Information scrambling and butterfly velocity in quantum spin glass chains

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Work done with:

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V. Lokesh, S. Bera & SB, **arxiv:2401.04772**
S. Bera, V. Lokesh K Y & SB, **PRL 128**, 115302 (2022)

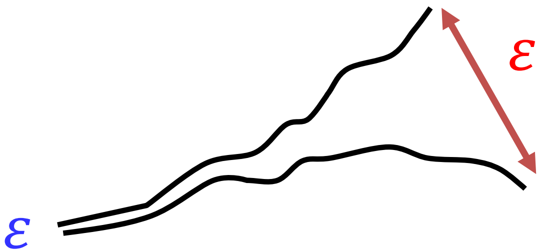
9th Indian Statistical Physics Community Meeting, ICTS, April 4, 2024

Classical and quantum Chaos

Single-particle chaos

Sensitivity to initial condition ⇒

$$\frac{\delta x(t)}{\delta x(0)} \sim e^{\lambda_L t}$$



$$\varepsilon e^{\lambda_L t} \lesssim \mathcal{O}(1)$$

λ_L, Lyapunov exponent

Quantum chaos

$$\frac{\partial x(t)}{\partial x(0)} = \{x(t), p(0)\} \quad \Rightarrow [x(t), p(0)]/i\hbar$$

Poisson bracket

Out-of-time order commutator

Larkin & Ovchinikov (1969)

$$\mathcal{D}(t) = -\langle [x(t), p(0)]^2 \rangle \sim \hbar^2\, e^{2\lambda_L t}$$

Many-body quantum chaos: Scrambling of quantum information

$\mathcal{D}(t) = -\langle [A(t), B(0)]^2 \rangle \sim e^{\lambda_L t}$ $\Rightarrow$ Quantum Lyapunov exponent λ_L

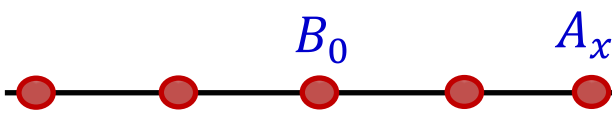
Out-of-time-order correlator (OTOC)

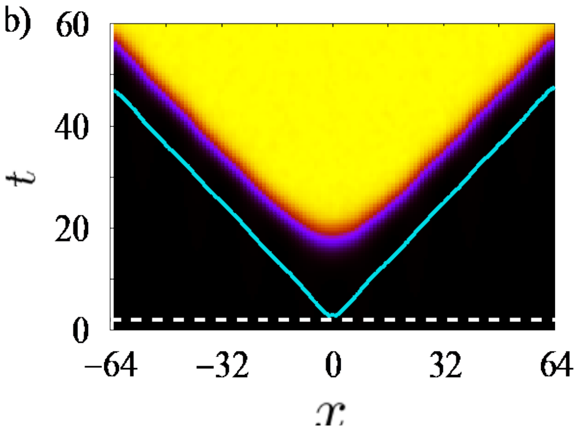
$$F(t) = \langle A(t)B(0)A(t)B(0) \rangle \sim \# - \mathcal{D}(t) \sim \# - \epsilon e^{\lambda_L t}$$

Spatial information propagation, Butterfly velocity v_B

$$\mathcal{D}(x,t) = -\langle [A_x(t), B_0(0)]^2 \rangle \sim e^{\lambda_L\left(t - \frac{|x|}{v_B}\right)}$$

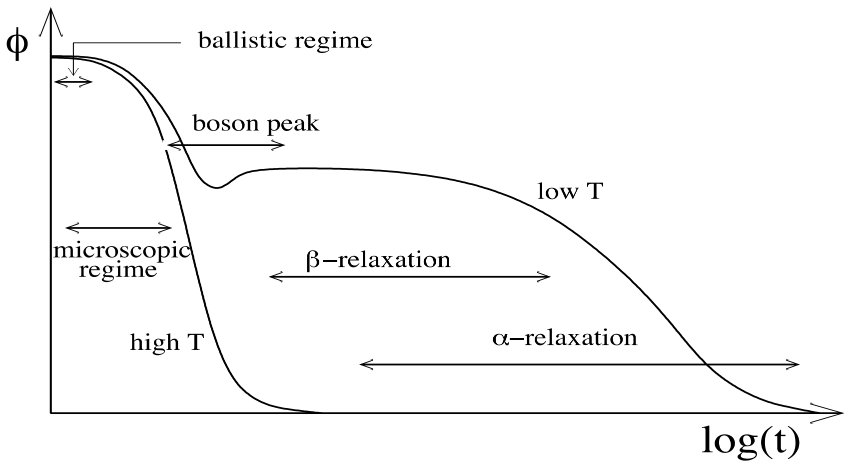
or $\sim e^{\lambda_L t\left(1 - \left(\frac{x}{v_B t}\right)^2\right)}$





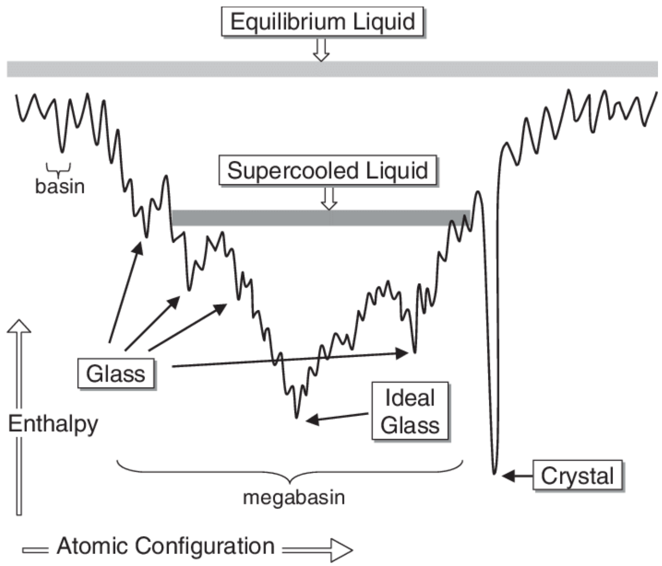
Why study chaos in glasses?

Kob, Les Houche (2002).



