

Tau Physics: hot topics and future prospects

Swagato Banerjee

A photograph of various spices and ingredients arranged on a dark surface. The items include a spoonful of yellow cardamom pods, a pile of star anise, a pile of yellow turmeric powder, and several cinnamon sticks. The ICTS logo is visible in the upper right corner of the image.

Future Flavours:
Prospects for Beauty,
Charm and Tau Physics

Flavour physics, the study of weak interactions of 25 April – 06 May 2022

Overview of this talk

<https://www.slac.stanford.edu/~mpeskin/Snowmass2021/BelleIIPhysicsforSnowmass.pdf>

Snowmass White Paper: Belle II physics reach and plans for the next decade and beyond

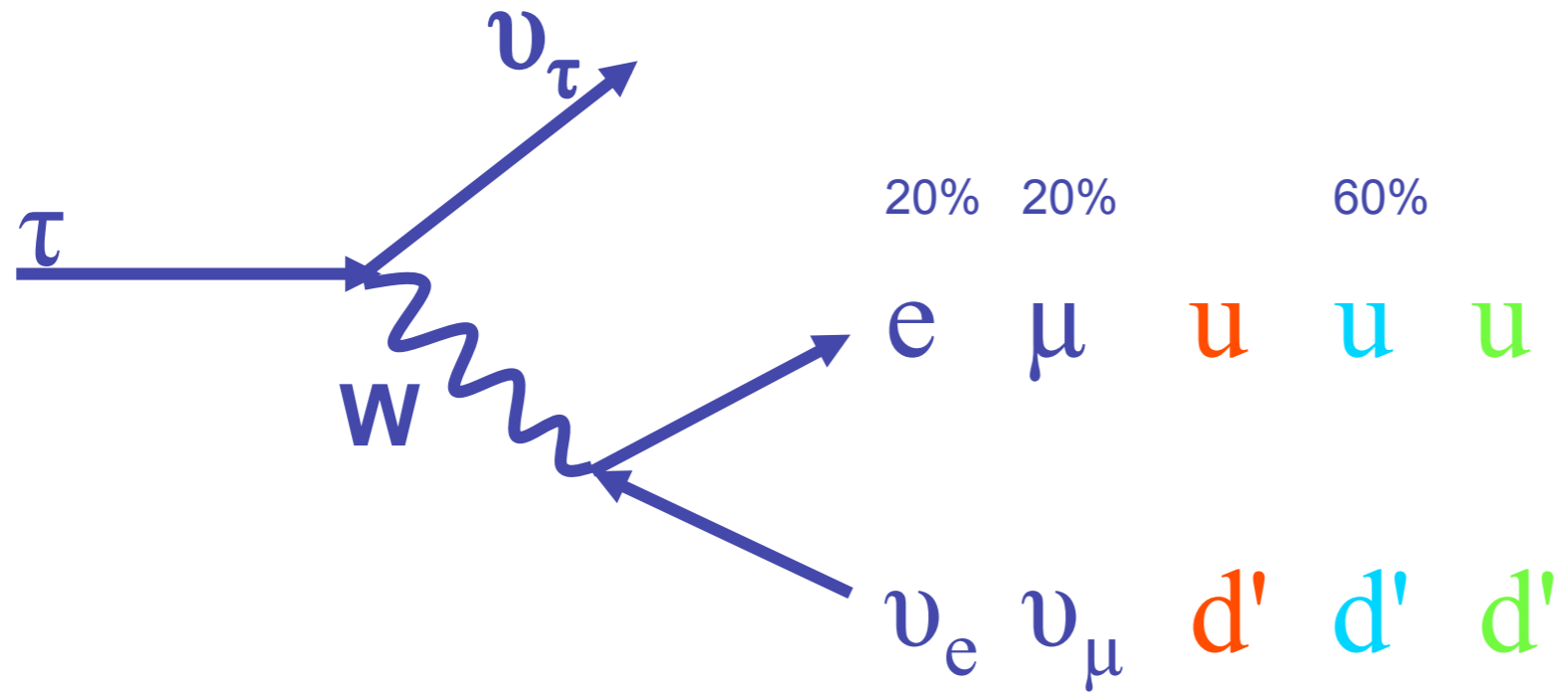
Belle II Collaboration



9	Tau lepton physics	27
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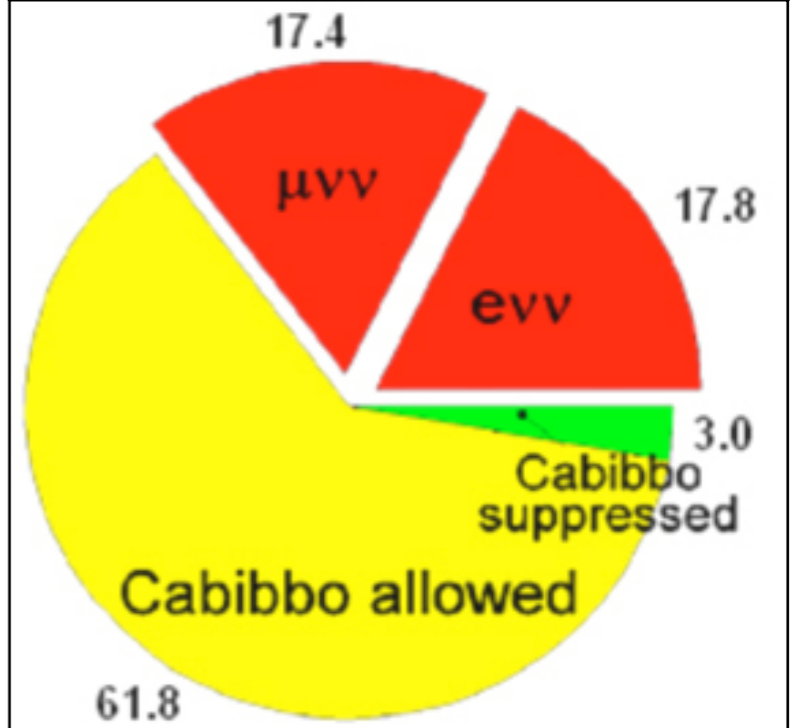
Tau Decays

Naive prediction:

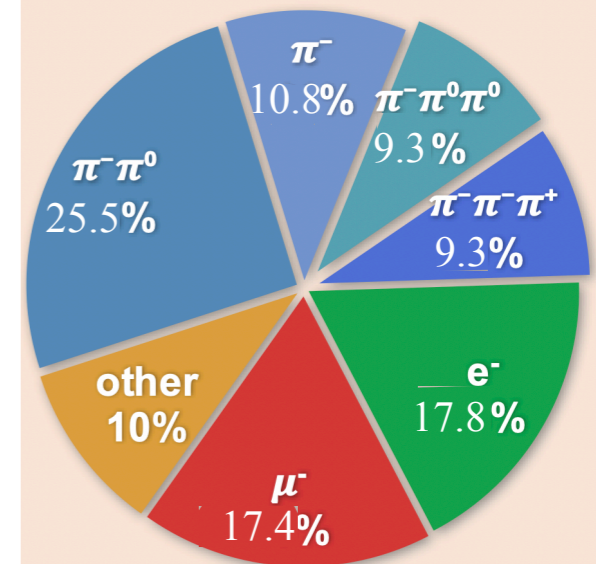


$$|d'\rangle = V_{ud}|d\rangle + V_{us}|s\rangle$$

Including QED & QCD corrections:



τ decay

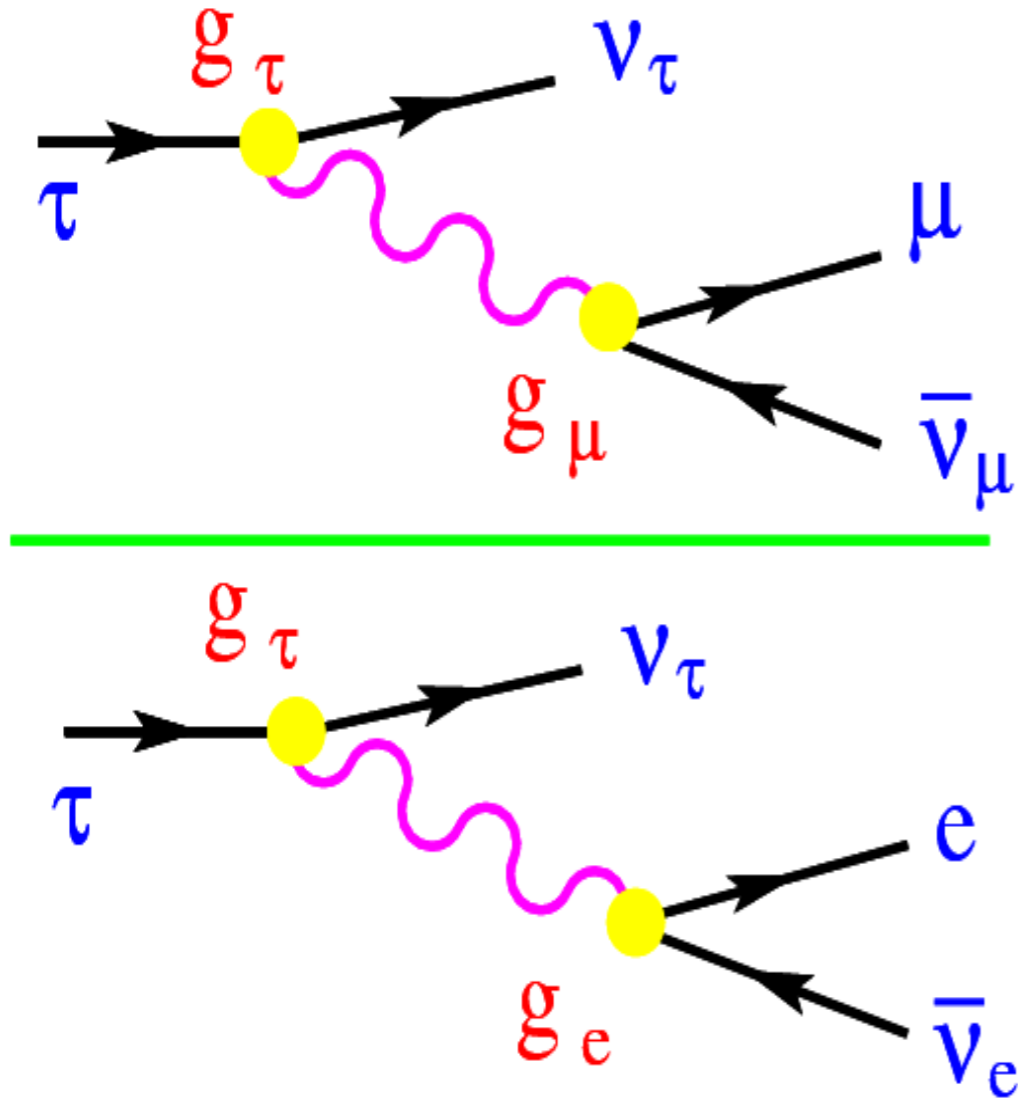


τ ⁻ DECAY MODES	Fraction (Γ _i /Γ)	Scale factor/ Confidence level	p (MeV/c)
Modes with one charged particle			
particle ⁻ ≥ 0 neutrals ≥ 0K ⁰ ν _τ ("1-prong")	(85.24 ± 0.06) %		-
particle ⁻ ≥ 0 neutrals ≥ 0K _L ⁰ ν _τ	(84.58 ± 0.06) %		-
μ ⁻ ν̄ _μ ν _τ	[g] (17.39 ± 0.04) %		885
μ ⁻ ν̄ _μ ν _τ γ	[e] (3.67 ± 0.08) × 10 ⁻³		885
e ⁻ ν̄ _e ν _τ	[g] (17.82 ± 0.04) %		888
e ⁻ ν̄ _e ν _τ γ	[e] (1.83 ± 0.05) %		888
h ⁻ ≥ 0K _L ⁰ ν _τ	(12.03 ± 0.05) %		883
h ⁻ ν _τ	(11.51 ± 0.05) %		883
π ⁻ ν _τ	[g] (10.82 ± 0.05) %		883
K ⁻ ν _τ	[g] (6.96 ± 0.10) × 10 ⁻³		820

The Review of Particle Physics (2021)

P.A. Zyla *et al.* (Particle Data Group), Prog. Theor. Exp. Phys. 2020, 083C01 (2020) and 2021 update.

Lepton Flavor universality: muon vs electron $\left(\frac{g_\mu}{g_e}\right)$



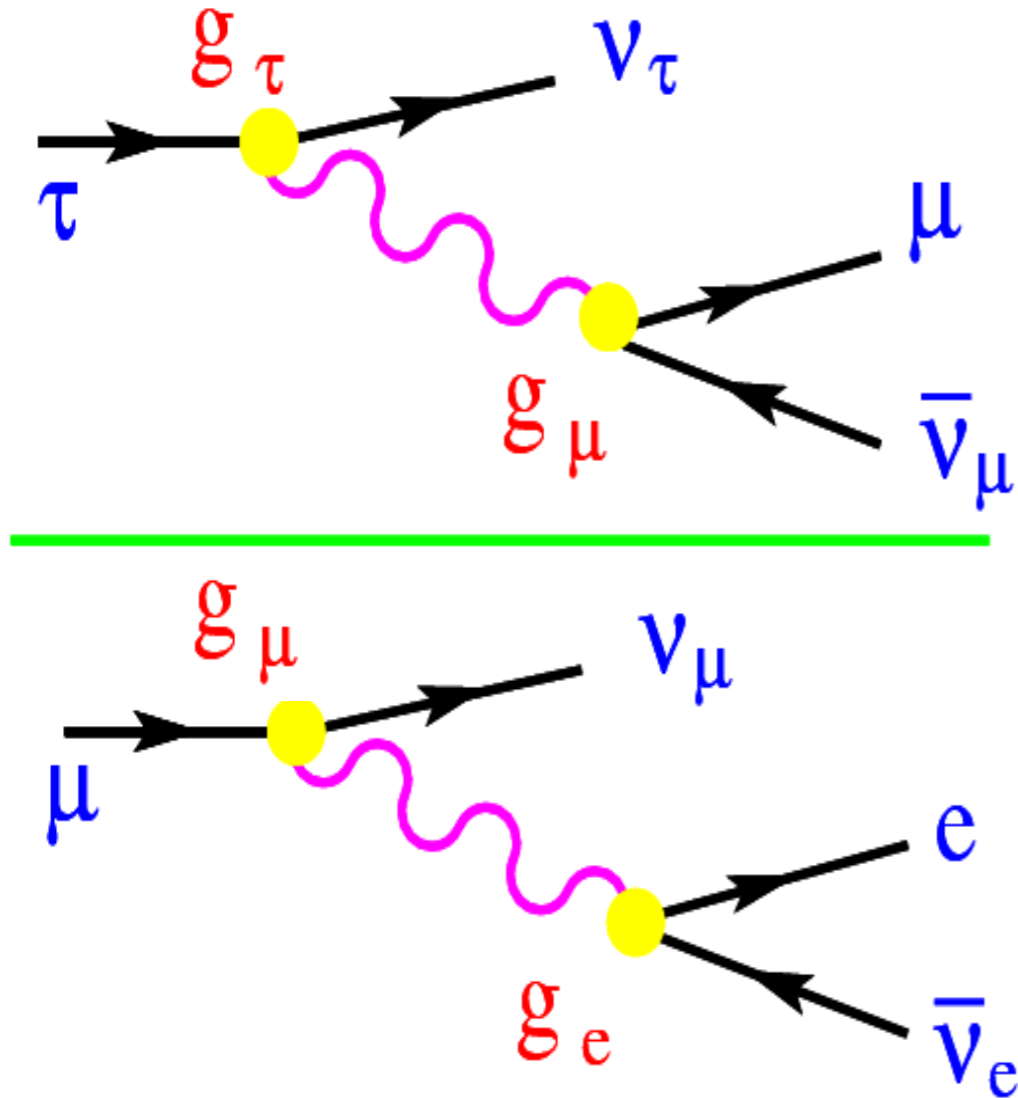
$$\left(\frac{g_\mu}{g_e}\right)^2 = \frac{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) f(m_e^2/m_\tau^2)}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) f(m_\mu^2/m_\tau^2)}$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x \ln x \quad (\text{approximating all } m_\nu = 0)$$

Measure:

$$R_\mu \equiv \frac{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)}$$

Lepton Flavor universality: tau vs electron $\left(\frac{g_\tau}{g_e}\right)$



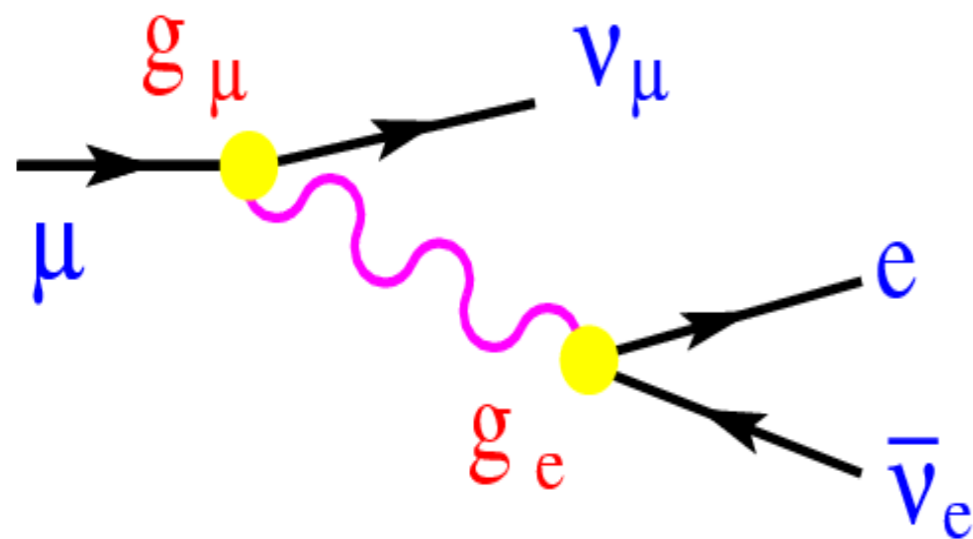
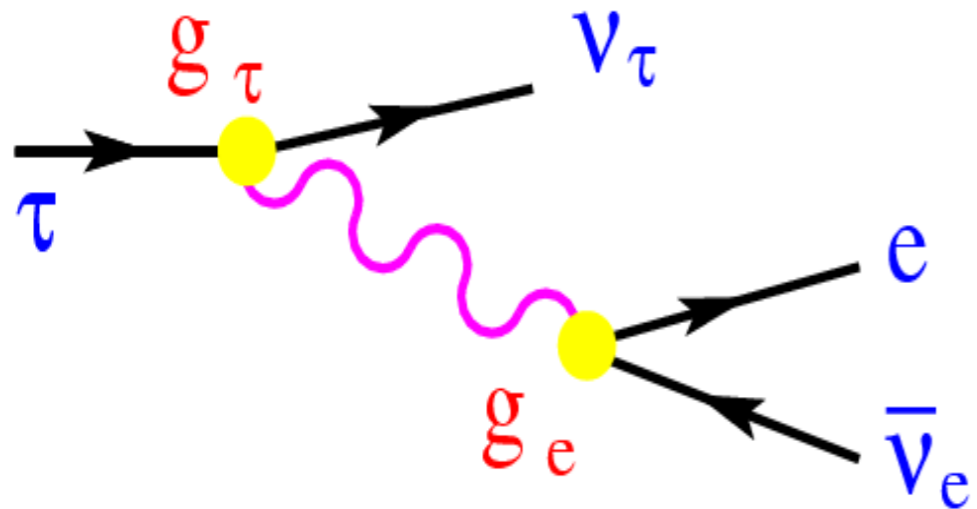
$$\left(\frac{g_\tau}{g_e}\right)^2 = \frac{\tau_\mu}{\tau_\tau} \mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau) \left(\frac{m_\mu}{m_\tau}\right)^5 \frac{f(m_e^2/m_\mu^2) r_{EW}^\mu}{f(m_\mu^2/m_\tau^2) r_{EW}^\tau}$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x \ln x \quad (\text{approximating all } m_\nu = 0)$$

