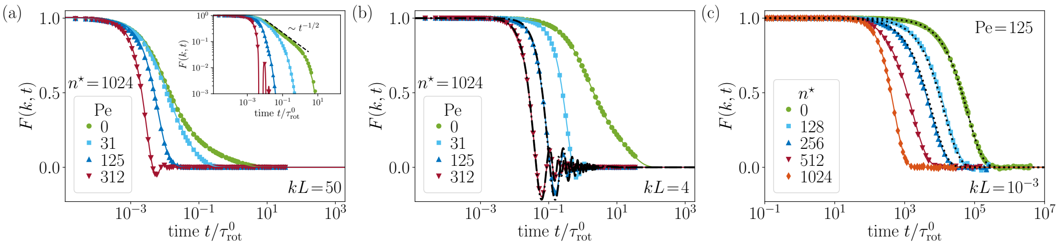
$$\frac{D_{\rm eff}}{D_{\rm eff}^0} \sim \frac{\tau_{\rm rot}}{\tau_{\rm rot}^0} \sim \frac{(n^*)^2}{\text{Pe}}$$

Corroborating the tube picture



- perpendicular displacement in comoving frame $\Delta\mathbf{r}_\perp(t)$
- plateau emerges for highly entangled systems → defines tube diameter d
- scaling $d/L \sim (n^*)^{-1}$ ✓

Spatio-temporal transport



- intermediate scattering function

$$F(k,t) = \langle \exp[-i\mathbf{k}\cdot\Delta\mathbf{r}(t)]\rangle = \left\langle \frac{\sin(k|\Delta\mathbf{r}(t)|)}{k|\Delta\mathbf{r}(t)|}\right\rangle$$

- agreement over 5 orders of magnitude → exact theory for high entanglement
- swimming motion approximately described by $|\Delta\mathbf{r}(t)| = vt$ → $F(k,t) = \sin(kvt)/kvt$

S. Mandal, C. Kurzthaler, T. Franosch, and H. Löwen, PRL **125**, 138002 (2020)

Mini-Résumé on Needles

Active Brownian particles in 3D

- intermediate scattering function can be solved analytically
- non-trivial oscillations as fingerprint of active propulsion

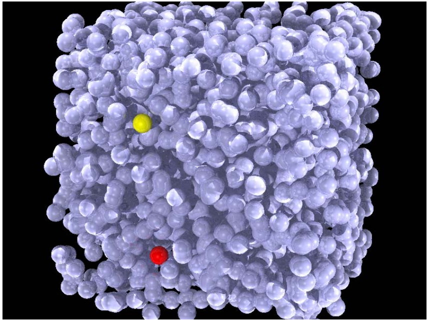
Active needles in suspension

- crowding rectifies the motion of active needles → crowded is faster
- Doi-Edwards tube model can be extended to active needles
- exact description on mesoscopic scales by renormalized orientational diffusion D_{rot}
- origin is a separation of time and length at mesoscopic scales

