Bangalore School on Statistical Physics XIV, Sept 2023

Statistical Mechanics of Complex Networks

Lecture 5: Evolution (Scale-free and Structural balance)

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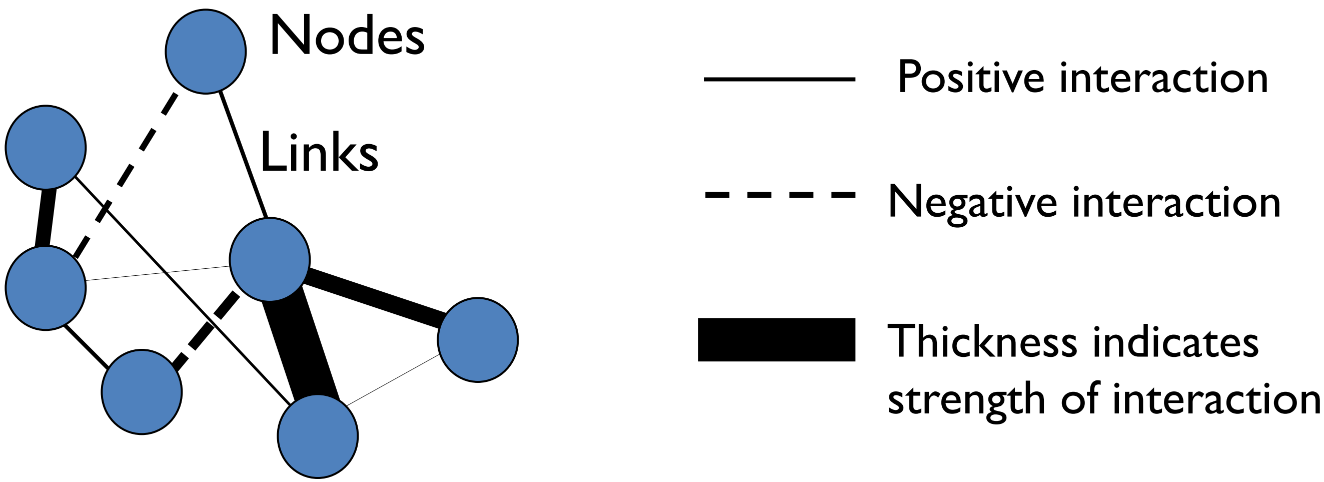
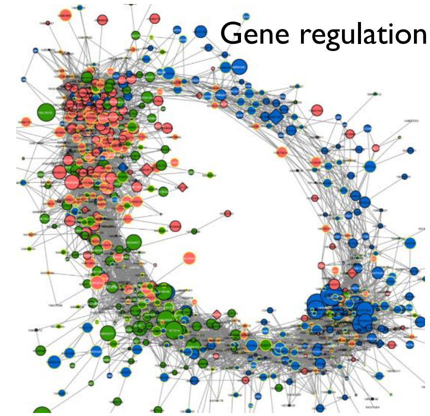
Networks can be disordered

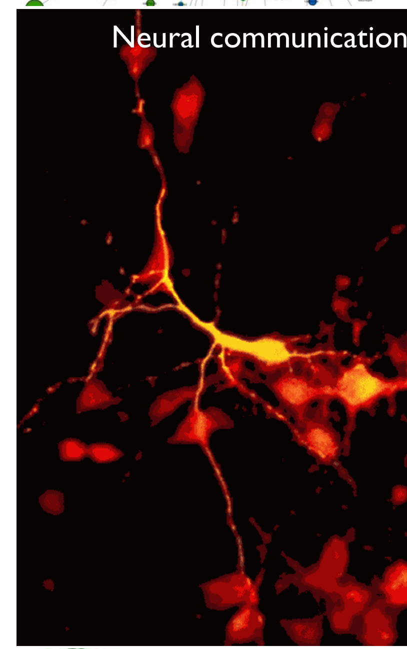


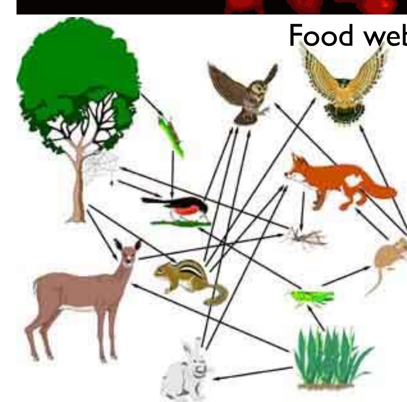
Many naturally occurring networks have links with heterogeneously distributed properties

Differences in links can be

- ❑ *Quantitative*: distribution of degree and/or link weights
- ❑ *Qualitative*: nature of interactions (+ve or –ve)





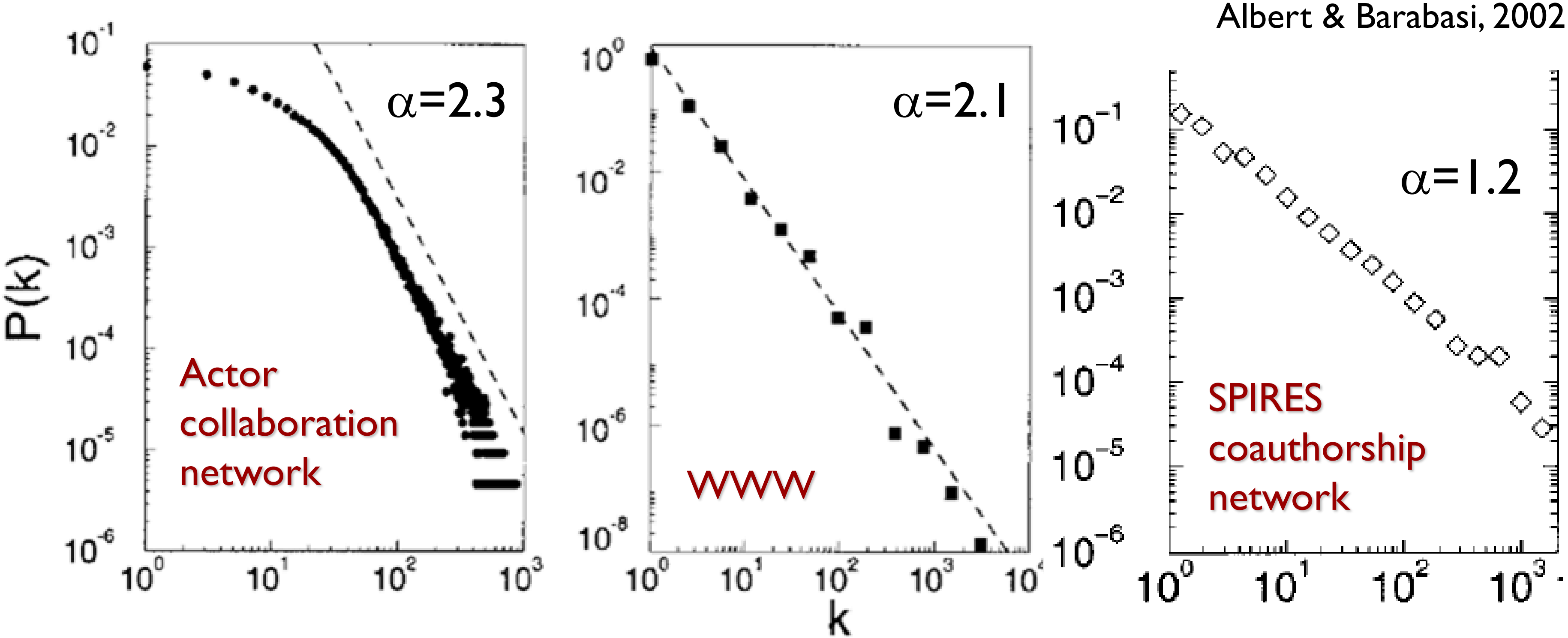


"Scale free" networks

Barabasi and Albert (1999): In many large networks node connections follow a scale-free distribution
⇒ degree distribution has a power law tail

$$P(k) \sim k^{-\alpha}$$

Albert & Barabasi, 2002



In contrast,
Nodes in Erdos-Renyi random networks, e.g., G(N,p), exhibit
Poisson degree distrn: $P(k) = e^{-\lambda} (\lambda^k/k!)$

Power laws, Pareto distributions and Zipf's law

"When the probability of measuring a particular value of some quantity varies inversely as a power of that value, the quantity is said to follow a power law, also known variously as Zipf's law or the Pareto distribution"

Mark E J Newman

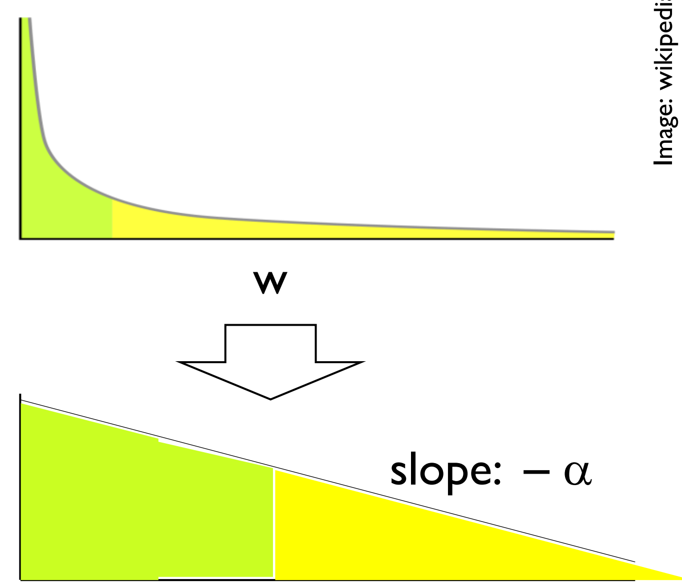
Contemporary Physics, **46** (2005) 323

Expresses a relation between 2 quantities that is independent of the scale one is looking at

$$\boxed{P\,(w) = C\,w^{-\alpha}}$$ exponent

$$\Rightarrow \underbrace{\log P(w)}_{Y} = -\alpha \underbrace{\log(w)}_{X} + \underbrace{\log(C)}_{B}$$

$$Y = -\alpha\ X + B$$



Appears as a linear relation in a log-log graph

(i.e., the axes of the graph are expressed logarithmically)