Measurement:

$$m_\tau, \tau_\tau, \mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)$$

Lepton Flavor universality: tau vs muon $\left(\frac{g_\tau}{g_\mu}\right)$



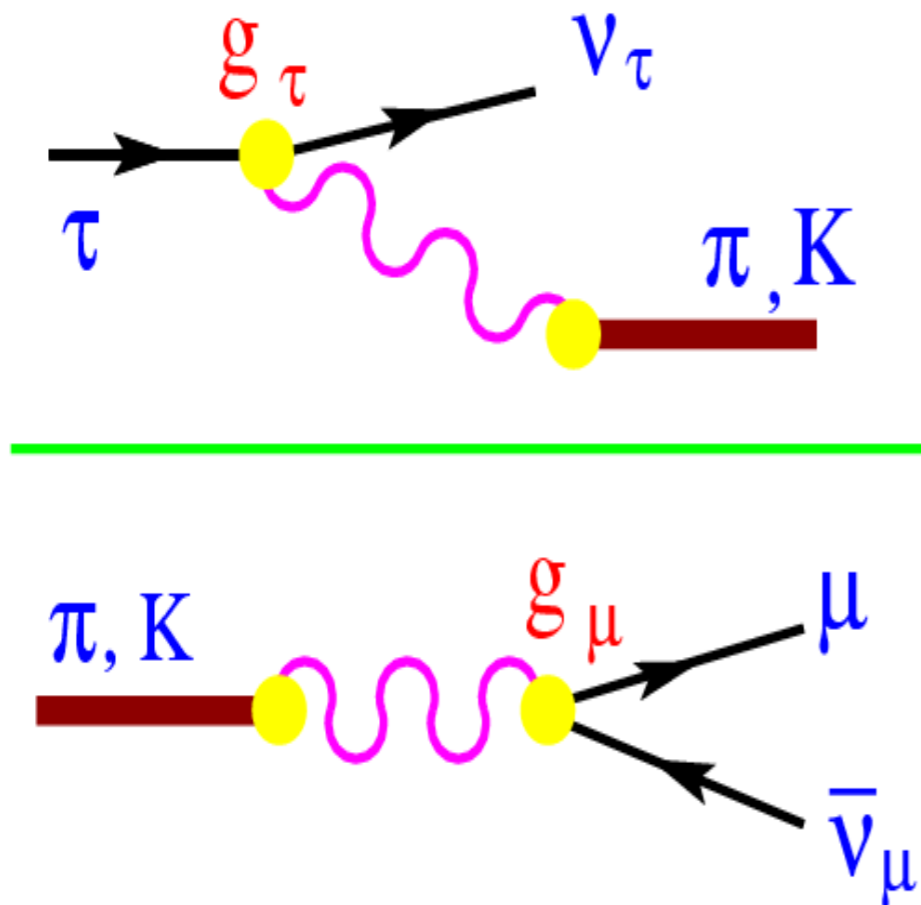
$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{\tau_\mu}{\tau_\tau} \mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau) \left(\frac{m_\mu}{m_\tau}\right)^5 \frac{f(m_e^2/m_\mu^2) r_{EW}^\mu}{f(m_e^2/m_\tau^2) r_{EW}^\tau}$$

$$f(x) = 1 - 8x + 8x^3 - x^4 - 12x \ln x \quad (\text{approximating all } m_\nu = 0)$$

Measurement:

$$m_\tau, \tau_\tau, \mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)$$

Lepton Flavor universality: tau vs muon $\left(\frac{g_\tau}{g_\mu}\right)$



$$\left(\frac{g_\tau}{g_\mu}\right)^2 = \frac{2m_h m_\mu^2 \tau_h}{(1 + \delta_{\tau/h}) m_\tau^3 \tau_\tau} \frac{\mathcal{B}(\tau^- \rightarrow h^- \nu_\tau)}{\mathcal{B}(h^- \rightarrow \mu^- \bar{\nu}_\mu)} \left(\frac{1 - m_\mu^2/m_h^2}{1 - m_h^2/m_\tau^2}\right)^2$$

Measurement:

$$m_\tau, \tau_\tau, \mathcal{B}(\tau^- \rightarrow h^- \nu_\tau) [h^- = \pi^- / K^-]$$

Lepton Flavor universality



$$\left(\frac{g_\mu}{g_e}\right)_\tau = 1.0019 \pm 0.0014$$

$$\left(\frac{g_\tau}{g_e}\right)_\tau = 1.0027 \pm 0.0014$$

$$\left(\frac{g_\tau}{g_\mu}\right)_\tau = 1.0009 \pm 0.0014$$

$$\left(\frac{g_\tau}{g_\mu}\right)_\pi = 0.9959 \pm 0.0038$$

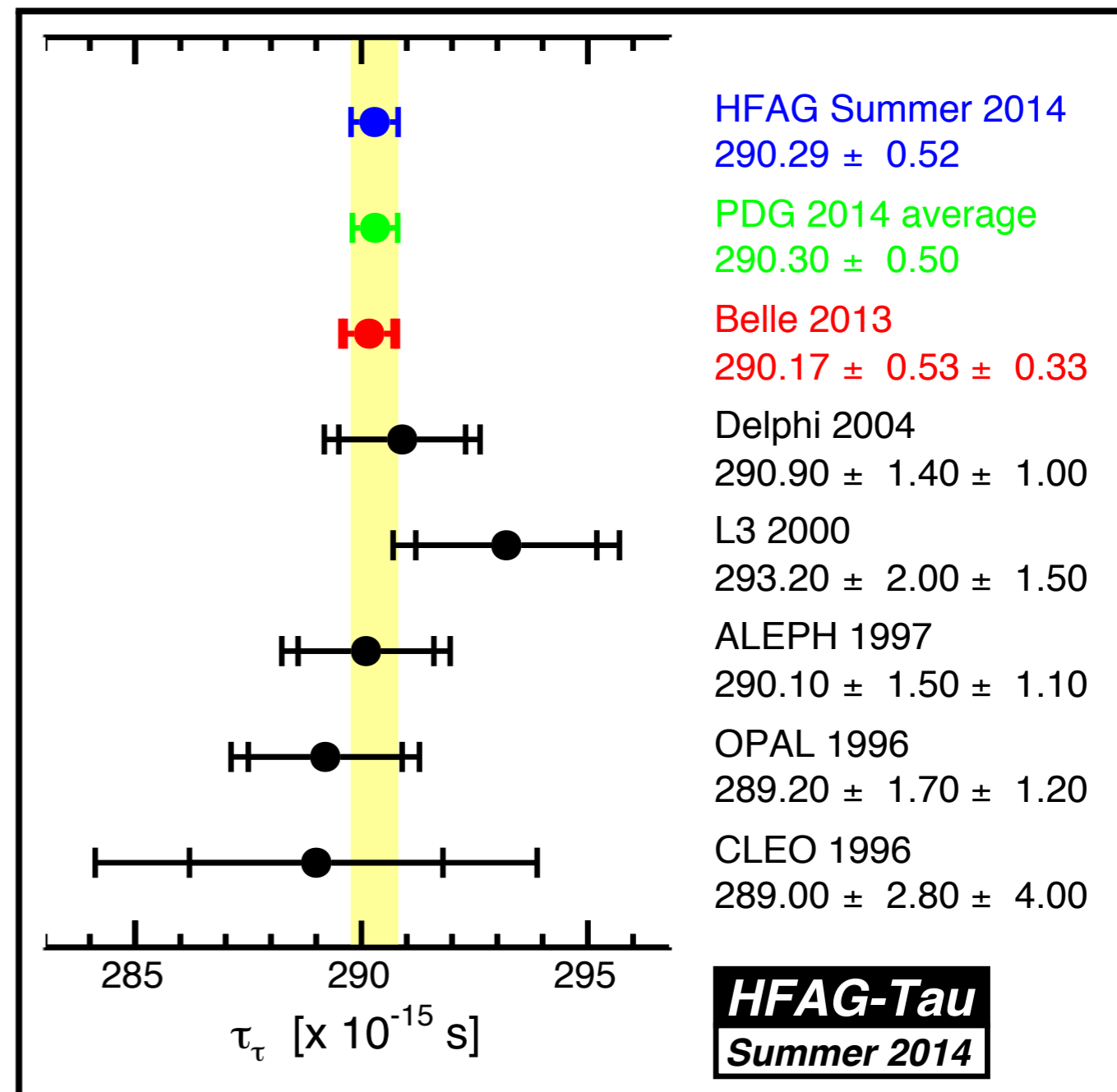
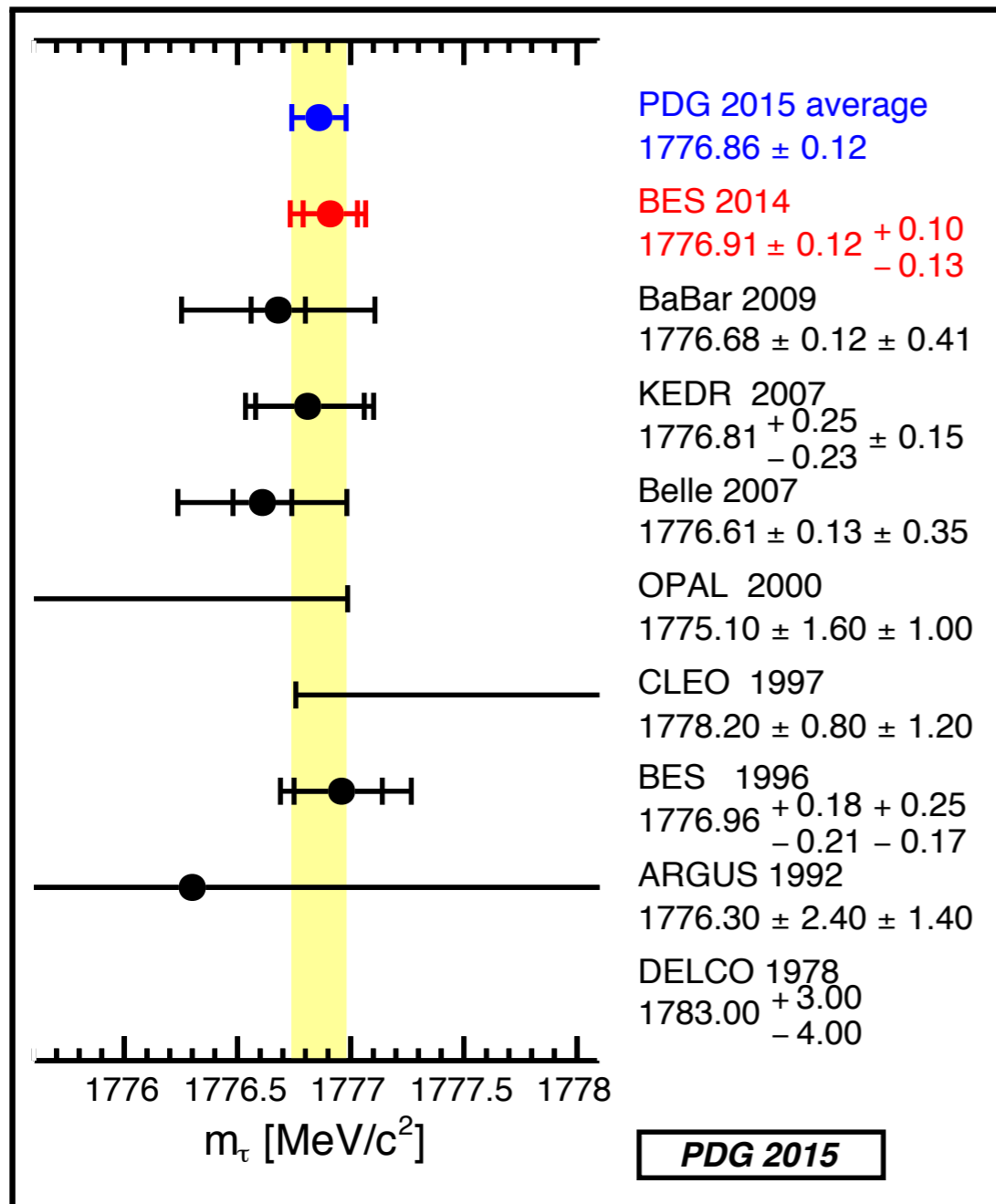
$$\left(\frac{g_\tau}{g_\mu}\right)_K = 0.9855 \pm 0.0075$$

$$\left(\frac{g_\tau}{g_\mu}\right)_{\tau+\pi+K} = 1.0003 \pm 0.0014$$

Table 12: Universality coupling ratios correlation coefficients (%)

$\left(\frac{g_\tau}{g_e}\right)_\tau$	51			
$\left(\frac{g_\mu}{g_e}\right)_\tau$	-50	49		
$\left(\frac{g_\tau}{g_\mu}\right)_\pi$	16	18	1	
$\left(\frac{g_\tau}{g_\mu}\right)_K$	12	11	-1	7
	$\left(\frac{g_\tau}{g_\mu}\right)_\tau$	$\left(\frac{g_\tau}{g_e}\right)_\tau$	$\left(\frac{g_\mu}{g_e}\right)_\tau$	$\left(\frac{g_\tau}{g_\mu}\right)_\pi$

τ mass and lifetime

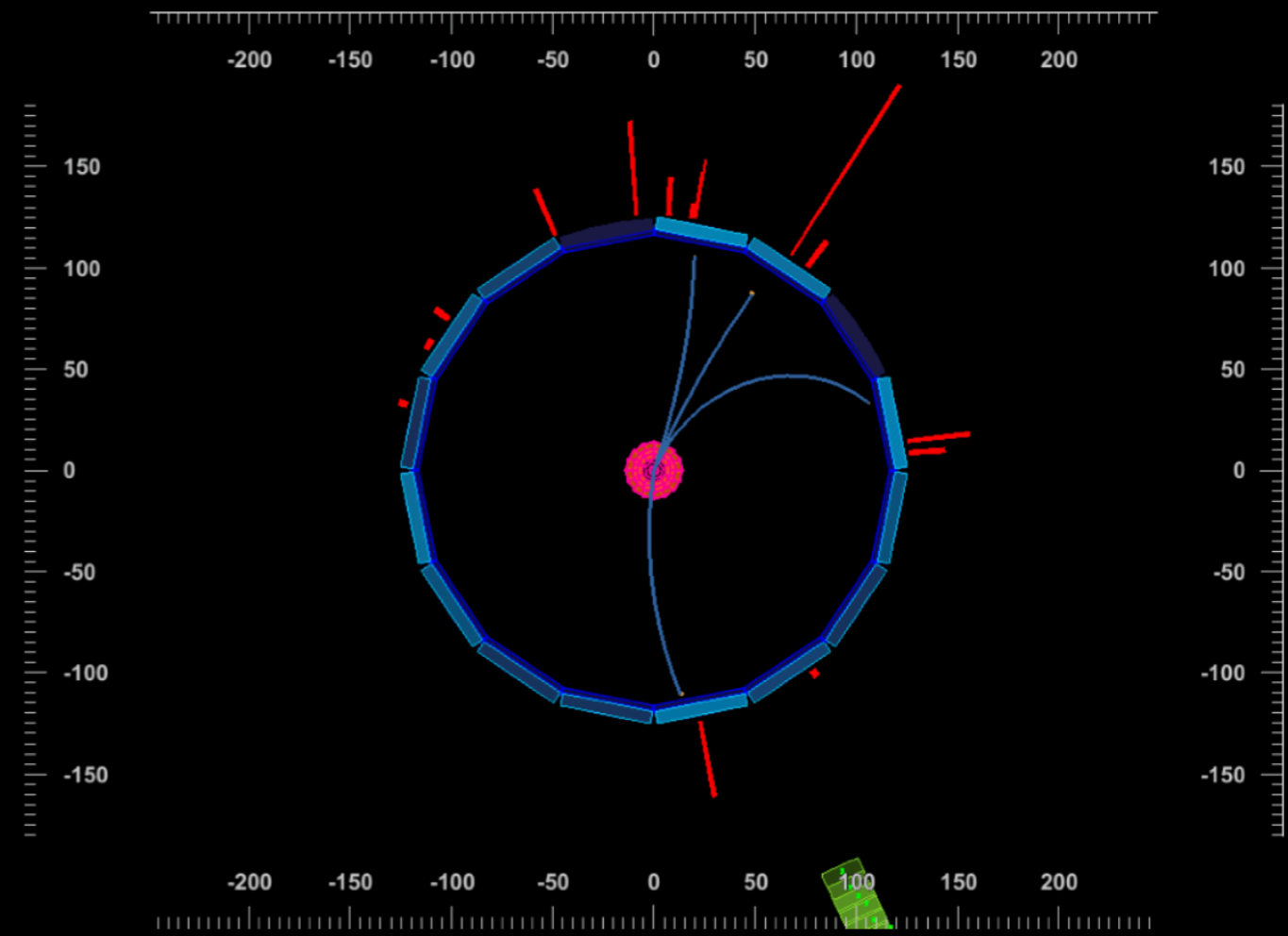
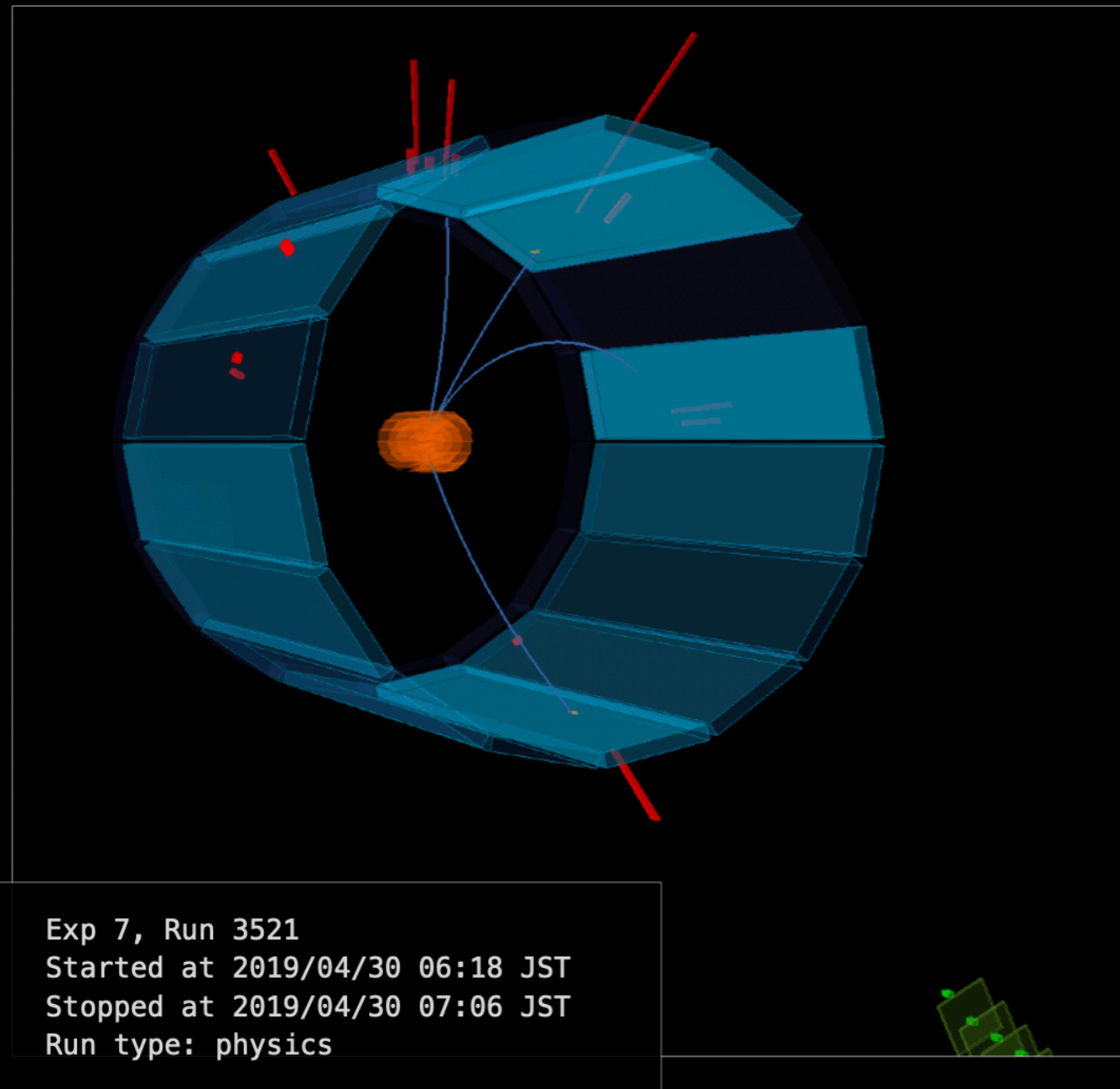


