Games, Networks and Self-Organization

Explaining the collective transition to social cooperation

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in collaboration with

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Games @ Complex Systems Group in IMSc

Shakti N Menon

Interacting Epidemics & Games

Raj Kumar Pan, Shakti N Menon

Games on Graphs: Coevolution of cooperation & communities

Minority Games & Information

V Sasidevan, A Kushal

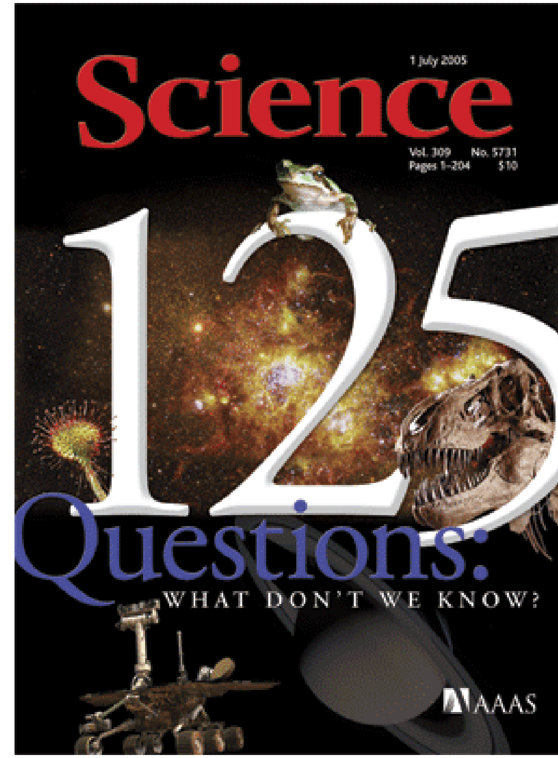
Solution concepts Co-action vs Nash

V Sasidevan

Why cooperate ?

Elizabeth Pennisi (2005):

"When Charles Darwin was working out his grand theory on the origin of species, he was perplexed by the fact that animals from ants to people form social groups in which most individuals work for the common good. This seemed to run counter to his proposal that individual fitness was key to surviving over the long term"

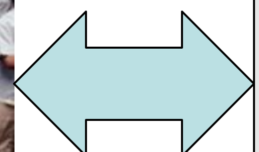


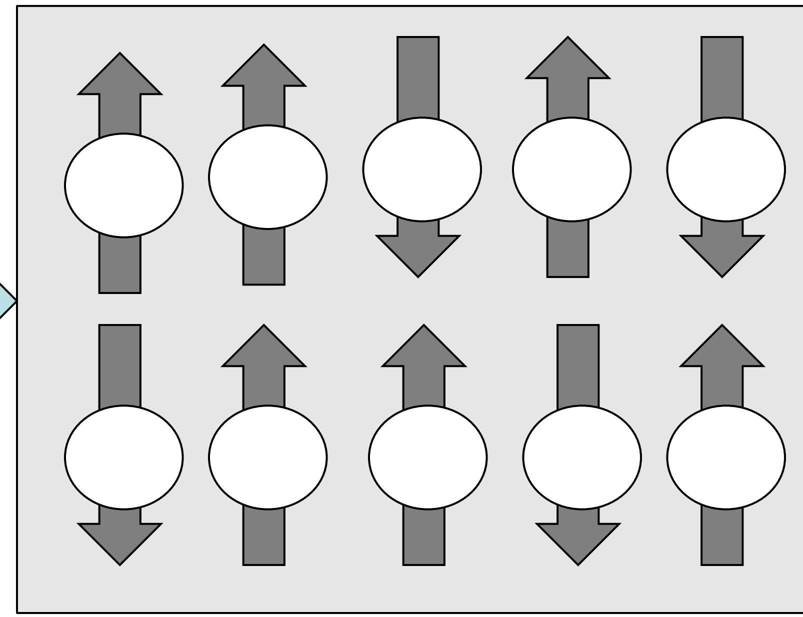
July 2005

J B S Haldane: "I will jump into the river to save two brothers or eight cousins" (precursor to Hamilton's Rule)

Natural selection may encourage altruistic behavior among kin as it improves the reproductive potential of the genetically related group as a whole, but unclear why unrelated individuals should help each other.

From the perspective of physicists…





Philip W Anderson, *Science* (1972)

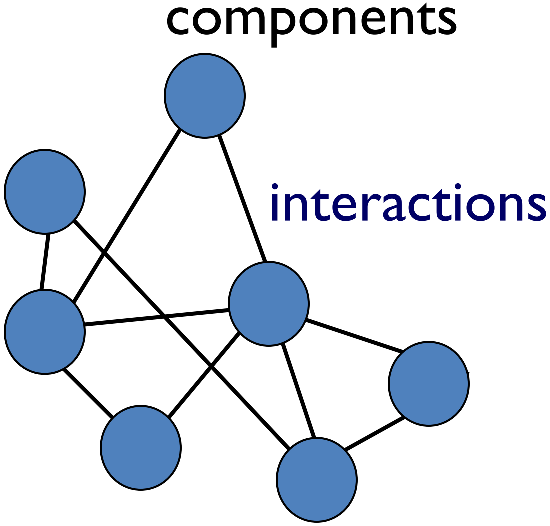
Emergence

"More is different"

A complex system comprisng

- many interacting components, and
- connected with each other in a network

→ results in emergence at the level of the system, of collective behavior qualitatively different from that of the individual components.

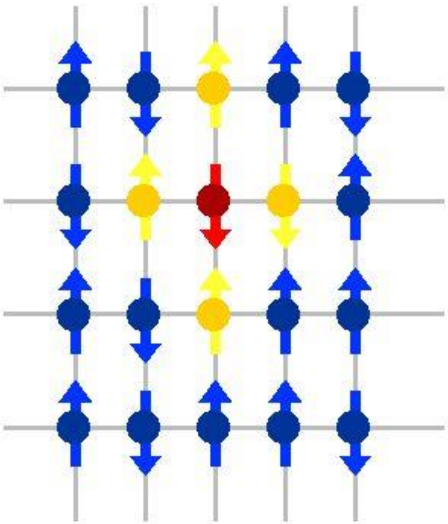


E.g., component = sensible individuals
system = mob
emergent behavior = riot

The Question:

how interactions between 'selfish' (rational) agents → unexpected systems-level coordination ('cooperation')

Spin models as a paradigm for Complex Systems



•Spin orientation: mutually exclusive choices

•Choice dynamics: decision based on information about choice of majority in local neighborhood

Simplest case: 2 possible choices

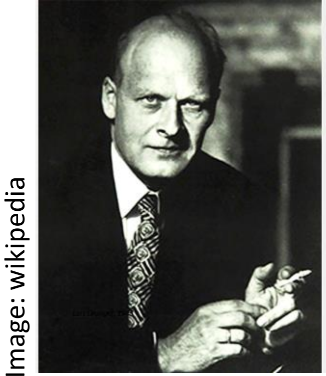
Ising model with Ferromagnetic interactions: each agent can be in one of 2 states (Yes/No , +/-)

The 2-dimensional Ising model (1944)

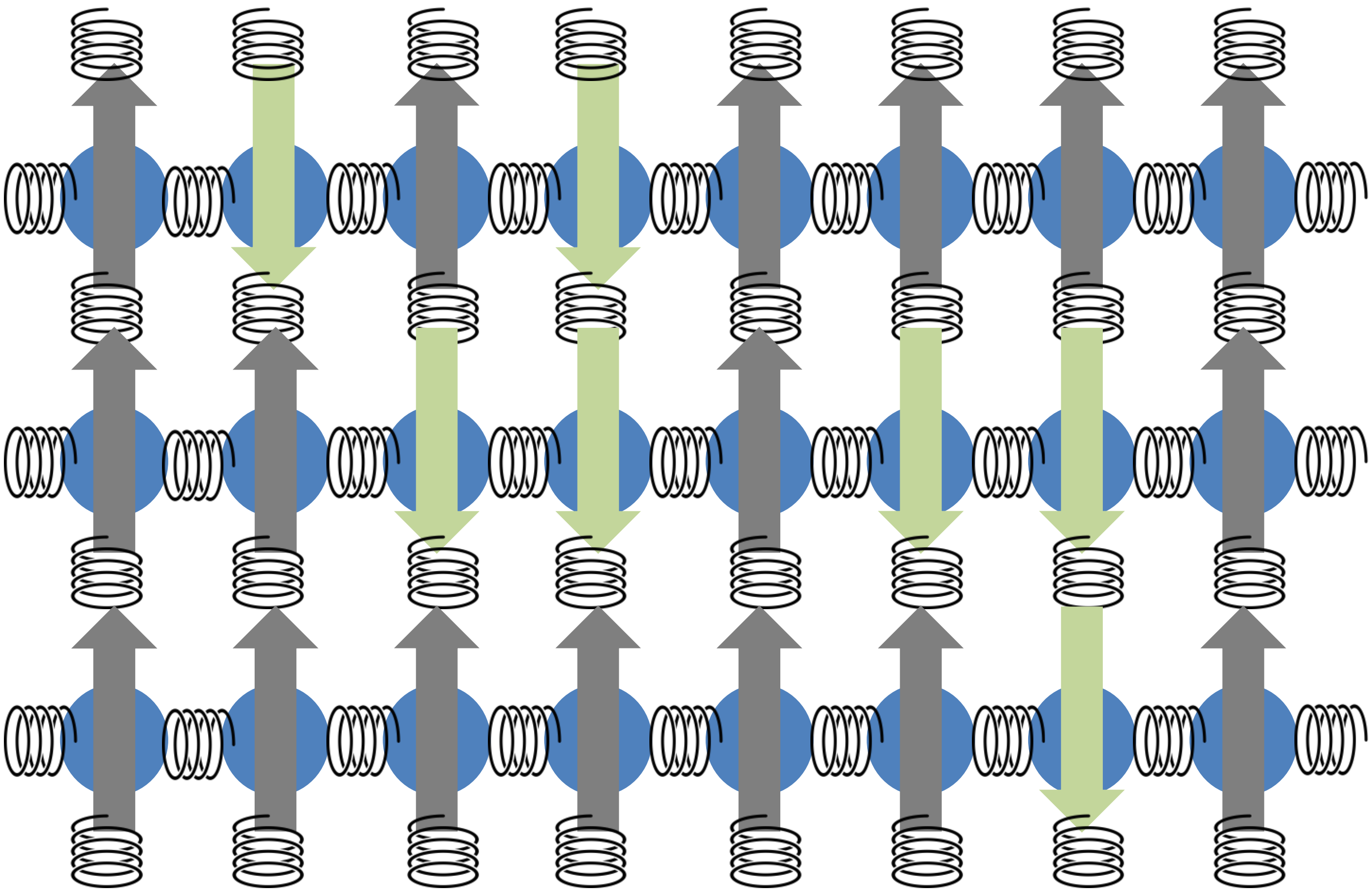
Emergence: transition to self-organized coordination

