

The allure of active matter

Sriram Ramaswamy

Centre for Condensed Matter Theory
Department of Physics
Indian Institute of Science
Bangalore



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INFOSYS - ICTS
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LECTURE SERIES

Subrahmanyan Chandrasekhar Lecture Series are delivered by eminent physicists on important new developments in their areas of speciality. The first lecture in any series is aimed at a general scientific audience, while the remaining are targeted at specialists.



Outline

- **Lecture 1: a self-contained tour**
 - early results & link to the latest
- **Lecture 2: active dynamics in a fluid**
 - and a connection to sedimentation
- **Lecture 3: non-reciprocal dynamics**
 - flocking without moving, odd mechanics, oscillation without inertia

Lecture 1

- **Introduction**
 - origins & prehistory; fundamentals
- **Dry active matter**
 - active nematics; motility in granular layers; chemotactic colloids
- **Wet active matter**
 - rheology, instability, active turbulence
- **Summary**

Lecture 2

- **Active fluids with inertia**
 - outswimming the instability
- **Many-particle sedimentation**
 - Kepler orbits; emergent elasticity
 - non-normal dynamics

Lecture 3

- **Non-reciprocal dynamics: fundamentals**
 - a world out of (detailed) balance
- **Flocking without moving**
 - order and defects; spontaneous waves on curved surfaces
- **Chiral active matter**
 - odd and odder elasticity
- **Summary and prospect**



Nisarg Bhat
(with Subroto Mukerjee)



Raushan Kant (with AK Sood)

Group

Ramita Mondal
(UG, w/ M Barma)



Karnpriya Pandey
(with Ambarish Ghosh)



Pankaj Popli
IoE postdoc



Narayan Dutt Sharma
(with C Dasgupta & S Mukerjee)



Alumni



- Chittrak Bhowmik (MS IISER Bpr --> Brandeis)
- Rahul Gupta: Sankhya Sutra Technologies
- Lokrshi Prawar Dadhichi: postdoc, U Leipzig
- Rahul Chajwa (w/ Rama Govindarajan and Narayanan Menon): postdoc, Stanford
- Rayan Chatterjee (w/ Prasad Perlekar): postdoc, Stanford
- Kuipou William (visitor): Univ of Yaounde, Cameroon
- Sabiha Majumder (w/ V Guttal): ING Group, Risk Management, Amsterdam
- Ranjan Krishna Modak (w/ Subroto Mukerjee): IIT Tiru
- Nitin Kumar (w/ AK Sood): IIT Bombay
- Harsh Soni: IIT Mandi
- Ananyo Maitra: CNRS, Cergy Univ Paris
- Suropriya Saha: group leader, MPI-DS Goettingen
- Tapan Chandra Adhyapak: IISER Tirupati



SJ Kole
Cambridge

- Sebastian Fuerthauer (p): TU Wien
- Saroj Kumar Nandi: TIFR Hyderabad
- K Vijay Kumar: ICTS-TIFR Bangalore
- Shradha Mishra: IIT-BHU
- Sumithra Sankararaman (p): Washington U St Louis
- Norio Kikuchi (p): Ibasei Ltd, Ibaraki, Japan
- Vijay Narayan (w/ N Menon): Evonetix, Cambridge
- Moumita Das (w/ G Ananthakrishna): Rochester Inst Tech
- Ronojoy Adhikari: DAMTP, Cambridge

R Aditi Simha: IIT Madras

Rangan Lahiri: TIFR Mumbai

Sujan Kumar Dhar (w/ Rahul Pandit): Abexome Technologies

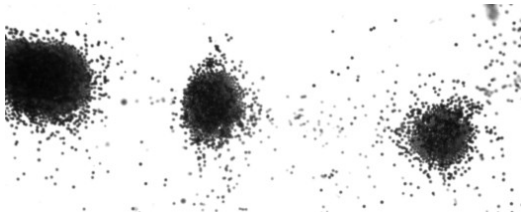
A V Indrani (w/ Chandan Dasgupta): Bruges, Belgium

- Yashodhan Hatwalne: Raman Research Institute
- Yatin Marathe (w/ N Kumar)

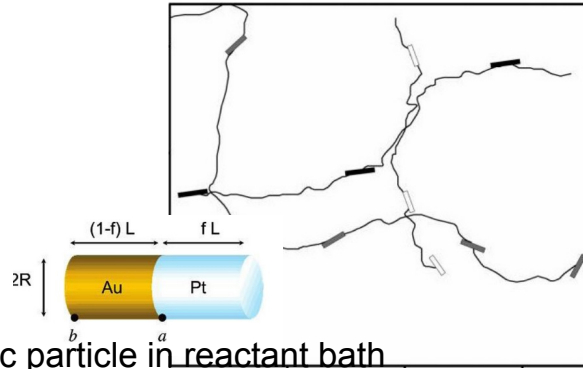
INTRODUCTION

- **Active particles:**
 - living creatures; their powered components; gadgets
 - Time's Arrow at particle scale: free-energy \longrightarrow work
 - collectively: active matter

The many kinds of active matter

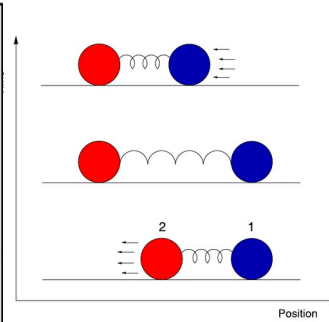


Scalar active matter
Chemically propelled droplets
S Thutupalli NCBS



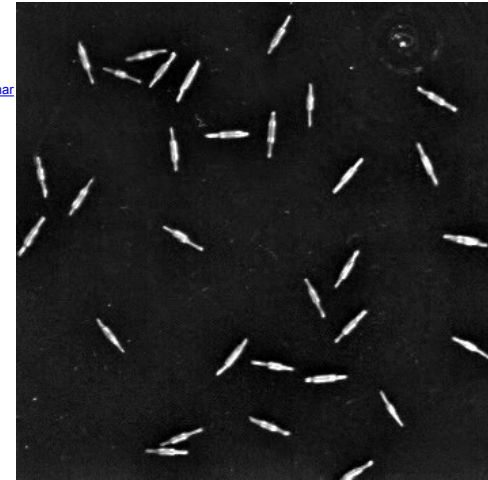
Catalytic particle in reactant bath
Sen, Mallouk, ... 2004; Golestanian et al. 2005

A motile rod transducing vertical shaking into horizontal motion: Nitin Kumar

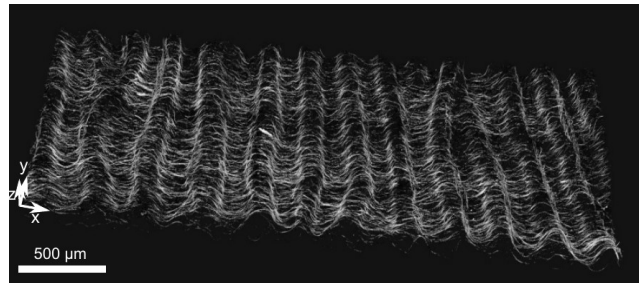


A Baule et al 2008
K V Kumar et al 2008

vector
a motile dimer: noise turned into directed movement



broken equipartition -- Vijay Narayan 2007 tensor



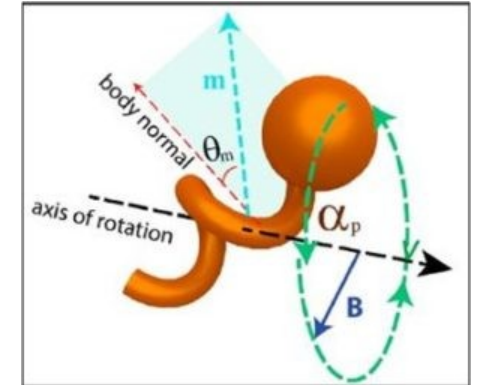
Extracts from a cell
Senoussi et al 2019

Zero-resistance states induced by electromagnetic-wave excitation in GaAs/AlGaAs heterostructures

Ramesh G. Mani[†], Jürgen H. Smet[‡], Klaus von Klitzing[†], Venkatesh Narayanamurti[†], William B. Johnson[§] & Vladimir Umansky^{||}

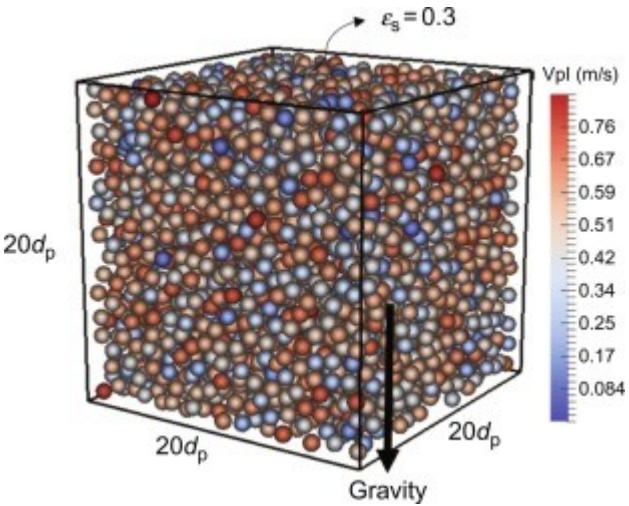
R.G. Mani et al. 2002
Alicea et al. PRB 2005: "... connection of our work to the well-studied phenomenon of 'flocking'"

quantum



chiral

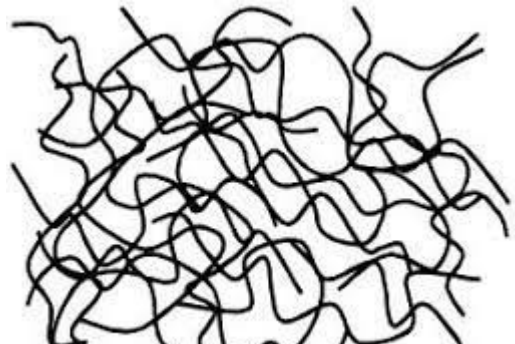
Magnetic nanopropellers: Ambarish Ghosh,



How do these differ structurally & dynamically?

https://t3.ftcdn.net/jpg/02/39/58/76/240_F_239587680_TmqOkI8wT5OVOyhBBJD4drE9ADmnnQZ6.jpg

<https://www.sciencedirect.com/topics/chemistry/hard-sphere-model>



<https://www.nytimes.com/2016/05/07/science/narcisse-snake-pits.html>

<http://franklin.chem.colostate.edu/szamel/group/sbrown/figs/melt.html>

Dynamical regimes

Wet*: suspended in a fluid



J Bertsch <http://www.thalassagraphics.com/blog/?p=167>

Dry*: with a passive momentum sink

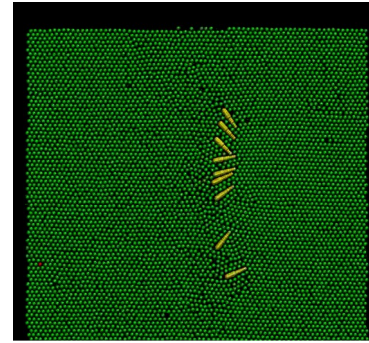


DARIUSZ PACIOREK/GETTY <https://www.wired.com/2013/03/powers-of-swarms/>

* not Feynman's wet & dry, sorry

Why should you care?

- **A grand challenge**
 - the emergent laws governing (self-)driven matter
- **Understand living matter as a physical material**
 - the role of mechanics in cell, tissue, organism and beyond
- **Imitate the functionalities of living matter**
 - smart active matter: motility, signalling, sensing



nine particles turning to find each other

Prehistory and history

self-electrophoresis 1956 fluids with energy sources 1969 hydrodynamics of swimmers 1975

animals/animation 1982/87 living LCs 1990s dry flocks 1995 active membranes '96

motile polymers, cytoskeleton 2001 flocks in fluid theory '02-04 dry active nematics: theory 2003

vibrated grains 2003 active colloids 04-05 Dry active nem sim/expt 06/07 rheo 2007+

bands '08 starling flocks '08 Dense active, tissue 2009+ microswimmers '11 MIPS '12

wet active nem expts 2012 Quinke expts 2013 pressure 2014 defect dynamics (wet)

thermo, entropy production morphogenesis turbulence active baths Command and control

non-reciprocal interactions interfaces chirality

Convective instability by active stress

BY B. A. FINLAYSON† AND L. E. SCRIVEN

Proc. R. Soc. Lond. A **310**, 183–189 (1969)

Motion that sets in suddenly and spontaneously in a previously still material, *without the intervention of outside forces*, is a dramatic kind of conversion of internal energy to kinetic energy. When the ensuing motion is smoothly circulatory and the material itself appears to be homogeneous, devoid of structure on the scale of the motion, the nature of the engine at work challenges understanding. There are indications that such engines operate at the cellular level in living systems, if not yet anywhere else. We are launching here a search for the types of material behaviour required for self-starting, continuous mechanochemistry in mechanically isolated

Before its time ... broken-symmetry hydrodynamics as yet unavailable ...
Still largely ignored in the literature

Mitchell: self-electrophoresis

P Mitchell, FEBS Lett **28** (1972) 1
see also Proc R Phys Soc Edin **25** (1956) 32

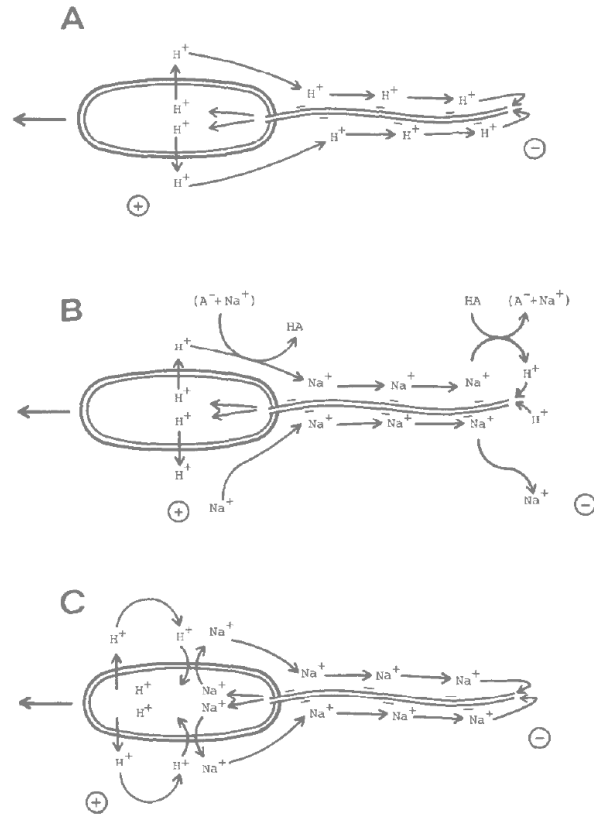


Fig. 1. Suggested mechanisms of self-electrophoretic bacterial locomotion. Protons are translocated outwards through the plasma membrane by the respiratory chain or ATPase system. The micro-tubular flagellum has a negative surface charge. In A and B, the flagellum is a specific H⁺ conductor. In C, proton translocation is transformed to Na⁺ translocation by an H⁺/Na⁺ antiporter system in the plasma membrane, and the flagellum is a specific Na⁺ conductor. The organism is driven to the left by the stream of water pulled to the right over the flagellum by the H⁺ or Na⁺ ions moving down their electrochemical potential gradient to the right.

Not a theory of bacterial swimming, but see
Lammert, Prost, Bruinsma J Theor Biol **178** (1996) 387-391

Not quite self-phoresis

J L Anderson, Annu Rev Fluid Mech **21** (1989) 61-99

Says: "... microfields established by active processes within a particle ... could self-propel ..."

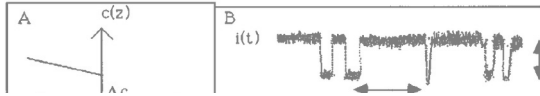
Does: small external gradient --> electric field --> slip velocity --> phoretic propulsion

$$U = -\frac{1}{3} \left(\frac{\epsilon \zeta}{4\pi\eta} \right) \frac{\chi}{k_c} \nabla C_\infty$$

For true self-phoresis see Golestanian, Liverpool, Ajdari PRL **94**, 220801(2005)

Membranes with active “force centres”

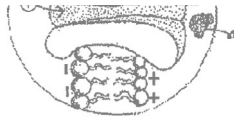
Prost & Bruinsma EPL **33** (1996) 321



$$\frac{\partial u(\mathbf{q}, t)}{\partial t} + \tau(q)^{-1} u(\mathbf{q}, t) = \eta(\mathbf{q}, t) + \Gamma \sum_k S_k(t) \exp[i\mathbf{q} \cdot \mathbf{R}_k(t)]$$

with a relaxation rate $\tau(q)^{-1}$

$$\tau^{-1}(q) = \frac{\kappa}{4\eta} q^3 + \kappa \lambda_p q^4$$



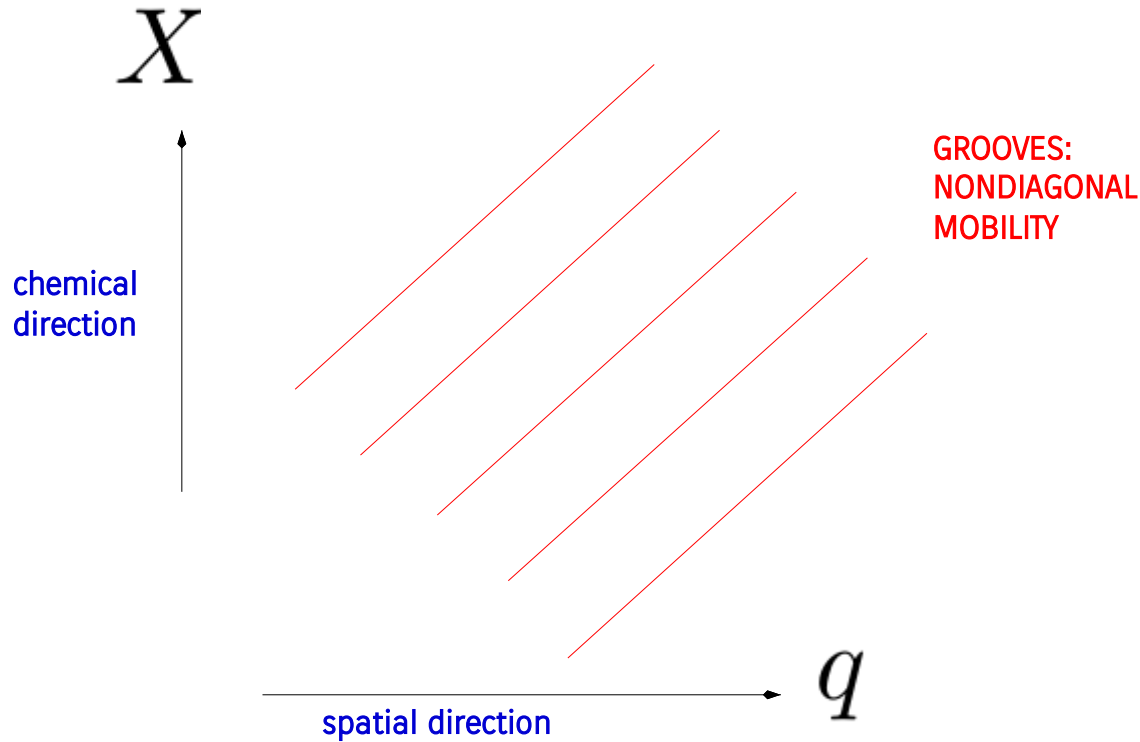
Enormous fluctuations if coupling one way.
Story changes if membrane acts back on pumps.
SR, Toner, Prost PRL **84** (2000) 3494