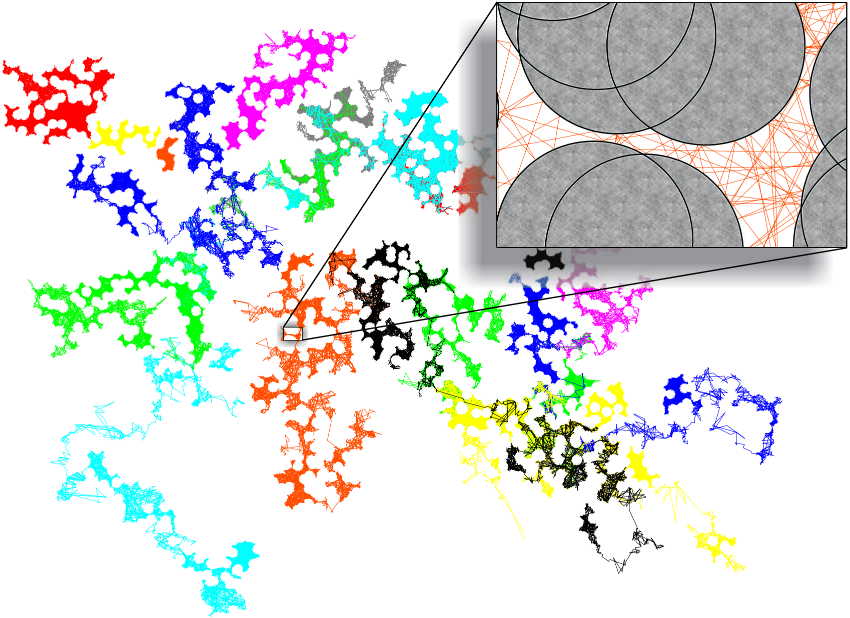
Lorentz Model

- classical gas of non-interacting, structureless particles
- randomly distributed, fixed obstacles: overlapping hard spheres *Swiss Cheese model*

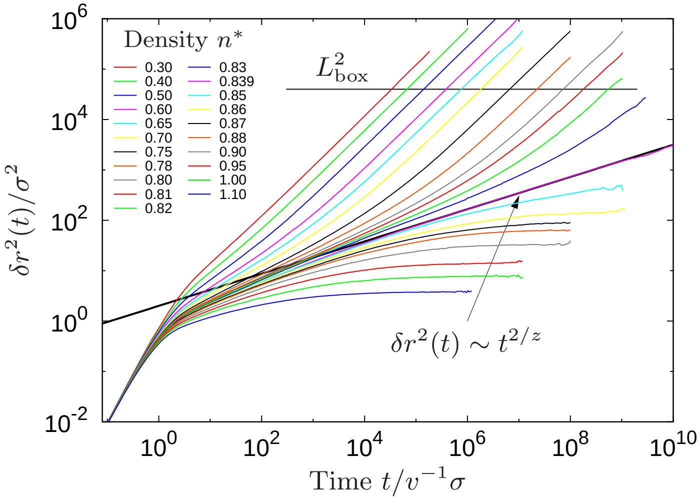
- ballistic motion, elastic scattering



- relevant for transport in disordered media
- single control parameter: reduced obstacle density $n^* = n\sigma^d$



Mean-Square Displacement (3D)



- ballistic motion, specular scattering
- critical density n_c
  subdiffusive transport $\delta r^2(t) \sim t^{2/z}$
- localization transition percolation of void space
  localization length
  $\ell^2 = \delta r^2(t \to \infty)$
  $\ell \sim (n_c^* - n^*)^{\nu + +\beta/2}$ geometric exponents of percolation ν, β
- scaling theory of critical phenomena

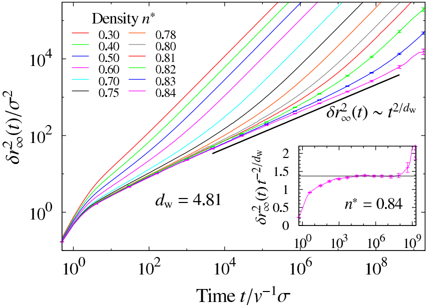
$$\delta r^2(t) = t^{2/z} \delta \hat{r}^2_{\pm}(\hat{t}), \qquad \hat{t} \propto t\ell^{-z}$$