Apparent diverging time scale $T > T_g$

$$\tau_\alpha \sim (T - T_d)^{-\gamma}$$



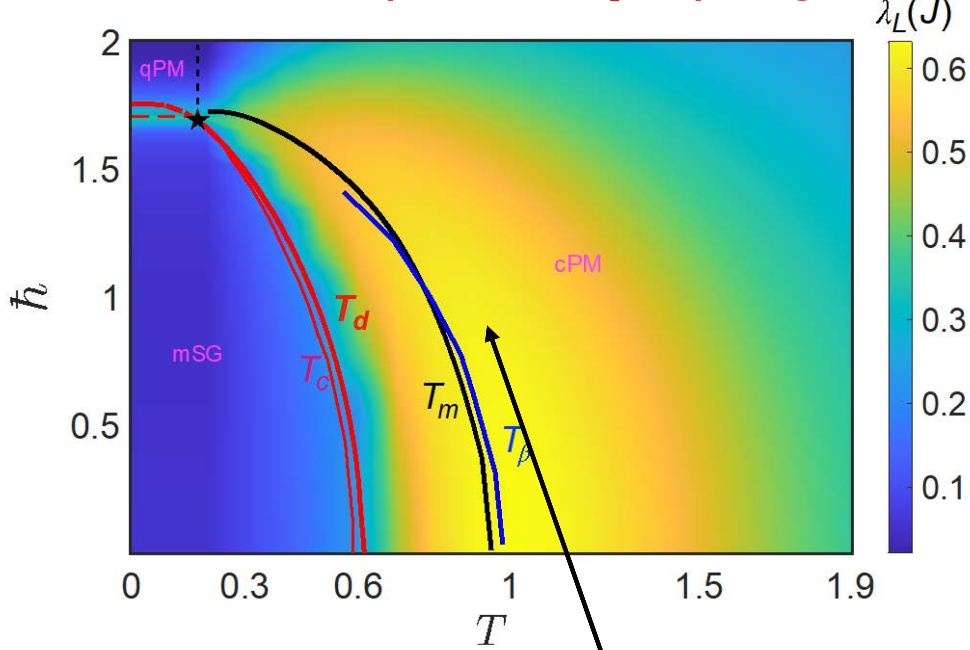
Classical potential energy Landscape

Many stationary points $\Rightarrow$

may lead to extreme sensitivity to initial condition

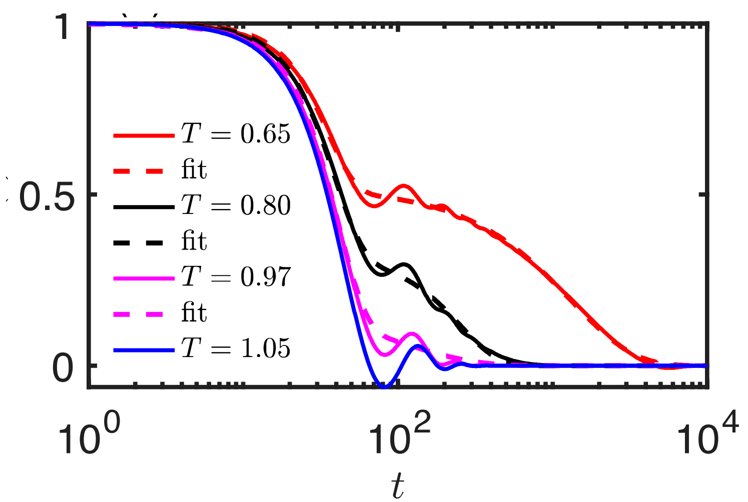
Shimono & Onodera,Mat.Trans. (2004).

Chaos in quantum p-spin glass



S. Bera, V. Lokesh K Y & SB, **PRL** (2022)

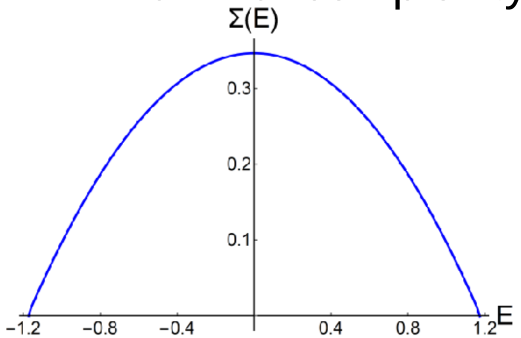
Complex two-step relaxation



Onset temperature T_β

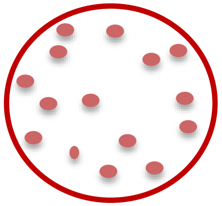
Maximum in Lyapunov exponent

$\Leftarrow$ Maximal complexity



Correale et al., **Sci. Post** (2023)

What happens to
spatial information propagation
or v_B?



Chaos in quantum spin glasses

Infinite range or zero dimensional models (like SYK)

Sachdev-Ye (SY) model
(parent of SYK model)

$$H = \frac{1}{2\sqrt{MN}} \sum_{i \neq j, \alpha\beta}^{N,M} J_{ij} S_{i\alpha\beta} S_{j\beta\alpha}$$

$SU(M)$ spins with spin S

Quantum fluctuations controlled by S

Quantum p-spin glass

$$H = \sum_{i=1}^{N} \frac{\pi_i^2}{2m} + \frac{1}{3!} \sum_{ijk} J_{ijk} s_i s_j s_k$$

$$[s_i, \pi_j] = i\hbar\delta_{ij}$$

Constraint $\sum_i s_i^2 = N$

Quantum fluctuations controlled by $\Gamma = \frac{\hbar^2}{mJ}$

⇒ Interplay of chaos with complex dynamics, non-trivial symmetry broken and unbroken phases, and phase transitions.

replica symmetry broken spin glass
spin liquid, local moments
spin glass transitions

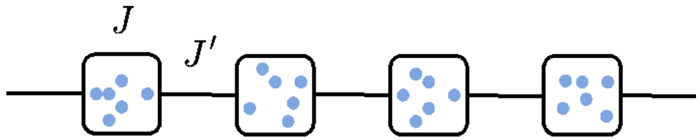
But, infinite-range or zero dimensional model.

How to study information propagation, like v_B?

$\Rightarrow$ Lattice generalization

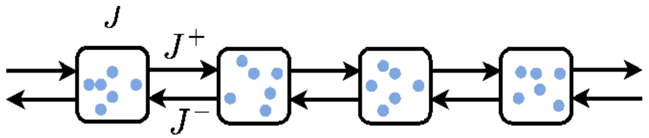
Lattice of spin glass quantum dots

Sachev-Ye chain



$$H = \frac{1}{2\sqrt{MN}} \sum_{x,i\neq j,\alpha\beta}^{N,M} J_{ij,x} S_{i\alpha\beta x} S_{j\beta\alpha x} + \frac{1}{\sqrt{MN}} \sum_{x,i\neq j,\alpha\beta}^{N,M} J_{ij,x} S_{i\alpha\beta x} S_{j\beta\alpha,x+1}$$

p-spin glass chain



$$H = \sum_{x,i=1}^{N} \frac{\pi_{ix}^2}{2m} + \frac{1}{3!} \sum_{x,ijk} J_{ijk,x} s_{i,x} s_{j,x} s_{k,x} + \frac{1}{2!} \sum_{x,ijk} s_{ix} (J_{ijk,x}^{+} s_{j,x+1} s_{k,x+1} + J_{ijk,x}^{+} s_{j,x-1} s_{k,x-1})$$