Contrast with

Distributions having a characteristic scale

Example: Exponential distribution

$$P\,(w) = C\, e^{-w/w^*}$$

Characteristic scale (e.g., time scale or length scale)

Mean : w^*

Standard deviation: w^*

For instance,

Radioactive decay

$$P\,(t) = C\, e^{-t\,/\,\tau}$$

τ : half-life

Or, waiting time between buses arriving

Essentially, exponential distribution of times between events seen when events are independent and occur at a constant average rate (i.e., a Poisson process)

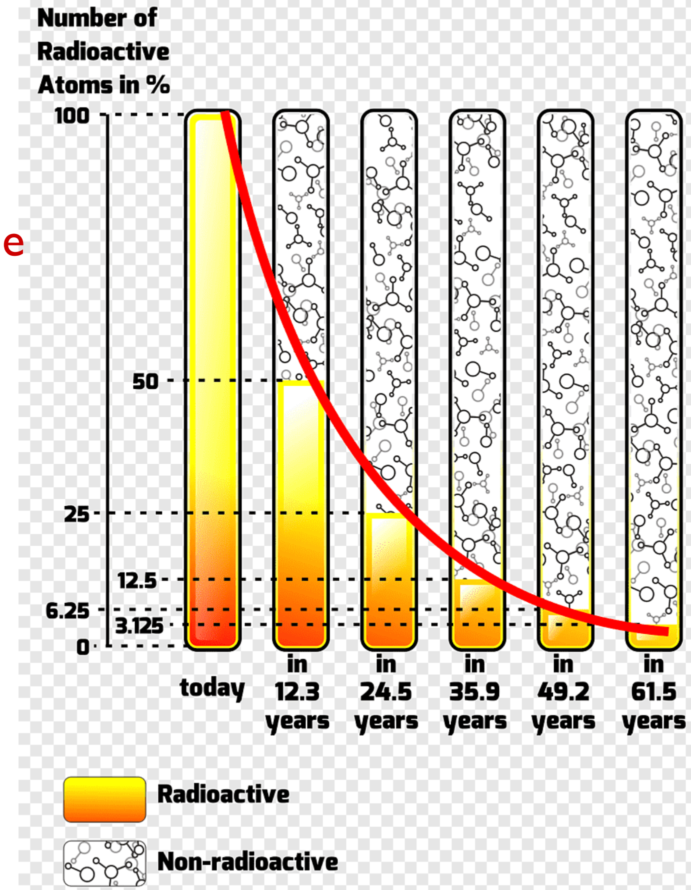


Image: www.pngwing.com

Power law distribution ⇒ "Long tails"

High probability of having values that are extremely large deviations from the mean, much larger than that expected from the variance

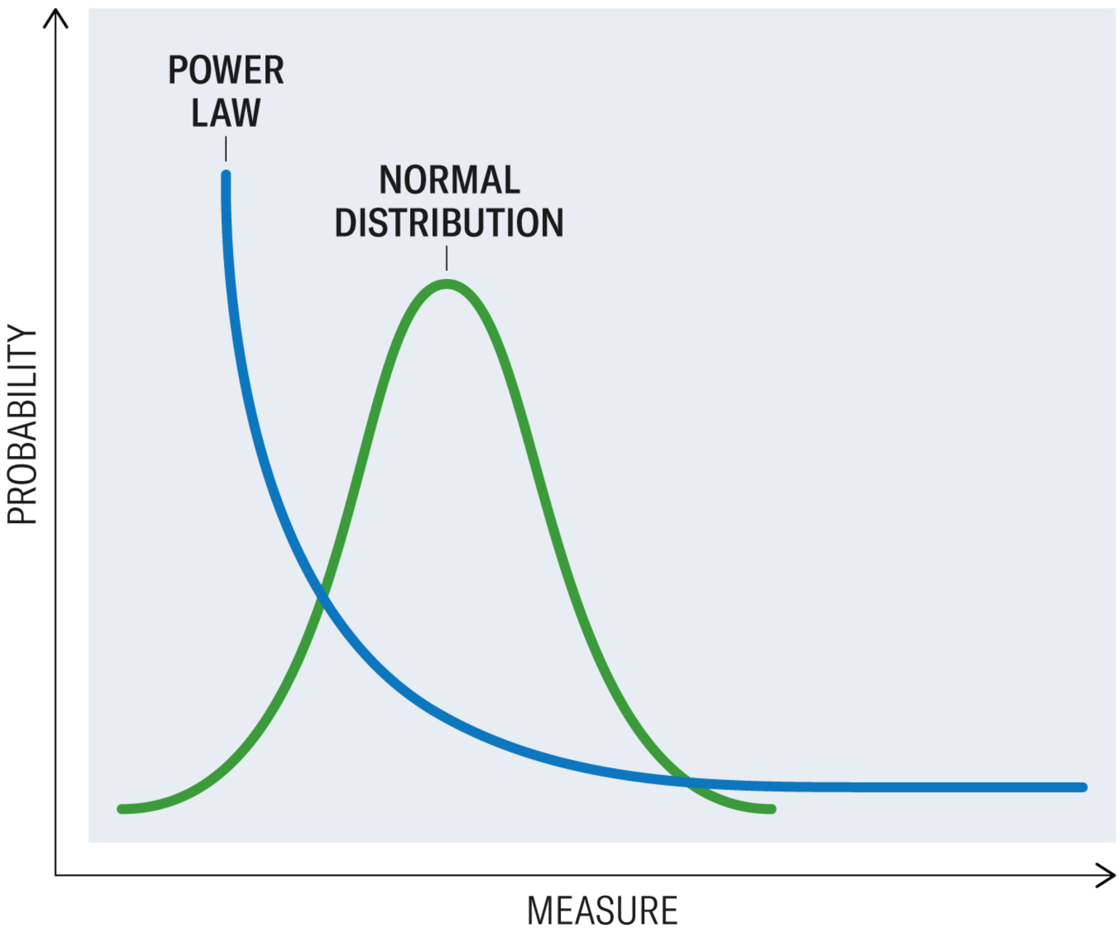


Image: Harvard Business Review

Networks with power-law degree distribution characterized by

Hubs

highly connected nodes which hold the network together

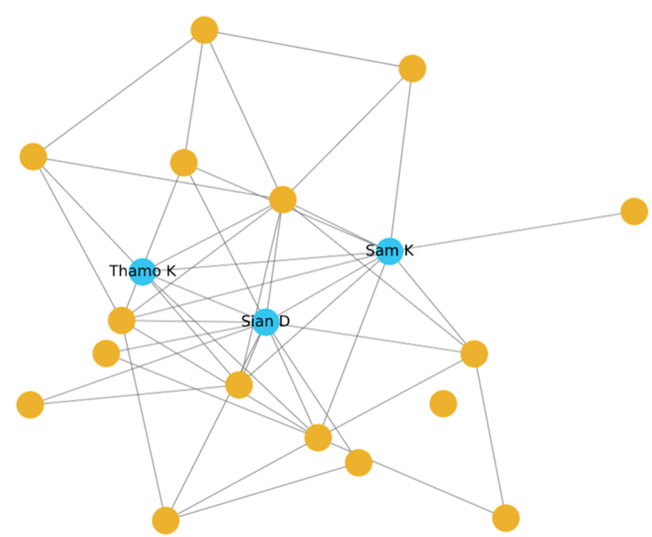


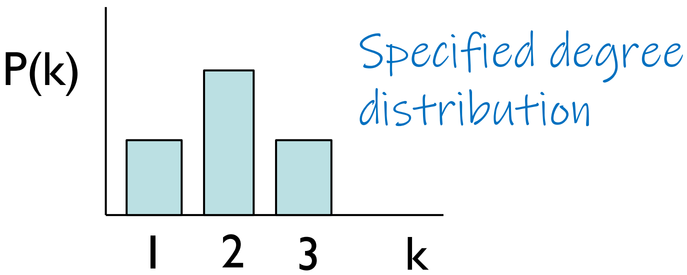
Image: towardsdatascience.com

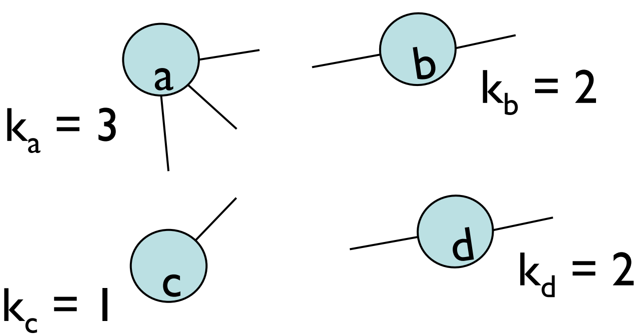
Random networks with arbitrary degree distributions

The configuration model

Generating random networks with any desired degree distribution (instead of Poisson, as in ER graphs), or rather, desired degree sequence

If the exact degree k_i of each individual node i=1,2,…,N in the network is specified a random network with these degrees is created by matching "stubs"





Recipe

- Assign to each node i a total of k_i *stubs* of edges ("half-edges") such that $\sum k_i = 2L$ where L is the total number of links
- Now choose a pair of stubs at random and connect to form a link
- Continue by choosing another pair of stubs randomly from the remaining 2L – 2 stubs, and so on until all stubs are used
- Results in a network in which each node i has exactly the assigned degree k_i

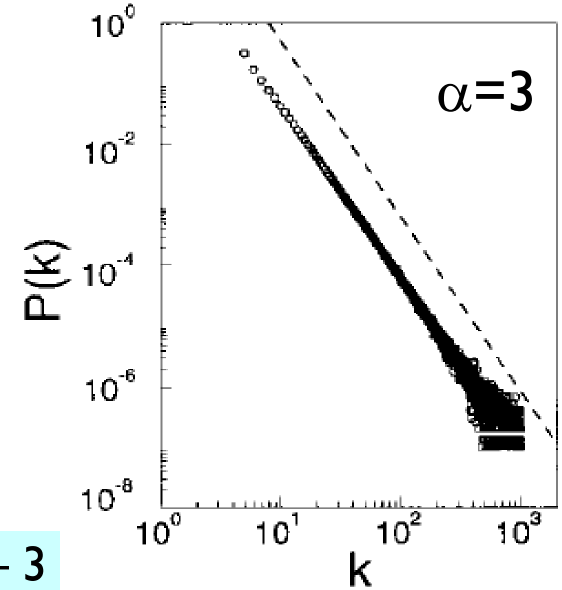
analogous to the G(N,L) random graph model, where the number of links is fixed

How can *scale-free* networks evolve ?

