# Non-perturbative determination of Spin-Spin interaction in finite temperature

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Work in preparation with
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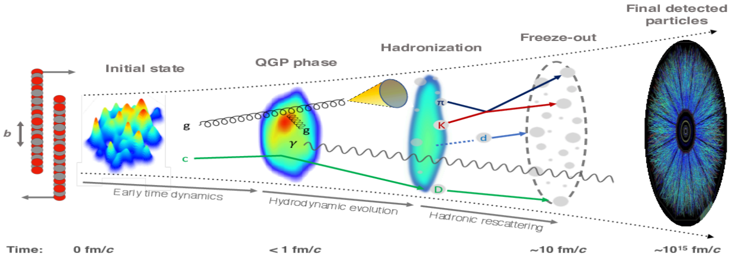
# Introduction




credit:CERN

- Quarkonia produces at very early times, $t_f < 0.1$ fm/c.
- Evolve through-out the evolution of the medium.
- Good probe to study properties of QGP medium.

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- Calculate $\mathcal{O}(1/M^2)$ correction to the potential on the lattice → non-perturbative spin interaction potential.

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- We want to study the spin interaction potential at finite temperature from first principles using Lattice QCD.

# Motivation

- Matsui and Satz proposed about sequential suppression of quarkonia in QGP $\rightarrow$ But, no distinction between pseudo-scalar (PS) and vector (V) quarkonia.

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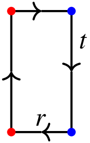
- More accurate charmonia spectral function for V states → Improves di-lepton production rate estimate in QGP.
- Long distance excitations in QGP receive large contributions due to spin interaction even at $T \sim 160$ GeV. [S.T et al. Phys.Rev.Lett. 135 (2025) 1, 012301]

# Thermal potential in real time

- In real time, the amplitude for the evolution of heavy $q\bar{q}$ system with separation $r$,

$$C_\Gamma(r,t) = \left\langle \Omega \middle| \mathcal{T}(r,t)\mathcal{T}^\dagger(r,0) \middle| \Omega \right\rangle_T$$

$$\mathcal{T}^\dagger(r,t) = \chi^\dagger(x)\, U(r,t)\, \Gamma\, \phi(y)$$



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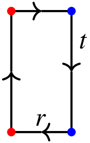


- Thermal potential can be defined as, [M. Laine et al. JHEP 0703:054,2007]

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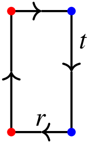


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- The NRQCD Lagrangian can be written as,

$$\begin{aligned} L_{\text{eff}} &= L_{\text{QCD}}(\psi,\bar{\psi},A_\mu) + \chi^\dagger\left(iD_0 - M + \frac{1}{2M}\left(D^2 + g_0\vec{\sigma}\cdot\vec{B}\right)\right)\chi \\ &+ \phi^\dagger\left(iD_0 + M - \frac{1}{2M}\left(D^2 + g_0\vec{\sigma}\cdot\vec{B}\right)\right)\phi + \mathcal{O}\left(\frac{1}{M^2}\right) \end{aligned}$$

# Analytic continuation-ill posed problem

- Full potential $\rightarrow V(r) = 2M + V_{\mathrm{s}}(r) + \frac{1}{4M^2} V_{\mathrm{BB}}(r) + \cdots$

$$\text{Static potential} \quad \longrightarrow \quad V_{\mathrm{s}}(r) = \lim_{t\to\infty} i\partial_t \log \langle \mathrm{Tr}_c(W(r,t)) \rangle$$

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- Constrain the infinite dimensional solutions space → Needs first principle physics motivated inputs.

# Spin-Spin and Self-Spin Correlator

- Spin-Spin and Self-Spin correlators are defined as,

$$
\begin{aligned}
W_{\rm BB}^{\rm int}(r,\tau) &= \mathcal{X}\int_0^\tau d\tau' \int_0^\tau d\tau'' \frac{\langle {\rm Tr}_c(\mathcal{T} W(r,\tau) g_0 B_i(\vec{y},\tau') g_0 B_i(\vec{x},\tau''))\rangle}{\langle {\rm Tr}_c(W(r,\tau))\rangle} \\
W_{\rm BB}^{\rm self}(r,\tau) &= \int_0^\tau d\tau' \int_0^\tau d\tau'' \frac{\langle {\rm Tr}_c(\mathcal{T} W(r,\tau) g_0 B_i(\vec{x},\tau') g_0 B_i(\vec{x},\tau''))\rangle}{\langle {\rm Tr}_c(W(r,\tau))\rangle}
\end{aligned}
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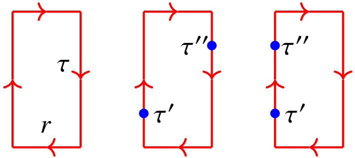
\begin{aligned}
\mathcal{X} &= 1(\text{Pseudo-scalar}) \\
&= -\frac{1}{3}(\text{Vector})
\end{aligned}
$$