Disorder averaging using replicas$\Rightarrow$

Solvable in the $N, M \to \infty$ limit using bosonic representation

$$S_{i\alpha\beta x} = b_{i\alpha x}^{\dagger} b_{i\beta x} - S\delta_{\alpha\beta}$$

$$G^{ab}(\tau,\tau') = -\frac{1}{M} \sum_{\alpha} \overline{\langle \mathcal{T}_\tau b_{i\alpha x}(\tau) b_{i\beta x}^{\dagger}(\tau') \rangle}$$

Bosonic Green's function

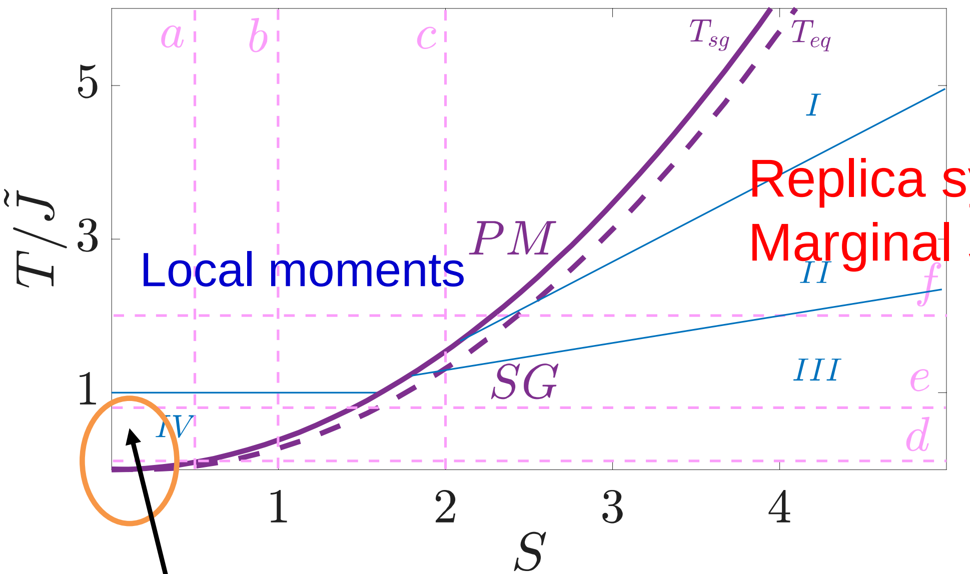
Solvable in the $N \to \infty$ limit

$$Q^{ab}(\tau,\tau') = \frac{1}{N} \sum_{i} \overline{\langle \mathcal{T}_\tau s_{ix}(\tau) s_{ix}(\tau') \rangle}$$

Dynamical spin correlation

Thermodynamic and dynamical phase diagram

Sachdev Ye chain



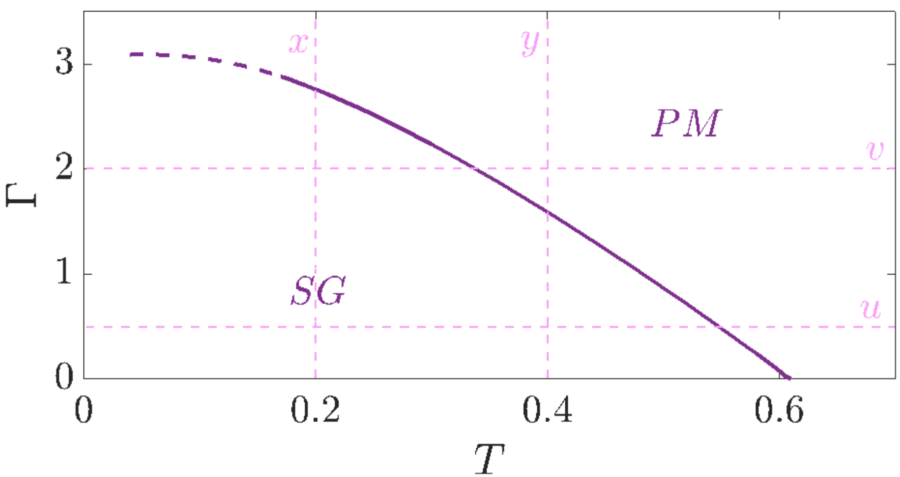
Local moments

Replica symmetry broken
Marginal spin glass

Spin liquid

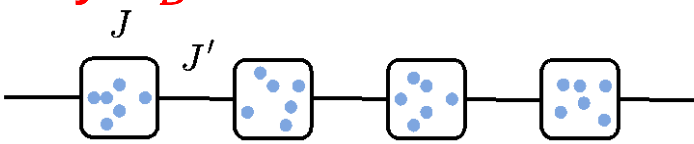
$S \to 0$,
Sachdev-Ye-Kitaev (SYK)
spin liquid