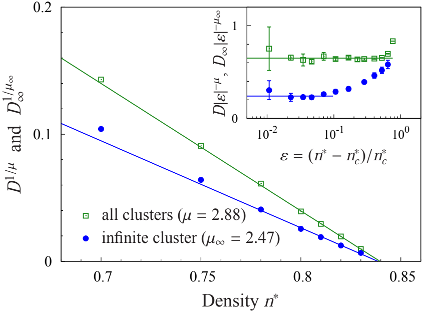
- corrections to scaling are relevant

F. Höfling, T. Franosch, E. Frey, PRL **96**, 165901 (2006)

Infinite cluster

Infinite cluster only



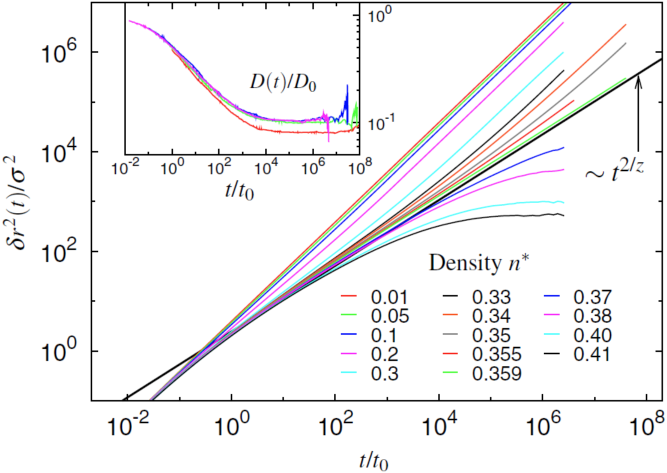


- Voronoi tesselation identifies infinite cluster only conducting side
- subdiffusive at critical point $\delta r^2_\infty \sim t^{2/d_w}$ walk dimension $d_w \approx 4.81$
- scaling behavior anticipated

- diffusion coefficient vanish $D(n) \sim \varepsilon^\mu$, $\varepsilon = (n - n_c)/n_c$ $D_\infty(n) \sim \varepsilon^{\mu_\infty}$ extrapolate to same point

M. Spanner *et al*, J.Phys.:Condens. Matt. (2011)

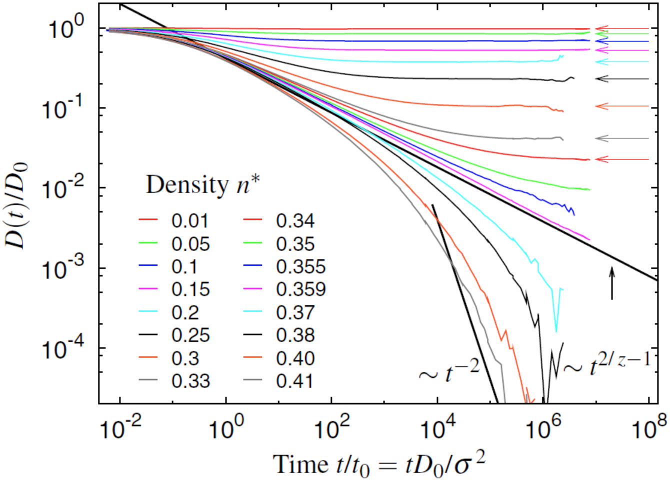
Mean-square displacement for Brownian tracers (2D)



- Brownian tracers
- critical density n_c
  subdiffusive transport
  $\delta r^2(t) \sim t^{2/z}$
  critical exponent $z = 3.036$
  (lattice value)
- localization transition percolation of void space
- scaling theory of critical phenomena

T. Bauer *et al*, EPJ-ST (2010)

Time-dependent diffusion (2D)



- time-dependent diffusion coefficient

$$D(t) := \frac{1}{2d}\frac{\mathrm{d}}{\mathrm{d}t}\delta r^2(t)$$

- diffusion constant

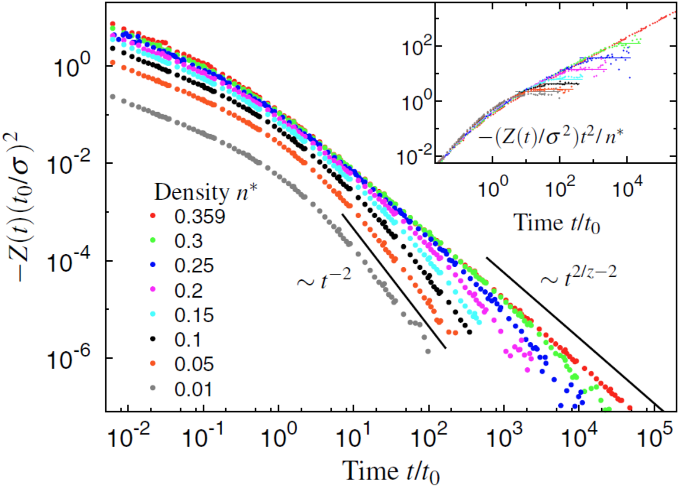
$$D = \lim_{t\to\infty} D(t)$$

slowing down of diffusion

- critical density n_c
  subdiffusive transport
  $D(t) \sim t^{2/z-1}$
  critical exponent $z = 3.036$
  (lattice value)
- localization transition $D(t) \to 0$

