Non-reciprocity and chirality in active matter

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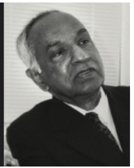

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OUTLINE

- Introduction: varieties of non-reciprocality
- Flocking without moving
- Chiral active matter and odd elasticity
- Summary and prospect

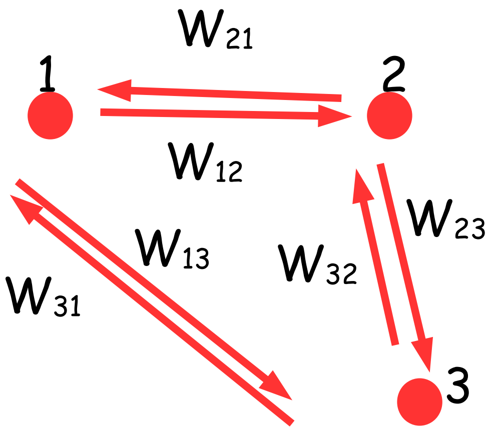
INTRODUCTION

varieties of non-reciprocal interactions

- Ubiquitous in the living world
  - A attracted to B, B repelled by A
  - locust flocking through cannibalistic pursuit & escape (Guttal et al. 2012)
  - sensing, signalling, ligand-receptor
- Ruled out at thermal equilibrium
  - detailed balance --> effective energy --> effective Newton’s III
- This talk: two kinds of non-reciprocality

Das, Rao, SR 2002; Gowrishankar & Rao 2016; Husain & Rao 2017, Saha et al. 2019, 2020
Loos & Klapp 2020, You et al. 2020, Fruchart et al 2020, Osat & Golestanian 2022

Thermal equilibrium is time-reversible: master equation



$$\frac{dP_a}{dt} = \sum_b (W_{ba}P_b - W_{ab}P_a)$$

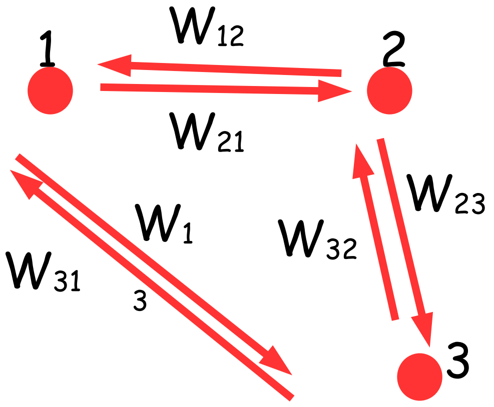
W_{ab} determined by system + environment

Thermal equilibrium: detailed balance
zero-current steady state P^s

$$P_1^s W_{12} = P_2^s W_{21} \quad \text{etc}$$

Possible only if $$\frac{W_{12}}{W_{21}} \frac{W_{23}}{W_{32}} \frac{W_{31}}{W_{13}} = 1$$

Easy to construct steady-state distribution



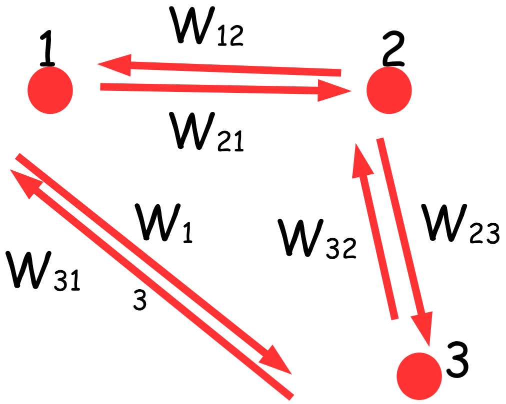
$$P_1^s = p$$

$$P_2^s = p \frac{W_{12}}{W_{21}}$$

$$P_3^s = p \frac{W_{23}}{W_{32}} \frac{W_{12}}{W_{21}} = p \frac{W_{13}}{W_{31}}$$

$$\frac{W_{12}}{W_{21}} \frac{W_{23}}{W_{32}} \frac{W_{31}}{W_{13}} = 1 \, \text{guarantees consistency}$$

Configurational energy U_a as emergent quantity



$$P_a^s \equiv C e^{-U_a}$$

$$\ln \frac{W_{ab}}{W_{ba}} = -(U_b - U_a)$$

velocity downhill along gradients of U

Equilibrium position-space dynamics

$$i, j, \text{ particle labels}$$

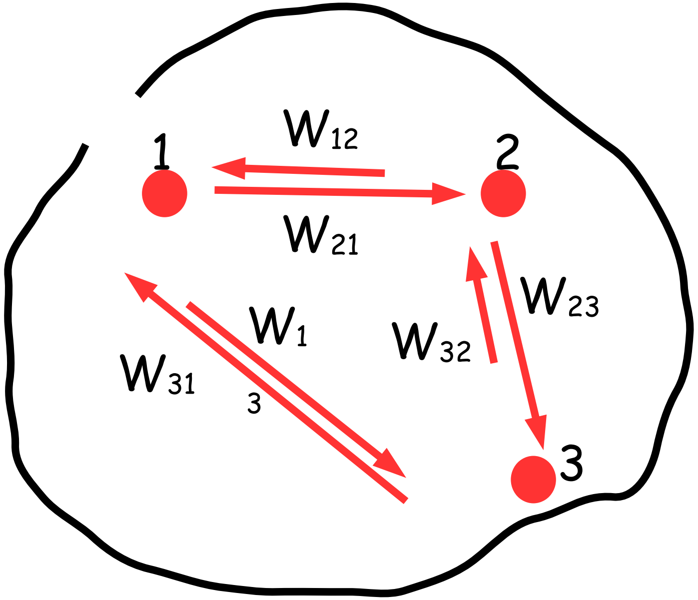
$$\mathbf{\Gamma}_{ij} \cdot \dot{\mathbf{X}}_i = -\frac{\partial U}{\partial \mathbf{X}_j}$$ + thermal fluctuations

$$U \text{ symmetric in } \mathbf{X}_i - \mathbf{X}_j$$

$$\text{force on } i \text{ due to } j = -\text{force on } j \text{ due to } i$$

Not suitable for directed one-way transmission of information

Active: sustained energy input: time-rev broken



detailed balance broken
currents in steady state

$$\frac{W_{12}}{W_{21}}\frac{W_{23}}{W_{32}}\frac{W_{31}}{W_{13}} \neq 1$$

Resulting dynamics generically non-reciprocal

Fruchart, Hanai, Littlewood, Vitelli
Nature 2021

$$\mathbf{\Gamma}_{ij} \cdot \dot{\mathbf{X}}_i = \sum_j \mathbf{F}_{ij} + \mathbf{f}_i$$

(noise)

$$\mathbf{F}_{ij} \neq -\mathbf{F}_{ji} \text{ in general}$$

Bowick, Marchetti, Fakhri, SR
PRX 2022

$$\text{also consider } \mathbf{\Gamma}_{ij} \neq \mathbf{\Gamma}_{ji}$$

directed one-way transmission of information possible

Non-reciprocality requires identity (species, distinguished locations, internal coordinate)

Early example: Das, Rao, SR EPL **60** 418 (2002); cond-mat:0404071 (Heisenberg magnet)

FLOCKING WITHOUT MOVING

orientation unit vector $\mathbf{p}_i$

turning inertia χ

classical "spin" angular momentum $\mathbf{s}_i$

interactions J_{ij}

Dadhichi et al. PRE 2020

$$\dot{\mathbf{p}}_i = \frac{\mathbf{s}_i}{\chi} \times \mathbf{p}_i$$

damping

aligning torque

$$\dot{\mathbf{s}}_i + \frac{\eta}{\chi}\mathbf{s}_i = \mathbf{p}_i \times \sum_j J_{ij}\mathbf{p}_j$$

$J_{ij} \neq J_{ji}$ if i in front of j; "front" defined by $\mathbf{p}_i$

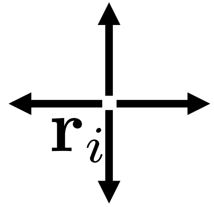
Torque not derived from potential energy

Antisymmetric part of J_{ij} breaks time-reversal

vision cone

Non-reciprocal XY rotors on a lattice

$$\dot{\mathbf{p}}_i = \frac{\mathbf{s}_i}{\chi} \times \mathbf{p}_i \qquad \dot{\mathbf{s}}_i + \frac{\eta}{\chi}\mathbf{s}_i = \mathbf{p}_i \times \sum_j J_{ij}\mathbf{p}_j$$

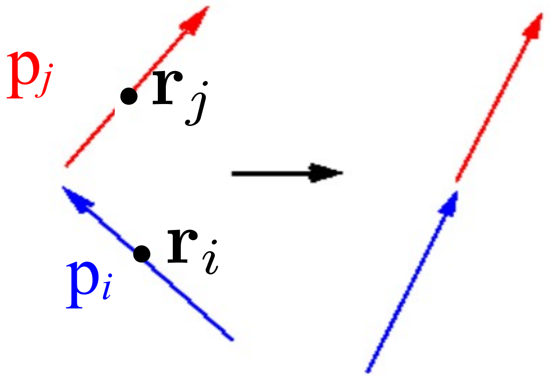


$$J_{ij} = J + \mathcal{A}\mathbf{p}_i \cdot \hat{\mathbf{r}}_{ij},\ j \in \text{nbhd of } i$$

$$\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i = \pm\hat{\mathbf{e}}_1,\ \pm\hat{\mathbf{e}}_2$$

$$J_- = J - \mathcal{A}$$

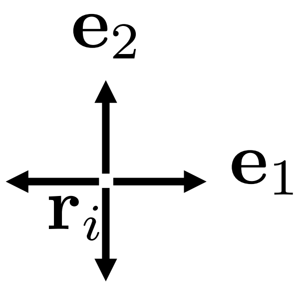
$$J_+ = J + \mathcal{A}$$



Order parameter contracted with lattice directions. Rotation symmetry now only discrete?