p-spin glass chain

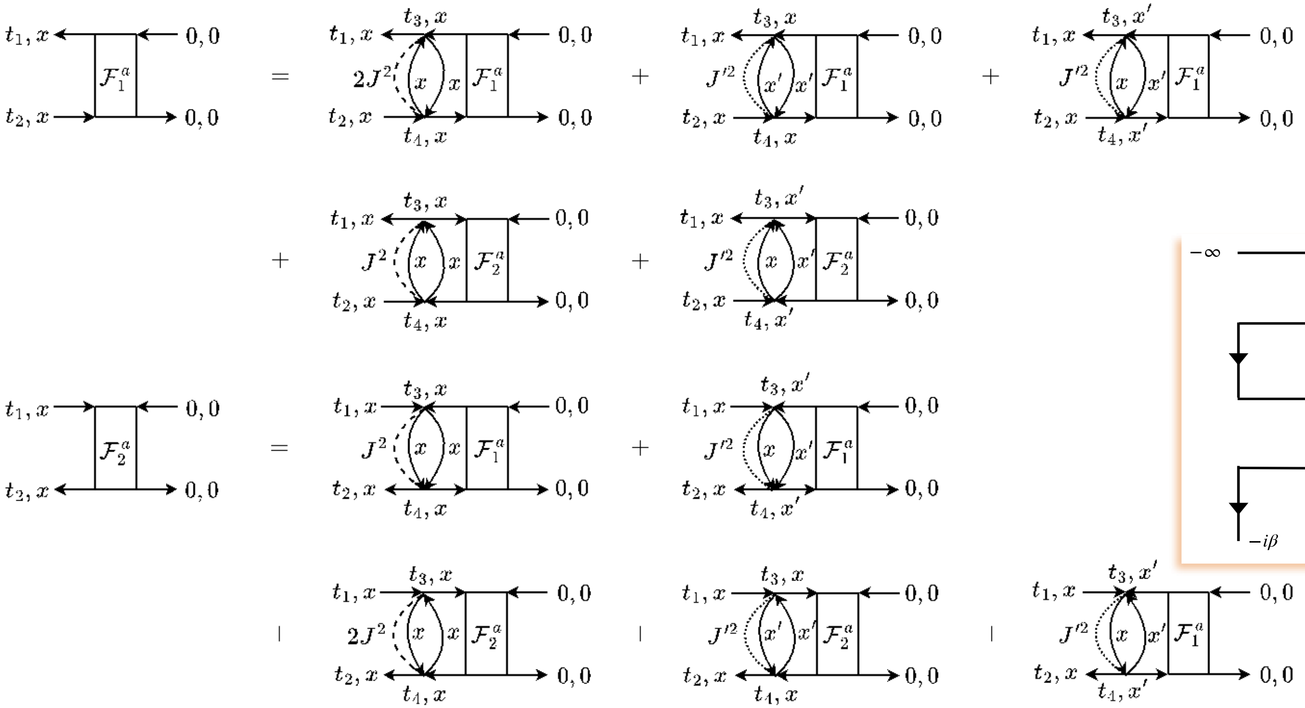


Information propagation and butterfly velocity v_B

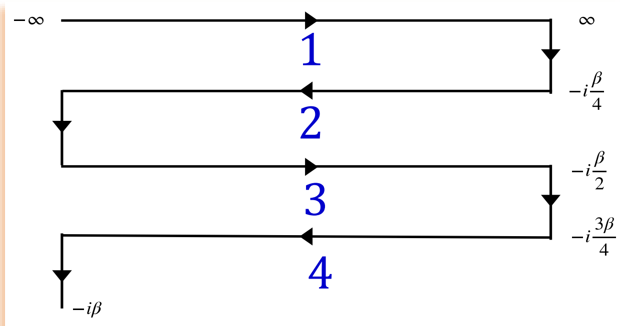
Non-local OTOCs

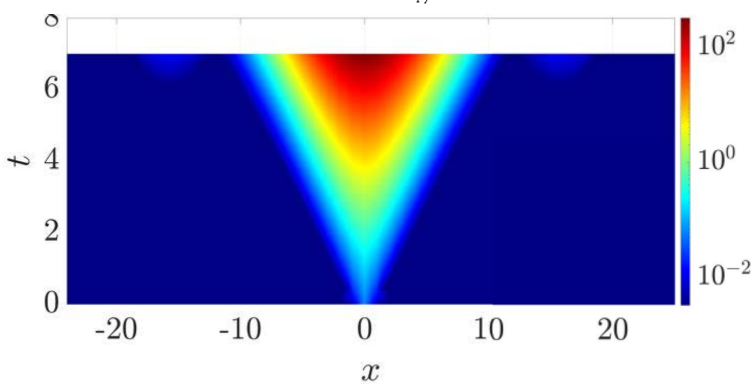


$$F_{1,x}(t_1,t_2) = \overline{\mathrm{Tr}[y b^\dagger_{i\alpha,x}(t_1) y b^\dagger_{j\beta,0}(0) y b_{i\alpha,x}(t_2) y b_{j\beta,0}(0)]} \qquad F_{2,x}(t_1,t_2) = \cdots$$