Most precise measurements
at $\tau^- \tau^+$ threshold

Most precise measurement by
Belle using 3-vs-3 topology

Tau-pair event at Belle II

1-vs-3 topology



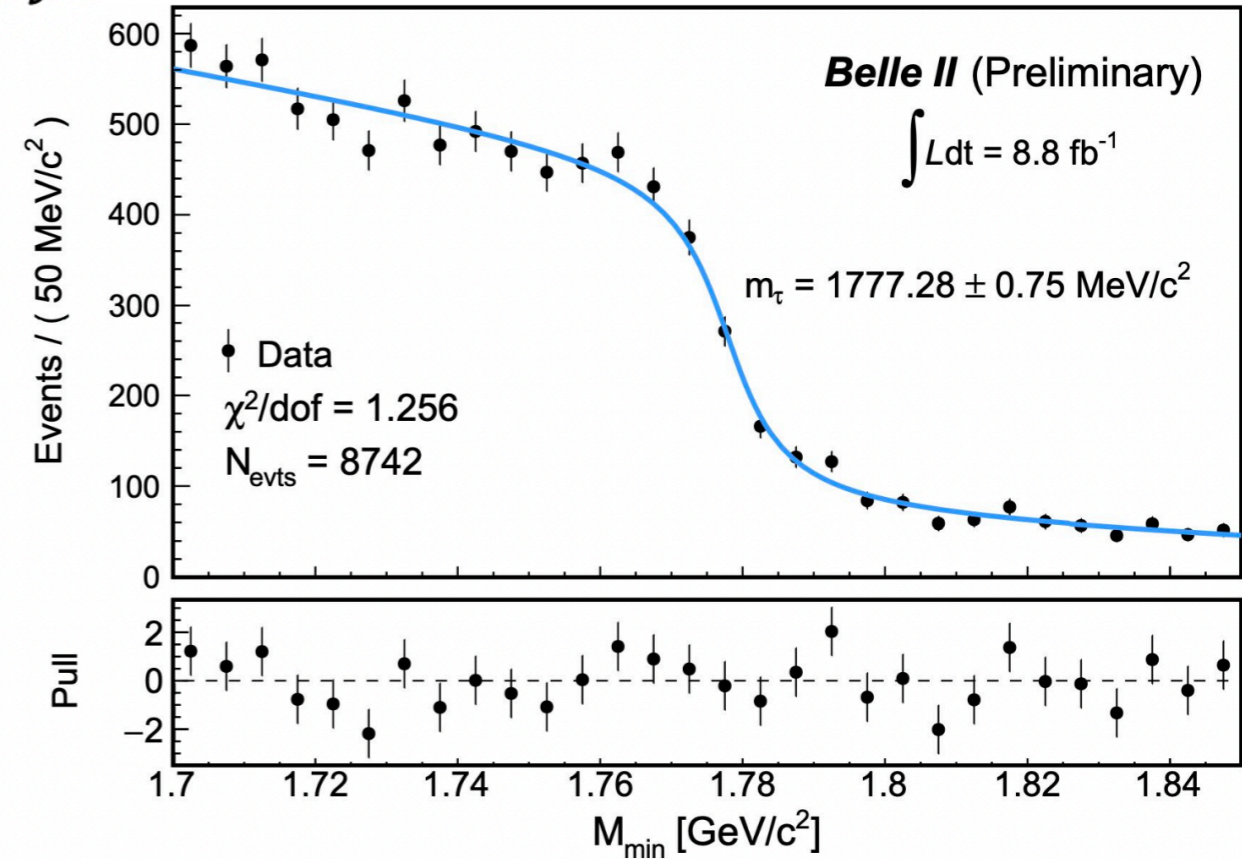
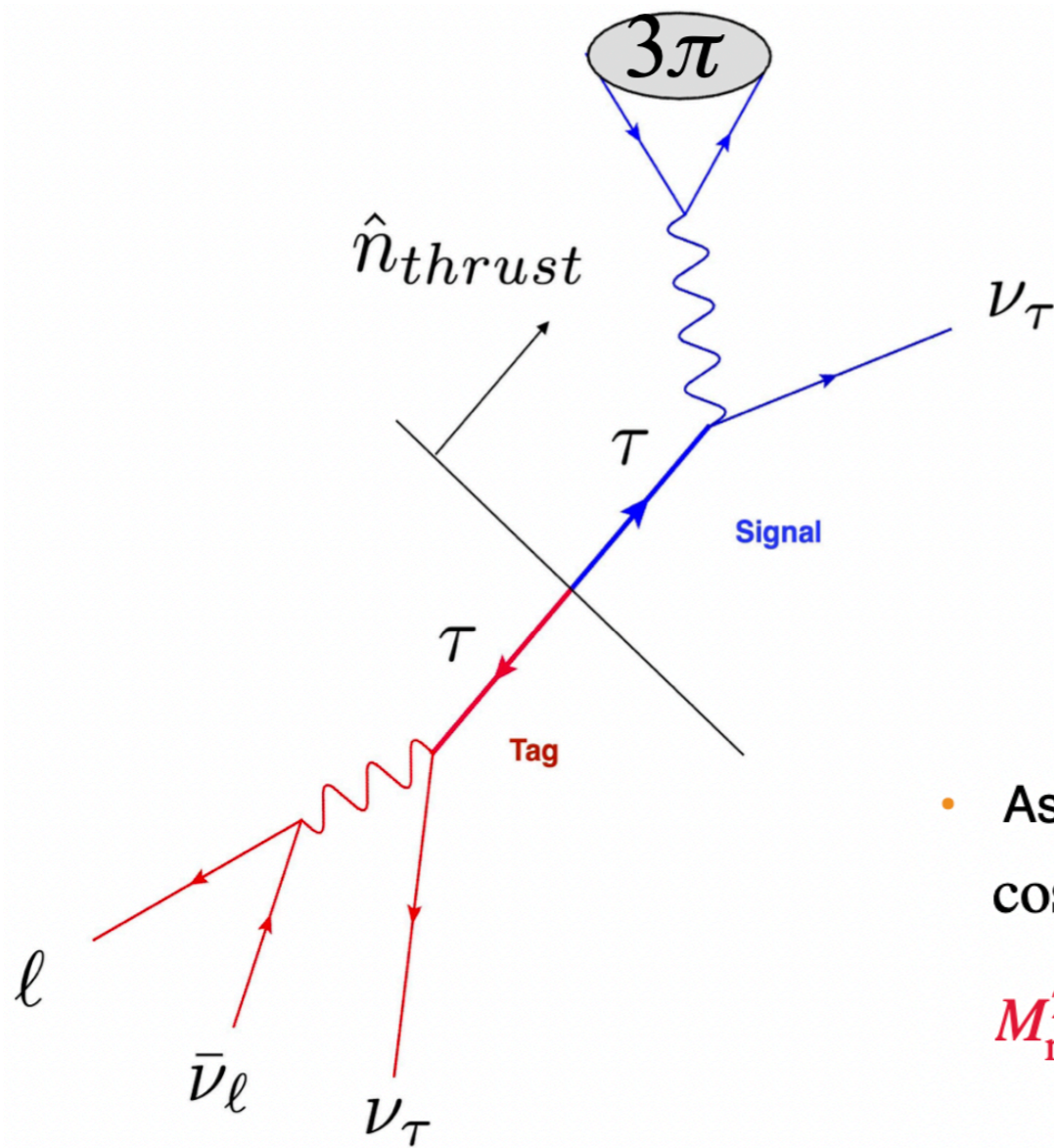
Tau Mass at Belle II

[arXiv:2008.04665](https://arxiv.org/abs/2008.04665) [hep-ex]

- The tau mass can be calculated as

$$m_\tau^2 = (p_h + p_\nu)^2$$

$$= 2E_h(E_\tau - E_h) + m_h^2 - 2|\vec{p}_h|(E_\tau - E_h) \cos(\vec{p}_h, \vec{p}_\nu)$$



- As the direction of the neutrino is not known, the approximation $\cos(\vec{p}_\nu, \vec{p}_h) = 1$ is taken, resulting in

$$M_{\min}^2 = 2E_h(E_\tau - E_h) + m_h^2 - 2|\vec{p}_h|(E_\tau - E_h) < m_\tau^2$$

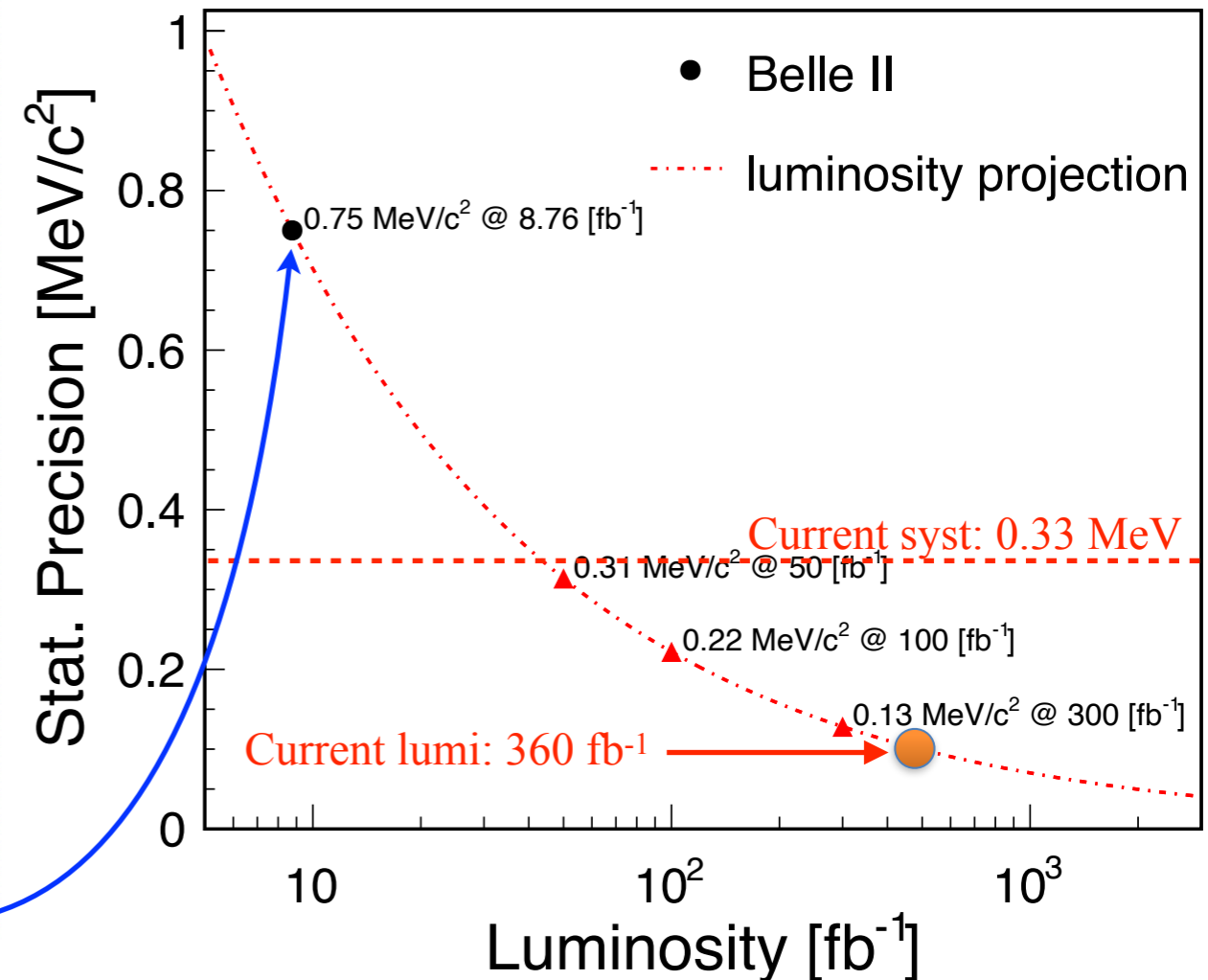
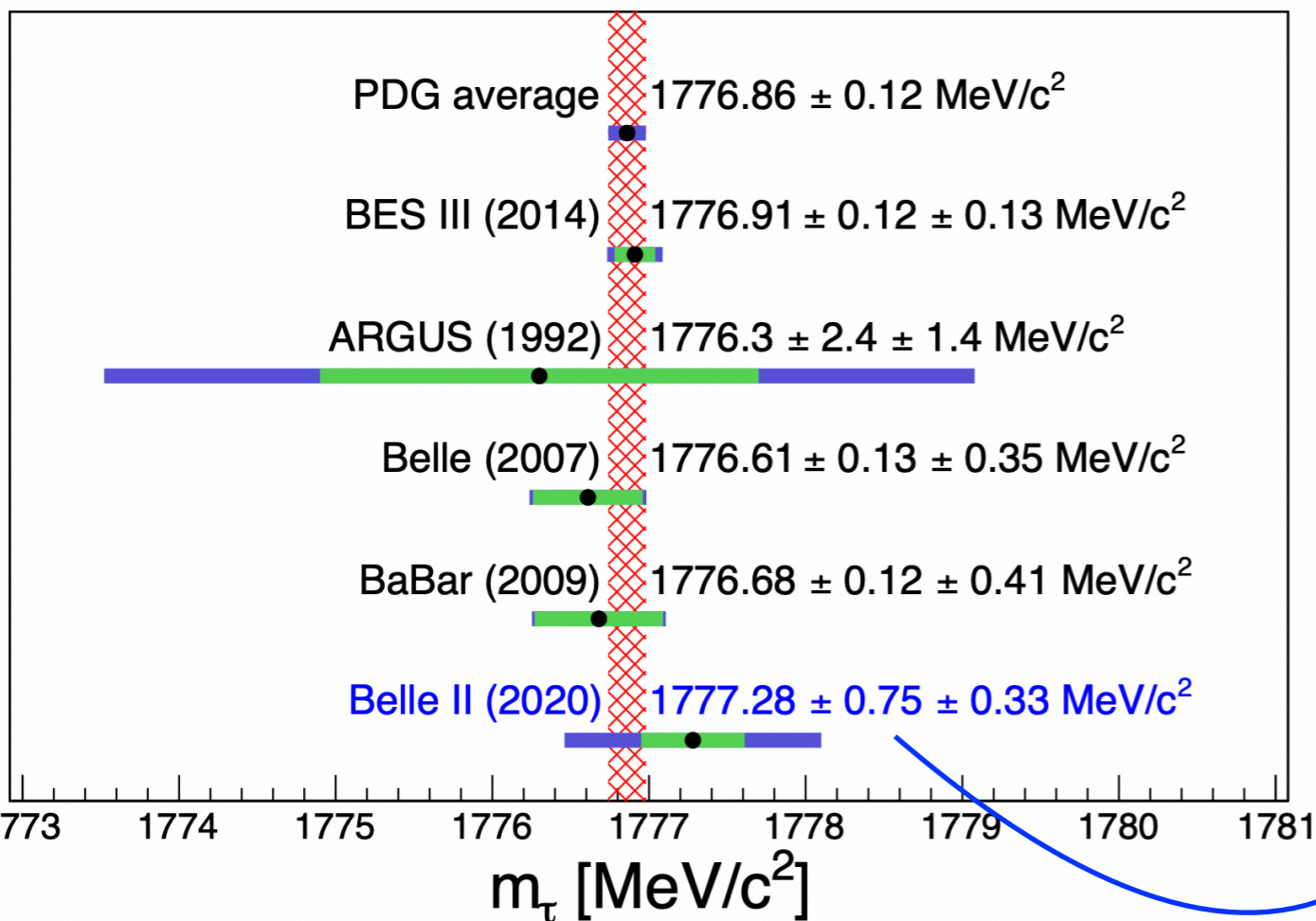
- Then, the distribution of the pseudomass is fitted to an empirical edge function, and the position of the cutoff indicates the value of the mass.

Tau Mass at Belle II

[arXiv:2008.04665](https://arxiv.org/abs/2008.04665) [hep-ex]

Our result is still dominated by statistical uncertainty, and consistent with previous measurements:

We expect significant reduction in the main systematic uncertainties.



Blue: statistical; Green: systematic

Projection towards high luminosity

Tau Lifetime at Belle II

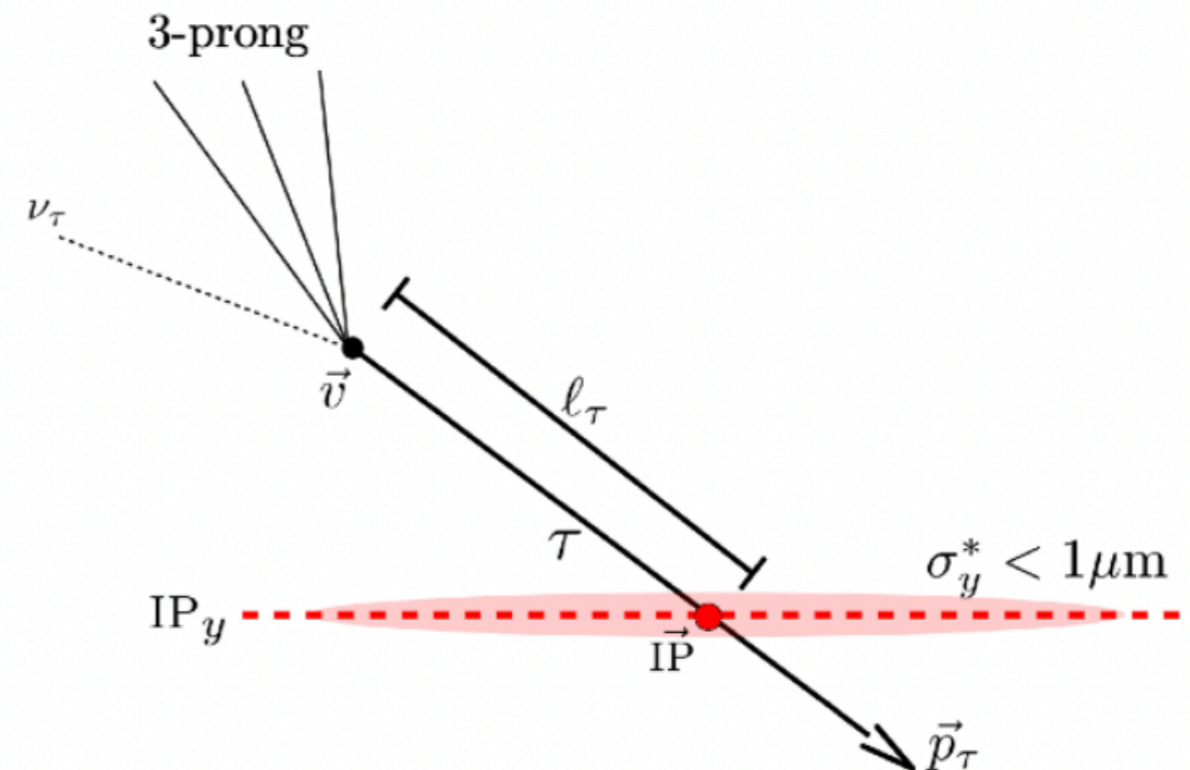
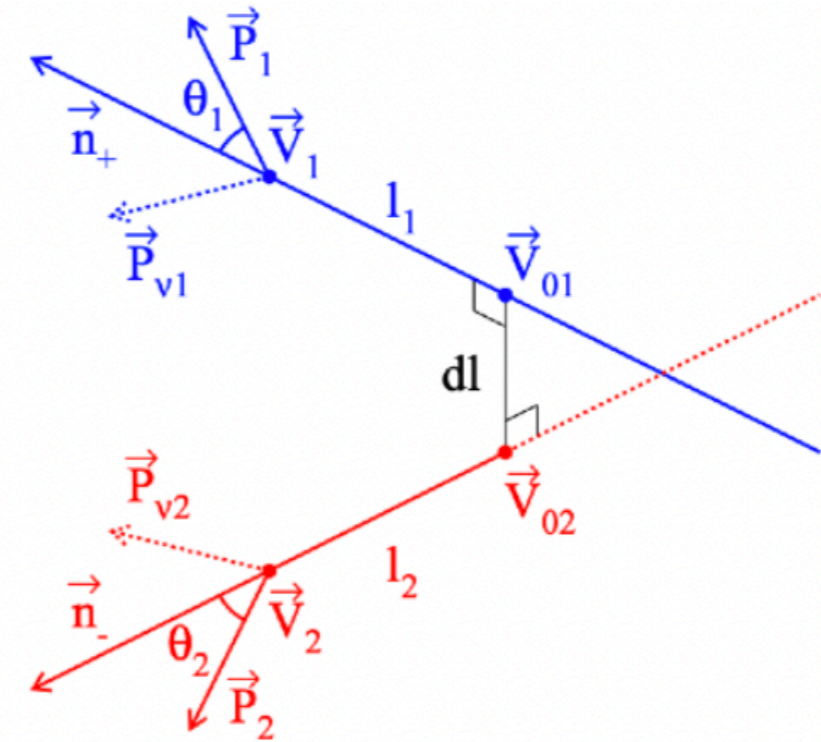
- The world-leading measurement by Belle¹ uses a **3x3 topology**, with both tau leptons decaying to $3\pi\nu_\tau$.

► $\tau_\tau = 290.17 \pm 0.53(\text{stat}) \pm 0.33(\text{syst}) \text{ fs}$

¹ PRL 112, 031801 (2014), arXiv:1310.8503 [hep-ex]

- Strategy at Belle II:**

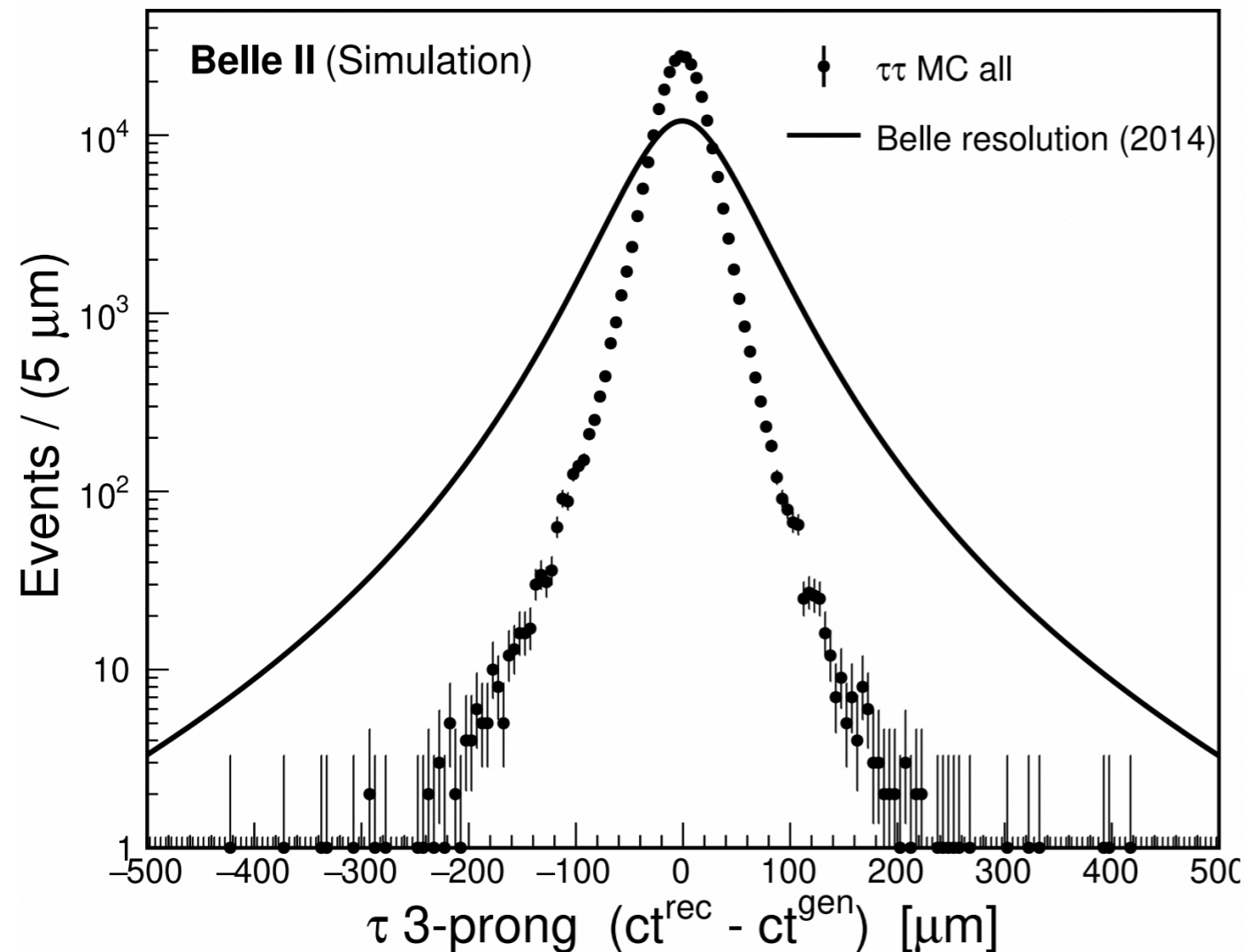
1. Reconstruct vertex for 3-prong τ .
Only one 3-prong = **higher statistics**.
2. Estimate the τ momentum \vec{p}_τ .
Hadronic decays in both sides.
3. Find the production vertex.
Intersection of \vec{p}_τ with the plane IP_y .



