Confined active matter

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Active matter in complex environments: ICTS Nov 2023

1: Fundamentals

- Systems, symmetries, dynamical regimes
- From equilibrium Langevin to active dynamics
- Active broken-symmetry hydrodynamics

2: Flocks in fluid

- Slow flocks in bulk fluid and in fluid films
- Fast flocks in fluid
  - flocking driven by fluid inertia
  - ordered and disordered “turbulence”

3: Motility in a medium of obstacles

- Interacting crawlers in a bead fluid
- Interacting crawlers in an elastic medium
- Trapping active particles

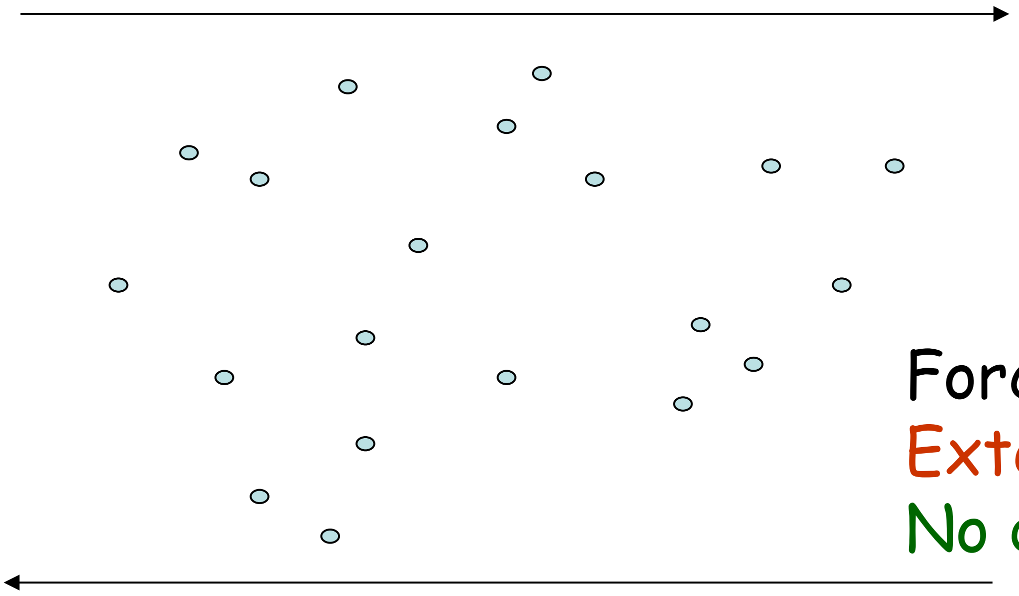
Fundamentals and dry flocks

INTRODUCTION

DRIVEN, ACTIVE, LIVING

How to drive

shear: from boundary



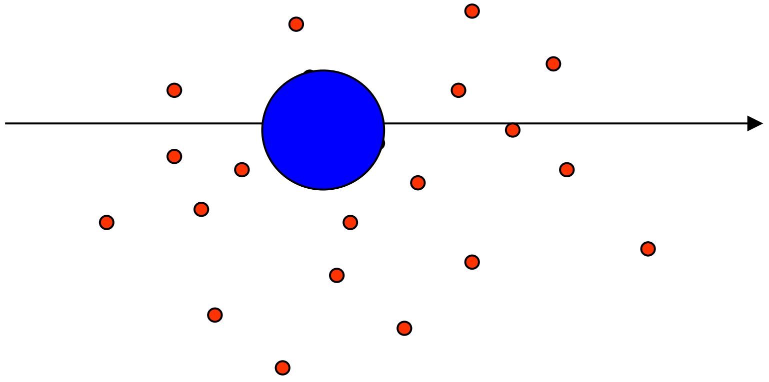
Force-free

Externally directed

No drift

How to drive

electric field, etc – “phoresis”: driven in bulk

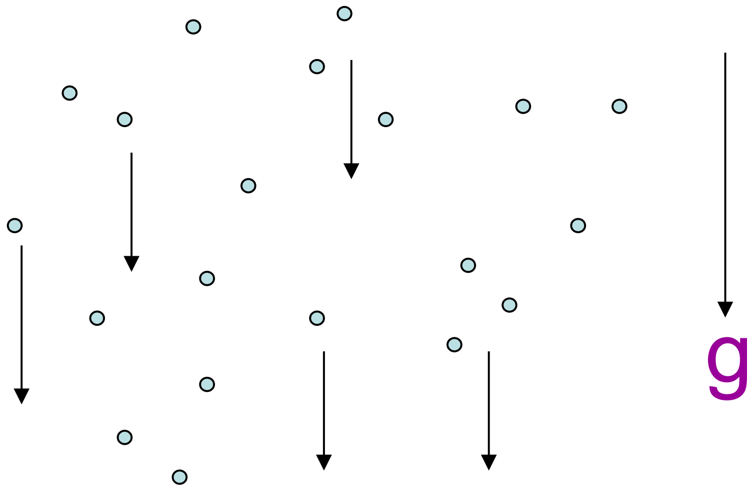


Force (co-ion + counterions) =0

Force-free
Externally directed
Nonzero drift

How to drive

sedimentation: in bulk



bound orbit

scattering orbit

tumbling orbit

rocking orbit

sedimenting disc array

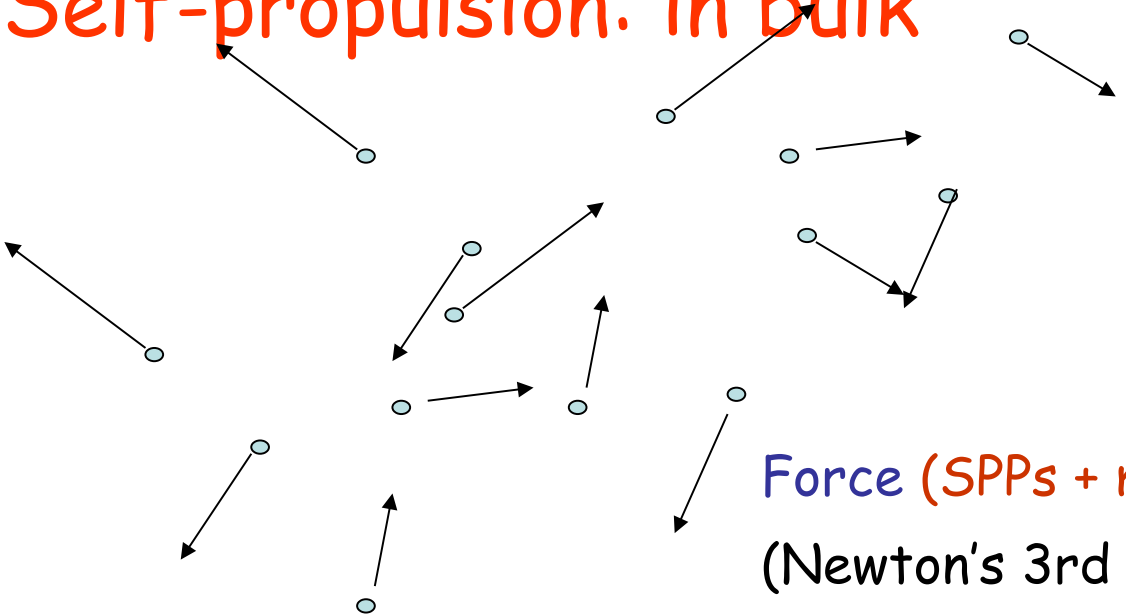
Chajwa, Menon, SR PRL 2018
Chajwa, Menon, SR, Govindarajan PRX 2020

Body force
Externally directed
Nonzero drift

How to drive

Living or active matter

Self-propulsion: in bulk



But force-free

Internally directed

Mean drift

depends on state

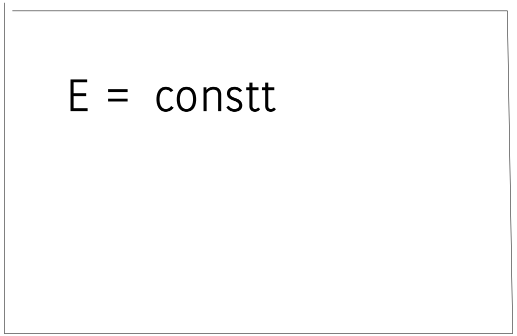
of order

Force (SPPs + medium) =0

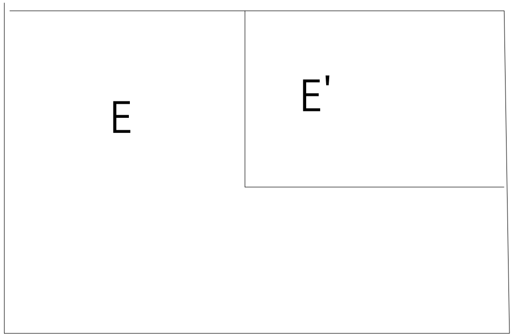
(Newton's 3rd Law)

Contrast with sedimentation, electrophoresis, shear

Thermal equilibrium: “closed” systems



$E = \text{constt}$



$E + E' = \text{constt}$

Temperature of subsystem = constt

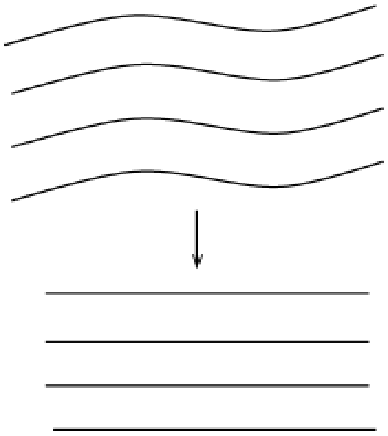
$t \leftrightarrow -t$

Know the rules

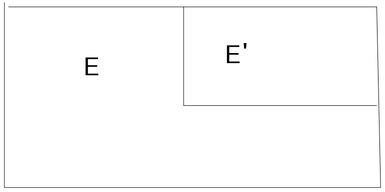
Isolated: probability uniform on constant-energy states

Almost isolated: probability ~ $\exp(-E/k_B T)$

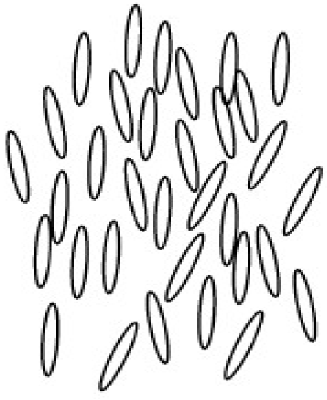
Thermal equilibrium: “closed” systems



E = constt



E + E' = constt
Temperature of subsystem = constt



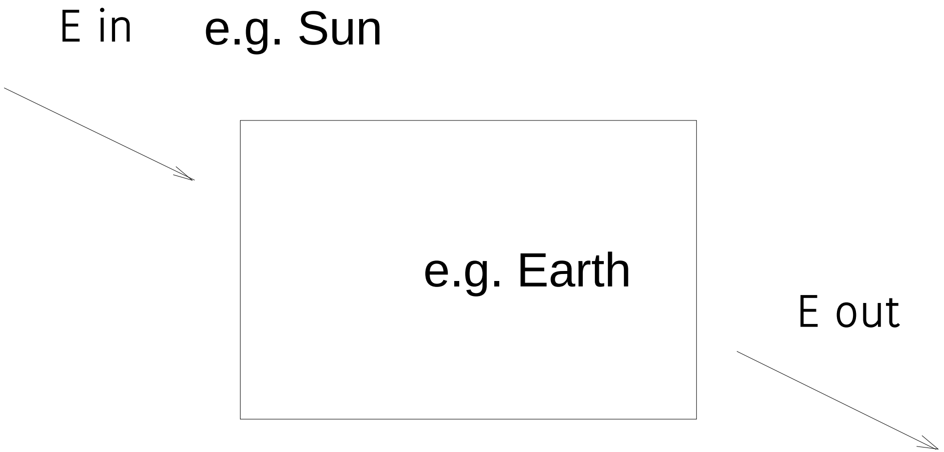
NEMATIC

- thermal equilibrium
  - isolated: max entropy; E, N fixed
  - almost: fix <E> and <N>
- temperature T, chem potl μ
- prob(C) ~ $\exp[-(E + \mu N)/k_B T]$

High density: order to *increase* entropy
Distortion lowers S: restoring force
Fluctuations: Prob(C) = $\exp[\Delta S(C)]$
Dynamics: derivatives of S --> ``forces''

Driven, active, living

open systems & open questions



What kinds of states can form?
Don’t know the general rules

~~$t \leftrightarrow -t$~~

Active matter

- Active particles are alive, or “alive”
  - living systems; their components; artificial realisations
  - Time’s Arrow at particle scale
  - steadily dissipate energy and produce work
  - collectively: active matter

Hydrodynamic description

- Slow variables
  - timescale $\rightarrow \infty$ as lengthscale $\rightarrow \infty$
  - densities of conserved quantities: k=0 mode is the conserved qty
  - Nambu-Goldstone modes*: restoring force = 0 at zero wavenumber
  - order parameter near onset if continuous (strictly), or in coarsening

* includes height field of interface, Rouse modes of polymer or membrane

Examples

- Simple fluid/suspension
  - mass, momentum, energy densities (+ species concentrations)
- Orientationally ordered fluid, e.g., nematic
  - and director field
- Density wave (solid, smectic, columnar)
  - and displacement field
- Heisenberg antiferromagnet
  - magnetisation (conserved), staggered magnetisation

Spatial variation of density of conserved quantity

excess deficit

+ + + + + + + + - - - - - - - -

wavelength λ

$$\text{time to remix} \rightarrow \infty \text{ as } \lambda \rightarrow \infty$$

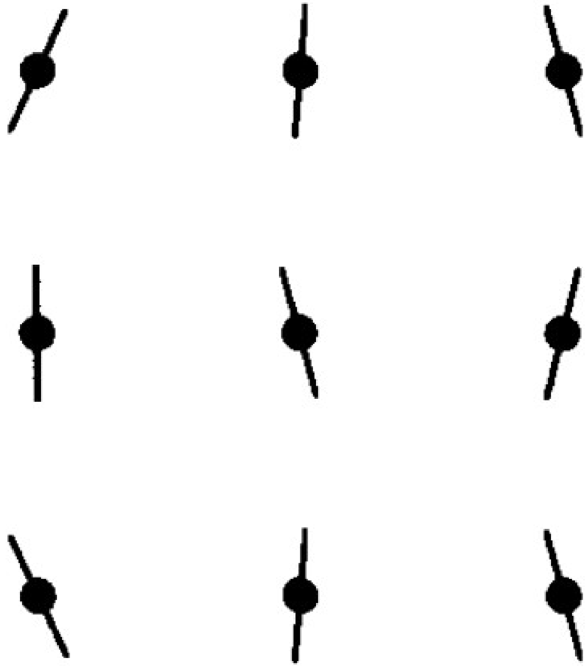
That's "hydrodynamic"

Hydrodynamic description of a simple fluid

Mass density	$\partial_t \rho = -\nabla \cdot \mathbf{g}$
Momentum density	$\partial_t g_j = -\nabla_i \Pi_{ij}$
energy density	$\partial_t \varepsilon = -\nabla \cdot \mathbf{J}_\varepsilon$

Express RHS in terms of variables on LHS by: microscopic theory or general principles

Broken-symmetry modes



In ordered phase: rotor angle field “hydrodynamic”

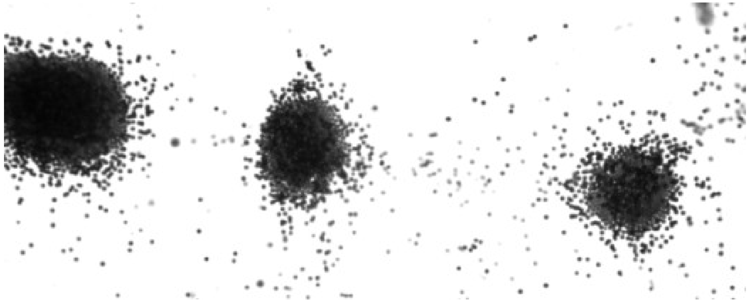
Rotate all by same amount: no restoring torque

Relaxation rate → 0 as wavelength → ∞

SYSTEMS, SYMMETRIES, DYNAMICAL REGIMES

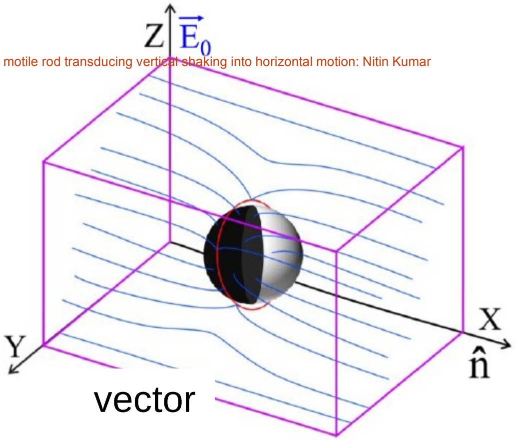
- Active particles are alive, or “alive”
  - each component powered; not wire + battery
  - each constituent carries dissipative Arrow of Time
  - steadily transduce free energy to movement
  - collectively: active matter

Systems



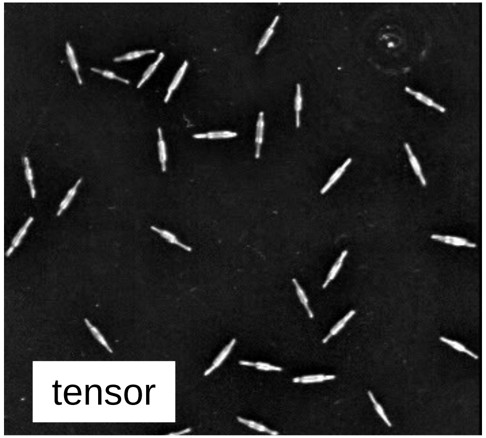
Scalar active matter
Chemically propelled droplets
S Thutupalli NCBS

A motile rod transducing vertical shaking into horizontal motion: Nitin Kumar

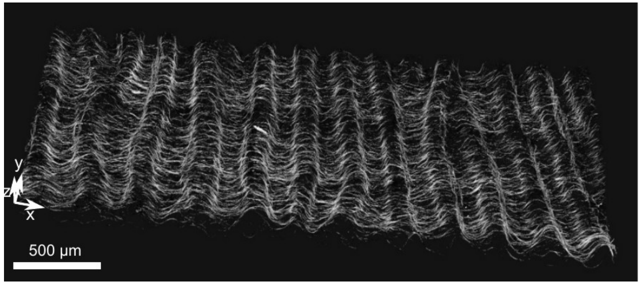


Sahu et al. 2020

a motile dimer: noise turned into directed movement



broken equipartition -- Vijay Narayan 2007



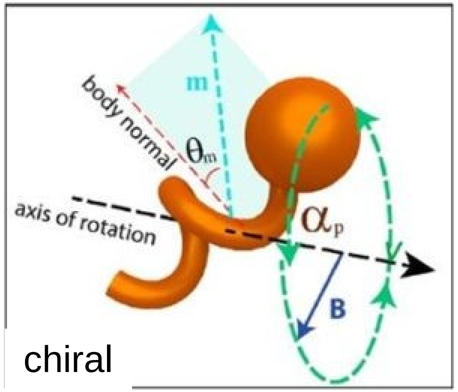
Extracts from a cell
Senoussi et al 2019

