Stable Mixing in Hawk–Dove Games under Best Experienced Payoff Dynamics

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Hawk-Dove Game: Motivating Example

- A ***buyer*** and ***seller*** bargain over the price of an asset (e.g., house)
- Two bargaining strategies: stubborn ***hawk*** / flexible ***dove***
  - *Two Doves*: Trade with a fair price
  - *Two Hawks*: Bargaining is likely to fail (low payoff)
  - *Hawk vs. Dove*: Trade with price favorable to the hawk

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Buyer \ Seller	h_2	d_2
h_1	0,0	$1+g$, $1-g$
d_1	$1-g$, $1+g$	1,1

- $g \in (0,1)$ = hawk's gain against a dovish opponent (=dove's loss; denoted by the ratio $\frac{v}{c}$ in other papers)

Hawk-Dove Applications in the Existing Literature

Hawk-Dove was employed in modeling various strategic situations:

- *Provision of public goods* (Lipnowski and Maital, 1983)
- *Nuclear deterrence* (Brams and Kilgour, 1987; Dixit et al., 2019)
- *Industrial disputes* (Bornstein, Budescu and Zamir, 1997)
- *Bargaining problems* (Brams and Kilgour, 2001)
- *International territorial conflicts* (Baliga and Sjostrom, 2020)
- *Task allocation problems* (Herold and Kuzmics, 2020)

Equilibria in Hawk-Dove Games

	h_2	d_2
h_1	$0,0$	$1+g,\ 1-g$
d_1	$1-g,\ 1+g$	$1,1$

- Hawk-Dove game admits **three** Nash equilibria:
  - ***Two pure (boundary) equilibria***
    (No costly conflicts, payoff inequality)
  - ***A mixed (interior) equilibrium***
    (Equal, yet relatively low payoff)
- *Which outcome is more likely to arise?*

Summary of Main Results

Earlier Results: *Global convergence* to *pure* equilibria
under *many* evolutionary dynamics
(i.e., *interior* states are *unstable*)

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Our Result: *Interior* states can be *stable* under *certain* evolutionary dynamics based on sampling (*Best Experienced Payoff Dynamics*)

Model (Hawk–Dove Game)

Table 1: Payoff Matrix of a Hawk–Dove Game $g, l \in (0, 1)$

		Player 2	
		h_2	d_2
Player 1	h_1	$0, 0$	$1+g, 1-l$
	d_1	$1-l, 1+g$	$1, 1$

Two *pure* equilibria (h_1, d_2) and (d_1, h_2) and one *mixed* equilibrium

Model (Evolutionary Dynamics)

	h_2	d_2
h_1	$0,0$	$1+g,\ 1-l$
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A unit-mass continuum of agents in each of two populations

Agents in population 1 are randomly matched with agents in population 2 to play hawk–dove game

Population state $p(t) = (p_1(t), p_2(t)) \in [0,1]^2$
$p_i(t)$ = share of agents playing action h_i at time t in population i

[$(0, 0.5)$ means everyone in population 1 plays d_1
and uniform play in population 2]

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$$\dot{p}_1(t) = w_1(p(t)) - p_1(t)$$
$$\dot{p}_2(t) = w_2(p(t)) - p_2(t)$$

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Goal: Characterize the limit points of the dynamics i.e., $\lim_{t\to\infty} p(t)$ for all initial states $p(0)$

Standard Definitions of Dynamic Stability

Definition

$p^* \in [0,1]^2$ is a *stationary* state if $w(p^*) = p^*$

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Definition

A stationary state is *unstable* if it is not Lyapunov stable

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Definition

A set P^* is *globally stable* if the population converges to P^* starting from any initial interior state

Illustration of Evolutionary Dynamics

Consider a hawk–dove game:

	h_2	d_2
h_1	$0, 0$	$1+g, 1-g$
d_1	$1-g, 1+g$	$1, 1$

Best Response Dynamics:

A new agent

- Knows the population state $p = (p_1, p_2)$
- Chooses a myopic best response

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Playing h_1 gives $(1+g)(1-p_2)$

Playing d_1 gives $(1-g)p_2 + 1 \cdot (1-p_2)$

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$$h_1 \succ d_1 \iff (1+g)(1-p_2) > (1-g)p_2 + 1 \cdot (1-p_2)$$
$$\iff p_2 < g$$