The system spontaneously orders at $T < T_c$

Image: wikipedia



Lars Onsager



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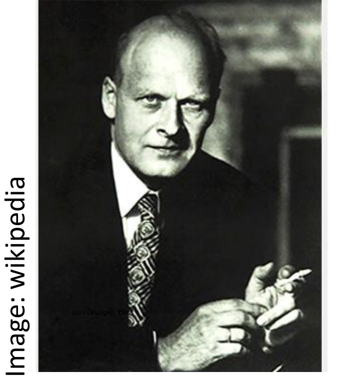

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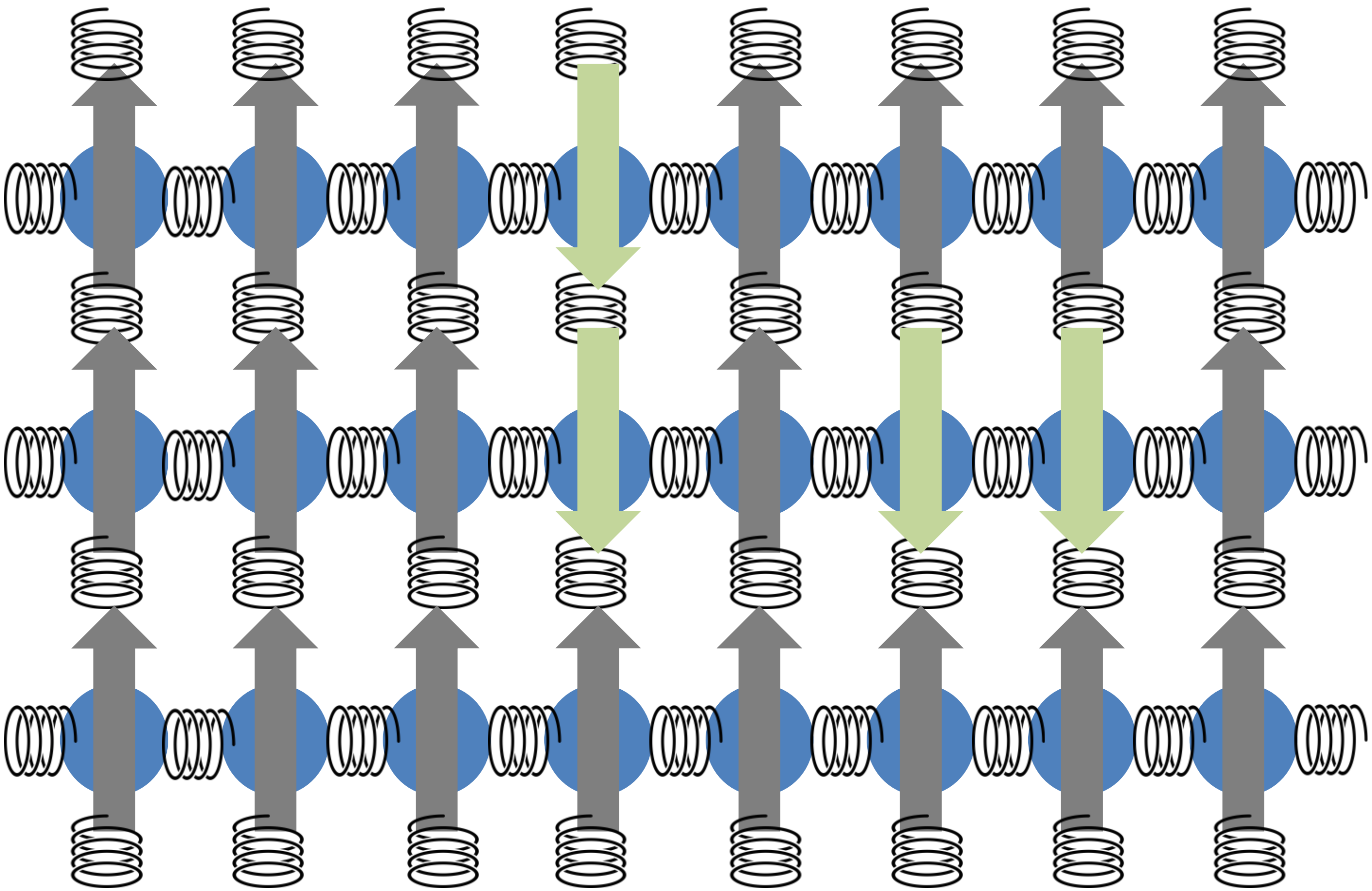

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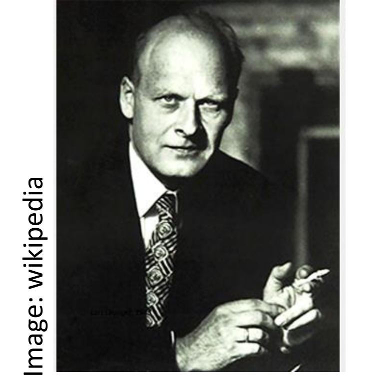

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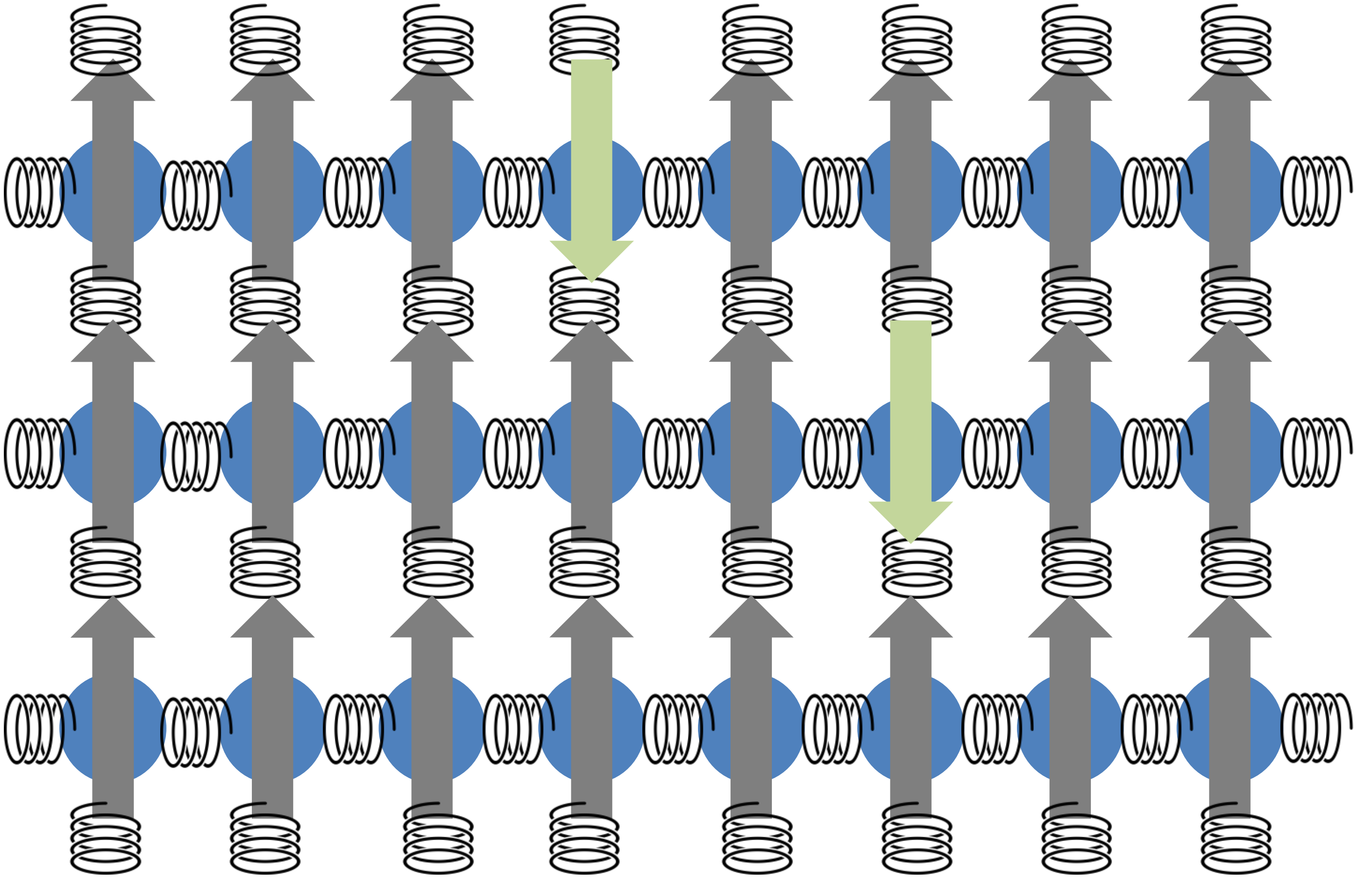

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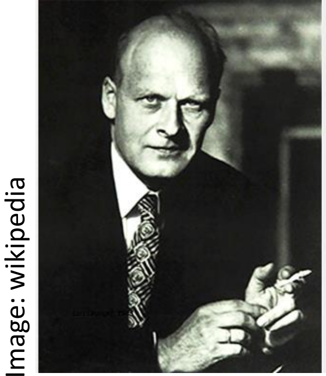