The Price-Barabasi-Albert preferential attachment scheme:

(A) Networks expand continuously by addition of new nodes

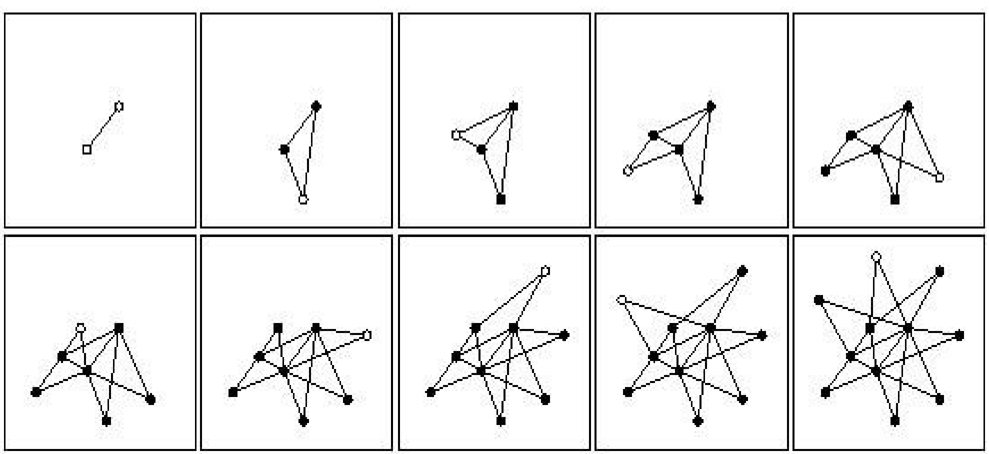
(B) New nodes attach preferentially to nodes already well-connected, i.e., probability that a new node is connected to a node of degree k_i is $\Pi(k_i)=k_i/\Sigma_j k_j$ (“*linear*” scheme)

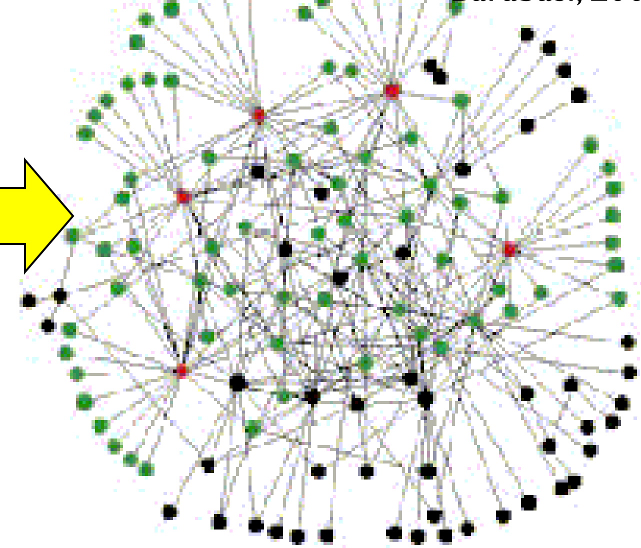


Albert, Jeong & Barabasi, 2000

Resulting network degree distribution: $P(k) \sim k^{-3}$

The birth of a BA scale-free network (H Jeong)





The Mechanism:
A Cumulative Advantage Process

Derek J de Solla Price : the earliest mathematically detailed mechanism by which scale-free degree distribution can arise in the context of networks of citations between scientific papers

Derek J de Solla Price

292 Journal of the American Society for Information Science—September-October 1976

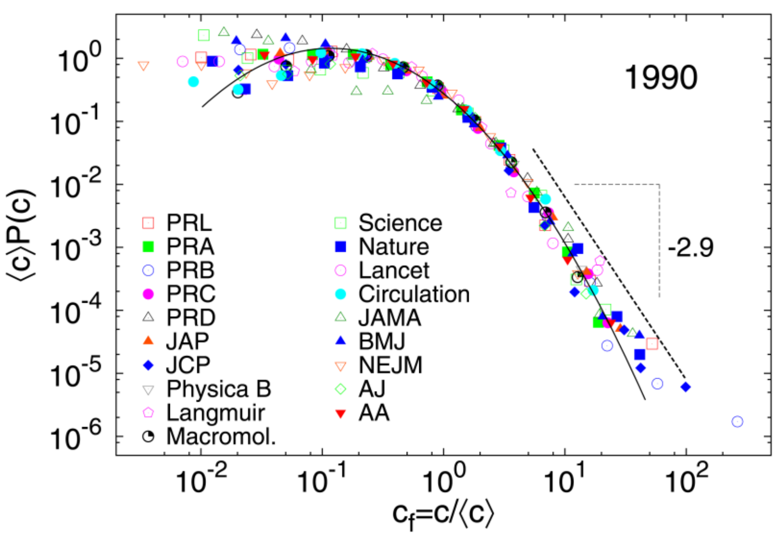
A General Theory of Bibliometric and Other Cumulative Advantage Processes*

Derek de Solla Price
Department of History of Science and Medicine
Yale University
New Haven, CT 06520

A Cumulative Advantage Distribution is proposed which models statistically the situation in which success breeds success. It differs from the Negative Binomial Distribution in that lack of success, being a non-event, is not punished by increased chance of failure. It is shown that such a stochastic law is governed by the Beta Function, containing only one free parameter, and this is approximated by a skew or hyperbolic distribution of the type that is widespread in bibliometrics and diverse social science phenomena. In particular, this is shown to be an appropriate underlying probabilistic theory for the Bradford Law, the Lotka Law, the Pareto and Zipf Distributions, and for all the empirical results of citation frequency analysis. As side results one may derive also the obsolescence factor for literature use. The Beta Function is peculiarly elegant for these manifold purposes because it yields both the actual and the cumulative distributions in simple form, and contains a limiting case of an inverse square law to which many empirical distributions conform.

The Rich get Richer

Distribution of citations to scientific papers



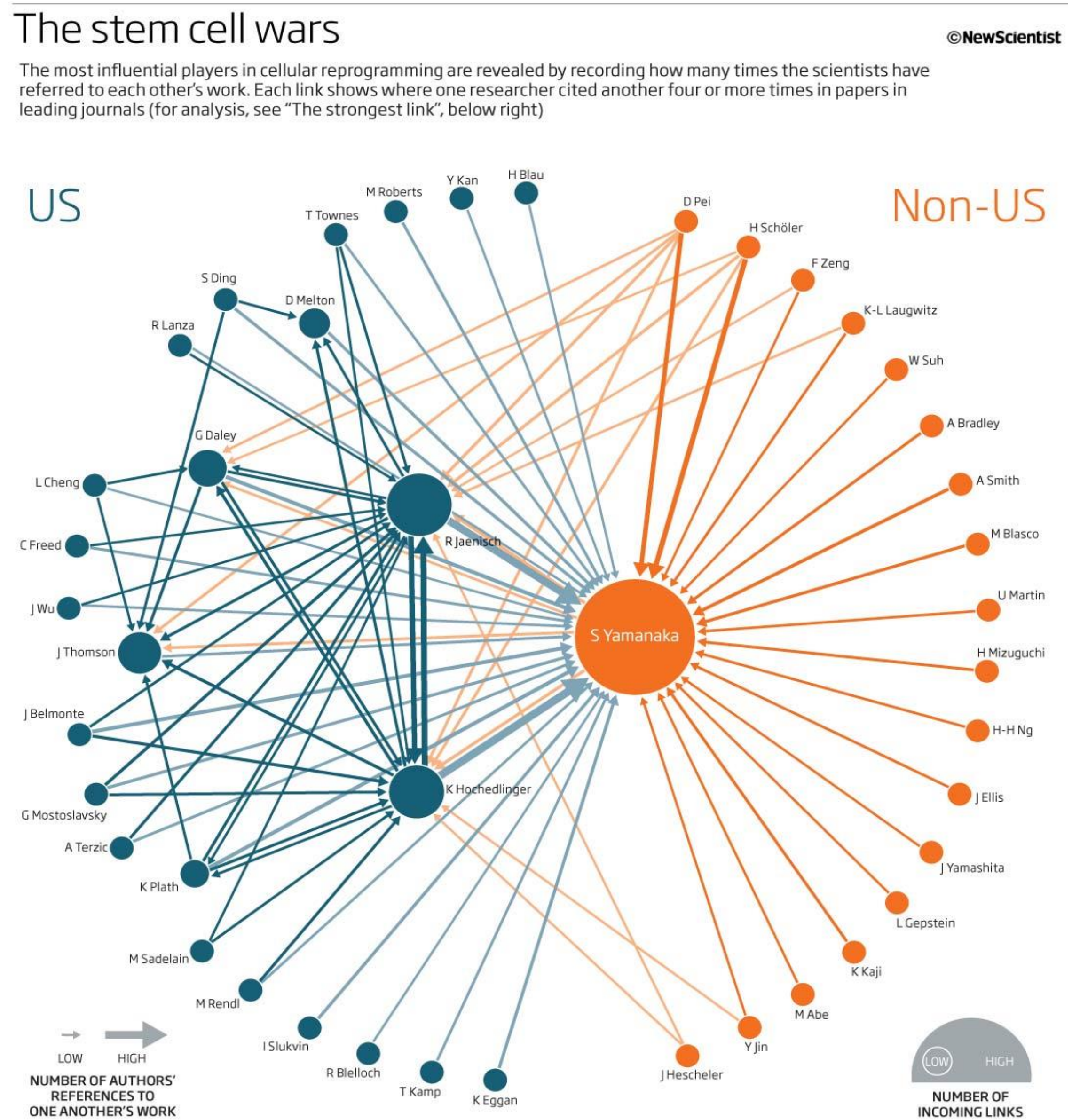
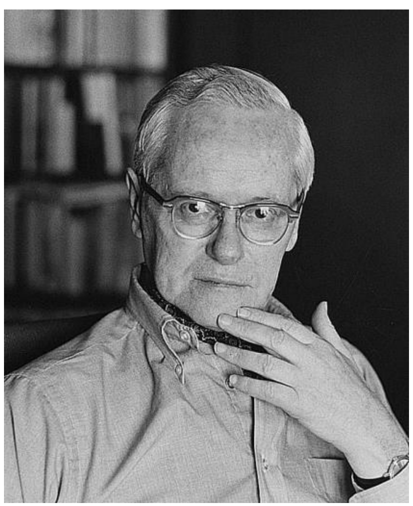


