Statistics

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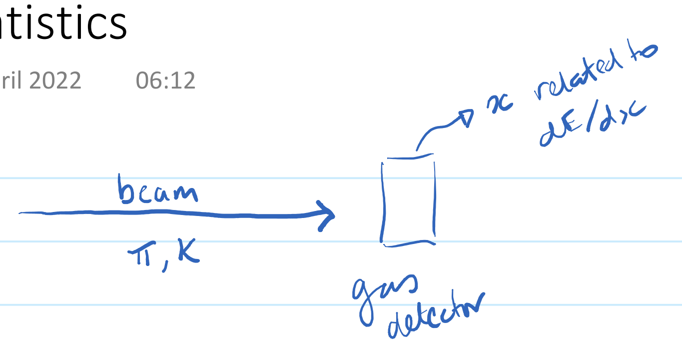
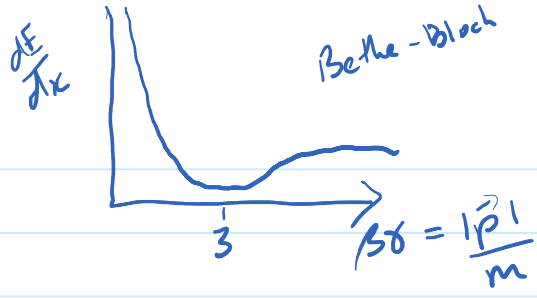
1)



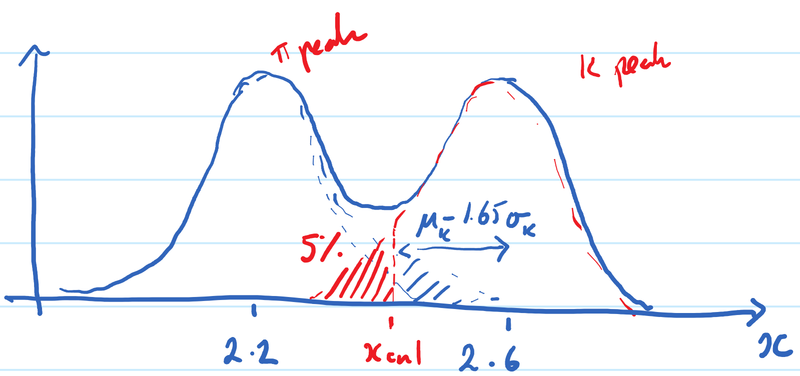


1) $$P_K(x|\sigma_K,\mu_K) = \frac{1}{\sigma_K\sqrt{2\pi}} e^{-(x-\mu_K)^2/2\sigma_K^2} \quad (\sigma_K = 0.2;\ \mu_K = 2.6)$$

Similarly $P_\pi(x|\sigma_\pi = 0.2, \mu_\pi = 2.2)$

2) x_0 new parameter f_K ≡ fraction K^+ in the beam

$$P_{TOT} = f_K P_K(x|\sigma_K,\mu_K) + (1-f_K)P_\pi(x|\sigma_\pi,\mu_\pi)$$



$f = 0.5$

$x > x_{cut}$

2. $$\underbrace{L = P_{TOT}(x_0|f_K)}_{\text{1 measurement}}$$

$$L(x_0, x_1, \ldots x_{N-1}|f_K) = \prod_{i=0}^{N-1} P_{TOT}(x_i|f_K)$$

3. $$\text{Significance} = S = \frac{N_K}{\sigma(N_K)}$$

$N_K = N_{TOT} - N_\pi$ $\quad \sigma(N_\pi) = 0$

$\sigma(N_K) = \sqrt{N_{TOT}}$

$$= \frac{N_K}{\sqrt{N_\pi + N_K}} \equiv \text{efficiency} \times \text{purity}$$

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2. Event with $x_0 > x_{cut}$. What is the probability K?