Non-reciprocal XY rotors on a lattice

$$\text{torque} = \mathbf{p}_i \times \sum_j J_{ij}\mathbf{p}_j \qquad J_{ij} = J + \mathcal{A}\mathbf{p}_i \cdot \mathbf{r}_{ij}$$



$$\mathbf{r}_{ij} = \mathbf{r}_j - \mathbf{r}_i = \pm\mathbf{e}_1,\ \pm\mathbf{e}_2$$

$$\mathbf{p}_j = \mathbf{p}(\mathbf{r}_i + \mathbf{e}_\alpha) \simeq \mathbf{p}_i + \mathbf{e}_\alpha \cdot \nabla\mathbf{p}_i + \ldots$$

$$\mathcal{A}\sum_j(\mathbf{p}_i \cdot \mathbf{r}_{ij})(\mathbf{p}_i \times \mathbf{p}_j) = \mathcal{A}\sum_\alpha(\mathbf{p}_i \cdot \mathbf{e}_\alpha)\mathbf{p}_i \times (\mathbf{p}_i + \mathbf{e}_\alpha \cdot \nabla\mathbf{p}_i + \ldots)$$

$$\to \mathcal{A}\sum_\alpha \mathbf{p} \cdot (\mathbf{e}_\alpha\mathbf{e}_\alpha) \cdot \nabla\mathbf{p} \times \mathbf{p} = \mathcal{A}(\mathbf{p} \cdot \nabla\mathbf{p}) \times \mathbf{p} + \ldots$$

Rotation-invariant! no lattice details upto Laplacian order

Continuum theory

ignore density, ignore motion

$$\partial_t \mathbf{p} = \frac{1}{\chi}\mathbf{s} \times \mathbf{p}; \;\; \mathbf{p} \cdot \mathbf{p} = 1$$

$$\partial_t \mathbf{s} = \mathcal{A}\mathbf{p} \times (\mathbf{p} \cdot \nabla \mathbf{p}) + J\mathbf{p} \times \nabla^2 \mathbf{p} - \frac{\eta}{\chi}\mathbf{s}$$

A = 0: Cavagna et al JSP2015, PRL2015; Attanasi et al. Nat Phys 2014; Yang & Marchetti PRL2015

Chaikin-Lubensky ch. 8 – rotor lattice

nonsymmetric Euclidean random Heisenberg model: Cavagna et al. PRL2017

Long-wavelength continuum theory

eliminate fast $\mathbf{s}$

$$\partial_t \mathbf{p} = \frac{1}{\chi} \mathbf{s} \times \mathbf{p}; \;\; \mathbf{p} \cdot \mathbf{p} = 1$$

$$\cancel{\partial_t} \mathbf{s} = \mathcal{A} \mathbf{p} \times (\mathbf{p} \cdot \nabla \mathbf{p}) + J \mathbf{p} \times \nabla^2 \mathbf{p} - \frac{\eta}{\chi} \mathbf{s}$$

$$\implies \partial_t \mathbf{p} = \frac{\mathsf{I} - \mathbf{pp}}{\eta} \cdot \left(J \nabla^2 \mathbf{p} + \mathcal{A} \underline{\mathbf{p} \cdot \nabla \mathbf{p}} \right)$$

$\mathbf{p}$ advects itself

Add noise: fixed-length version of “Malthusian” Toner-Tu

Self-advection: long-range order in 2D low-noise phase

Also: turning instability of ordered phase

Dadhichi, Kethapelli, Chajwa, SR, Maitra Phys Rev E **101** 052601 (2020)

Non-reciprocality drives bend instability

Eliminate s systematically: expand in powers of $1/\eta$

Perturb about uniformly ordered steady state

Ordering direction = || Transverse directions = $\perp$

$$\mathbf{p} \simeq (1, \delta p_\perp)$$

$$\partial_t \delta p_\perp - \frac{\mathcal{A}}{\eta} \partial_{||} \delta p_\perp = \frac{J}{\eta} \left(\nabla^2 - \frac{\chi \mathcal{A}^2}{J \eta^2} \partial_{\|}^2 \right) \delta p_\perp$$

Instability criterion $\mathcal{A}/\eta > \sqrt{J/\chi}$

Long-wavelength kinematic wave speed > short-wavelength turning wave speed

Turning-wave regime

$$\omega \simeq \pm\sqrt{\frac{J}{\chi}}k_{||} - \frac{i}{2}\left(\frac{\eta}{\chi} \pm \frac{\mathcal{A}}{\sqrt{\chi J}}\right)$$

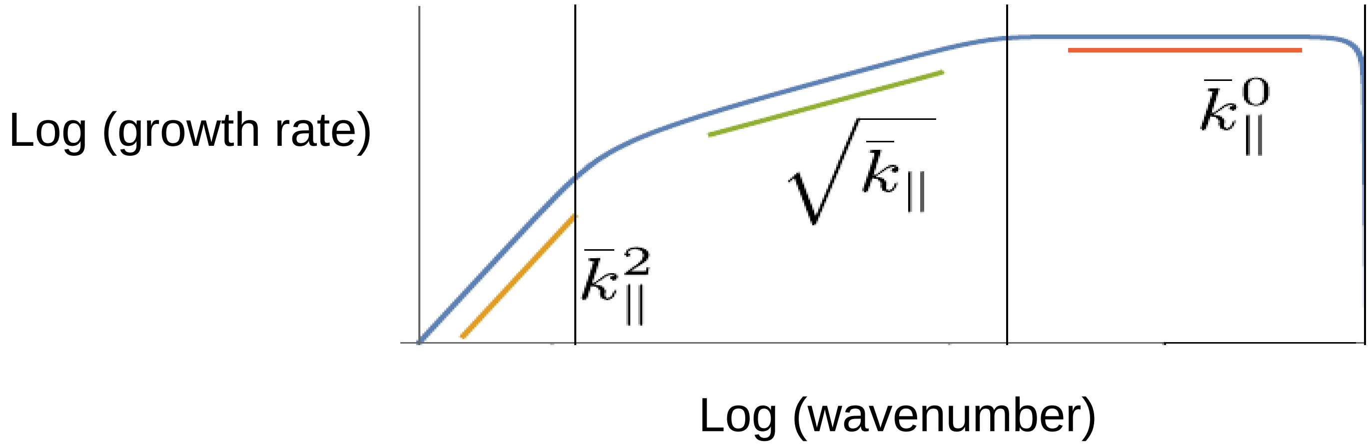
Asymmetry threshold for instability

$$\mathcal{A}_c \equiv \eta\sqrt{J/\chi}$$

$$\mathcal{A} > 0$$

Bird sees forward neighbour better
Unstable disturbance travels to rear
Makes sense

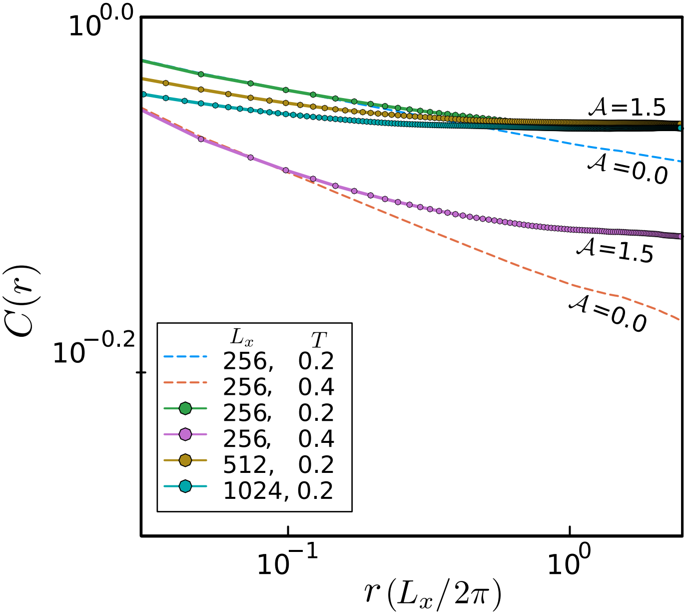
Instability regimes



Numerical studies: complex ordered turbulent states (Pankaj Popli, Ananyo Maitra)

Non-reciprocity --> long-range 2D XY order

Pankaj Popli



But (Besse, Chaté, Solon, PRL 2022)
this order is metastable:
explosion of asters + shock lines
Can we understand this?

$$\implies \partial_t \mathbf{p} = \frac{\mathsf{I} - \mathbf{pp}}{\eta} \cdot \left(J\nabla^2\mathbf{p} + \mathcal{A}\mathbf{p}\cdot\nabla\mathbf{p}\right)$$

Ananyo Maitra

Defects screened on length scales $> J/\mathcal{A}$?

Analogy: motility screens elastic displacement

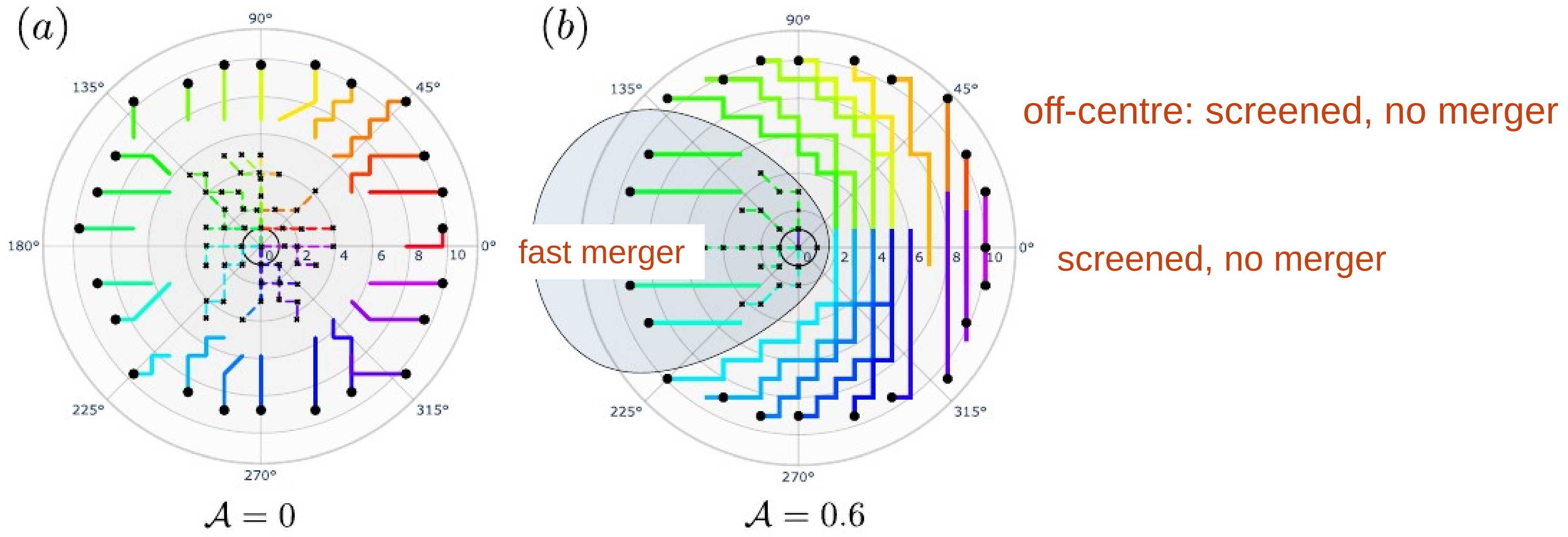
$$\partial_t \mathbf{p} = \frac{\mathsf{I} - \mathbf{pp}}{\eta} \cdot \left(J\nabla^2 \mathbf{p} + \mathcal{A}\mathbf{p} \cdot \nabla \mathbf{p}\right)$$