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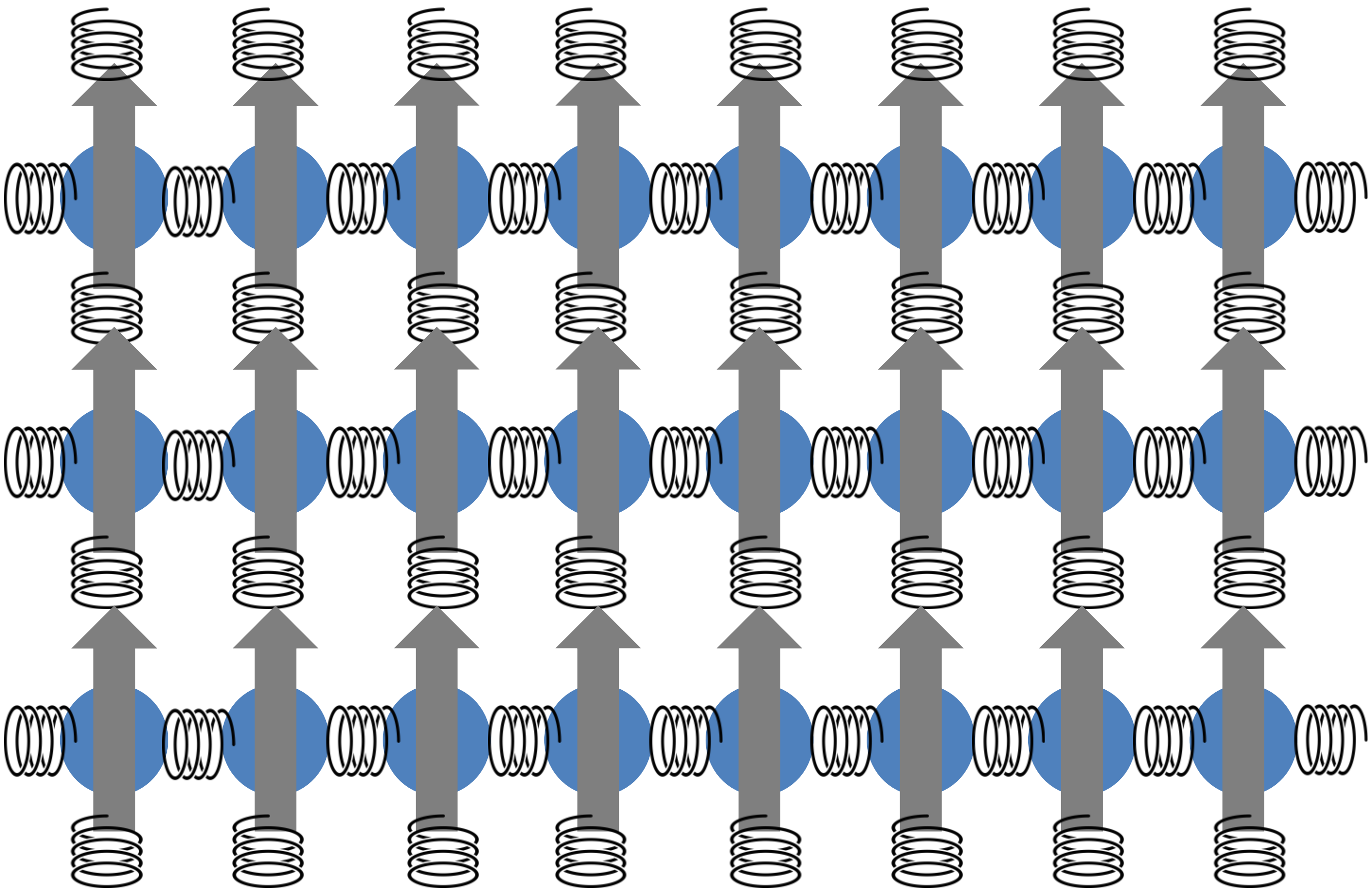
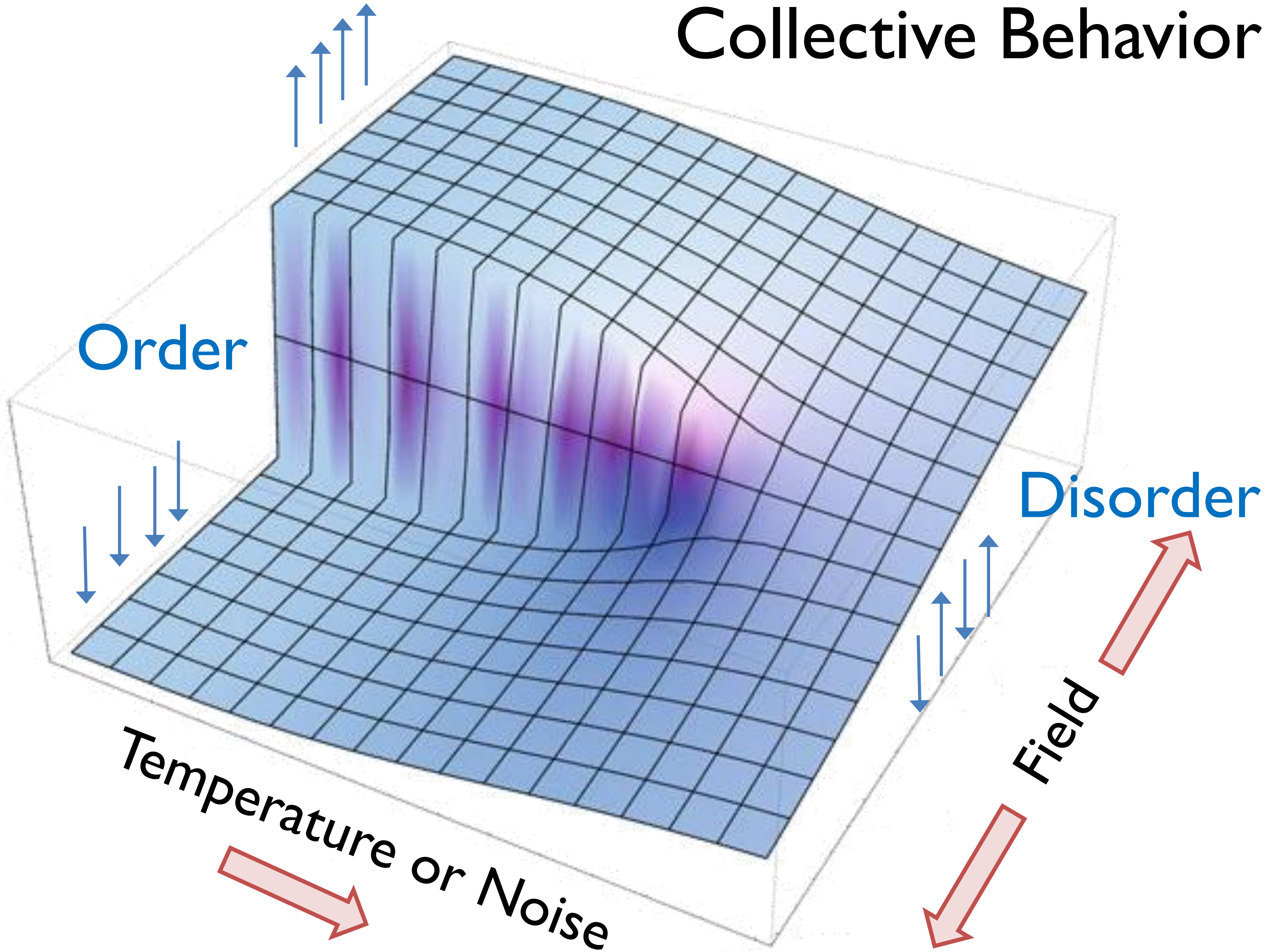
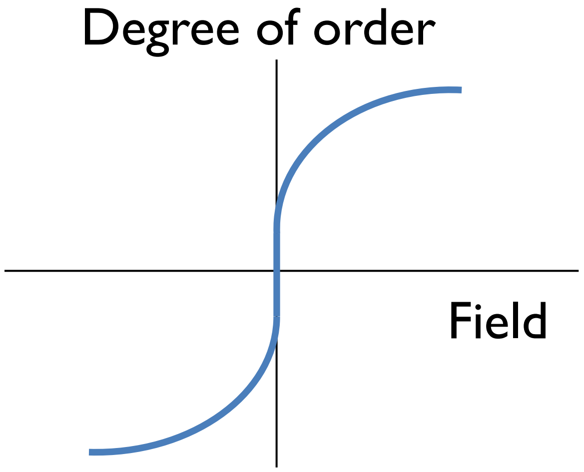


Collective Behavior



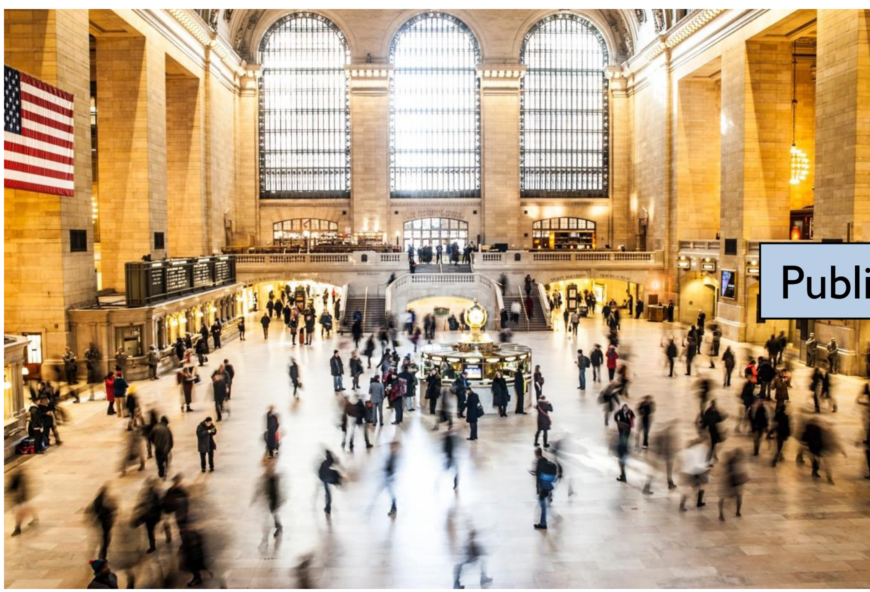
Selinger, Introduction to the Theory of Soft Matter, Springer