Image: Chatterjee et al, PLoS One (2016)

Matthew Effect

Image: Sutori

Image: ThoughtCo



Robert Merton

Already well-known scientists receive disproportionate credit for their contributions, while less-known scientists receive less credit than their works merit

Why the degree distribution is *scale-free*

Start with a small number (m_0) of nodes, at every time step, add a new node with $m(\leq m_0)$ links to nodes already present in the system.

From

Statistical mechanics of complex networks

Reka Albert and Albert-Laszlo´Barabasi

Rev Mod Phys 74 (2002) 47-97

Continuum theory: The continuum approach introduced by Barabási and Albert (1999) and Barabási, Albert, and Jeong (1999) calculates the time dependence of the degree k_i of a given node i. This degree will increase every time a new node enters the system and links to node i, the probability of this process being $\Pi(k_i)$. Assuming that k_i is a continuous real variable, the rate at which k_i changes is expected to be proportional to $\Pi(k_i)$. Consequently k_i satisfies the dynamical equation

$$\frac{\partial k_i}{\partial t} = m\Pi(k_i) = m\frac{k_i}{\sum_{j=1}^{N-1} k_j}. \tag{79}$$

The sum in the denominator goes over all nodes in the system except the newly introduced one; thus its value is $\Sigma_j k_j = 2mt - m$, leading to

$$\frac{\partial k_i}{\partial t} = \frac{k_i}{2t}. \tag{80}$$

The solution of this equation, with the initial condition that every node i at its introduction has $k_i(t_i) = m$, is

$$k_i(t) = m\left(\frac{t}{t_i}\right)^{\beta} \quad \text{with} \quad \beta = \frac{1}{2}. \tag{81}$$

Equation (81) indicates that the degree of all nodes evolves the same way, following a power law, the only difference being the intercept of the power law.

Using Eq. (81), one can write the probability that a node has a degree $k_i(t)$ smaller than k, $P[k_i(t)<k]$, as

$$P[k_i(t)<k] = P\left(t_i > \frac{m^{1/\beta}t}{k^{1/\beta}}\right). \tag{82}$$

Assuming that we add the nodes at equal time intervals to the network, the t_i values have a constant probability density

$$P(t_i) = \frac{1}{m_0 + t}. \tag{83}$$

Substituting this into Eq. (82) we obtain

$$P\left(t_i > \frac{m^{1/\beta}t}{k^{1/\beta}}\right) = 1 - \frac{m^{1/\beta}t}{k^{1/\beta}(t+m_0)}. \tag{84}$$

The degree distribution $P(k)$ can be obtained using

$$P(k) = \frac{\partial P[k_i(t)<k]}{\partial k} = \frac{2m^{1/\beta}t}{m_0+t}\frac{1}{k^{1/\beta+1}}, \tag{85}$$

predicting that asymptotically ($t\to\infty$)

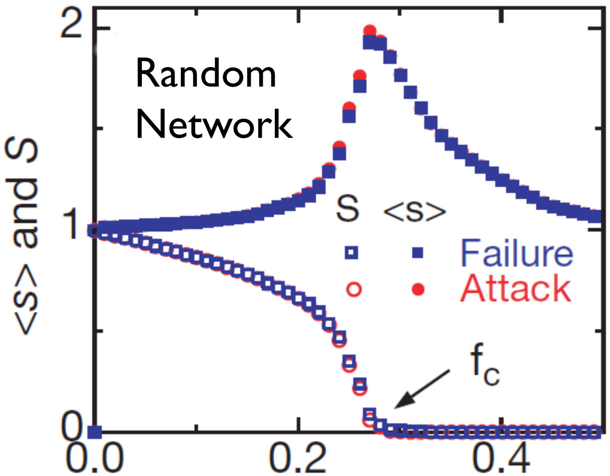
$$P(k) \sim 2m^{1/\beta}k^{-\gamma} \quad \text{with} \quad \gamma = \frac{1}{\beta} + 1 = 3 \tag{86}$$

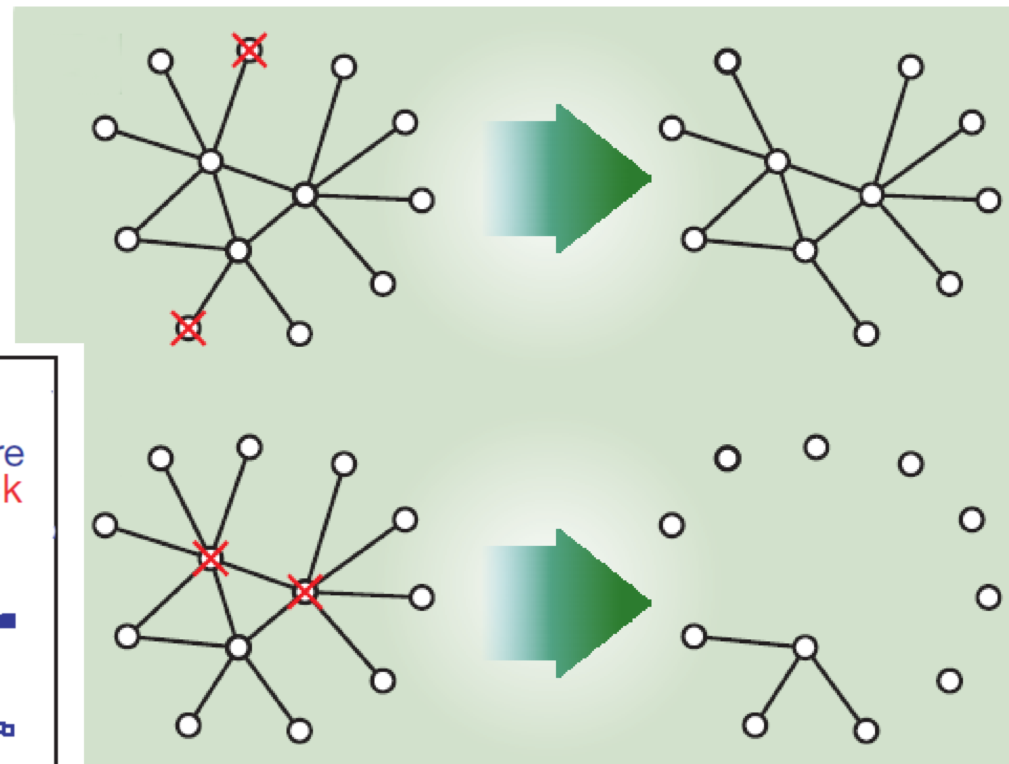
being independent of m, in agreement with the numerical results.

Importance of “hubs”

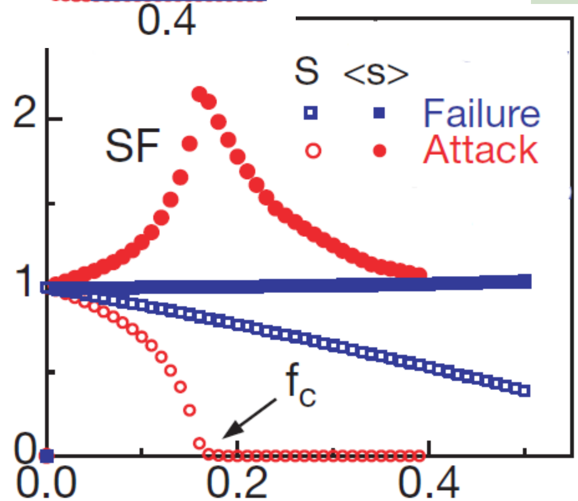
Random failure of nodes typically has little effect on scale-free network as most nodes connect only to a few other nodes: *Robustness to random node removal*

Newman 2008





S: relative size of largest cluster, and <s> : average size of the isolated clusters, shown as a function of the fraction of removed nodes



However targeting the highest-degree nodes (hubs) has devastating effect on the network – most nodes become isolated on removing a few hubs: *Vulnerability to targeted removal of hubs*

How does a heterogeneous degree distribution affect dynamical processes on the network – e.g., epidemic propagation?