$\text{Defects screened on length scales } > J/\mathcal{A}?$ Ananyo Maitra

Recall Oseen modification of Stokes solution or my 1st lecture

$$\zeta \partial_t \mathbf{u} = -\delta F/\delta \mathbf{u} + f\mathbf{n}(\mathbf{t})\delta(\mathbf{r} - \mathbf{R}(t))$$

$$[-\zeta v_0 \partial_x - (\mu\nabla^2 + \lambda\nabla\nabla\cdot)]\mathbf{U} = f\delta(\mathbf{r})\hat{\mathbf{x}}$$

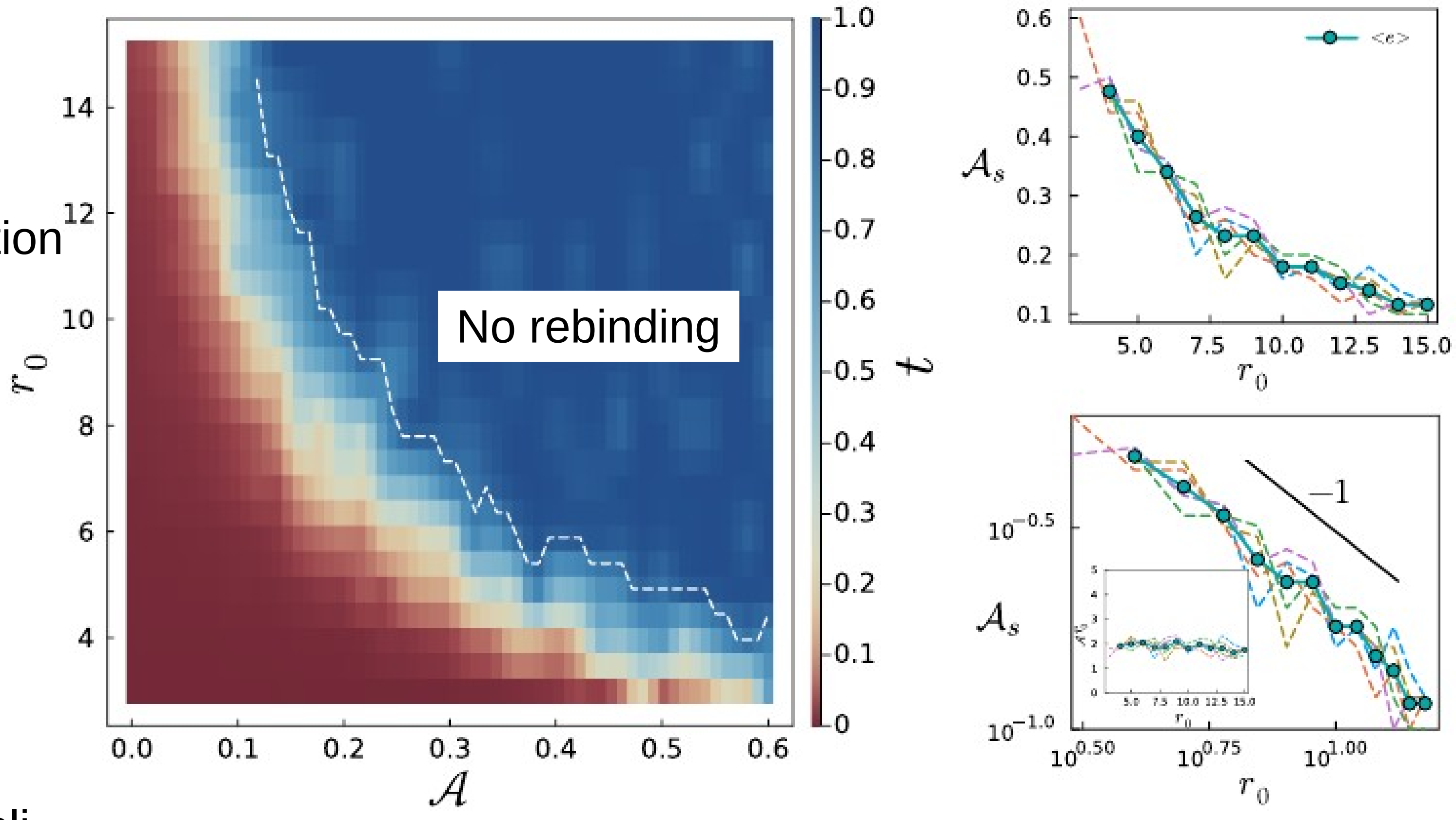


Pankaj Popli

FIG. 6. Defect trajectories for -1 (solid lines) and $+1$ (dashed lines) charges. **a)** XY model i.e $\mathcal{A} = 0$, both the charge interact symmetrically and annihilate with each other. **b)** In presence of nonreciprocity $\mathcal{A} = 0.6$ the saddles remains screened if initially positioned outside the shaded cone but merge otherwise. At this value of nonreciprocity, screening length is order of lattice spacing.

Non-reciprocity prevents defect annihilation

initial separation



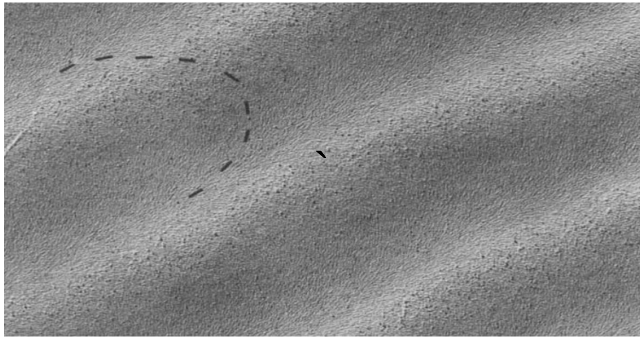
non-reciprocality

Pankaj Popli

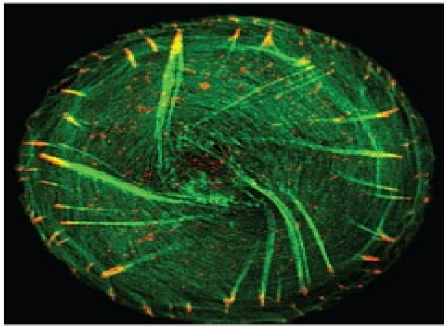
Summary: non-reciprocal XY

- Lattice XY model with 2D long-range order
- Directional flow of information
- Advection without motion
- Non-reciprocity cloaks defect interactions
  - the likely explanation for aster eruption, metastability (Besse et al 2022)

CHIRAL ACTIVE MATTER

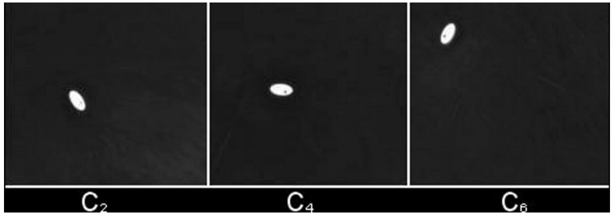


DNA cholesterics (Dinoflagellate),
Livolant et al. 1992



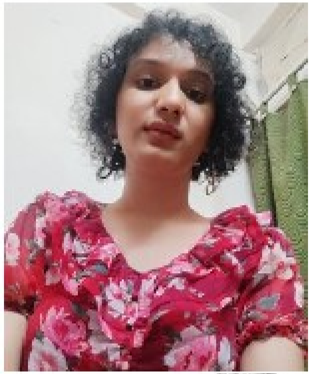
Cellular chirality Tee et al. 2015

Self-shearing cell layer Duclos et al. 2018



Arora, Sood, Ganapathy 2021

Chirality: not superposable on mirror image




SJ Kole, GP Alexander, SR, A Maitra, PRL 2021 and arXiv:2306.03695

2D chirality (needs up-down distinction in 3D)

https://upload.wikimedia.org/wikipedia/commons/9/95/Shaken.JPG

https://upload.wikimedia.org/wikipedia/commons/9/95/Shaken.JPG

https://en.wikipedia.org/wiki/Sinistral_and_dextral

Chiral active hydrodynamics

translationally ordered system: density-wave of any scalar/pseudoscalar ψ

$$\partial_t \psi + \nabla \cdot \mathbf{J} = 0 \qquad \mathbf{J} = \psi \mathbf{v} + \mathbf{J}_{passive} + \mathbf{J}_{active}$$

$$\rho(\partial_t + \mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \cdot \boldsymbol{\sigma}$$

Currents $\boldsymbol{J}$ solute transport

stresses $\boldsymbol{\sigma}$ momentum transport

Hohenberg & Halperin Rev Mod Phys 1977

Active model B* (Wittkowski ...., Andreev, Son, Spivak) chiral currents
Active model H* Kole et al. PRL **126** (2021) 248001 Chiral active stresses

Kole et al. PRL 2021
and arXiv:2306.03695

(Pseudo)scalar active hydrodynamics

translationally ordered phases

Neglect inertia   Solve for $\boldsymbol{v}$

$$\rho(\partial_t + \boldsymbol{v}\cdot\nabla)\boldsymbol{v} = -\nabla\Pi + \mu\nabla^2\boldsymbol{v} + \nabla\cdot(\boldsymbol{\Sigma}^a + \boldsymbol{\Sigma}^r)$$

$$\partial_t\psi + \nabla\cdot(\psi\boldsymbol{v}) = \Lambda\nabla^2\delta F/\delta\psi + \nabla\cdot(\sqrt{2k_BT\Lambda}\mathbf{f})$$