Zero-resistance states induced by electromagnetic-wave excitation in GaAs/AlGaAs heterostructures

Ramesh G. Mani*†, Jürgen H. Smet†, Klaus von Klitzing†, Venkatesh Narayanamurti*‡, William B. Johnson§ & Vladimir Umansky||

R.G. Mani et al. 2002
Alicea et al. PRB 2005: "... connection of our work to the well-studied phenomenon of 'flocking'"

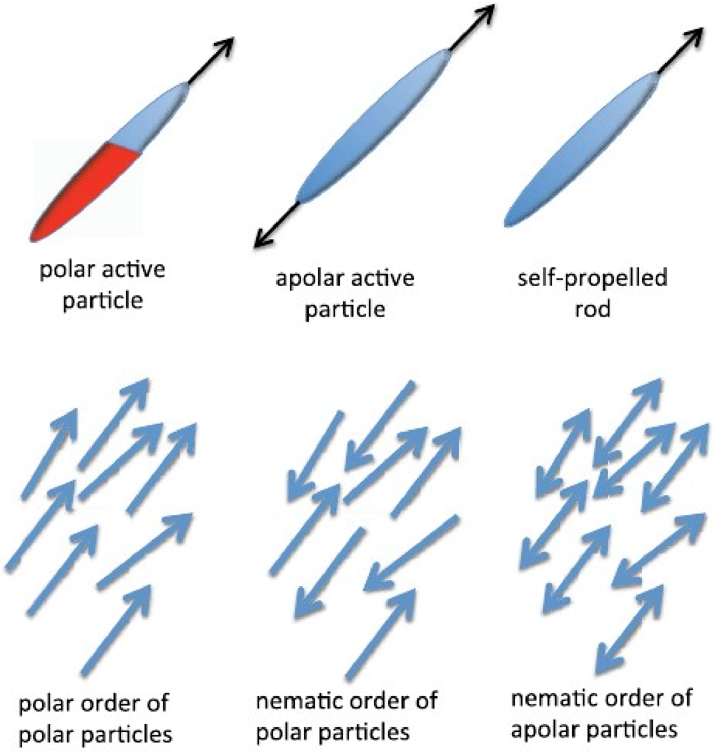
quantum



Magnetic nanopropellors: Ambarish Ghosh, IISc

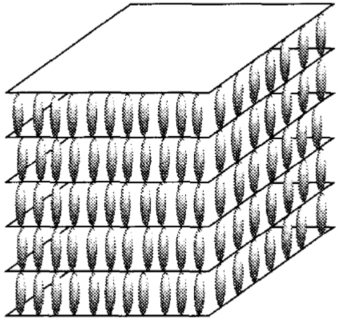
Broken symmetries: orientational

Polar and apolar uniaxial
Particles and phases

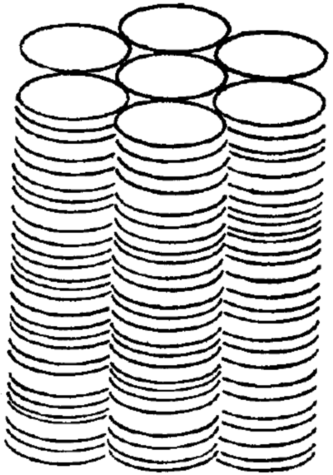


Marchetti et al., Rev Mod Phys 2013

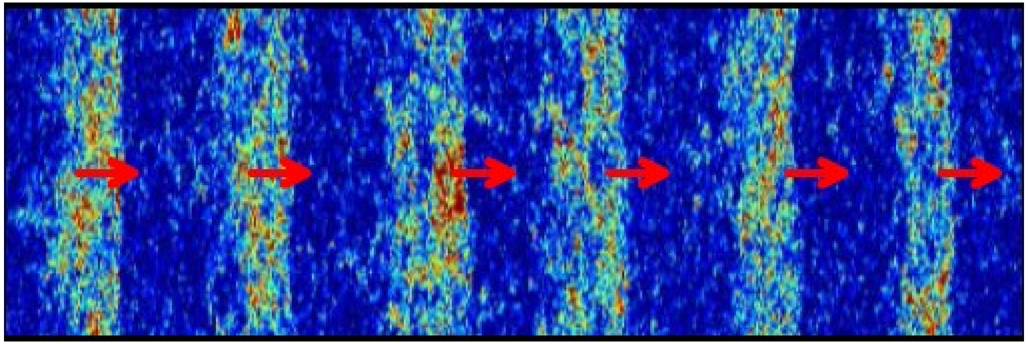
Broken symmetries: translational



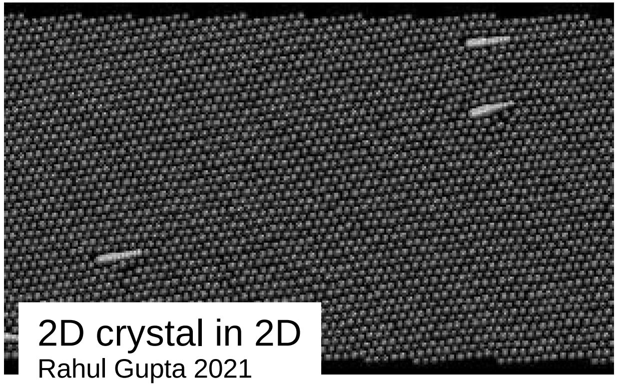
1D crystal in 3D
Chaikin & Lubensky 1994



2D crystal in 3D
Chandrasekhar, Sadashiva & Suresh 1977

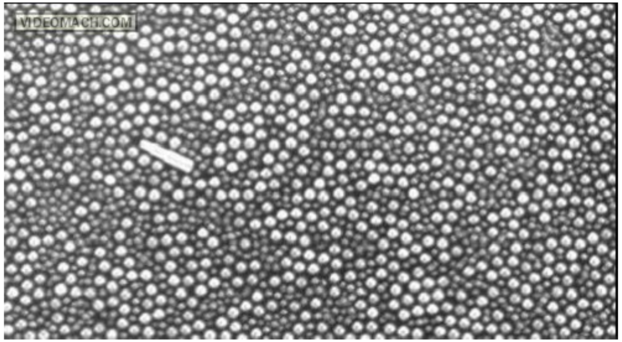


Moving 1D crystal in 2D
Solon et al. 2015



2D crystal in 2D
Rahul Gupta 2021

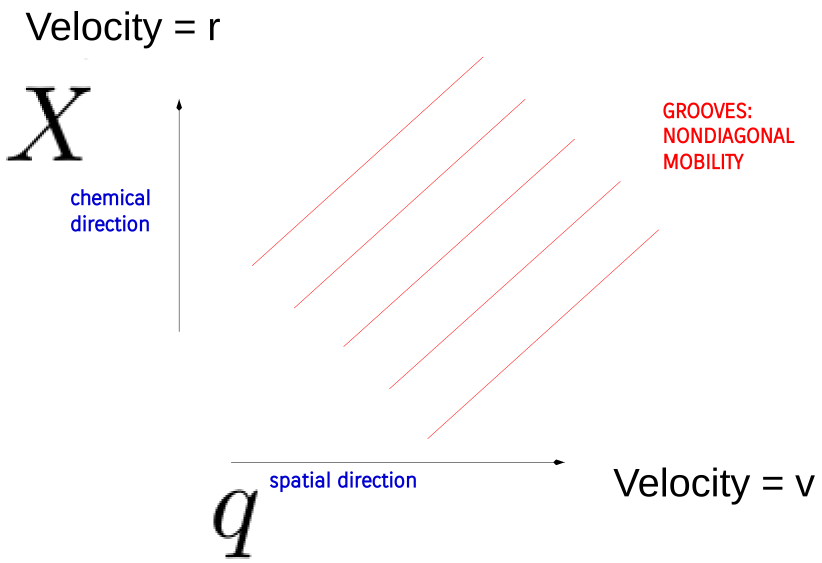
Chiral 1D crystal in 3D
Whitfield et al. 2017



active amorp
h

Active amorphous solid in 2D
Nitin Kumar 2014

FROM EQUILIBRIUM LANGEVIN TO ACTIVE DYNAMICS



SR JSTAT 2017
Dadhichi, Maitra, SR JSTAT 2018
Jülicher, Ajdari, Prost RMP Colloq 1993
Marchetti et al. RMP 2013

Motor: catalyst for fuel breakdown

Include chemical direction in configuration space

Driving force $\Delta\mu = \mu_{\text{reactant}} - \mu_{\text{product}}$ in *chemical* direction

Mobility nondiagonal: vel = Mob*Force has *spatial* component

Formalise this: build active dynamics; discover “new” terms

From Langevin equations to active dynamics

SR JSTAT 2017
Dadhichi, Maitra, SR JSTAT 2018

Temperature T; effective Hamiltonian $H(q,p,X,\Pi)$

q (time-rev even), p (odd); X, Π: extra coord, momentum

Off-diagonal q-dependent Onsager coefficients

$$\dot{q} = \partial_p H$$

$$\dot{p} + \Gamma_{11}\partial_p H + \Gamma_{12}(q)\partial_\Pi H = -\partial_q H + \eta$$

$$\dot{\Pi} + \Gamma_{21}(q)\partial_p H + \Gamma_{22}\partial_\Pi H = -\partial_X H + \xi$$

noises η, ξ

$$\dot{X} = \partial_\Pi H$$

$$\langle \eta(0)\xi(t)\rangle = 2k_B T\Gamma_{12}(q)\delta(t)$$

eliminate $\dot{X}$ from the p equation

From Langevin equations to active dynamics

Temperature *T*; effective Hamiltonian $H(q,p,X,\Pi)$

q (time-rev even), *p* (odd); X, Π: extra coord, momentum

Off-diagonal q-dependent Onsager coefficients

$$\begin{aligned}
\dot{q} &= \partial_p H \\
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\dot{\Pi} + \Gamma_{21}(q)\partial_p H + \Gamma_{22}\partial_\Pi H &= -\partial_X H + \xi \\
\dot{X} &= \partial_\Pi H
\end{aligned}$$

eliminate $\dot{X}$ from the p equation

noises η, ξ

$$\langle \eta(0)\xi(t)\rangle = 2k_B T\Gamma_{12}(q)\delta(t)$$

SR JSTAT 2017

From Langevin equations to active dynamics

$$\dot{q} = \partial_p H$$

$$\dot{p} + \Gamma \partial_p H - \frac{\Gamma_{12}(q)}{\Gamma_{22}} \partial_X H = -\partial_q H + f$$

$$\dot{X} + \frac{\Gamma_{12}(q)}{\Gamma_{22}} \partial_p H = -\frac{1}{\Gamma_{22}} \partial_X H + \frac{\xi}{\Gamma_{22}}$$

$f \equiv \eta - (\Gamma_{12}/\Gamma_{22})\xi$ has variance $\propto$ $\Gamma \equiv \Gamma_{11} - \Gamma_{12}^2(q)/\Gamma_{22}$

Equilibrium: $\partial_X H = 0$

simplest

Active? Hold $-\partial_X H \equiv -\Delta\mu \neq 0$ fixed

From Langevin equations to active dynamics

$$\dot{q} = \partial_p H$$

$$\dot{p} + \Gamma \partial_p H - \frac{\Gamma_{12}(q)}{\Gamma_{22}} \Delta\mu = -\partial_q H + f$$

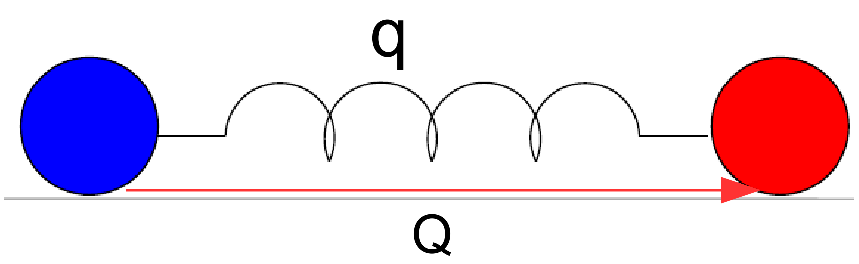
"New" terms, ruled out in equilibrium dynamics.
In general can't hide by redefining H, temperature....

$$\dot{q} + \Gamma^{-1} \partial_q H = \frac{\Delta\mu}{\Gamma_{22}\Gamma} \Gamma_{12}(q) + \Gamma^{-1} f$$

No inertia: q-only equation of motion

Build all(?) active-matter dynamics this way (SR JSTAT 2017, Dadhichi, Maitra, SR 2018)

Active Brownian or active Ornstein-Uhlenbeck from dimers



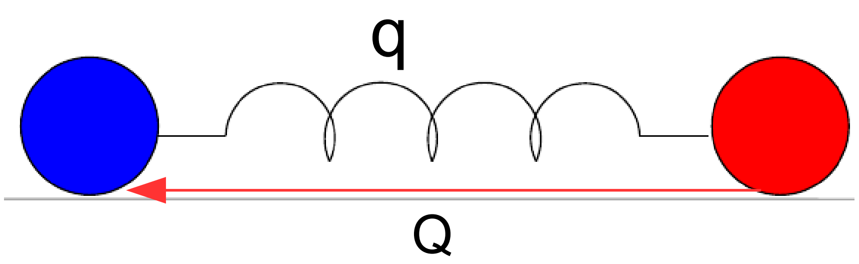
Joint & relative *(q,p)* & *(Q,P)*; "chemical" *(X,Π)*

$$\dot{p} + \gamma_{11}\partial_p H + \boxed{\gamma_{12}(Q)}\partial_\Pi H \;=\; -\partial_q H + \eta$$

$$\dot{\Pi} + \boxed{\gamma_{21}(Q)}\partial_p H + \gamma_{22}\partial_\Pi H \;=\; -\partial_X H + \xi$$

$$\dot{P} + \Gamma_{11}\partial_P H \;=\; -\partial_Q H + \bar{\eta}$$

Active Brownian or active OU* from dimers



Joint & relative *(q,p)* & *(Q,P)*; "chemical" *(X,Π)*

$$\dot{p} + \gamma_{11}\partial_p H + \gamma_{12}(Q)\partial_\Pi H = -\partial_q H + \eta$$

$$\dot{\Pi} + \gamma_{21}(Q)\partial_p H + \gamma_{22}\partial_\Pi H = -\partial_X H + \xi$$

$$\dot{P} + \Gamma_{11}\partial_P H = -\partial_Q H + \bar{\eta}$$

*Ornstein-Uhlenbeck

Active Brownian or active OU from dimers

$$\dot{q} + \frac{\gamma_{22}}{\mathcal{D}}\partial_q H = \boxed{\frac{\gamma_{12}(Q)}{\mathcal{D}}}\Delta\mu + \frac{\gamma_{22}}{\mathcal{D}}\eta - \boxed{\frac{\gamma_{12}(Q)}{\mathcal{D}}}\xi$$

$$\dot{Q} + \frac{1}{\Gamma_{11}}\partial_Q H = \bar{\eta}/\Gamma_{11}$$

$$\mathcal{D} = \gamma_{11}\gamma_{22} - \gamma_{12}(Q)^2$$

$$\gamma_{12}(Q) \propto Q \text{ propels particle}$$

$$\text{Active Brownian: } H \sim -Q\cdot Q + (Q\cdot Q)^2$$

$$\text{Active OU: } H \text{ harmonically binds } Q$$

Additive white noise in q dynamics inevitable.

Apply to a simple field theory

Order-parameter field $\boldsymbol{p} = (p_x, p_y)$
Free-energy functional $F[\boldsymbol{p}]$ favours order at $a<0$
"Model A" dynamics: passive

$$\partial_t \mathbf{p} = -\Gamma^{-1}\frac{\delta F}{\delta \mathbf{p}} + \sqrt{2k_B T/\Gamma}\,\mathbf{f}$$

Unit strength spacetime white noise

$$F = \int d^d x [\frac{a}{2}|\mathbf{p}|^2 + \frac{b}{4}|\mathbf{p}|^4 + \frac{c}{2}|\nabla \mathbf{p}|^2]$$

Apply to a simple field theory

Order-parameter field $\boldsymbol{p} = (p_x, p_y)$
Free-energy functional $F[\boldsymbol{p}]$ favours order at $a<0$
"Model A" dynamics: active

$$\partial_t \mathbf{p} = -\Gamma^{-1}\frac{\delta F}{\delta \mathbf{p}} + \sqrt{2k_B T/\Gamma}\,\mathbf{f}$$

$$+\frac{\Delta\mu}{\Gamma_{22}\Gamma}\Gamma_{12}(\mathbf{p}, \nabla\mathbf{p}, \ldots)$$

Unit strength spacetime white noise

Apply to a simple field theory

Order-parameter field ($\boldsymbol{p} = p_x, p_y$)
Free-energy functional $F[\boldsymbol{p}]$
“Model A” dynamics: active, simplest “new term”

$$\partial_t \mathbf{p} = -\Gamma^{-1}\frac{\delta F}{\delta \mathbf{p}} + \sqrt{2k_B T/\Gamma}\mathbf{f}$$

$$-\lambda \mathbf{p}\cdot\nabla\mathbf{p}$$

The Toner-Tu model
for a flock with only orientation
and no concentration

$$\lambda \propto \Delta\mu$$

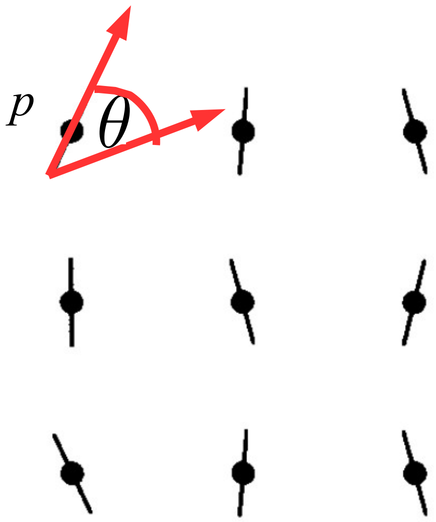
So: planar rotors out of equilibrium self-advect

broader context -->

Broken-symmetry hydrodynamics

slow variables (passive)

- Conserved, broken symmetry, critical
  - timescale --> infinity as length-scale --> infinity
- e.g., rotor lattice: $\boldsymbol{p}$, θ, energy density, “spin” ang mom S



Time-independent Hamiltonian H so energy conserved
Rotations commute with H so S conserved