Illustration of Evolutionary Dynamics

$$\dot{p}_1 = \begin{cases} 1 - p_1 & \text{if} \quad p_2 < g \\ -p_1 & \text{if} \quad p_2 > g \end{cases}$$

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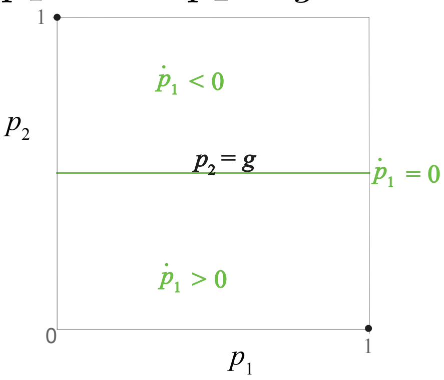

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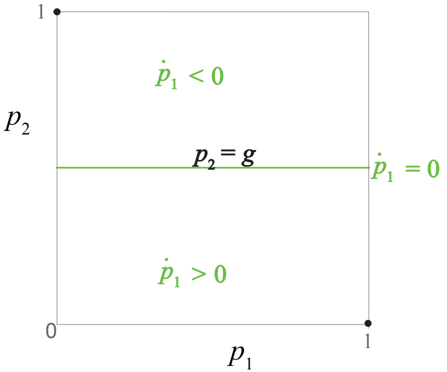

$\dot{p}_2 > 0$ if $p_1 < g$
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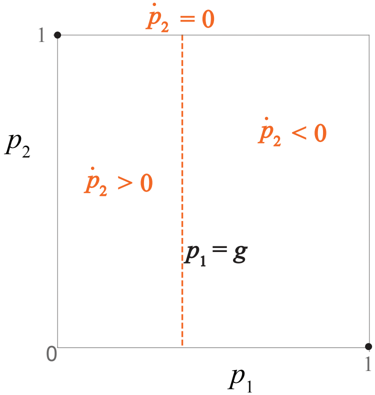

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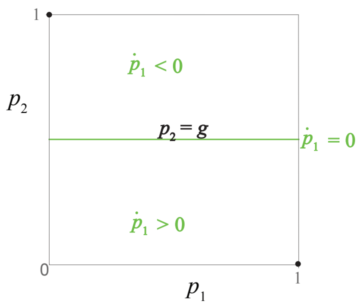

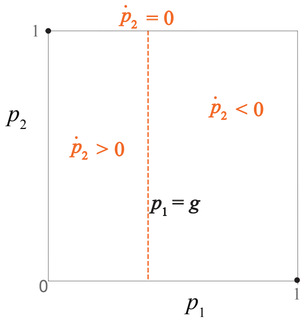

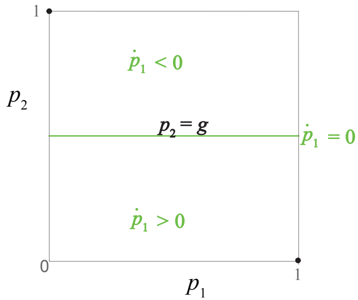

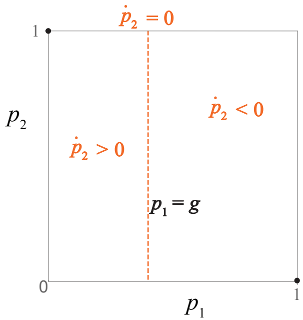

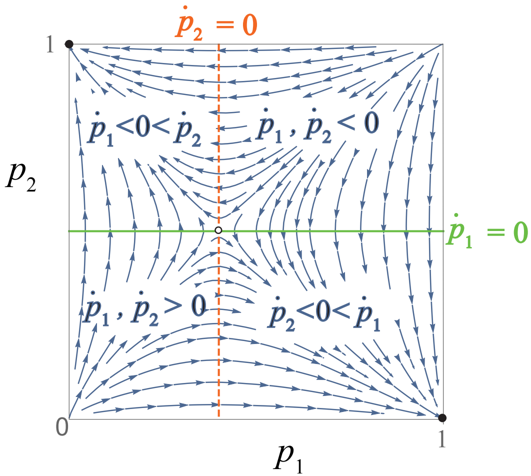

Figure 1: Best response dynamics in hawk–dove game

Model (Best Experienced Payoff Dynamics)