+ time-rev-breaking (“active”) currents

$$\psi: \text{ scalar/pseudoscalar}$$

pseudoscalar: changes sign under space inversion

choose $F[\psi]$ to favour density wave

build $\boldsymbol{\Sigma}^a$, $\boldsymbol{\Sigma}^r$, in terms of ψ

1D: layers
2D: columns
3D: unit cells

Achiral, active, layered

translationally ordered system: density-wave of any scalar/~~pseudoscalar~~ ψ

$$\partial_t \psi + \nabla \cdot \mathbf{J} = 0 \qquad \mathbf{J} = \psi \mathbf{v} + \mathbf{J}_{passive} + \mathbf{J}_{active}$$

$$\rho(\partial_t + \mathbf{v} \cdot \nabla)\mathbf{v} = -\nabla \cdot \boldsymbol{\sigma}$$

achiral active stress $\zeta \Delta\mu \nabla\psi\nabla\psi$

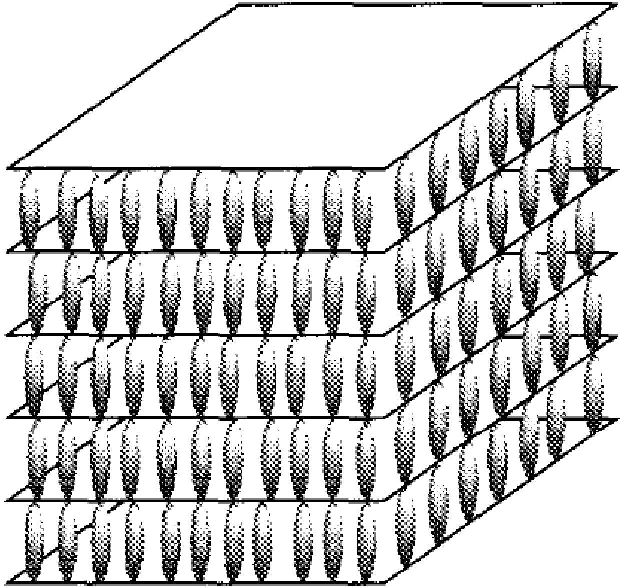
Active B & H
Wittkowski et al.
NComms **5** (2014) 4351
Tiribocchi et al.
PRL **115** (2015) 188302

$\zeta\Delta\mu =$ chemomechanical coupling $\times$ chemical driving

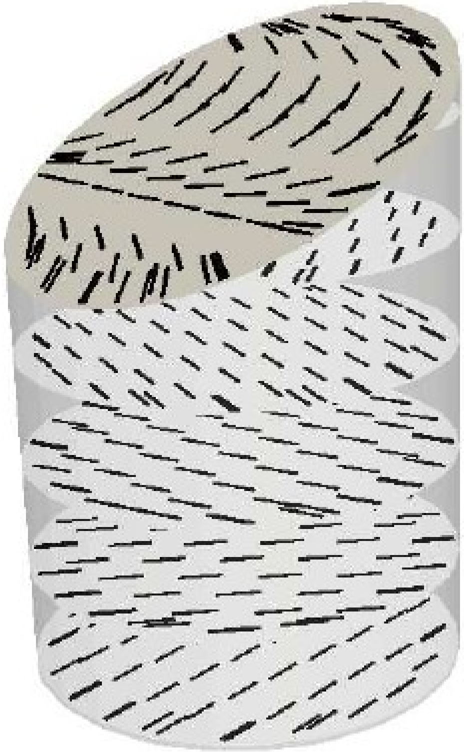
$\psi = Ae^{iq_0(z-u)}$ $\quad u =$ displacement field $\quad q_0 =$ ordering wavenumber along z

Two kinds of 1D periodic matter

Smectic liquid crystal: density wave
layers = density maxima, achiral
Chaikin & Lubensky
Principles of Condensed Matter Physics



Cholesteric liquid crystal
orientation wave
no true layers, chiral



in equilibrium:
no hydrodynamic distinction.
Lubensky 1972, Radzihovsky & Lubensky 2011

If active?

Adhyapak, SR, Toner 2012; Whitfield, Adhyapak, ... 2017
SJ Kole, GP Alexander, SR, A Maitra, PRL 2021

Spontaneous undulation instability: achiral

Smectics: Adhyapak et al. PRL 2013; cholesterics: Whitfield et al. EPJE 2017

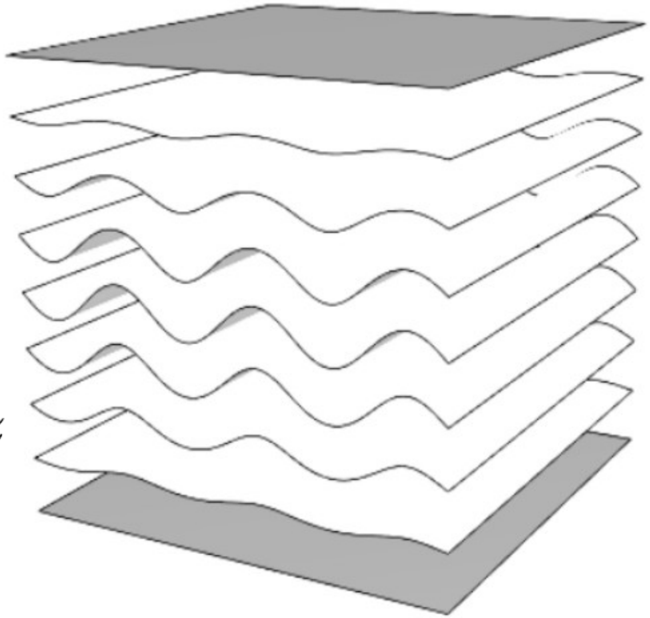
Fluid layers + active stresses normal to layers
ζ<0: pull in, equiv to system under tension
Layers buckle to maintain spacing

$$\mathbf{\Sigma}^a = \zeta \Delta\mu \nabla\psi\nabla\psi$$

$$\psi = Ae^{iq_0(z-u)}$$

$q_0 =$ ordering wavenumber along z

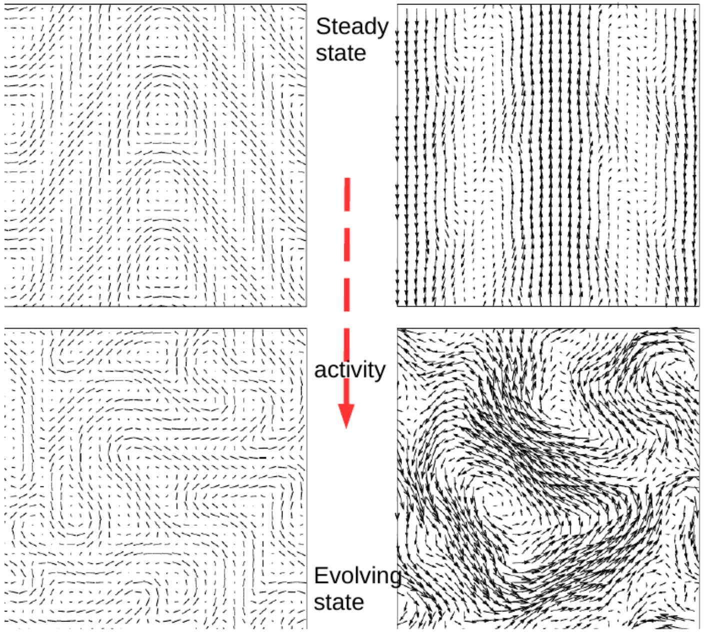
$u =$ displacement field



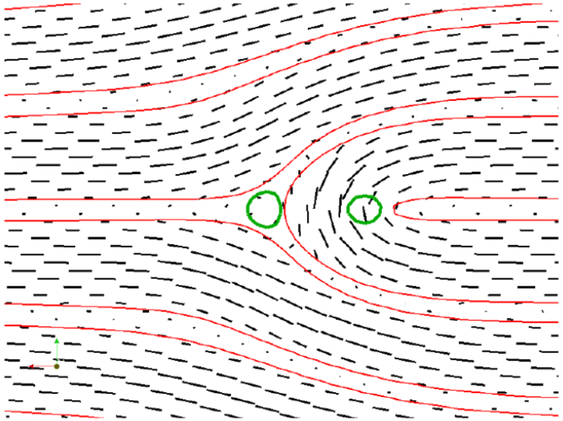
Active force density

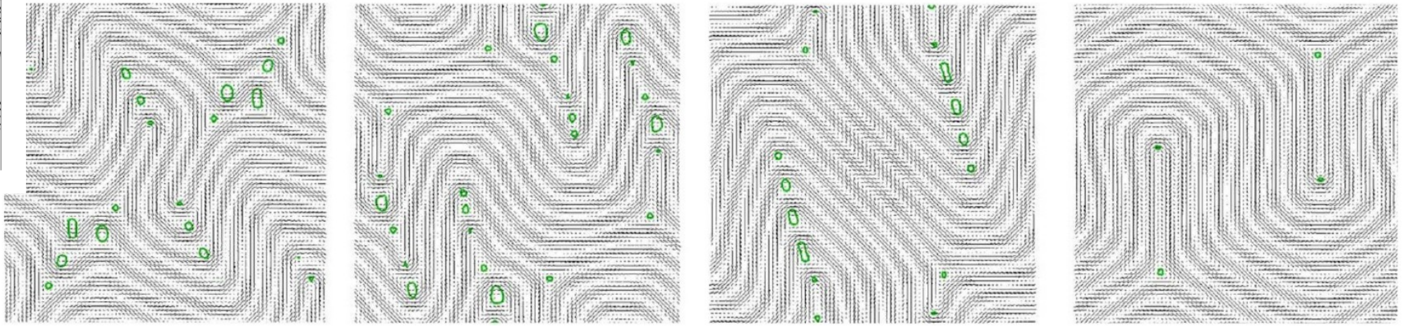
$-\zeta\Delta\mu\nabla^2 u\hat{z}$

Achiral instabilities of active cholesterics



director velocity





Patterns of lambda lines

Cholesteric + achiral active stresses Whitfield, Adhyapak, Tiribocchi, Alexander, Marenduzzo, SR EPJE 2017

Building chiral active stresses

chiral translationally ordered system: density-wave of any pseudoscalar ψ

$$\psi = Ae^{iq_0(z-u)}$$

$q_0 =$ ordering wavenumber along z

$u =$ displacement field

achiral active stress $\zeta\nabla\psi\nabla\psi$

3D chiral active stress

$$\sigma_{ij}^{\chi} = z_{\chi}\epsilon_{ijk}\nabla_l(\nabla_j\psi\nabla_k\psi)$$

$$f_i = \epsilon_{ij}\nabla_j\nabla^2 u$$

$$\epsilon_{ij} = \epsilon_{ijz}$$

Active chiral force density tangent to contours of constant mean curvature

2D chiral active stress

$$\boldsymbol{\sigma}_{\chi} = \zeta_c\boldsymbol{\varepsilon}\cdot\nabla\psi\nabla\psi$$