Emergence of
Order-disorder transition



"Field" induced transition from

"Disorder"



Creative Commons

Public announcement

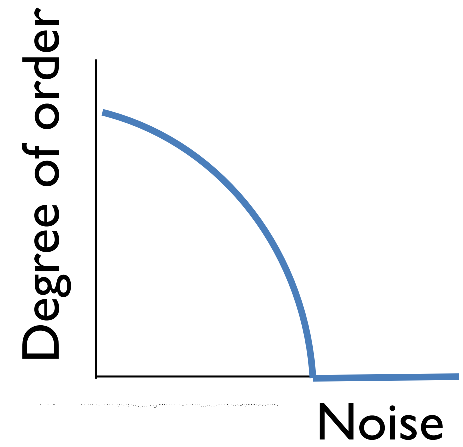
"Order"

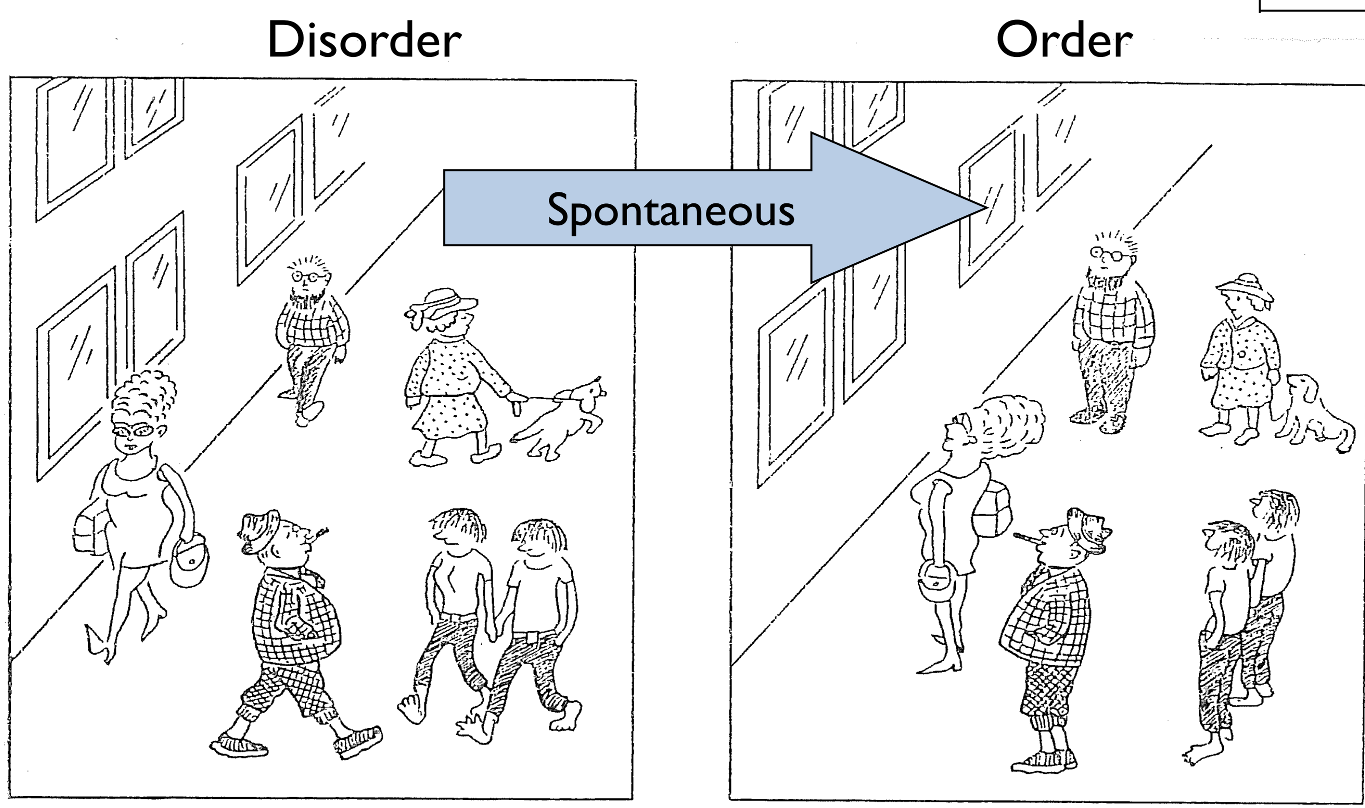
Jake Wyman/Corbis

Discontinuous or First-order phase transition

Emergence of
Cooperative phenomenon

Self-organized transition from





Kikoin & Kikoin, Molecular Physics, Mir

Continuous phase transition

But... Agents are not particles!

Not simply responding inertly to external forces!

Future goals are the driving force for behavior

Teleodynamics

Telos "end"	+	Dunamis "power"

Agents are necessarily "forward-looking",
$\Rightarrow$ current actions are influenced by perception of future events.

perception may of course be influenced by what has happened in the past …
but also by biases and prejudices of the individual actors

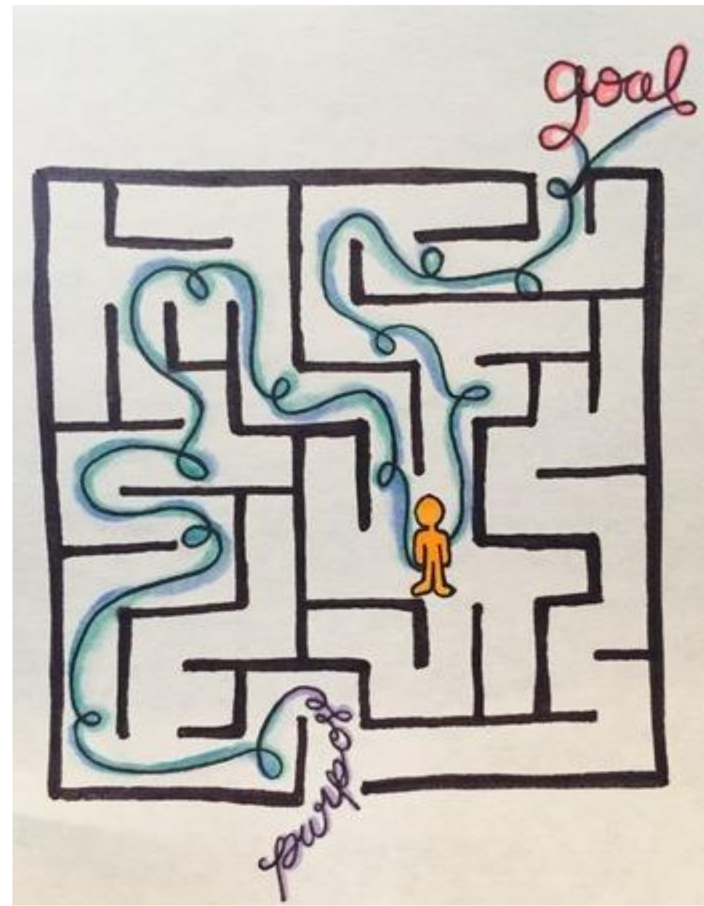


Image: www.adlerpedia.org

From spins to games

"I think that I will pick A"

But "she thinks that I think that I will pick A"

But "she thinks that I think that she thinks that I think that I will pick A"

And so on…

An infinite regress of *theories of mind* two opponents use to guess the action that the other will choose

How to choose course of action ?

Should rational[*] agents cooperate ?

[*] agents are non-altruistic, only interested in maximizing their own benefit

Should rational(*) agents cooperate ?

Enter…. Game Theory

- *Games*: Strategic interactions between agents
- *Agents*: Variety of entities, ranging from human beings/ animals/ cells to organizations and nations, or even, computer programs.
- Each agent receives a *payoff* depending upon the strategy choice made by all agents including herself
- Agents aim to maximize their payoff by choosing optimal *strategies*, taking into account that other agents will also be doing the same

(*) agents are non-altruistic, only interested in maximizing their own benefit

2-person symmetric single-stage games

Each agent (*A* and *B*) has two possible strategies (Actions 1 and 2)

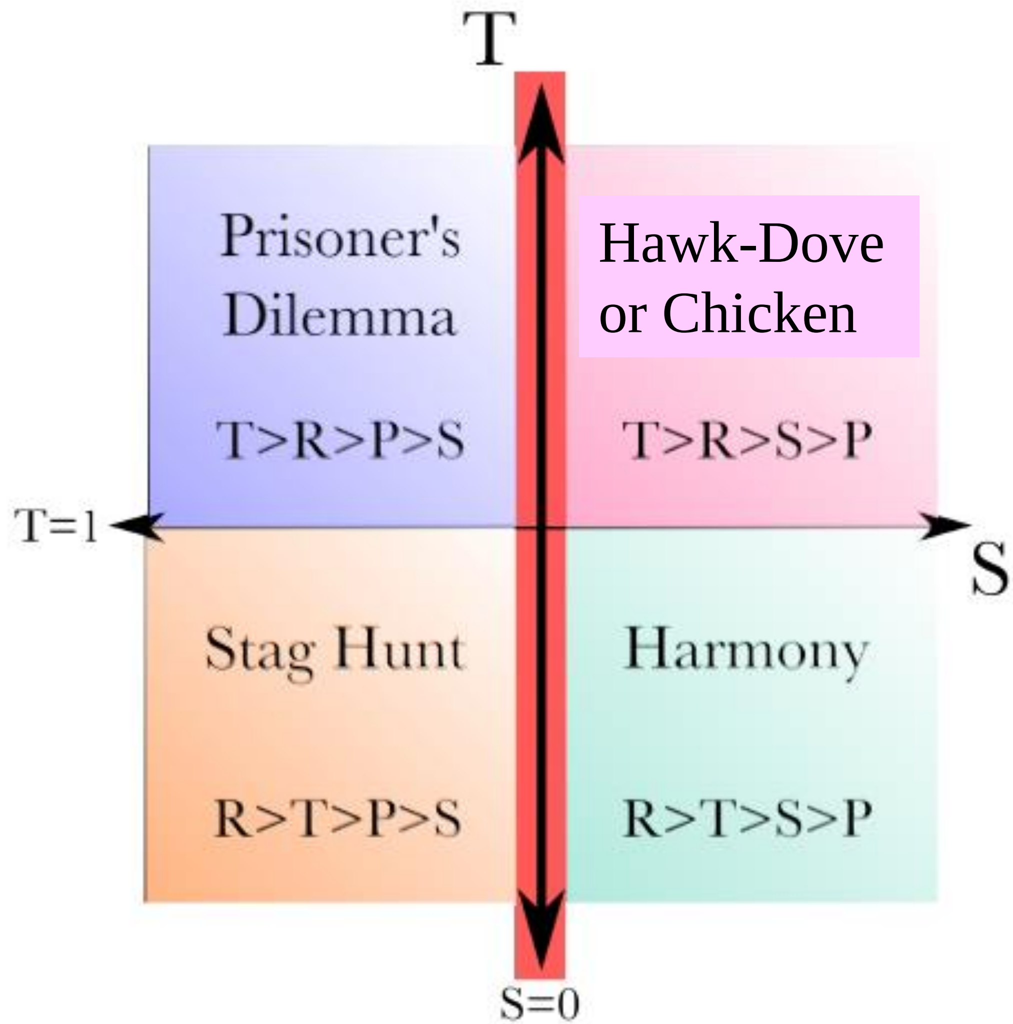
Each agent receives a payoff corresponding to the pair of choices made by them:

Agent A \ Agent B	Action 1	Action 2
Action 1	(R,R)	(S,T)
Action 2	(T,S)	(P,P)

An agent may employ a mixed strategy, in which she randomly selects her options, choosing Action 1 with some probability p and Action 2 with probability $(1 - p)$.
A pure strategy corresponds to p=0 or p=1

Varying the relative ordering of the payoffs yields different 2-person symmetric games

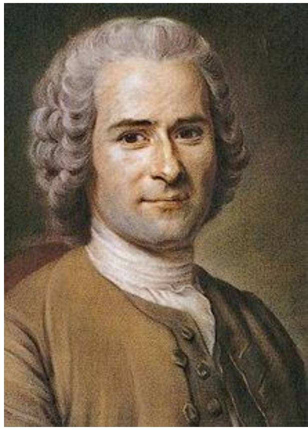
Two of the payoffs P and R can be fixed w/o loss of generality to set origin and scale of the payoff values

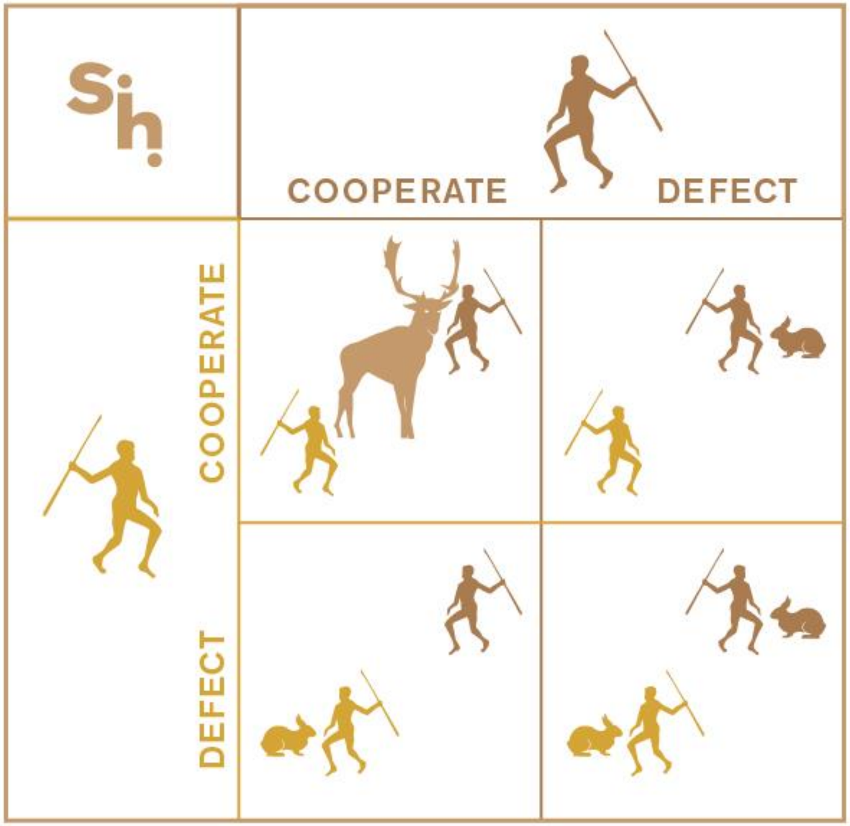


Example: Stag-Hunt

$R > T \geq P > S$

Jean-Jacques Roussea (1755)
Describes social situations where cooperation required to achieve best possible outcome





Evolutionary Games Infographics Project

Strategy defined by probability p of Action 1 [probability of Action 2 is(1-p)]

Strategic interaction between 2 agents choosing

- Action 1: a high-risk strategy having potentially large reward, viz., hunting for stag
- Action 2: a relatively low-risk, but poor-yield, strategy, viz., hunting for hare.

Three Nash equilibria:

- $p^*=1$ a pure strategy (also ESS)
- $p^*=0$ a pure strategy (also ESS)
- $p^* = P/(P - R + T)$ a mixed strategy [assuming S=0]

The Nash equilibrium of a game may sometimes be inferior to an alternate choice of actions by the agents in which all the parties get higher payoff … gives rise to "social dilemmas" such as

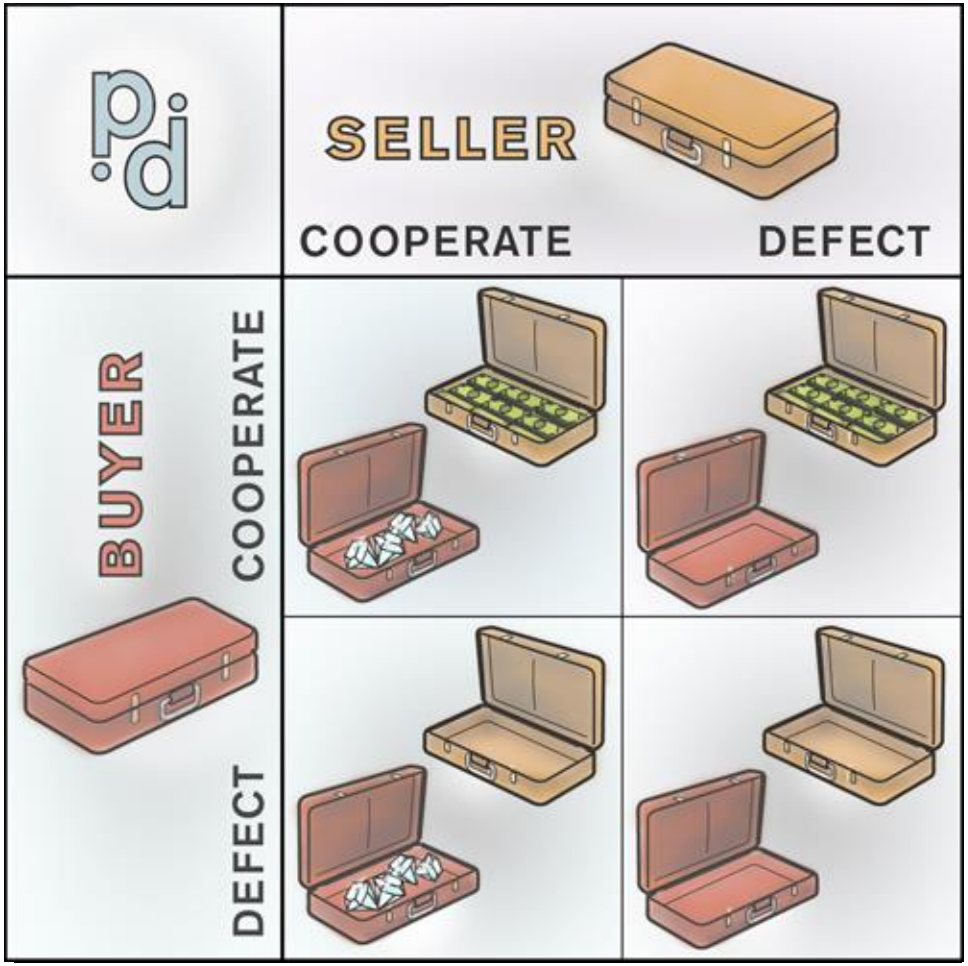
Prisoners Dilemma

originally framed by Merrill Flood and Melvin Dresher at RAND (1950)

$T>R > P \geq S$

Nash equilibrium for PD : both defect

But mutual cooperation will result in higher payoffs R (> P) for both.

