Lattice Quantum Gravity

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Asymptotic Safety

Weinberg proposed idea that gravity might be Asymptotically Safe in 1976 [Erice Subnucl. Phys. 1976:1]. This scenario would entail:

- Gravity is effectively renormalizable when formulated non-perturbatively. Problem lies with perturbation theory, not general relativity.
- In a Euclidean lattice formulation the fixed point would show up as a continuous phase transition point, the approach to which would define a continuum limit.

Lattice gravity

- Euclidean dynamical triangulations (EDT) is a lattice formulation that was introduced in the '90's. [Ambjorn, Carfora, and Marzuoli, The geometry of dynamical triangulations, Springer, Berlin, 1997] Lattice geometries are approximated by triangles with fixed edge lengths. The dynamics is contained in the connectivity of the triangles, which can be added or deleted.
- Key new idea that inspired this study is that a fine-tuning of bare parameters in EDT is necessary to recover the correct continuum limit (Laiho, *et al.* PRD 96 (2017) 6, 064015). This is in analogy to using Wilson fermions in lattice gauge theory to study quantum chromodynamics (QCD) with light or massless quarks. Strong similarities are seen.

Einstein Hilbert Action

Continuum Euclidean path-integral:

$$Z = \int \mathscr{D}g \; e^{-S[g]}, \tag{1}$$

$$S[g_{\mu\nu}] = -\frac{k}{2}\int d^d x \sqrt{\det g}(R - 2\Lambda), \tag{2}$$

where $k = 1/(8\pi G_N)$.

Discrete Euclidean (Regge) action is

$$S_E = k\sum 2V_2\delta - \lambda \sum V_4, \tag{3}$$

where $\delta = 2\pi - \sum\theta$ is the deficit angle around a triangular face, V_i is the volume of an i-simplex, and $\lambda = k\Lambda$.

Measure term

Continuum calculations suggest a form for the measure

$$Z = \int \mathscr{D}g \prod_x \sqrt{\det g}^{\beta} \; e^{-S[g]}, \tag{4}$$

Going to the discretized theory, we have

$$\prod_x \sqrt{\det g}^{\beta} \to \prod_{j=1}^{N_2} \mathscr{O}(t_j)^{\beta}, \tag{5}$$

where $\mathscr{O}(t_j)$ is the order of triangle t_j, i.e. the number of 4-simplices to which a triangle belongs. Can incorporate this term in the action by taking exponential of the log. β is a free parameter in simulations. Can interpret as an ultra-local measure term, since it looks like a product over local 4-volumes. Must fine-tune β to recover physical geometries.

Main problems to overcome

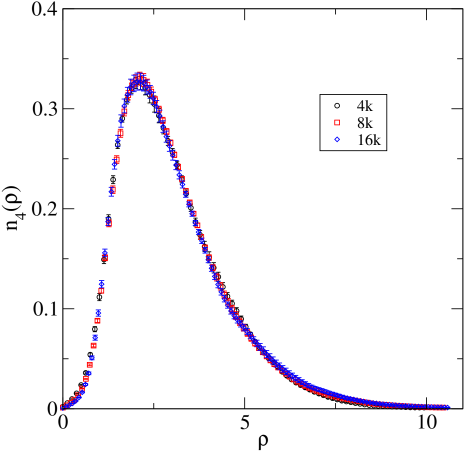
- Must show recovery of semiclassical physics in 4 dimensions, reproducing general relativity in the appropriate limit.
- Must show existence of continuum limit at continuous phase transition.

Simulations

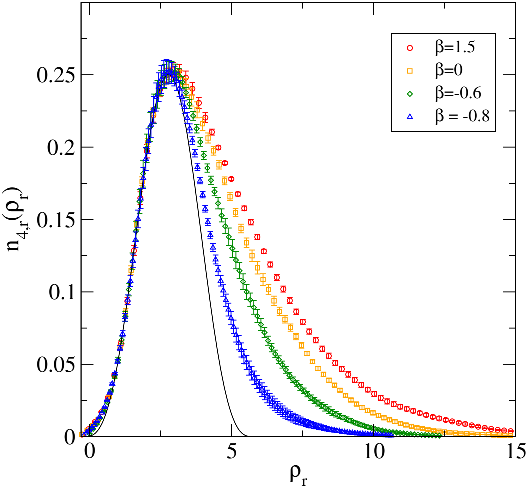
Methods for doing these simulations were introduced in the 90's. We wrote new code from scratch.