Tau Lifetime at Belle II

- In MC simulations, the Belle II proper time resolution is **~2x better than Belle**.
 - Due to PXD and smaller beam pipe diameter.

Proper decay time resolution:

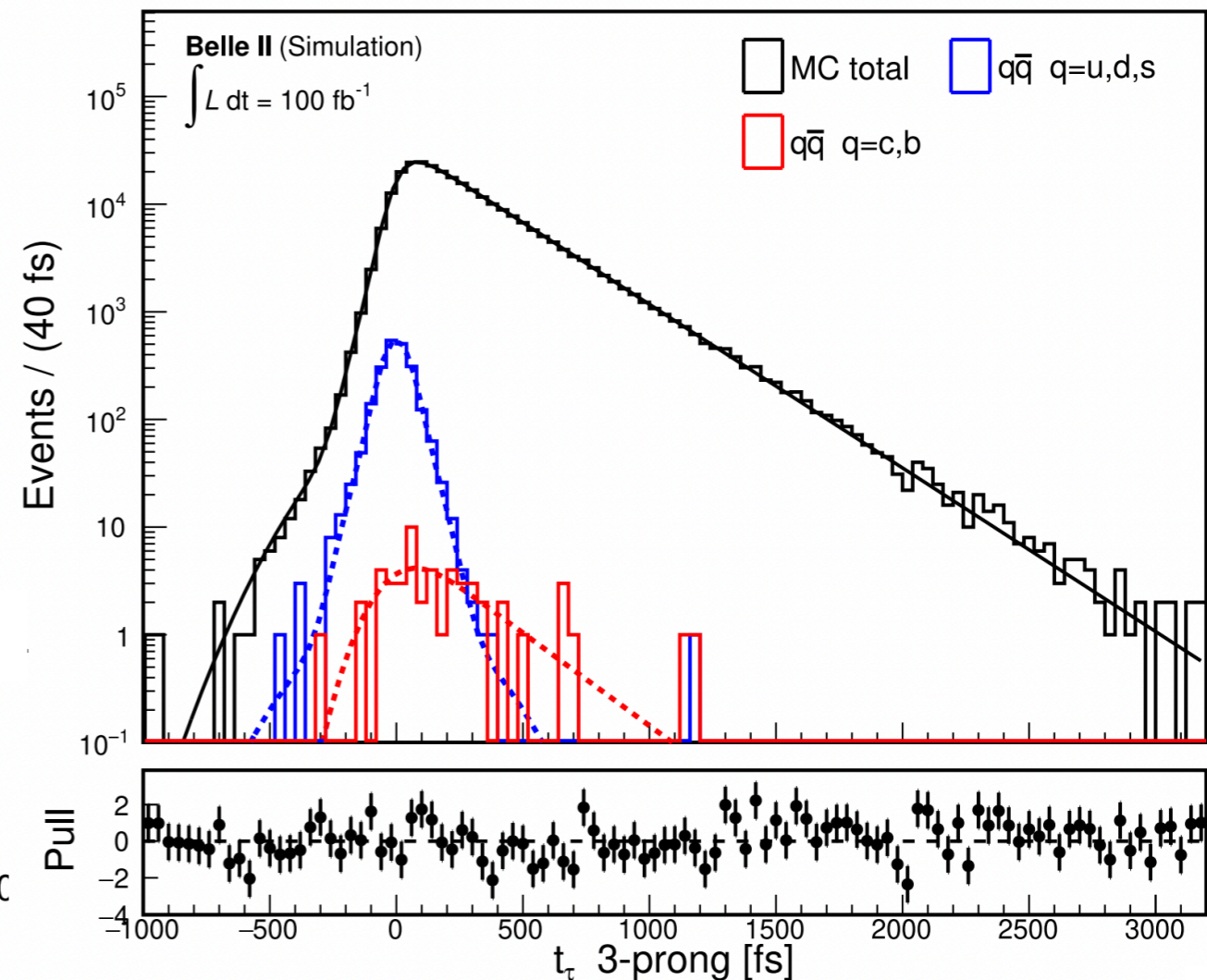


Fit proper time distribution, subtracting $q\bar{q}$ backgrounds

- Lifetime extraction:

- $\tau_\tau = 287.2 \pm 0.5$ (stat) fs

- Same statistical uncertainty of Belle. (200 fb^{-1} vs 711 fb^{-1})

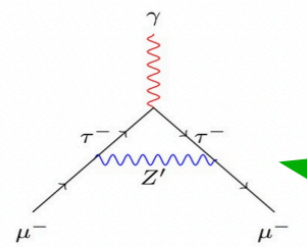


Lepton Flavor universality at Belle II

Lepton flavor violating Z' explanation of the muon anomalous magnetic moment

Wolfgang Altmannshofer¹, Chien-Yi Chen^{2,3}, P. S. Bhupal Dev⁴, Amarjit Soni⁵

[arXiv:1607.06832](https://arxiv.org/abs/1607.06832) [hep-ph]



Z' contribution to the anomalous magnetic moment of the muon

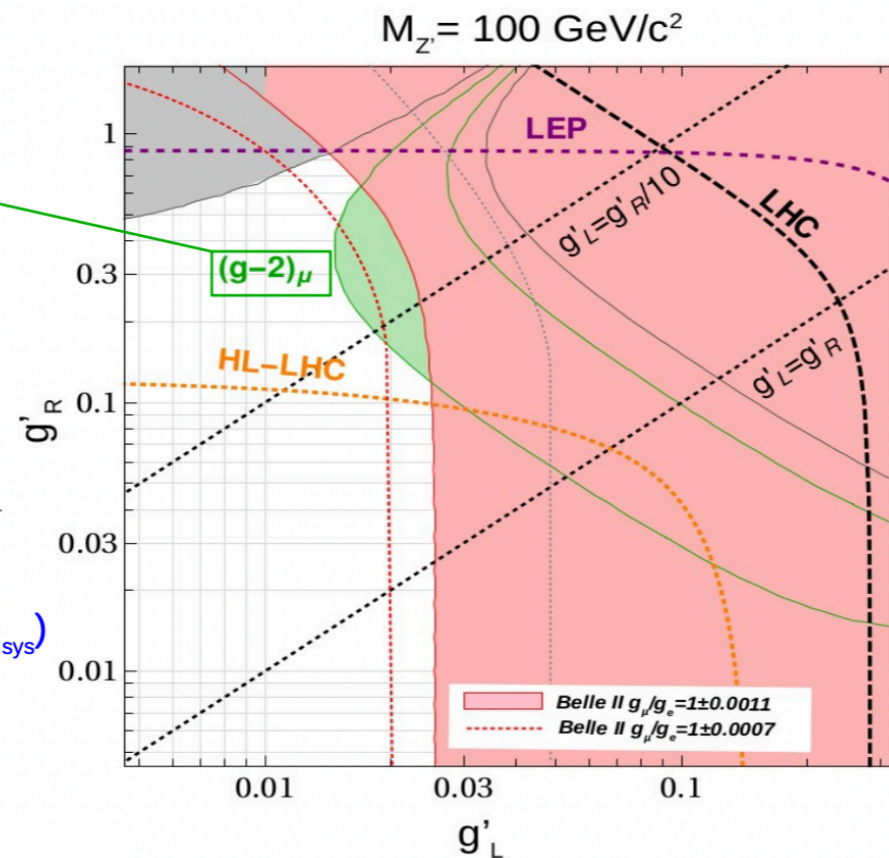
$$\frac{R_{\mu e}}{R_{\mu e}^{\text{SM}}} = 1 + \frac{|g'_L|^2}{g_2^2} \frac{4m_W^2}{m_{Z'}^2} + \left(\frac{|g'_L g'_R|^2}{g_2^4} + \frac{|g'_L|^4}{g_2^4} \right) \frac{8m_W^4}{m_{Z'}^4}$$

BaBar: $R_{\mu e} = 0.976 \pm 0.004 (= 0.0016_{\text{stat}} \pm 0.0036_{\text{sys}})$

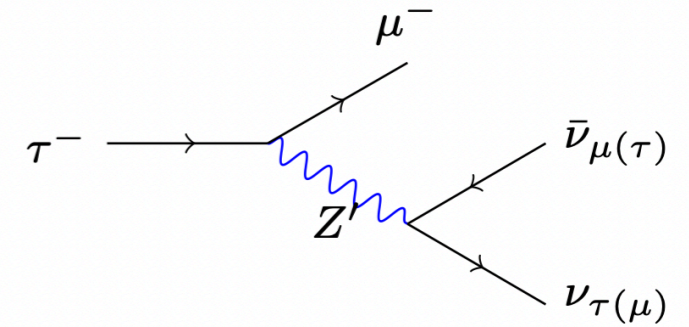
SM: $R_{\mu e} = 0.972559 \pm 0.00005$ (see here)

PDG: $R_{\mu e} = 0.979 \pm 0.004$

$R_{\mu e}^{\text{PDG}}/R_{\mu e}^{\text{SM}} - 1 = 0.0064 \pm 0.004$



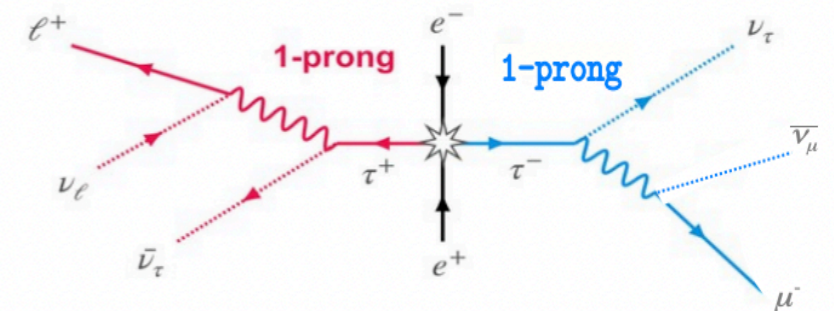
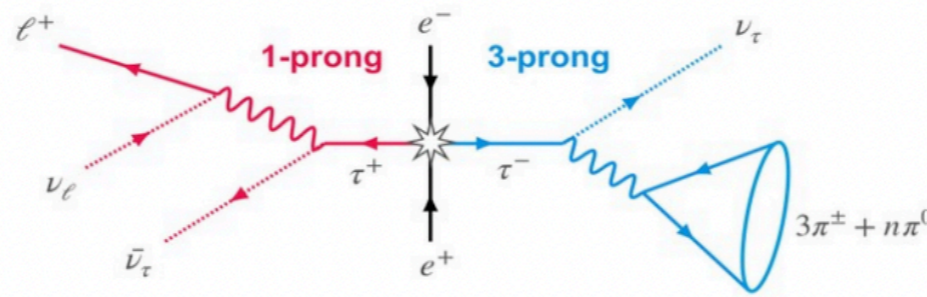
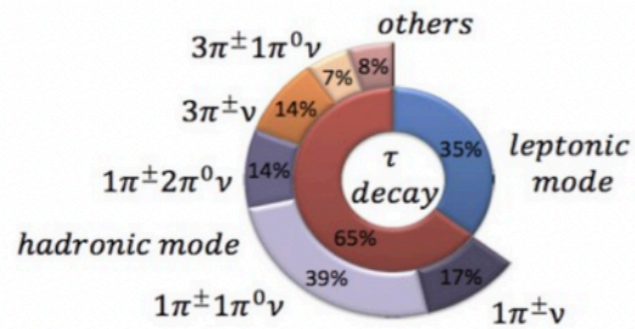
g'_R
1
0.3
0.1
0.03
0.01
 10^{-1}
 10^{-2}
 10^{-3}
 10^{-4}
 10^{-5}
 10^{-6}



Feynman diagram for the Z' contribution to the lepton flavor universality violating tau decay.

Leptonic branching fractions

We plan to use both 3-prong (LFU+BF) and 1-prong (LFU) tag side



$$R_\mu \equiv \frac{\mathcal{B}(\tau^- \rightarrow \mu^- \bar{\nu}_\mu \nu_\tau)}{\mathcal{B}(\tau^- \rightarrow e^- \bar{\nu}_e \nu_\tau)}$$

Belle II will significantly improve the precision on inputs to lepton-flavor universality-violating quantities yielding some of the most stringent constraints on non-SM deviations from charged current lepton universality.

The $|V_{us}|$ element of CKM Matrix

V_{ij} : Mixing between Weak and Mass Eigenstates

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

$$|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1$$

- $|V_{ud}| = 0.97373 \pm 0.00031$ (from nuclear β decays)

J.C.Hardy & I.S.Towner, PRC 102 (2020) 045501

- $|V_{ub}| = (3.82 \pm 0.24) \times 10^{-3}$ (from $B \rightarrow X_u \ell \nu$ decays)

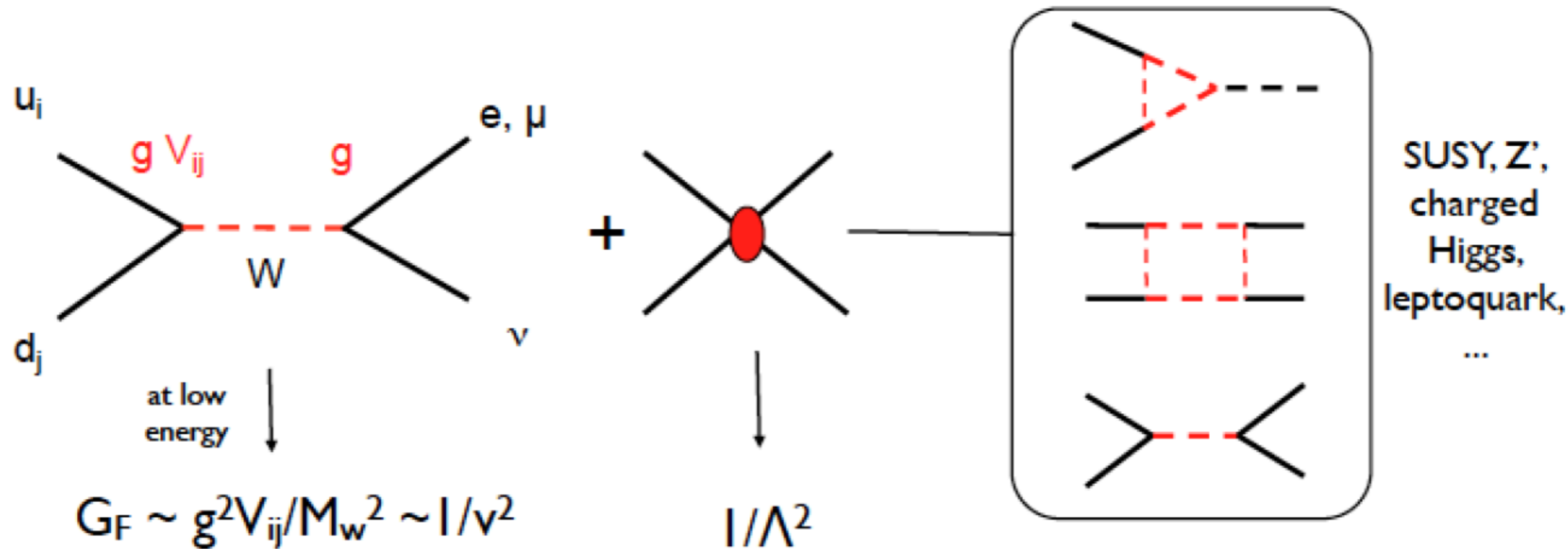
Particle Data Group 2021

$$\Rightarrow |V_{us}|^{\text{CKM}} = 0.2277 \pm 0.0013$$

Precision measurement of $|V_{us}|$ is a test of CKM unitarity

CKM Unitarity

V-A interaction via W-exchange with quarks have V_{ij}



Standard Model

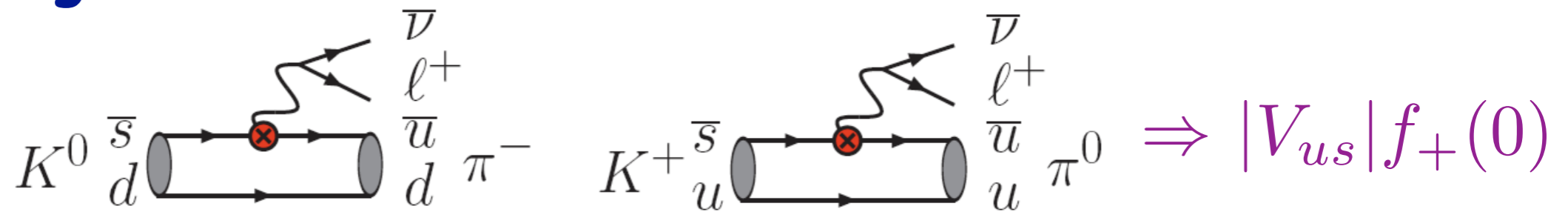
New Physics

$\Delta_{CKM} \sim (v/\Lambda)^2$ sensitive to new physics in large class of models

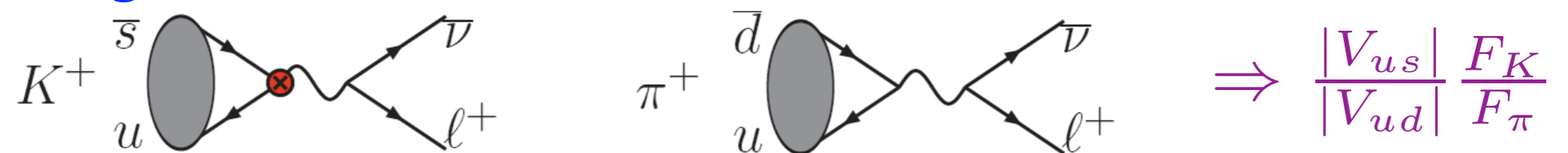
CKM Unitarity violation: $|V_{ud}|^2 + |V_{us}|^2 + |V_{ub}|^2 = 1 + \Delta_{CKM}$

Approaches to $|V_{us}|$

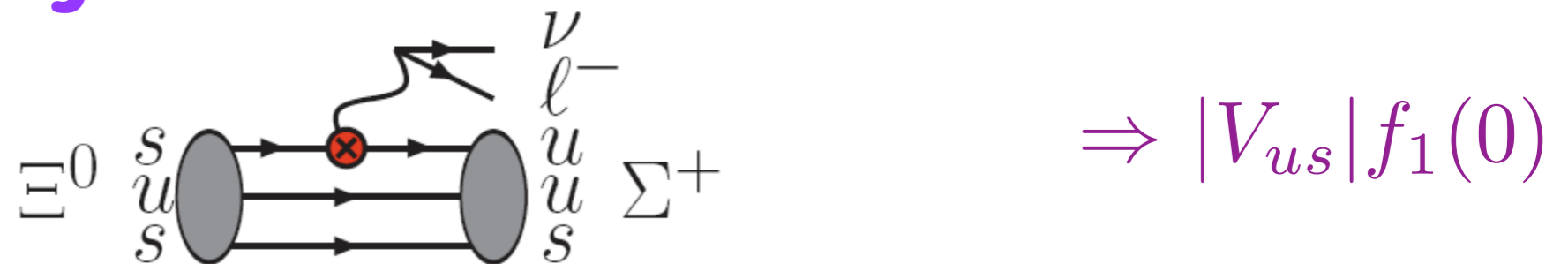
K13 decays:



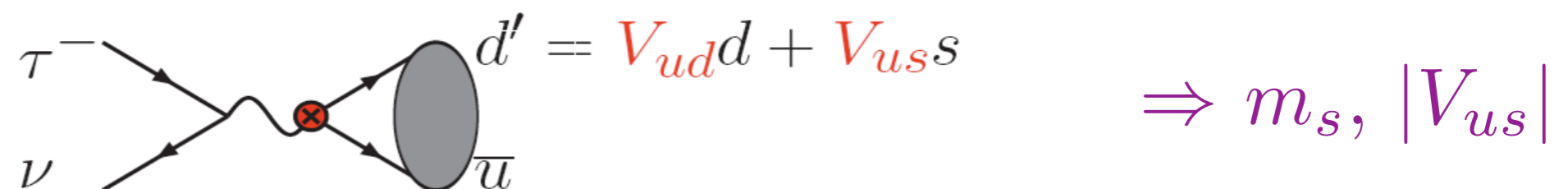
K12 decays:



Hyperon decays:



τ decays:



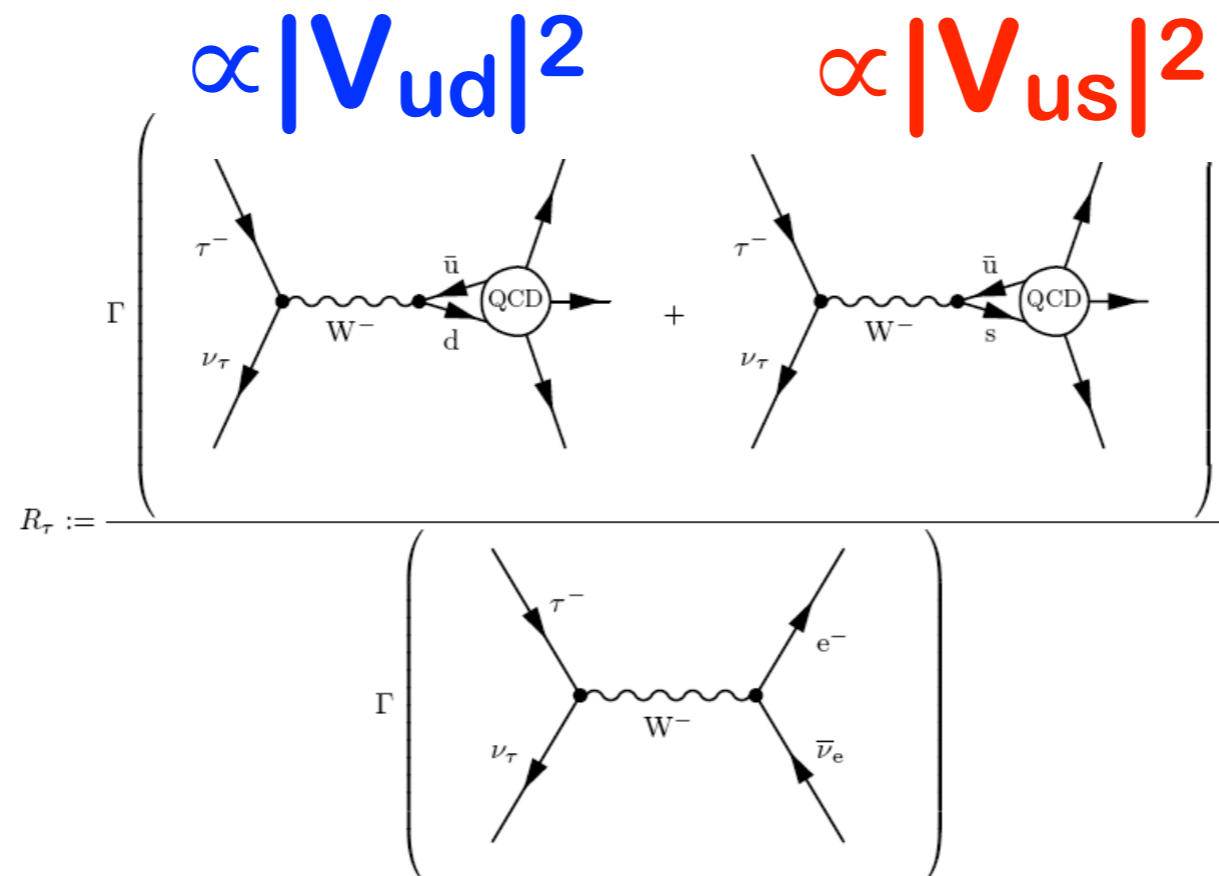
Hadronic width of the τ lepton

Parton model:

$$R_\tau \equiv \frac{\Gamma(\tau^- \rightarrow \nu_\tau + \text{hadrons})}{\Gamma(\tau^- \rightarrow \nu_\tau e^- \bar{\nu}_e)} \approx N_C$$

QCD:

$$R_\tau = R_\tau^{NS} + R_\tau^S \approx |V_{ud}|^2 N_C + |V_{us}|^2 N_C$$



Hadronic width of the τ lepton

QCD corrections : $R_\tau = |V_{ud}|^2 N_C + |V_{us}|^2 N_C + \mathcal{O}(\alpha_s)$

Spectral Moments: $R_\tau^{kl} = \int_0^1 dz (1-z)^k z^l \frac{dR_\tau}{dz}$, $z = \frac{q^2}{m_\tau^2}$

Zeroth order moments are simply the τ branching fractions

Finite energy sum rules \Rightarrow SU(3) breaking sensitive to m_s :

$$\delta R_\tau^{kl} = \frac{R_{\tau, non-strange}^{kl}}{|V_{ud}|^2} - \frac{R_{\tau, strange}^{kl}}{|V_{us}|^2}$$

$$\delta R_{\tau, th}^{00} = 0.1544(37) + 9.3(3.4) m_s^2 + 0.0034(28) = 0.238 \pm 0.033$$

$$m_s = 93.00 \pm 8.54 \text{ MeV} \quad [\text{PDG2020}]$$

E.Gamiz, M.Jamin, A.Pich, J.Prades & F. Schwab, arXiv 0709.0282 [hep-ph]

Truncation errors studied with QCD lattice inputs in terms of weights:

$$|V_{us}| = \sqrt{R_{V+A;us}^w(s_0) / \left[\frac{R_{V+A;ud}^w(s_0)}{|V_{ud}|^2} - \delta R_{V+A}^{w,OPE}(s_0) \right]}$$

R. J. Hudspith, R. Lewis, K. Maltman, and J. Zanotti, arXiv:1702.01767 [hep-ph]

$|V_{us}|$ from inclusive strange decays

[Preliminary]

Table 13: HFLAV 2021 τ branching fractions to strange final states.

Branching fraction	HFLAV 2021 fit (%)
$K^- \nu_\tau$	0.6957 ± 0.0096
$K^- \pi^0 \nu_\tau$	0.4322 ± 0.0148
$K^- 2\pi^0 \nu_\tau$ (ex. K^0)	0.0634 ± 0.0219
$K^- 3\pi^0 \nu_\tau$ (ex. K^0, η)	0.0465 ± 0.0213
$\pi^- \bar{K}^0 \nu_\tau$	0.8375 ± 0.0139
$\pi^- \bar{K}^0 \pi^0 \nu_\tau$	0.3810 ± 0.0129
$\pi^- \bar{K}^0 2\pi^0 \nu_\tau$ (ex. K^0)	0.0234 ± 0.0231
$\bar{K}^0 h^- h^- h^+ \nu_\tau$	0.0222 ± 0.0202
$K^- \eta \nu_\tau$	0.0155 ± 0.0008
$K^- \pi^0 \eta \nu_\tau$	0.0048 ± 0.0012
$\pi^- \bar{K}^0 \eta \nu_\tau$	0.0094 ± 0.0015
$K^- \omega \nu_\tau$	0.0410 ± 0.0092
$K^- \phi (K^+ K^-) \nu_\tau$	0.0022 ± 0.0008
$K^- \phi (K_S^0 K_L^0) \nu_\tau$	0.0015 ± 0.0006
$K^- \pi^- \pi^+ \nu_\tau$ (ex. K^0, ω)	0.2924 ± 0.0068
$K^- \pi^- \pi^+ \pi^0 \nu_\tau$ (ex. K^0, ω, η)	0.0387 ± 0.0142
$K^- 2\pi^- 2\pi^+ \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$K^- 2\pi^- 2\pi^+ \pi^0 \nu_\tau$ (ex. K^0)	0.0001 ± 0.0001
$X_s^- \nu_\tau$	2.9076 ± 0.0478

$$|V_{us}|_{\tau S} = \sqrt{R_s / \left[\frac{R_{VA}}{|V_{ud}|^2} - \delta R_{\text{theory}} \right]}$$

$$B_s = (2.908 \pm 0.048)\%$$

$$B_{VA} = B_{\text{hadrons}} - B_s = (61.83 \pm 0.10)\%$$

To get R, we normalize by

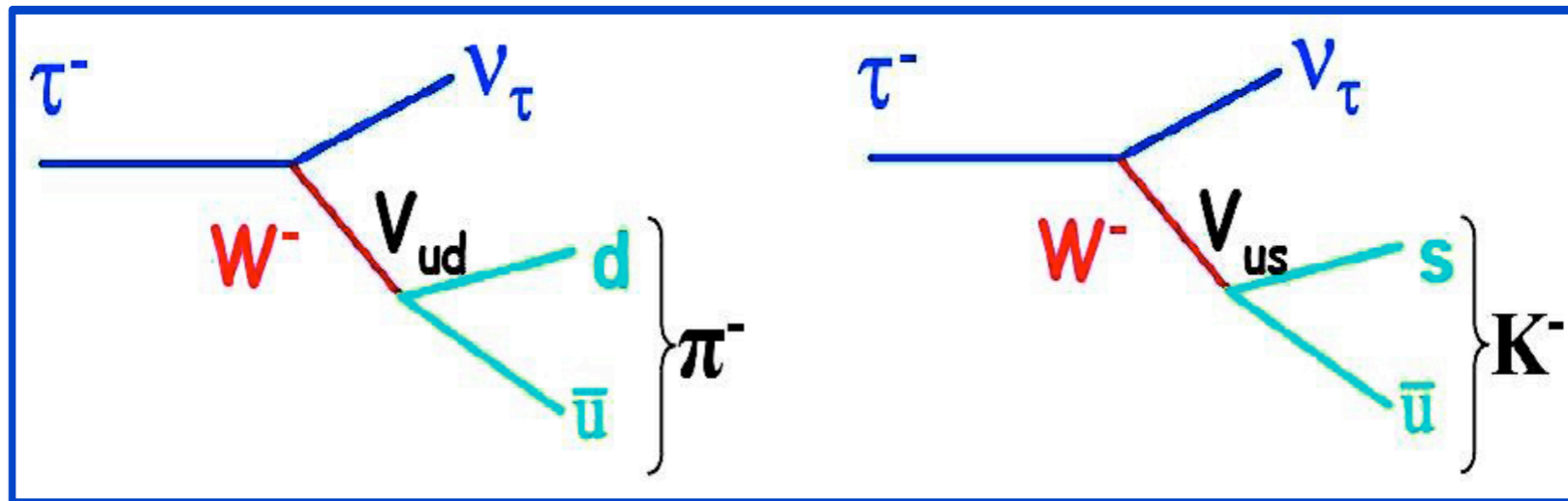
$$(B_e)^{\text{univ}} = (17.812 \pm 0.022)\%$$

The error on B_e is improved using lepton universality & improved measurements of mass (m_τ) and lifetime (τ_τ).

$$\Rightarrow |V_{us}| = (0.2184 \pm 0.0021)$$

Dominant contribution to error on $|V_{us}|$ comes from error on the measured B_s . δR_{theory} contributes to $\Delta|V_{us}| = 0.0011$.

$|V_{us}|$ from exclusive τ decays

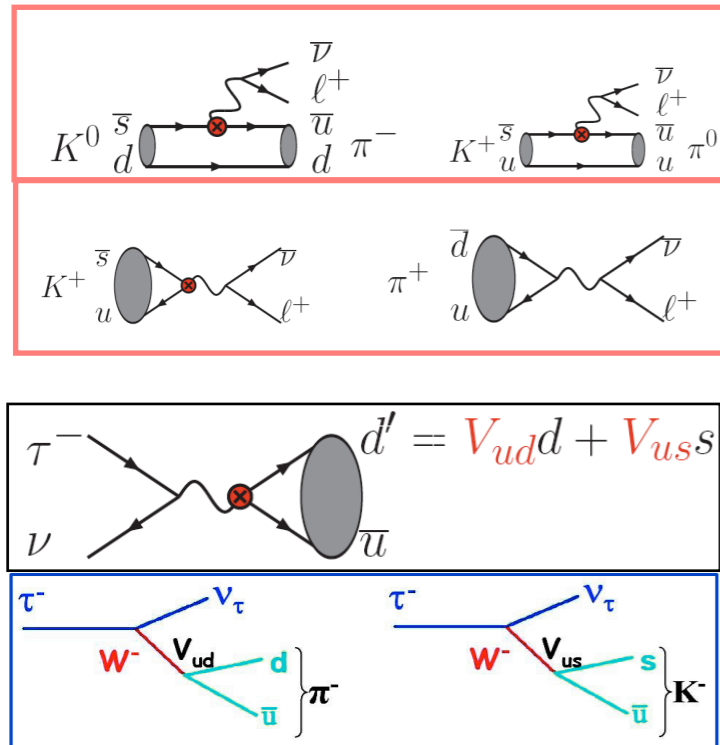


$$\frac{\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau)}{\mathcal{B}(\tau^- \rightarrow \pi^- \nu_\tau)} = \frac{f_{K^\pm}^2 |V_{us}|^2 (m_\tau^2 - m_K^2)^2}{f_{\pi^\pm}^2 |V_{ud}|^2 (m_\tau^2 - m_\pi^2)^2} (1 + \delta R_{\tau K/\tau\pi})$$

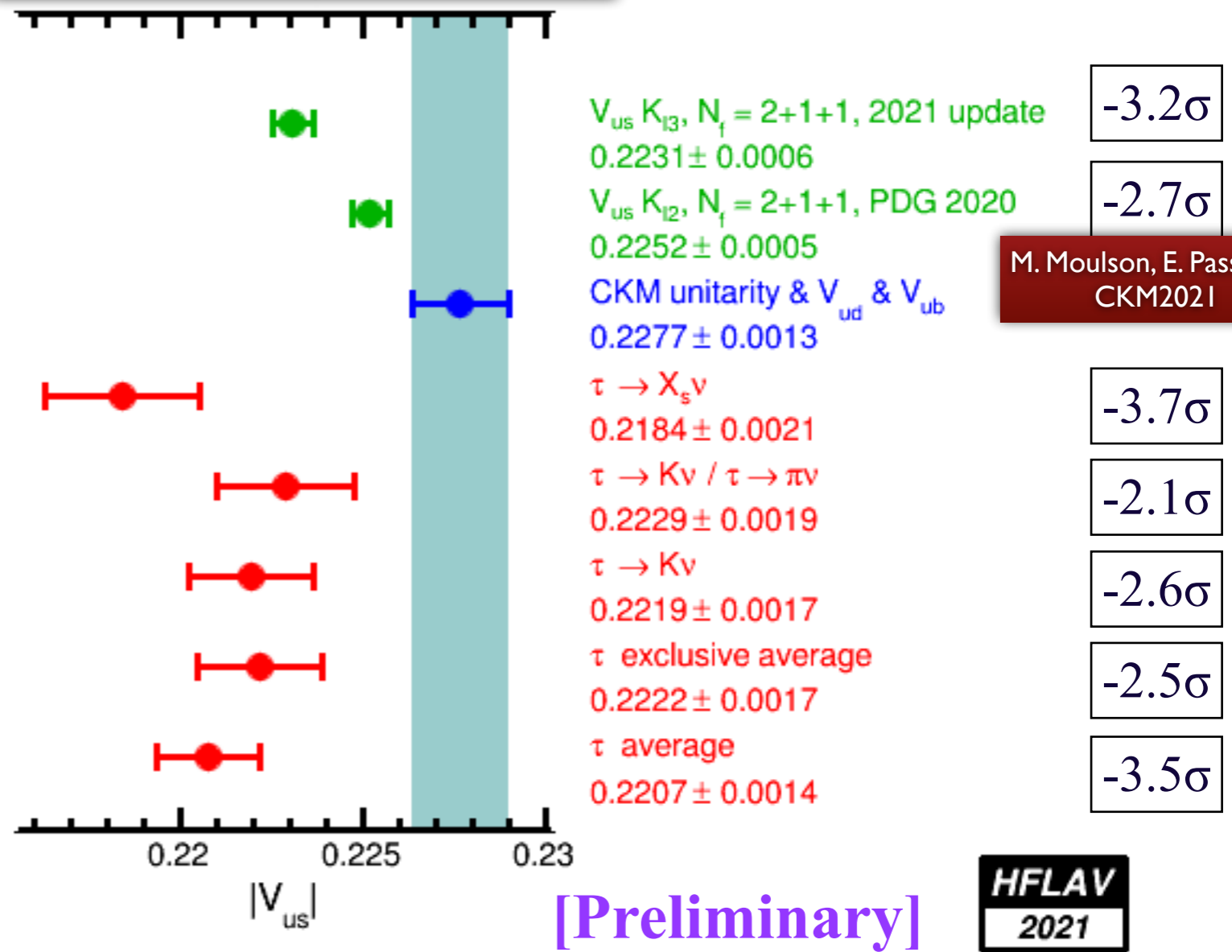
$$\mathcal{B}(\tau^- \rightarrow K^- \nu_\tau) = \frac{G_F^2}{16\pi\hbar} f_{K^\pm}^2 |V_{us}|^2 \tau_\tau m_\tau^3 \left(1 - \frac{m_K^2}{m_\tau^2}\right)^2 S_{EW} (1 + \delta R_{\tau K})$$

- Independent of convergence of OPE, as electroweak corrections cancel
- Radiative corrections $S_{EW} = 1.02320 \pm 0.00030$ [Erler 2004]
- Long Distance effects ($R_{\tau K/\tau\pi}$) known [Decker & Finkmeier 1995, Marciano 2004]
- All non-perturbative QCD effects encapsulated as ratio of meson decay constants:
 $f_K/f_\pi = 1.1932 \pm 0.0021$, $f_K = 155.7 \pm 0.3$ MeV [FLAG 2019 Lattice Averages]

Summary of $|V_{us}|$ results



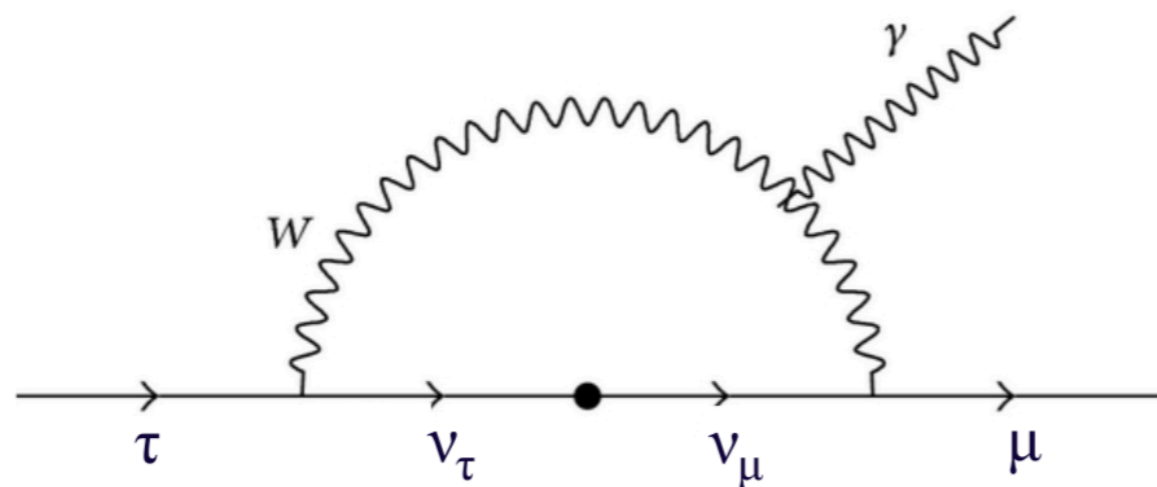
Cabibbo-angle anomaly



M. Moulson, E. Passemar
CKM2021

- $|V_{us}|$ from kaon and tau falls short of CKM unitarity value by $\sim 3\sigma$
- $|V_{us}|$ from inclusive tau decays independent of Lattice errors used for kaons
- New physics affecting 3rd generation only affects $|V_{us}|$ from taus
- Tau decays at Belle II offers unique and complementary insight

Search for lepton number/flavor violation in τ decays



$$\mathcal{B}(\tau^\pm \rightarrow \mu^\pm \gamma) \text{ [Lee-Shrock, Phys. Rev. D 16, 1444 (1977)]}$$

$$= \frac{3\alpha}{128\pi} \left(\frac{\Delta m_{23}^2}{M_W^2} \right)^2 \sin^2 2\theta_{\text{mix}} \mathcal{B}(\tau \rightarrow \mu \bar{\nu}_\mu \nu_\tau)$$

With $\Delta \sim 10^{-3} \text{ eV}^2$, $M_W \sim \mathcal{O}(10^{11}) \text{ eV}$
 $\approx \mathcal{O}(10^{-54})$ ($\theta_{\text{mix}} : \text{max}$)

many orders below experimental sensitivity!