A new agent:

- Tests each action k times, with each trial against a newly drawn opponent
- Then chooses the action whose mean payoff was highest during the testing phase

(Osborne and Rubinstein 1998, Sethi 2000, Sandholm et al. 2019)

Model (Best Experienced Payoff Dynamics)

	h_2	d_2
h_1	$0, 0$	$1 + g, 1 - l$
d_1	$1 - l, 1 + g$	$1, 1$

h_1-sample: the sample against which h_1 is tested
d_1-sample: the sample against which d_1 is tested

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X_k^q, Y_k^q are i.i.d. Binomial(k, q)
$X_k^{p_2}$ = number of h_2 players in the h_1-sample
$Y_k^{p_2}$ = number of h_2 players in the d_1-sample

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Playing h_1 gives $(1+g)(k - X_k^{p_2})$
Playing d_1 gives $(1-l)Y_k^{p_2} + 1 \cdot (k - Y_k^{p_2}) = k - lY_k^{p_2}$

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Assume when there is a tie, the new agent plays d_1

$$\begin{aligned} \text{New agent plays } h_1 &\iff (1+g)(k - X_k^{p_2}) > k - lY_k^{p_2} \\ &\iff (1+g)X_k^{p_2} - lY_k^{p_2} < gk \end{aligned}$$

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$$\dot{p}_1 = w_1(p_2) - p_1 \tag{1}$$

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$$\dot{p}_1 = w_1(p_2) - p_1 \tag{1}$$

$w_1(p_2)$ is the probability that new agent in population 1 chooses h_1
$w_1(p_2) = Pr\left((1+g)X_k^{p_2} - lY_k^{p_2} < gk\right)$

Model (Best Experienced Payoff Dynamics)

Best Experienced Payoff Dynamics in the Hawk–Dove Game:

$$\dot{p}_1 = w_1(p_2) - p_1$$
$$\dot{p}_2 = w_2(p_1) - p_2,$$

where, $w_1(q) = w_2(q) = Pr\left((1+g)X_k^q - lY_k^q < gk\right)$

Results

Let $p(t) = (p_1(t), p_2(t))$

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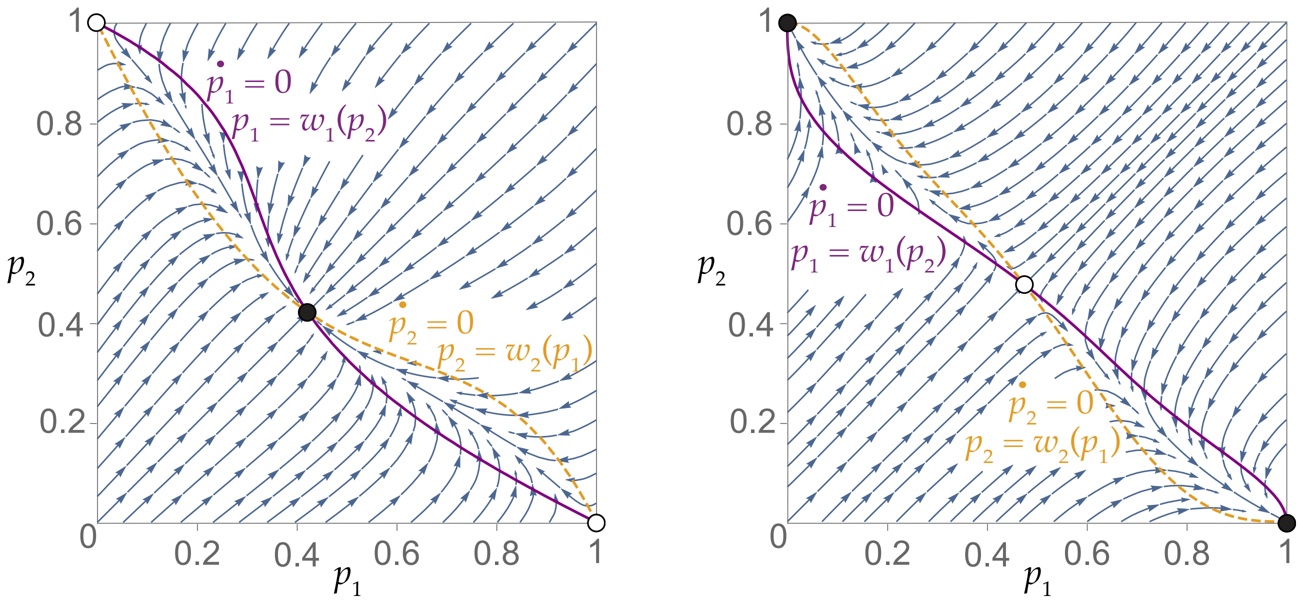
The paper provides *sufficient* conditions under which *pure* stationary states are *unstable*