- The Metropolis Algorithm is implemented using a set of local update moves.
- We introduce a new algorithm for parallelizing the code, which we call parallel rejection. Exploits the low acceptance of the model, and partially compensates for it. Checked that it reproduces the scalar code configuration-by-configuration. Buys us a factor of ~ 10.

Three volume distribution



Three volume distribution



Diffusion process and the spectral dimension

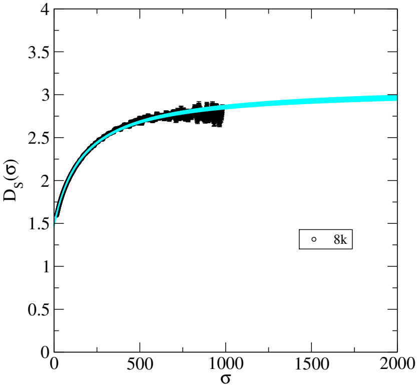
Spectral dimension is defined by a diffusion process

$$D_S(\sigma) = -2\frac{d\log P(\sigma)}{d\log\sigma}, \tag{6}$$

where σ is the diffusion time step on the lattice, and $P(\sigma)$ is the return probability, i.e. the probability of being back where you started in a random walk after σ steps.

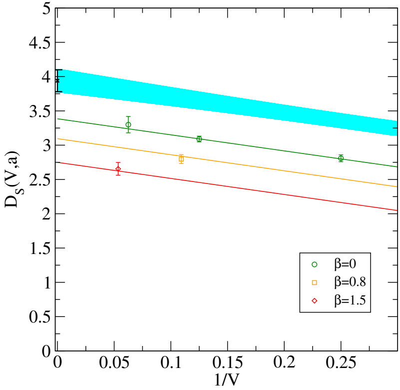
Spectral Dimension

χ^2/dof=1.25, p-value=17%
$D_S(\infty) = 3.090 \pm 0.041$, $D_S(0) = 1.484 \pm 0.021$

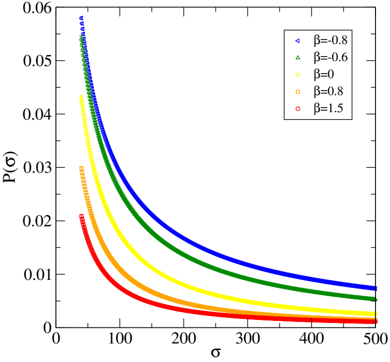


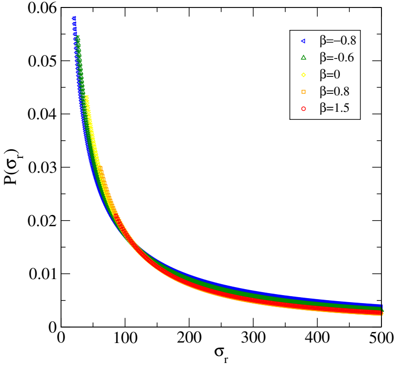
Infinite volume, continuum extrapolation

χ^2/dof=0.52, p-value=59%
$D_S(\infty) = 3.94 \pm 0.16$



Relative lattice spacing





Return probability left and rescaled return probability right.

de Sitter Instanton

Work with Scott Bassler and Marc Schiffer.

If the de Sitter solution dominates the partition function, we would have

$$Z(\kappa_4, \kappa_2) = \sum_{N_4} e^{-(\kappa_4 - \kappa_4^c)N_4 + k(\kappa_2)\sqrt{N_4}} \tag{7}$$