Microbial hydrodynamics

Pedley & Kessler, Annu Rev Fluid Mech **24** 313 (1992)

- Coarse-graining: PDEs for concentration, orientation
- Gravity, imposed flow: bioconvection, focusing
- Don't discuss instability of aligned swimmers

From equilibrium Langevin equations to active dynamics



Jülicher, Ajdari, Prost RMP Colloq 1993
Marchetti et al. RMP 2013
SR JSTAT 2017
Dadhichi, Maitra, SR JSTAT 2018

Apply chemical force, get physical (i.e. spatial) motion
Formalise this: build active dynamics; discover “new” terms

From equilibrium Langevin equations to active dynamics

“physical” position, momentum q, p chemical X

Hamiltonian $H(p, q, X)$, temperature T

$$\dot{q} = \partial_p H$$

$$\dot{p} + \Gamma \partial_p H - \Omega(q) \partial_X H = -\partial_q H + f \quad \langle ff \rangle \propto 2k_B T \Gamma$$

$$\dot{X} + \Omega(q) \partial_p H = -\mathcal{M} \partial_X H + \xi \quad \langle \xi \xi \rangle \propto 2k_B T \mathcal{M}$$

$\Omega(q)$: chemomechanical coupling

From equilibrium Langevin equations to active dynamics

“physical” position, momentum q, p chemical X

Hamiltonian $H(p, q, X)$, temperature T

$$\dot{q} = \partial_p H$$

$$\dot{p} + \Gamma \partial_p H - \Omega(q) \partial_X H = -\partial_q H + f \quad \langle ff \rangle \propto 2k_B T \Gamma$$

$$\dot{X} + \Omega(q) \partial_p H = -\mathcal{M} \partial_X H + \xi \quad \langle \xi \xi \rangle \propto 2k_B T \mathcal{M}$$

Active? Hold chemical force $-\partial_X H \equiv \Delta\mu = \text{constant}$

From equilibrium Langevin equations to active dynamics

$$\dot{q} = \partial_p H$$

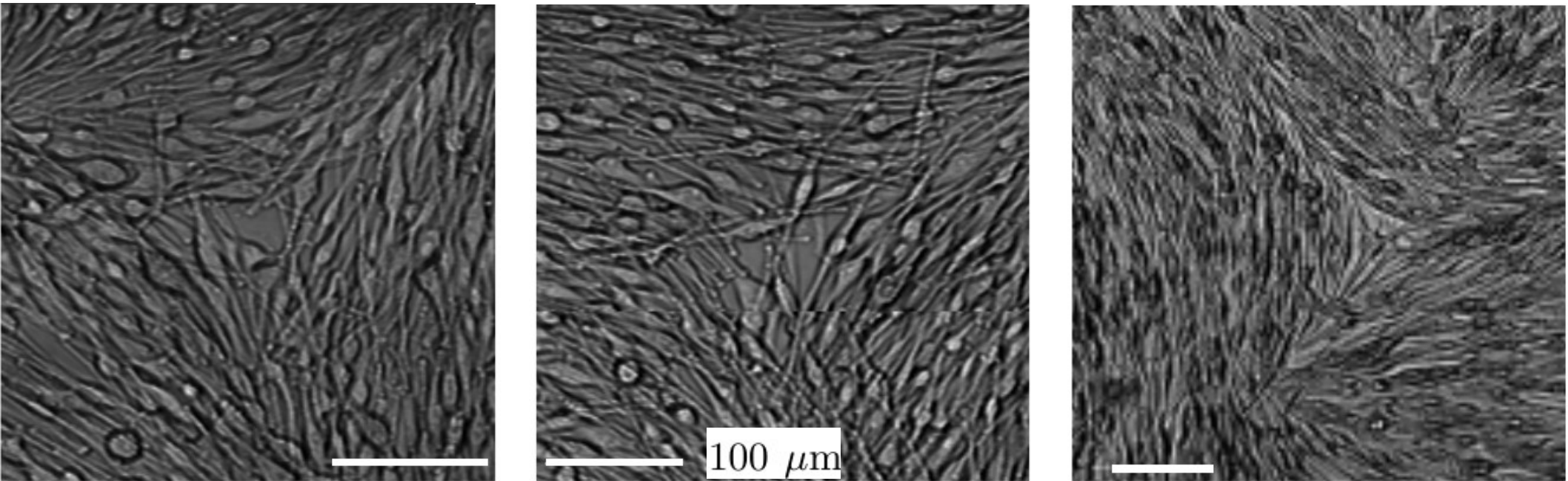
$$\dot{p} + \Gamma \partial_p H + \Delta \mu \Omega(q) = -\partial_q H + f$$

$\Omega(q)$: “new term” in equation of motion

Elastic properties of nematoid arrangements formed by amoeboid cells

R. Kemkemer¹, D. Kling¹, D. Kaufmann², and H. Gruler^{1,a}

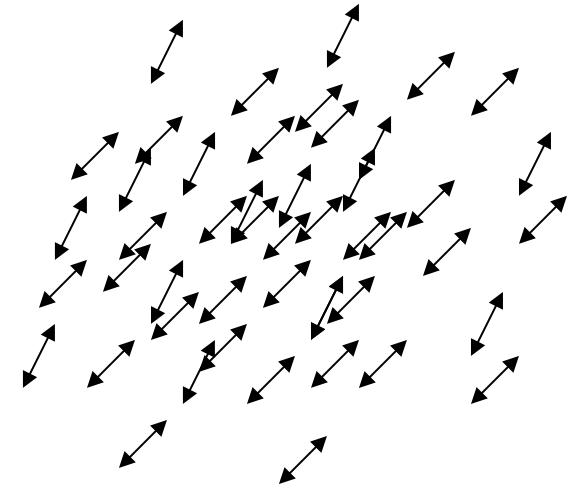
Eur. Phys. J. E **1**, 215–225 (2000)



Strength one-half defects: living proof of nematic nature

DRY ACTIVE MATTER

the nematic phase



Apolar ordering: no macroscopic velocity

On substrate: forget momentum

Slow variables: concentration $c(\mathbf{x}, t)$

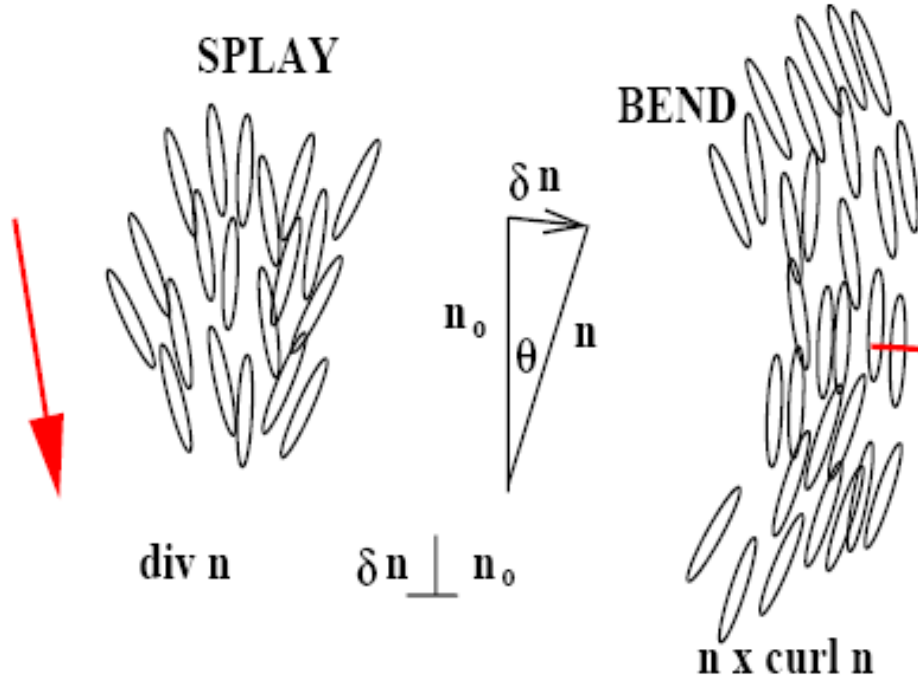
traceless symmetric order parameter Q_{ij}

SR, Simha, Toner EPL 2003; Mishra, Simha, SR JSTAT 2010

Giant density fluctuations in active nematics

SR, Simha, Toner 2003

Langevin PDEs for concentration c (conserved) and angle θ (Nambu-Goldstone) fields



At wavevector \mathbf{q} :

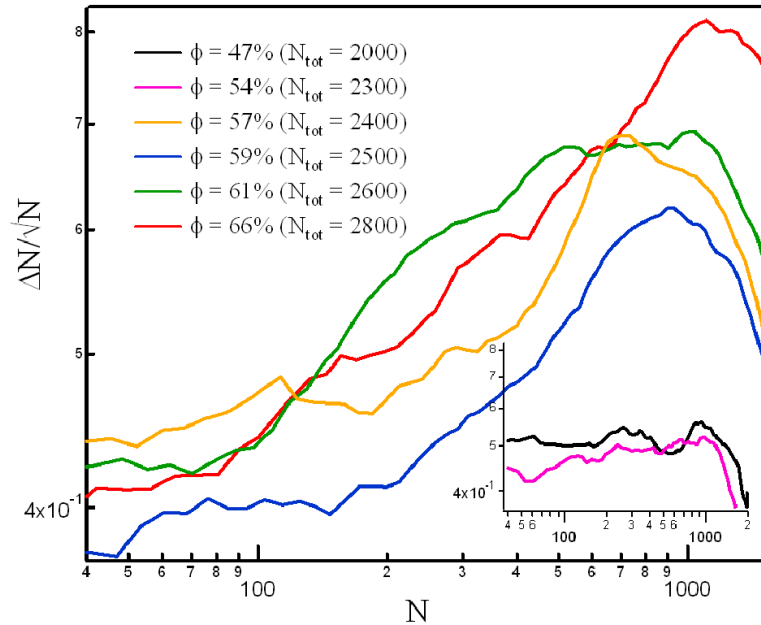
angle fluctuation $\langle |\theta_{\mathbf{q}}|^2 \rangle \sim 1/q^2$

diffusive current $\sim \nabla c$

$J_x^{active} \sim \partial_y \theta$; $J_y^{active} \sim \partial_x \theta$;

giant number-density fluctuations: $\langle |c_{\mathbf{q}}|^2 \rangle \sim 1/q^2$

Experiments: giant number fluctuations
in a nonliving active nematic
V Narayan, SR, N Menon 2007



Computer experiments

Ngo et al 2014

Vicsek-style model

$\Delta n \sim n^a$, $1/2 < a < 1$

Linear theory inadequate?

Shankar et al 2018

Active current $\sim \text{div } Q$

Magnitude of $Q \sim L^{-\eta(\Delta)}$

Number fluctuations

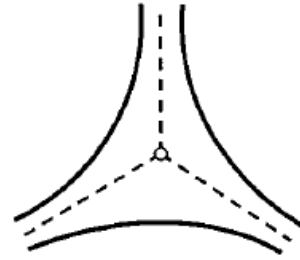
non-universal, weakened by
quasi-long-range order?

Topological defects in a nematic



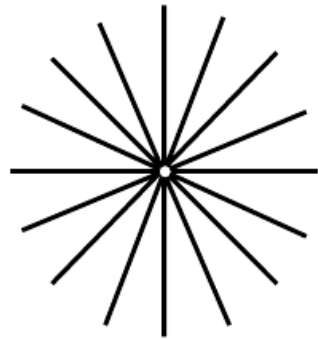
$$m = \frac{1}{2}, \quad \phi_0 = 0$$

(a)



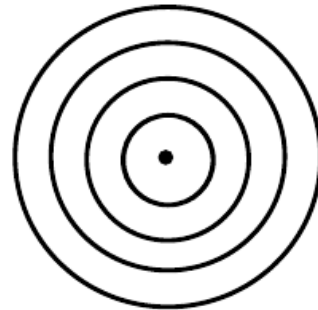
$$m = -\frac{1}{2}, \quad \phi_0 = 0$$

(b)



$$m = 1, \quad \phi_0 = 0$$

(c)

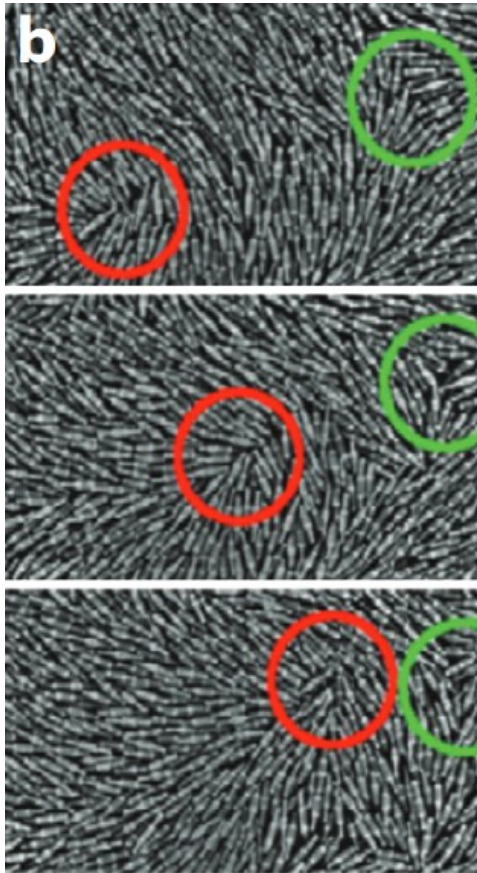


$$m = 1, \quad \phi_0 = \frac{\pi}{2}$$

(d)

Self-propulsion of $+1/2$ defects in active nematics

prediction & observation



The symmetry of the field around the strength $-1/2$ defect will result in no net motion, while the curvature around the $+1/2$ defect has a well-defined polarity and hence should move in the direction of its “nose” as shown in the figure.

V Narayan et al., Science **317** (2007) 105

motile $+1/2$ defect, static $-1/2$ defect

Defects as particles:

$+1/2$ motile, $-1/2$ not

$+1/2$ velocity $\sim \text{div}Q$

Giomi, Bowick, Ma, Marchetti PRL 2013

Thampi, Golestanian, Yeomans PRL 2014

DeCamp et al NMat 2015

Defect-unbinding theory: Suraj Shankar, M C Marchetti, SR, MJ Bowick

Defect unbinding in active nematics

Shankar
et al. PRL 2018

Recall equil BKT transition:

but +1/2 defect is motile!

Like insulator in a field? Finite barrier?

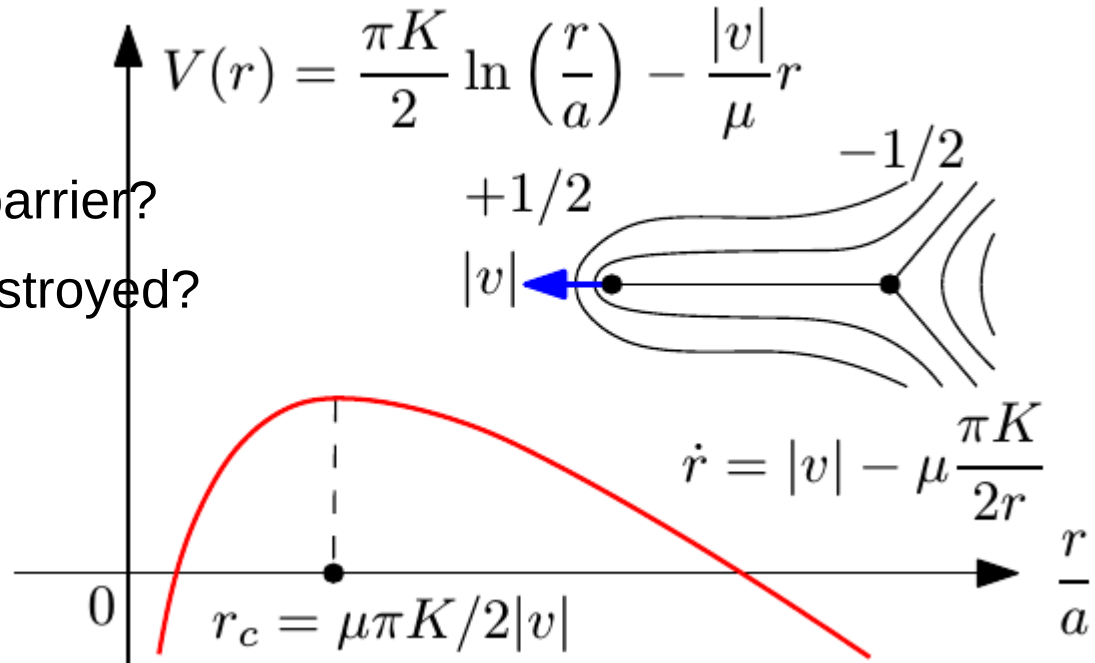
Active nematic order always destroyed?

But active nematics exist!