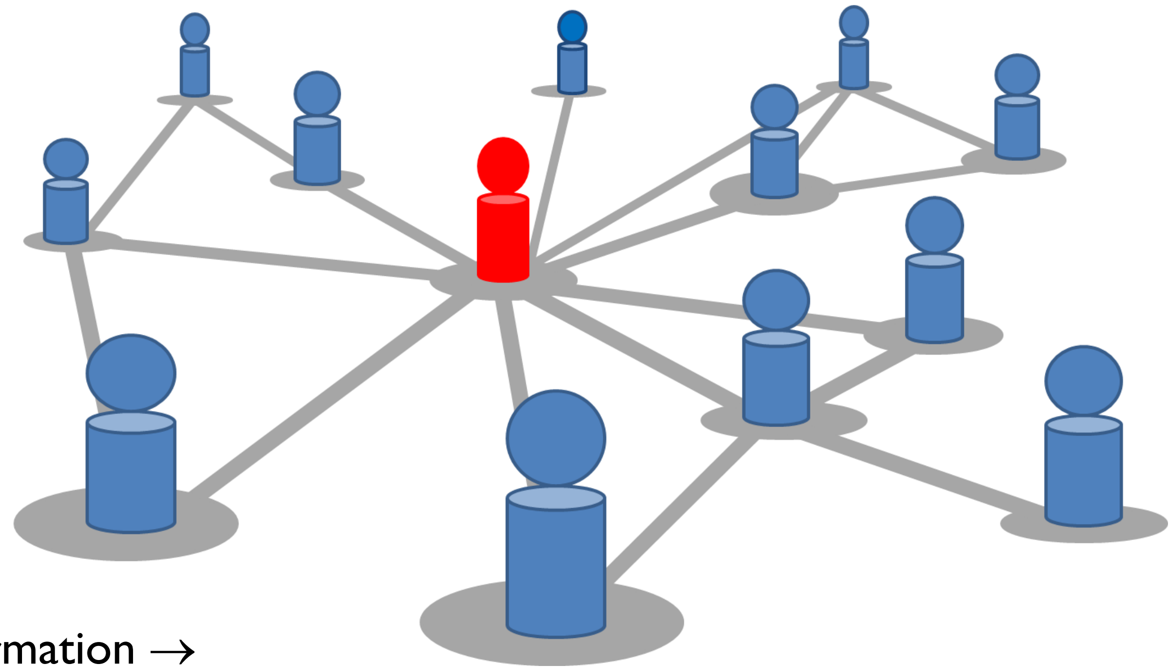


Evolutionary Games Infographics Project

Results of experimental realizations of PD and other non-cooperative games incorporating such dilemmas do show deviation from Nash solutions…
Humans tend to be much "nicer" than what rational game-theoretic models would tend to suggest

Games on Graphs

2-player games between rational players well-understood



But games between players connected on a social network – with agents interacting only with their nearest neighbors – Spatial constraints on information communication →

Bounded Rationality

Each agent has incomplete information → only aware of actions of their neighbors

In iterated games, where players choose a new action at subsequent time steps complex collective behavior arises depending on

- the type of game (payoff matrix),
- uncertainty about information of other's actions (noise), and,
- the nature of the connection topology (network).

Example:

Voluntary vaccination during epidemic

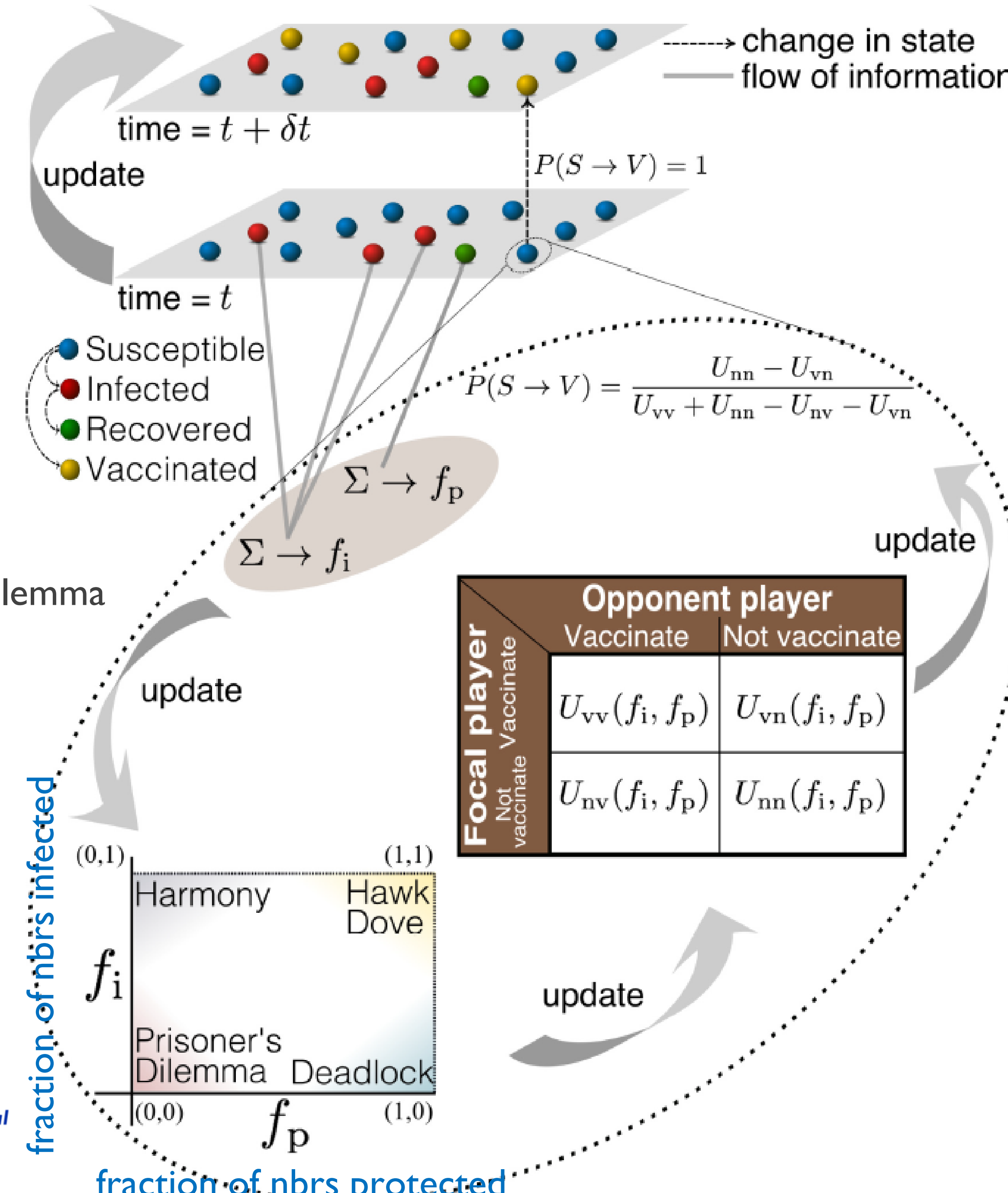
$U_{vv} > U_{vn} > U_{nv} > U_{nn}$: Harmony
$U_{nv} > U_{vv} > U_{vn} > U_{nn}$: Hawk-Dove
$U_{nv} > U_{vv} > U_{nn} > U_{vn}$: Prisoner's Dilemma
$U_{nv} > U_{nn} > U_{vv} > U_{vn}$: Deadlock

$$f_i = \alpha(I/N) + (1-\alpha)(k_{inf}/k).$$

By using parameter α, we tune the nature of information being used in making decision about vaccination.

$\alpha = 0$ → $\alpha = 1$

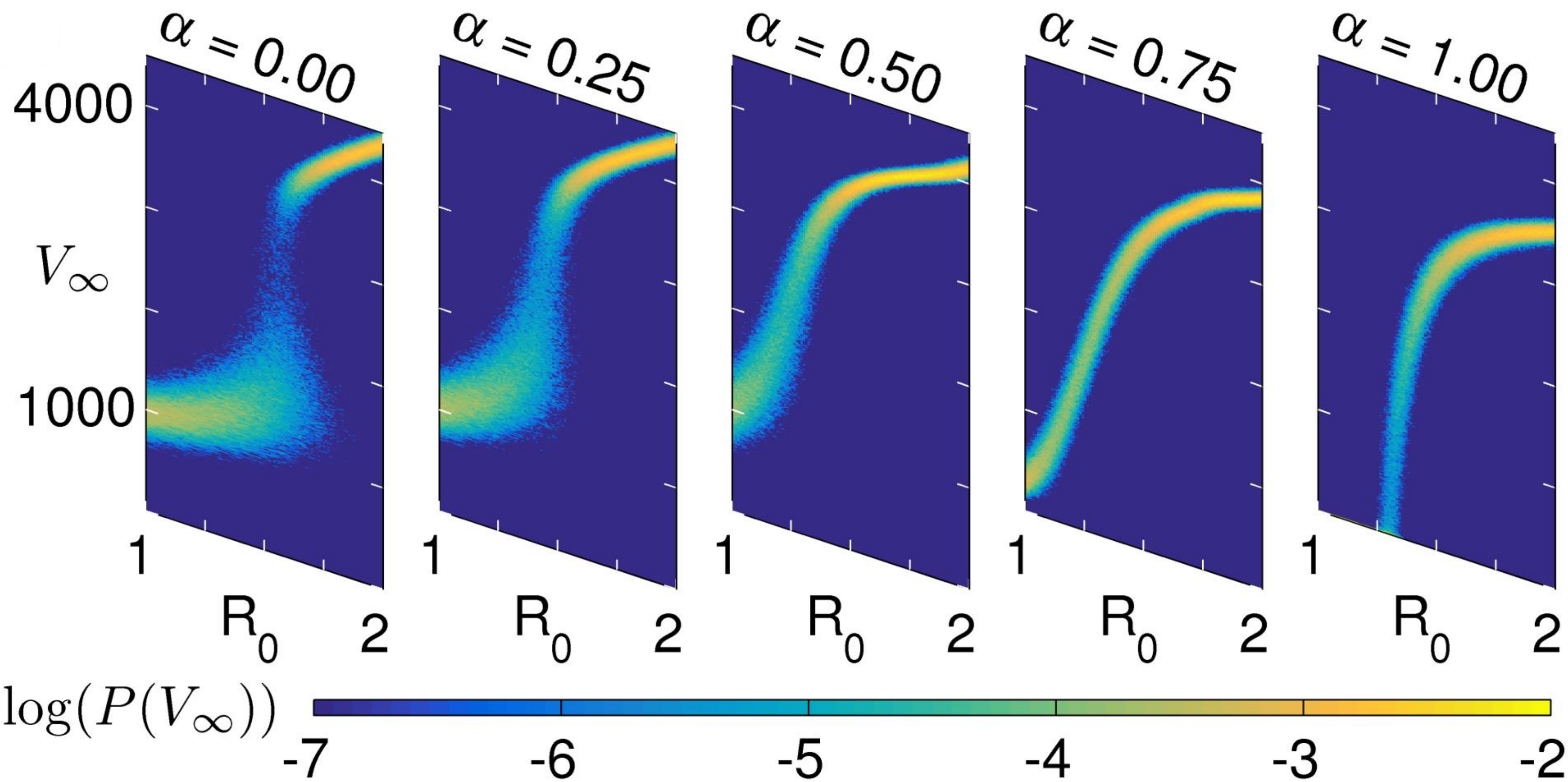
Entirely local information → *Entirely global information*