where

$$k(\kappa_2) \propto \frac{a^2}{G} \tag{8}$$

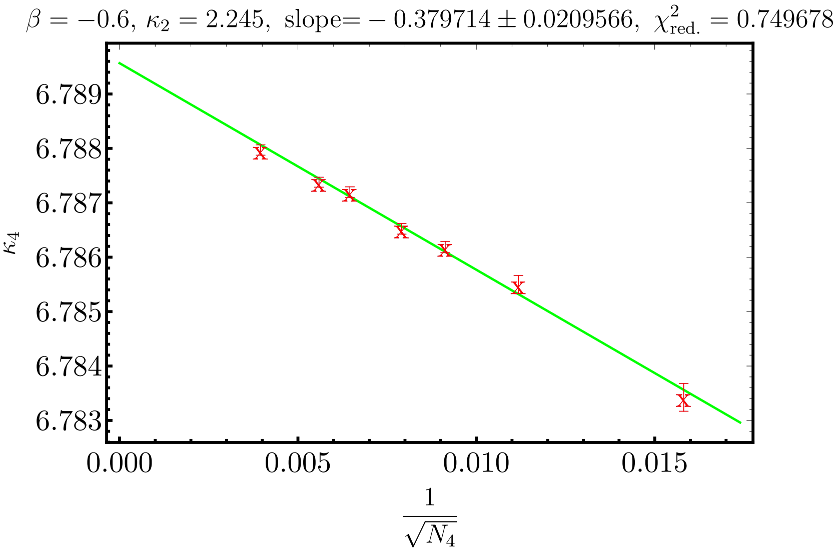
de Sitter Instanton

$$\langle N_4 \rangle = \frac{\sum_{N_4} N_4 e^{-(\kappa_4 - \kappa_4^c)N_4 + k(\kappa_2)\sqrt{N_4}}}{\sum_{N_4} e^{-(\kappa_4 - \kappa_4^c)N_4 + k(\kappa_2)\sqrt{N_4}}} \approx \frac{k^2(\kappa_2)}{4(\kappa_4 - \kappa_4^c)^2} \tag{9}$$

Thus,

$$k = 2|\kappa_4 - \kappa_4^c|\sqrt{N_4} \tag{10}$$

Extracting G



Hawking-Moss instanton solution

$$Z(\kappa_4, \kappa_2) \approx \exp\left(\frac{k^2(\kappa_2)}{4(\kappa_4 - \kappa_4^c)}\right) = \exp\left(\frac{3\pi}{G\Lambda}\right) \tag{11}$$

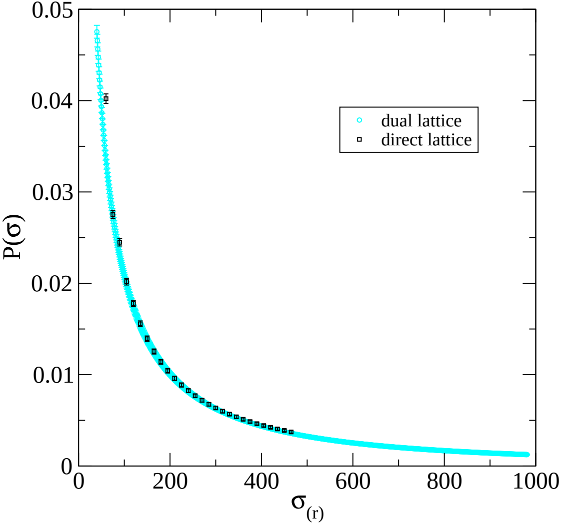
Can use

$$(\kappa_4 - \kappa_4^c)N_4 = \frac{\Lambda}{8\pi G}V_4, \tag{12}$$

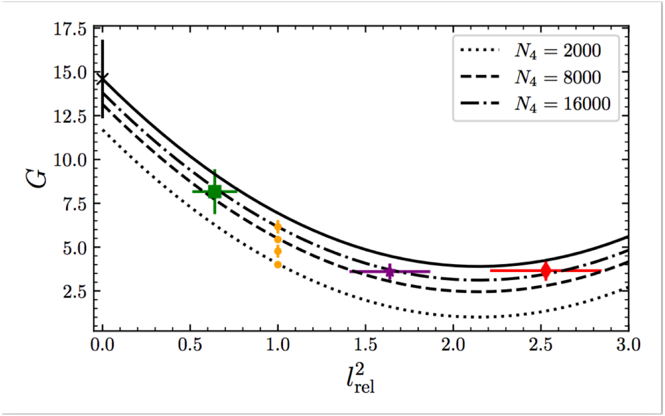
where $V_4 = C_4 N_4 a^4$, with C_4 a geometric factor. These relations can be combined to allow us to write k in terms of G, giving us an absolute lattice spacing (in link units a).

For many applications, we would like to know the lattice spacing in simplex units, for example the relative lattice spacing from the return probability is measured in simplex units. We can also determine G in principle from the bound states of scalar particles. Natural to use simplex units in that case.

Converting link and simplex distance



Absolute lattice spacing



Can get absolute lattice spacing once we know G. Our "coarse" lattice spacing is around 1/4 the Planck length, and our fine lattice spacing is around 1/6 the Planck length, so pushing into sub-Planckian regime.

Adding matter

We have looked at adding both scalar and fermion matter fields.

Adding scalars and U(1) gauge fields dynamically (unquenched) was done already over twenty years ago.

For now we are revisiting things in the quenched approximation, where matter loops are neglected, since this allows us to reuse existing lattice ensembles.