The Kermack-McKendrick S-I-R Model (1927)

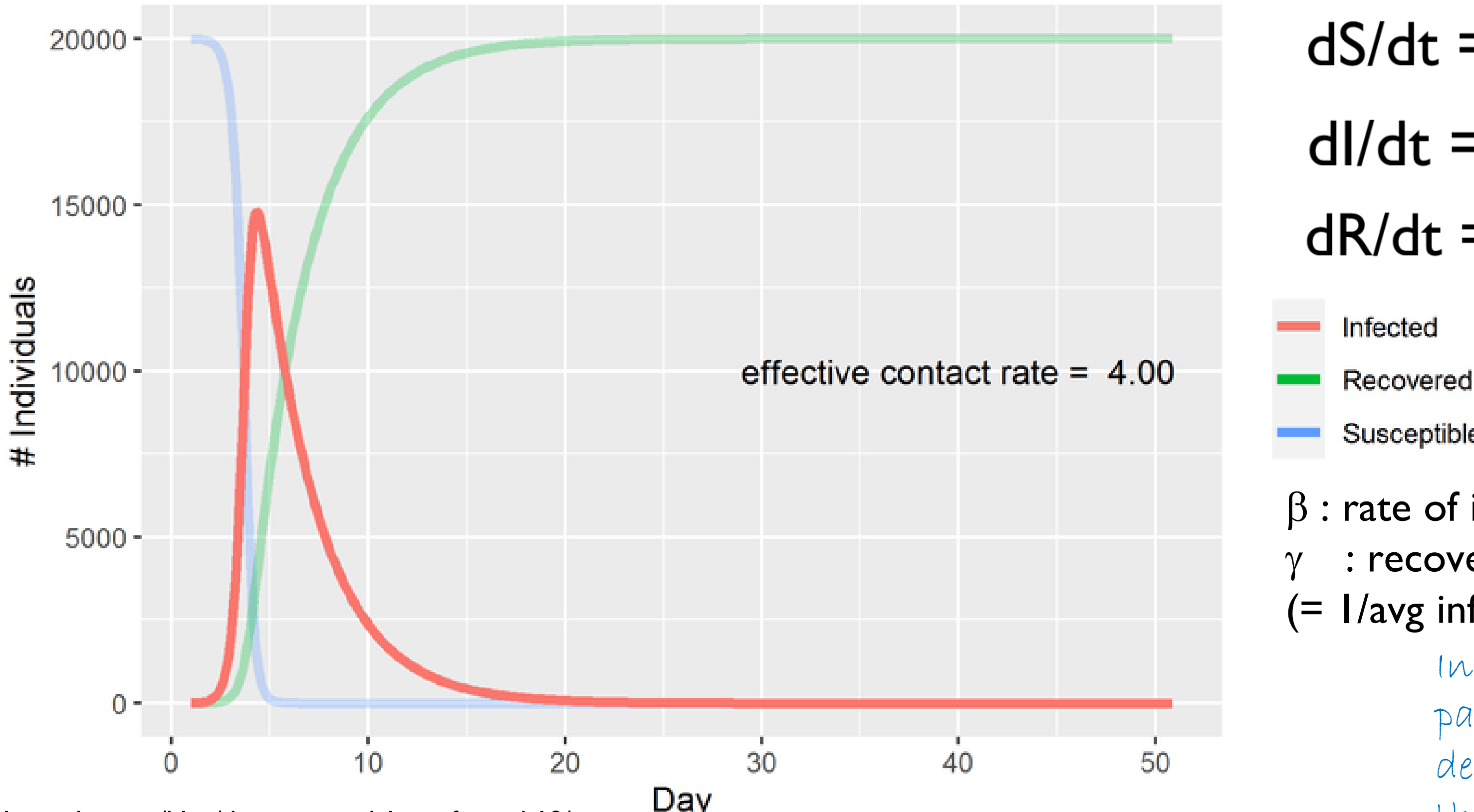


Image:kx.com/blog/dynamic-modeling-of-covid-19/

$dS/dt = -\beta\ S\ I$

$dI/dt = \beta\ SI - I / \tau$

$dR/dt = I / \tau$

β : rate of infection spreading

γ : recovery rate

(= 1/avg infectious period, τ)

In reality β has two parts – average degree (social) and the ease with which a pathogen can be transmitted across (biological)

Epidemic occurs if the number of infections increases with time

$\Rightarrow$ Change in infected population $dI/dt > 0$, i.e., $\beta\ S\ I > I / \tau$

$\Rightarrow$ Condition for epidemic $S > 1 / \beta\ \tau$

As in the initial stage of an epidemic S = N, total population…

An epidemic will occur if $N\ \beta\ \tau > 1$

$\boxed{R_0 = N\ \beta\ \tau > 1}$

Minimum immunization coverage required to stop epidemic (obvious policy implications)

SIR model equation: $dI/dt = \beta SI - I/\tau$

$\Rightarrow$ To stop epidemic we need to make $dI/dt < 0$, i.e., $S(t=0) < 1 / \beta\tau$

where

β : rate of infection spreading

τ : average infectious period

Let total population be N

Thus, proportion of the population that is susceptible, $s = S(t=0)/N$ needs to be made smaller than $1/(N\beta\tau) = 1/R_0$ (because $R_0 = N \beta\tau$)

$\Rightarrow$ The fraction of population that needs to be immunized to stop the epidemic (assuming homogeneous mixing) is $p > 1 - (1/R_0)$

For $R_0 \approx 2$, $p_{min} \approx 50\%$, while for $R_0 \approx 3$, $p_{min} \approx 66\%$

No threshold for epidemics in *scale-free networks*

Networks of sexual relations have been claimed to be scale-free !

A few highly promiscuous individuals act as "hub" nodes

plays crucial role in spreading sexually transmitted diseases !

VOLUME 90, NUMBER 2 PHYSICAL REVIEW LETTERS week ending 17 JANUARY 2003

Absence of Epidemic Threshold in Scale-Free Networks with Degree Correlations

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[3]*Laboratoire de Physique Théorique (UMR 8627 du CNRS), Bâtiment 210, Université de Paris-Sud, 91405 Orsay Cedex, France*

(Received 8 August 2002; published 15 January 2003)

Random scale-free networks have the peculiar property of being prone to the spreading of infections. Here we provide for the susceptible-infected-susceptible model an exact result showing that a scale-free degree distribution with diverging second moment is a sufficient condition to have null epidemic threshold in unstructured networks with either assortative or disassortative mixing. Degree correlations result therefore irrelevant for the epidemic spreading picture in these scale-free networks. The present result is related to the divergence of the average nearest neighbor's degree, enforced by the degree detailed balance condition.

If the contact structure of a disease is network with heterogeneous degree distribution, the condition for occurrence of an epidemic is:

$$R = N\beta\tau \quad > \langle k \rangle / \{\langle k^2 \rangle - \langle k \rangle\}$$

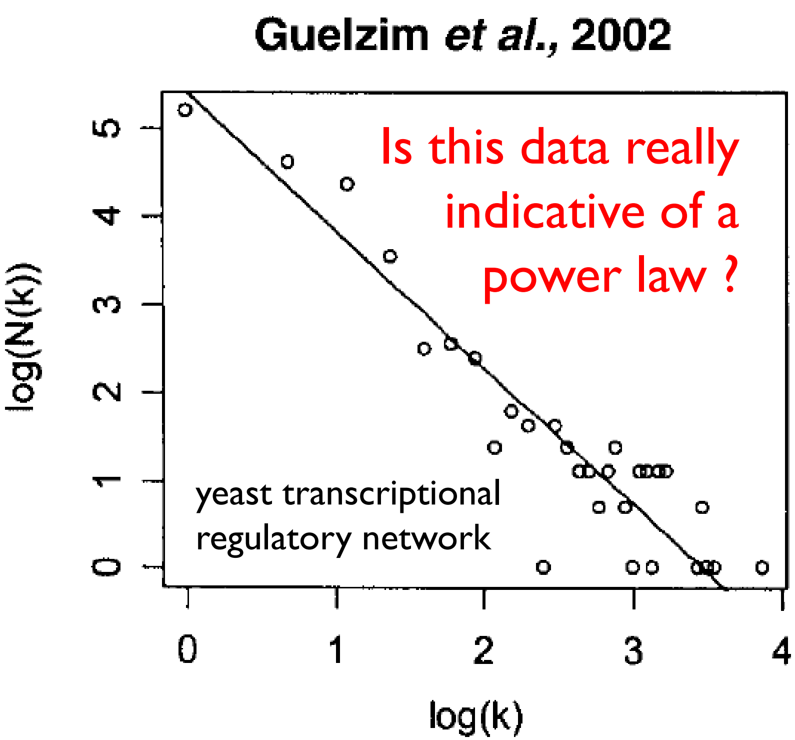
For a scale-free network having degree exponent $2<\alpha \leq 3$, $<k^2> \rightarrow \infty$

$\Rightarrow$ There is no epidemic threshold !

Even diseases with extremely low transmission probabilities are likely to cause a major outbreak involving a significant fraction of population

But are most real networks "scale free" ?

- Scale-free networks characterized by long-tailed degree distribution (power laws) have been proposed as unifying concept for biological complex systems – have been reported in metabolic, protein, and gene interaction networks.
- But many of these reports of scale-free networks are possibly just a result of bad statistics (a combination of extremely limited data and faulty analysis) !
- Almost any distribution seen over a small enough range in a double logarithmic scale would appear linear – and wrongly interpreted as power law



- To establish power laws from finite data one has to use unbiased techniques such as *maximum likelihood estimation*.
- Rigorous re-analysis of many of the data sets used by earlier studies that claimed power-law degree distributions have shown little evidence for scale-free nature!
  (E.g., R Khanin & E Wit, *J Comp Biol* **13** (2006) 810)

Can other processes yield scale-free degree distribution?