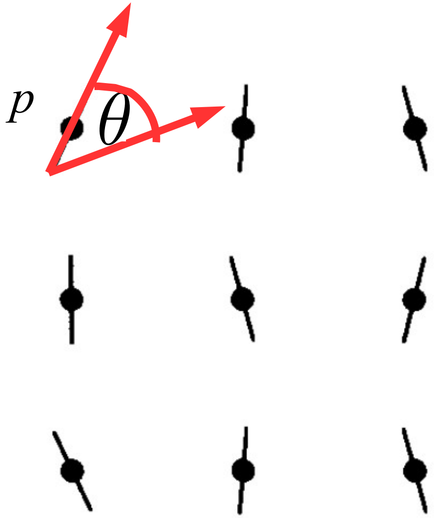
Chaikin & Lubensky
ch. 8

Broken-symmetry hydrodynamics

slow variables (active)

- Conserved, broken symmetry, critical
  - timescale --> infinity as length-scale --> infinity
- e.g., rotor lattice: $\boldsymbol{p}$, θ

Sustained energy throughput, not conservation
Rotation invariance but no H so S not conserved



Chaikin & Lubensky
ch. 8

Flock = active polar liquid crystal

Reynolds 1987: movie stampedes

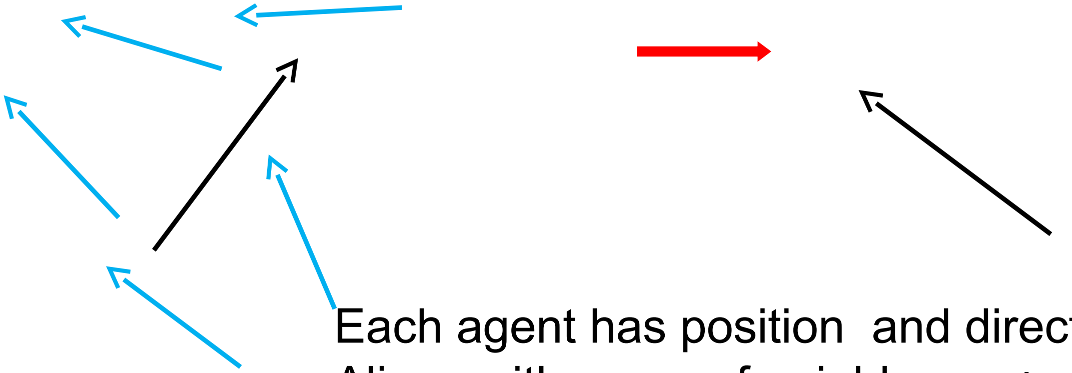
Vicsek et al 1995: agent-based simulations

- each particle: an arrow
- orient parallel to neighbours + noise
- move in direction of arrow

Toner-Tu 1998: field theory

- long-range order in $d = 2$

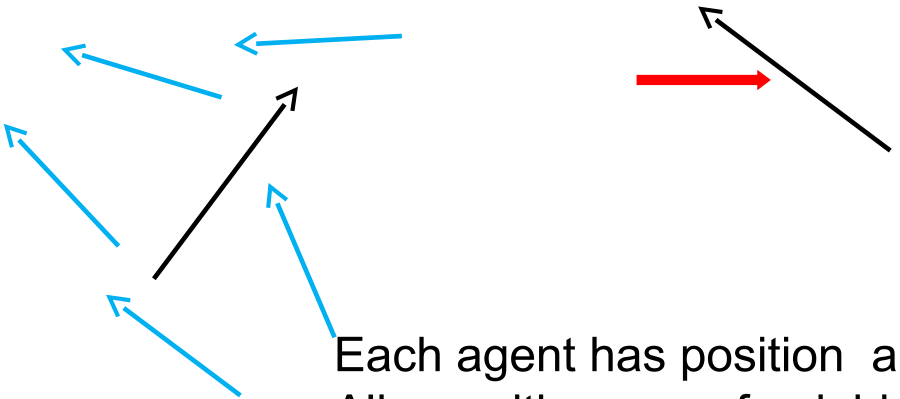
Interacting agents and flocking models



Each agent has position and direction
Aligns with mean of neighbours + noise

Reynolds 1987
Vicsek *et al.* 1995

Interacting agents and flocking models



Each agent has position and direction
Aligns with mean of neighbours + noise
Follows its nose

Low noise, high density: ordered flock
High noise, low density: isotropic state
Reynolds 1987
Vicsek *et al.* 1995

Coarse-grain →

Continuum field theory: Toner-Tu 1995

Chimera of fluid and magnet

$$\partial_t \boldsymbol{p} + v_0 \boldsymbol{p} \cdot \nabla \boldsymbol{p} = \Gamma \boldsymbol{h} + \boldsymbol{f} \qquad \boldsymbol{h} = -\delta F / \delta \boldsymbol{p}$$

noise

$$F = \int d^3 r [\frac{1}{4}(\boldsymbol{p} \cdot \boldsymbol{p} - 1)^2 + \frac{K}{2}(\nabla \boldsymbol{p})^2 - E \boldsymbol{p} \cdot \nabla c]$$

Local moment | elasticity | Gradients orient $\boldsymbol{p}$

$$\partial_t c + \nabla \cdot (v_1 \boldsymbol{p} c) = 0$$

LRO in d=2, anomalous density fluctuations, wavelike excitations
But no real fluid, no momentum conservation
What does fluid do? Simha & SR 2002

Flocks in fluid

SLOW FLOCKS IN BULK FLUID

Simha-SR 2002
Hatwalne et al 2004

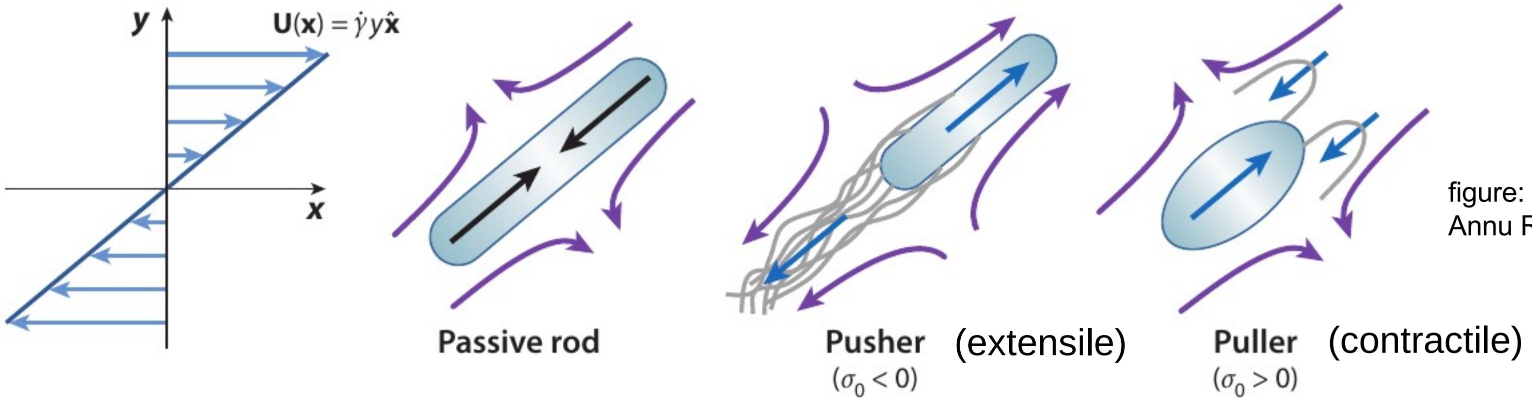


figure: Saintillan
Annu Rev Fluid Mech 2018

The active particles of a bulk living liquid crystal

orientable objects with permanent force dipoles

Build nematic or polar hydrodynamics from these

Work with orientation tensor $\mathbf{Q}$ or polar order parameter $\boldsymbol{p}$,

and velocity field $\mathbf{u}$ or momentum density $\mathbf{g} = \rho\mathbf{u}$

Apply our general approach

Liquid crystal elastic torques — Viscous damping — Active force from “chemistry”

$$\partial_t g = [g, p]\frac{\delta H}{\delta p} - \hat{\eta}\frac{\delta H}{\delta g} + \frac{\Delta\mu}{\Gamma_{22}\Gamma}\Gamma_{12} + \text{noise}$$

$$\Gamma_{12} = \nabla \cdot (pp)$$

Stokesian active nematic* hydrodynamics

Comoving co-rotating derivative

Extensional flow orients

Thermodynamic relaxation

$$\mathcal{D}_t \boldsymbol{p} = \lambda \boldsymbol{S} \cdot \boldsymbol{p} - \Gamma \frac{\delta F}{\delta \boldsymbol{p}}$$

$$-\mu \nabla^2 \mathbf{u} = -\sigma_a \nabla \cdot (\boldsymbol{pp}) + \text{elasticity} + \text{pressure}$$

Viscosity

Active stress $\propto$ $\boldsymbol{pp}$

F = free energy favouring alignment; $\mathbf{u}$ = incompressible velocity field
$\boldsymbol{p}$ = orientation (*not quite, but $\boldsymbol{p} \rightarrow -\boldsymbol{p}$ invariant), $\boldsymbol{S}$= deformation rate

Simha and SR PRL 2002; Hatwalne, SR, Rao, Simha PRL 2004
Kruse, Juelicher, Joanny, Prost, Voituriez, Sekimoto PRL 2004 – cytoskeleton

Stokesian active nematic hydrodynamics

$$\mathcal{D}_t \boldsymbol{p} = \lambda \boldsymbol{S} \cdot \boldsymbol{p} - \Gamma \frac{\delta F}{\delta \boldsymbol{p}}$$

$$-\mu \nabla^2 \mathbf{u} = -\sigma_a \nabla \cdot (\boldsymbol{pp}) + \text{elasticity} + \text{pressure}$$

Solve Stokes for $\mathbf{u}$ in terms of $\boldsymbol{p}$, perturb at wavevector $\boldsymbol{q}$ about mean aligned state $\boldsymbol{p}_0$

Frank constant

$$\partial_t \delta p = \frac{\sigma_a}{\mu} \cos 2\theta (1 + \lambda \cos 2\theta) \delta p - K \left(\Gamma + \frac{1}{\mu} \right) q^2 \delta p$$

Simha-SR PRL 2002

θ = angle between $\boldsymbol{q}$ and $\boldsymbol{p}_0$

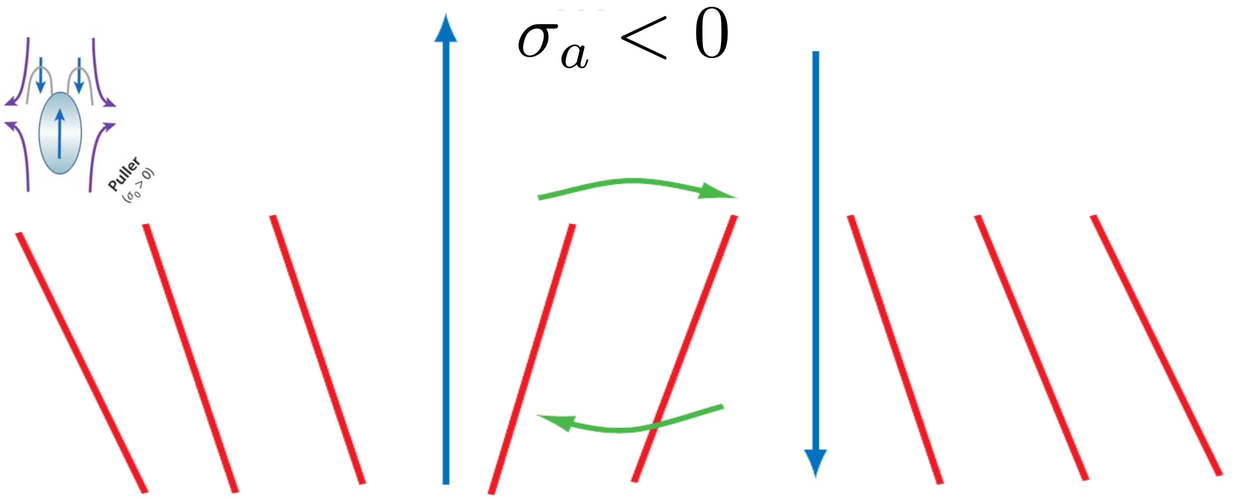
Splay-bend mode

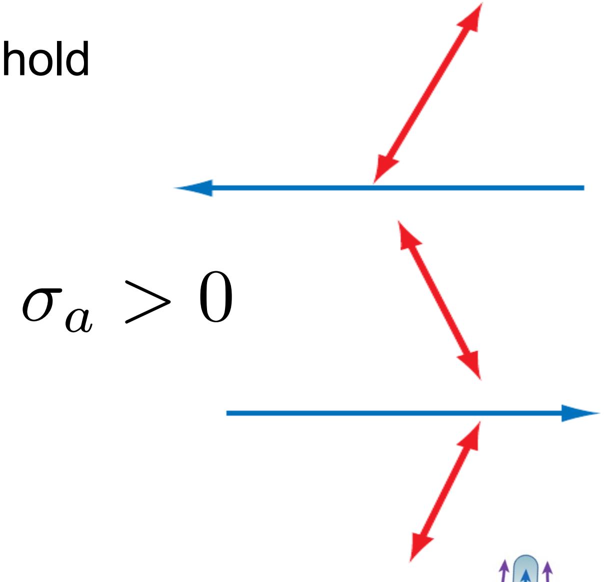
$\sigma_a > 0$: extensile, bend unstable ($0 < \theta < \pi/4$)

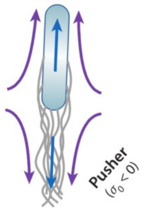
$\sigma_a < 0$: contractile, splay unstable ($\pi/4 < \theta < \pi/2$)

Consequence: bulk active nematic/polar always unstable

viscous hydrodynamics, neglect inertia: unstable without threshold







Growth-rate → σ_a/μ for length scales > $\xi=(K/\sigma_a)^{1/2}$, K = Frank constt
viscosity/active stress: a single timescale

Simha & SR PRL 2002, Voituriez et al 2005, SR & Rao NJP 2007; polar: Giomi, Marchetti, Liverpool 2008
Active turbulence: Saintillan-Shelley/Yeomans/Dogic/Bausch/Sagues/Doostmohammadi ....

Stable Stokesian flocks
A Maitra arXiv: 2110.15633

$\mathbf{n}\nabla \cdot \mathbf{n}$

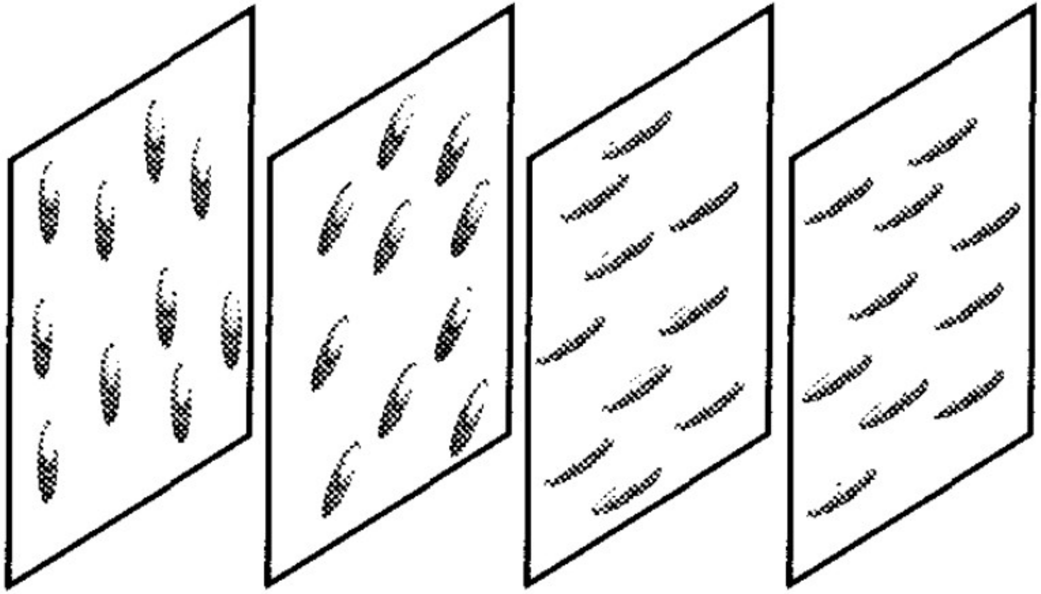
splay

(a)

$\mathbf{n} \cdot \nabla\mathbf{n}$

bend

(b)