Film of 3D chiral + distinguished normal

Shear strain from dilation

Chiral + active: non-reciprocal stresses

3D

$$\text{Stress}\, \sigma_{ij} = \zeta w_{ij} + z_\chi \epsilon_{ijk} \nabla_l w_{kl}$$

$$w_{z\perp} = w_{\perp z} = \nabla_\perp u;\; w_{zz} = 2\partial_z u$$

“elastic” force in direction with no displacement field
Even when strain = 0

2D

$$\sigma_{ij} = \zeta W_{ij} + z_c \epsilon_{ik} W_{kj}$$

$$W_{zz} = -W_{xx} = \partial_z u$$

$$W_{xz} = -W_{zx} = \partial_x u$$

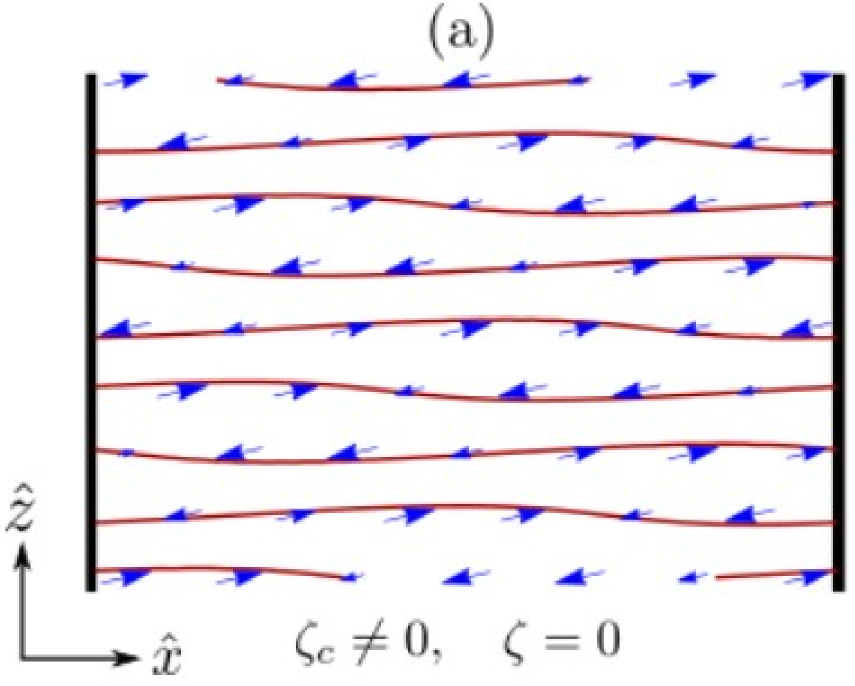
Shear strain from dilation

Odder than breaking of Maxwell-Betti reciprocity (Scheibner et al. 2020, Braverman et al. 2020)

$$\sigma_{ij} = C_{ijkl} U_{kl}$$

$$C_{ijkl} \neq C_{klij}$$

2D chirality + stripes + activity: instabilities



Can understand as layers of perpetually rotating wheels

Final state: spirals? Ordered? Turbulent?

Spontaneous vortex lattice

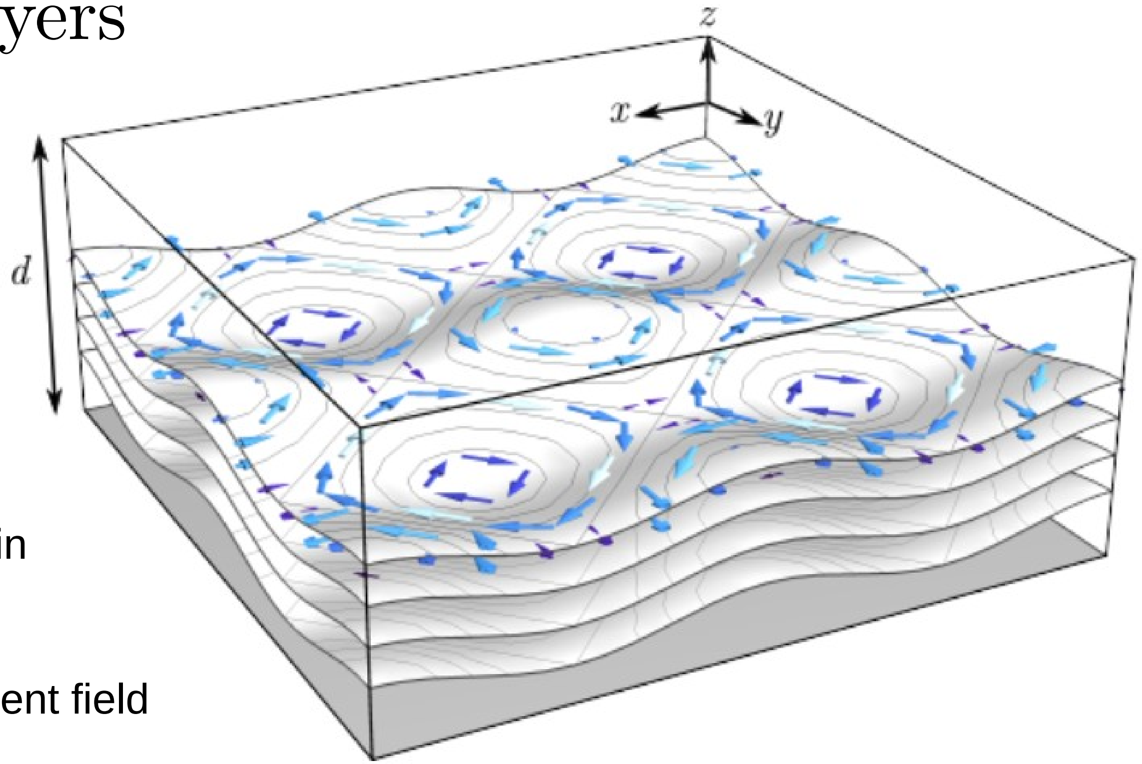
SJ Kole, GP Alexander, SR, A Maitra, PRL **126**, 248001 (2021)

$$u = \text{displacement field of layers}$$

Spontaneous undulations and

$$f_i = \epsilon_{ij} \nabla_j \nabla^2 u$$

Active chiral force tangent to contours of constant mean curvature



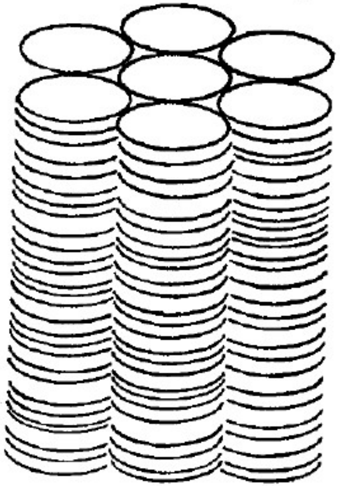
Control undulations by *imposed* uniaxial strain
On-off switch for flow- vortex lattice!

“Elastic” force in direction with no displacement field
Highly broken reciprocity

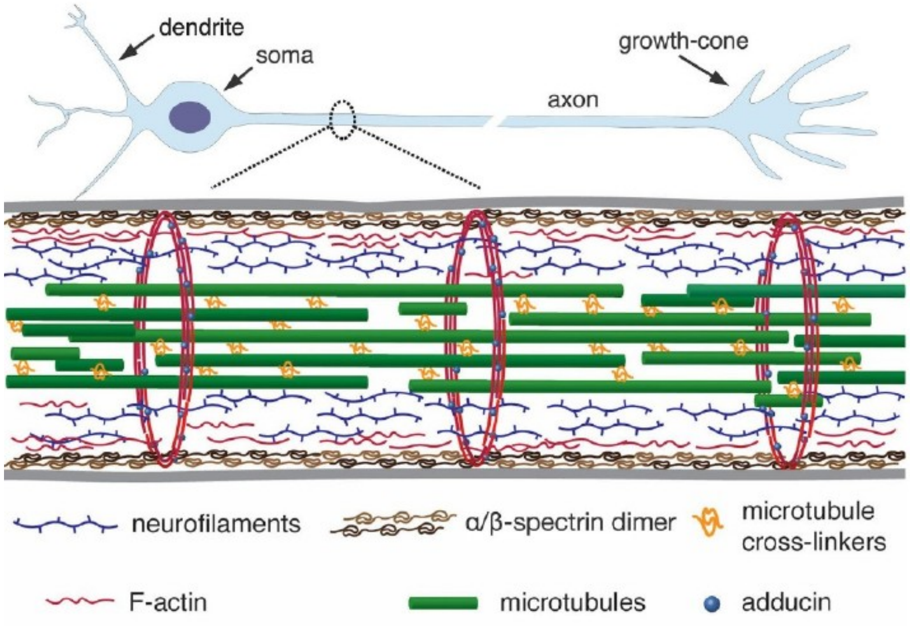
Doesn’t destroy structure; contrast with typical active-stress effect

2D translational order in 3D: columns

Richer than lamellar
column direction + recip lattice



Chandrasekhar, Sadashiva, Suresh
Pramāṇa 1977



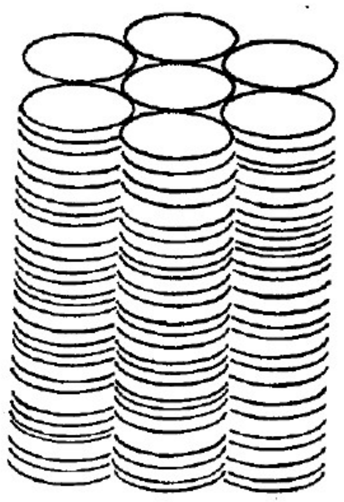
Dubey et al. eLife 2020

Dynamics of Ordered Active Columns: Flows, Twists, and Waves
SJ Kole, A Maitra, G P Alexander, SR
arXiv:2306.03695