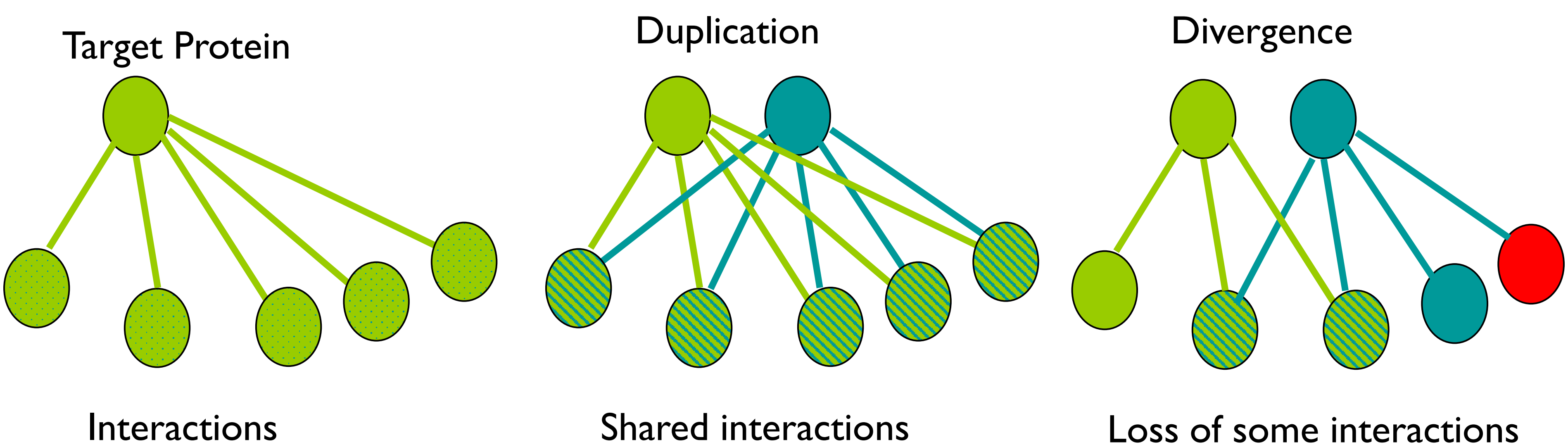
The Case of Duplication & Divergence

preferential attachment appropriate for explaining scale-free character of WWW
Less clear how it might play a role in biological systems,
e.g., protein-protein interaction network that has been claimed to be scale-free

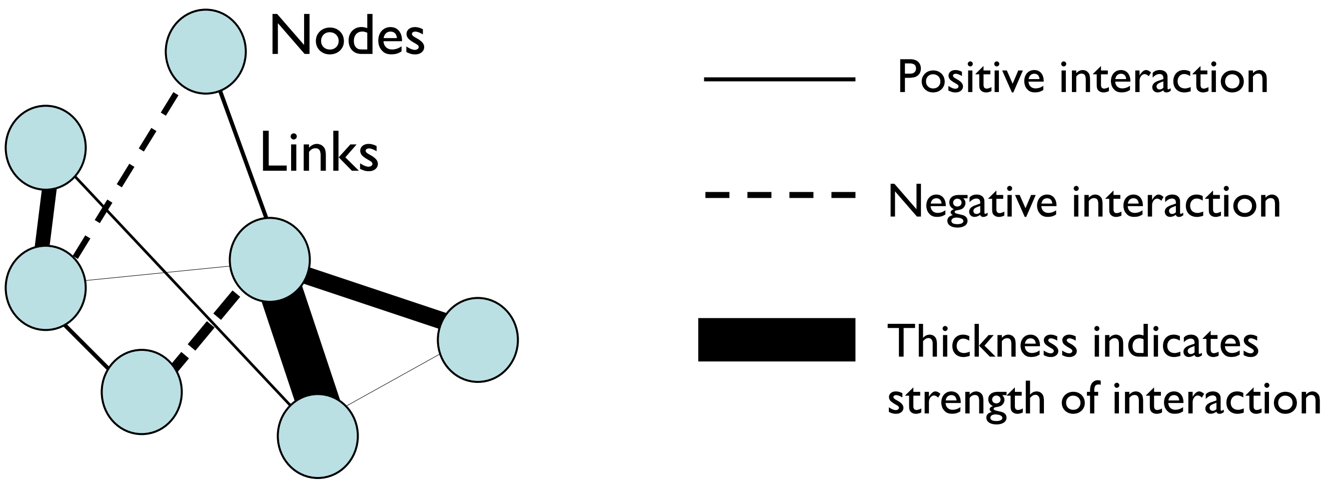
As most biological systems have emerged through a long history of evolution, can evolutionary processes give rise to a network with scale-free property ?

Vazquez et al, ComPlexUs (2003)

In the Duplication-Divergence mechanism, a node along with all its interactions are duplicated with probability p and then some of the interactions mutated with probability q
$\Rightarrow$ claimed to yield networks with scale-free degree distribution but is it true?



Networks can be disordered



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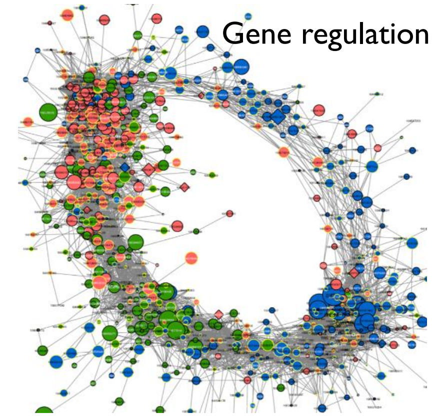
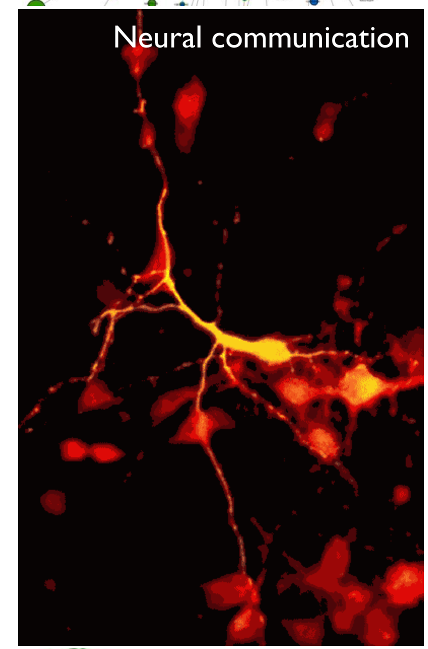
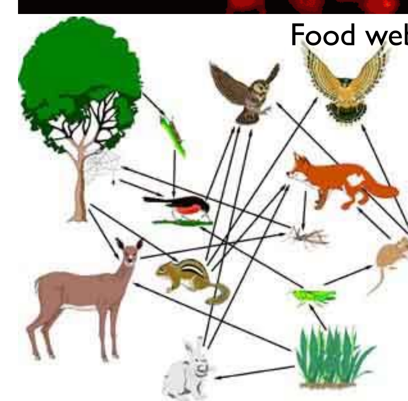
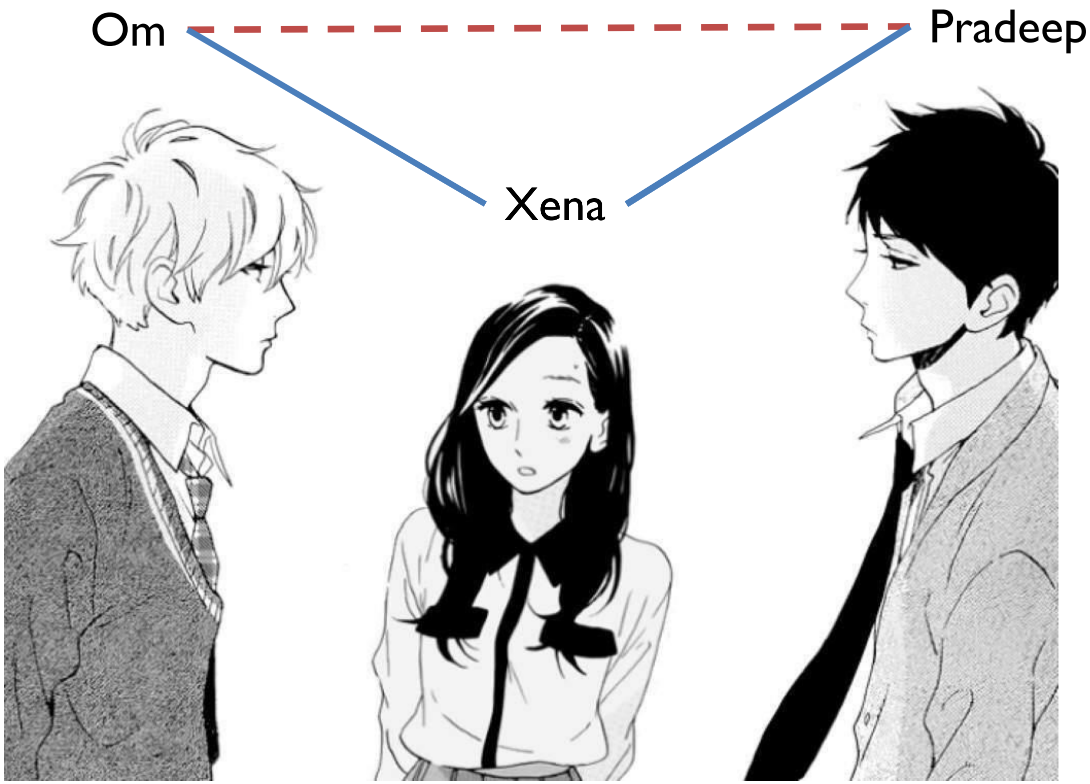






Heider's "Three-Body Problem"