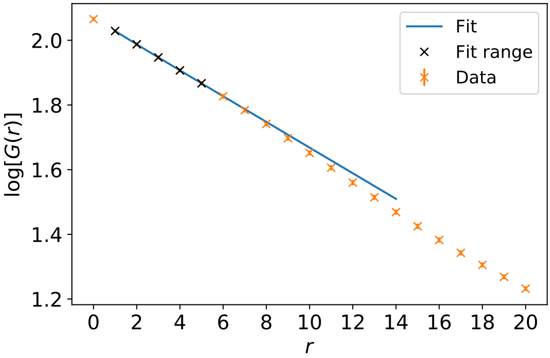
Scalar field

In the continuum we can add to the Einstein Hilbert action the action for the scalar field:

$$S[g,\phi] = \int d^4x\sqrt{g}\left(\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi + \frac{1}{2}m_0^2\phi^2\right), \qquad (13)$$

Lattice discretization is straightforward. Looking at scalar propagators on quenched configurations. This work is done in collaboration with Judah Unmuth-Yockey, Marc Schiffer, and Mingwei Dai.

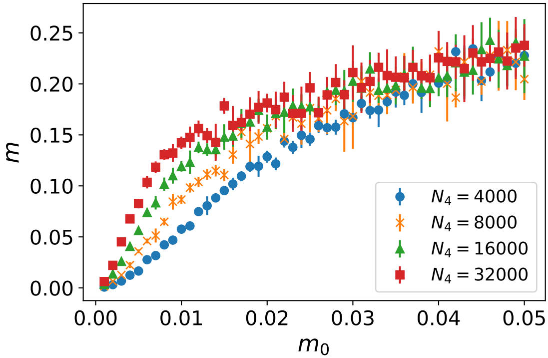
Scalar propagator



$$G(r) = (-\Box + m_0^2)^{-1}_{0r} \tag{14}$$

Functional form of the correlator is $B \exp(Ar)/r^C$.

Mass dependence



The shift symmetry of the action ensures that $m_r \to 0$ as $m_0 \to 0$.

Binding energy

We follow the work of de Bakker and Smit, Nucl.Phys. B484 (1997) 476. They looked at gravitational binding on EDT near the transition, at what was effectively a single coarse lattice spacing. We revisit this work with our current understanding and ensembles.

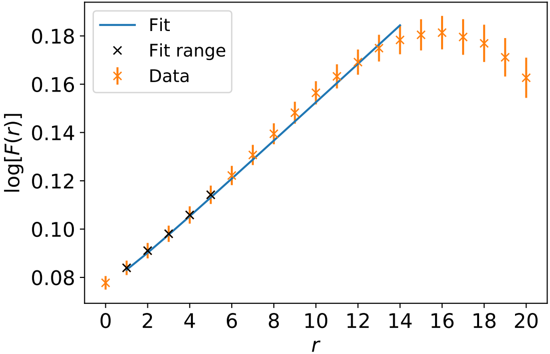
We are looking to calculate the binding energy:

$$E_b \equiv 2m - M = \frac{1}{4} G^2 m^5. \tag{15}$$

This is just the familiar energy of the hydrogen atom, $\alpha^2 m_{\mathrm{red}}/2$ but with $\alpha \to Gm^2$ and $m_{\mathrm{red}} \to m/2$. Presumably applies in the non-relativistic limit.

Also worth noting that we can calculate this for different dimensions. In three dimensions, $E_b \propto m^2$.

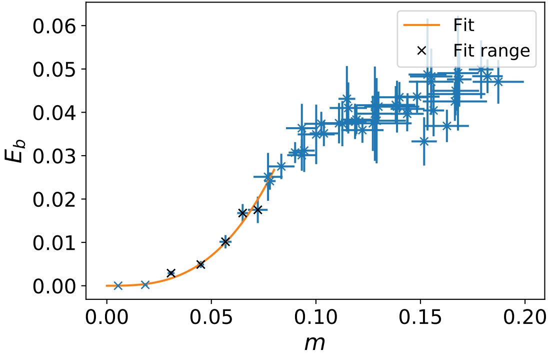
Binding Energy



$$F(r) = \frac{G^{(2)}(r)}{G(r)^2} \tag{16}$$

where G and $G^{(2)}$ are the one and two particle scalar propagators, respectively. Can extract $2m - M$ from exponential of this ratio.