J Prost

2D translational order in 3D: columns

Richer than lamellar
column direction + recip lattice



3D Active model H
+ free energy favouring 2D order

Expand about ordered phase

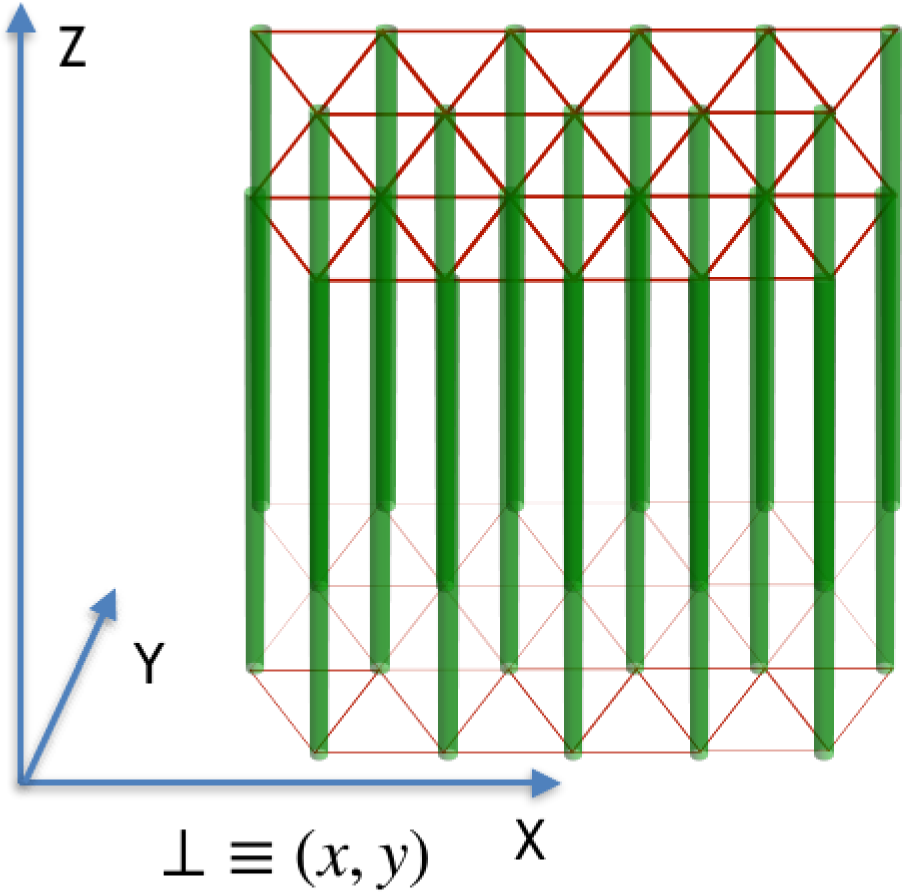
Chandrasekhar, Sadashiva, Suresh
Pramāṇa 1977

Dynamics of Ordered Active Columns: Flows, Twists, and Waves
SJ Kole, A Maitra, G P Alexander, SR
arXiv:2306.03695

Active chiral columnar phases

2d displacement field $\mathbf{u}_\perp(x, y, z)$

Strain: $E_{ij} = \partial_i u_j + \partial_j u_i$ (linearised)
$(i, j \in \{x, y\})$



Active columnar phases: achiral stresses

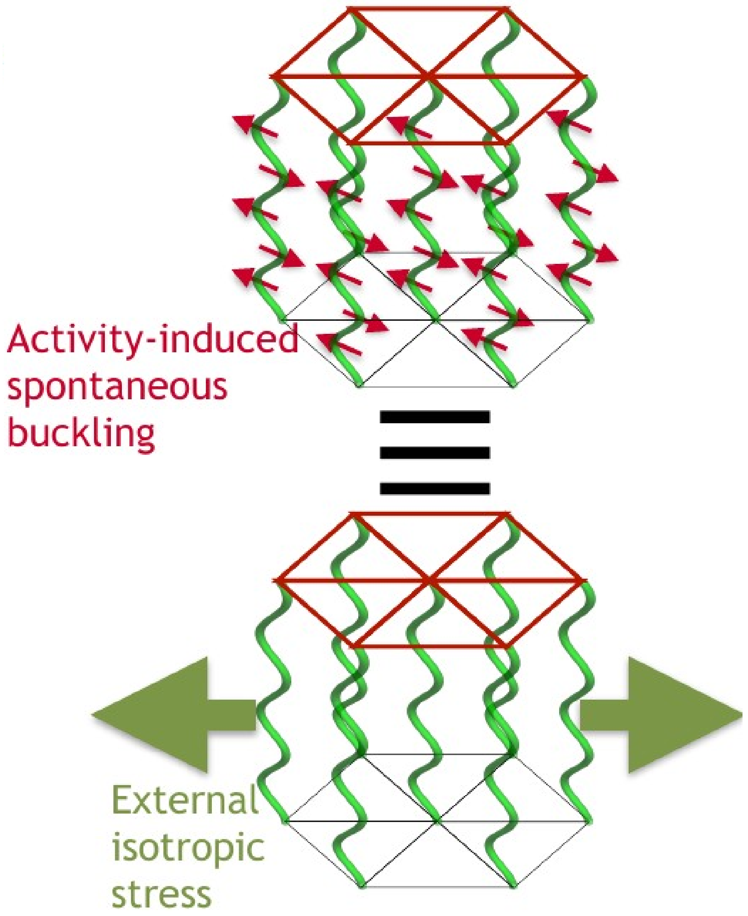
$$\mathbf{f}^a = \zeta \Delta\mu \nabla^2 \mathbf{u}_\perp$$

$\zeta > 0$ active stabilisation of columns

$\zeta < 0$ Instability

Exact mapping to Helfrich-Hurault Instability under isotropic stress.

Achiral active stress ≡ External stress



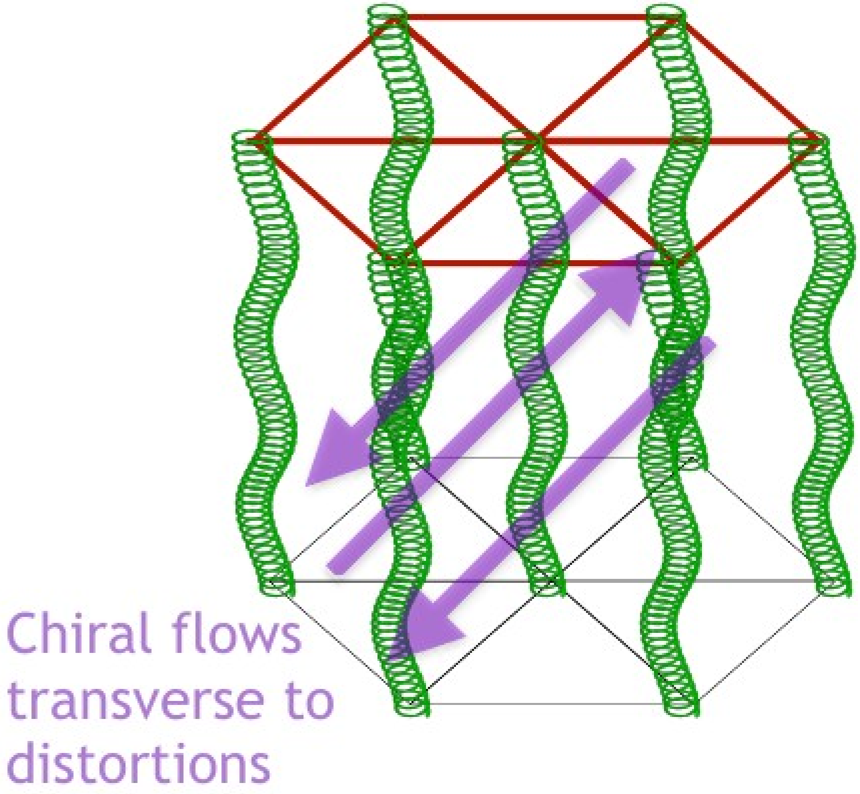
Active columnar phase: chiral force density

$$\zeta_c \Delta\mu \nabla \times \nabla^2 \mathbf{u}_\perp$$

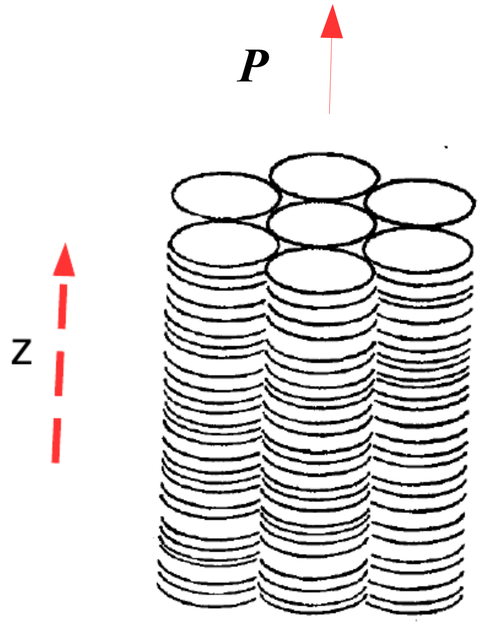
APOLAR

Subleading correction to growth-rate
of spontaneous Helfrich-Hurault instability
or column tension.

Flows leading to column twist



Active columnar phase: chiral polar force density



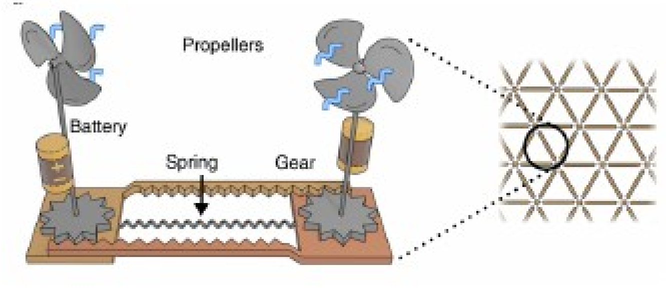
$$\zeta_{pc}\Delta\mu\nabla_{\perp}^{2}\boldsymbol{\epsilon}\cdot\mathbf{u}_{\perp}$$

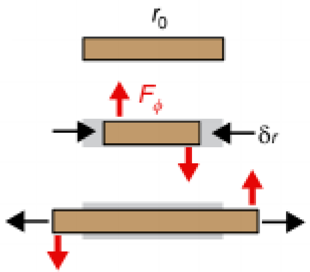
Balance against viscous force:
wave*number* drops out

$$\mu\nabla^{2}\mathbf{v}$$

2d odd elasticity in a 3d material!

cf. Scheibner et al. 2020: motorised odd bonds

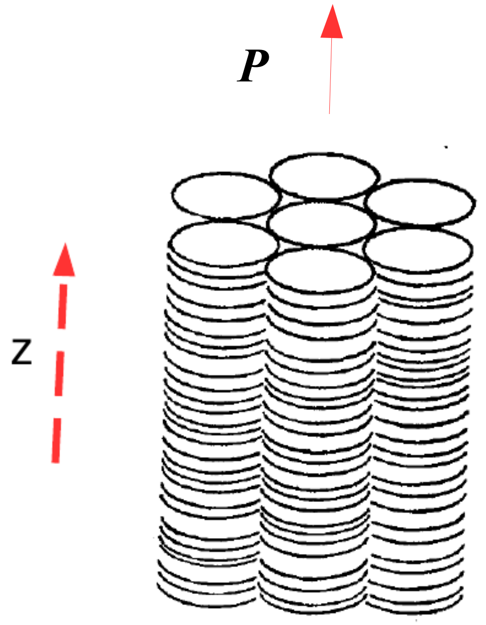




Need polarisation vector $\boldsymbol{P}$ in addition to pseudoscalar ψ

Active columnar phase: chiral polar force density

$$\mu \dot{\mathbf{u}}_\perp = \zeta_{pc} \Delta\mu \nabla_\perp^2 \boldsymbol{\epsilon} \cdot \mathbf{u}_\perp$$



Plasmon-like oscillation in Stokes fluid

$\omega_\pm \sim c_\pm(\theta) - iD_\pm(\theta)$

$c_\pm(\theta), D_\pm(\theta)$ functions of wavevector directions, activity and elasticity.