Keldysh contour

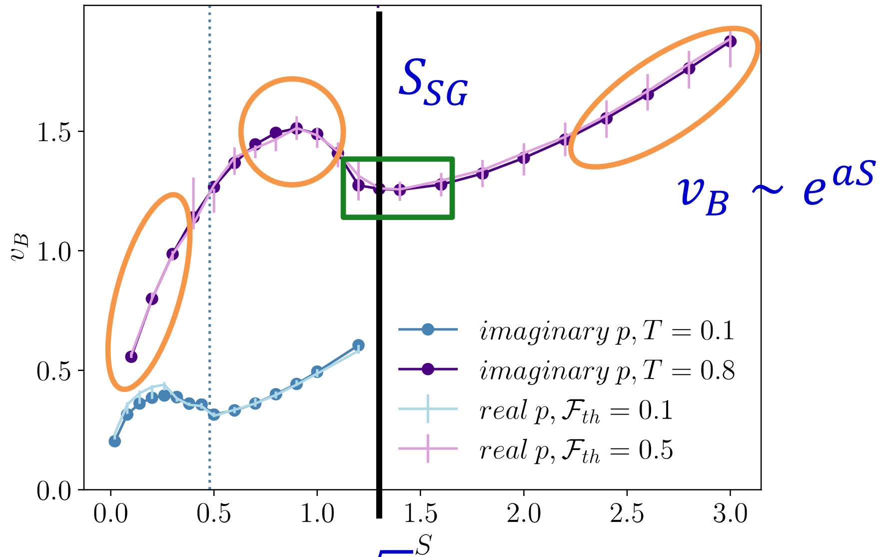




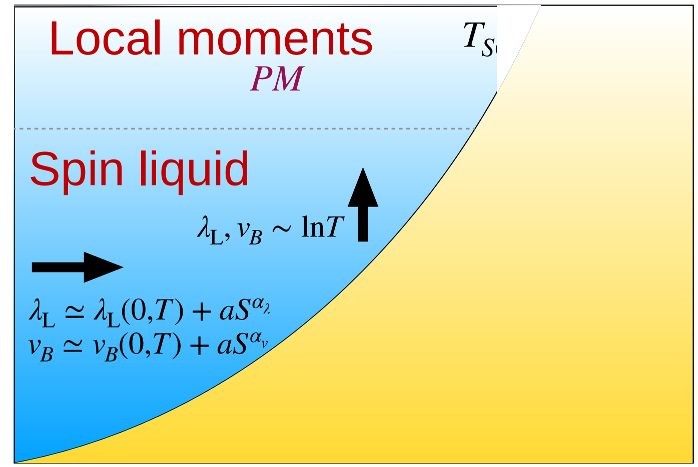
v_B extracted from light cone

Effect of quantum fluctuations on butterfly velocity

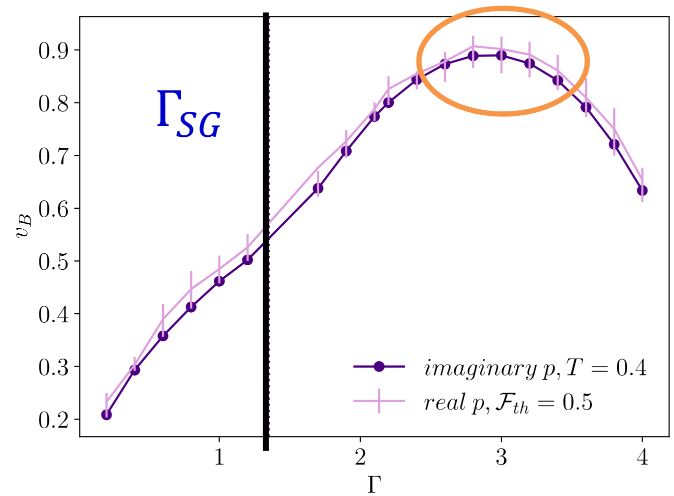
SY chain



$v_B \sim v_B(0,T) + a\sqrt{S}$



p-spin glass chain

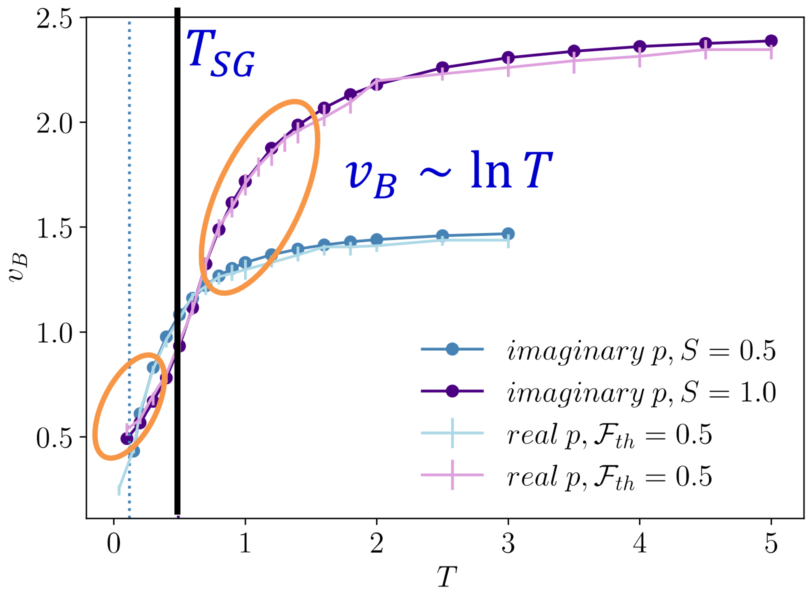


Similar maximum in $v_B(\Gamma)$

- Maximum in $v_B(S)$ in the PM phase $\Rightarrow v_B$ non-monotonic with quantum parameter.
- $v_B(S \to 0) \to$ very small in the spin liquid
- More or less smooth crossover across spin glass transition, dip in v_B at the transition

Effect of thermal fluctuations on butterfly velocity

Sachdev-Ye chain

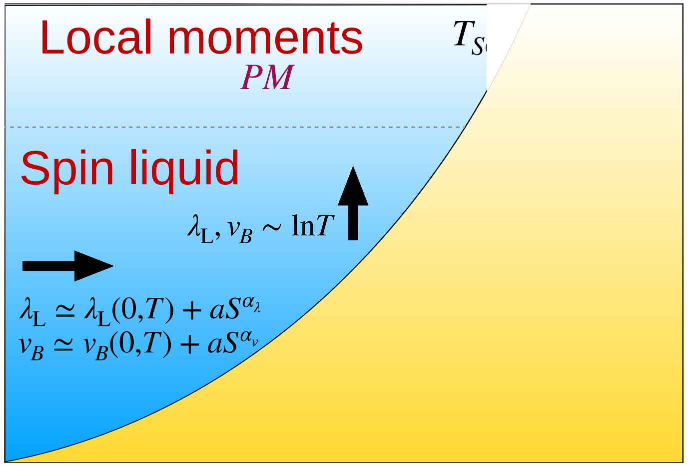


$$v_B \simeq v_B(0,S) + aT^{1.5}$$

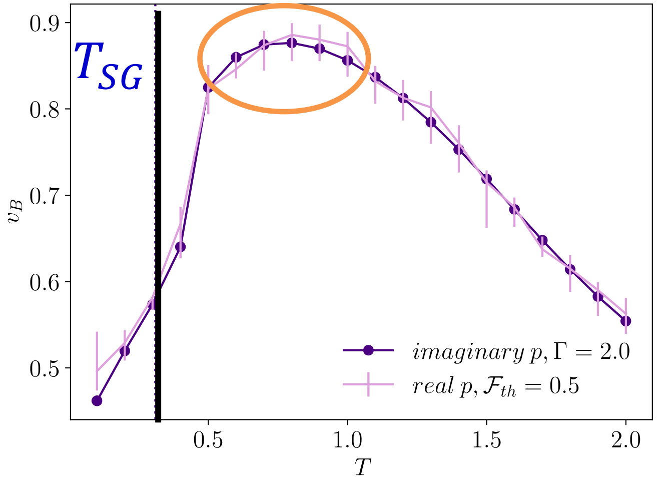
o No maximum in $v_B(T)$ in the PM phase

o $v_B(T \to 0)$ is finite $\Rightarrow$ Finite speed of information for spin glass ground state.

o More or less smooth crossover across spin glass transition



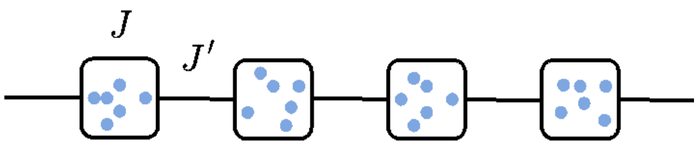
p-spin glass chain



Maximum in $v_B(T)$

Conclusions

o Chaos in two quantum spin glasses,
Sachdev-Ye (SY) and quantum p-spin glass



SY chain

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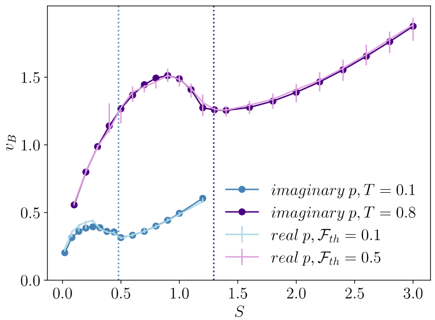
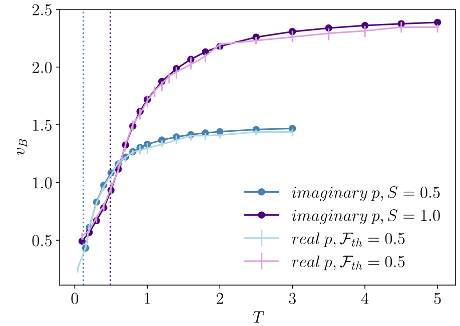
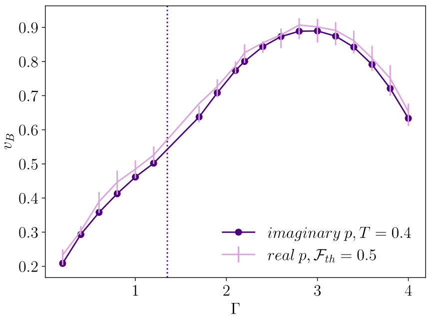
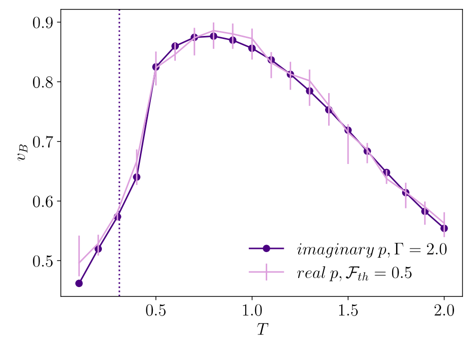




Relation between information scrambling and glassy complexity
(complex dynamics, complex potential energy landscape)

p-spin glass chain





Different evolutions of glassy complexity with quantum and thermal parameters the two models

⇒ need to compute glassy complexity or number of stationary states (quantum TAP equations)

Characterization of phases and phase transitions of quantum many-body systems in terms of chaos?