# Finding analytical structure

- In Hard-thermal-loop pert. theory at tree level, the correlators can be calculated.

$$\langle W_{\rm BB}(r,\tau)\rangle \quad = \quad -F(r)\,\tau - G(r)\,\frac{\beta}{\pi}\log\left(\sin\frac{\pi\tau}{\beta}\right) + \cdots$$

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- After analytical continuation the thermal potential can be written as,

$$\mathrm{V}_{\rm BB}(r) = \lim_{t\to\infty} i\partial_t \log\left(W_{\rm BB}(r,\tau\to it)\right) = F(r) - i\,G(r)$$

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- Lattice correlators can be fitted with the analytical exp. → Extract real and imaginary parts of the thermal potential.

# Origin of Imaginary potential

- In the static limit ($k_0 = 0, \vec{k} \rightarrow 0$) at LO,

  $\Pi_{00} = m_D^2 \sim g^2 T^2 \longrightarrow$ Color-screening of chromo-electric fields.

  $\Pi_{ij} = 0 \longrightarrow$ No color-screening of chromo-magnetic fields $\longrightarrow$ Non-perturbative generation of $m_G \sim g^2 T/\pi$. [A.D. LINDE, Phys. Lett. B 96, 289 (1980)]

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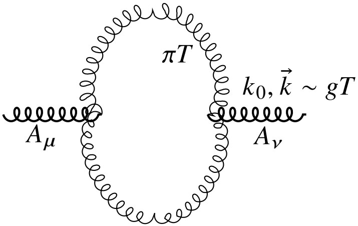
- 


  Energy loss of gauge fields by scattering with hard particles $|\vec{p}| \sim \pi T$.

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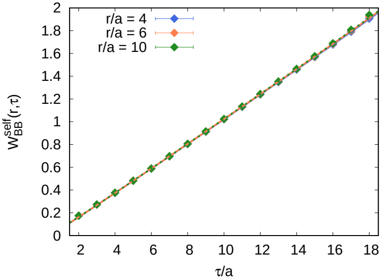
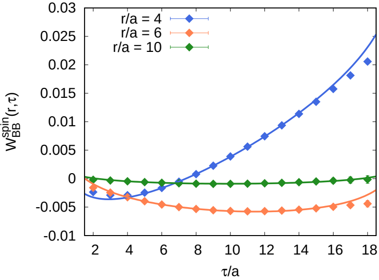
- Noise due to ultraviolet fluctuations reduced using gradient flow.
- We have analyzed $N_{\text{conf}} = 2000$ configurations on $N_s = 68$, $N_\tau = 16, 20$ lattice at $T = 1.5\, T_d$ ($T_d \sim 312$ MeV) for quenched QCD.

# Spin Correlators

Fitting of the lattice data with determined functional form:

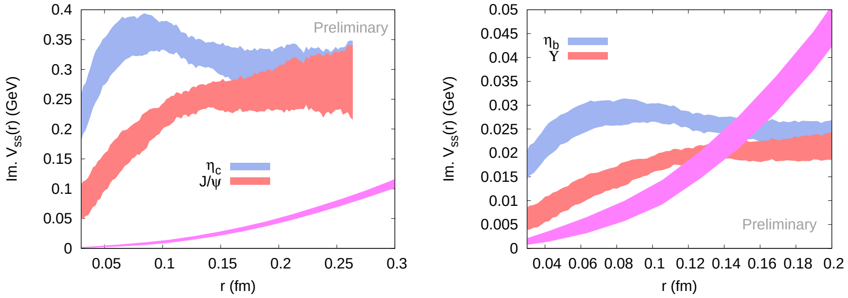



Self-spin and Spin-spin correlator for $N_\tau = 20$ lattice at a fixed flow-time.