T. Bauer *et al*, EPJ-ST (2010)

Long-time tails (2D)



- velocity-autocorrelation function

$$Z(t) := \frac{1}{2d}\frac{\mathrm{d}^2}{\mathrm{d}t^2}\delta r^2(t)$$

- persistent anticorrelations for $n^* < n_c^*$

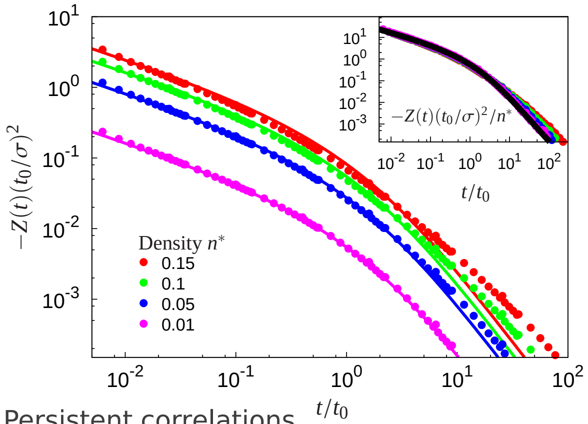
$$Z(t) \simeq -At^{-2}$$

repeated scattering with the same obstacle

- rectification shows universality approximately $\propto n^*$ → can be calculate to first order in n^*

T. Bauer *et al*, EPJ-ST (2010)

Brownian tracers in the 2D Lorentz model



Formal scattering theory

borrowed from quantum many-body problems

Lorentz model analogous to Anderson model:

Localization transition ⇔ Metal-Insulator transition

Long-time tails ⇔ weak localization

Multiple collision expansion to first order in density

Velocity autocorrelation function (VACF)

- explicit expression for $Z(\omega)$
- diffusion coefficient
  $D = Z(\omega = 0) = D_0(1 - \pi n^*) + \mathcal{O}(n^{*2})$
- algebraic long-time tail

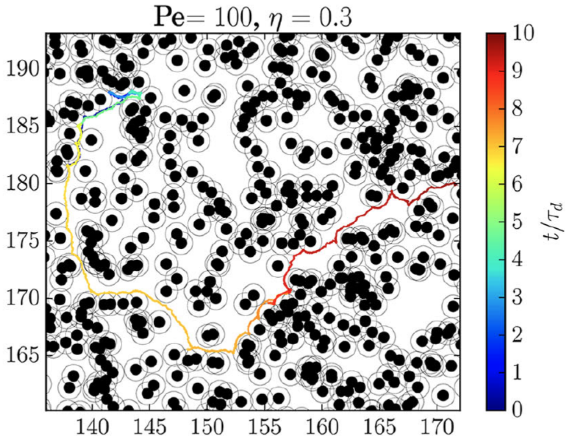
$$Z(t) \simeq \frac{-\pi n^* \sigma^2}{2t^2} \qquad \text{for } t \to \infty$$

Franosch *et al* Chem. Phys. (2010)

Persistent correlations

- theory describes nicely the data at low densities ✓
- Long-time tails persists at all densities

ABP in disordered environement



- Soft spheres: Weeks-Chandler-Anderson potential
- diameter of ABP: $2R_s$
- definition of Péclet number $\mathrm{Pe} = 2R_s v/D$
  persistent length $L = v\tau_{\mathrm{rot}}$
  time scale $\tau_{\mathrm{rot}} = 1/D_{\mathrm{rot}}, \tau_{\mathrm{diff}} = (2R_s)^2/D$
  here $\tau_{\mathrm{rot}}/\tau_{\mathrm{diff}} = 1/3$

$$\frac{L}{2R_s} = \mathrm{Pe}\,\frac{\tau_{\mathrm{rot}}}{\tau_{\mathrm{diff}}} = 100 \times \frac{1}{3}$$