Bertin et al. NJP 2013

NGO et al. PRL 2014

Shi et al NJP 2014



Langevin equations for +/- 1/2 defects: positions and polarization

Shankar et al. 2018:

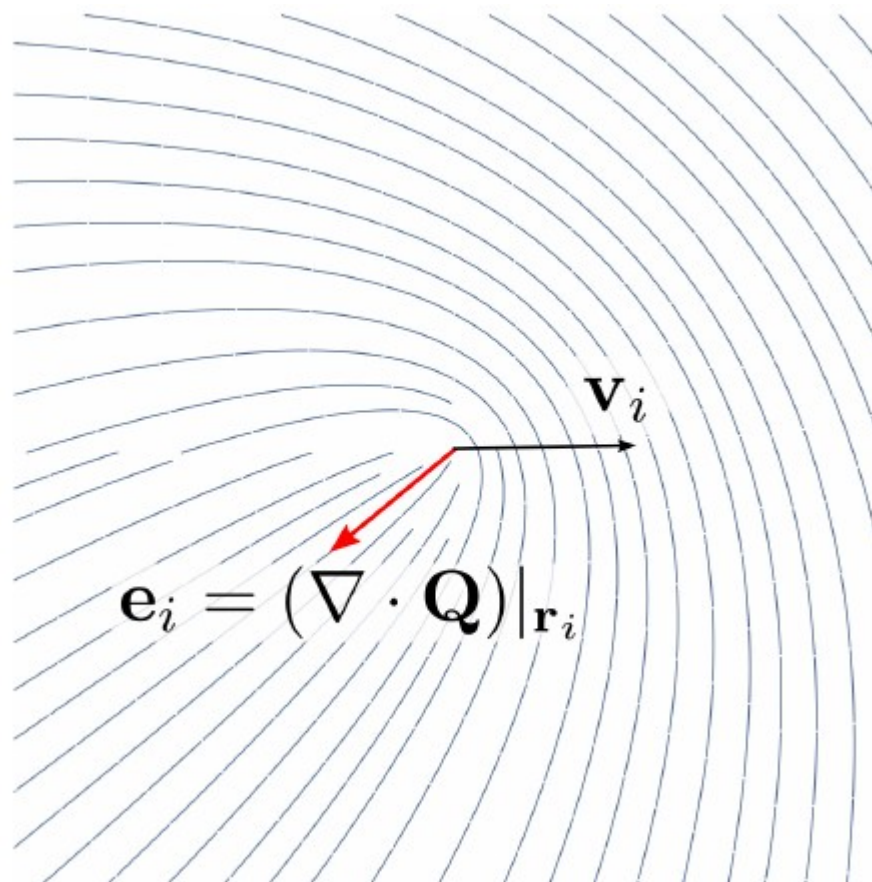
From active nematic dynamics

+1/2 self-velocity \propto polarization

$$\dot{\mathbf{r}}_i^+ = \mathbf{v}\mathbf{e}_i + \mu\nabla_{\mathbf{r}_i}\mathcal{U} + \sqrt{2\mu T}\boldsymbol{\xi}_i(t)$$

$$\dot{\mathbf{r}}_i^- = -\mu\nabla_{\mathbf{r}_i}\mathcal{U} + \sqrt{2\mu T}\boldsymbol{\xi}_i(t)$$

$$\mathcal{U} = -2\pi K \sum_{i \neq j} q_i q_j \ln \left| \frac{\mathbf{r}_i - \mathbf{r}_j}{a} \right|$$



Langevin equations for +/- 1/2 defects: positions and polarization

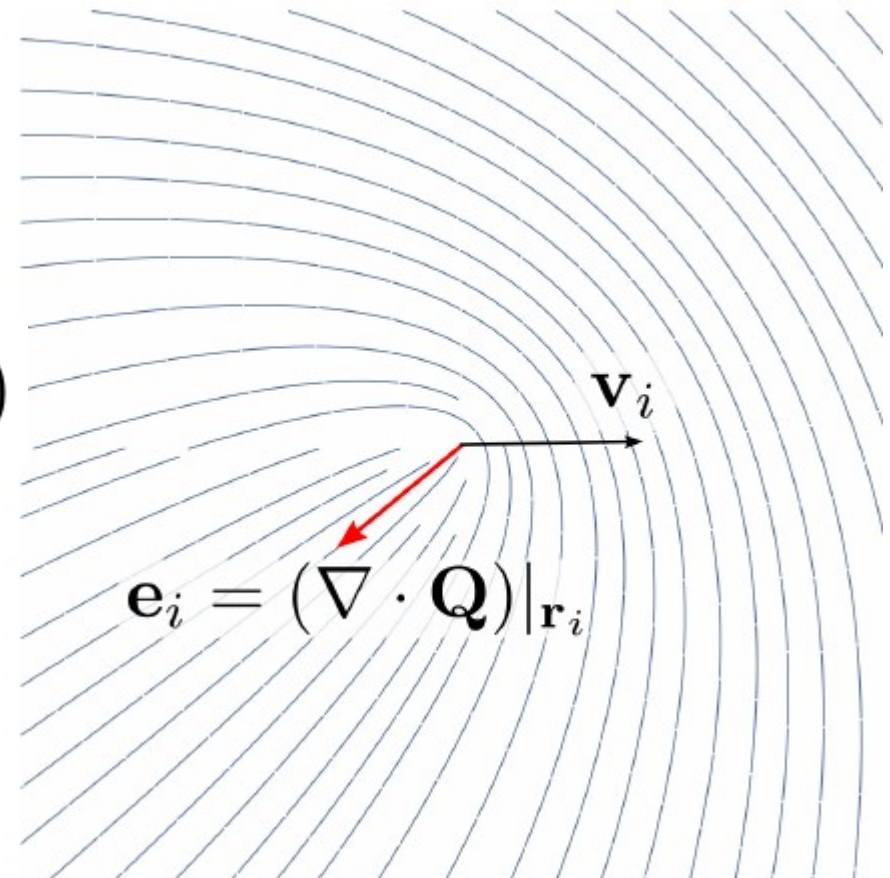
Shankar et al. 2018:

$$\mathbf{e}_i = |\mathbf{e}_i|(\cos \theta_i, \sin \theta_i)$$

$$\mathbf{F}_i \equiv -\nabla_i \mathcal{U} = |\mathbf{F}_i|(\cos \psi_i, \sin \psi_i)$$

$$\partial_t \theta_i = v |\mathbf{F}_i| \times \text{const.} \sin(\theta_i - \psi_i)$$

+ angular noise

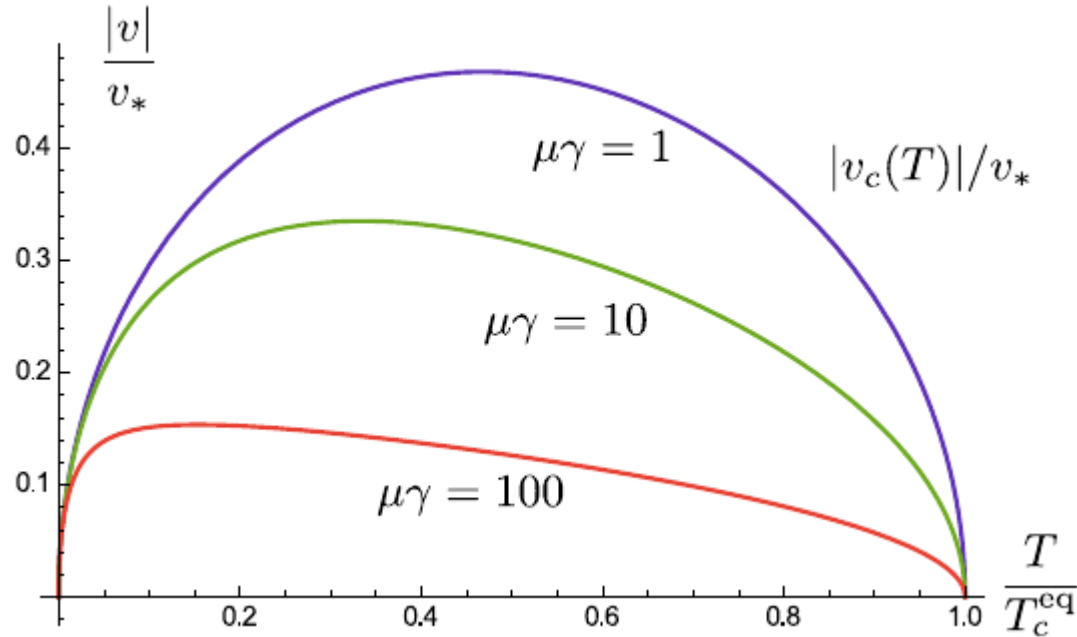


Alignment torque: $v < 0$: alignment; $v > 0$: anti-alignment

Re-entrance!

Shankar et al. 2018:

Threshold activity

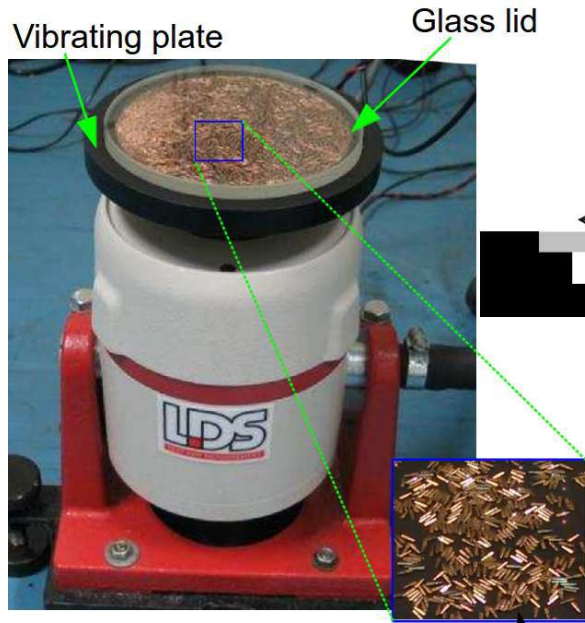


$$\frac{|v_c(T)|}{v_*} = \sqrt{\frac{16 \tilde{T}(1 - \tilde{T})}{\pi \left[1 + (3\pi/32)\mu\gamma\tilde{T} \right]}}$$

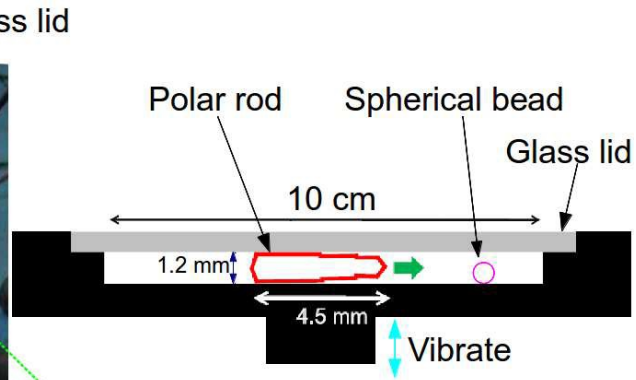
At low enough T , D_R goes to zero, i.e., persistence length grows
Directed motion of $+1/2$ wins, defects liberated, order destroyed
(A Maitra)

Defect ordering:
Shankar & Marchetti
PRX 2019

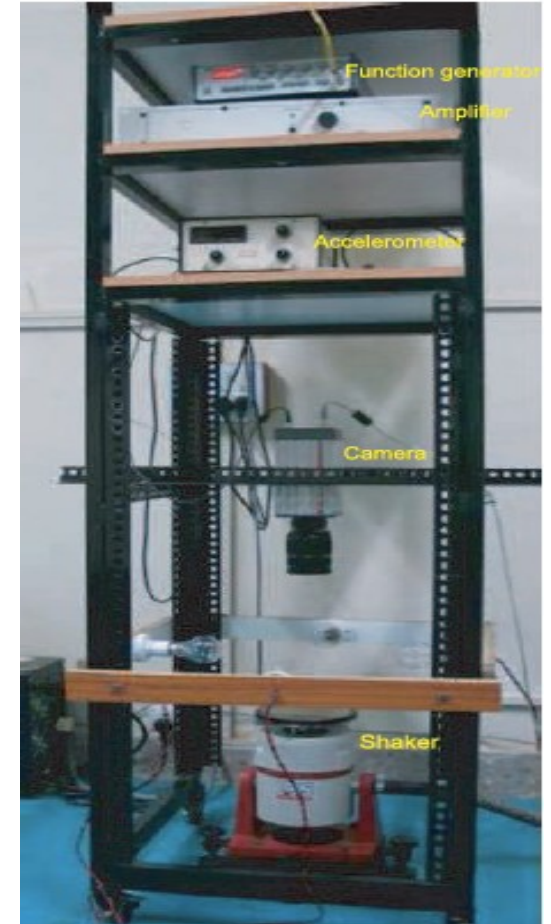
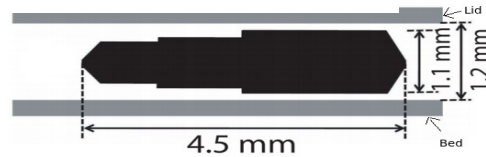
Flocking at a distance



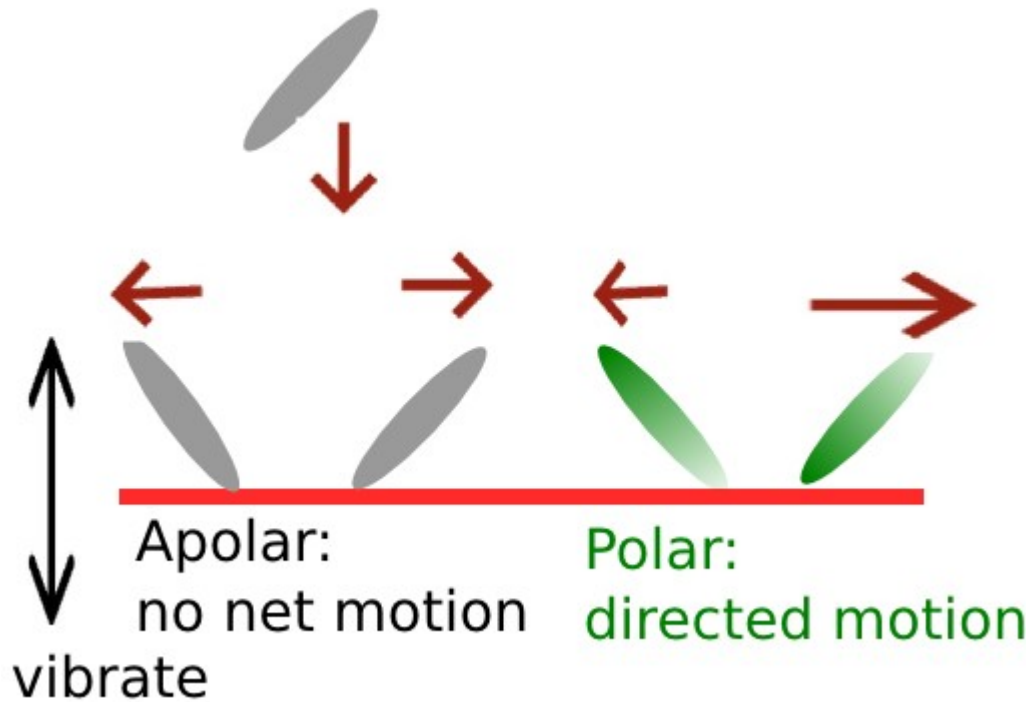
Real experimental setup



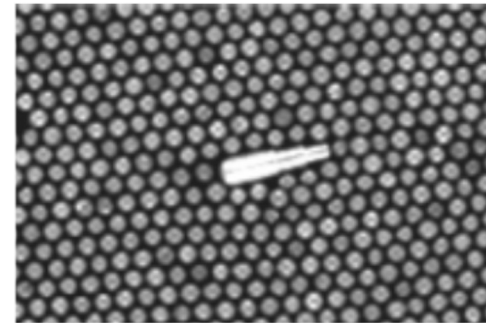
Schematic diagram



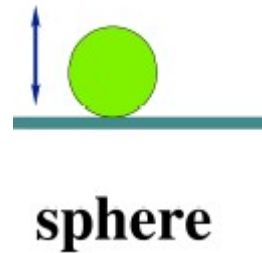
Horizontal motility from vertical shaking



static friction \Rightarrow centre of mass moves



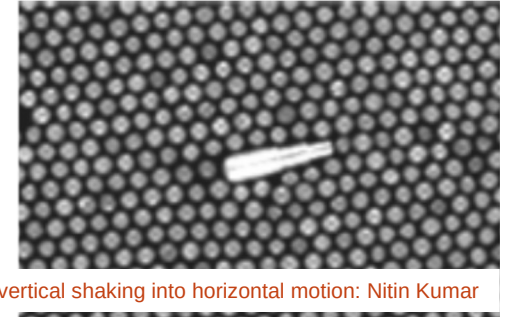
A motile rod transducing vertical shaking into horizontal motion: Nitin Kumar



sphere

First look at a fluid bead-layer

1-particle rendition of Kumar, Soni, SR, Sood Nature Comm 2014



$$\dot{\mathbf{R}}(t) = v_0 \mathbf{n}(t) \quad \text{motility}$$

$$\partial_t(\rho \mathbf{v}) + (\zeta - \eta \nabla^2) \mathbf{v} = f \mathbf{n}(t) \delta [\mathbf{r} - \mathbf{R}(t)] - \nabla P$$

Substrate drag, viscosity Motile rod pushes beads pressure

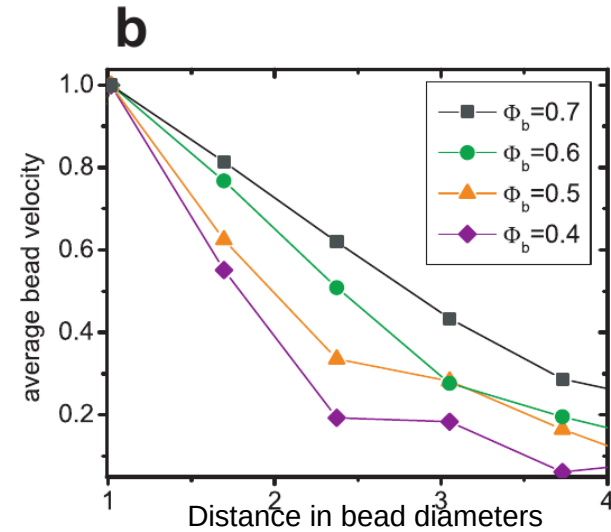
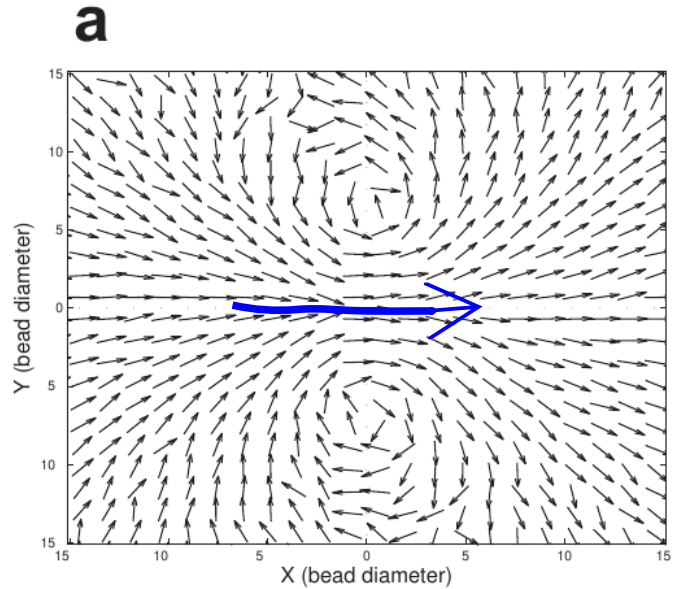
$$\dot{\mathbf{n}} = (\mathbf{I} - \mathbf{nn}) \cdot (\mathbf{v} + \nabla \mathbf{v} \cdot \mathbf{n} + \dots) \quad \text{(schematically)}$$

flow reorients \mathbf{n} parallel to \mathbf{v}

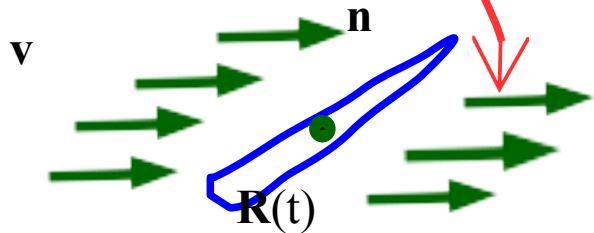
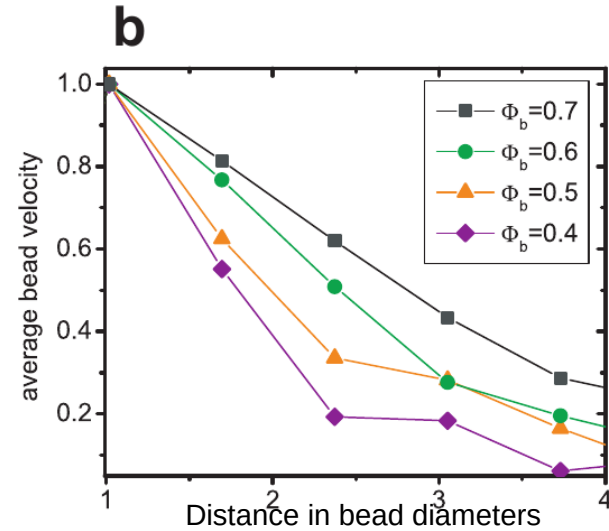
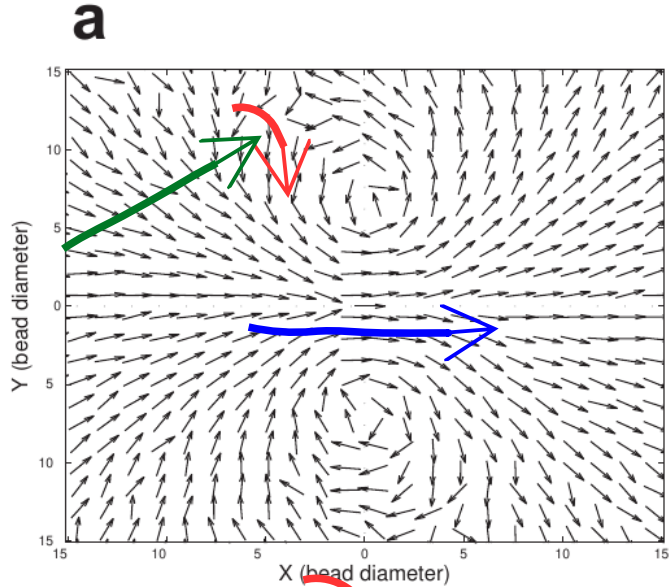
Gradients rotate & align \mathbf{n}

in pictures:

Flow-field around a mover in a fluid layer



An emergent aligning interaction



Nonuniform drag: flow reorients \mathbf{n} parallel to \mathbf{v}
The weathercock effect

A granular flock at very low concentration

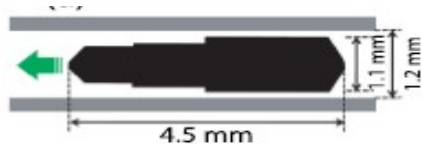
Kumar, Soni, Sood, SR Nature Communications 2014; arXiv:1402.4262



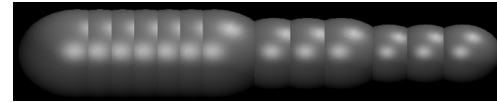
Nitin Kumar (student of A K Sood, IISc)

</home/sriram/talks/activemattertalks/current/Video1.avi>

</home/sriram/talks/activemattertalks/current/Video2.avi>



Confined quasi-2d geometry



/home/sriram/talks/activemattertalks/current/vdo_liquid.mpg

Granular dynamics simulation: Harsh Soni

</home/sriram/talks/activemattertalks/current/Video3.avi>

</home/sriram/talks/activemattertalks/current/Video4.avi>

</home/sriram/talks/activemattertalks/current/Video5.avi>

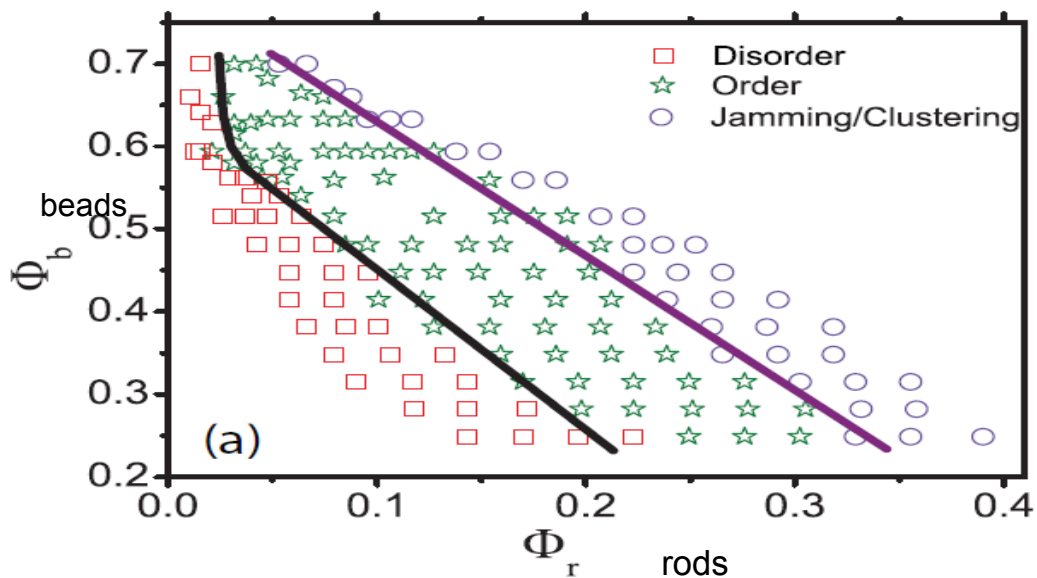
</home/sriram/talks/activemattertalks/current/Video6.avi>

cf Deseigne et al PRL 2010

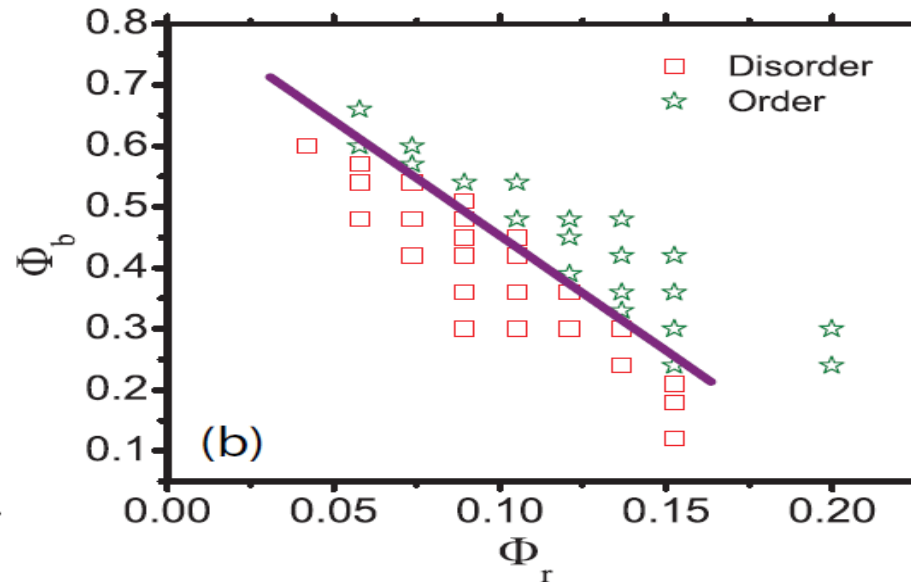
Weber et al PRL 2013

Phase diagram

Flocking by increasing inert-particle concentration



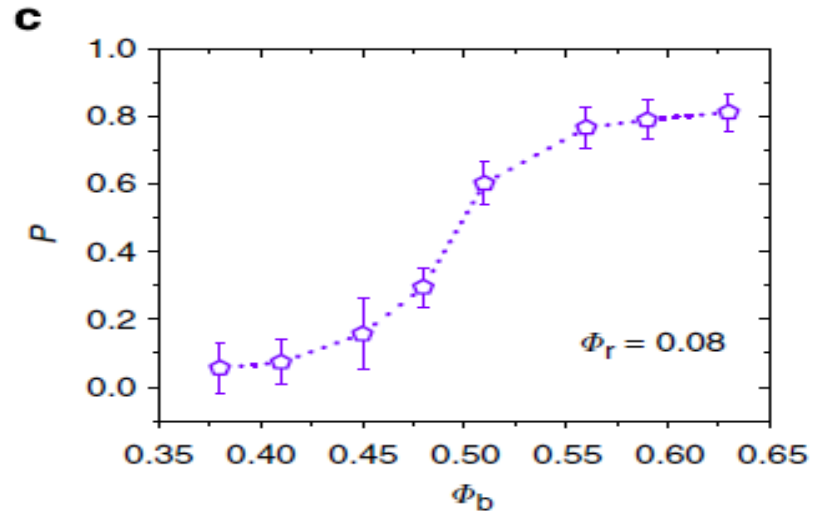
Experiment



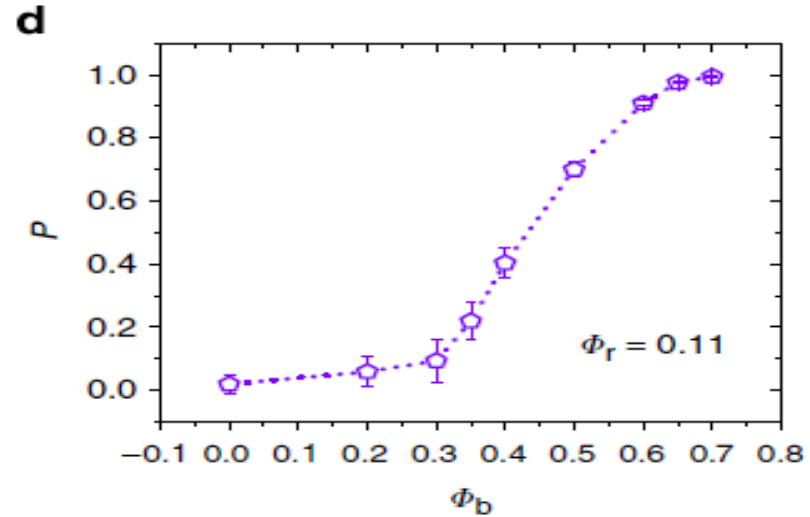
Simulation

A phase transition

Amount of order as function of inert-particle concentration



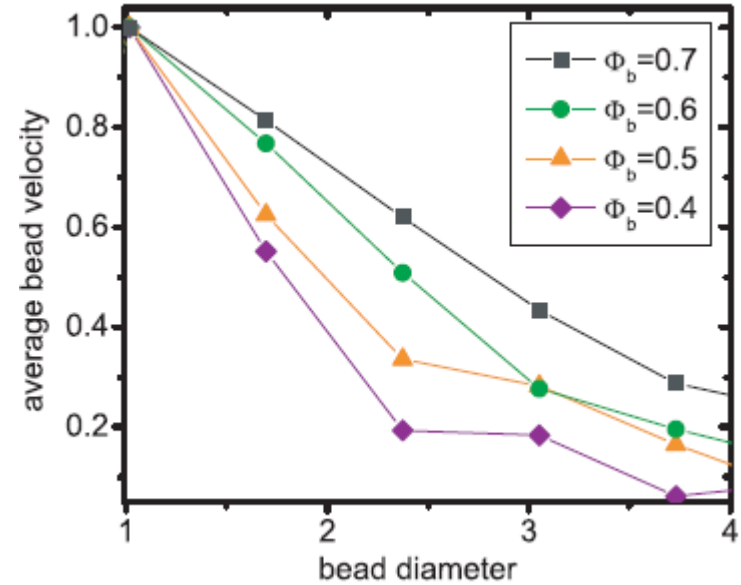
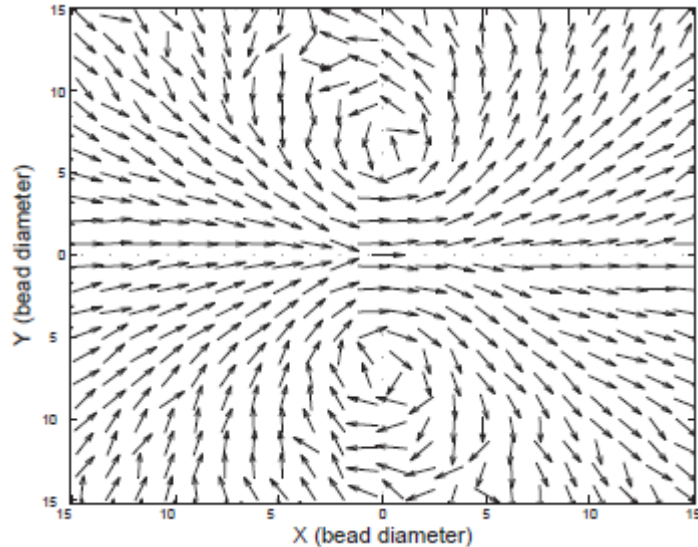
Experiment



Simulation

The mechanism: moving polar rod creates flow

Simulation: H Soni



Increase Φ_b --> increase decay length of velocity

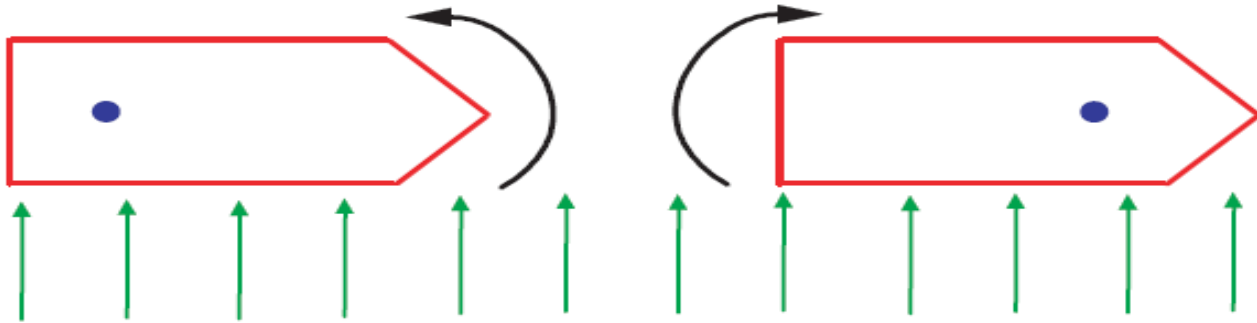
Screened monopole
cf Brotto et al. PRL 2013

The mechanism: flow orients polar rod

Flow rotates polar particles to point the right way: the weathercock effect
Need a substrate

</home/sriram/talks/activemattertalks/current/Video7.avi>

Could have been either way
Design problem



qualitatively similar to Bricard et al. colloidal rollers Nature 2013

- flow field simpler, medium compressible
- single-rod motility from solid contact mechanics
- Crucial difference: non-motile-bead concentration is control parameter
- purely 2d system

Theory of flocking at a distance

Kumar, Soni, Sood, SR arXiv:1402.4262, Nat Comm 2014

continuity $\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$

Order parameter \mathbf{P} ,
velocity \mathbf{v} ,
number density ρ

$$\rho \partial_t \mathbf{v} = - (\Gamma - \eta \nabla^2) \mathbf{v} + \alpha \mathbf{P} - B \nabla \rho + \dots$$

damping forcing

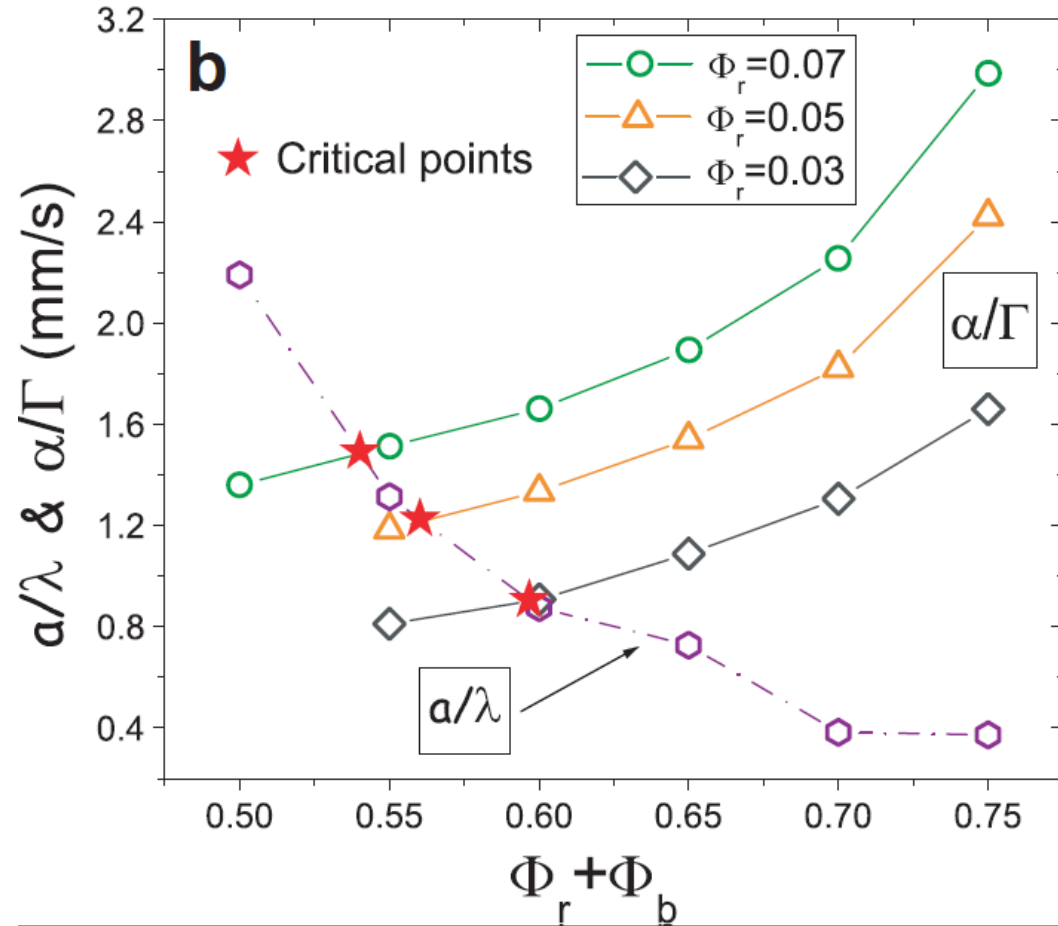
$$\partial_t \mathbf{P} = \lambda \mathbf{v} - (a - K \nabla^2) \mathbf{P} - A \nabla \rho + \dots$$

Flow coupling Rotational relaxation

Transition determined by effective coupling $\bar{a} = a - \lambda \alpha / \Gamma$

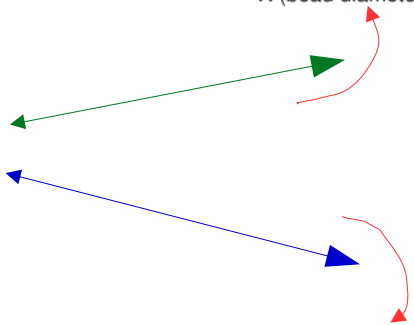
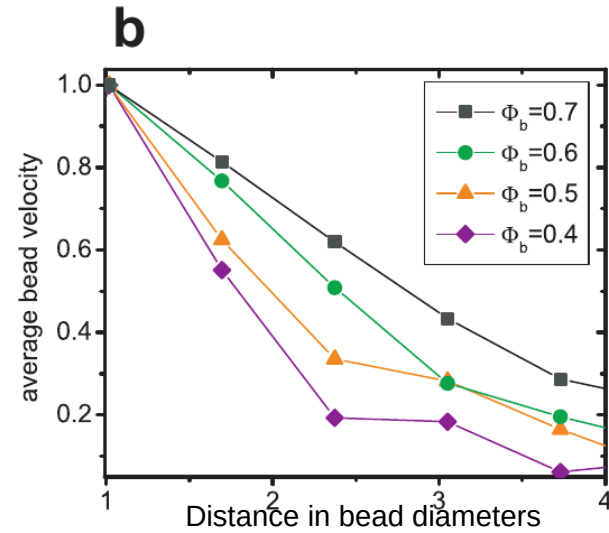
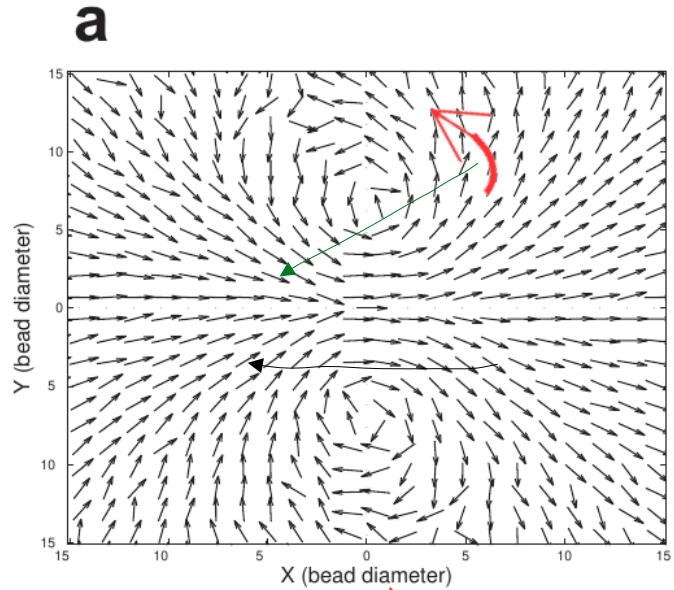
- Independent measurements in simulation (H Soni)
- $\alpha > 0$, $\lambda > 0$ and increases with ρ
- So: increase ρ : get transition to ordered state of \mathbf{P}