$\mathbf{n} \cdot \nabla \times \mathbf{n}$

twist

Chaikin & Lubensky
1995

3D: bend-splay and bend-twist

$$\omega = i\frac{\sigma_a}{\mu}\cos 2\theta(1+\lambda\cos 2\theta)$$

bend-splay

$$\omega = i\frac{\sigma_a}{\mu}\cos^2\theta(1+\lambda)$$

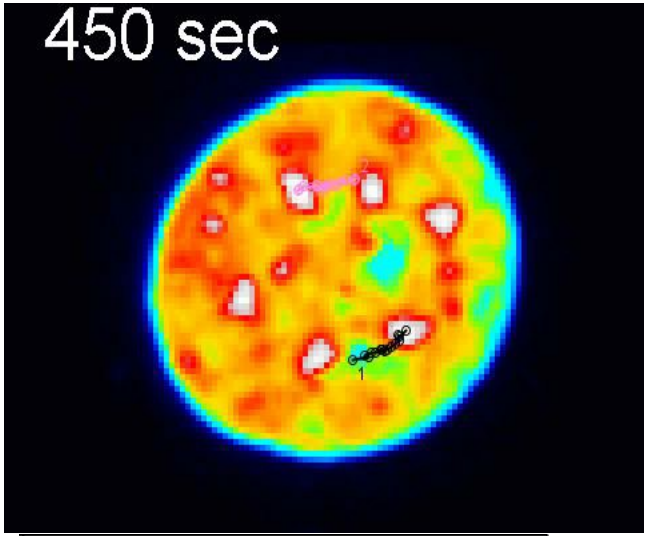
bend-twist

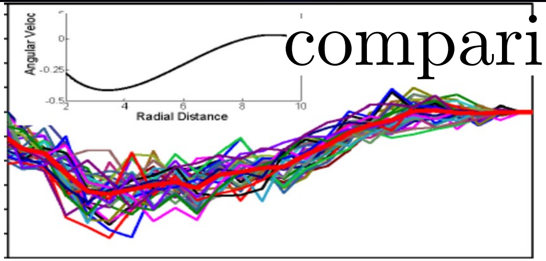
Bend-Twist not mitigated by interpolation to splay; should dominate in 3D extensile

see Shendruk, Thijssen, Yeomans, Doostmohammadi, PRE 2018

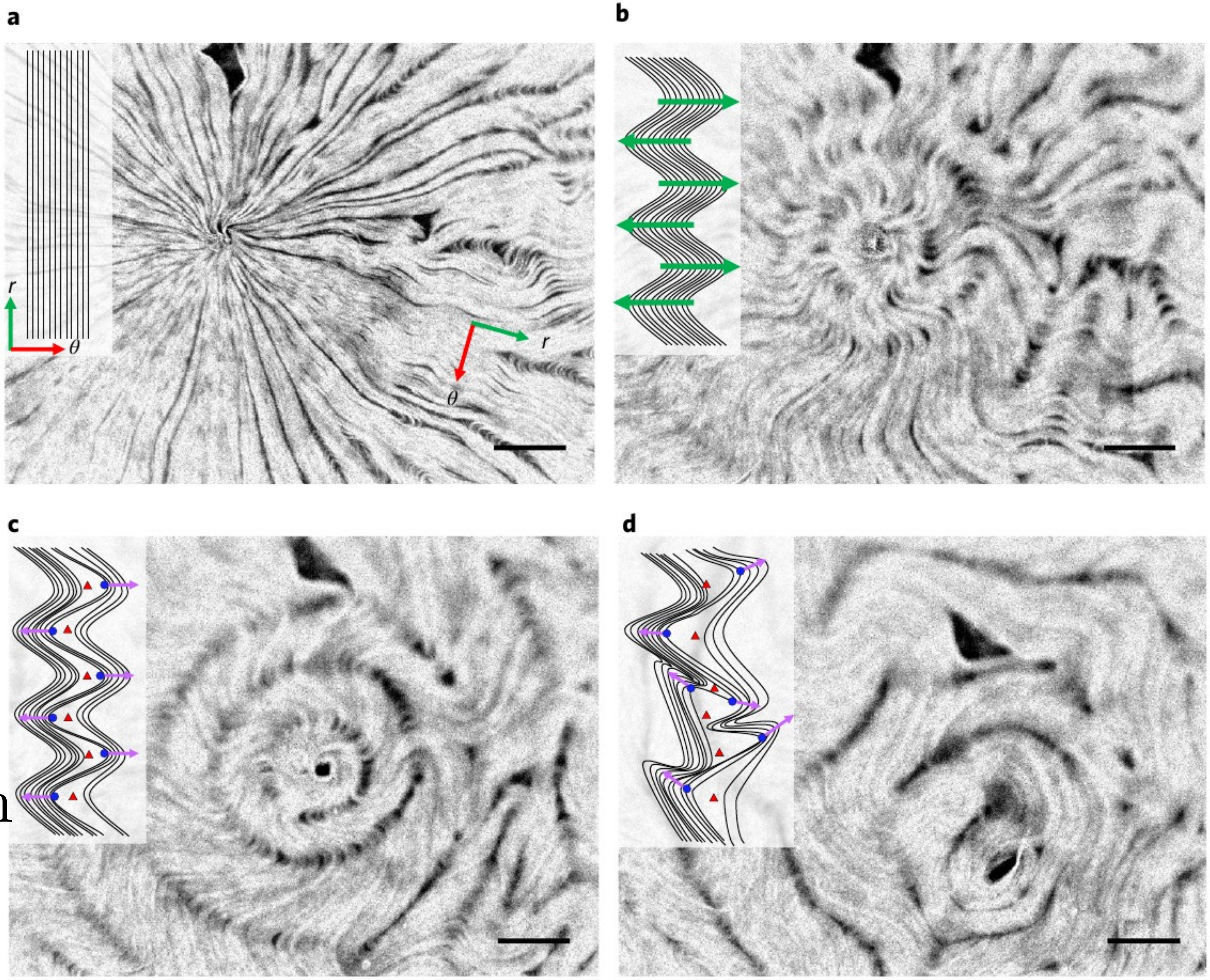
Detailed confirmation: active nematic of microtubules + motors + ATP
Martínez-Prat, Ignés-Mullol, Casademunt & Sagués, Nat Phys 2019

Other evidence, e.g. nuclear rotation in cell:
Kumar, Maitra, Sumit, SR, Shivashankar 2014





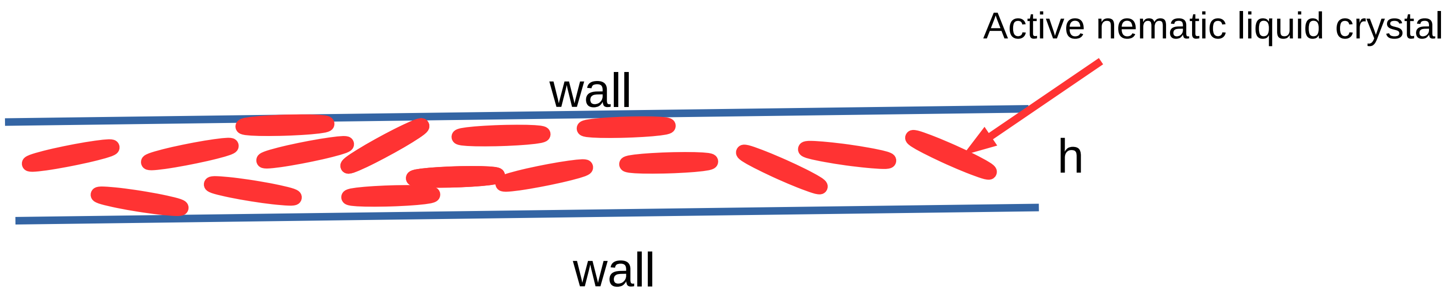
comparison



Slow flocks in fluid films

apolar

Maitra et al. PNAS 2018



Degenerate planar alignment on both walls

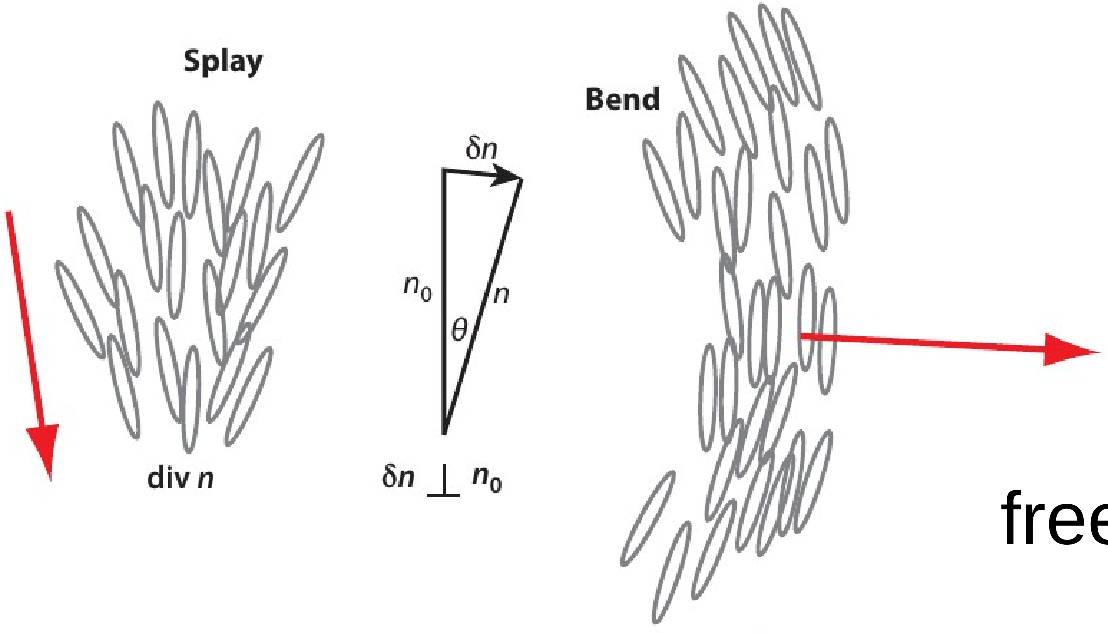
K = Frank elastic constant of underlying liq crystal

Length scale $\xi = (K/\sigma_a)^{1/2}$; stable if $h < \xi$

Fix h, increase activity σ_a: diffusive instability?

No: much more interesting

confined active nematics



Orientation θ
concentration c
velocity field v

free-energy functional

$$\mathcal{H} = \int d^2\mathbf{r} \left[\frac{K}{2} (\nabla\theta)^2 + g(c) \right]$$

confined active nematics

$$\dot{\theta} = \frac{1-\lambda}{2}\partial_x v_y - \frac{1+\lambda}{2}\partial_y v_x - \Gamma_\theta \frac{\delta \mathcal{H}}{\delta \theta}$$

$|\lambda|$ > 1 flow-aligning, < 1 flow-tumbling

Γ = viscosity/h^2

$$\Gamma \mathbf{v} = -\nabla \Pi + \mathbf{f}^p + \mathbf{f}^a$$

2d pressure Passive Active

$$\nabla \cdot \mathbf{v} = 0$$

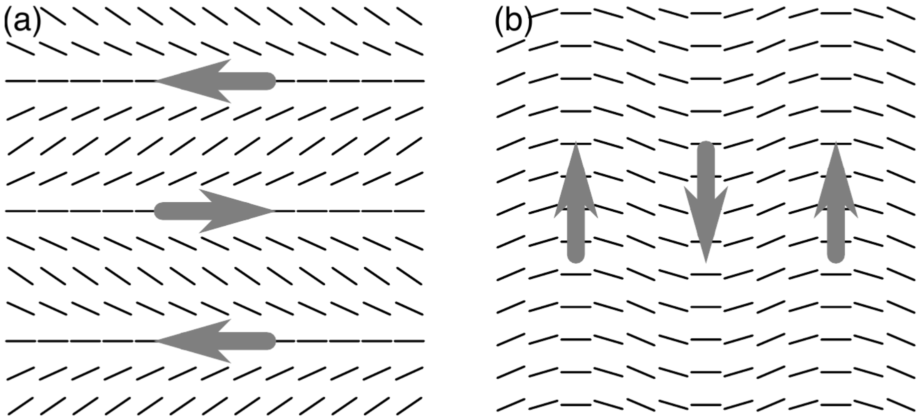
Solve for v, get effective equations for θ

Need force densities f^p, f^a

Force densities

passive

$$\mathbf{f}^p = -\frac{1+\lambda}{2}\partial_y\left(\frac{\delta\mathcal{H}}{\delta\theta}\right)\hat{\mathbf{x}} + \frac{1-\lambda}{2}\partial_x\left(\frac{\delta\mathcal{H}}{\delta\theta}\right)\hat{\mathbf{y}}$$



not momentum-conserving
that's OK: walls
can derive from 3D hydro

active

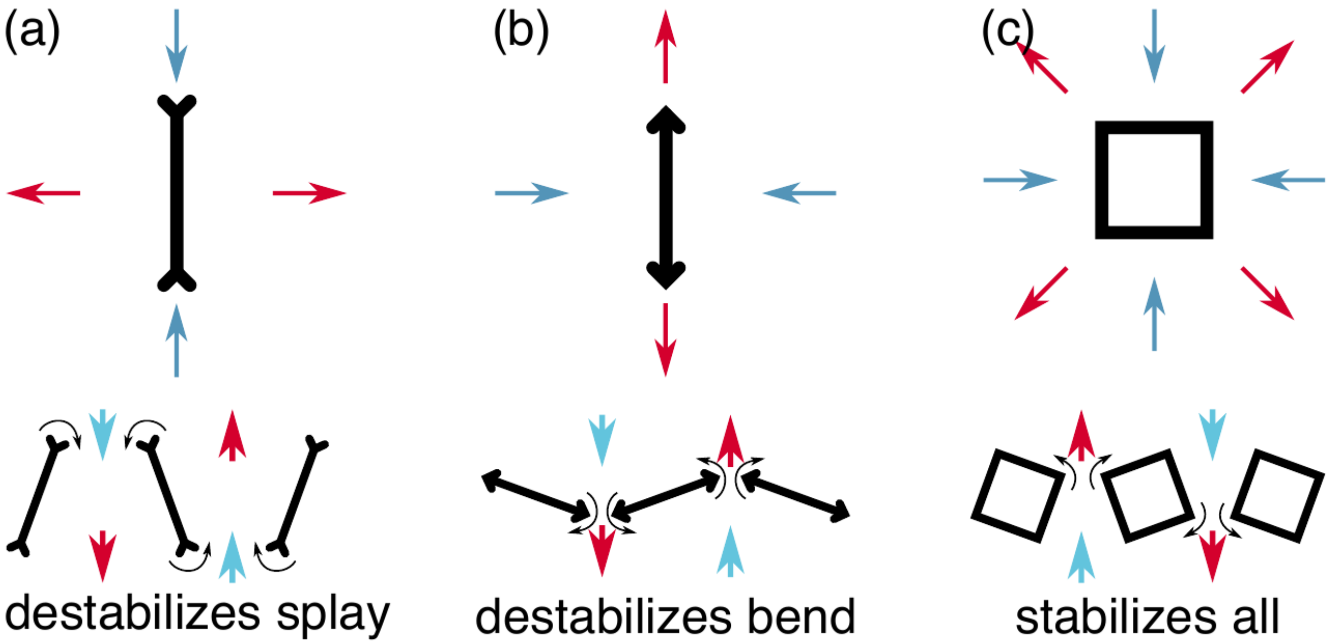
$$f_x^a = -(\zeta_1+\zeta_2)\Delta\mu\,\partial_y\theta$$
$$f_y^a = -(\zeta_1-\zeta_2)\Delta\mu\,\partial_x\theta$$

$\mathbf{n}(\nabla\cdot\mathbf{n})$

Splay ≠ bend

$\mathbf{n}\cdot\nabla\mathbf{n}$

cf flexoelectricity:
Lavrentovich,
Prost & Marcerou,
Meyer



Higher multipole for bulk hydrodynamics, but competes on substrate.
Can stabilise for all activity levels!

Consequence: effective θ dynamics

$$\partial_t \theta_{\mathbf{q}} \;=\; -D(\phi) q^2 \theta_{\mathbf{q}}$$

$$D(\phi) = \Gamma_\theta K + \frac{\Delta\mu}{2\Gamma}(1 - \lambda \cos 2\phi)(-\zeta_1 \cos 2\phi + \zeta_2)$$

Without ζ_2 instability inevitable at large $\Delta\mu$

Not any more!

$|\lambda| < 1$ (flow-tumbling): large +ve ζ_2 always stable. Stability diagram -->

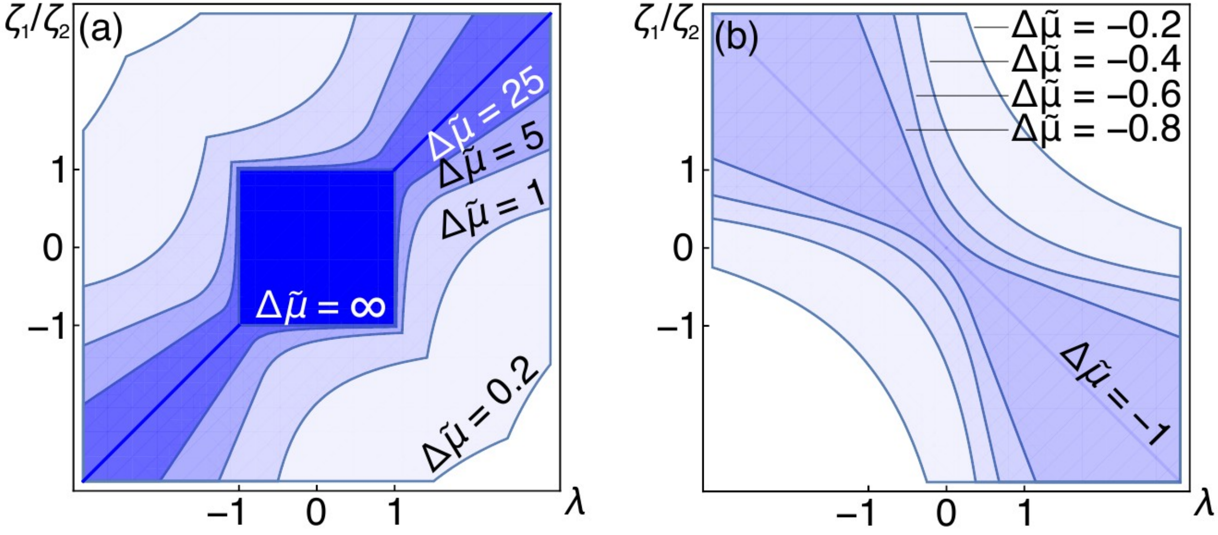
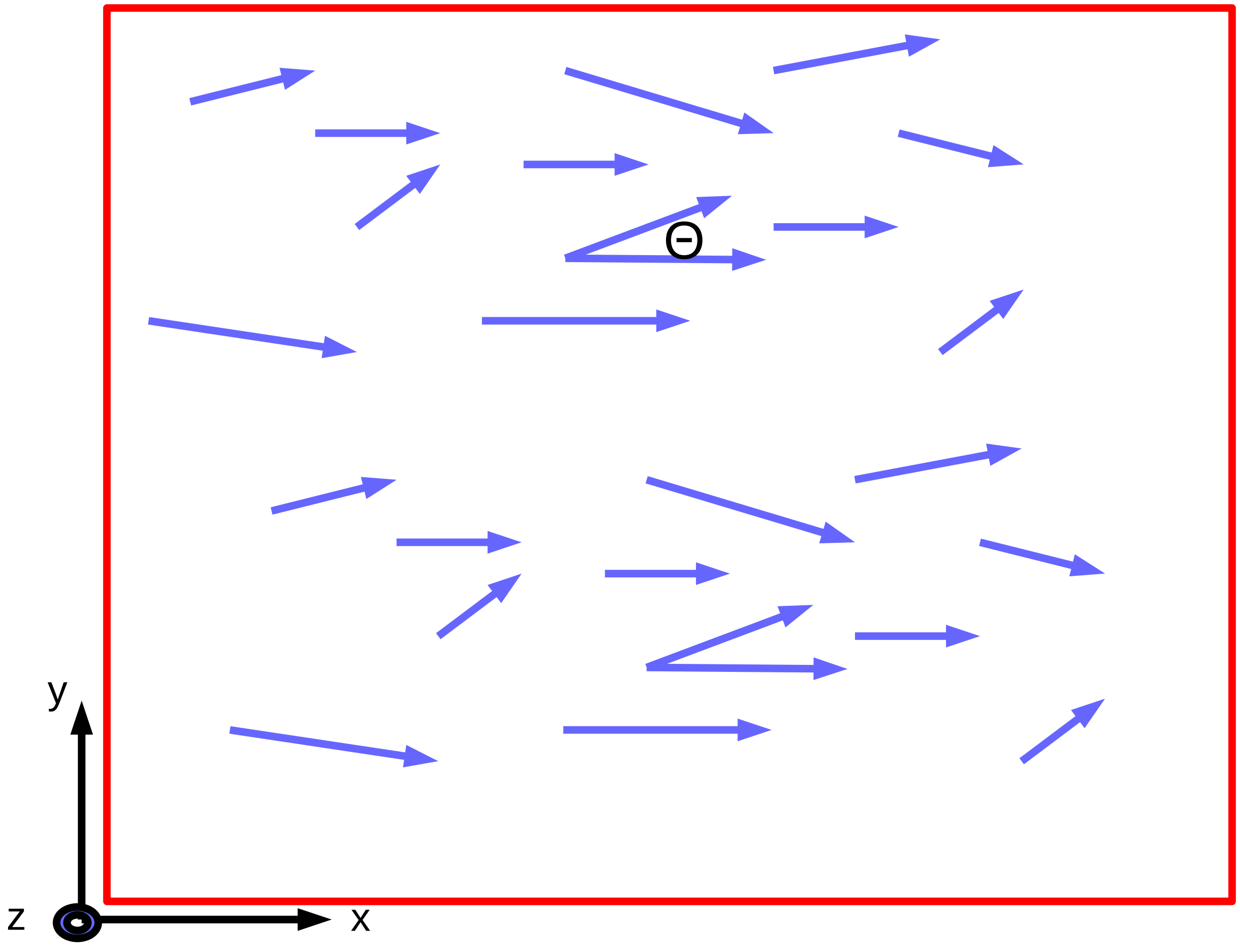


FIG. 2: Regions of stability of the ordered phase as a function of the flow-alignment parameter λ, the ratio ζ_1/ζ_2 of the old and new active forces and the overall magnitude of activity relative to passive friction $\Delta\tilde{\mu} = \zeta_2 \Delta\mu / 2K\Gamma\Gamma_\theta$. (a) For $\Delta\tilde{\mu} > 0$, the region of linear stability of the ordered phase (shades of blue) shrinks with increasing activity, yet the central dark blue square is stable for arbitrary high activity. (b) For $\Delta\tilde{\mu} < 0$, stability is abolished for large enough activity, namely $\Delta\tilde{\mu} < -1$.