- packing fraction $\eta = \pi R_0^2/A$

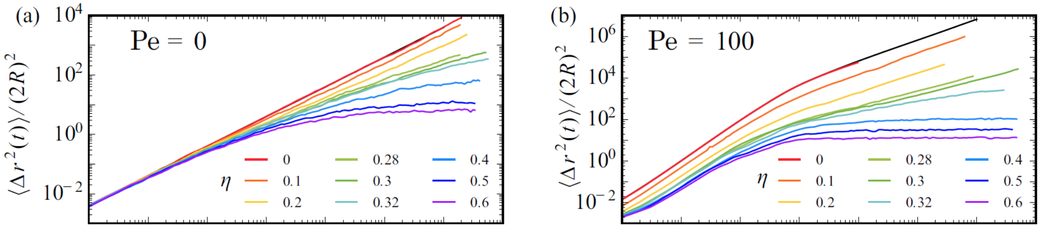
  area fraction $\phi_0 = 1 - \exp(-\eta)$

- **sliding** along the boundary

M. Zeitz, K. Wolff, and H. Stark, Eur. Phys. J. E (2017) **40**: 23

ABP in disordered environement

Mean-square displacement

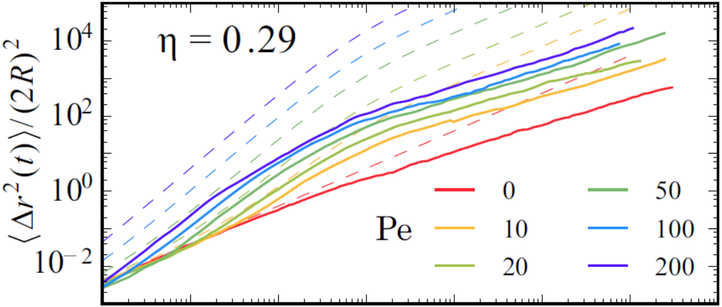


- crossover from persistent to diffusive motion $L/2R_s = 33.33$ for $\eta = 0$
- suppression of persistent motion already at small obstacle density since $L \gg 2R_s$
- confining space are explored more rapidly by **sliding**
- subdiffusive transport $\propto t^{2/z}$ at critical packing fraction
- localized at high packing fraction

M. Zeitz, K. Wolff, and H. Stark, Eur. Phys. J. E (2017) **40**: 23

ABP in disordered environement

Critical density

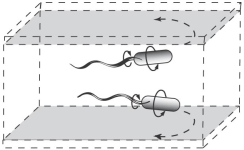


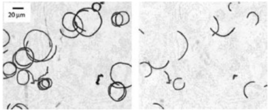
- subdiffusion for all Péclet numbers at critical packing fraction
- critical exponent $z = z_{\text{lat}} \doteq 3.036$ is universal in 2D ✓ M. Spanner *et al* PRL **116**, 060601 (2016)
- prefactor of asymptotic behavior depends strongly on Péclet number