A basic characterization of relationships between mutual acquaintances



aminoapps.com

psychspace.com

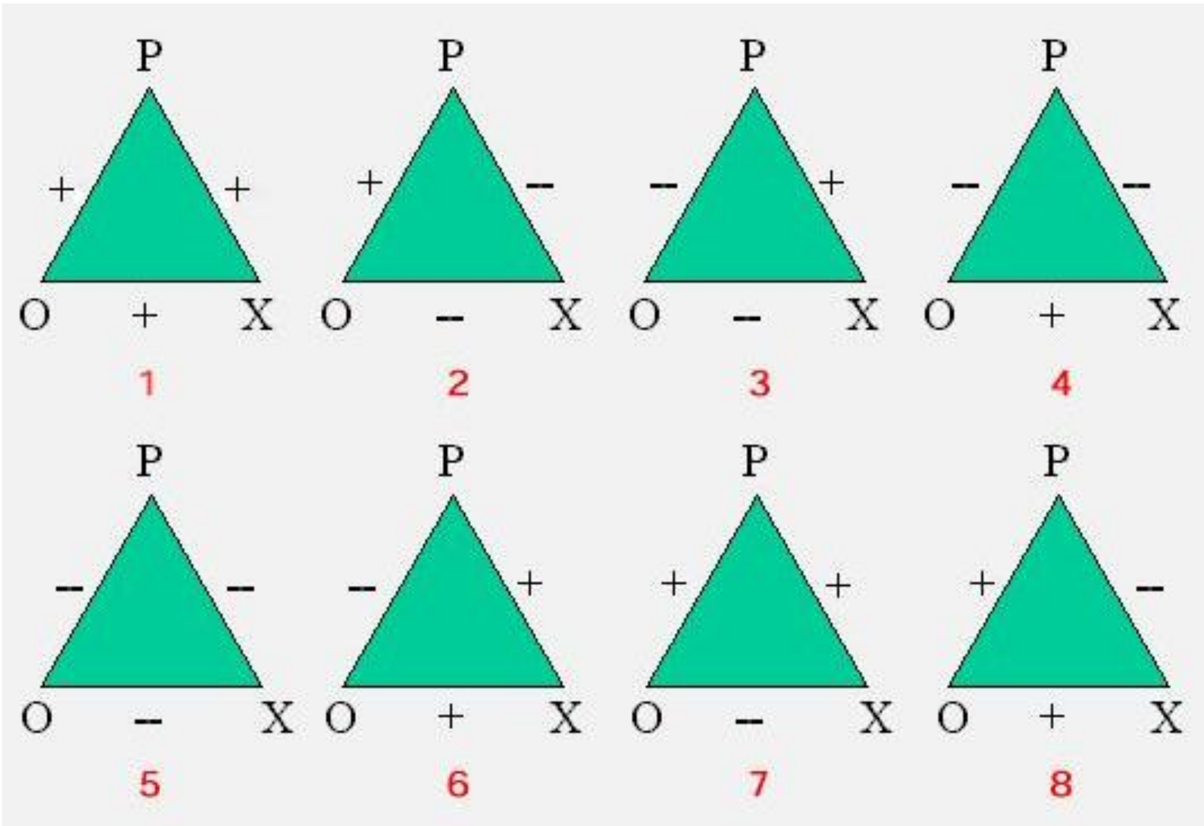
Fritz Heider (1896-1988)

F Heider (1946) Attitudes and cognitive organization. *J Psychol* **21**:107–112.

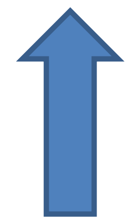
(1) Om and Xena are friends ⇒ OX: +ve interaction (link)
(2) Pradeep and Xena are friends ⇒ PX: +ve interaction (link)
(3) Om and Pradeep are enemies ⇒ OP: –ve interaction (link)

Tension

Structural Balance



Balance



No Balance

Relationship triangles containing exactly 2 friendships are prone to transition to triangles with either 1 or 3 friendships ⇒ friend of my enemy is my enemy

A single friendship may appear in a relationship triangle that initially had none ⇒ enemy of my enemy is my friend

Structural Balance from triads to networks

Dorwin Cartwright (1915-2008)

Frank Harary (1921-2005)

Carwright & Harary (1956)

Generalization of Heider's theory to network of N nodes

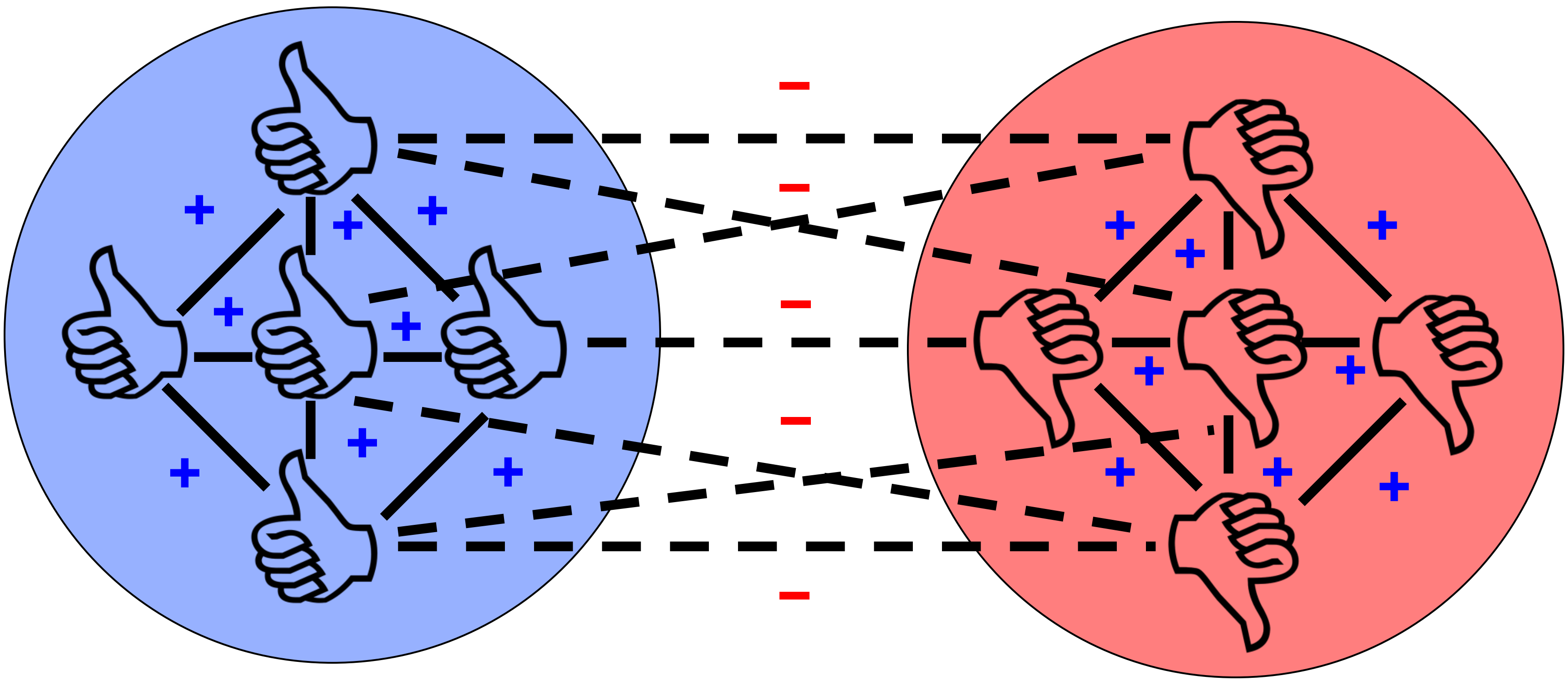
Psychol Rev **63**:277–293

In a balanced network, *every* cycle (closed loop) is *balanced*, i.e., product of the signs of the links in the loop is +ve

A <u>complete</u> graph (a network where all pairs of nodes are connected) is balanced if each constituent triad is balanced

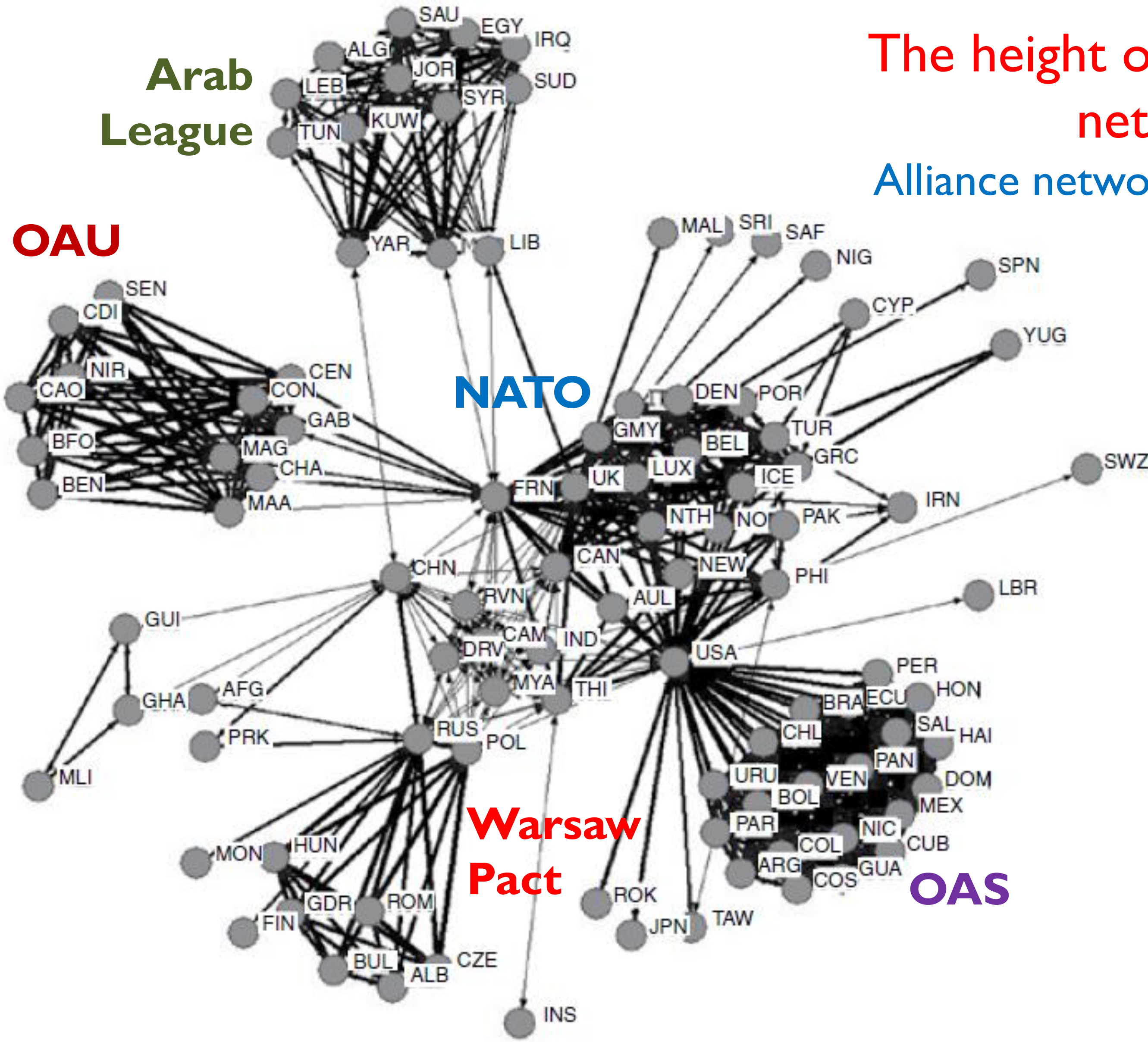
The local concept of balance results in non-trivial network structure