Slow flocks in fluid films

polar



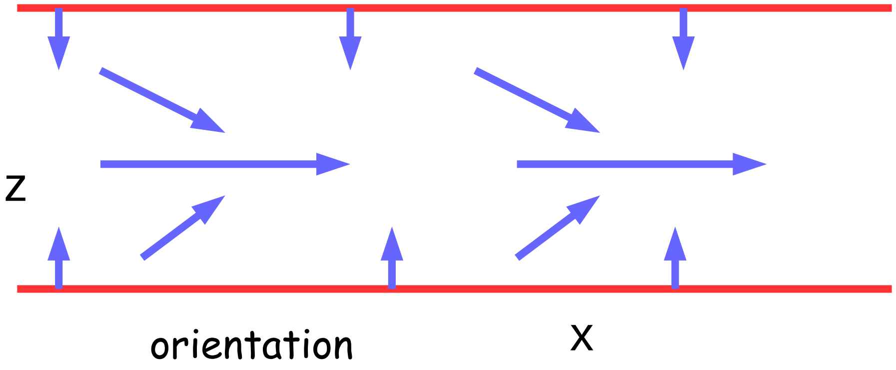
Extended in XY, confined in z

- base & lid: preferred frame
- forget momentum conservation?
- 2d confined = Toner-Tu model?

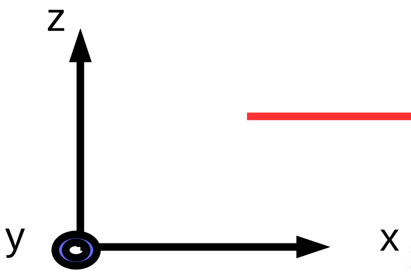
NO

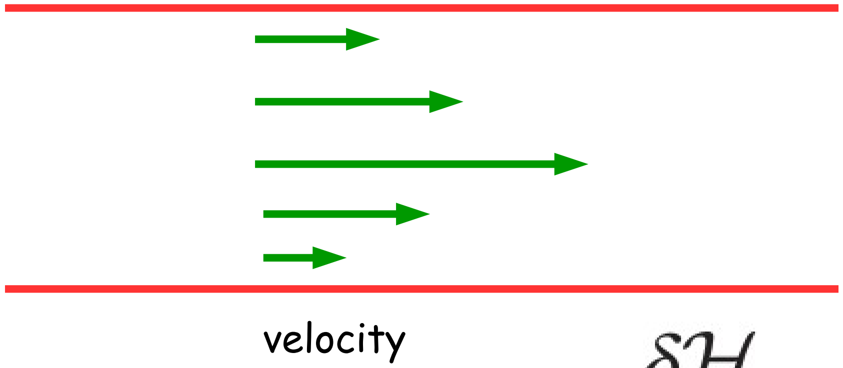
Maitra, Srivastava, Marchetti, SR, Lenz PRL 2020

Confinement: fluid velocity irrelevant?



Side view





- Polar vs apolar: how different?
- Confined geometry
  - bounding walls: momentum sink
  - velocity field relaxes “fast”
- But: incompressibility
  - can’t forget velocity field
  - “dry” flocking models not enough

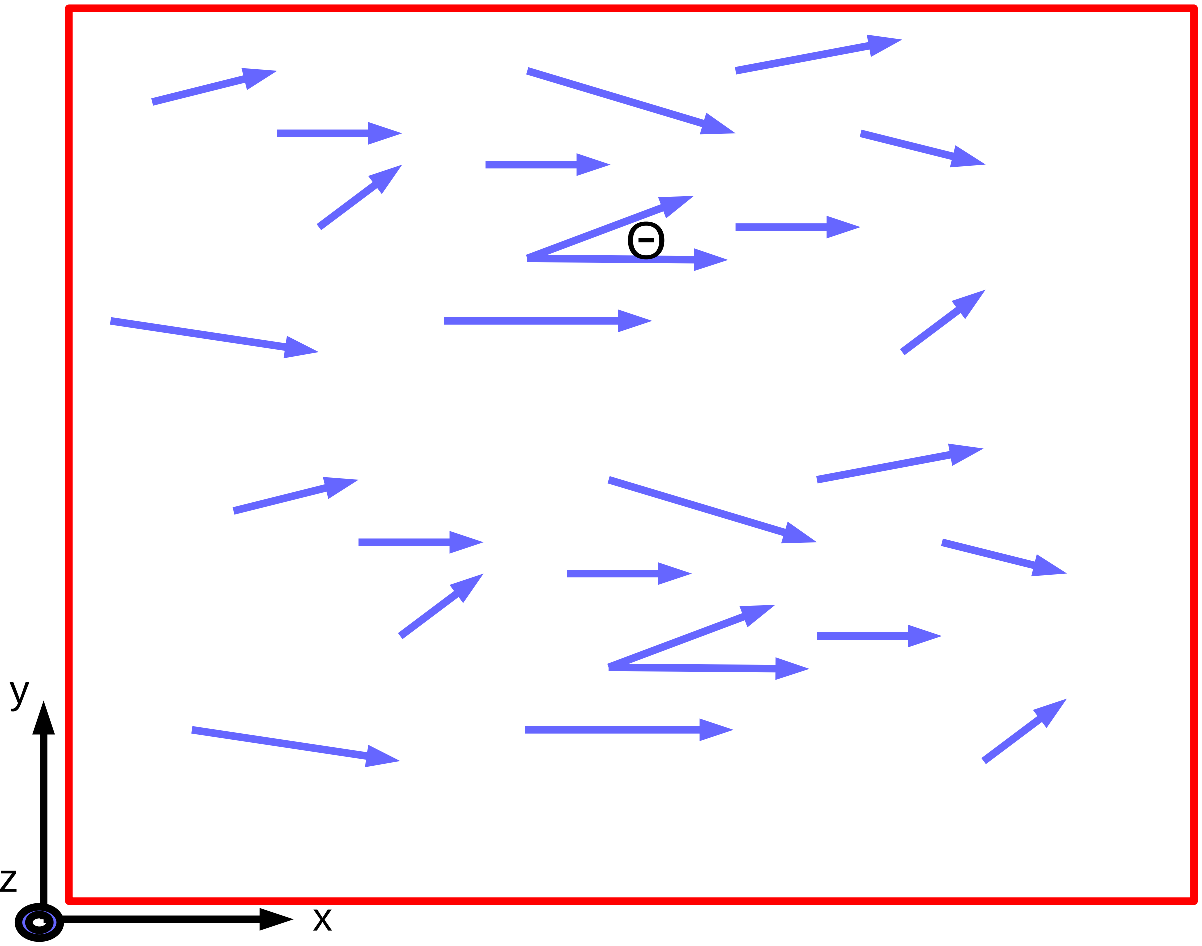
Brotto et al. PRL 2013
Kumar et al. Nat Comm 2014

$$\partial_t \mathbf{p} = \Lambda \mathbf{u} - \frac{\delta \mathcal{H}}{\delta \mathbf{p}}$$

$$\Gamma \mathbf{u} = \upsilon \mathbf{p} - \nabla \Pi - \Lambda \frac{\delta \mathcal{H}}{\delta \mathbf{p}}$$

$$\nabla \cdot \mathbf{u} = 0$$

Robust long-range order



$$\partial_t \Theta^q = -\frac{\Lambda v}{\Gamma}(q_y^2/q^2)\Theta^q$$

Incompressible velocity game-changing

For the case $\Lambda > 0$

Nambu-Goldstone gets anisotropic “mass”

velocity acts like Coulomb field

angle fluctuations *finite* in d=2, LRO

Not Toner-Tu

If $\Lambda < 0$: inescapable instability

END LECTURE 1

Maitra, Srivastava, Marchetti, SR, Lenz PRL 2020

End lecture 1

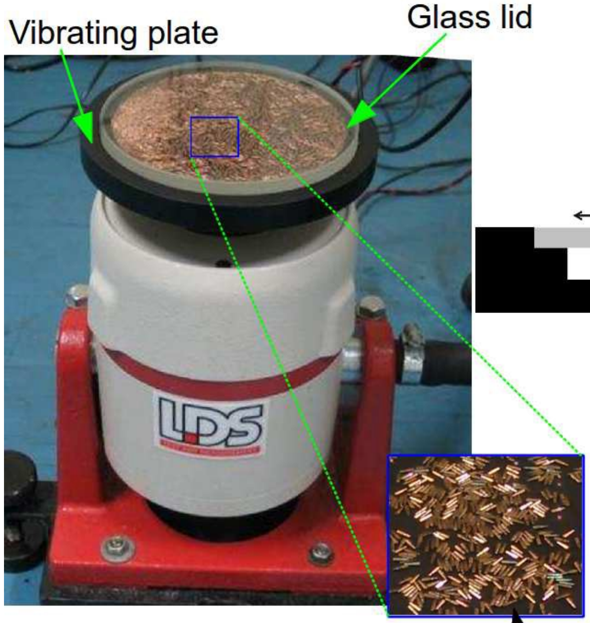
CRAWLING THROUGH AN ELASTIC MEDIUM

- What if the medium is elastic, not viscous?
- Strain-rate fields --> strain
- This work: on a substrate
  - velocity field "fast", damped elastodynamics

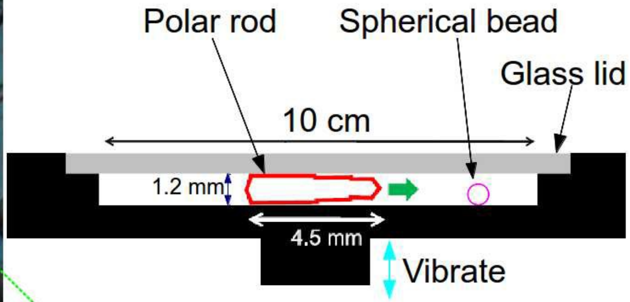
cell/tissue mechanics

Rahul Gupta, Raushan Kant,
Harsh Soni, Ajay Sood, SR
PRE 2022

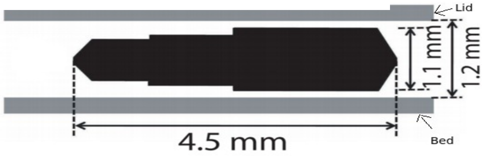
Experimental setup



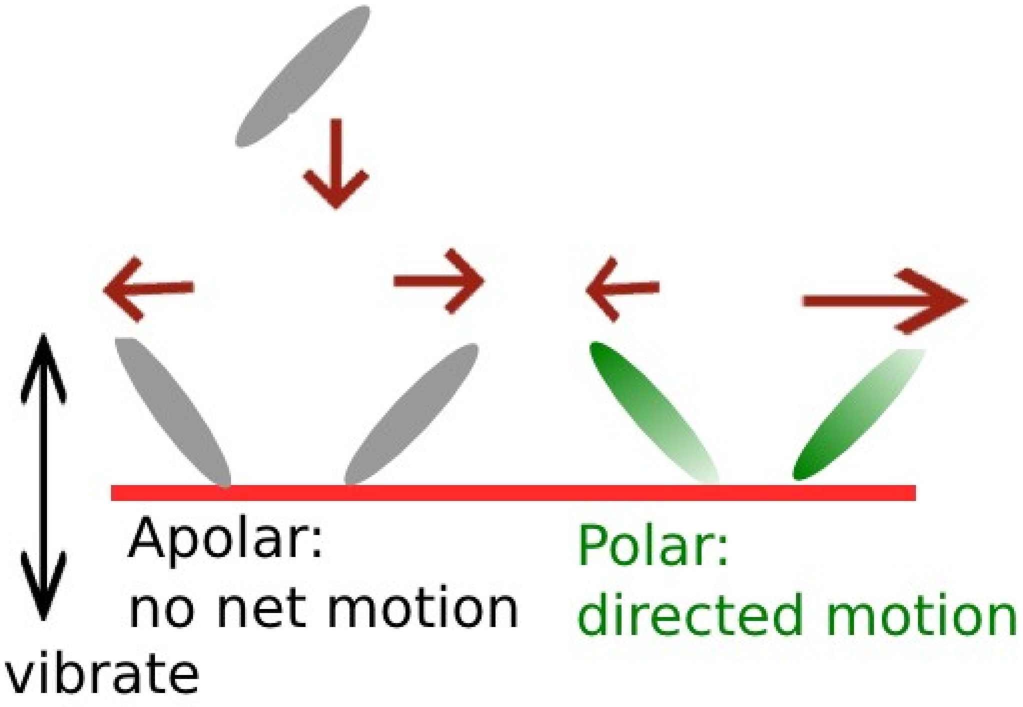
Real experimental setup

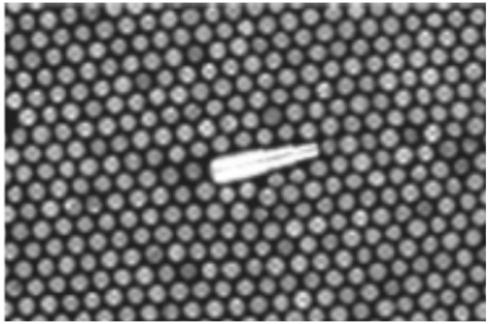


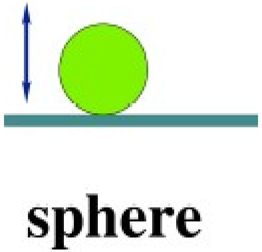
Schematic diagram



Horizontal motility from vertical shaking







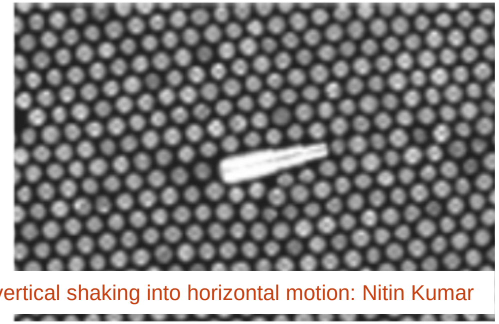
A motile rod transducing vertical shaking into horizontal motion: Nitin Kumar

static friction ⇒ centre of mass moves

Yamada, D., Hondou, T. & Sano, M. Phys. Rev. E **67**, 040301 (2003)

First look at a fluid bead-layer

1-particle rendition of Kumar, Soni, SR, Sood Nature Comm 2014



A motile rod transducing vertical shaking into horizontal motion: Nitin Kumar

$$\dot{\mathbf{R}}(t) = v_0 \mathbf{n}(t)$$ motility

$$\partial_t(\rho \mathbf{v}) + (\zeta - \eta \nabla^2)\mathbf{v} = f\mathbf{n}(\mathbf{t})\delta\left[\mathbf{r} - \mathbf{R}(t)\right] - \nabla P$$

Substrate drag, viscosity   Motile rod pushes beads   pressure

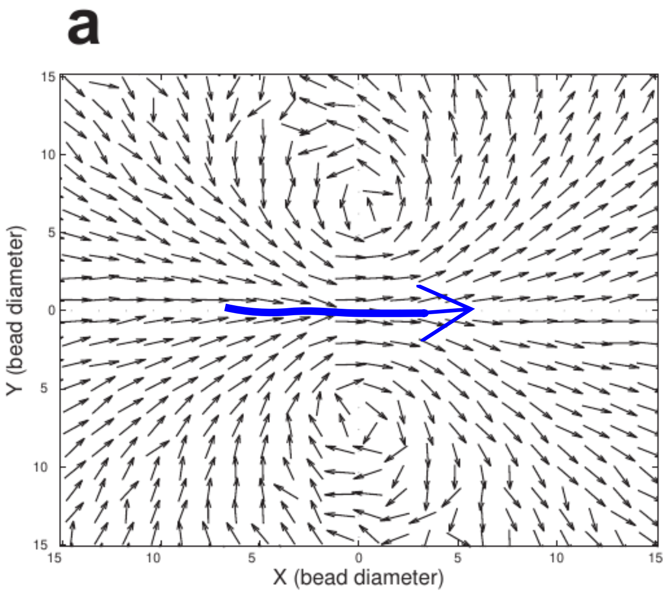
$$\dot{\mathbf{n}} = (\mathbf{I} - \mathbf{n}\mathbf{n}) \cdot (\mathbf{v} + \nabla \mathbf{v} \cdot \mathbf{n} + \ldots)$$ (schematically)

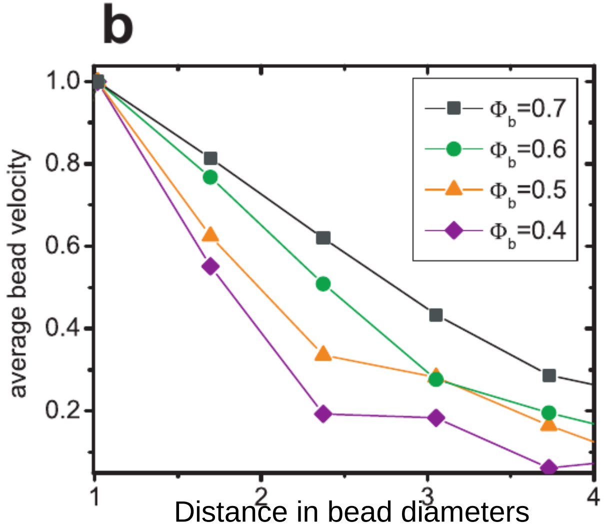
flow reorients $\mathbf{n}$ parallel to $\mathbf{v}$

Gradients rotate & align $\mathbf{n}$

in pictures:

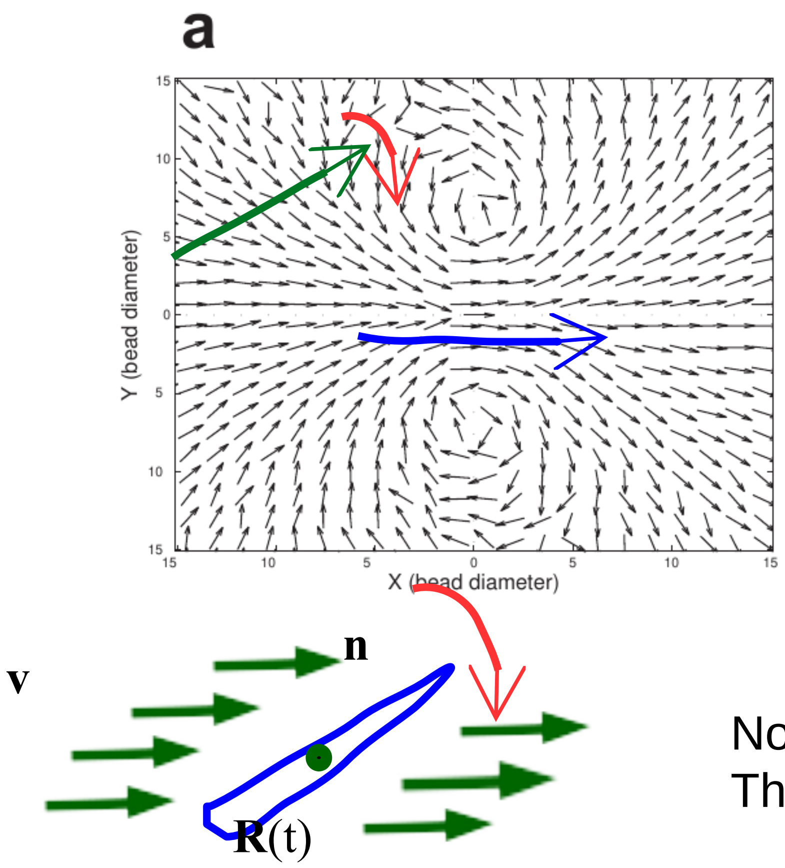
Flow-field around a mover in a fluid layer

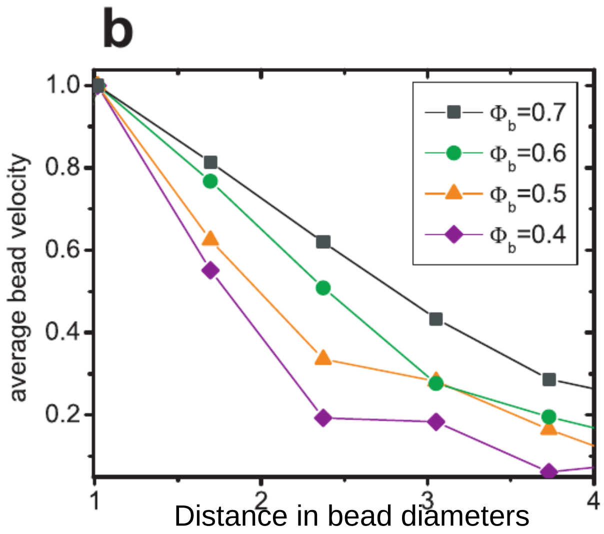




Kumar, Soni, SR, Sood 2014

An emergent aligning interaction





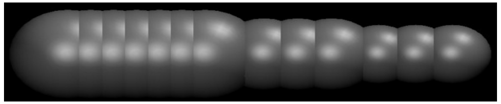
Nonuniform drag: flow reorients **n** parallel to **v**

The weathercock effect

A granular flock at very low concentration

Kumar, Soni, Sood, SR Nature Communications 2014; arXiv:1402.4262

Nitin Kumar (student of A K Sood, IISc)



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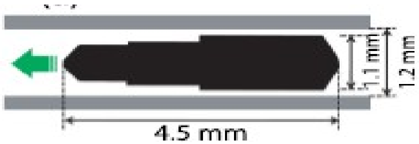
/home/sriram/talks/activemattertalks/current/vdo_liquid.mpg



Granular dynamics simulation: Harsh Soni

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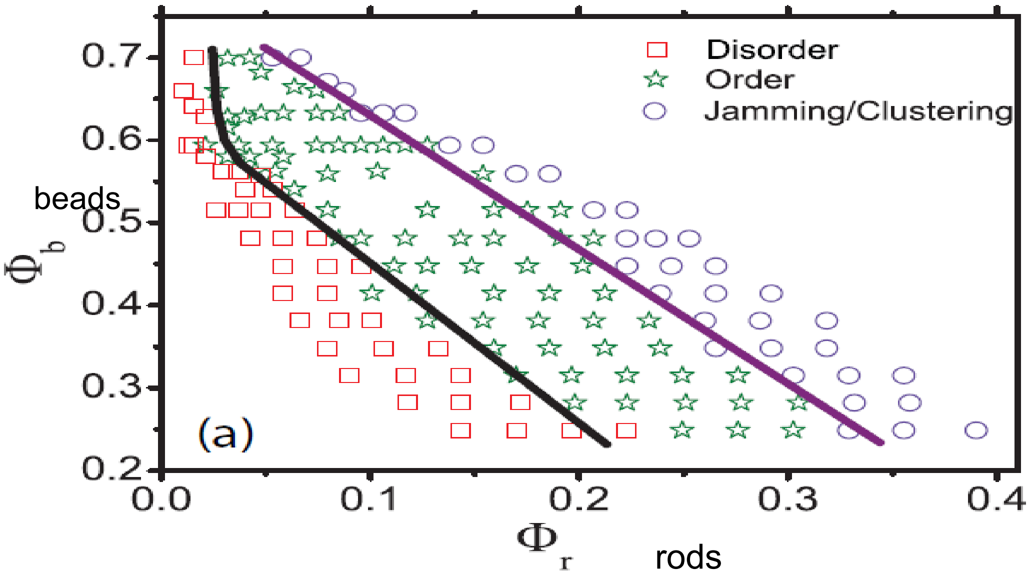
Confined quasi-2d geometry

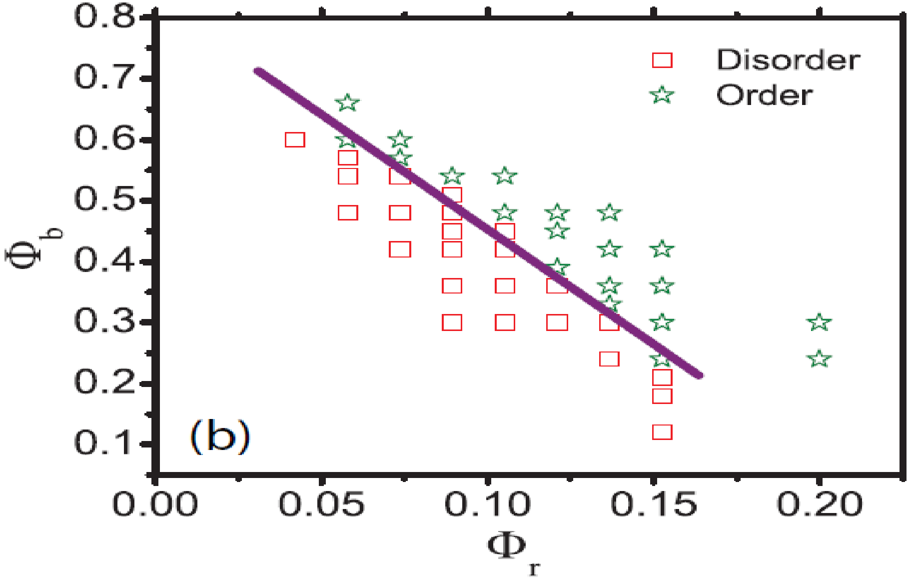
cf Deseigne et al PRL 2010
Weber et al PRL 2013

Phase diagram

Flocking by increasing inert-particle concentration



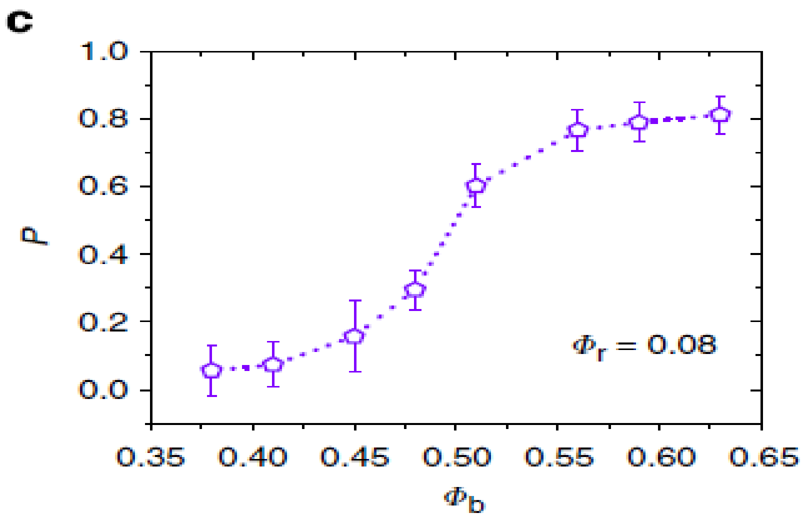
Experiment



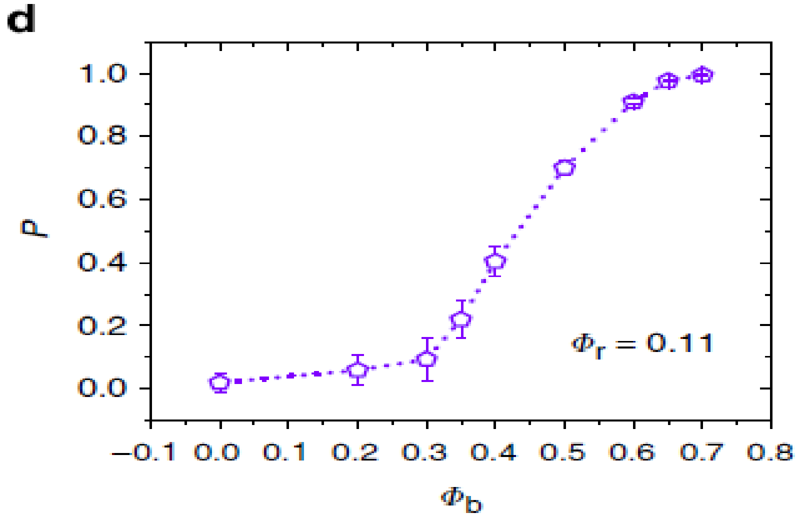
Simulation

A phase transition

Amount of order as function of inert-particle concentration



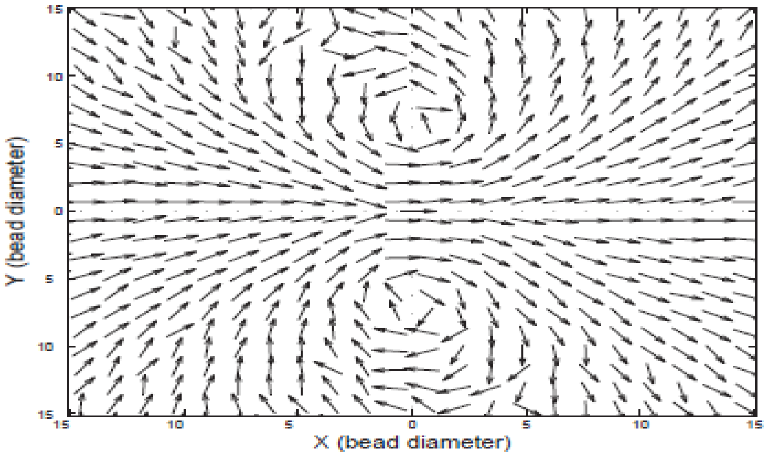
Experiment

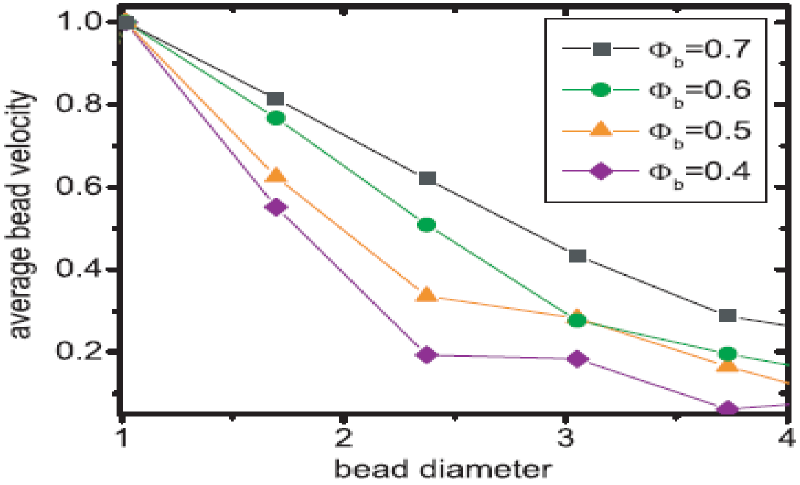


Simulation

The mechanism: moving polar rod creates flow

Simulation: H Soni





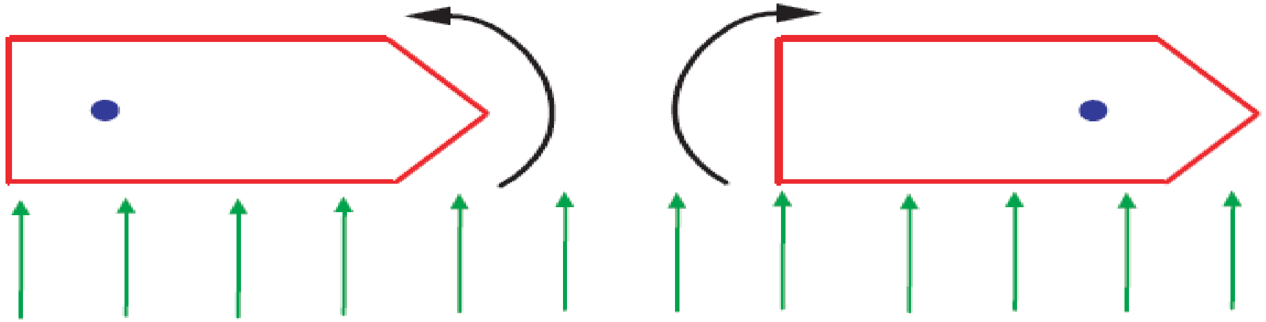
Increase Φ_b --> increase decay length of velocity

Screened monopole
cf Brotto et al. PRL 2013

The mechanism: flow orients polar rod

Flow rotates polar particles to point the right way: the weathercock effect
Need a substrate
/home/sriram/talks/activemattertalks/current/Video7.avi

Could have been either way
Design problem



qualitatively similar to Bricard et al. colloidal rollers Nature 2013
- flow field simpler, medium compressible
- single-rod motility from solid contact mechanics
- Crucial difference: non-motile-bead concentration is control parameter
- purely 2d system

Theory of flocking at a distance

Kumar, Soni, Sood, SR arXiv:1402.4262, Nat Comm 2014

Order parameter P,
velocity v,
number density ρ

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

$$\rho \partial_t \mathbf{v} = -(\Gamma - \eta \nabla^2)\mathbf{v} + \alpha \mathbf{P} - B \nabla \rho + \ldots$$

$$\partial_t \mathbf{P} = \lambda \mathbf{v} - (a - K \nabla^2)\mathbf{P} - A \nabla \rho + \ldots$$

Transition determined by effective coupling $\bar{a} = a - \lambda\alpha/\Gamma$

- Independent measurements in simulation (H Soni)
- $\alpha > 0$, $\lambda > 0$ and increases with ρ
- So: increase ρ: get transition to ordered state of P

Theory of flocking at a distance

Kumar, Soni, Sood, SR arXiv:1402.4262, Nat Comm 2014

Order parameter $\mathbf{P}$,
velocity $\mathbf{v}$,
number density ρ

continuity

$$\partial_t \rho + \nabla \cdot (\rho \mathbf{v}) = 0$$

damping forcing

$$\rho \partial_t \mathbf{v} = -(\Gamma - \eta \nabla^2)\mathbf{v} + \alpha \mathbf{P} - B \nabla \rho + \ldots$$

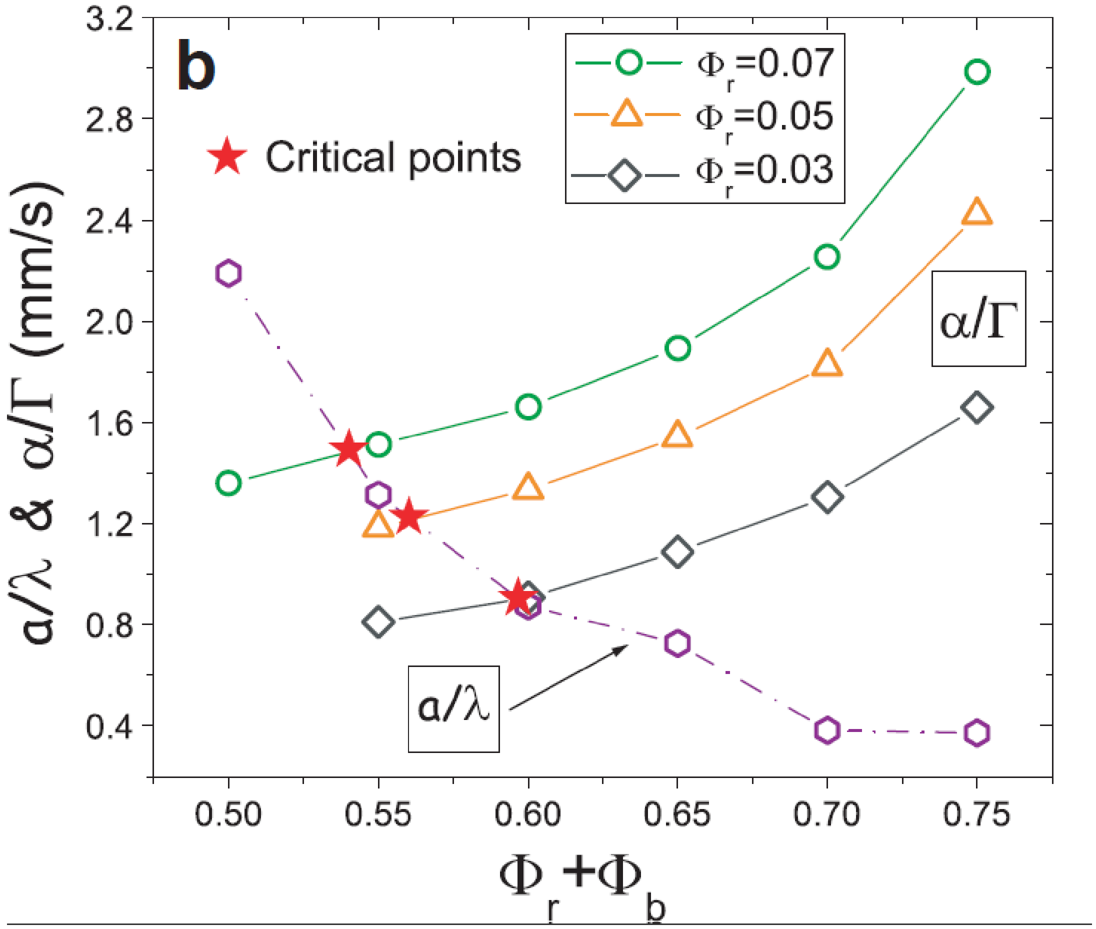
$$\partial_t \mathbf{P} = \lambda \mathbf{v} - (a - K \nabla^2)\mathbf{P} - A \nabla \rho + \ldots$$