M. Zeitz, K. Wolff, and H. Stark, Eur. Phys. J. E (2017) **40**: 23

Circle swimmers

Escherichia coli



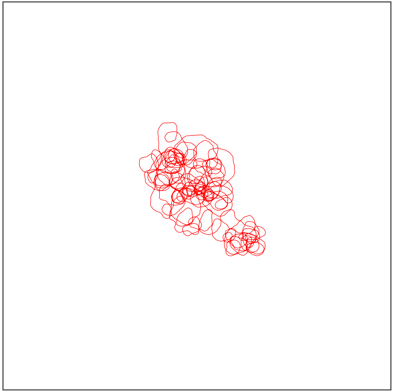


DiLuzio   Lauga

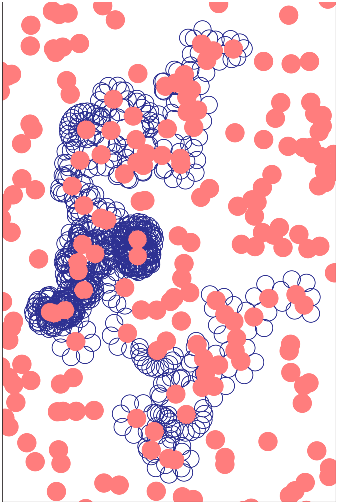
- Chirality of flagellar motion
- Hydrodynamic coupling close to boundaries
  → circular motion
- angular drift velocity ω

$$\frac{\mathrm{d}}{\mathrm{d}t}\vartheta(t) = \omega + \zeta(t)$$

$$\langle \zeta(t)\zeta(t') \rangle = 2D_{\mathrm{rot}}\delta(t-t')$$



Disordered environment



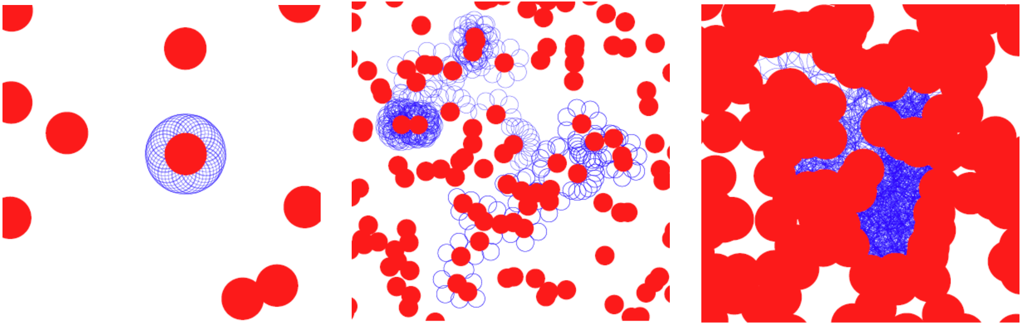
Skipping orbits

- independently distributed obstacles
  exclusion radius σ
  dimensionless scatterer density $n^* = n\sigma^2$
  → **Lorentz model**
- idealize to pure circular motion
  → deterministic orbits
  dimensionless trajectory curvature $B = \sigma/R$
- **skipping orbits** along edges of scatterer clusters
- model originally designed for electron transport
  in a **magnetic field** → 2DEG, obstacle by nanofabrication

long-range transport
by edge percolation

W. Schirmacher *et al*, Phys. Rev. Lett. (2015)

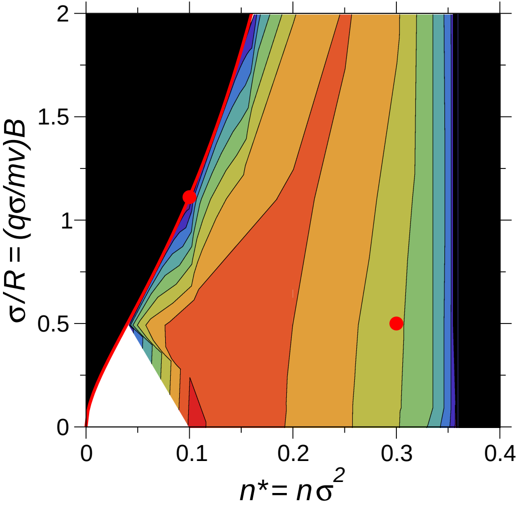
Phases



- low scatterer density:
  skipping orbit around isolated clusters only → insulating phase
- intermediate density:
  skipping orbits percolate through entire system → diffusive phase
- high scatterer density:
  void space consists only of finite pockets → localized phase