If $f_K \approx 1$: 100% whereas if $f_K \sim 10^{-4}$ $P_K \sim 0\%$

3. $f_K = 10\%$

Recall Bayes

$$p(a|b) = \frac{p(b|a)\, p(a)}{p(b)}$$

$$= \frac{p(b|a)\, p(a)}{p(b|a)\, p(a) + p(b|\bar{a})\,[1 - p(a)]}$$

a : K b: $x > x_{cut} = 2.27$

$p(x > x_{cut} \,|\, K) = 0.95$

$p(K) = 0.1$

$p(x > x_{cut} \,|\, \bar{K}) = p(x > x_{cut} \,|\, \pi) = 0.36$ $\Rightarrow$

$\Rightarrow p(K \,|\, x > x_{cut}) = 23\%$

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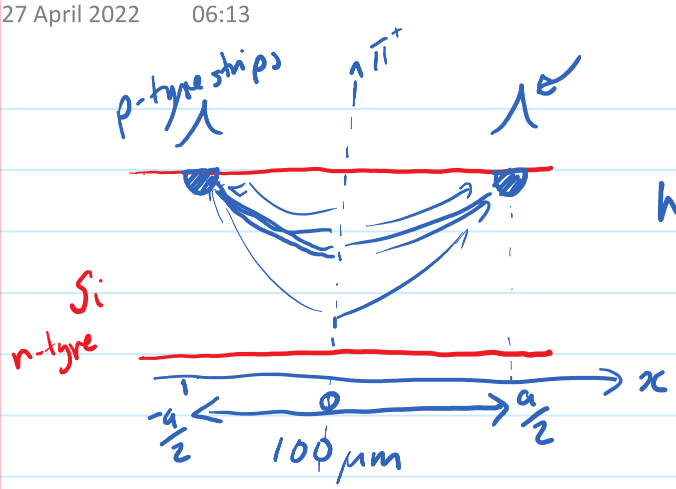
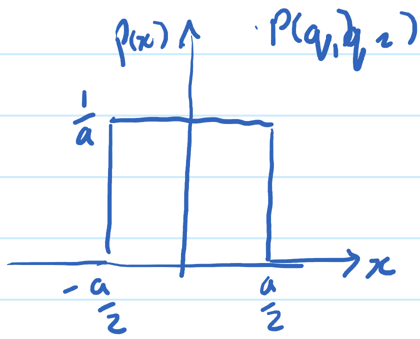
2.1)



What is the position uncertainty?

$a = 100\,\mu m$



$P(x < -\frac{a}{2}) = 0$ similarly $P(x > \frac{a}{2}) = 0$

$$\text{Variance} = \langle x^2 \rangle - \langle x \rangle^2$$

$$\langle x \rangle = \int_{-\frac{a}{2}}^{\frac{a}{2}} x\, P(x)\, dx = 0 \; ; \quad \langle x^2 \rangle = \int_{-\frac{a}{2}}^{\frac{a}{2}} \frac{x^2}{a}\, dx = \left[\frac{x^3}{3a}\right]_{-\frac{a}{2}}^{\frac{a}{2}}$$

$$= \frac{a^2}{12}$$

$$\sigma = \sqrt{\text{Var}} = \frac{a}{\sqrt{12}} = 30\,\mu m \;//$$

2.2 10 events with a large value of x
(subset) 2 events " " " " " have μ's too.

$L_n(p)$? where p is the probability that high x has muons

Follows a binomial distribution

$$P(r; p, n) = p^r (1-p)^{n-r} \frac{n!}{r!(n-r)!}$$

successes $\mu s = 2$; n: 10

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$$\Rightarrow \mathcal{L} = p^2(1-p)^8 \frac{10!}{2!8!} = 45p^2(1-p)^8$$

$$\Rightarrow \boxed{-2\ln L = -2\ln 45 - 2\ln p - 8\ln(1-p)}$$

$$L \propto e^{-(\hat{m}-m)^2/2\sigma^2}$$

$$-2\ln L \propto \frac{(\hat{m}-m)^2}{\sigma^2}$$

$$\left\{ \left.\frac{\partial L}{\partial p}\right|_{\hat{p}} = 0 \Rightarrow \hat{p} = \frac{2}{10} = 0.2 \right.$$