LNv/LFV is NOT forbidden by any continuous symmetry
 \Rightarrow most New Physics (NP) models naturally include such processes

**Any observation of LNv/LFV
 \Rightarrow unambiguous signature of NP**

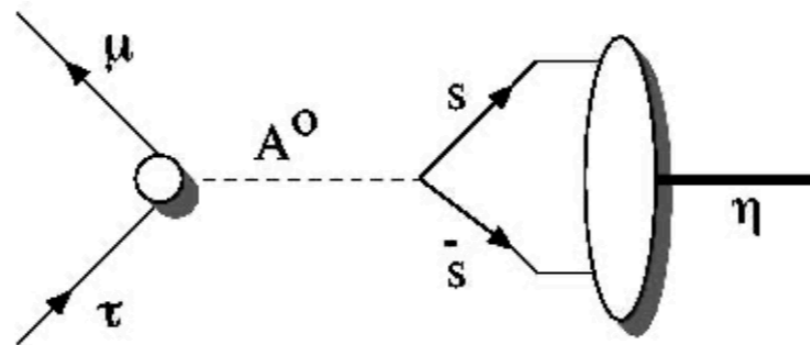
- Mass dependent couplings enhance tau LFV w.r.t. lighter leptons
- Some models predict LFV up to existing experimental bounds
- eg. SUSY models: non-diagonal slepton mass matrix \Rightarrow LFV
- Normal (Inverted) hierarchy for slepton $\Rightarrow \tau \rightarrow \mu \gamma$ ($\tau \rightarrow e \gamma$)

New Physics expectations

- Neutrinoless 2 and 3 body τ decays have different sensitivity

	$\mathcal{B}(\tau \rightarrow l\gamma)$	$\mathcal{B}(\tau \rightarrow lll)$
mSUGRA+seesaw (EPJC14(2000)319, PRD66(2002)115013)	10^{-8}	10^{-9}
SUSY SO(10) (NPB649(2003)189, PRD68(2003)033012)	10^{-8}	10^{-10}
SUSY Higgs (PLB549(2002)159, PLB566(2003)217)	10^{-10}	10^{-8}
Non-Universal Z' (PLB547(2002)252)	10^{-9}	10^{-8}
SM+Heavy Majorana ν_R (PRD66(2002)034008)	10^{-9}	10^{-10}

- η final state enhanced due to color factor and Higgs- $s\bar{s}$ vertex



- MSSM + seesaw: (M.Sher, PRD66 (2002) 057301)

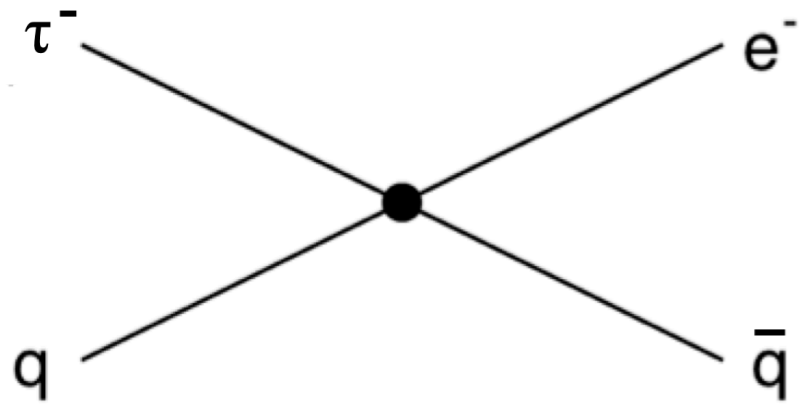
$$\mathcal{B}(\tau \rightarrow \mu\eta) = 0.84 \times 10^{-6} \times \left(\frac{\tan\beta}{60}\right)^6 \left(\frac{100 \text{ GeV}}{m_A}\right)^4$$

where m_A is the pseudoscalar Higgs mass and $\tan\beta = \langle H_u \rangle / \langle H_d \rangle$

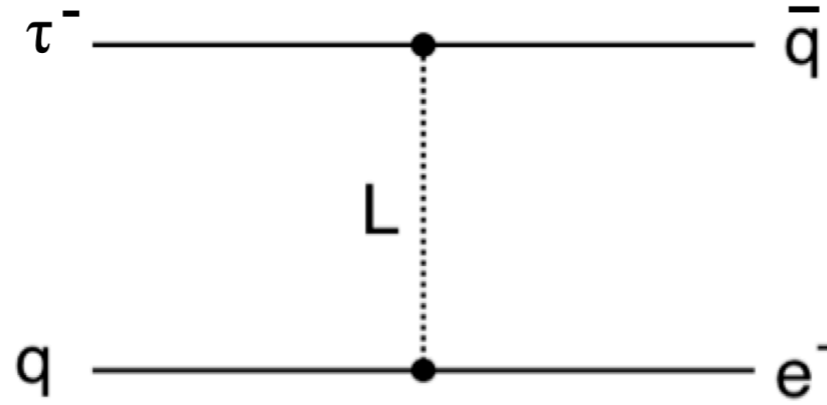
$$\Rightarrow \mathcal{B}(\tau \rightarrow \mu\eta) : \mathcal{B}(\tau \rightarrow \mu\gamma) : \mathcal{B}(\tau \rightarrow \mu\mu\mu) = 8.4 : 1.5 : 1$$

New Physics expectations

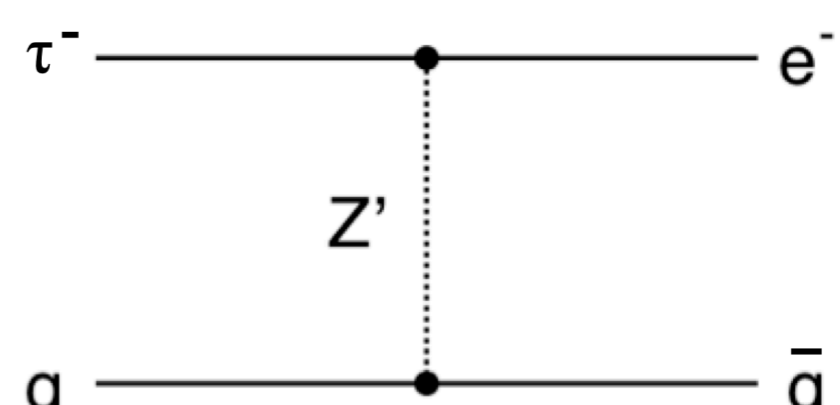
Tree level :



Compositeness

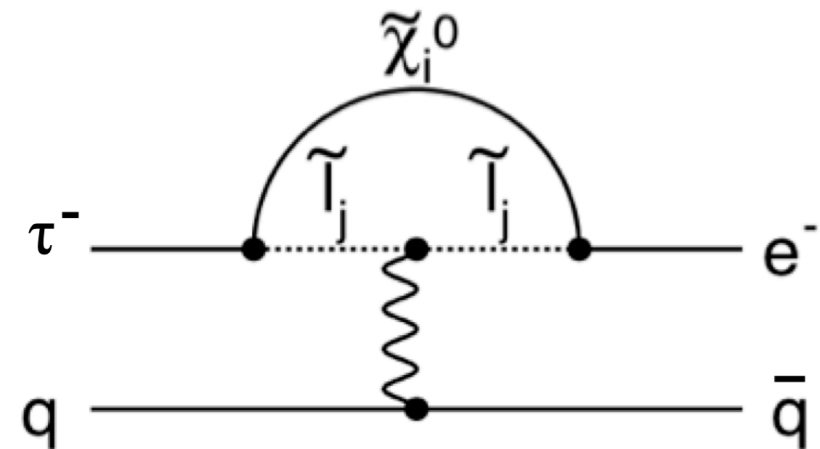


Leptoquarks

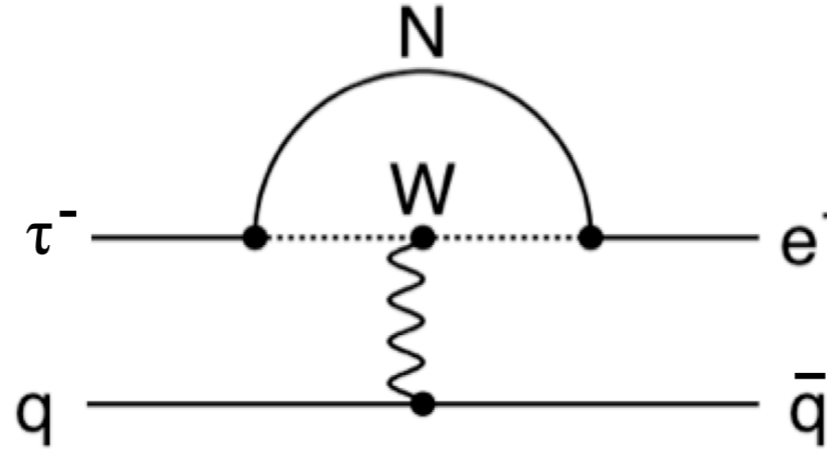


Heavy gauge bosons

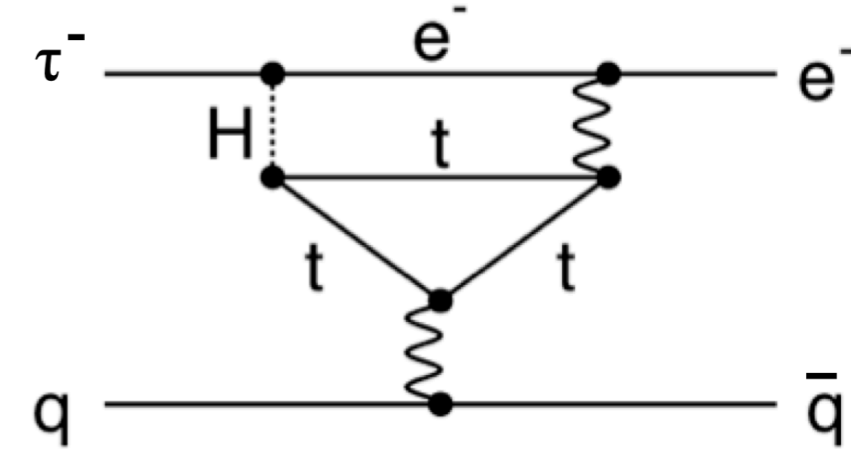
Loop induced :



Supersymmetry



Heavy neutrinos



Extended Higgs models

$$e^-e^+ \rightarrow \tau^-\tau^+$$

- Known initial conditions (beam energy constraint)
- Clean environment (less backgrounds)

$\tau \rightarrow l\gamma$

Signal-Side **Tag-Side**

Backgrounds:

- $\tau \rightarrow e\gamma$ ($\tau \rightarrow \mu\gamma$):
- Radiative Bhabha (di-muon)
- $\tau^+\tau^-\gamma$ ($\tau \rightarrow l\nu\bar{\nu}$)
- $q\bar{q}$ (γ)

$\tau \rightarrow lll$ ($\tau \rightarrow lhh'$)

Signal-Side **Tag-Side**

Backgrounds:

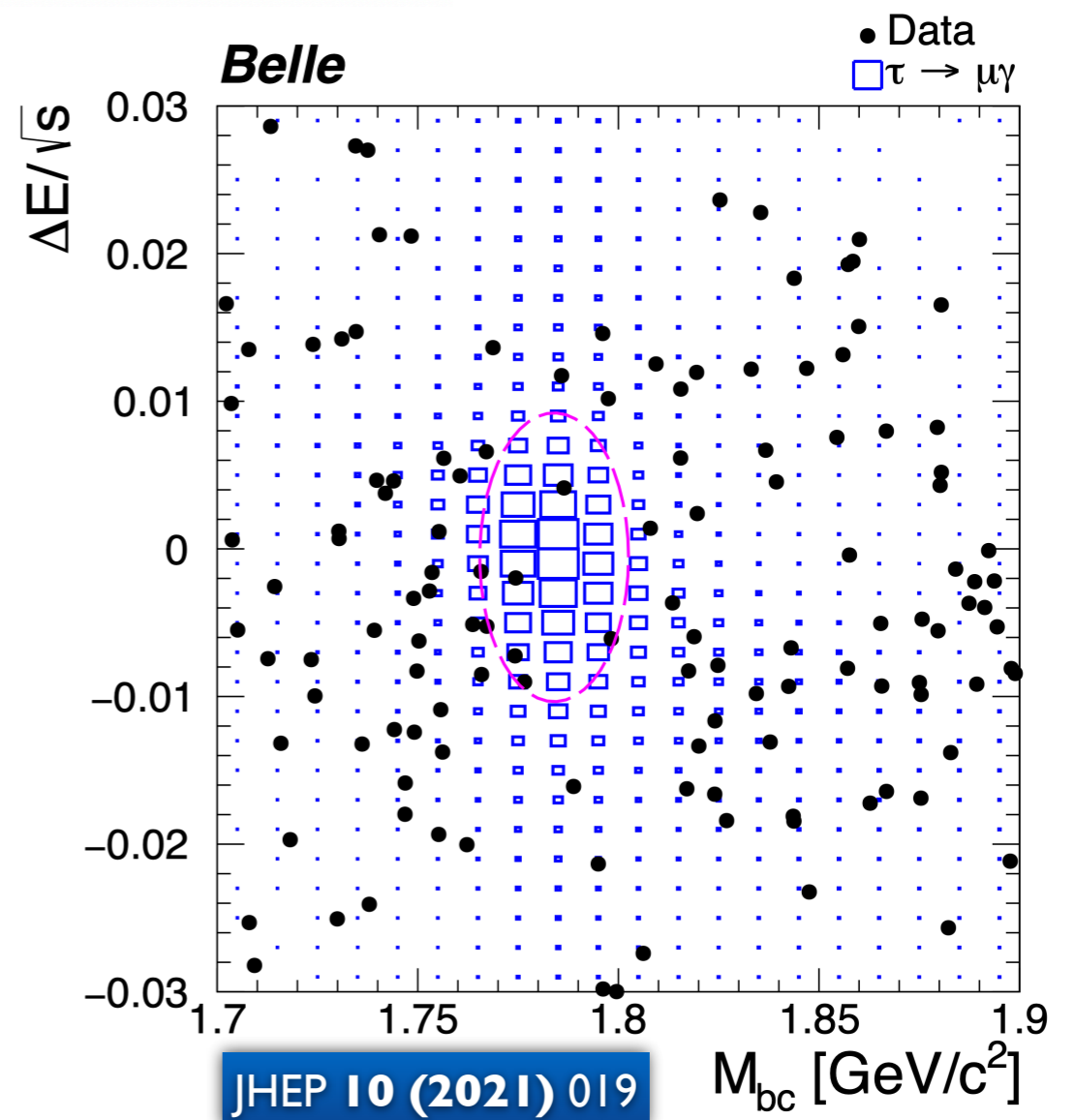
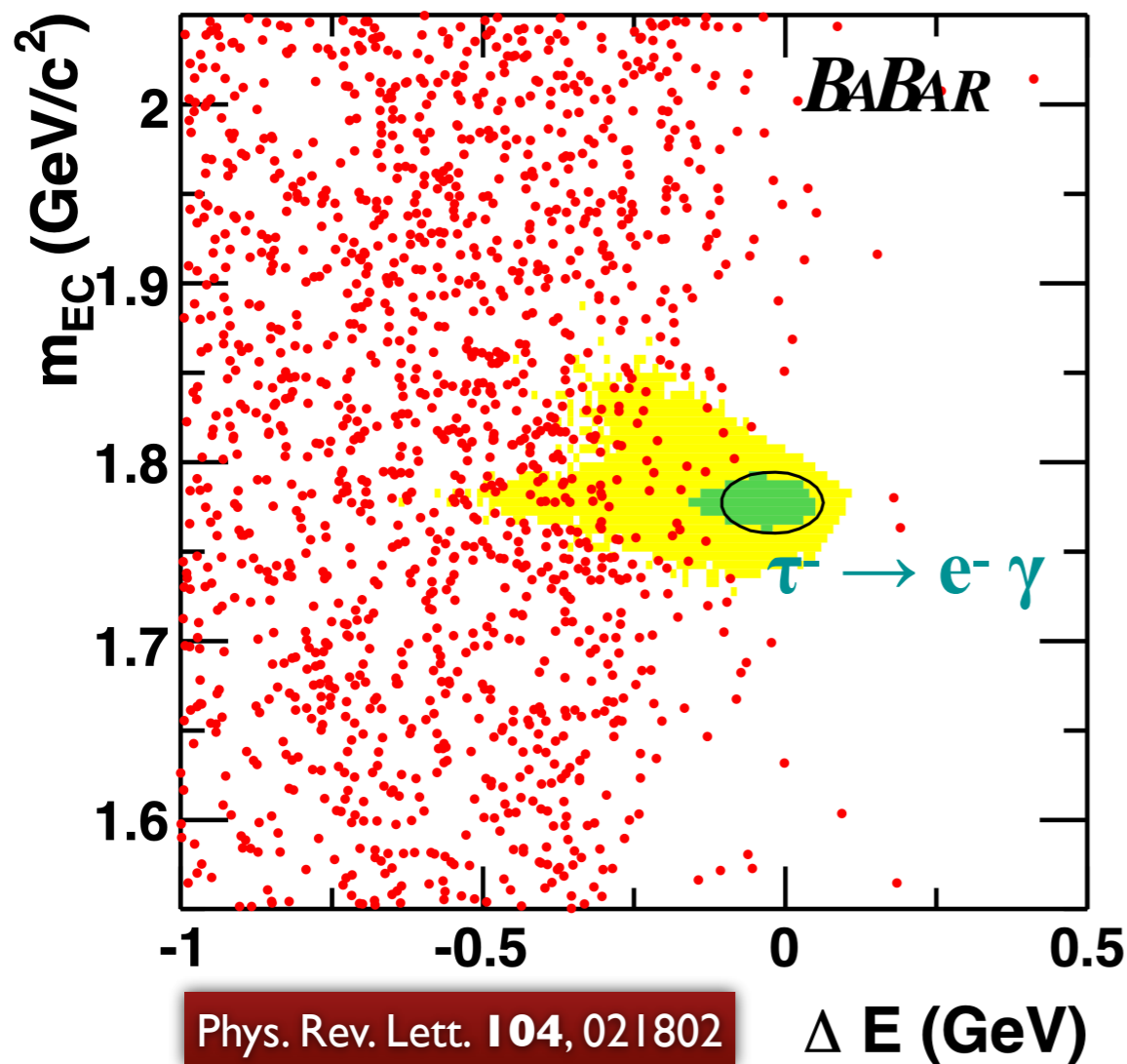
- $\tau^- \rightarrow l'^-l^+l^-$:
- Bhabha, di-muon
- $\tau^- \rightarrow l^+l'^-l'^-$, $\tau \rightarrow lhh'$:
- $\tau^+\tau^-$, $q\bar{q}$

# of ν (s) in Signal-side	Signal: 0	$\tau^+\tau^-$: 1-2	Bhabha, di-muon, $q\bar{q}$: 0
# of ν (s) in Tag-side	Signal: 1-2	$\tau^+\tau^-$: 1-2	Bhabha, di-muon, $q\bar{q}$: 0

Signal characteristics

$$M_{bc} = \sqrt{(E_{\text{beam}}^{\text{CM}})^2 - |\vec{p}_{l\gamma}^{\text{CM}}|^2} \simeq m_{\tau}$$

$$\Delta E / \sqrt{s} = (E_{l\gamma}^{\text{CM}} - \sqrt{s}/2) / \sqrt{s} \simeq 0$$



Upper limit estimation

$$B_{UL}^{90} = N_{UL}^{90} / (N_{\tau} \times \epsilon)$$

- ϵ : high statistics signal MC simulated for different Data-taking periods

$\epsilon =$ Trigger . Reco . Topology . PID . Cuts . Signal-Box

90% 70% 70% 50% 50% 50%

Cumulative:

90% 63% 44% 22% 11% ~5%

Decay modes	obs	2σ signal ellipse		ϵ (%)	UL ($\times 10^{-8}$)	
		obs	exp		obs	exp
$\tau^{\pm} \rightarrow e^{\pm} \gamma$	0	1.6 \pm 0.4	3.9 \pm 0.3	3.9 \pm 0.3	9.8	3.3
$\tau^{\pm} \rightarrow \mu^{\pm} \gamma$	2	3.6 \pm 0.7	6.1 \pm 0.5	6.1 \pm 0.5	8.2	4.4