Purely geometric transition

Phase diagram



- Transition line to localized phase independent of curvature
  → conventional percolation of void space

$$n_c^* = 0.359081\ldots$$

  critical exponents correspond to conventional percolative transport on lattices

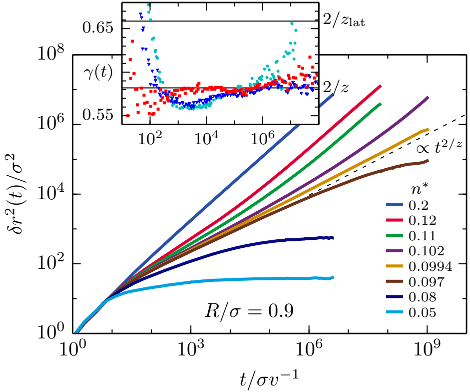
- Delocalization transition

$$n_m(R) = n_c^* \sigma^2 / (\sigma + R)^2$$

  → percolation of disks+halo
  exact result!, 2 significant digits

Kuzmany & Spohn, PRE (1998)

Mean-square displacements



local exponent $\gamma(t) = \dfrac{\mathrm{d}\log \delta r^2(t)}{\mathrm{d}\log t}$

anamalous transport at $n_m^* = n_m^*(R)$

- mean-square displacement (MSD)

$$\delta r^2(t) = \langle [\mathbf{R}(t) - \mathbf{R}(0)]^2 \rangle$$

- *subdiffusive* close to $n_m^* = n_m^*(R)$

$$\delta r^2(t \to \infty) \sim t^{2/z}$$

critical exponent $z = 2/\gamma$ measured value
$z = 3.44 \pm 0.03$

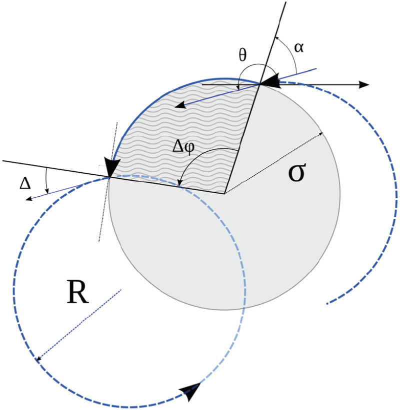
- different from random walkers on percolating lattices $z \neq z_{\mathrm{lat}} = 3.036 \pm 0.001$

New dynamic universality class

W. Schirmacher *et al*, Phys. Rev. Lett. (2015)

Circle swimmer in disordered environment

A more realistic scattering rule

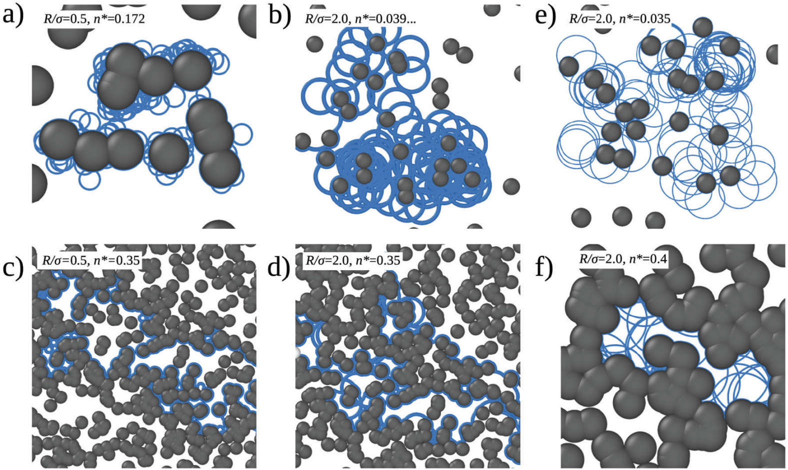


- ideal circle swimmer $D_{\mathrm{rot}} = 0$
- circle radius $R = v/\omega$
- obstacle diameter σ, dimensionless obstacle density $n^* = N\sigma^2/L^2$
- swimmer **slides** along the edge
- random exit angle $\Delta \in [-\pi/2, \pi/2]$ orientation ϑ remains fixed