Flow coupling Rotational relaxation

Transition determined by effective coupling $\bar{a} = a - \lambda\alpha/\Gamma$

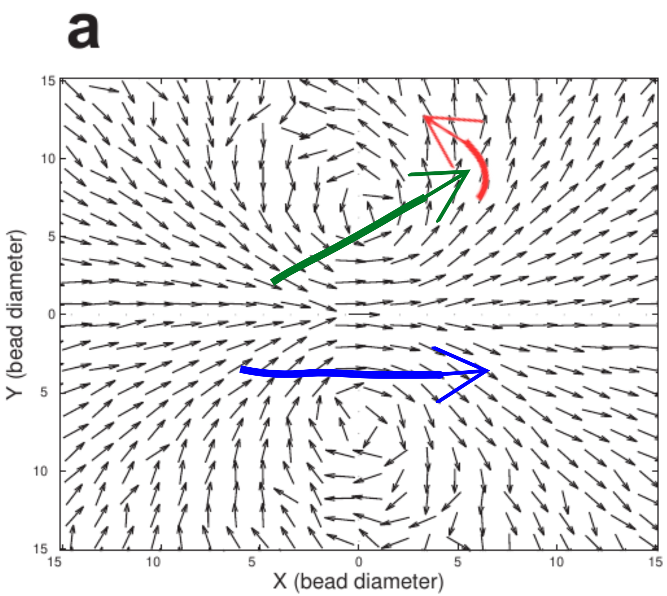
- Independent measurements in simulation (H Soni)
- $\alpha > 0$, $\lambda > 0$ and increases with ρ
- So: increase ρ: get transition to ordered state of $\mathbf{P}$

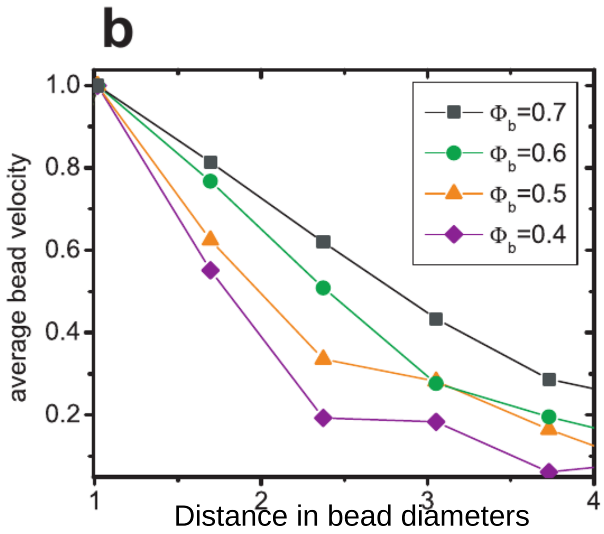
Estimating mean-field critical point from simulation

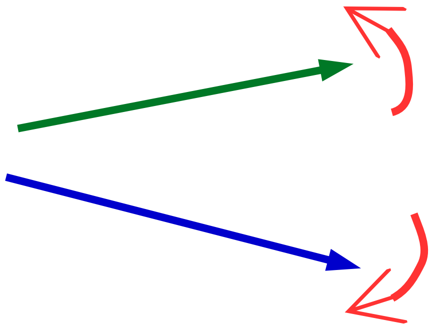


Side-by-side: rotation by vorticity

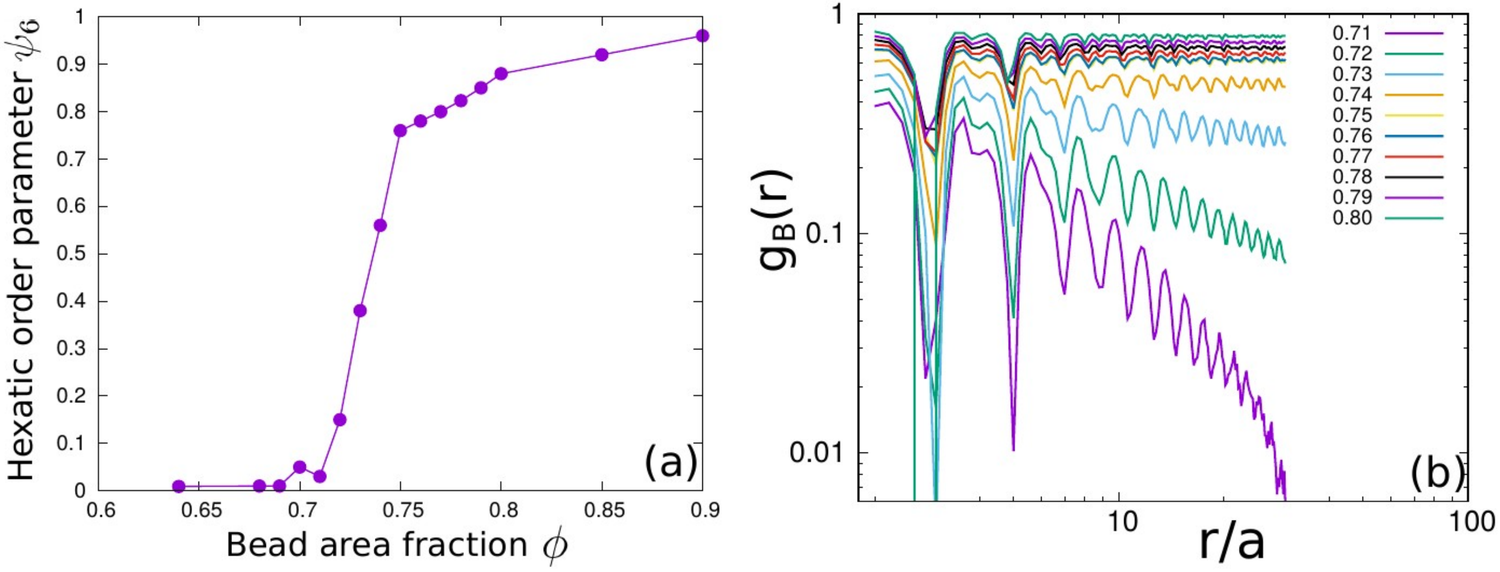
negative taxis: “repulsion”





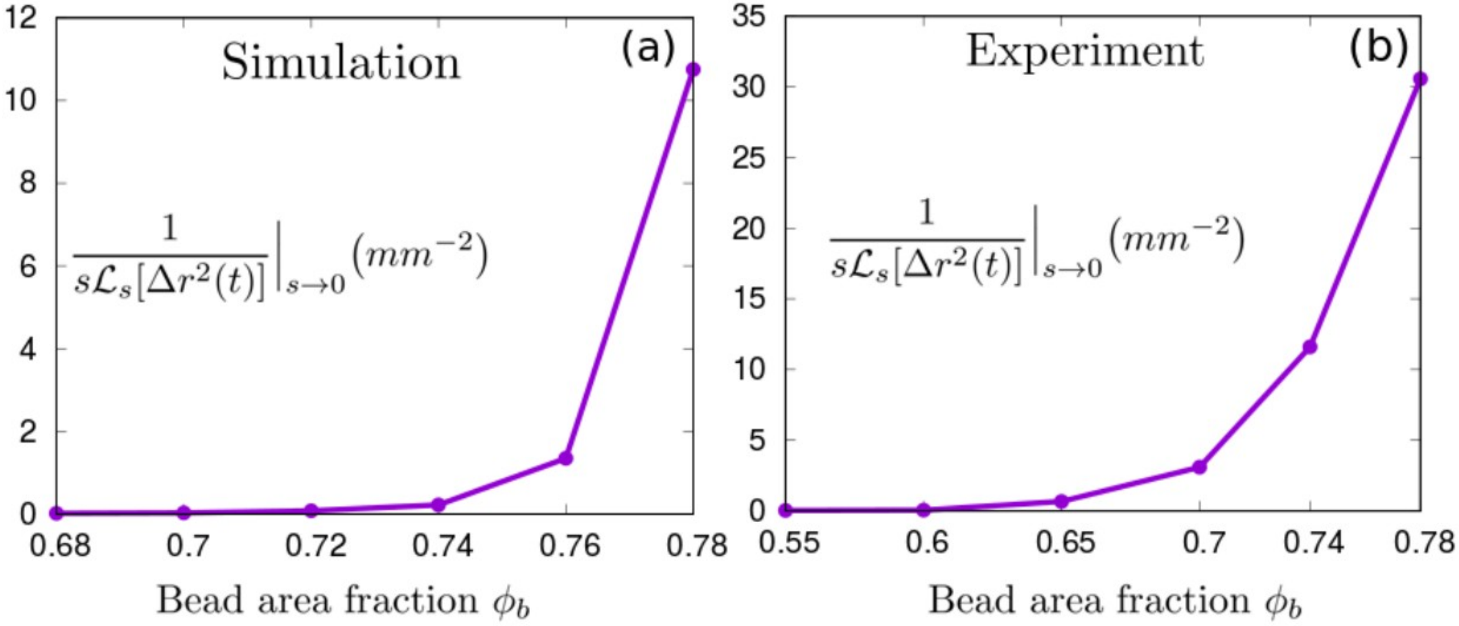


Dense bead layer: crystalline



Increase bead packing, transition to crystal
Long-range 6-fold order as proxy

Onset of rigidity



Single-particle microrheology in real and numerical experiments

Contrast between elastic and fluid media

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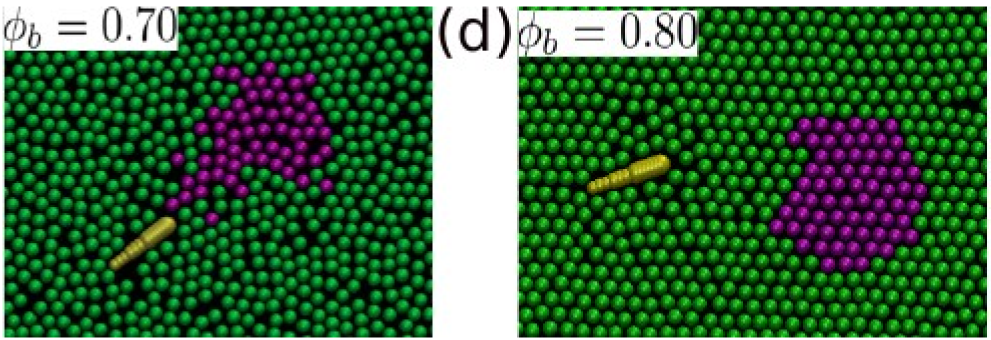
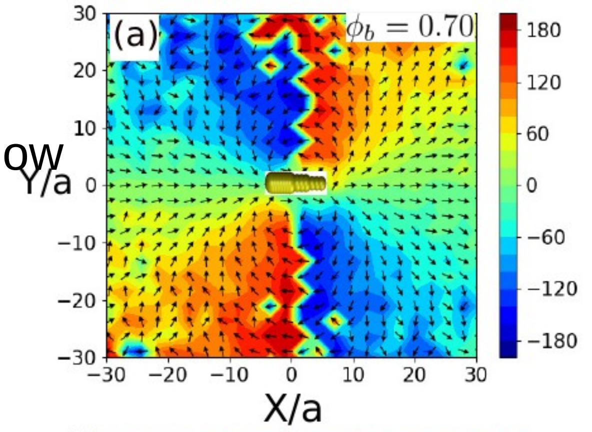
Bead flow promotes flocking at a distance
Kumar et al Nat Comm 2014
Increase density, crystallize, what happens?



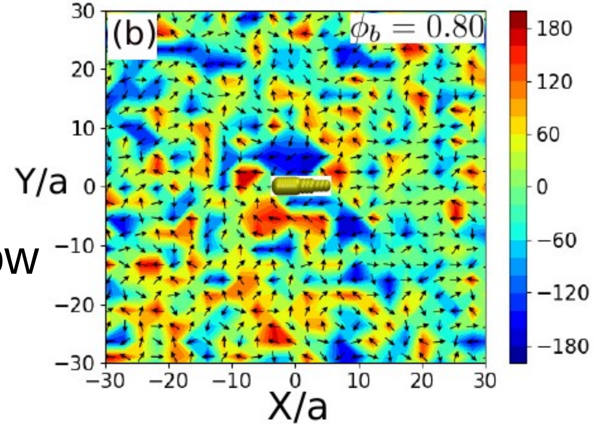
fluid

crystal

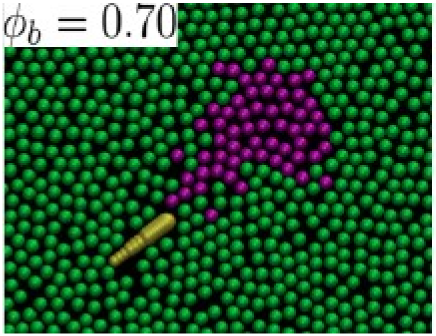
Fluid: bead flow

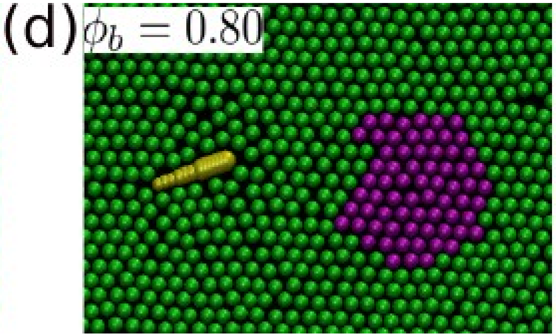


crystal: no flow



Comparison: crystal vs fluid





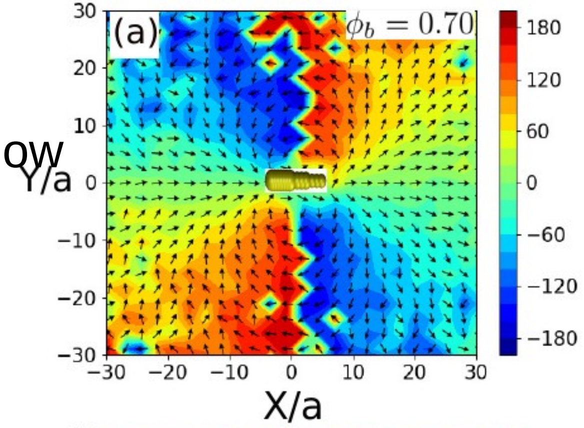
fluid

fluid phase: motile rod drags beads

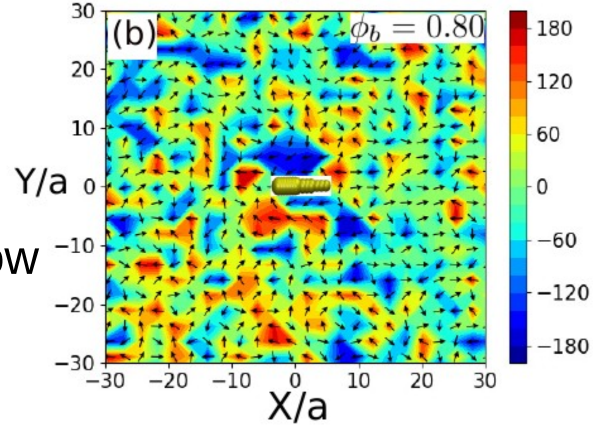
crystal

crystal: no dragging

Fluid: bead flow



crystal: no flow



Crawling through a crystal: theory?