Estimating mean-field critical point from simulation

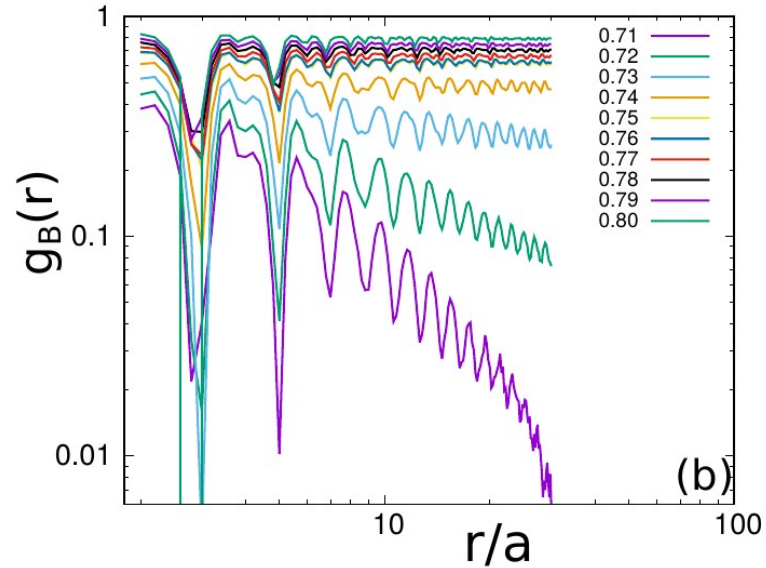
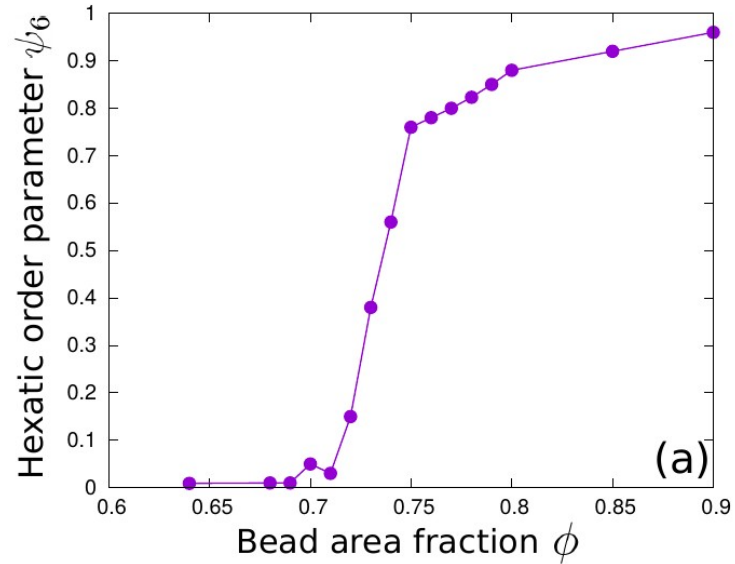


Side-by-side: rotation by vorticity

negative taxis: “repulsion”

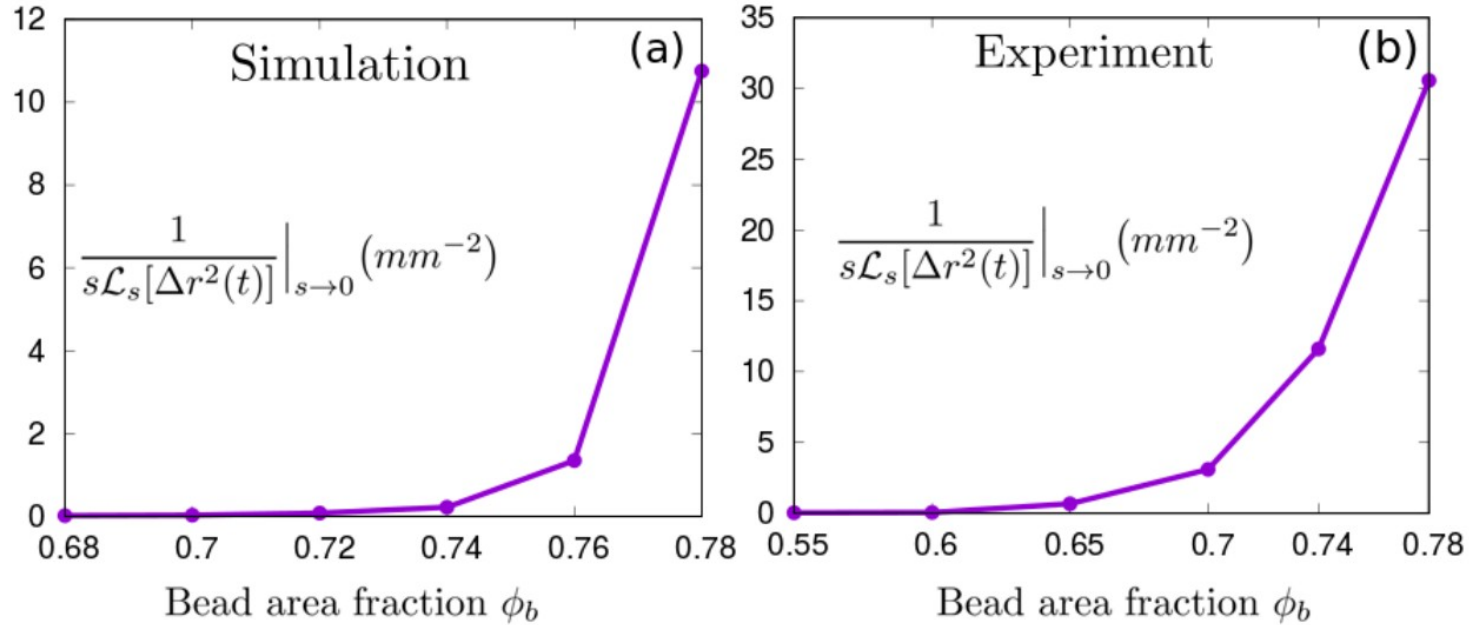


Dense bead layer: crystalline



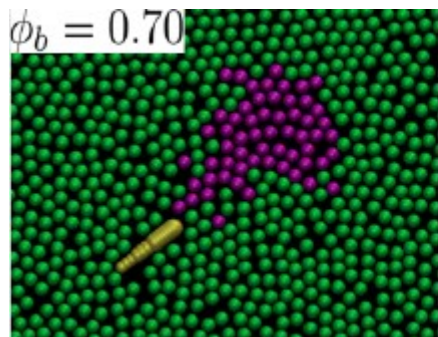
Increase bead packing, transition to crystal
Long-range 6-fold order as proxy

Onset of rigidity



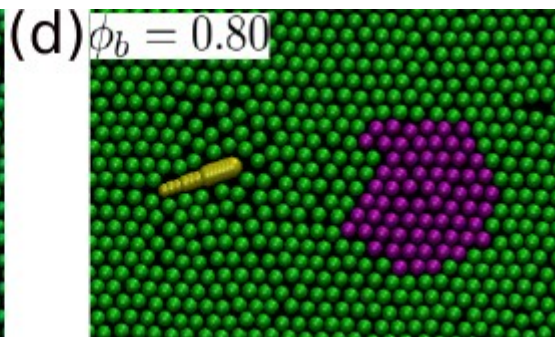
Single-particle microrheology in real and numerical experiments

Comparison: crystal vs fluid



fluid

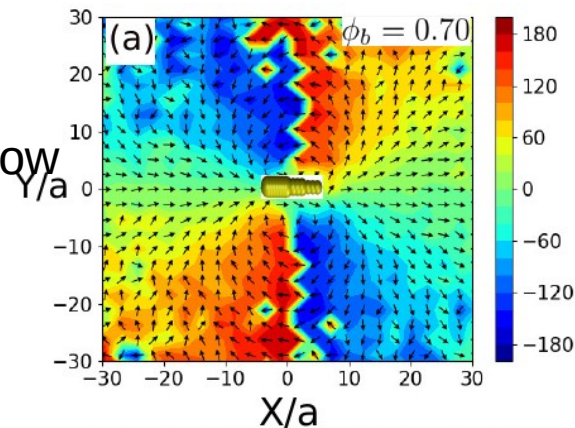
fluid phase: motile rod drags beads



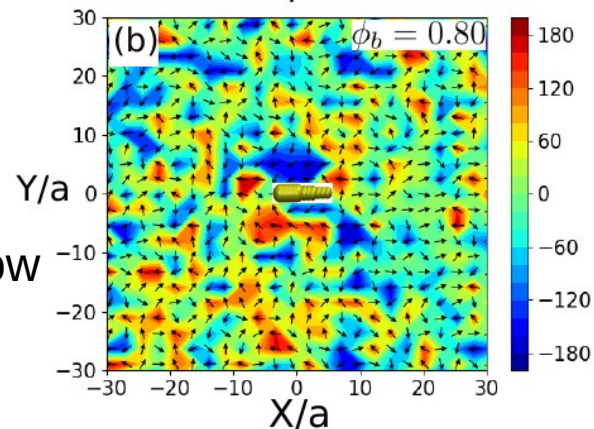
crystal

crystal: no dragging

Fluid: bead flow



crystal: no flow



Crawling through a crystal: theory?

- **Safran et al: force dipoles in elastic medium**
 - motility ignored cell/tissue mechanics
- **Henkes et al. 2020: elastic medium made of ABPs**
 - active forcing + repulsive pair potential, no reorienting by medium
- **This work: coupled dynamics**
 - motile particles strain medium, **strain reorients particles**
 - naturally non-reciprocal dynamics

Rahul Gupta, Raushan Kant,
Harsh Soni, Ajay Sood, SR
PRE 2022

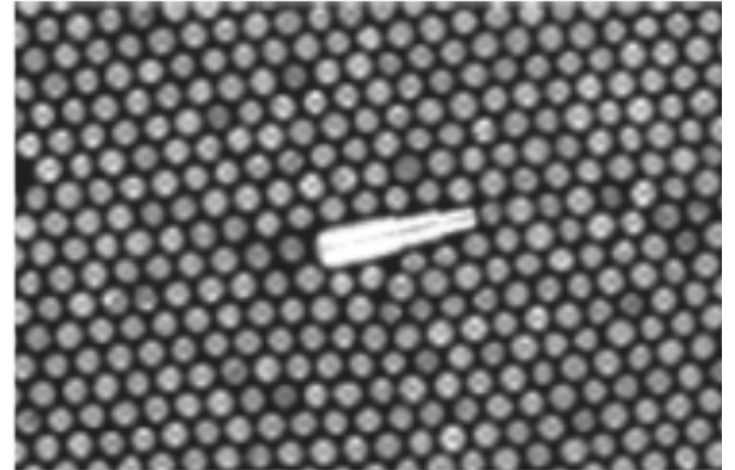
Motile particles in elastic medium on substrate

Particle position $\mathbf{R}(t)$, orientation $\mathbf{n}(t)$

Displacement field of medium $\mathbf{u}(\mathbf{r}, t)$

Lamé elastic free energy F

Friction ζ , self-prop force f , speed v_0

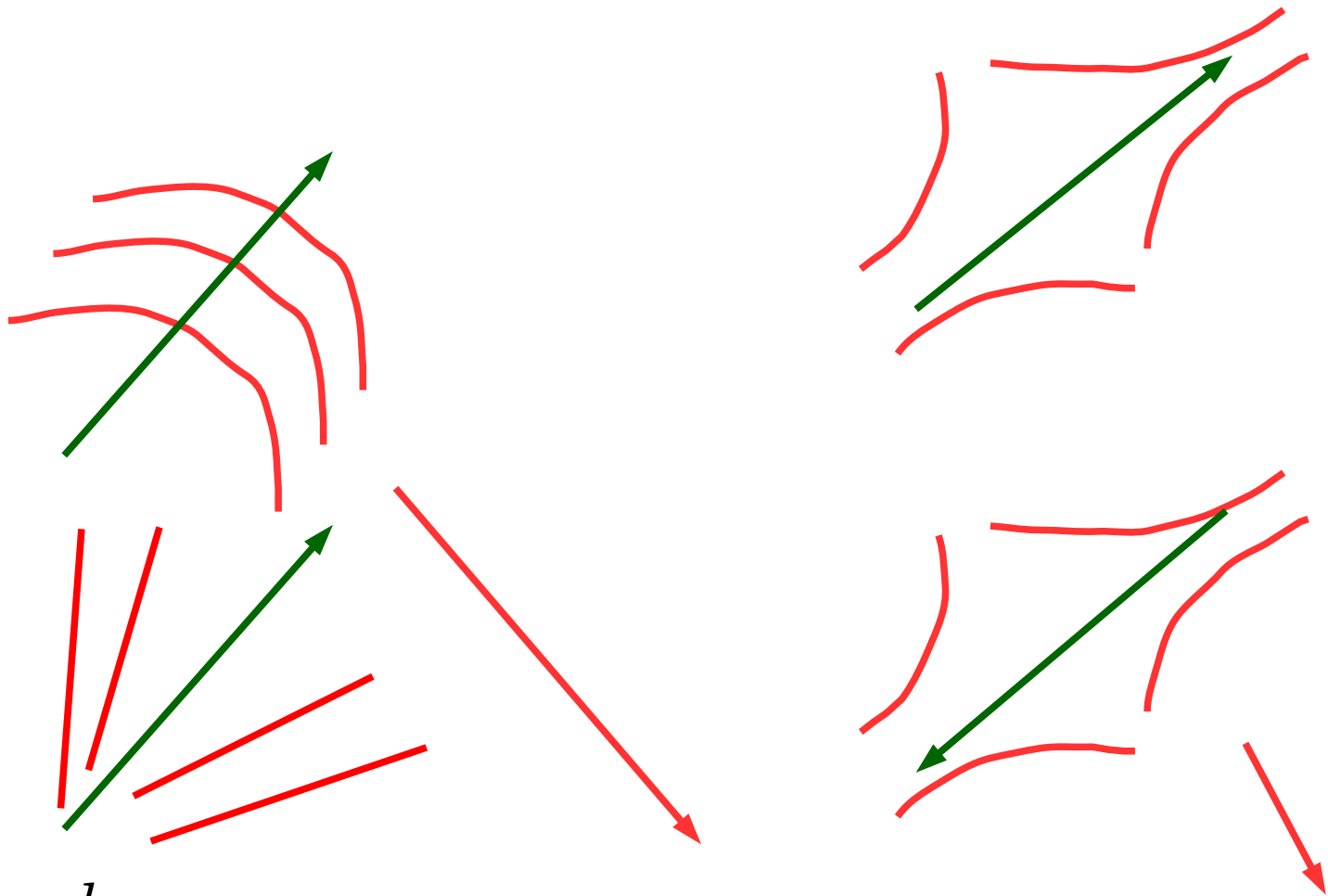


A motile rod transducing vertical shaking into horizontal motion: Nitin Kumar

$$\dot{\mathbf{R}}(t) = v_0 \mathbf{n}(t)$$

$$\zeta \partial_t \mathbf{u} = -\delta F / \delta \mathbf{u} + f \mathbf{n}(t) \delta(\mathbf{r} - \mathbf{R}(t))$$

driving through a crystal



$$\frac{d\mathbf{n}}{dt} = (\mathbf{I} - \mathbf{n}\mathbf{n}) \cdot (\gamma_1 \nabla^2 \mathbf{u} + \gamma_2 \nabla \nabla \cdot \mathbf{u} + \kappa \boldsymbol{\varepsilon} \cdot \mathbf{n})$$

Strain field of a motile particle

\mathbf{U} = displacement field in frame comoving and corotating with particle

$$[-\zeta v_0 \partial_x - (\mu \nabla^2 + \lambda \nabla \nabla \cdot)] \mathbf{U} = f \delta(\mathbf{r}) \hat{\mathbf{x}}$$

Screening

$$\alpha = \zeta v_0 / \mu$$

$$U_x = \frac{f}{4\pi\mu} \left\{ \left[K_0\left(\frac{\alpha r}{2}\right) - \frac{x}{r} K_1\left(\frac{\alpha r}{2}\right) \right] e^{-\frac{\alpha x}{2}} \right.$$

$$\beta = \zeta v_0 / \lambda$$

$$\left. + \frac{\beta}{\alpha} \left[K_0\left(\frac{\beta r}{2}\right) + \frac{x}{r} K_1\left(\frac{\beta r}{2}\right) \right] e^{-\frac{\beta x}{2}} \right\}$$

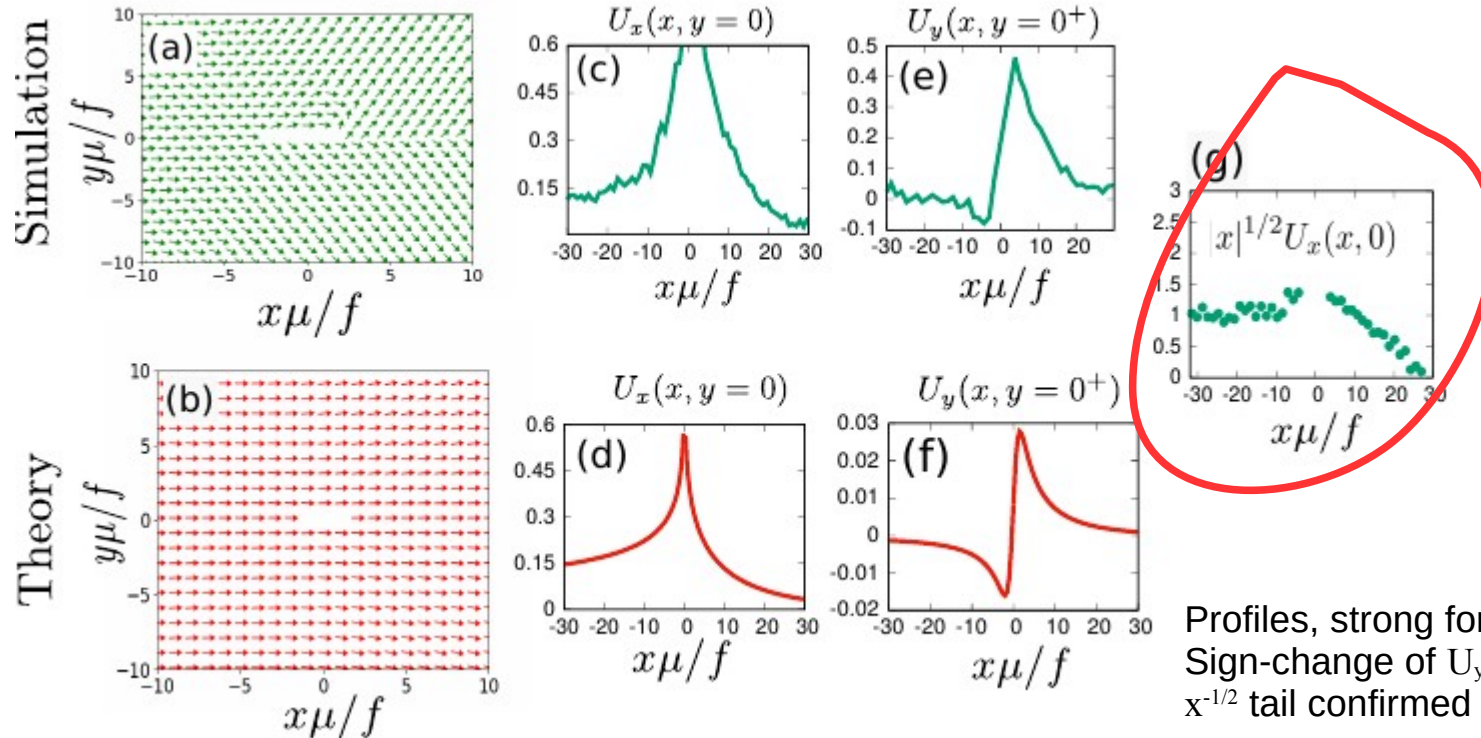
and similarly U_y

Crucial: asymp forms of $K_0 \Rightarrow$ exponential decay for $x > 0$, $|x|^{-1/2}$ for $x < 0$