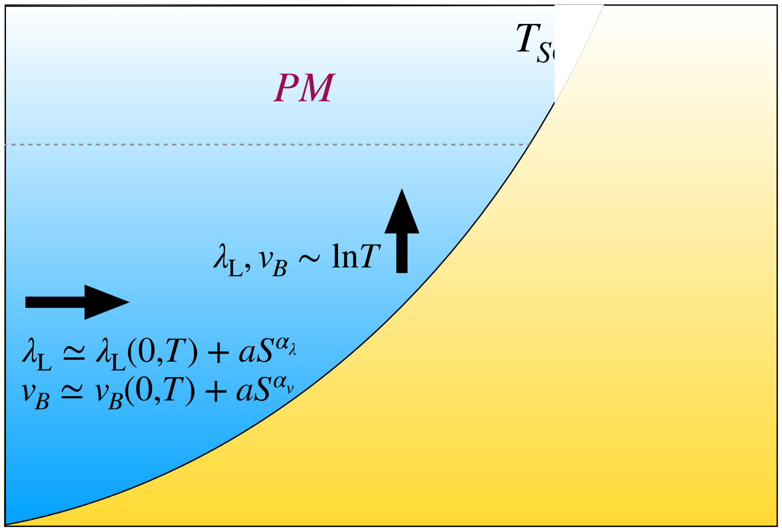
Chaos as an "Order Parameter"

Models, phases, phase transitions	λ_L	v_B	D
SYK quantum dots, interacting diffusive metal without quasiparticles	$2\pi T$	$\sqrt{T}$	$D_E = \frac{v_B^2}{\lambda_L}$
Weakly interacting diffusive metal	$T^{\frac{d}{2}}$	$T^{\frac{d}{4}} \sim T^{1-1/z}$ *at critical point	$D = \frac{v_B^2}{4\lambda_L}$
Critical Fermi surface for fermions coupled to U(1) gauge field	T	$T^{1/3}$	$D_E \sim \frac{v_B^2}{\lambda_L}$
2+1 d $O(N)$ non-linear sigma model Quantum critical Ordered phase Disordered phase	 T/N T^3 $e^{-E_g/T}$	 c $\sim c$ $\sqrt{T}$	
Classical spin liquid	$\sqrt{T}$	$T^{1/4}$	$D_S \sim \frac{v_B^2}{\lambda_L}$
Classical XXZ model Ordered or power law phase Disordered phase	 T^3 $\sqrt{T}$	 Non power law Power law	$D_S \sim \frac{v_B^2}{\lambda_L}$ At high T

Gu et al. JHEP (2017); Guo et al. PRB (2019); Patel et al. PNAS (2017); Patel et al. PRX (2017)
; Swingle at l. PRB (2017); Chowdhury et al. PRD (2017); Sahu et al. PRB (2020);
Bilitewski et al. PRL (2018); Ruidas & SB, SciPost Phys (2021), ..

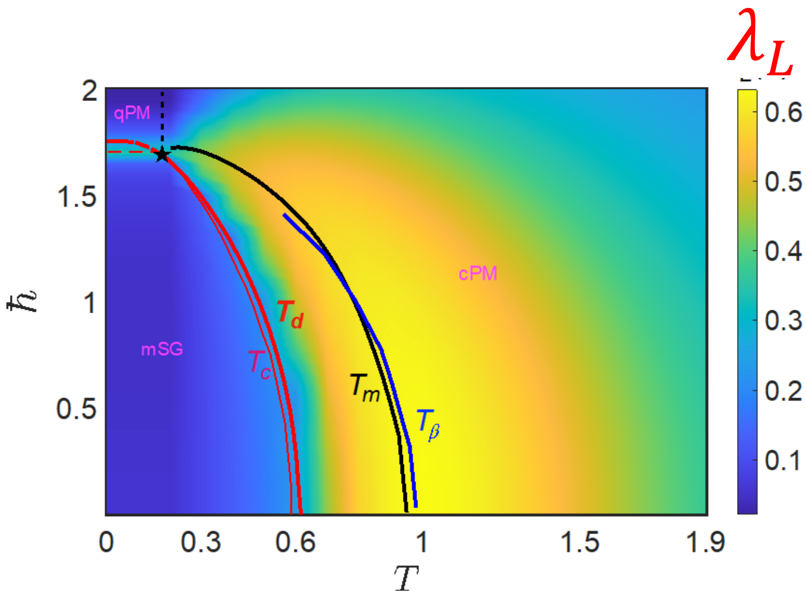
Chaos phase diagram

SY chain



V. Lokesh, S. Bera & SB,
arxiv:2401.04772

p-spin glass chain



S. Bera, V. Lokesh K Y & SB, **PRL** (2022)

OTOC in the paramagnetic and marginal Spin glass phase

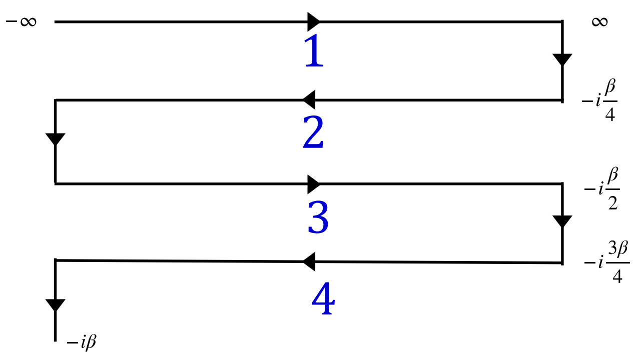
Regularized OTOC $F_1(t_1,t_2) = \overline{\mathrm{Tr}[y b^{\dagger}_{i\alpha}(t_1) y b^{\dagger}_{j\beta}(0) y b_{i\alpha}(t_2) y b_{j\beta}(0)]}$ $\qquad y^4 = \dfrac{e^{-\beta H}}{\mathrm{Tr}[e^{-\beta H}]}$

$F_2(t_1,t_2) = \cdots$

Schiwnger-Keldysh contour for generating function

$$\mathcal{Z} = \frac{1}{\mathrm{Tr}[e^{-\beta H}]} Tr[e^{-\frac{\beta H}{4}} U(t_0,t_f) e^{-\frac{\beta H}{4}} U(t_f,t_0) e^{-\frac{\beta H}{4}} U(t_0,t_f) e^{-\frac{\beta H}{4}} U(t_0,t_f)]$$