a-c) last page

d) Poisson distribution $\Longleftarrow$ binomial

λ_x = number of events expected

$= \mathcal{L}\sigma_x$

$\lambda_{\mu x} = \mathcal{L}\sigma_{x\mu}$

$$P_x = \frac{e^{-\overset{=\lambda_x}{\mathcal{L}\sigma_x}}(\mathcal{L}\sigma_x)^{10}}{10!}$$

$$P_{\mu x} = \frac{e^{-\mathcal{L}\sigma_{x\mu}}(\mathcal{L}\sigma_{x\mu})^2}{2!}$$

$\sigma_{x\mu} = p\,\sigma_x$

$$L_n(p) = p(\mu\,|\,\text{large } x) = \frac{p(\text{large } \overset{1}{x}\,|\,\mu)\; p(\mu)}{p(x)}$$

$$= e^{\mathcal{L}\sigma_x(1-p)}\, p^2 \left(\frac{10!}{2!\,(\mathcal{L}\sigma_x)^8}\right)$$

t

$$\hat{p} = \frac{2}{\mathcal{L}\sigma_x} \qquad \mathcal{L}\sigma_x = 10, 11, 11.2, 8$$

$$\Delta\ln L = -2\ln L + 2\ln L(\hat{p})$$

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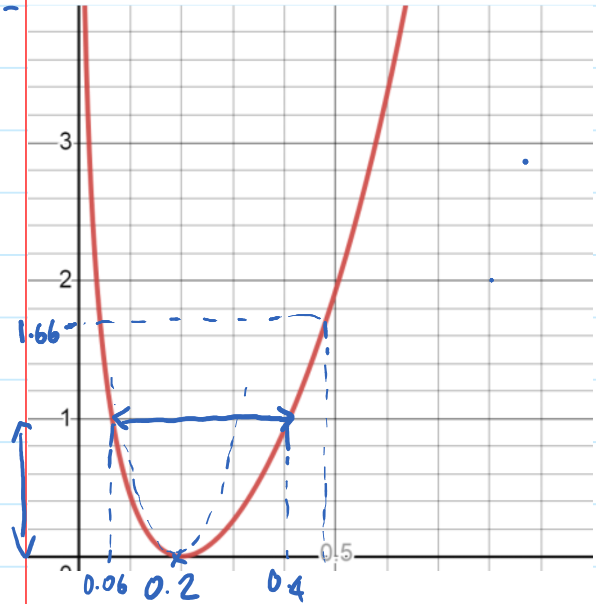
2σ 4



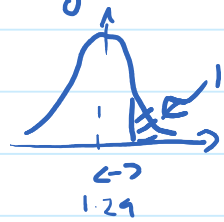
$-2\ln L = 1\sigma$

$\hat{p}$

$-2\ln L + 2\ln L(\hat{p})$

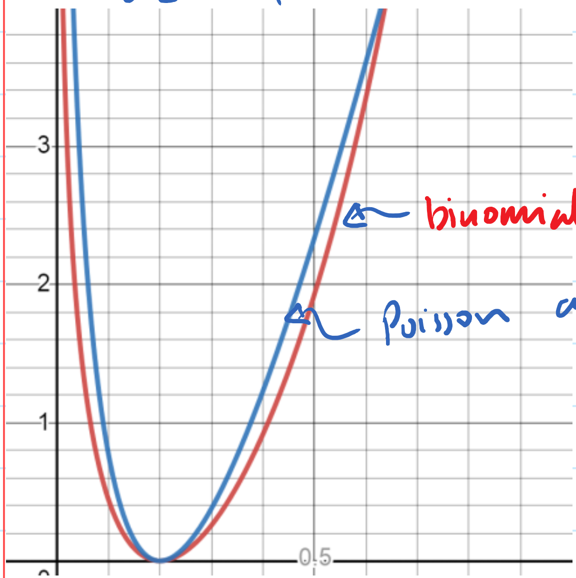
$$\chi^2 = \sum_i \frac{(x_i - \hat{x})^2}{\sigma^2}$$