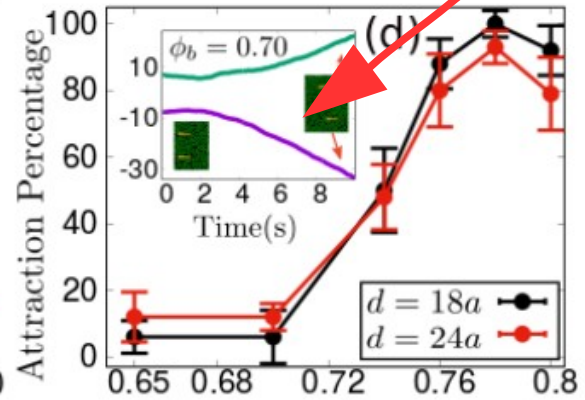
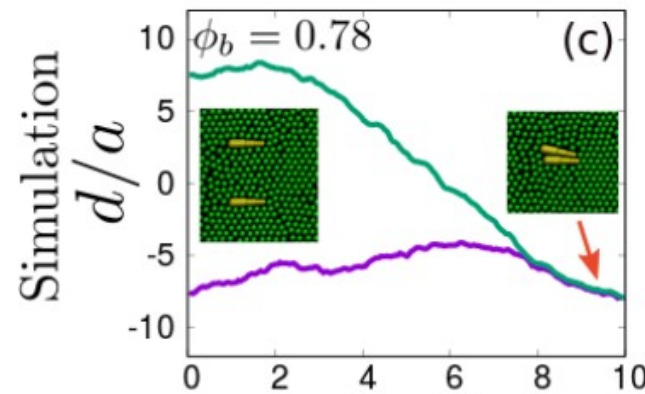
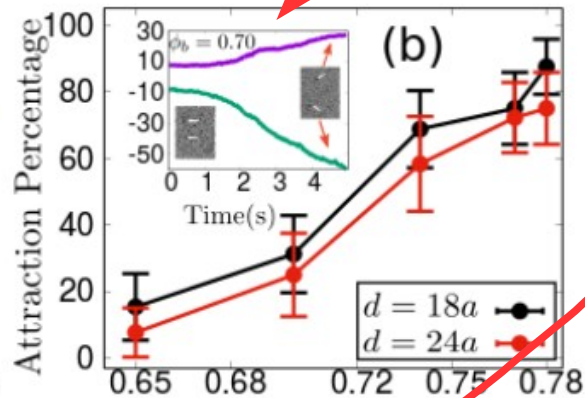
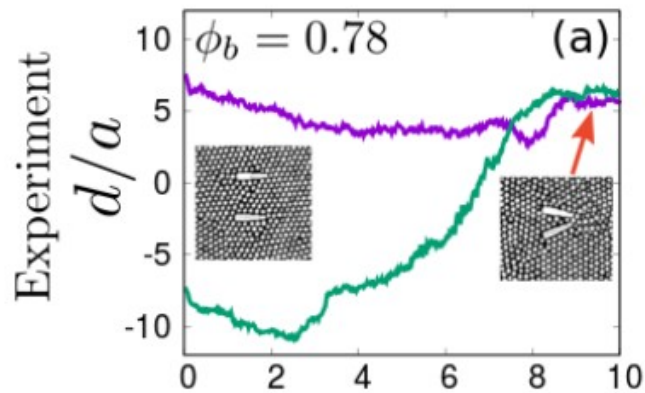
Overdamped elastic wake

Comparison with measured fields

Numerical experiment on vibrated layer of grains
Inelasticity, static friction, base, lid all included



Capture in experiment and simulation

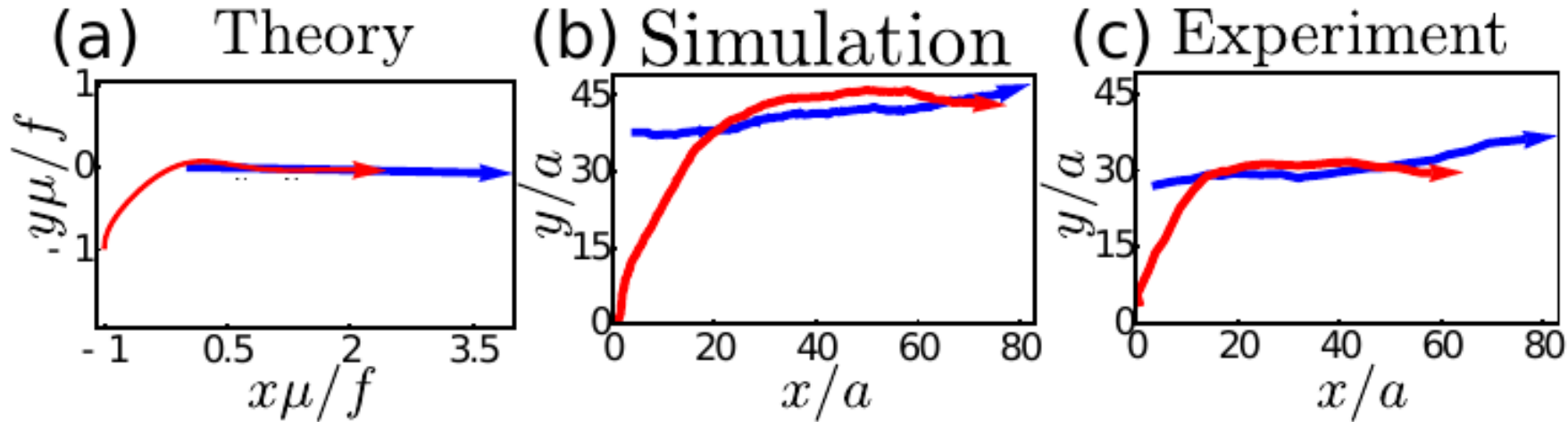


Repel when beads are fluid

$$\frac{dn}{dt} = (\mathbf{I} - \mathbf{nn}) \cdot (\gamma_1 \nabla^2 \mathbf{u} + \gamma_2 \nabla \nabla \cdot \mathbf{u} + \kappa \boldsymbol{\varepsilon} \cdot \mathbf{n})$$

fraction ϕ_b

Non-reciprocal interaction



(arXiv:2007.04860)

Particle in front gets no indication of particle behind
Pursuer particle senses distortion field of pursued particle!

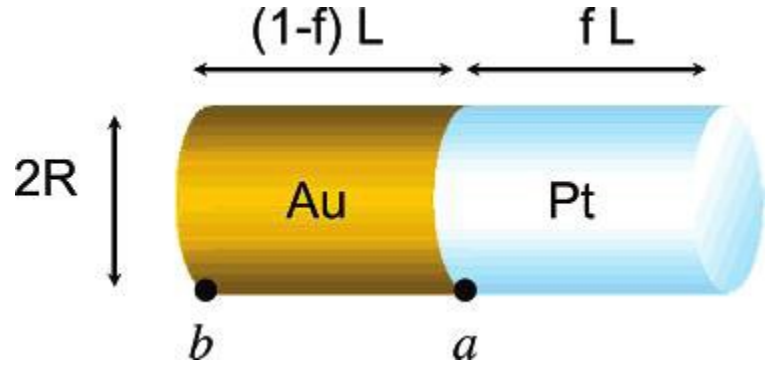
pursuit and capture: laboratory experiment

pursuit and capture: numerical experiment

Currently: bead-rod segregation; motile minority condenses non-motile majority
Rausan Kant / Rahul Gupta / Ajay Sood, Soft Matt 2020, PRE 2022 and in prep

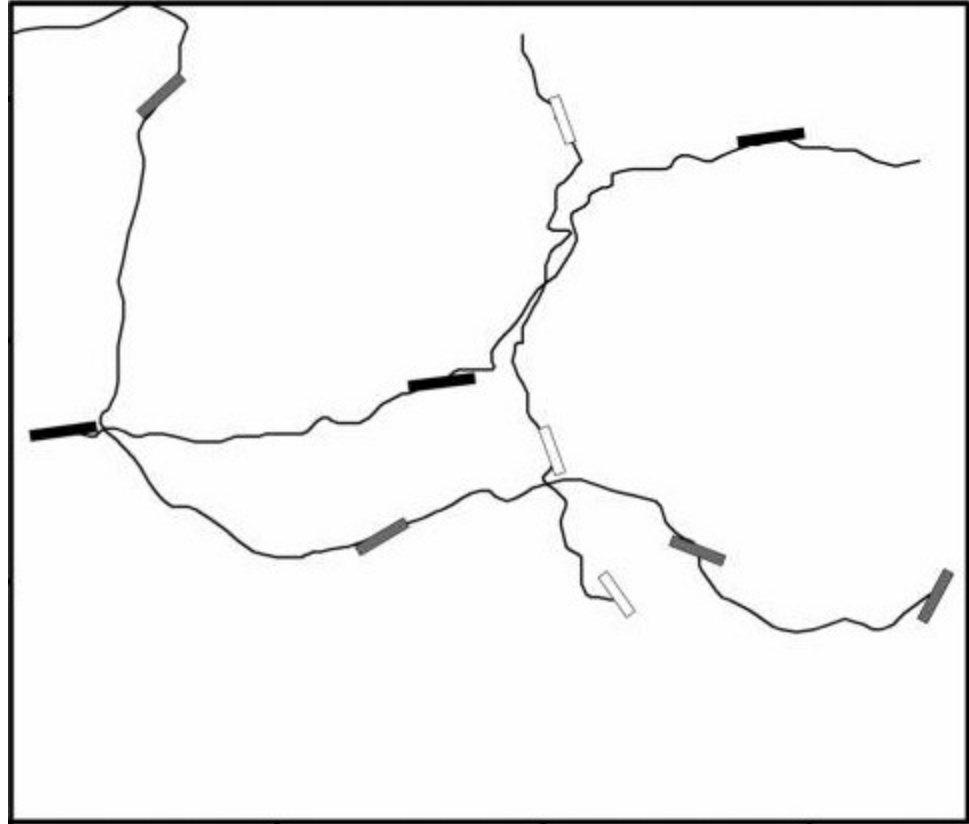
Chemotactic colloids*

*not “dry” exactly, but ...



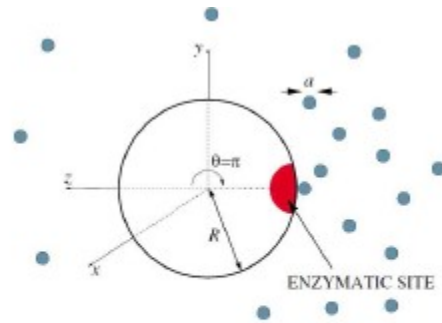
Paxton et al. JACS 2004

Golestanian, Liverpool, Ajdari PRL 2005
Howse Golestanian PRL 2007



How?

Force-free chemical self-propulsion



Sphere with enzyme coat $\sigma(\theta, \phi)$ in a reactant bath.

Particle makes its own gradient

Asymmetric distribution of reaction products

Asymmetric stresses

Flow

Propulsion

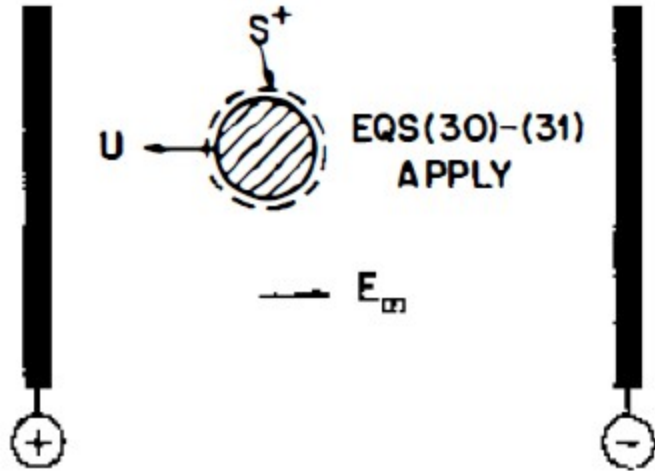
Properties of motile organisms

- Directed force-free motion
- Flocking
- Gradient-sensing
- Signalling
- Clumping
- Patterns



Suropriya Saha, Ramin Golestanian, SR
Phys Rev E 2014

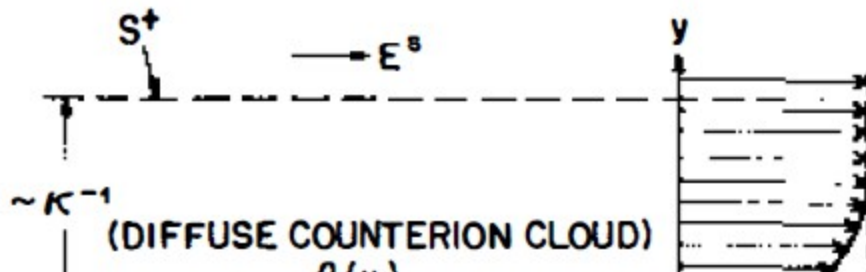
VIEW FROM OUTER REGION



Phoresis:
field-driven force-free propulsion

J.L. Anderson
Annu Rev Fluid Mech 1989

VIEW FROM INNER REGION

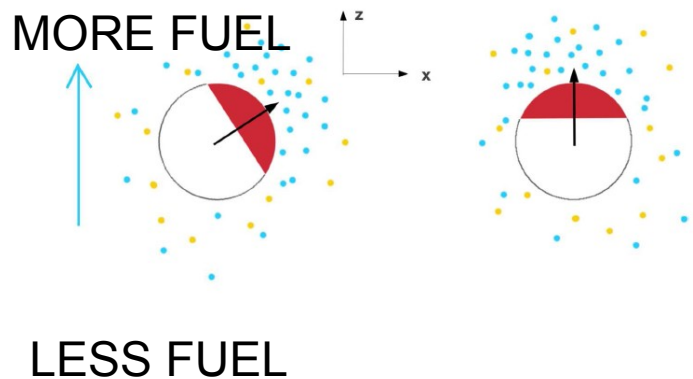


Apparent slip velocity v_s

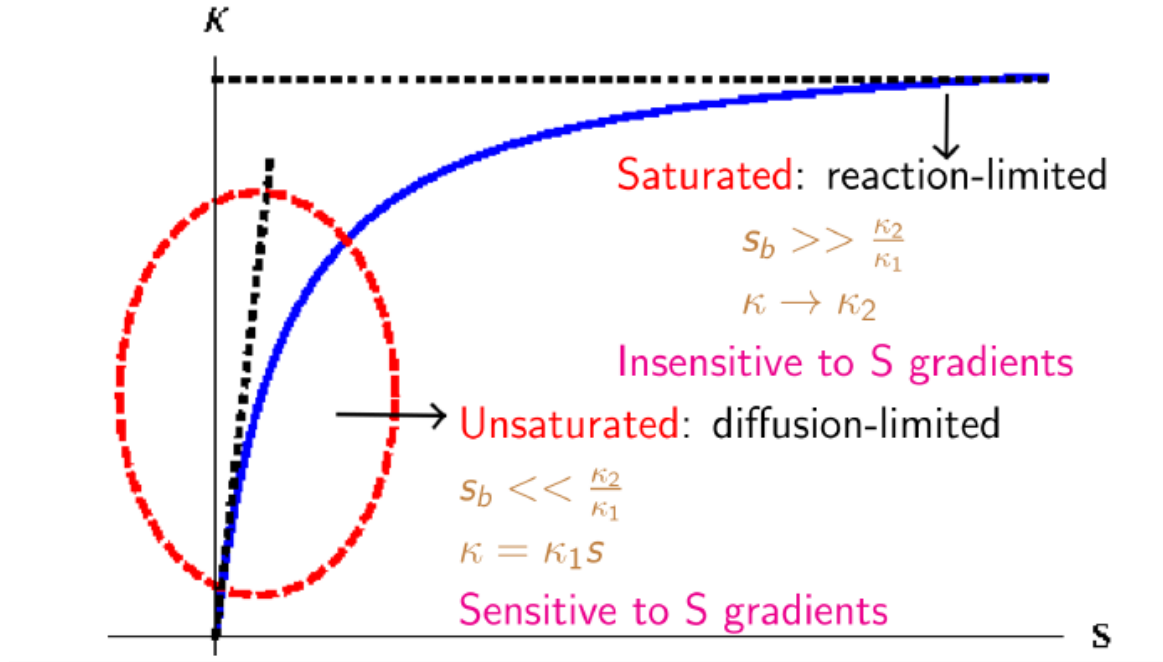
Imitate chemotaxis?

Suropriya Saha, R Golestanian, SR
Phys Rev E 2014

- Polar profile of reactivity, uniform medium --> motion
- Can particle orient in response to gradient?
- Yes! Can design chemotactic or antichemotactic particles
- Need $l=1$ in interaction with surface and $l=2$ in reactivity or v.v.