$t_0 \to -\infty, \qquad t_f \to \infty$



Typically, replicas are not needed in Keldysh for disorder averaging

Works for paramagnetic phase

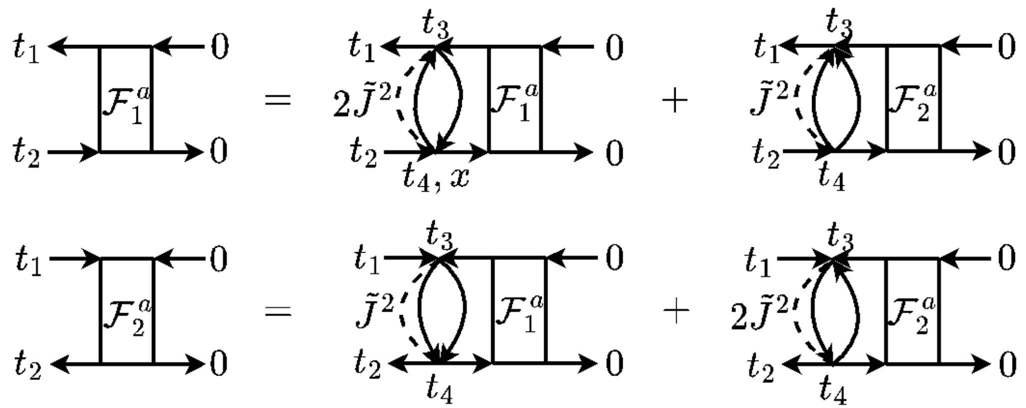
For dynamical correlations in spin glass phase, need replicas even in Keldysh

$\Rightarrow \mathcal{Z}^{\mathrm{n}}$ Houghton, Jain, Young (1983)
Cugliandolo et al. (2019)

$$F_1^a(t_1,t_2) = \overline{\left\langle y\, \bar{b}^{(4)}_{i\alpha a}(t_1) y\, \bar{b}^{(3)}_{j\beta a}(0) y b^{(2)}_{i\alpha a}(t_2) y b^{(1)}_{j\beta a}(0) \right\rangle} = \# - \left(\frac{1}{N}\right) \mathcal{F}_1^a(t_1,t_2)$$

$$\mathcal{F}_1^a(t,t) \sim e^{\lambda_L t}$$

How does replica symmetry breaking enter in chaos?



Kernel equations

$$\mathcal{F}_1^a(t_1,t_2) = \int dt_3 dt_4 K_{11}^a(t_1,t_2,t_3,t_4)\mathcal{F}_1^a(t_3,t_4) + \int dt_3 dt_4 K_{12}^a(t_1,t_2,t_3,t_4)\mathcal{F}_2^a(t_3,t_4)$$

$$\mathcal{F}_2^a(t_1,t_2) = \cdots$$

$$K_{11}^a(t_1,t_2,t_3,t_4) = 2\tilde{J}^2 G_A(t_3-t_1)G_R(t_2-t_4)G_{lr}^{+}(t_4-t_3)G_{lr}^{-}(t_3-t_4) \qquad K_{12}^a, \cdots$$

Wightmann correlation $\quad G_{lr}^{\pm}(\omega) = 2\pi\sqrt{q_{EA}}\,\delta(\omega)\epsilon_{ab} + \pi(\rho(\omega)/\sinh(\beta\omega/2))\,\delta_{ab}$

$q_{EA} = 0$ for paramagnetic phase, $q_{EA} \neq 0$ for spin glass

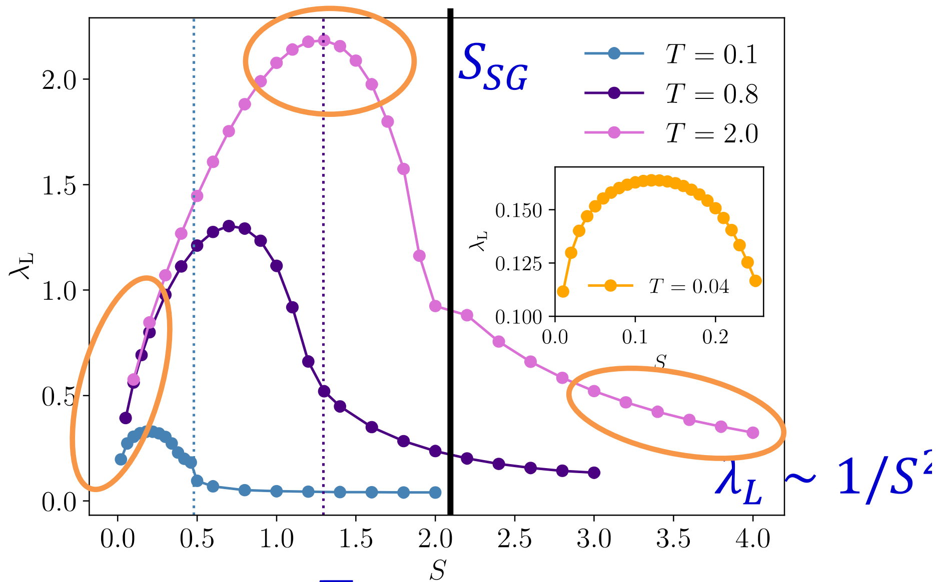
Solve the Kernel equations with growth ansatz

$$\mathcal{F}_\mu^a(t_1,t_2) = e^{\frac{\lambda_L(t_1+t_2)}{2}} f_\mu^a(t_1-t_2)$$

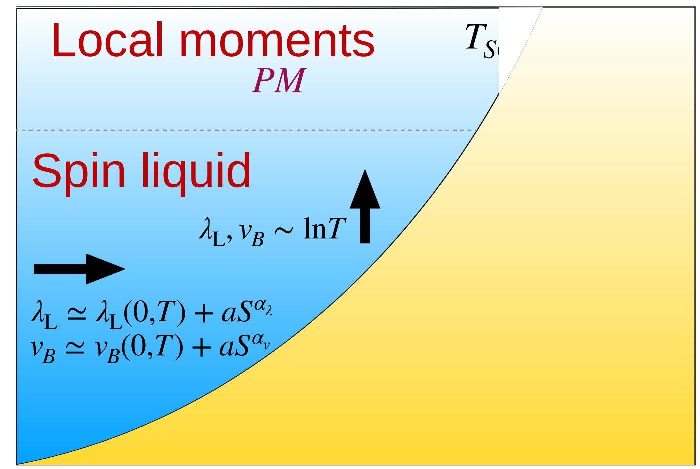
Self consistently find λ_L

Effect of quantum fluctuations on Lyapunov exponent

SY chain

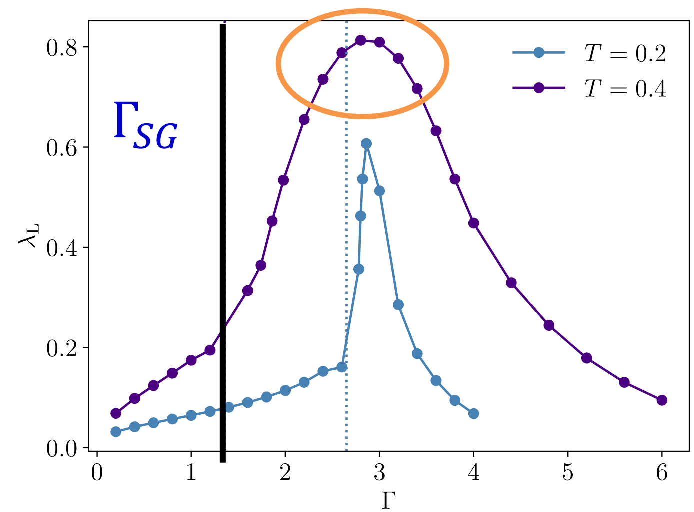


$\lambda_L \sim \lambda_L(0,T) + a\sqrt{S}$



- o Maximum in $\lambda_L(S)$ in the PM phase $\Rightarrow \lambda_L$ non-monotonic with quantum parameter.
- o $\lambda_L(S \to 0) \ll 2\pi T$
- o More or less smooth crossover across spin glass transition