Phys. Rev. Lett.
104 (2010) 021802

$N_{\tau} = 963$ M

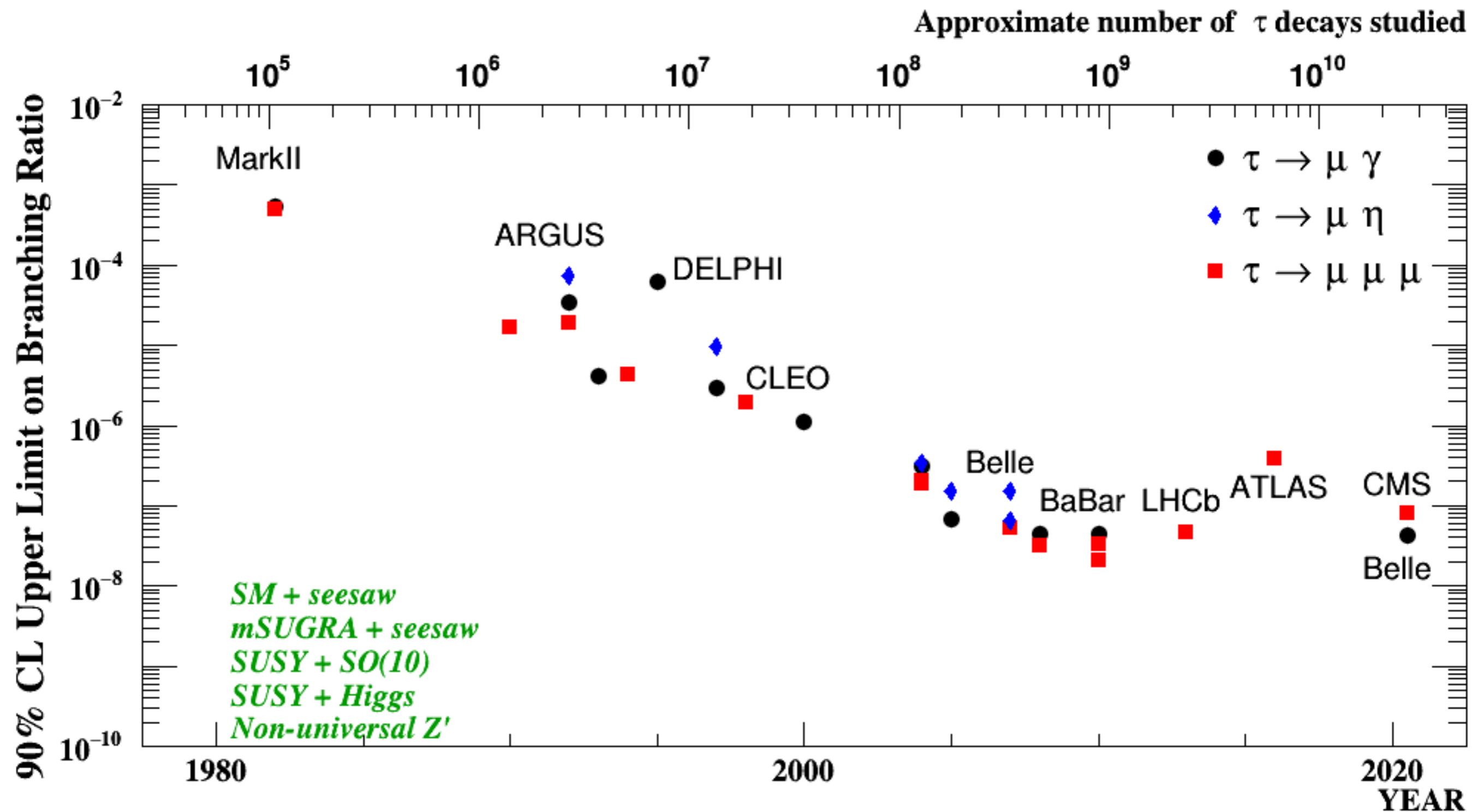
Mode	ϵ (%)	N_{BG}	σ_{syst} (%)	N_{obs}	\mathcal{B} ($\times 10^{-8}$)
$\tau^{-} \rightarrow e^{-} e^{+} e^{-}$	6.0	0.21 \pm 0.15	9.8	0	< 2.7
$\tau^{-} \rightarrow \mu^{-} \mu^{+} \mu^{-}$	7.6	0.13 \pm 0.06	7.4	0	< 2.1
$\tau^{-} \rightarrow e^{-} \mu^{+} \mu^{-}$	6.1	0.10 \pm 0.04	9.5	0	< 2.7
$\tau^{-} \rightarrow \mu^{-} e^{+} e^{-}$	9.3	0.04 \pm 0.04	7.8	0	< 1.8
$\tau^{-} \rightarrow e^{+} \mu^{-} \mu^{-}$	10.1	0.02 \pm 0.02	7.6	0	< 1.7
$\tau^{-} \rightarrow \mu^{+} e^{-} e^{-}$	11.5	0.01 \pm 0.01	7.7	0	< 1.5



Phys. Lett.
B687 (2010) 139

$N_{\tau} = 1438$ M

Current status of LFV limits



Current and future prospects

Background limited search

Background free search

N_{UL}^{90}

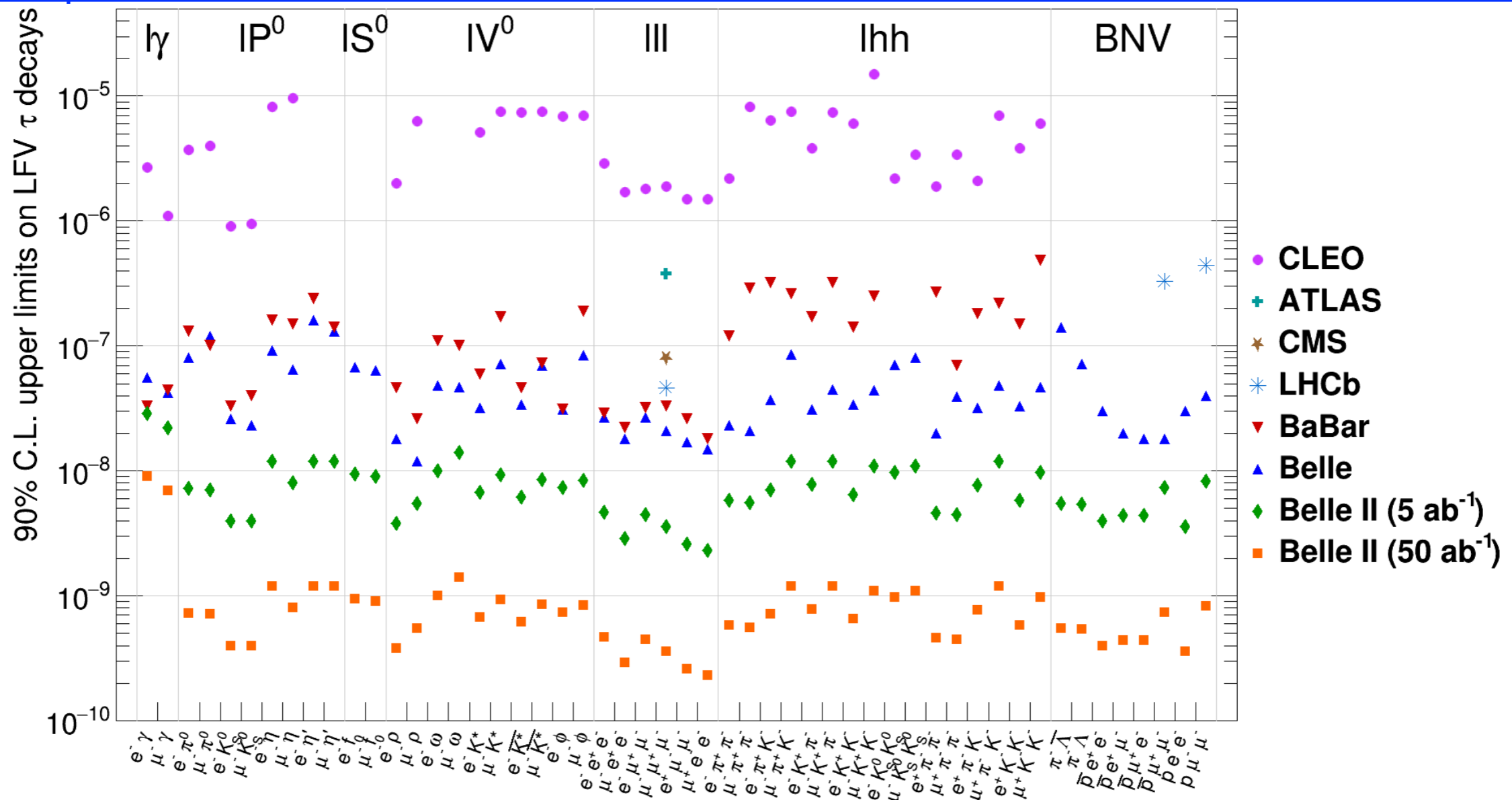
$$\sqrt{\mathcal{L}}$$

2.44 [Feldman – Cousins for $N_{obs} = 0$]

B_{UL}^{90}

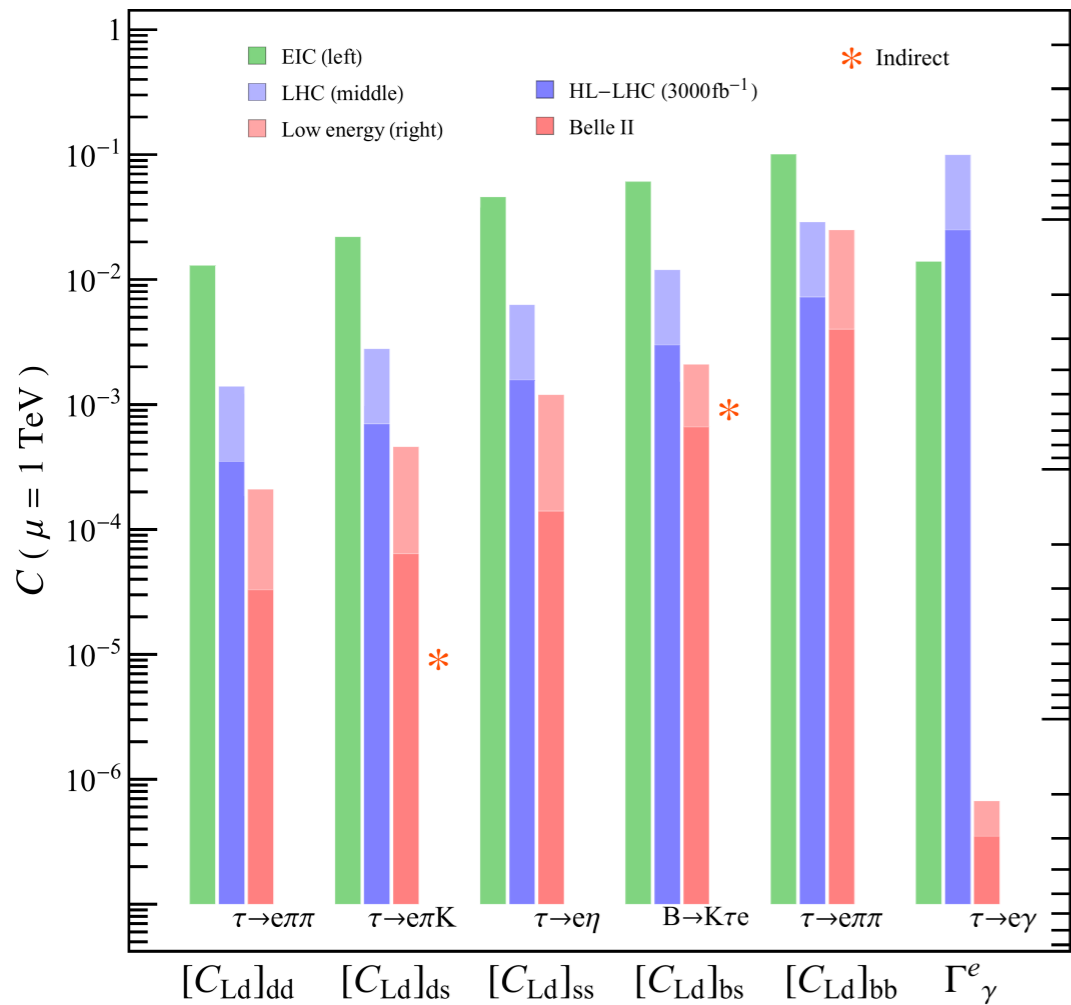
$$\propto 1/\sqrt{\mathcal{L}}$$

$$\propto 1/\mathcal{L}$$

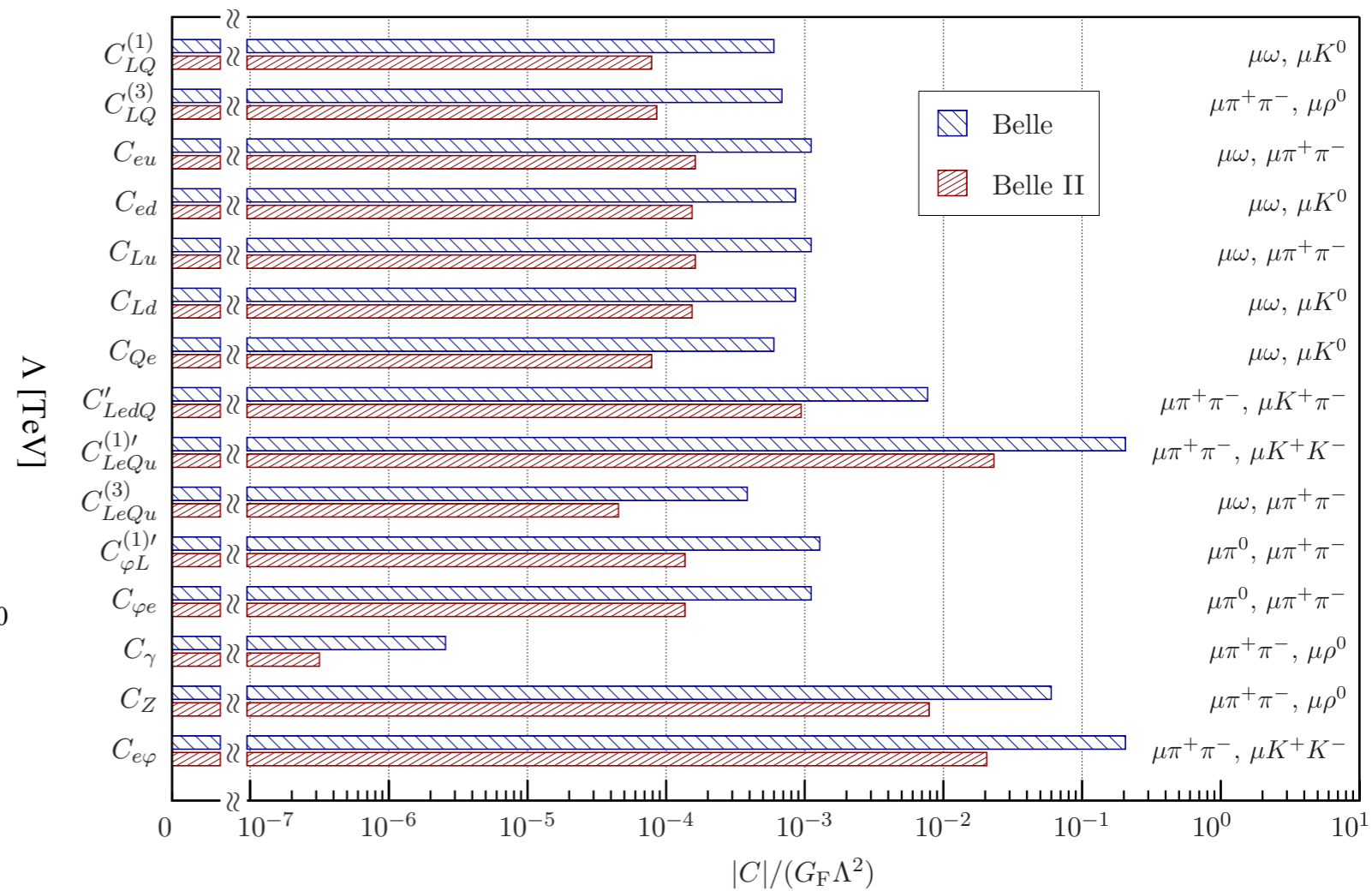


Synergy with other experiments

Model-independent probes of new physics at scale (Λ) encoded as Wilson coefficients (C_n) via EFT approach



Tau to electron transitions

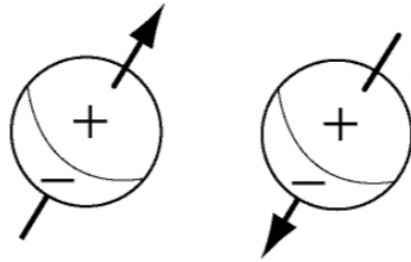


Tau to muon transitions

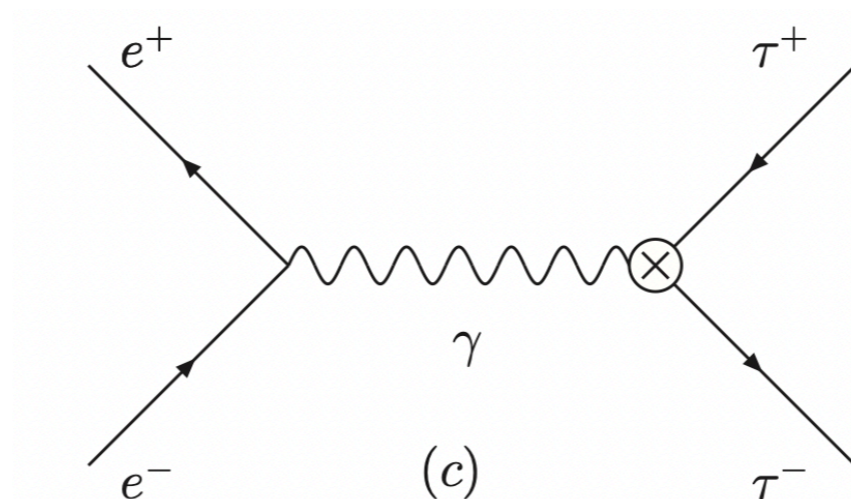
e-Print: [2203.14919](https://arxiv.org/abs/2203.14919) [hep-ph]

Electric dipole moment of τ

- Charge asymmetry along spin direction



- $\text{EDM} \neq 0 \Rightarrow \text{P, T violation}$. Search for CP violation in $\tau^- \tau^+ \gamma$ vertex.



- SM prediction $\simeq \mathcal{O}(10^{-37} e \cdot \text{cm})$ far below experimental sensitivity
- New Physics contributions in loops can enhance EDM $\simeq \mathcal{O}(10^{-19} e \cdot \text{cm})$

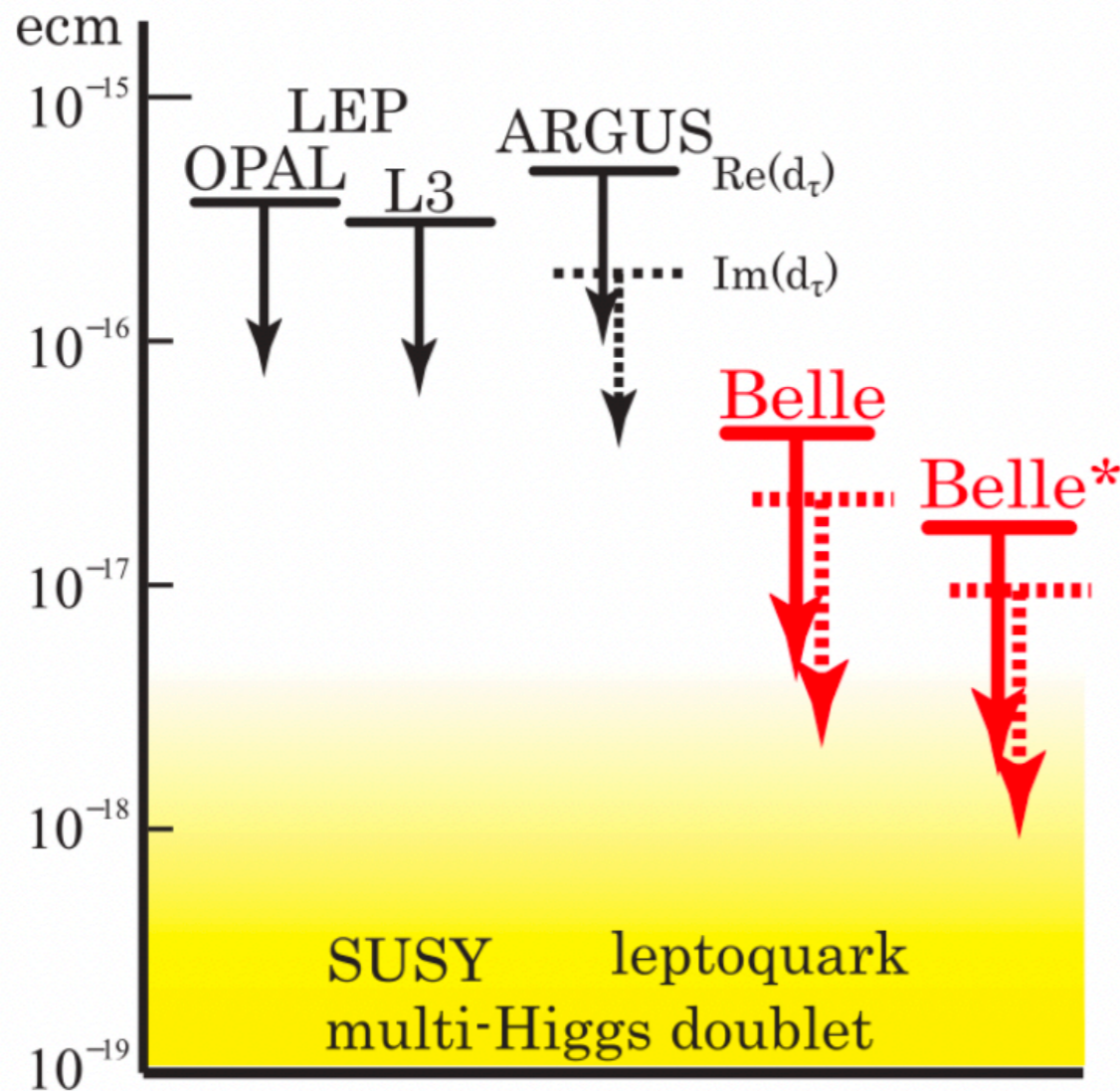
W. Bernreuther, A. Brandenburg, and P. Overmann, Phys. Lett. B 391, 413 (1997).