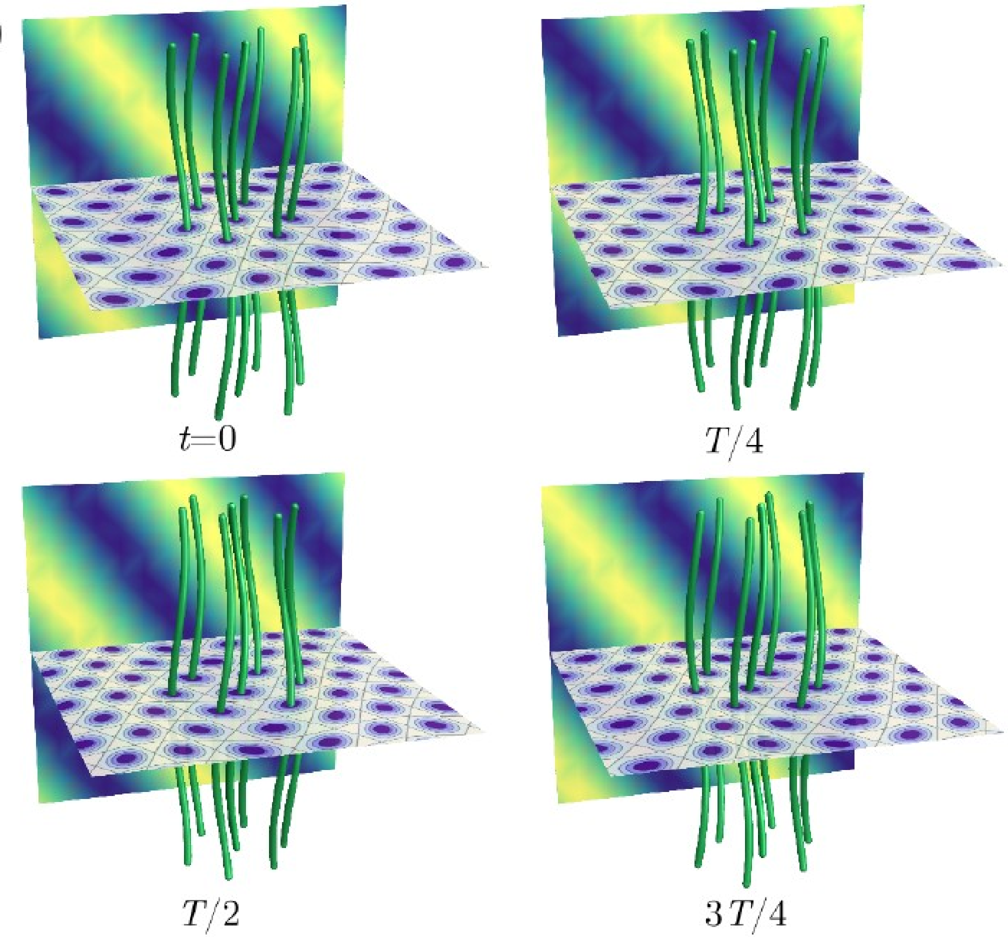
Crucially, not of wavenumber!

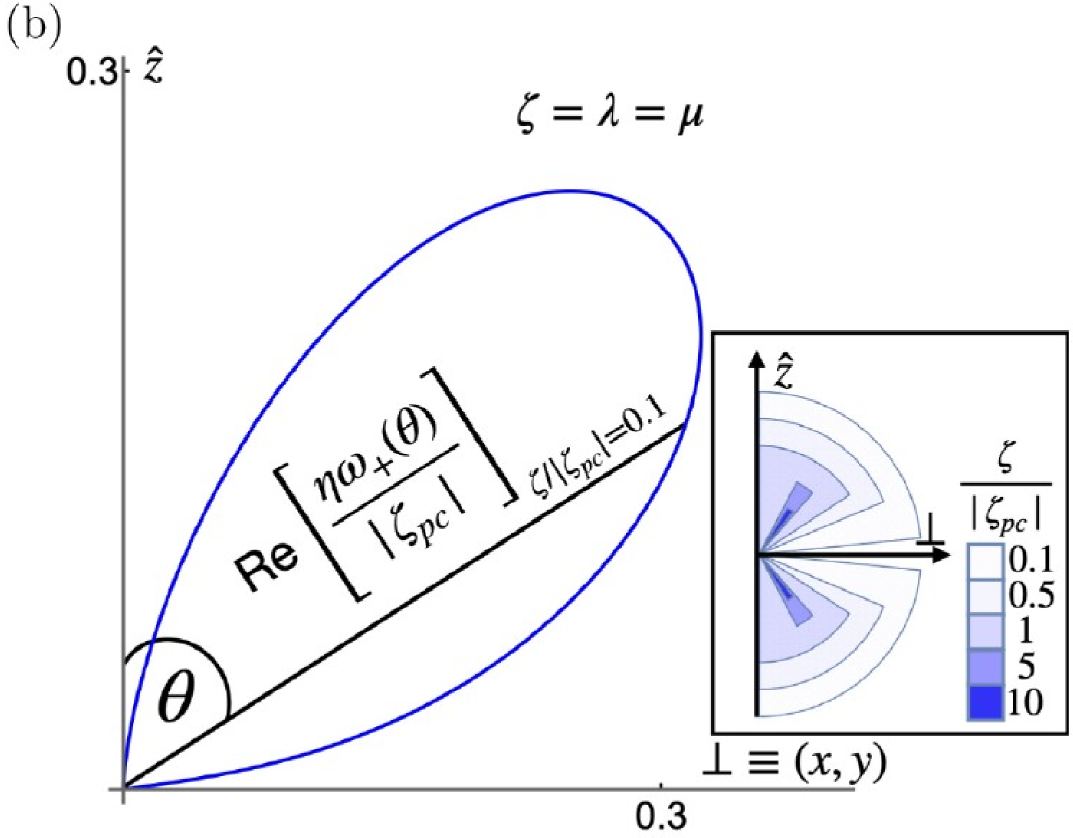
3d character important:
Oscillation *suppressed* for perturbations purely in the x-y plane or along z

$\rightarrow$ No oscillation in 2d odd gels

Displacement fields beating against each other like position and momentum

Active, polar, chiral: plasmon-like oscillations





Odder still

Chirality + Polarity + Activity
→ Three-dimensional, polar
odd viscosity

$$\mathbf{f}^{ov}_{\text{lin}} = \eta_{o1}\Delta\mu\,\nabla^2_\perp \epsilon \cdot \mathbf{v}_\perp + \eta_{o2}\Delta\mu\,\partial_z[(\nabla \mathbf{v}_\perp)^S_{yz}, -(\nabla \mathbf{v})^S_{xz}, \nabla_\perp \times \mathbf{v}_\perp]$$

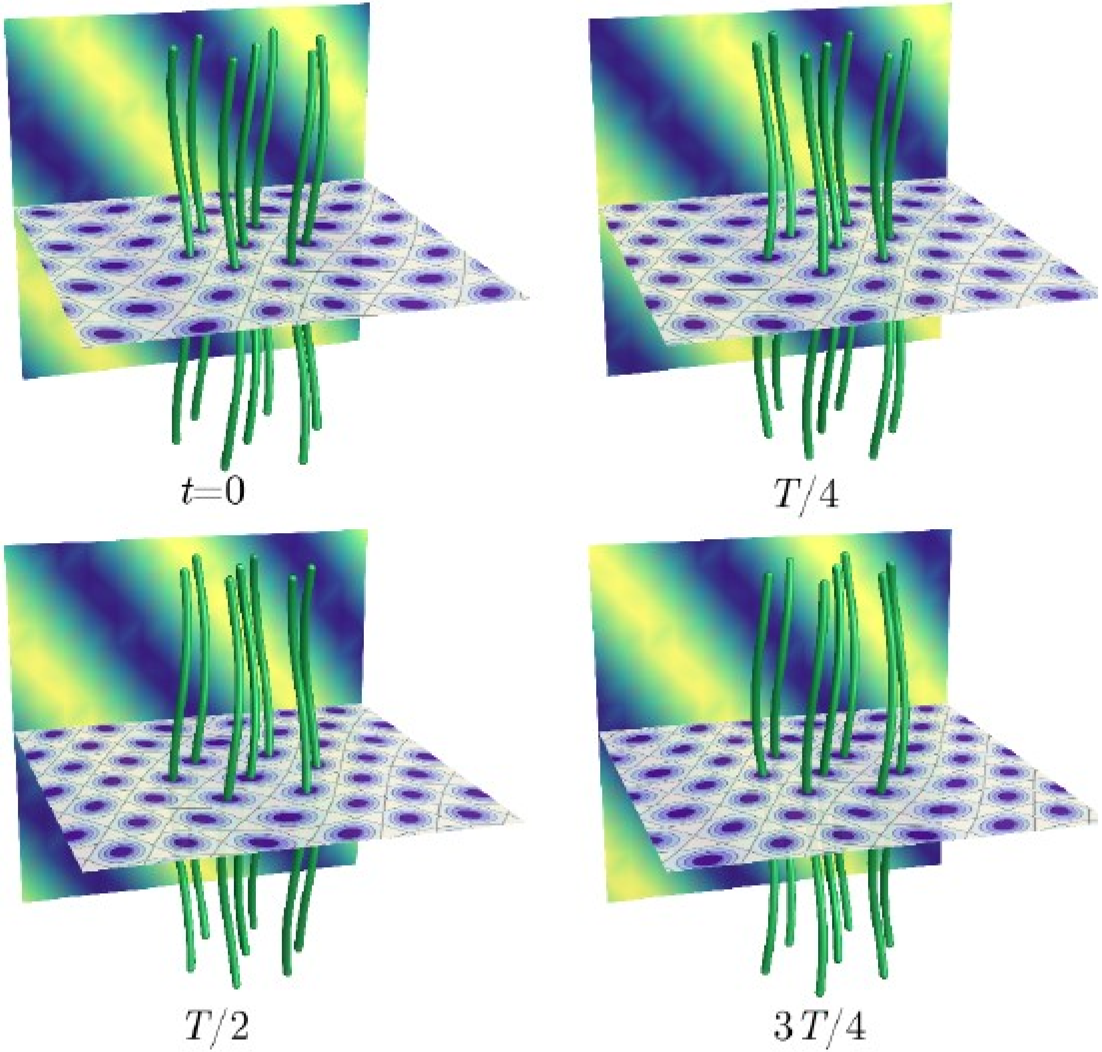
Odd viscosity+regular (even) elasticity
→ Oscillations from displacements beating against each other