Phase transition

from low vaccination coverage to high coverage as R_0 is increased

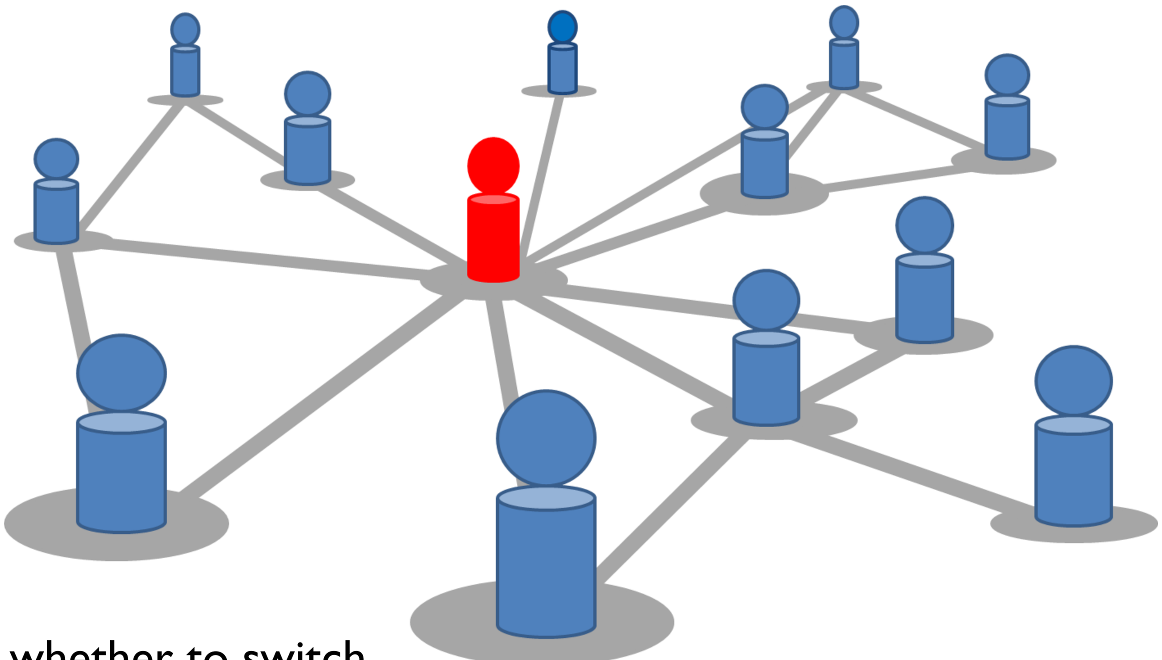
Discontinuous transition for low α, Continuous transition for high α



Probability distribution of V_∞ as a function of R_0 for different values of α

Can connection topology alone drive transition to cooperation ?

Each agent on a node plays a 2-person game with each of its neighbors



Each agent i plays a 2-person game – decided by a payoff matrix with P=S=0, R=1 and T, a free parameter – with each of its neighbors, and collects a net payoff π_i at the end of one iteration of the game

Noise or uncertainty in deciding whether to switch between actions arising from incomplete information results in the stochastic action update rule

each agent i randomly picks a neighbor j and copies its action with a probability

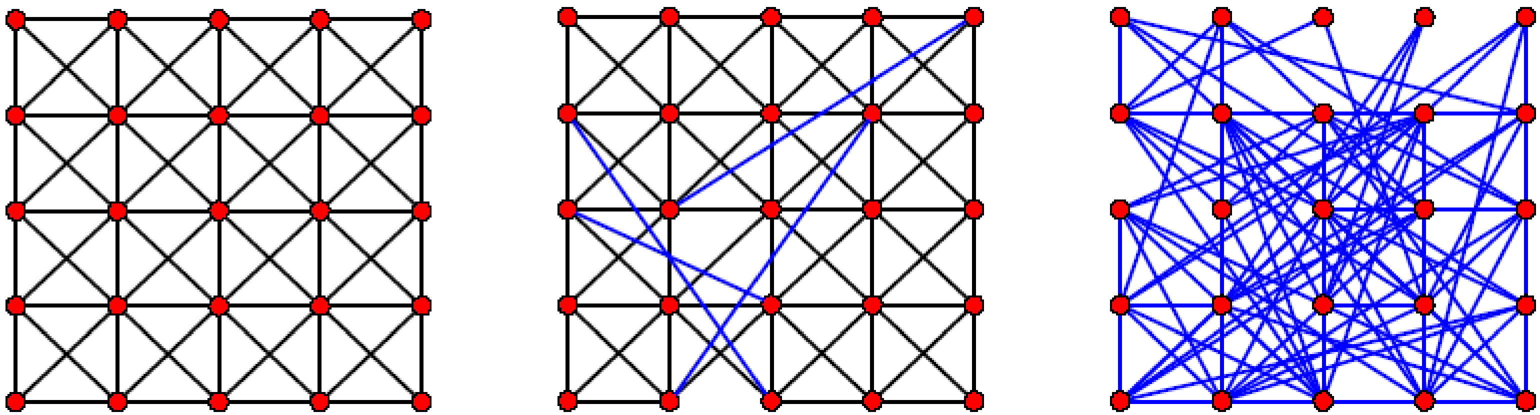
Fermi distribution function

$$\Pi_{i \to j} = \frac{1}{1+\exp(-(\pi_j - \pi_i)/K)} ,$$

K is a temperature, a measure of the "noise" in the decision-making process

The connection topology of interactions is systematically altered using the paradigm of

"Small world" networks



Regular Network
p = 0

"Small-world" Network
0 < p < 1

Random Network
p = 1

→

Increasing Randomness

p: fraction of random, long-range connections

The connection topology of interactions is systematically altered using the paradigm of

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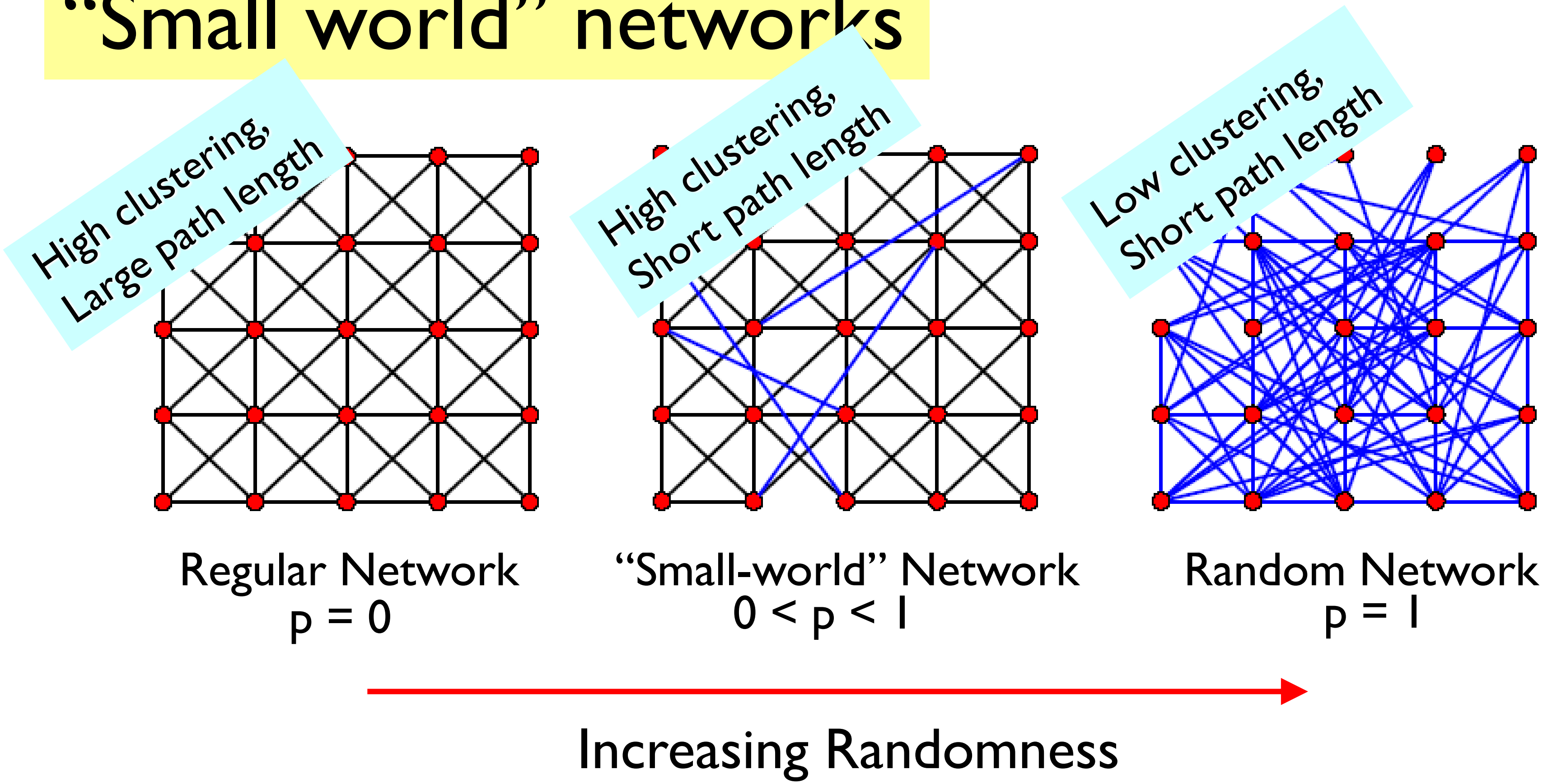
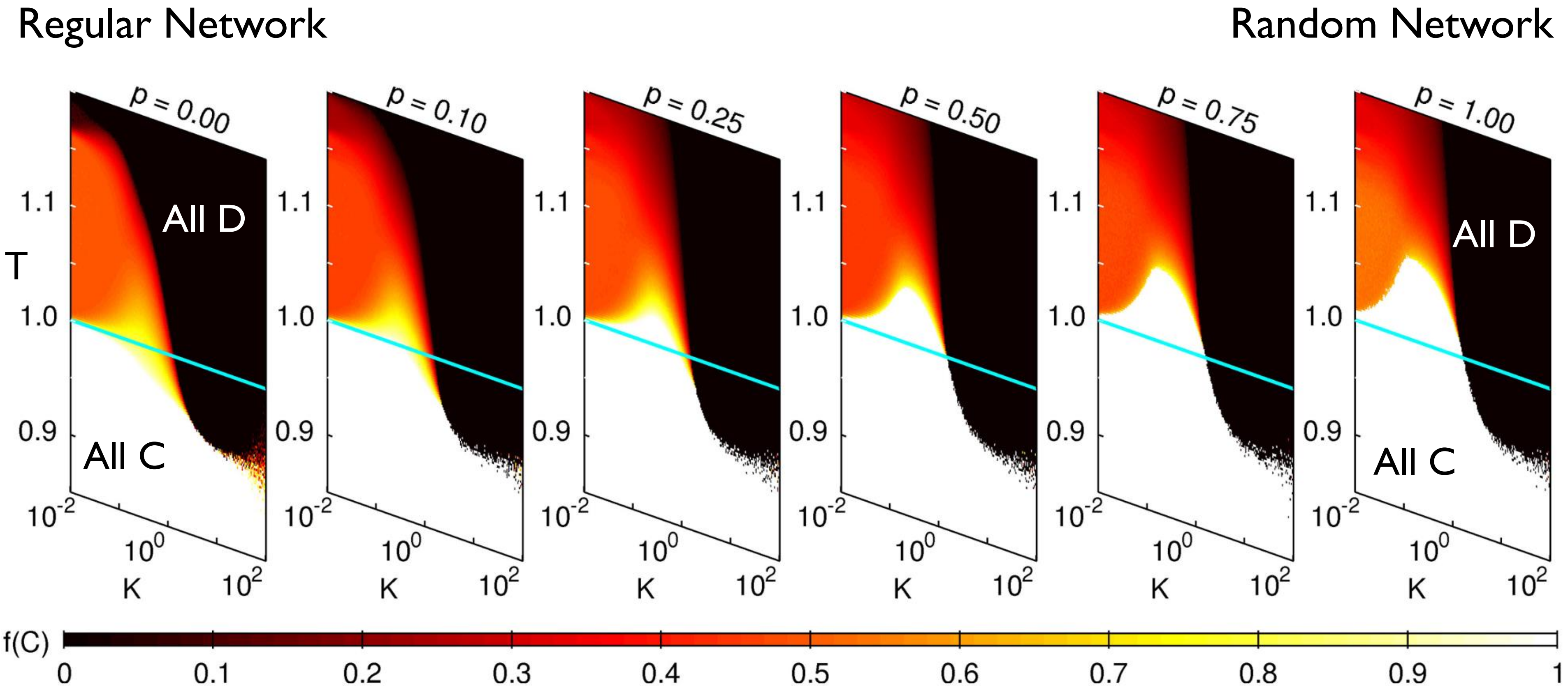