- Safran et al: force dipoles in elastic medium
  - motility ignored

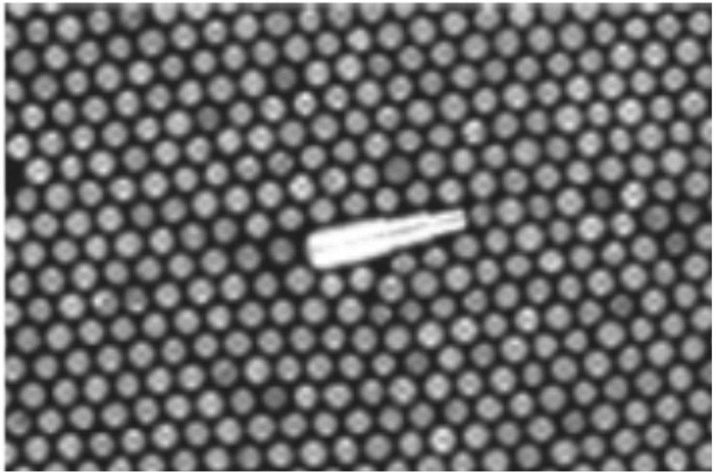
cell/tissue mechanics

- Henkes et al. 2020: elastic medium made of ABPs
  - active forcing + repulsive pair potential, no reorienting by medium
- This work: coupled dynamics
  - motile particles strain medium, strain reorients particles
  - naturally non-reciprocal dynamics

Rahul Gupta, Raushan Kant,
Harsh Soni, Ajay Sood, SR
PRE 2022

Motile particles in elastic medium on substrate

Particle position $\mathbf{R}(\mathrm{t})$, orientation $\mathbf{n}(\mathrm{t})$
Displacement field of medium $\mathbf{u}(\mathbf{r},\mathrm{t})$
Lamé elastic free energy F
Friction ζ, self-prop force f, speed v_0



A motile rod transducing vertical shaking into horizontal motion: Nitin Kumar

$$\dot{\mathbf{R}}(t) = v_0 \mathbf{n}(t)$$

$$\zeta \partial_t \mathbf{u} = -\delta F / \delta \mathbf{u} + f \mathbf{n}(\mathbf{t}) \delta(\mathbf{r} - \mathbf{R}(t))$$

driving through a crystal

Motile particles in elastic medium on substrate

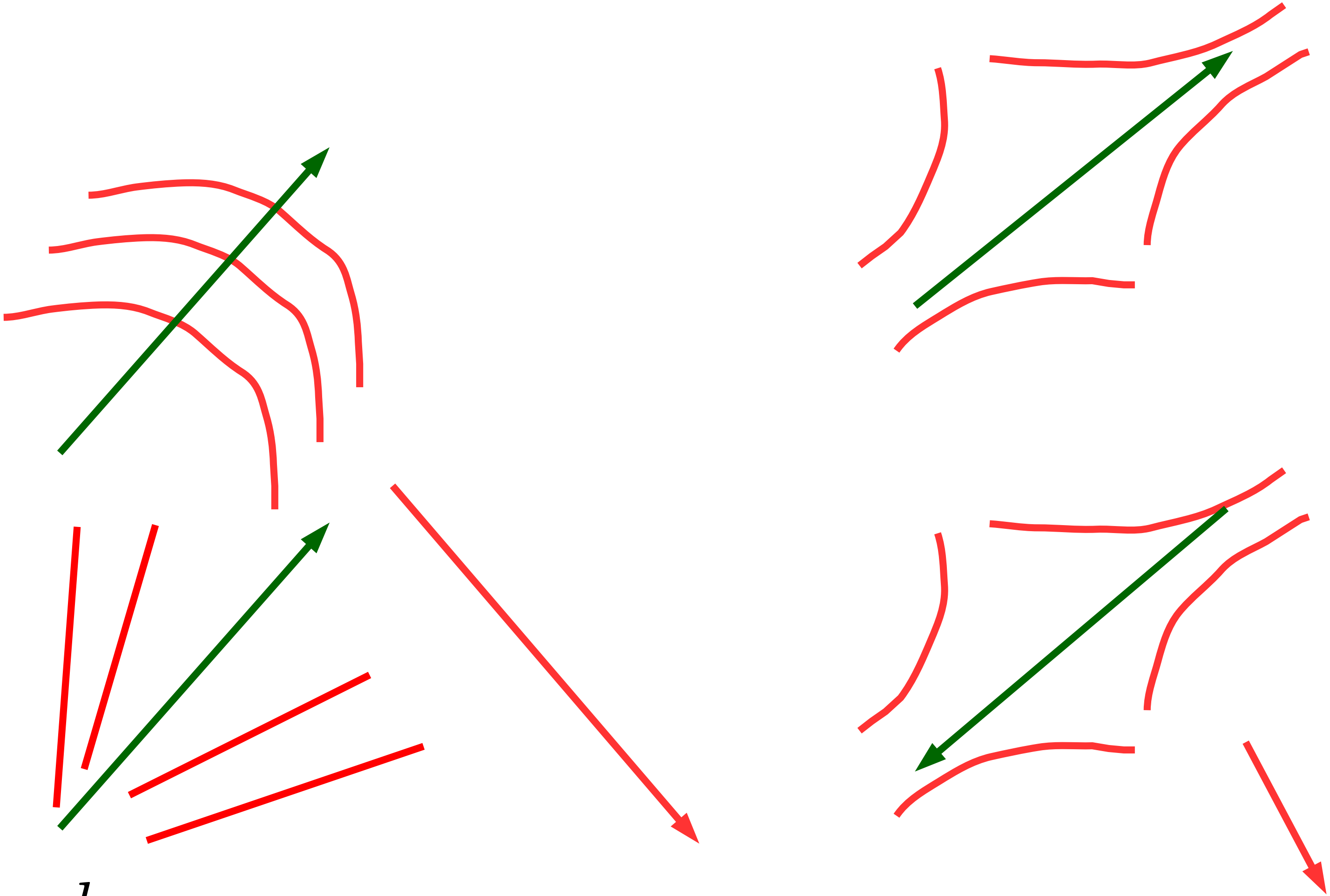
$$\dot{\mathbf{R}}(t) = v_0 \mathbf{n}(t)$$

$$\zeta \partial_t \mathbf{u} = -\delta F / \delta \mathbf{u} + f \mathbf{n}(\mathbf{t}) \delta(\mathbf{r} - \mathbf{R}(t))$$

$$\frac{d\mathbf{n}}{dt} = (\mathbf{I} - \mathbf{nn}) \cdot \left(\gamma_1 \nabla^2 \mathbf{u} + \gamma_2 \nabla\nabla \cdot \mathbf{u} + \kappa \varepsilon \cdot \mathbf{n} \right)$$

$$\varepsilon = (\nabla \mathbf{u} + \nabla \mathbf{u}^{\mathrm{T}})/2$$

Curvature: polar orientation
Strain: apolar orientation



$$\frac{d\mathbf{n}}{dt} = (\mathbf{I} - \mathbf{nn}) \cdot \left(\gamma_1 \nabla^2 \mathbf{u} + \gamma_2 \nabla\nabla \cdot \mathbf{u} + \kappa \varepsilon \cdot \mathbf{n}\right)$$

Strain field of a motile particle

$\mathbf{U}$= displacement field in frame comoving and corotating with particle

$$[-\zeta v_0 \partial_x - (\mu \nabla^2 + \lambda \nabla\nabla\cdot)]\mathbf{U} = f\delta(\mathbf{r})\hat{\mathbf{x}}$$

Screening

$$\alpha = \zeta v_0/\mu$$

$$\beta = \zeta v_0/\lambda$$

$$U_x = \frac{f}{4\pi\mu}\Big\{\Big[K_0\Big(\frac{\alpha r}{2}\Big) - \frac{x}{r}K_1\Big(\frac{\alpha r}{2}\Big)\Big]e^{-\frac{\alpha x}{2}} + \frac{\beta}{\alpha}\Big[K_0\Big(\frac{\beta r}{2}\Big) + \frac{x}{r}K_1\Big(\frac{\beta r}{2}\Big)\Big]e^{-\frac{\beta x}{2}}\Big\}$$

and similarly U_y

Crucial: asymp forms of $K_0 \Rightarrow$ exponential decay for x>0, $|x|^{-1/2}$ for x<0

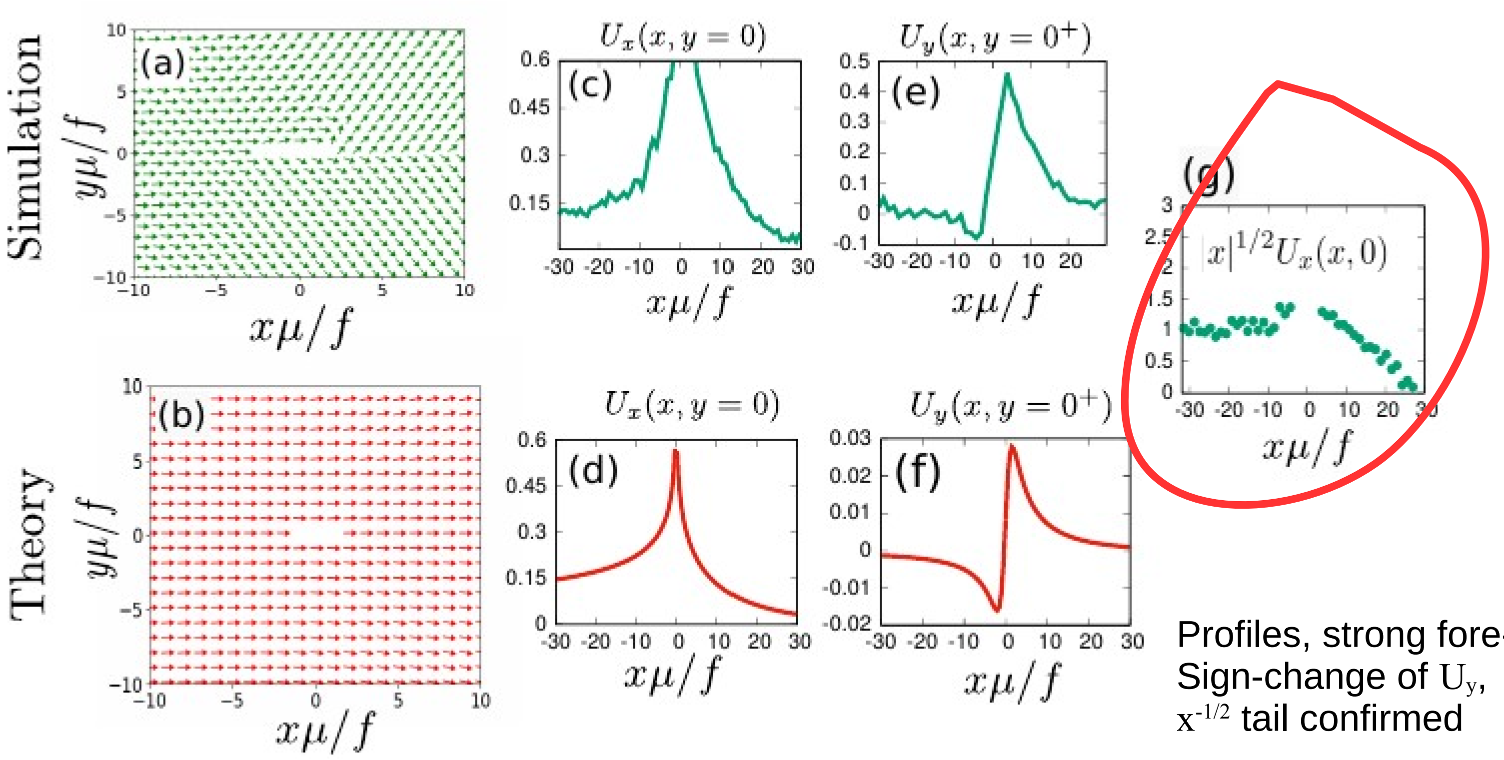
Overdamped elastic wake

Banerjee, Mondal, Banerjee, Thutupalli, Rao arXiv 2109.10438

Same behaviour for density field in a compressible fluid layer

Comparison with measured fields

Numerical experiment on vibrated layer of grains
Inelasticity, static friction, base, lid all included



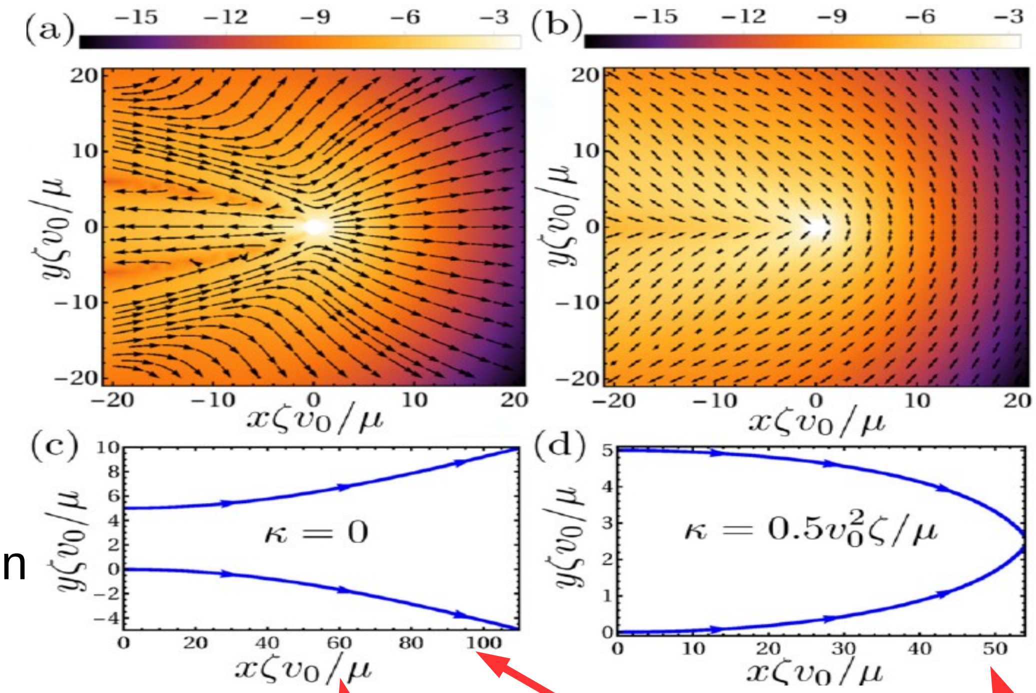
Profiles, strong fore-aft asymmetry,
Sign-change of U_y,
$x^{-1/2}$ tail confirmed

One particle in strain field of other

align with extension axis ⇒ “attraction”

Streamline plot of lattice curvature

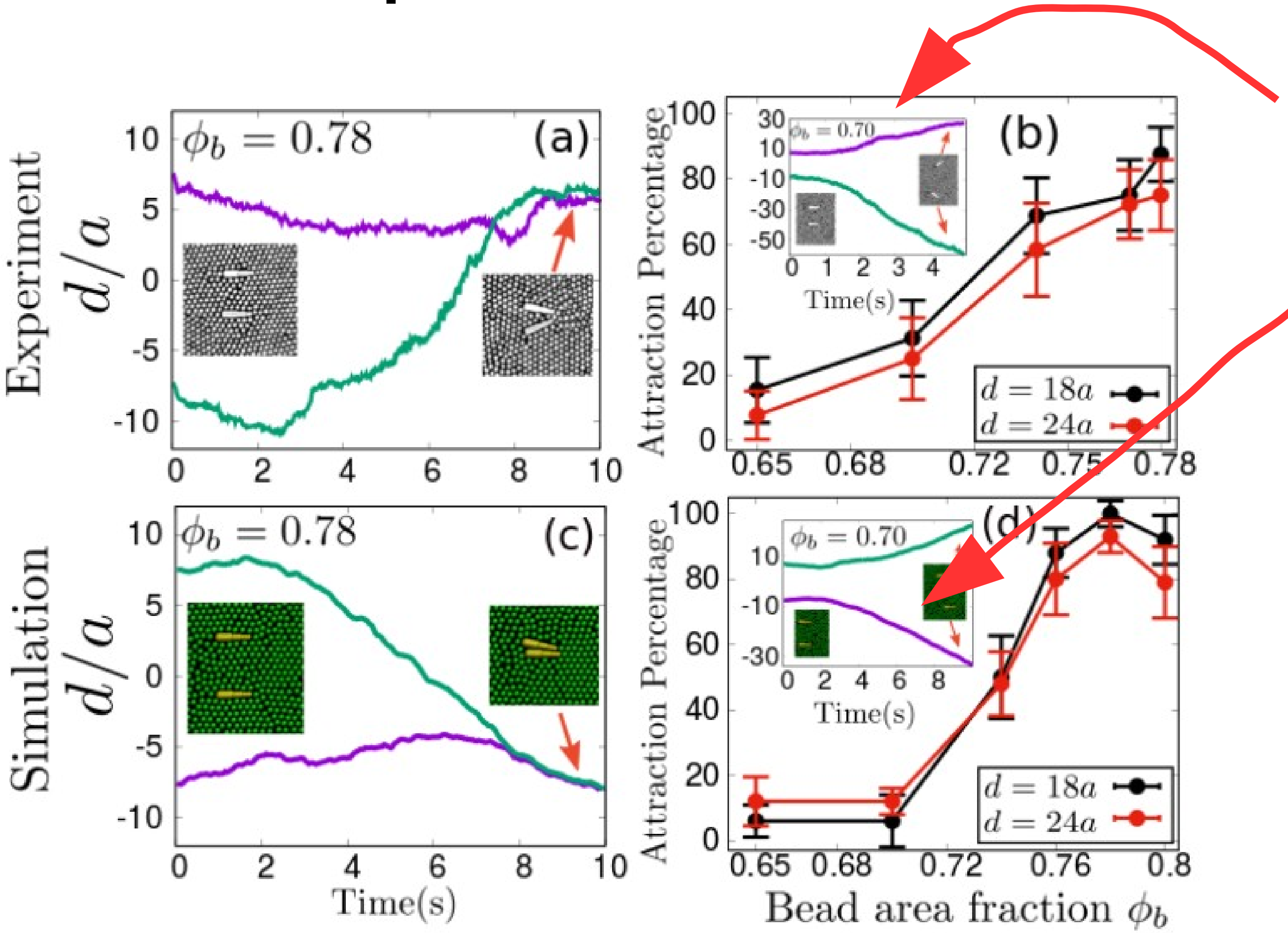
Principal axis of strain



Particle trajectories with and without effect of strain

$$\frac{d\mathbf{n}}{dt} = (\mathbf{I} - \mathbf{n}\mathbf{n}) \cdot \left(\gamma_1 \nabla^2 \mathbf{u} + \gamma_2 \nabla\nabla \cdot \mathbf{u} + \kappa \varepsilon \cdot \mathbf{n}\right)$$

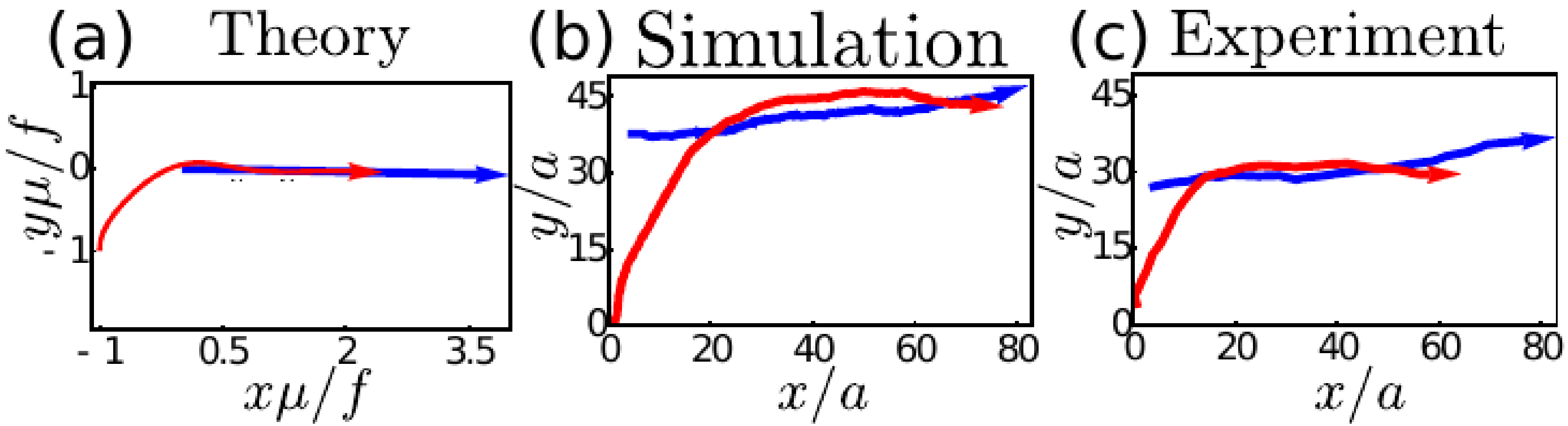
Capture in experiment and simulation



Repel when beads are fluid

Non-reciprocal interaction

(arXiv:2007.04860)



Particle in front gets no indication of particle behind
Pursuer particle senses distortion field of pursued particle!

pursuit and capture: laboratory experiment

pursuit and capture: numerical experiment

SUMMARY

- General framework for “powered” matter
  - from equilibrium Langevin to active dynamics
  - 1-particle models, broken-symmetry hydrodynamics
- Instability and superstability
  - flocks in fluid unstable without inertia; surprises in confinement
- Motility in dense media – fluid and elastic
  - flow and strain as signals; non-reciprocal pursuit and capture