Oscillations:

Even elasticity

$$\dot{u}_i = M_{ij} F_j$$

Odd mobility



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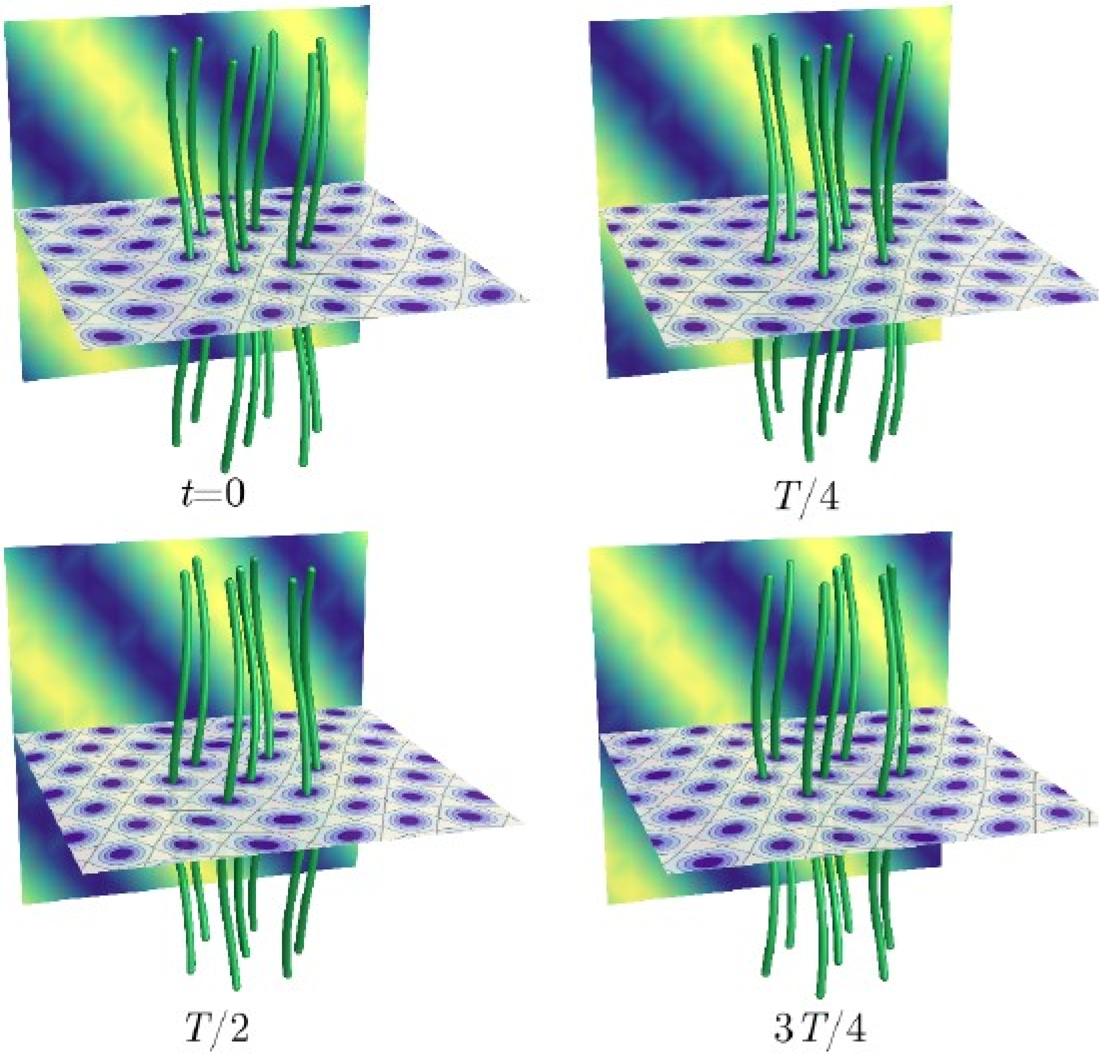
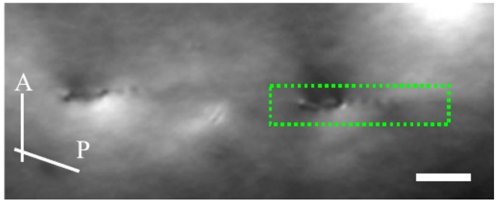
Odd viscosity+regular (even) elasticity
→ Oscillations from displacements beating against each other

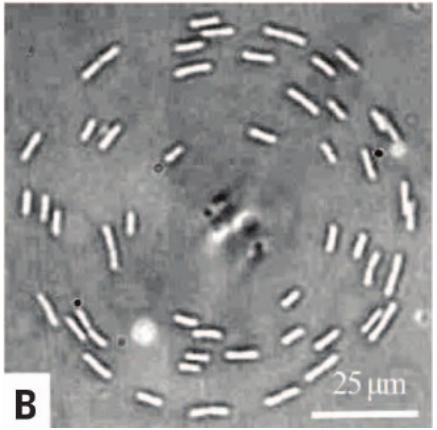
3d odd viscosity $\propto \eta_{o2}$
→ Oscillations even for perturbations purely along $\hat{z}$

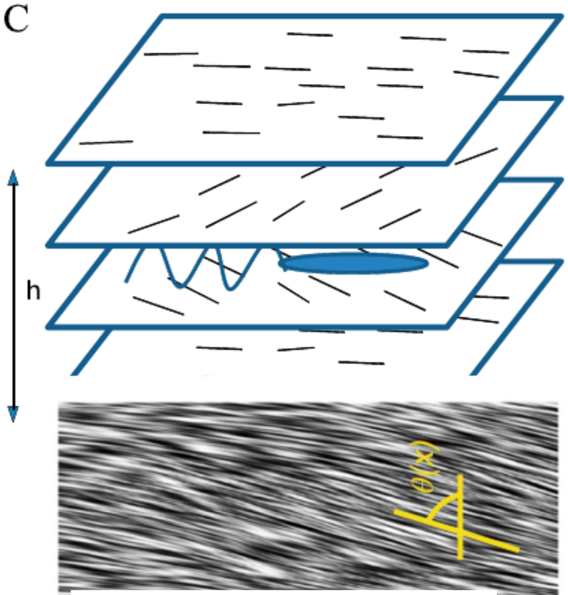


Possible realisations

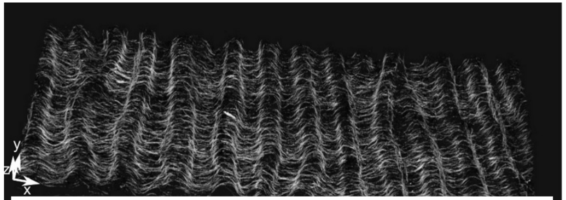


Bacteria in a biocompatible liquid crystal
Zhou et al., PNAS **111**, 1265 (2014)
Peng et al., Science **354**, 882 (2016)

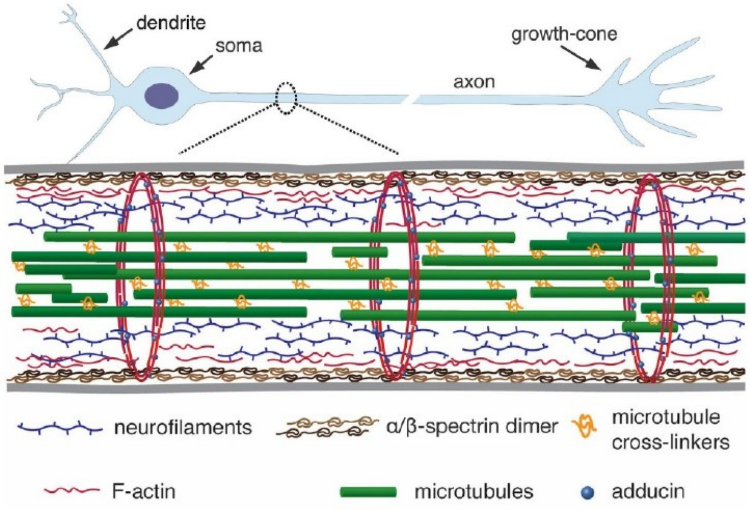




epithelium (Duclos et al. 2018)



MT-kinesin sheet (Senoussi et al. 2019)



Dubey et al. eLife 2020

J Prost

Active chiral matter: summary

- Chirality in passive matter
  - makes amazing structures, hides away from mechanics
- Chirality + activity
  - breaks symmetries unbreakable in equilibrium matter
- Activity + spatial asymmetries
  - greater freedom to transform dynamics

SUMMARY

- Non-reciprocal XY: flocking without moving
- NR --> cloaking --> aster eruption, metastability
- Very odd mechanics from NR in chiral matter

OUTLOOK

- Predictive hydrodynamics for powered matter
  - instability, non-reciprocity, chiral effects, sensing?
- When does small-scale broken T affect large scales*?
- New principles for slow variables in living matter**?
- Relation with driven quantum matter?

"..... headful of ideas that's drivin' me insane ..." -- Bob Dylan

*Relation to classic pattern-formation questions; role of noise

**Self-adjust to critical point? Other ideas?

Bowick et al. PRX 2022