90%



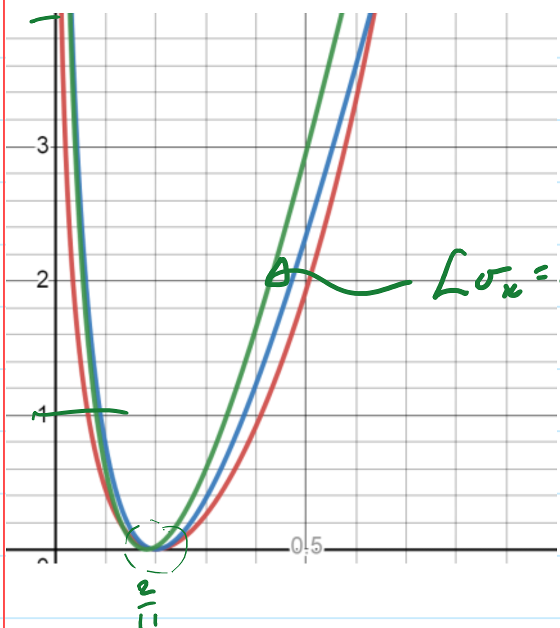
0.47 90% C.L. upper limit

1σ range $(0.06, 0.4) \equiv 0.2^{+0.20}_{-0.14}$



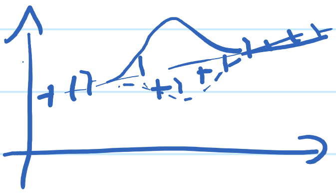
binomial

Poisson assuming $L\sigma_x = 10$

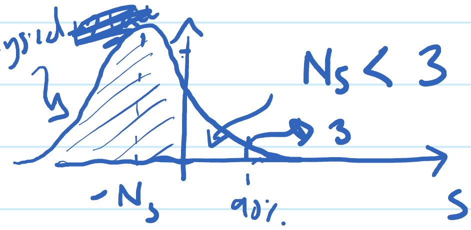


$L\sigma_x = \lambda_x = 11$

$-N_s$



unphysical



$N_s < 3$ 90%

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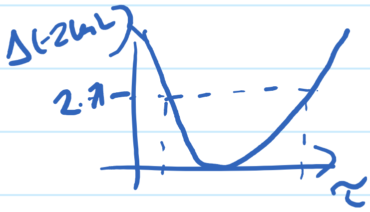
3.1 $t_0 = 3\times10^{-13}\,s$

$$P(t|\tau) = \frac{1}{\tau} e^{-t/\tau}$$

$$L = \frac{1}{\tau} e^{-t_0/\tau} \Rightarrow \ln L = -\ln\tau - \frac{t_0}{\tau}$$

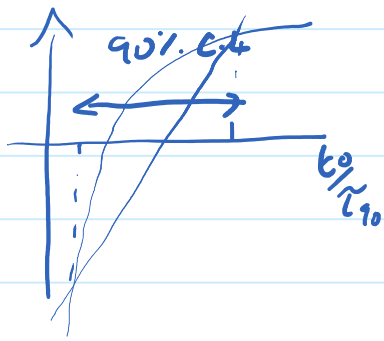
$$\left.\frac{\partial \ln L}{\partial \tau}\right|_{\hat\tau} = -\frac{1}{\hat\tau} + \frac{t_0}{\hat\tau^2} = 0 \Rightarrow \hat\tau = t_0$$



$$-2\ln L = 2\ln\tau + 2\frac{t_0}{\tau}$$

$$\Delta(-2\ln L) = 2\ln\tau_{90} + 2\frac{t_0}{\tau_{90}} \underbrace{-2-2\ln t_0}_{-2\ln L(\hat\tau)} = 2.71$$

$$\Rightarrow \ln\left(\frac{t_0}{\tau_{90}}\right)^2 = 2\frac{t_0}{\tau_{90}} - 4.71$$



$0.3t_0 < \tau_{90} < 10t_0$