Interaction range from MM

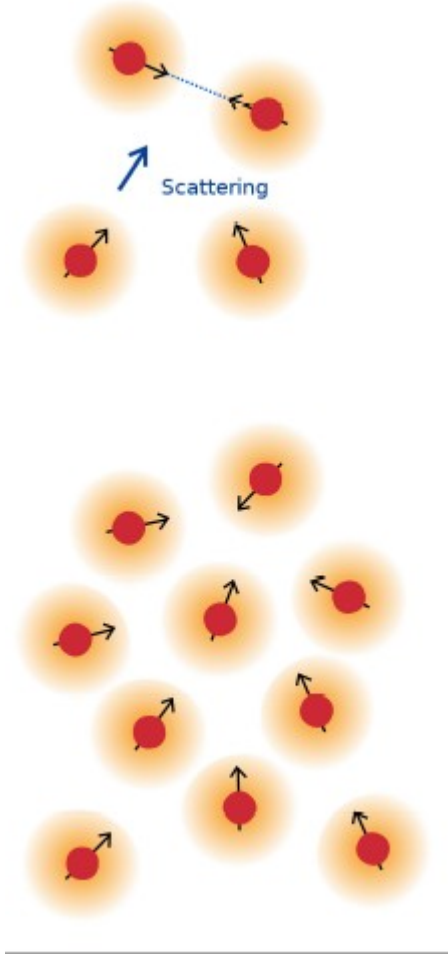


$$\xi_s = [N\rho_0\kappa'(s_0)/D_s]^{-1/2}$$

Abundant reactant: long-range
 Sparse reactant; screened

Interactions between active colloids

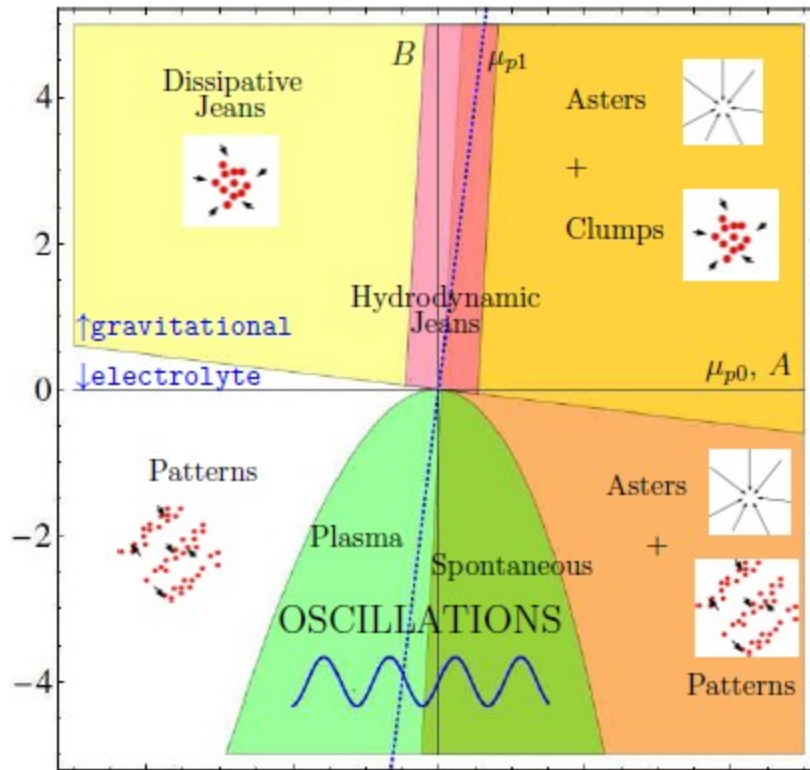
- Active colloid modifies cloud
- Another senses the resulting gradient
- Reorients, moves towards or away
- Diffusion: $1/r$ interaction, long ranged



Collective behaviour

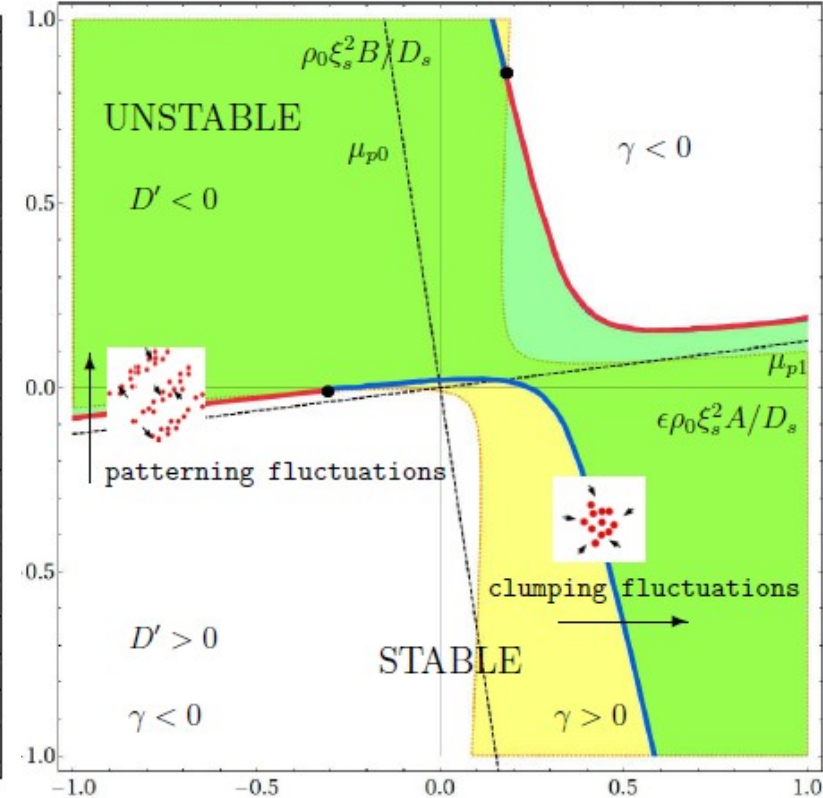
Reorienting tendency

Signs of coefficients
gravity-like or Coulomb-like
Oscillations seen
in computer expts
– Stark et al



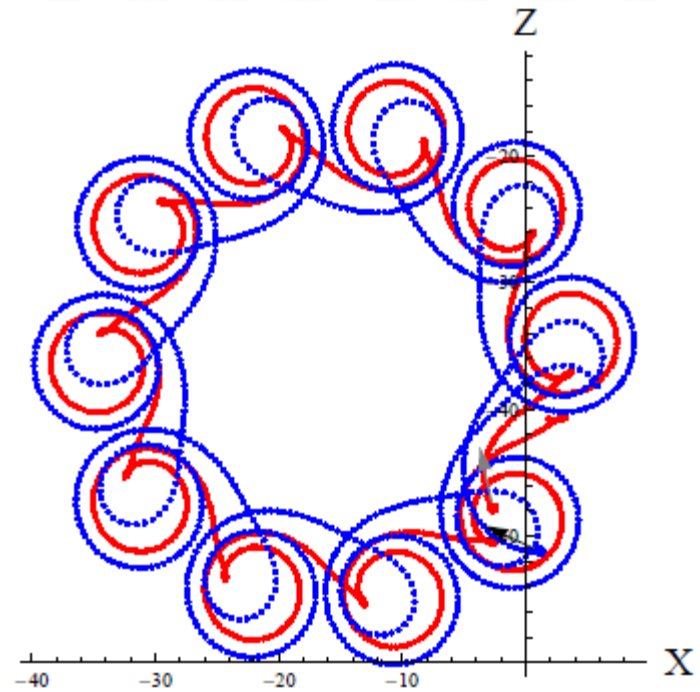
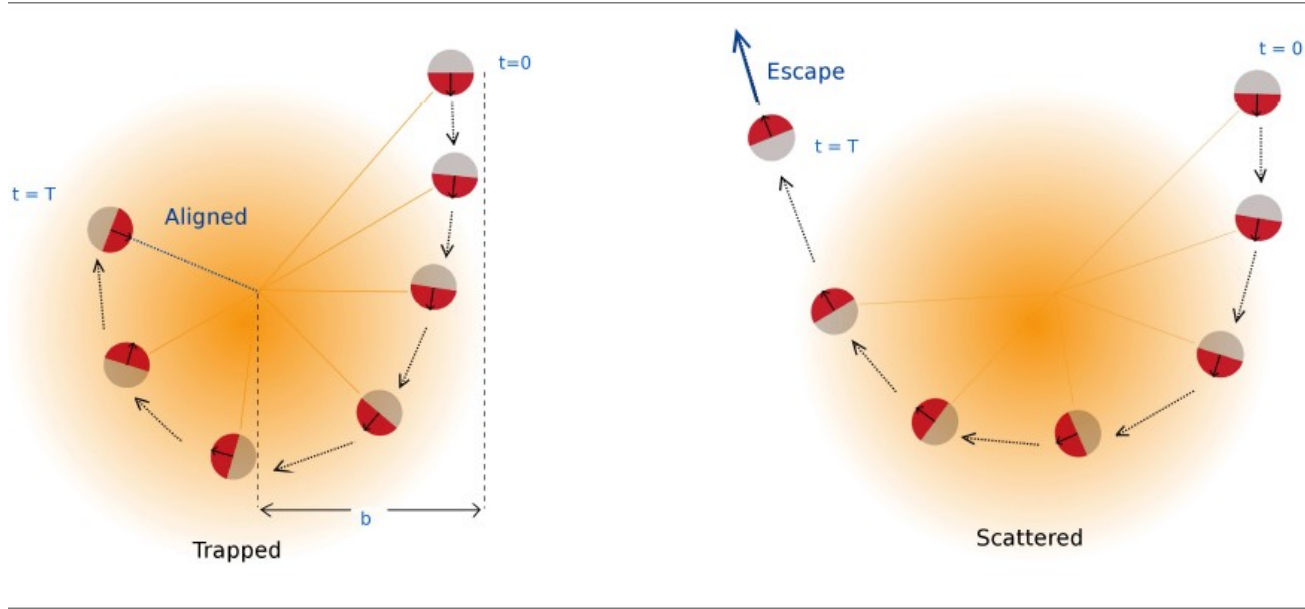
Reaction-limited
Fuel abundant
Long-range effects

Gradient-climbing tendency



Diffusion-limited
Fuel scarce
screening

Scattering or trapping, pair dances



More recent: Saha, SR, Golestanian NJP 2019
Pairing, waltzing Non-reciprocity

WET ACTIVE MATTER

flock in fluid

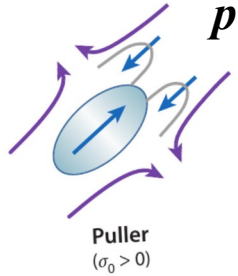
Comoving co-rotating derivative

Extensional flow orients

Thermodynamic relaxation

orientation

$$\mathcal{D}_t \mathbf{p} = \lambda \mathbf{S} \cdot \mathbf{p} - \Gamma \frac{\delta F}{\delta \mathbf{p}}$$



flow

$$-\mu \nabla^2 \mathbf{u} = -\sigma_a \nabla \cdot (\mathbf{p}\mathbf{p}) + \text{elasticity} + \text{pressure}$$

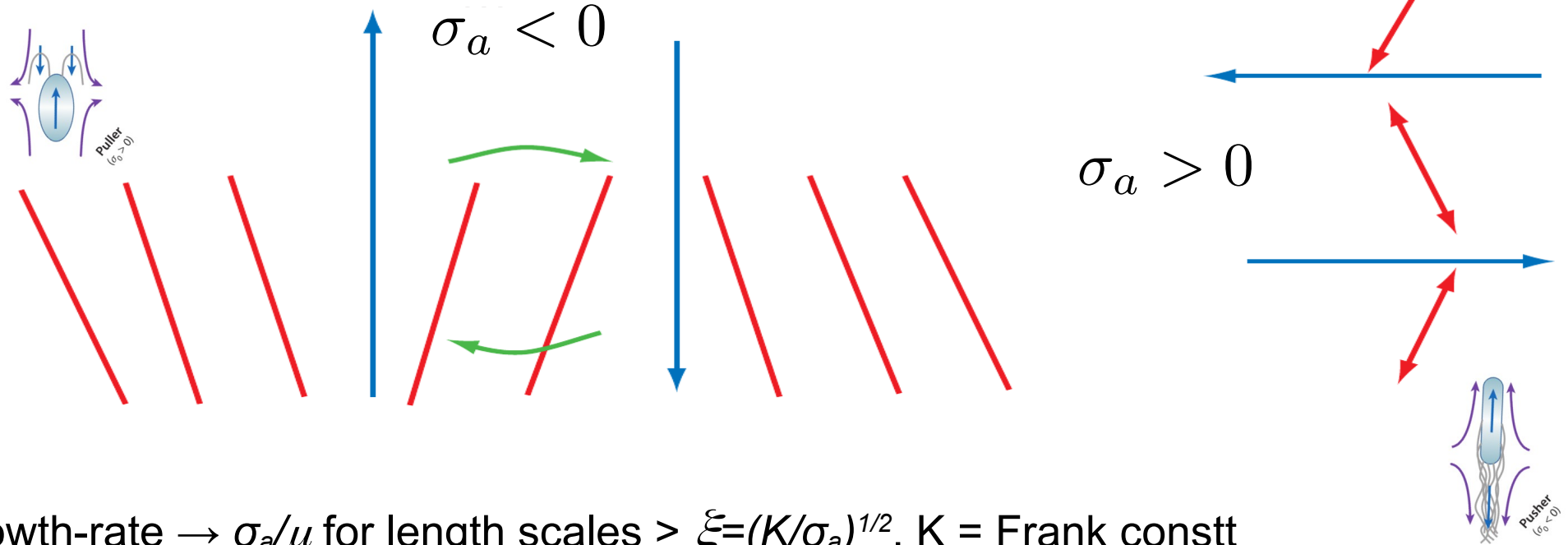
Viscosity

$$\text{Active stress} \propto \mathbf{p}\mathbf{p}$$

F = free energy favouring alignment; \mathbf{u} = incompressible velocity field
 \mathbf{p} = orientation (* not quite, but $\mathbf{p} \rightarrow -\mathbf{p}$ invariant), \mathbf{S} = deformation rate

Swimmers are force dipoles

viscous hydrodynamics, neglect inertia: unstable without threshold



Growth-rate $\rightarrow \sigma_a/\mu$ for length scales $> \xi = (K/\sigma_a)^{1/2}$, K = Frank constt
 viscosity/active stress: a single timescale

A single timescale, direction dependent

$$\omega = i \frac{\sigma_a}{\mu} \cos 2\theta (1 + \lambda \cos 2\theta) \quad \text{bend-splay}$$

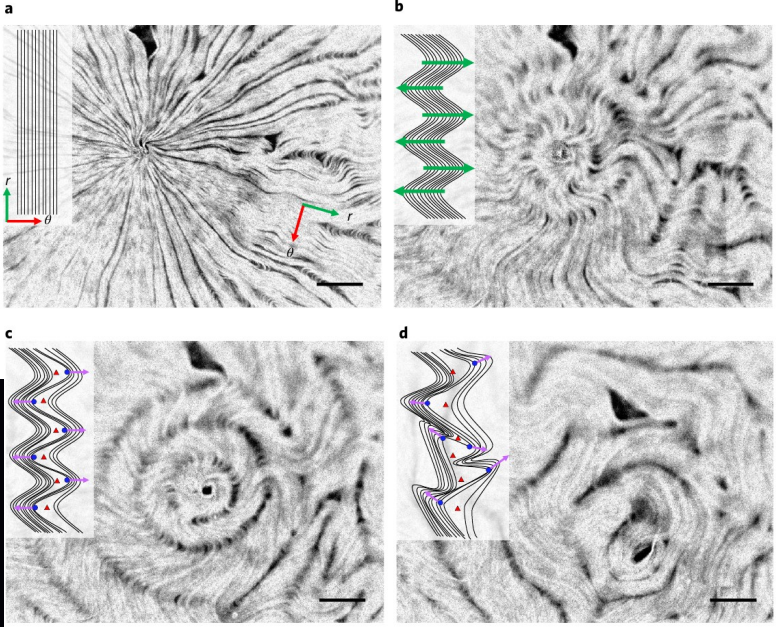
$$\omega = i \frac{\sigma_a}{\mu} \cos^2 \theta (1 + \lambda) \quad \text{bend-twist}$$

Bend-Twist not mitigated by interpolation to splay; should dominate in 3D extensile

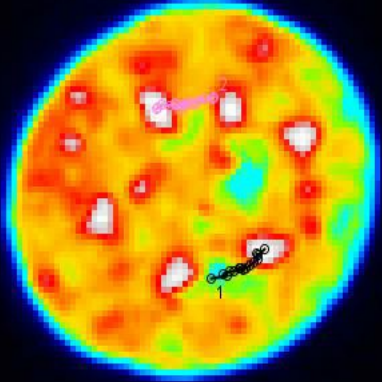
see Shendruk, Thijssen, Yeomans, Doostmohammadi, PRE 2018

Detailed confirmation: active nematic of microtubules + motors + ATP
 Martínez-Prat, Ignés-Mullol, Casademunt & Sagués, Nat Phys 2019

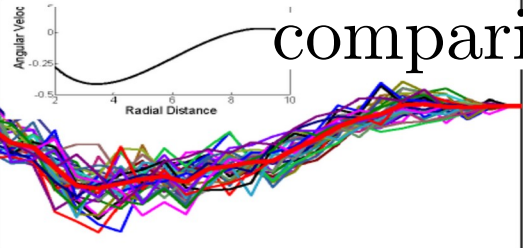
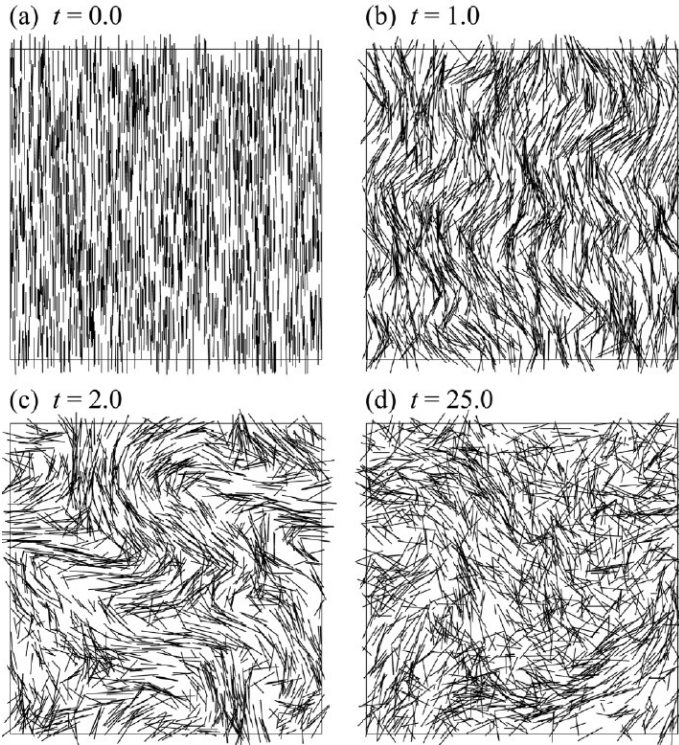
Other evidence, e.g. nuclear rotation in cell:
 Kumar, Maitra, Sumit, SR, Shivashankar 2014



450 sec



aintillan & Shelley 2007
 active turbulence
 alert, Casademunt, Joanny ARCMP 2023

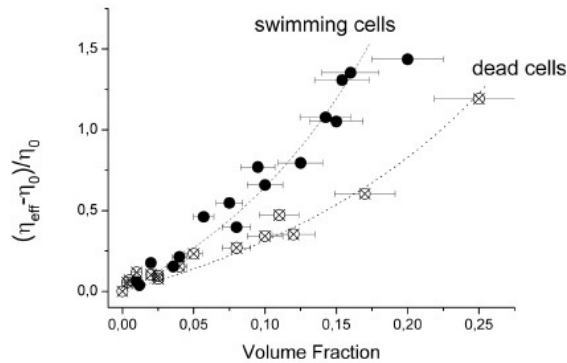
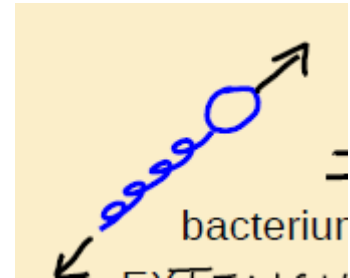
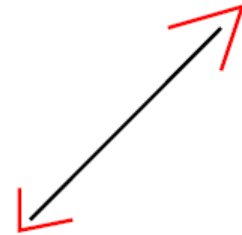