⊛ Renormalization of color-magnetic field is non-trivial → Renormalization of the coupling is not sufficient. [M. Laine JHEP 06 (2021) 139, G. Moore et al. arXiv:2410.01578]

# Imaginary part of Spin potential

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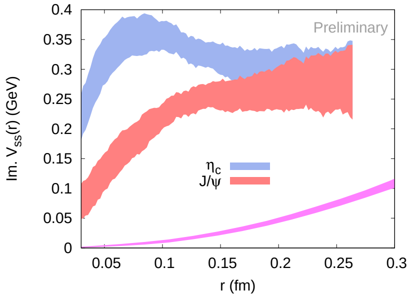


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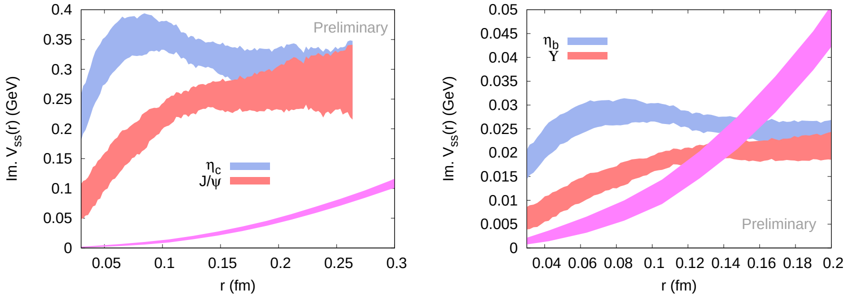
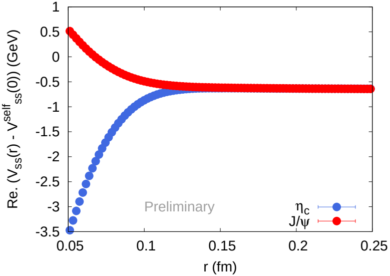
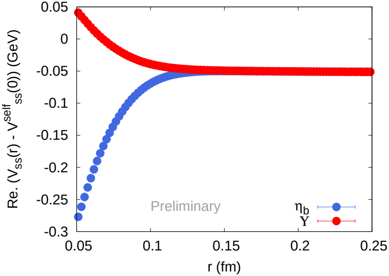



- Non-perturbative study confirms the pert. prediction of imag. part→ But, the magnitude is much larger than pert. prediction.
- Spin potential dominates at short distances than static potential → Need to take into account to construct realistic spectral function.
- At short distances, strength of PS and V potential is quite different → Different thermal decay widths → "PS will get suppressed at earlier *T* than V".

# Real part of Spin potential

- $\text{Re}\, V^{\text{self}}_{\text{BB}}(r,T)|_{\text{latt}} = \underbrace{\text{Re}\, V^{\text{self}}_{\text{BB}}(0,0)}_{\text{UV div.}} + \underbrace{\text{Re}\, V^{\text{self}}_{\text{BB}}(r,T)}_{\text{finite T cont.}}.$



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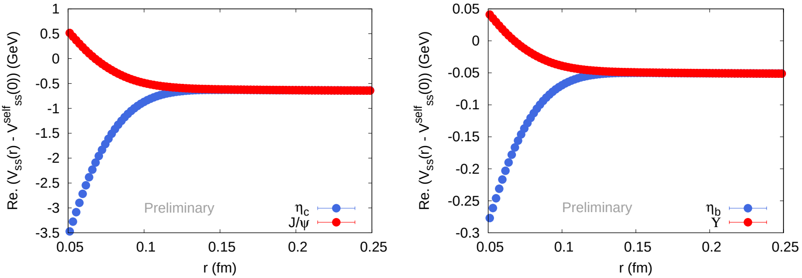


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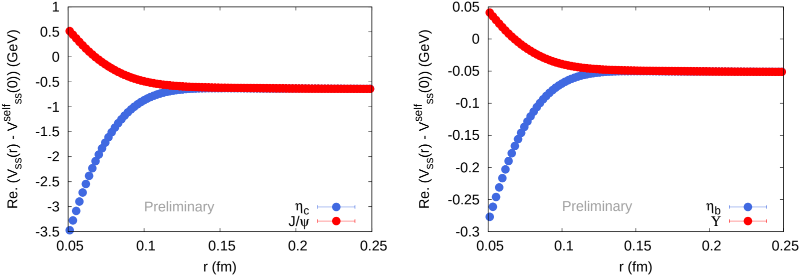


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- Spin-spin potential can be explained in terms of lattice regulated delta function $\rightarrow$ Functional form remains same as $T = 0$ potential, calculated using lattice pert. theory.

# Conclusion

- For the first time, we have calculated non-perturbative spin interaction potential in finite temperature QGP medium from first principles.
- Spin potential develops an imaginary part at finite temperature, similar to static potential.
- At short distances around the size of quarkonia bound state, spin potential gives dominant contribution to thermal potential → Need to take into account to construct realistic spectral function.
- Qualitatively pseudo-scalar states will get suppressed early than vector states.
- To predict the suppression temperature of PS and V quarkonia, the temperature dependence of the spin-dep potential needs to be studied in detail.