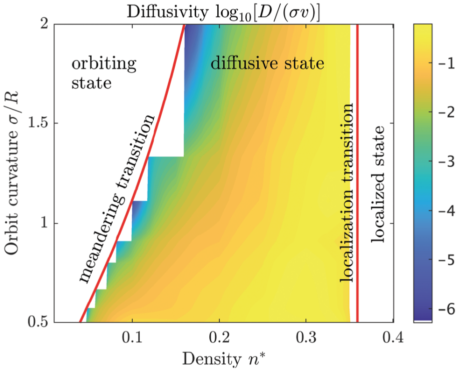
O. Chepizhko and T. Franosch, Soft Matter **15**, 452 (2019)

Circle swimmer in disordered environment

Sample trajectories



Circle swimmer in disordered environment



Phase diagram

- phase diagram pure **geometric!**
- percolation to localization at

$$n_c^* = 0.359081\ldots$$

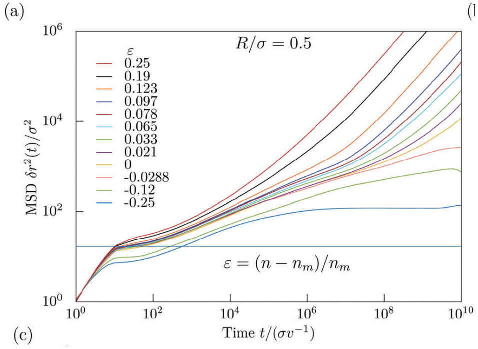
- **meandering transition** at

$$n_m^*(\sigma, R) = n_c^* \frac{\sigma^2}{(\sigma + R)^2} = 0.35908\ldots \frac{\sigma^2}{(\sigma + R)^2}$$

O. Chepizhko and T. Franosch, Soft Matter **15**, 452 (2019)

Circle swimmer in disordered environment

Mean-square displacements



- critical dynamics

$$\delta r^2(t) \propto t^{2/z} \qquad \text{for } t \to \infty$$

  at the meandering transition n_m^*
- dynamics drastically slower than **lattice** or **magnetotransport**

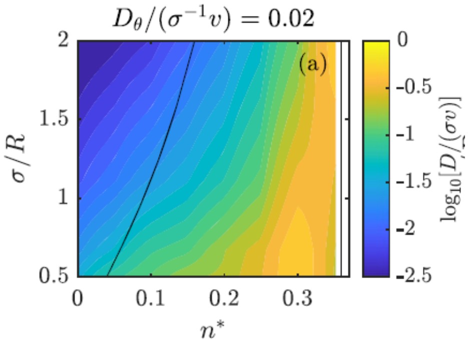
$$z = 5.17 \pm 0.48$$

- various scaling relations hold $D \propto (n^* - n_m^*)^\mu$
- attributed ot weak links in percolating networks
  → **new universality class**

O. Chepizhko and T. Franosch, Soft Matter **15**, 452 (2019)

Circle swimmer in disordered environment

Adding orientational noise



- angular diffusion

$$\frac{\mathrm{d}}{\mathrm{d}t}\vartheta(t) = \omega + \zeta(t)$$
$$\langle\zeta(t)\zeta(t')\rangle = 2D_{\text{rot}}\delta(t-t')$$

- Localization transition is unaffected geometric blocking ✓
- smearing of **meandering transition**
- **enhancement** of diffusion by meandering along boundaries for small obstacle density n^* most **efficient** for small radii $R \ll \sigma$
- strong suppression for large n^*

O. Chepizhko and T. Franosch, New J. Phys. **22**, 073022 (2019)

Conclusion on random environments

Passive particles ballistic/Brownian particles

- Lorentz model as paradigm for disorder
  randomly distributed overlapping obstacles
- localization transition at critical obstacle density
- **percolative transport:** critical phenomenon
  → **universal exponents** ✓

Active Brownian particles and circle swimmers

- ABP displays same universal localization transition ✓
- Meandering transition for ideal circle swimmer/magneto-transport
  new universality class for percolative transport
- small or moderate density of obstacles promotes diffusion
  → **crowded is faster**