Swimming affects viscosity: theory and experiment

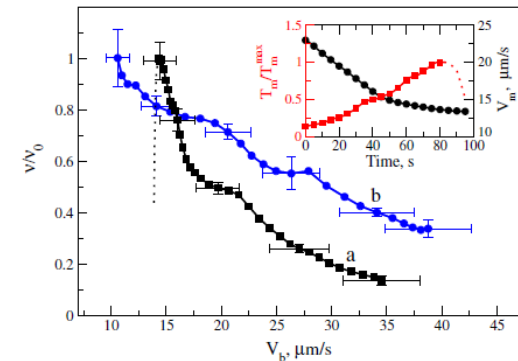
Hatwalne et al. PRL 2004

more viscous alive than dead

less viscous alive than dead



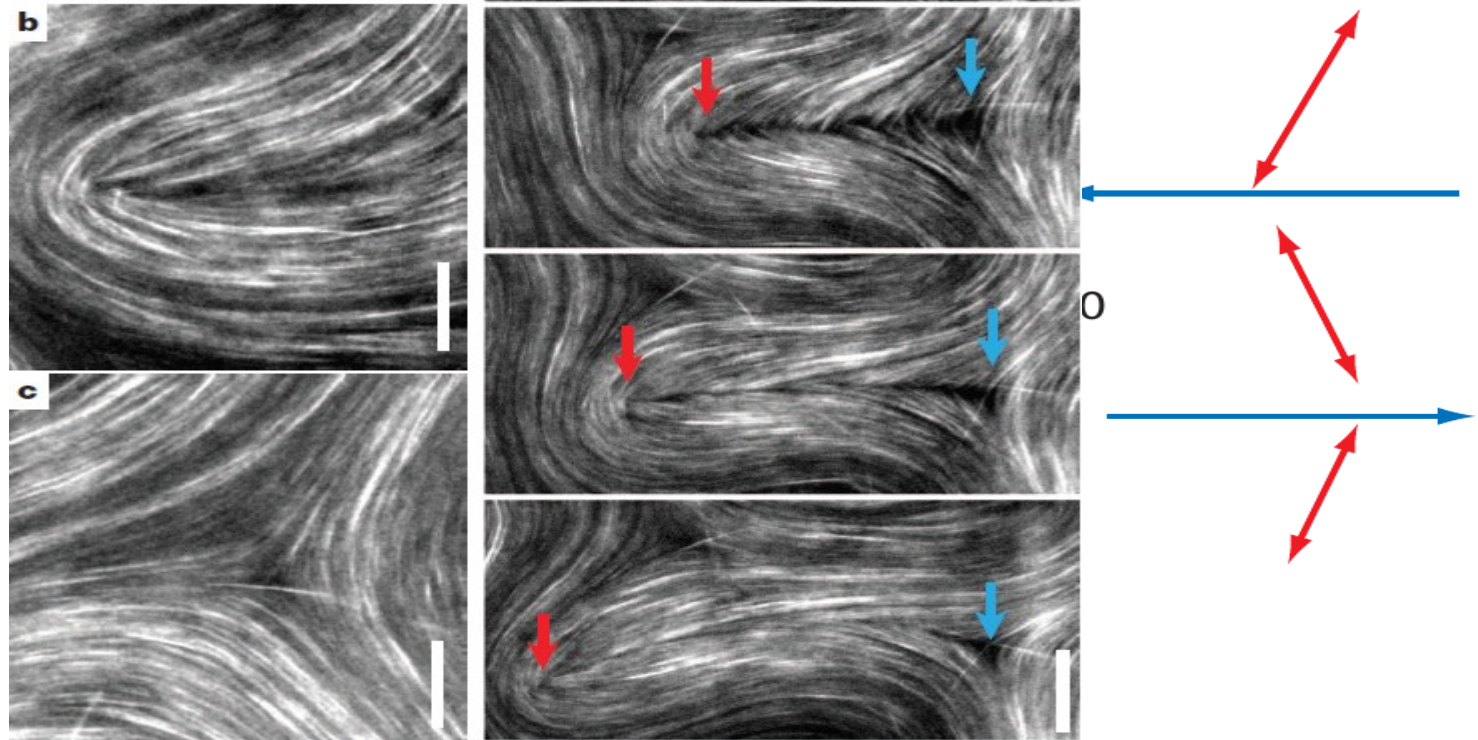
Rafai et al 2010
Sokolov & Aranson 2009
Gachelin et al 2012



Bulk active LCs are unstable, turbulent

Simha & SR PRL 2002

Sanchez et al 2012
(Dogic group)



Strength 1/2 defects take over; active turbulence (Yeomans, Marenduzzo, Cates, Marchetti, Dunkel, Giomi
Experiments: Sagues Also in systems on substrate: epithelia (Sano, Silberzan....); bacteria (Dombrowski, Shuang, Aranson, Lavrentovich....); surprises in confined active nematic & polar systems: Maitra et al PNAS 2018, PRL 2019...

But large fast flocks in fluid are stable



J Bertsch <http://www.thalassagraphics.com/blog/?p=167>

Chisholm, Legendre, Lauga, Khair JFM 2016
Wang and Ardekani JFM 2012; Li, Ostace, Ardekani PRE 2016
Dombrowski, Jones, Katsikis, Bhalla, Griffith, Klotsa PR Fluids 2019
Klotsa, Baldwin, Hill, Bowley, Swift, PRL 2015

Becker et al. NComms 2015
Filella et al. PRL 2018

How much inertia do you need to stabilise a flock in fluid?

With inertia: outswim the viscous instability?

velocity

$$\underline{\rho(\partial_t \mathbf{u} + \mathbf{u} \cdot \nabla \mathbf{u})} = -\nabla P + \mu \nabla^2 \mathbf{u} + \nabla \cdot (\boldsymbol{\Sigma}^a + \boldsymbol{\Sigma}^r)$$

pressure
viscosity
Active and passive order-parameter stresses

orientation

$$\partial_t \mathbf{p} + (\mathbf{u} + v_0 \mathbf{p}) \cdot \nabla \mathbf{p} = \lambda \mathbf{S} \cdot \mathbf{p} + \boldsymbol{\Omega} \cdot \mathbf{p} + \Gamma \mathbf{h}$$

Advection
shear-alignment
rotation
relaxation

$$\boldsymbol{\Sigma}^a \equiv -\sigma_a \mathbf{p} \mathbf{p}$$

$$\nabla \cdot \mathbf{u} = 0$$

$$\mathbf{h} = -\delta F / \delta \mathbf{p}$$

F free-energy functional
favouring nonzero $\langle p \rangle$

The key dimensionless number

$$R = \rho v_0^2 / 2\sigma_a$$

self-propulsion speed v_0

instability invasion speed $\sqrt{\sigma_a/\rho}$

What's that?

Recall Stokesian limit: scale-independent growth rate σ_a/μ

Include inertia (linearised Navier-Stokes)
growth rate linear in wavenumber q

$$\omega \propto q \left(v_0^2 - 2\sigma_a \frac{1 + \lambda}{\rho} \right)^{1/2}$$

self-propulsion speed v_0

instability invasion speed $\sqrt{\sigma_a/\rho}$

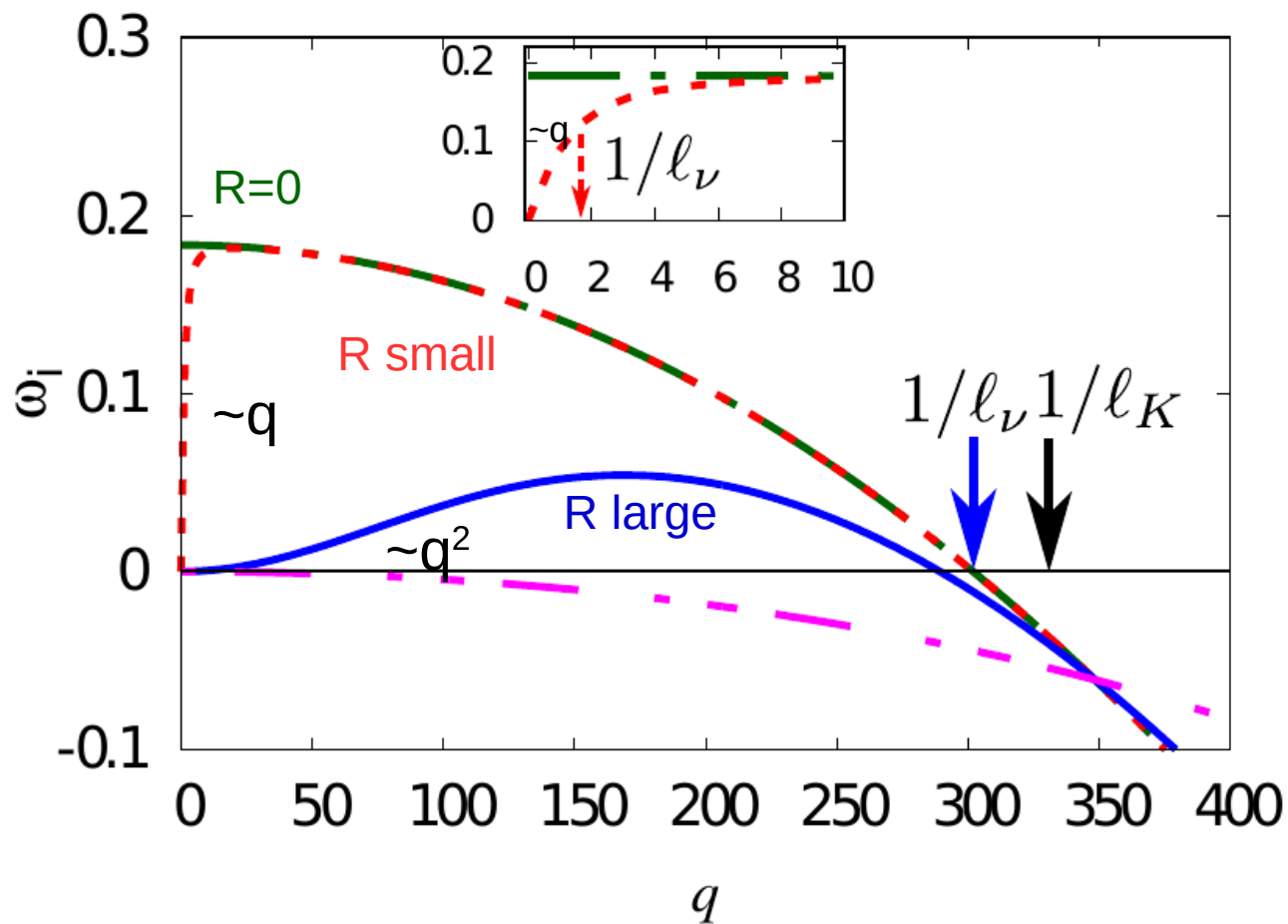
growth/decay of twist-bend mode

$$R \equiv \rho v_0^2 / 2\sigma_0$$

$$\omega \propto q \left(v_0^2 - 2\sigma_a \frac{1+\lambda}{\rho} \right)^{1/2} + i q^2 \frac{\mu}{2\rho} \frac{v_0}{\left(v_0^2 - 2\sigma_a \frac{1+\lambda}{\rho} \right)^{1/2}}$$

for $R \gtrsim 1 + \lambda$

$$q \ll \rho v_0 / \mu, \sqrt{\rho \sigma_a} / \mu$$

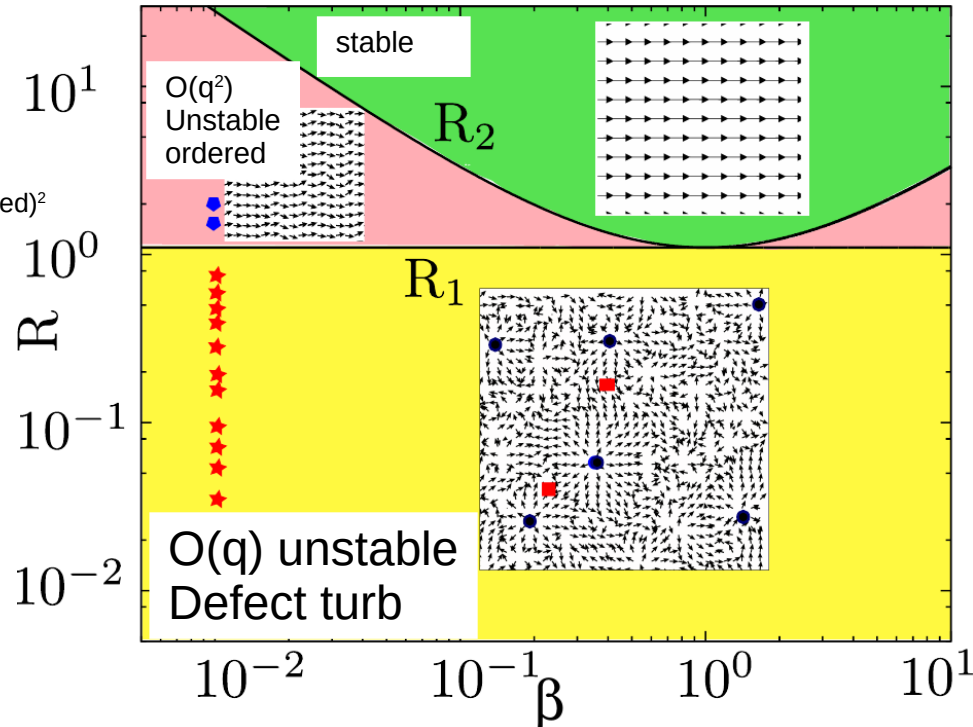


Inertia: outrun Stokesian instability

defect to phase turbulence: disorder to flock

$$R \equiv \rho v_0^2 / 2\sigma_0$$

(swim speed/instab-invasion speed)²



Inertia Drives a Flocking Phase Transition in Viscous Active Fluids

Rayan Chatterjee, Navdeep Rana, R. Aditi Simha, Prasad Perlekar, and Sriram Ramaswamy

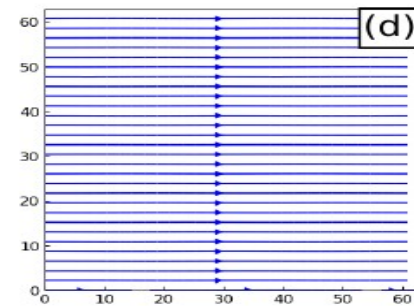
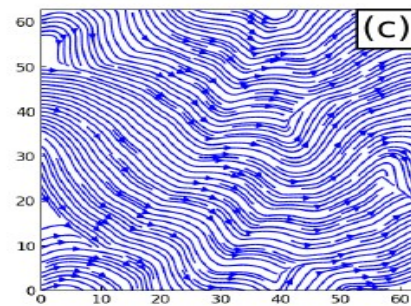
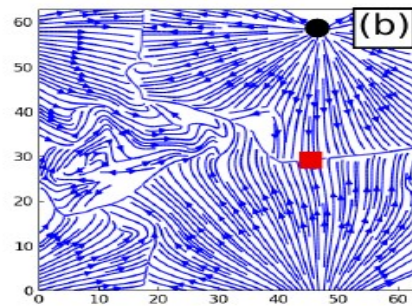
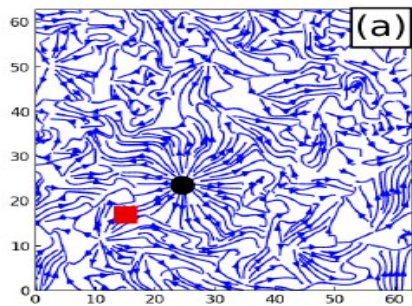
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$$\beta \equiv \rho \Gamma K / \mu$$

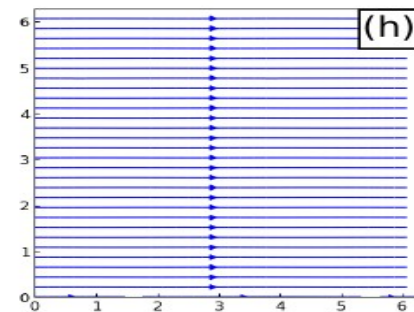
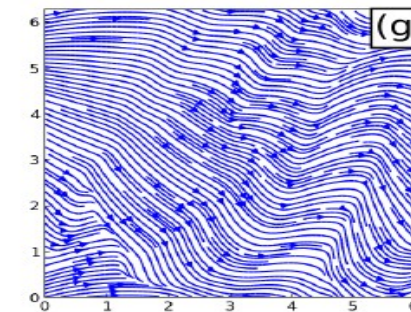
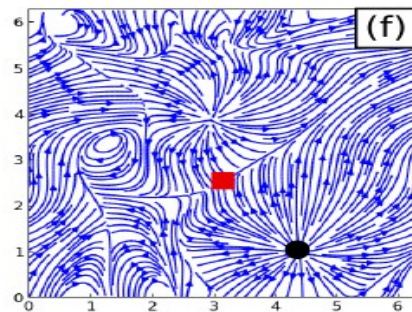
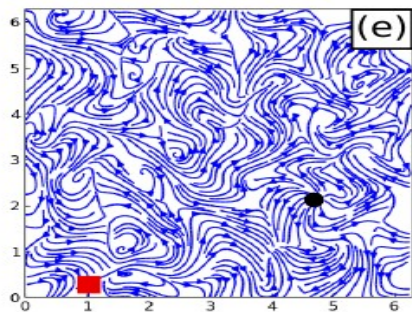
Orientational diffusion / vorticity diffusion

From defect turbulence to phase turbulence

2D



3D



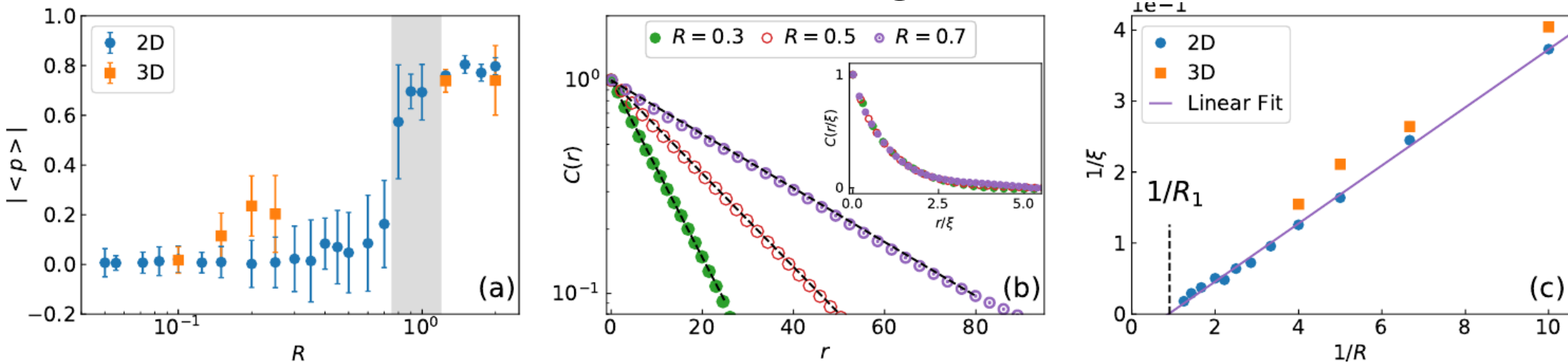
defect turbulence in the $O(q)$ growth regime

phase turbulence in the $O(q^2)$ growth regime

Polar system: defects are hedgehogs (normal and hyperbolic)

Phase turbulence: ordered state

Inertial flocking transition



- Order parameter $|\langle \mathbf{p} \rangle| = \left\langle \sqrt{\langle p_x \rangle^2 + \langle p_y \rangle^2 + \langle p_z \rangle^2} \right\rangle_t$ onset near $R_I \simeq 1$.

- correlation fn. $C(r) = \frac{\langle p(\mathbf{x}) \cdot p(\mathbf{x} + \mathbf{r}) \rangle}{\langle p(\mathbf{x})^2 \rangle}$ shows scaling collapse.

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- Correl length \sim inter-defect distance ξ grows near $R_I \simeq 1$

SUMMARY

- General framework for “powered” matter
- Phase diagram depends on dynamical regime
- Imitations of motility; complex media
- Active turbulence and escaping it

EXPERIMENTAL
SYSTEM?