p: fraction of random, long-range connections

Watts and Strogatz (1998): Many biological, technological and social networks have connection topologies that lie between the two extremes of completely regular and completely random.

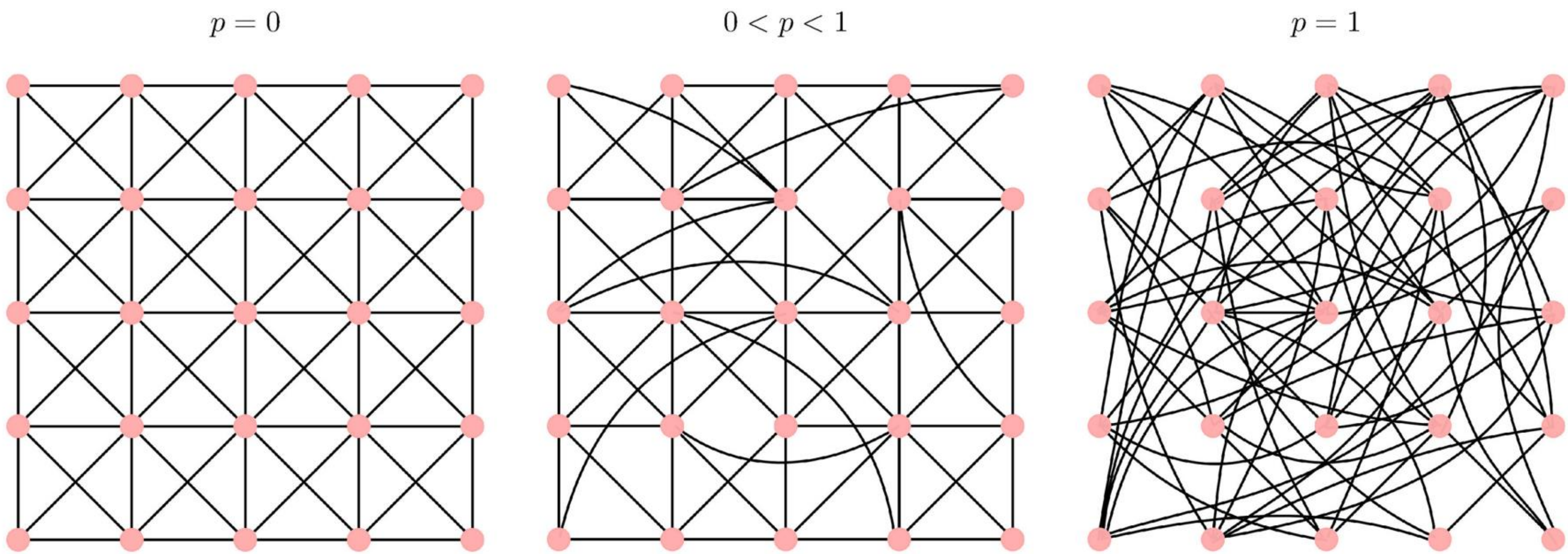
Three collective dynamical states

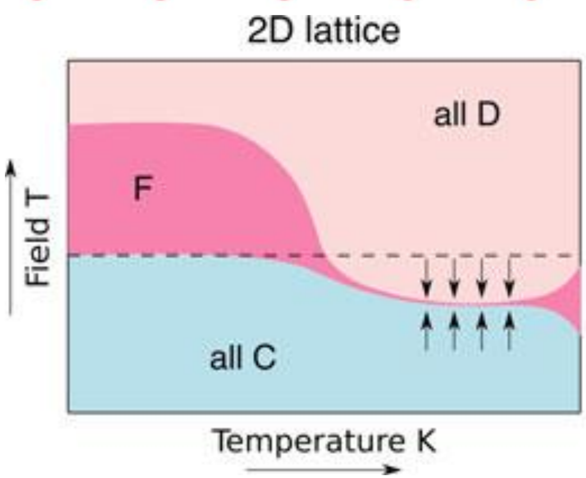
For each value of game dynamics parameters T (temptation payoff) and K (noise), the system converges to one of two possible collective dynamical states: (i) all C (absorbing state), (ii) all D (absorbing state) and (iii) a fluctuating state F where each agent intermittently switches between cooperation and defection



But as the connection topology (governed by p) changes, the phase diagram also alters

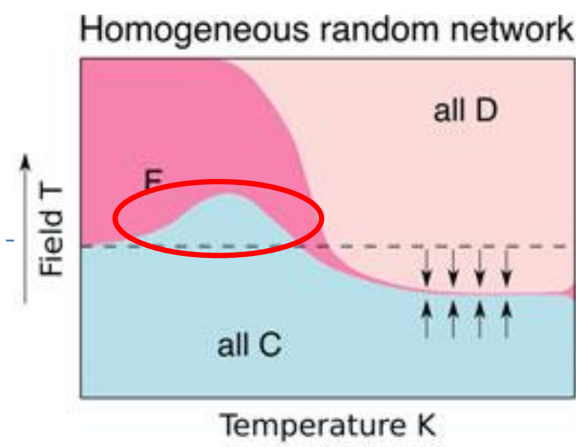
Topological transition: Cooperation as a consequence of nature of connections among agents





Prisoners Dilemma regime

Stag Hunt regime



The temptation T can be viewed as a field, in analogy to spin systems, as its value biases an agent's preference for which action to choose. agents update their action at every round by choosing a neighbor at random and copying their action with a probability that is given by a Fermi function, where the level of temperature (noise) is controlled by the parameter K.

Conclusions

- When played in a spatially extended setting, iterated games provide a framework for the investigation of the process of collective decision-making under conditions of bounded rationality (incomplete information)
- While payoff matrices are known and are identical for all agents, individuals only have knowledge of the choice of actions of that subset of agents with whom they had previously interacted (i.e., their topological neighbors).
- As T decreases below R= 1, the game changes from the Prisoner's Dilemma (characterized by an unique equilibrium that is given by the dominant strategy of mutual defection) to the Stag Hunt (characterized by multiple equilibria).
- The transition between all D and all C is discontinuous, as it involves an abrupt change in the order parameter f (C) [asymptotic fraction of cooperating agents]. The transition from these absorbing states to the fluctuating regime is a continuous one.
- The all C regime moves up the K − T plane as p is increased explaining the sudden emergence of complete cooperation in the PD regime when moving from a regular lattice to a random network keeping average number of neighbors fixed.

THANKS

- V. Sasidevan & S. Sinha, *Scientific Reports* **5**, 13071 (2015)
- V. Sasidevan & S. Sinha, *Scientific Reports* **6**, 30831 (2016)
- S. N. Menon, V. Sasidevan & S. Sinha, *Front. Phys.* **6**, 34 (2018)
- A. Sharma, S. N. Menon, V. Sasidevan & S. Sinha, *PLoS Comput. Biol.* **15**, e1006977 (2019)
- S. N. Menon, V. Sasidevan & S. Sinha, in *Network Theory and Agent-Based Modeling in Economics and Finance* (Springer, 2019) pp 265-281. (arxiv1906.11683)