Power law mass dependence

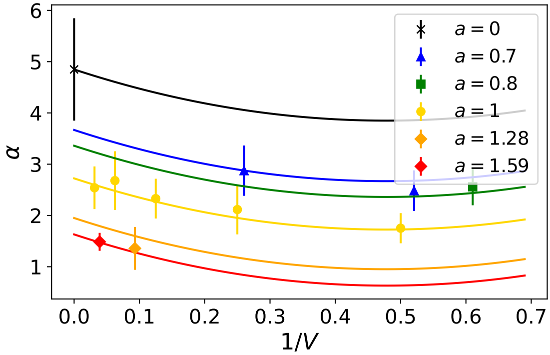


Fit to form

$$E_b = A \times m^{\alpha} \tag{17}$$

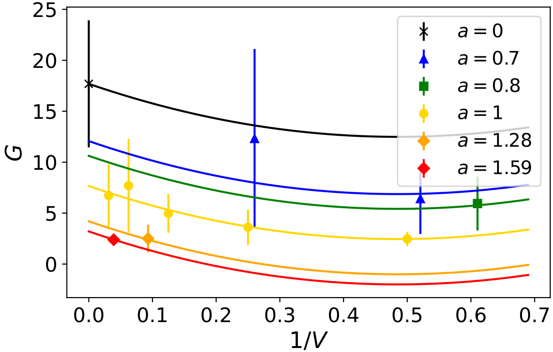
α should be 5, A is proportional to Newton's constant squared.

Exponent result



Note: $d = 3$ corresponds to $\alpha = 2$,
Sensitive dependence of α on d means there is a big extrapolation to $d = 4$, $\alpha = 5$.

Result for G



The same fit gives a value for Newton's constant. Should be compared to the de Sitter value, 14.5 ± 2.0.

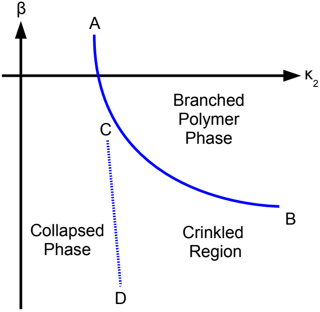
Conclusions

Two different ways to compute Newton constant. One makes contact with quantum cosmology and involves semiclassical expansion of partition function around de Sitter space. The other involves the gravitational interaction of scalar particles. Nontrivial agreement suggests that we are describing quantum gravity. There are non-perturbative effects in the strongly coupled regime used to set the relative lattice spacing. They must be scaling or else this picture wouldn't hold together, suggesting we are not "merely" reproducing the effective theory.

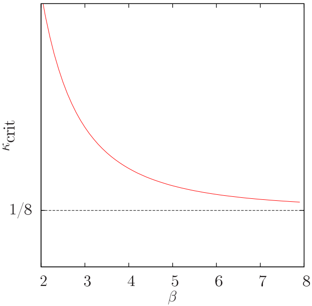
Unphysical behavior gets smaller as the continuum limit is approached for a number of different types of observables. Evidence for a continuum limit and thus asymptotic safety.

Back-up Slides

Phase diagram EDT vs. QCD with Wilson fermions

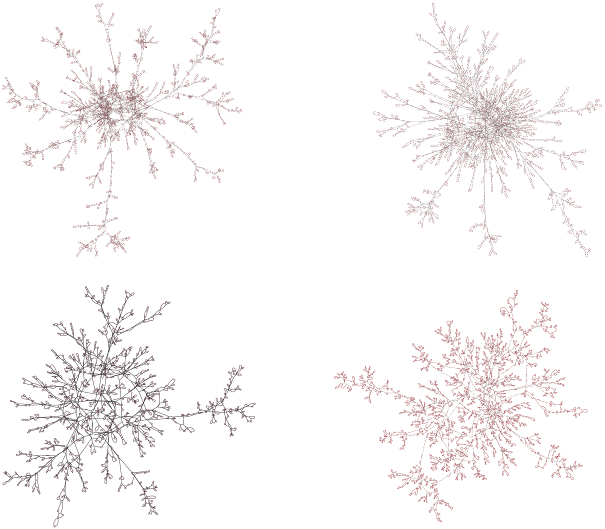


EDT



QCD

Visualization of geometries



Coarser to finer, left to right, top to bottom.

Kähler-Dirac fermions on dynamical triangulations

$$S_{KD} = \int d^4x \sqrt{g}\, \overline{\omega}(\mathrm{d} - \delta + m_0)\omega. \quad (18)$$

where ω is a collection of Grassman valued p-form fields, which are coupled through the exterior derivative d and its adjoint δ, and m_0 is the mass.