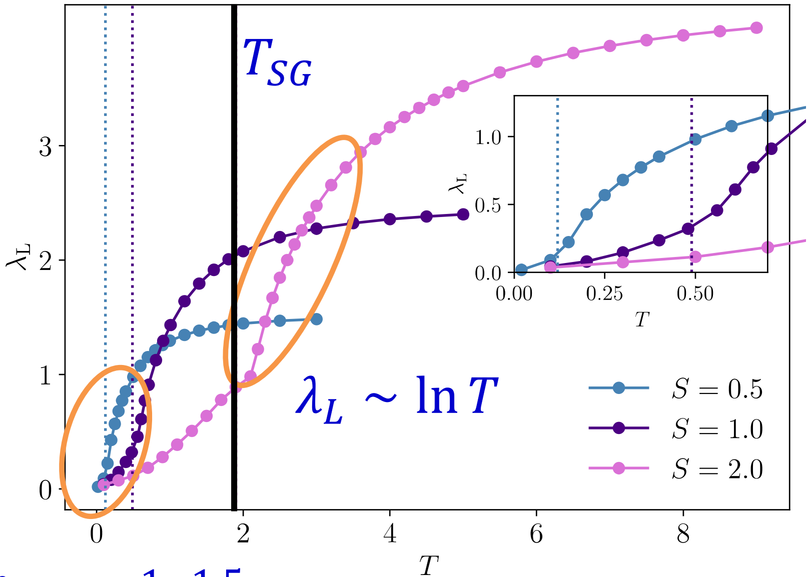
p-spin glass chain



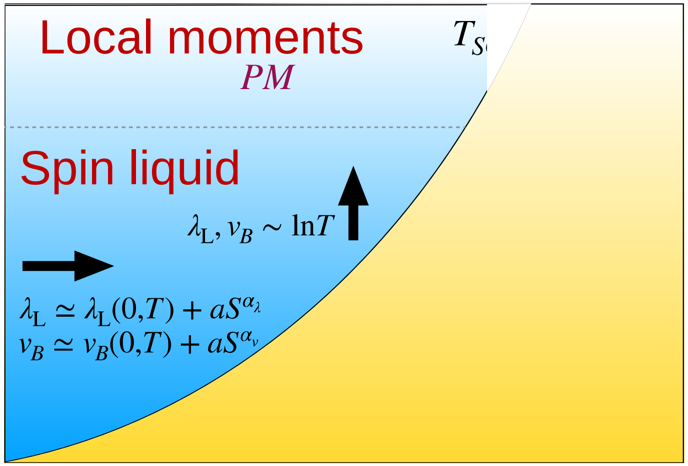
Similar maximum in $\lambda_L(\Gamma)$

Effect of thermal fluctuations on Lyapunov exponent

SY chain



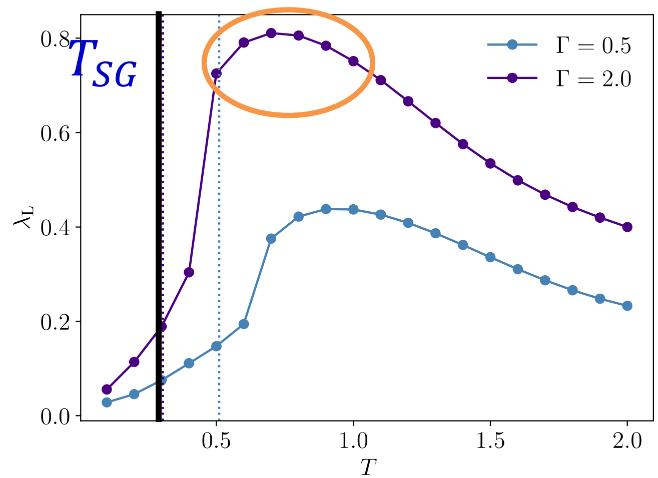
$\lambda_L \sim T^{1-1.5}$



- o No maximum in $\lambda_L(T)$ in the PM phase

- o More or less smooth crossover across spin glass transition

p-spin glass chain



Maximum in λ_L(T)

Large N (M) saddle points

Lattice saddle point same as zero-dimensional saddle point with effective coupling $\tilde{J}^2 = J^2 + J'^2$

Sachdev & Ye, PRL (1993); Georges et al., PRL (2000); Georges et al., PRB (2001) Cugliandolo et al, PRB (2001)

SY chain

Large N (M) fields $\Rightarrow$

p-spin glass chain

$$G^{ab}(\tau,\tau') = -\frac{1}{M}\sum_{\alpha}\overline{\langle \mathcal{T}_\tau b_{i\alpha x}(\tau) b^{\dagger}_{i\beta x}(\tau')\rangle}$$

Bosonic Green's function

$$Q^{ab}(\tau,\tau') = \frac{1}{N}\sum_{i}\overline{\langle \mathcal{T}_\tau s_{ix}(\tau) s_{ix}(\tau')\rangle}$$

Dynamical spin correlation

$$(G^{-1})^{ab}(i\omega_k) = (i\omega_k + \lambda)\,\delta_{ab} - \Sigma^{ab}(i\omega_k)$$
$$\Sigma^{ab}(\tau) = \tilde{J}^2[G^{ab}(\tau)]^2 G^{ba}(-\tau)$$

$$(Q^{-1})^{ab}(i\omega_k) = \left(\omega_k^2/\Gamma + z\right)\delta_{ab} - \Sigma^{ab}(i\omega_k)$$
$$\Sigma^{ab}(\tau) = \frac{3\tilde{J}^2}{2}[Q_{ab}(\tau)]^2,$$

Replica index $a = 1, \cdots, n$, eventually take $n \to 0$

- Paramagnetic phase, replica symmetric and diagonal $G^{ab}(\tau) = G(\tau)\delta_{ab}$
- Spin glass phase, one step replica symmetry breaking (1RSB)

Break $n \times n$ matrix into n/m diagonal $m \times m$ blocks ($\epsilon_{ab} = 1$)

$$G^{ab}(\tau) = \tilde{G}(\tau)\delta_{ab} - g\epsilon_{ab}$$

Edward-Anderson order parameter $q_{EA} = g^2$