Results (Interior state can be globally stable)

Figure 2: Phase Plots for various values of k



The figure illustrates the phase plots of BEP dynamics for two environments: left panel, $g = l = 0.4, k = 2$; right panel, $g = l = 0.4, k = 5$.

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$g \in (0, 1)$ and $k \geq 2$

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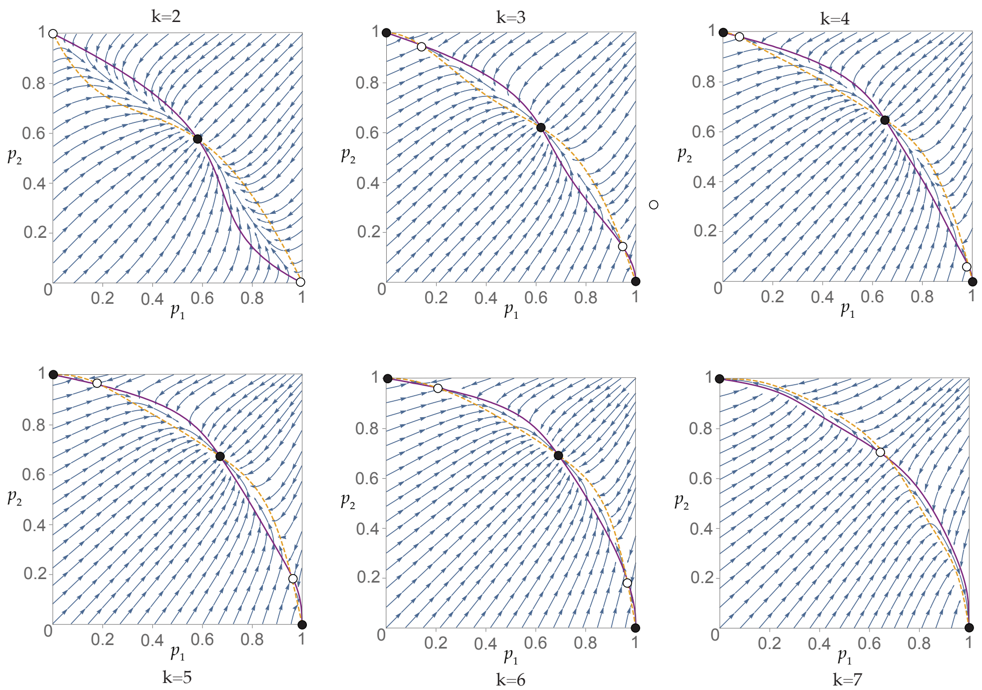
$$d_1 \succ h_1 \iff 1-g+(k-1)\cdot 1 > (k-1)(1+g) \iff kg < 1$$

Results (Boundary and Interior states can be stable)

Figure 3: Phase Plots for Various Values of k ($g = l = 0.85$)



Related Literature and Important Themes

Sampling dynamics in games:

Public goods game (Mantilla et al., JPET 2018)
Centipede game (Sandholm et al., TE 2019)
Finitely repeated games (Sethi, JET 2021)
Prioner's dilemma (Arigapudi et al., JET 2021)
Traveler's dilemma (Berkemer et al., GEB 2023)
Trust Game (Arigapudi and Lahkar, EL 2024)
Coordination games (Arigapudi et al., AEJ-Micro (forthcoming))

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Two themes:

- ***Non-Nash*** outcomes can be ***stable***
- ***Strict equilibria*** can be ***unstable***

Conclusion

Hawk–Dove games admit two types of equilibria:
boundary and *interior*

Existing literature: Interior states are *unstable* under *many* evolutionary dynamics

Our Result: Interior states can be *stable* under *certain* evolutionary dynamics

THANK YOU