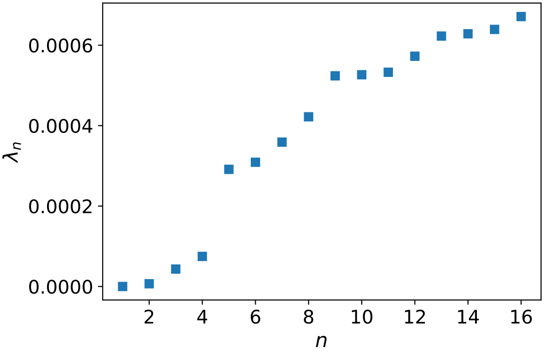
In flat space (in 4d) this corresponds to four copies of Dirac fermions.

There is a straightforward mapping from the exterior derivatives to operators acting on discrete space, without the need to introduce vielbeins or spin connections.

In flat space the lattice Kähler-Dirac action leads to staggered fermions. We will see that this approach leads to a natural extension of staggered fermions to random geometries.

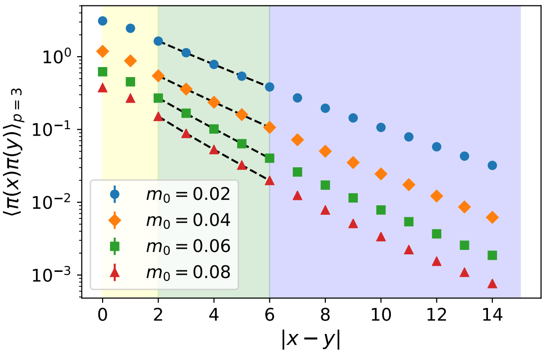
Work done with Simon Catterall and Judah Unmuth-Yockey. PRD 98 (2018) 11, 114503.

Kähler-Dirac eigenvalue spectrum



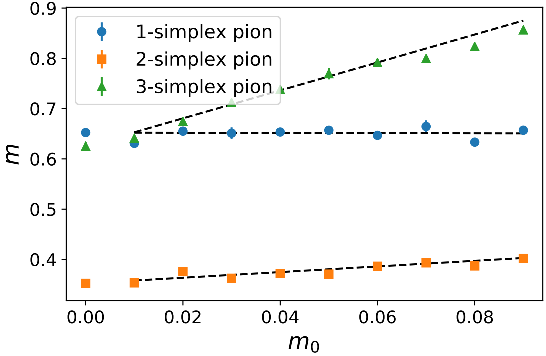
Approximate four-fold degeneracy of (quenched) eigenvalue spectrum

“Meson" propagators



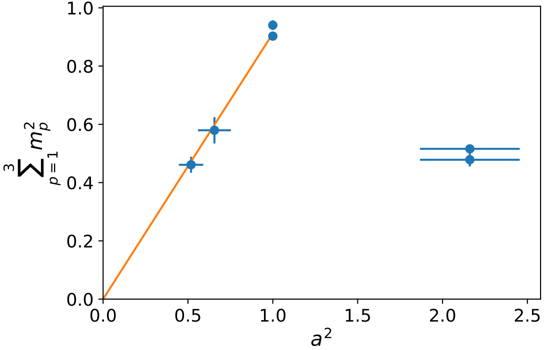
We can construct scalar "pion" correlators and extract their masses

“Taste" breaking in chiral limit



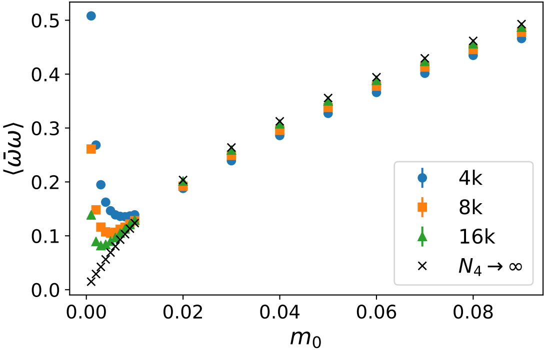
Different flavor (or “taste") pions extrapolated to the chiral limit at fixed lattice spacing.

Taste breaking mass splittings



In the continuum limit the taste breaking appears to vanish. Very similar to QCD with staggered fermions.

Chiral condensate



Chiral condensate appears to vanish in the chiral limit at infinite volume, so no spontaneous chiral symmetry breaking. Important, since we do not want chiral symmetry breaking at the strong coupling scale (the Planck scale!). This would be in disagreement with phenomenology since fermion bound states are not of order the Planck mass.