Huang, W. Lu, and Z. Tao, Phys. Rev. D 55, 1643 (1997).

- $\text{EDM} \neq 0 \Rightarrow$ unambiguous signature of New Physics

Electric dipole moment of τ

• Current Status:



- Belle; 29.5fb⁻¹ data [PLB 551(2003)16]
 - $-2.2 < \text{Re}(d_\tau) < 4.5$ ($10^{-17} e \text{ cm}$)
 - $-2.5 < \text{Im}(d_\tau) < 0.8$ ($10^{-17} e \text{ cm}$)

• Belle; 833 fb⁻¹ data ([arXiv:2108.11543](https://arxiv.org/abs/2108.11543) [**hep-ex**])

- 95% confidence intervals

$$-1.85 \times 10^{-17} < \text{Re}(d_\tau) < 0.61 \times 10^{-17} \text{ ecm},$$

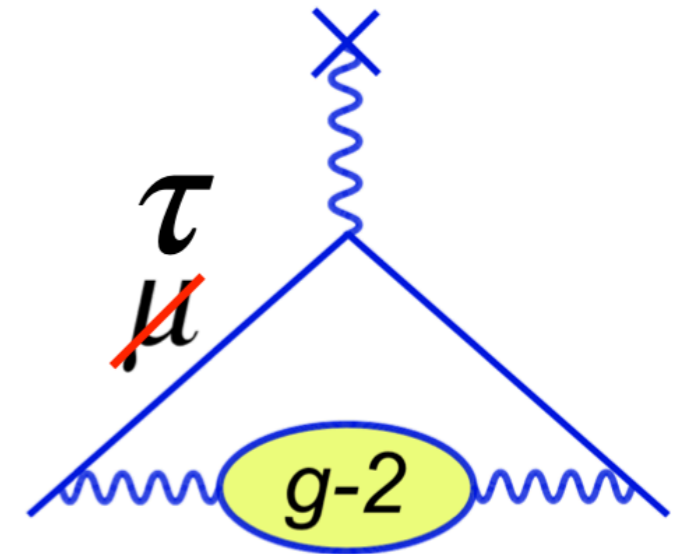
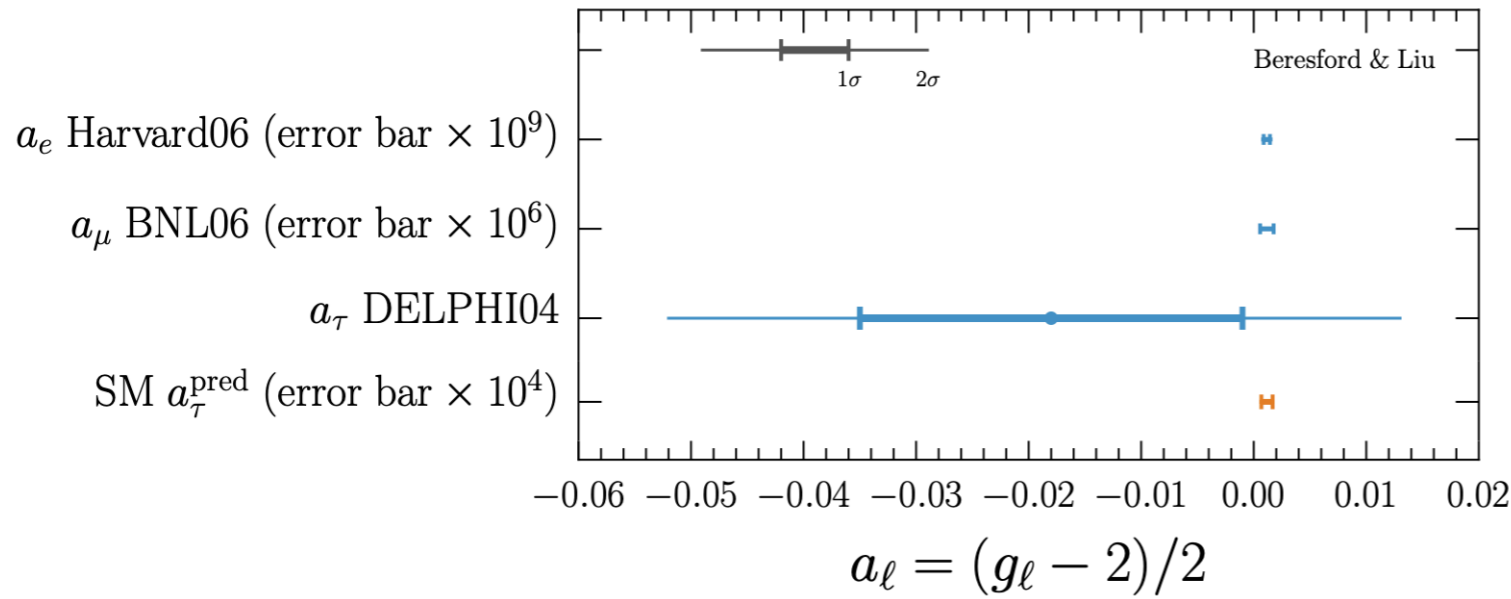
$$-1.03 \times 10^{-17} < \text{Im}(d_\tau) < 0.23 \times 10^{-17} \text{ ecm}.$$

- Consistent with zero EDM
- Systematic errors similar to statistical
- Dominant systematics: Data-MC mismatch in momentum/angular distributions

• Future Projections:

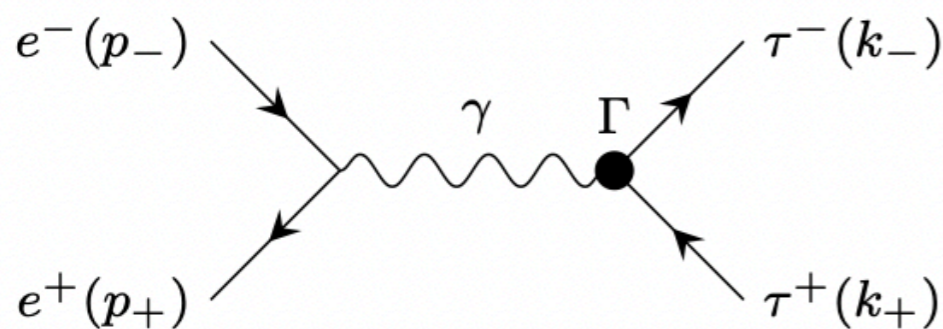
- Preliminary studies at Belle II show much better control of data-MC mismatch
- After improved control of systematics, extrapolation based on statistical errors only
- Probe $\simeq \mathcal{O}(10^{-19} e \cdot \text{cm})$ with 50 ab⁻¹ data at Belle II.
- Further improvement expected from proposed upgrade of polarized e- beams.

Magnetic dipole moment of τ



- Tensions are seen in electron and muon.
- Current bound in tau $\sim 10^{-2}$ [DELPHI, Eur. Phys. J. C 35, 159 (2004)].
- Belle II will explore $(g-2)_\tau$. Polarized beam can enhance sensitivity.

EFT Extension for τ -Pair Production



$$q = k_+ + k_-; \quad q^2 \geq 4m_\tau^2 \quad e > 0; \quad \sigma^{\mu\nu} = \frac{i}{2} [\gamma^\mu, \gamma^\nu]$$

$$\langle \tau^-(k_-), \tau^+(k_+), \text{out} | J_{\text{em}}^\mu | 0 \rangle = -e \bar{u}(k_-) \Gamma^\mu v(k_+)$$

$$\Gamma^\mu = \underbrace{F_1(q^2) \gamma^\mu}_{\text{radiative corrections}} + \underbrace{F_2(q^2) \frac{1}{2m_\tau} i\sigma^{\mu\nu} q_\nu}_{\text{MDM}} + \underbrace{F_3(q^2) \frac{1}{2m_\tau} \sigma^{\mu\nu} q_\nu \gamma_5}_{\text{EDM}}$$

$$F_1(q^2), F_2(q^2) \text{ are called the Dirac and Pauli; } \quad F_1(0) = 1; \quad F_2(0) = a_\tau$$

Magnetic dipole moment of τ

4.1. Transverse asymmetry

To get an observable sensitive to the relevant signal define the azimuthal transverse asymmetry as

$$A_T^\pm = \frac{\sigma_R^\pm|_{\text{Pol}} - \sigma_L^\pm|_{\text{Pol}}}{\sigma} = \mp \alpha_\pm \frac{3\pi}{8(3-\beta^2)\gamma} [|F_1|^2 + (2-\beta^2)\gamma^2 \text{Re}\{F_2\}], \quad (29)$$

where

$$\begin{aligned} \sigma_L^\pm|_{\text{Pol}} &\equiv \int_{\pi/2}^{3\pi/2} d\phi_\pm \left[\frac{d\sigma^S}{d\phi_\pm} \Big|_{\text{Pol}(e^-)} \right] \\ &= \pm \text{Br}(\tau^+ \rightarrow h^+ \bar{\nu}_\tau) \text{Br}(\tau^- \rightarrow h^- \nu_\tau) \\ &\quad \times \alpha_\pm \frac{(\pi\alpha)^2 \beta}{8s} \frac{1}{\gamma} [|F_1|^2 + (2-\beta^2)\gamma^2 \text{Re}\{F_2\}], \end{aligned} \quad (30)$$

$$\sigma_R^\pm|_{\text{Pol}} \equiv \int_{-\pi/2}^{\pi/2} d\phi_\pm \left[\frac{d\sigma^S}{d\phi_\pm} \Big|_{\text{Pol}(e^-)} \right] = -\sigma_L^\pm|_{\text{Pol}}. \quad (31)$$

4.2. Longitudinal asymmetry

Then, we define the longitudinal asymmetry as

$$A_L^\pm = \frac{\sigma_{\text{FB}}^\pm(+)|_{\text{Pol}} - \sigma_{\text{FB}}^\pm(-)|_{\text{Pol}}}{\sigma} = \mp \alpha_\pm \frac{3}{4(3-\beta^2)} [|F_1|^2 + 2 \text{Re}\{F_2\}], \quad (34)$$

where

$$\begin{aligned} \sigma_{\text{FB}}^\pm(+)|_{\text{Pol}} &\equiv \int_0^1 d(\cos\theta_\pm^*) \frac{d\sigma_{\text{FB}}^S}{d(\cos\theta_\pm^*)} \Big|_{\text{Pol}(e^-)} \\ &= \mp \alpha_\pm \text{Br}(\tau^+ \rightarrow h^+ \bar{\nu}_\tau) \text{Br}(\tau^- \rightarrow h^- \nu_\tau) \frac{\pi\alpha^2}{4s} \beta [|F_1|^2 + 2 \text{Re}\{F_2\}], \end{aligned} \quad (35)$$

$$\sigma_{\text{FB}}^\pm(-)|_{\text{Pol}} \equiv \int_{-1}^0 d(\cos\theta_\pm^*) \frac{d\sigma_{\text{FB}}^S}{d(\cos\theta_\pm^*)} \Big|_{\text{Pol}(e^-)} = -\sigma_{\text{FB}}^\pm(+)|_{\text{Pol}}. \quad (36)$$

Tau anomalous magnetic moment form factor at super B/flavor factories

J. Bernabéu^{a,b}, G.A. González-Sprinberg^c, J. Papavassiliou^{a,b}, J. Vidal^{a,b,*}

Combining Eq. (29) and Eq. (34) one can determine the real part of $F_2(s)$.

$$\text{Re}\{F_2(s)\} = \mp \frac{8(3-\beta^2)}{3\pi\gamma\beta^2} \frac{1}{\alpha_\pm} \left(A_T^\pm - \frac{\pi}{2\gamma} A_L^\pm \right).$$

[Nucl.Phys.B790:160-174,2008](#)

TABLE II: Contributions to $\text{Re} F_2^{\text{eff}}(s)$ in units of 10^{-6} .

	$s = 0$	$s = (10 \text{ GeV})^2$
1-loop QED	1161.41	-265.90
e loop	10.92	-2.43
μ loop	1.95	-0.34
2-loop QED (mass independent)	-0.42	-0.24
HVP	3.33	-0.33
EW	0.47	0.47
total	1177.66	-268.77

Andreas Crivellin,
Martin Hoferichter,
J. Michael Roney

[arXiv:2111.10378 \[hep-ph\]](#)

Precision $\simeq \mathcal{O}(10^{-5})$
with 40 ab^{-1} of data
with polarized beam

Second class currents

Hadronic current in τ decays: $J_h^\mu = \langle 0 | V^\mu - A^\mu | u\bar{d} \rangle$

Charge Conjugation: $\langle 0 | V^\mu | u\bar{d} \rangle \xrightarrow{C} -\langle 0 | V^\mu | u\bar{d} \rangle$ $\langle 0 | A^\mu | u\bar{d} \rangle \xrightarrow{C} \langle 0 | A^\mu | u\bar{d} \rangle$

Isospin Rotation: $R(u\bar{d}) : e^{i\pi I_2} |\pi^+\rangle = (-1)^I |\pi^-\rangle = -|\pi^-\rangle$

G-Parity combines C & R: $\langle 0 | V^\mu | u\bar{d} \rangle \xrightarrow{G} \langle 0 | V^\mu | u\bar{d} \rangle$ $\langle 0 | A^\mu | u\bar{d} \rangle \xrightarrow{G} -\langle 0 | A^\mu | u\bar{d} \rangle$

Classification of weak currents according to their G parity

Current Class	Vector	Axial Vector
First	$G = +1$	$G = -1$
Second	$G = -1$	$G = +1$

S. Weinberg, Phys. Rev. 112, 1375 (1958).

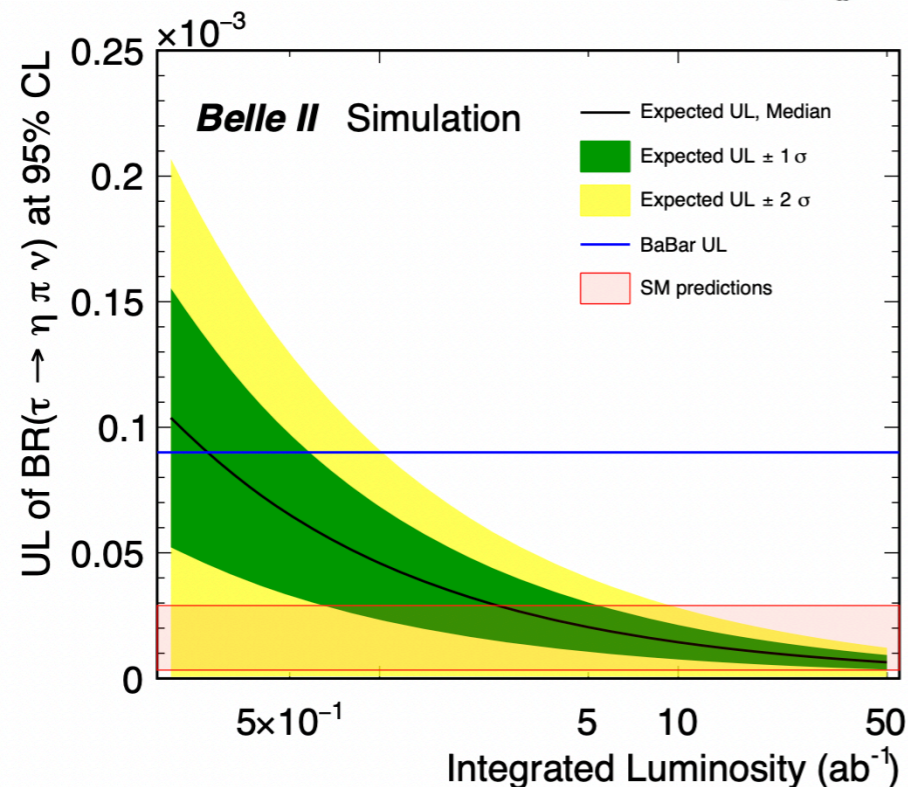
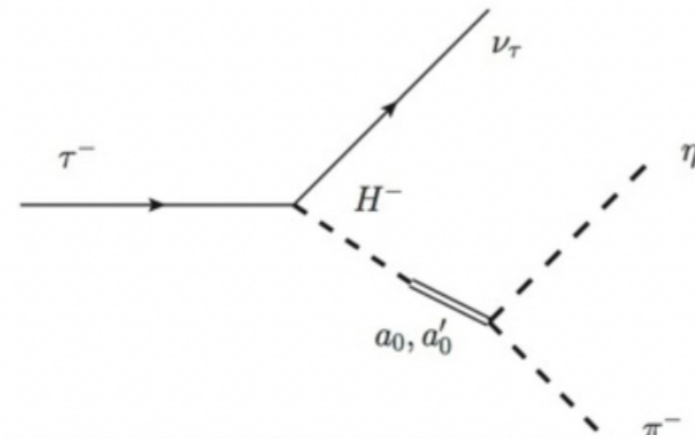
C. Leroy and J. Pestieau, Physics Letters B 72, 398 (1978).

Isospin violating Second Class Current (SCC) in $\tau^- \rightarrow \pi^- \eta \nu_\tau$ decays:

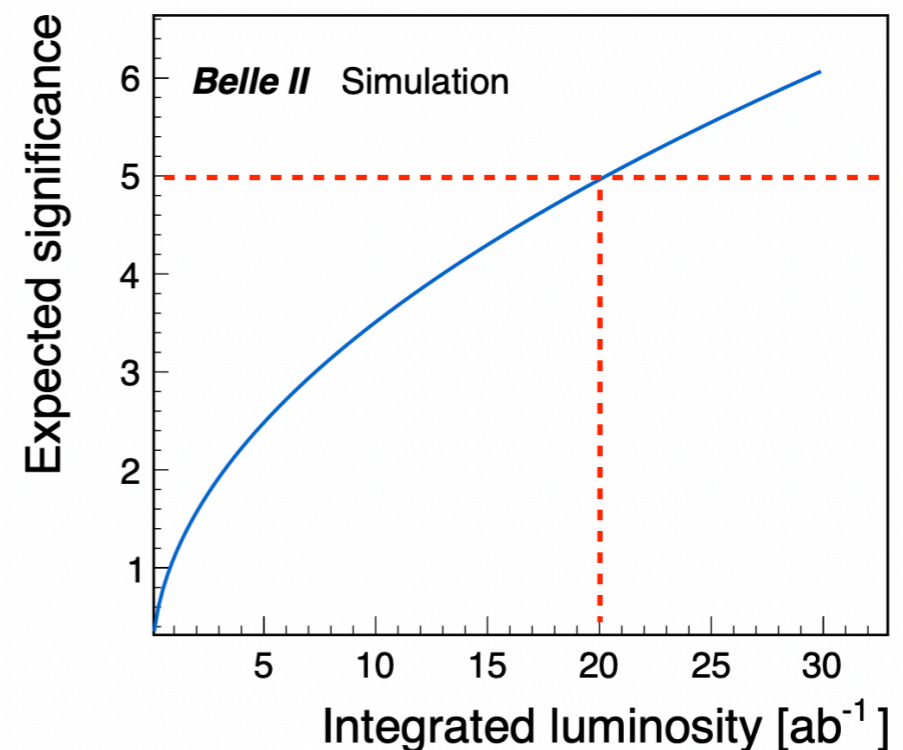
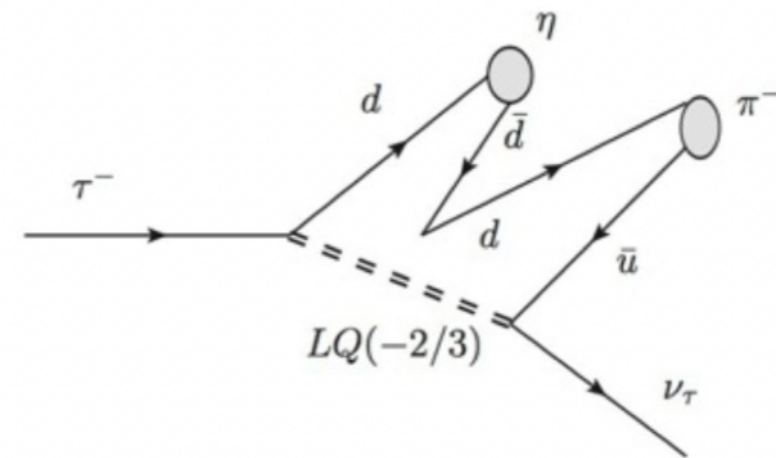
- expected at the level of $(m_u - m_d) \sim 10^{-5}$
- enhanced by new physics contributions

Scalar contributions from extended Higgs/Leptoquark sector

Charged Higgs exchange



Leptoquark exchange



A precision measurement, accompanied by improved theoretical knowledge of the scalar form factor, will set stringent bounds on charged Higgs exchange competitive to those obtained from $B^- \rightarrow \tau^- \nu_\tau$ data, even if no excess is seen over second class current predictions.

E. A. Garcías, M. H. Villanueva, G. L. Castro, and P. Roig, J. High Energy Phys. 12, 027 (2017).

Lots of interesting physics with tau's