Any balanced network can be partitioned into two communities such that all edges inside each community are positive and all edges between nodes in opposite communities are negative (one of these communities may be empty)



In absence of any external influence or noise, the two communities are unified and opposed in their response to any issue

Balance and International Relations



The height of cold war from a network perspective

Alliance network of nations in 1962

As bipartite relations among countries that comprise major alliances change through events such as war, triads become unbalanced

$\Rightarrow$ creates tension

$\Rightarrow$ Reorganization into a balanced state involving new blocs and alliances

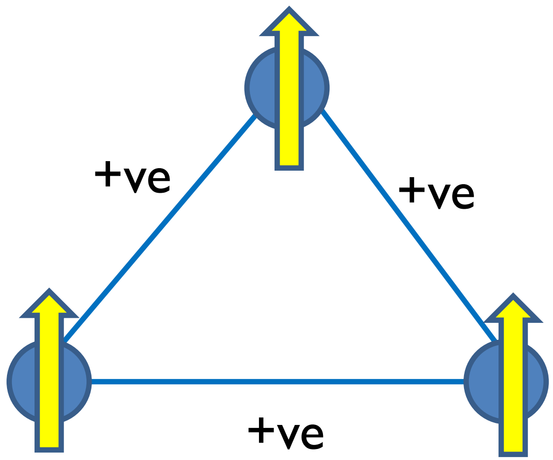
(Evolution to balance)

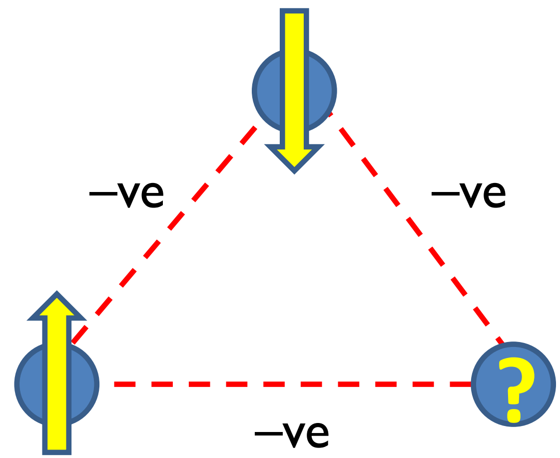
Z Maoz, *Networks of Nations* (Camb Univ Press, 2010)

For physicists

Structural balance ≡ No Frustration

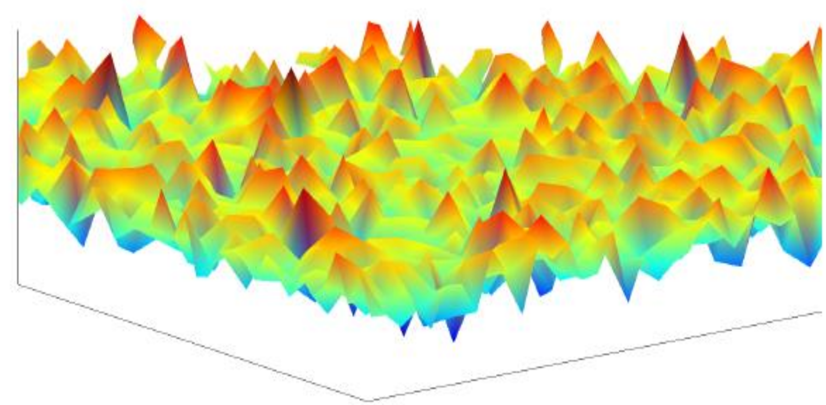
E.g., Ising spin systems with exchange interactions of FM or AFM type





Absence of structural balance would result in a rugged energy landscape, with the system trapped in any one of a large number of local minima

e.g., Spin Glasses

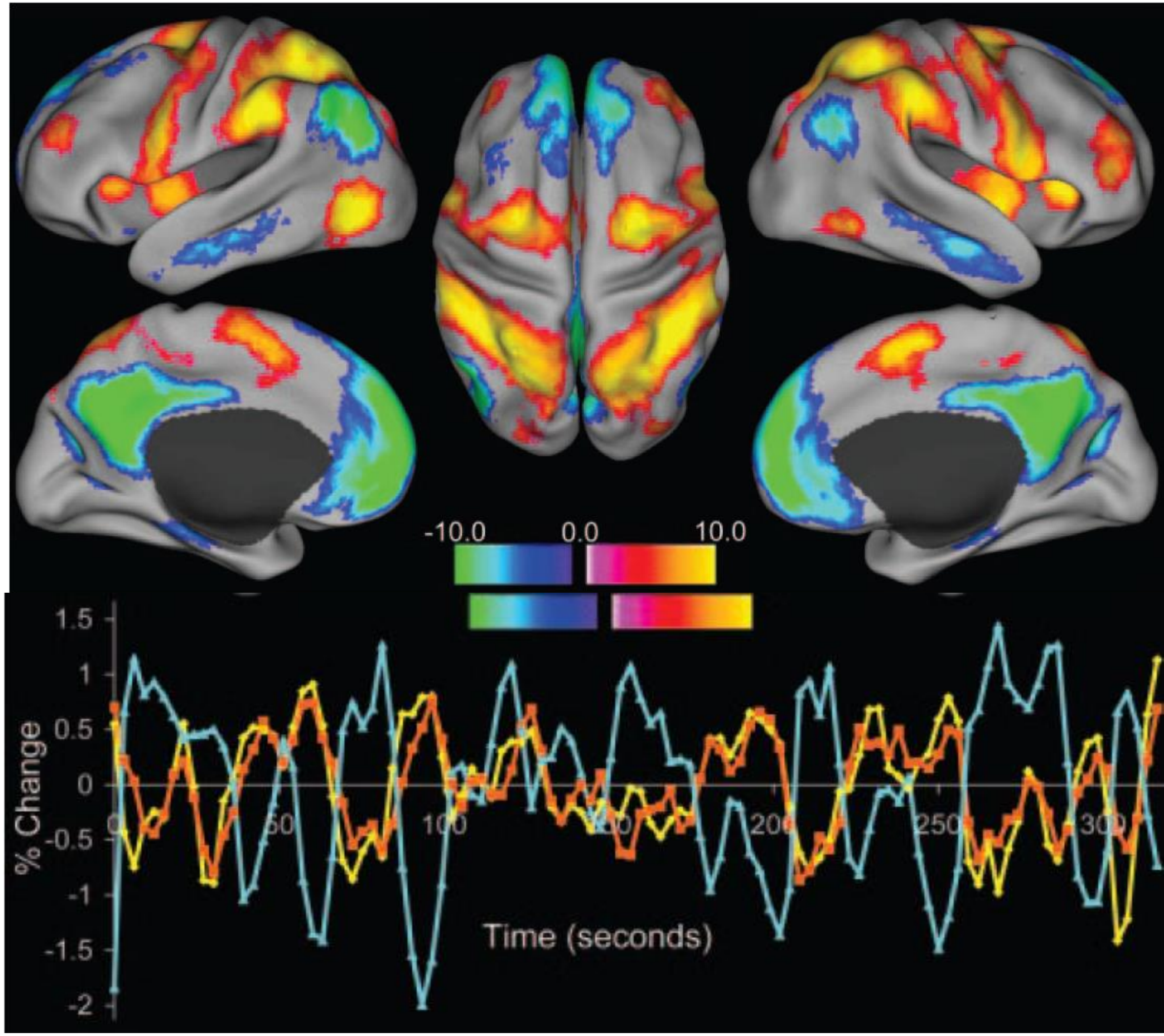


A balanced network would have smooth energy landscape

Structural balance in the brain ?

On the basis of spontaneous correlations & anti-correlations of fluctuations in fMRI between different brain regions, two "diametrically opposed" widely distributed brain regions identified.

One network consists of regions routinely exhibiting task-related activations;
the other of regions routinely exhibiting task-related deactivations.



Fox et al, PNAS 102 (2005) 9673

But how is balance achieved ?

Most studies on structural balance have been carried out in the context of social networks

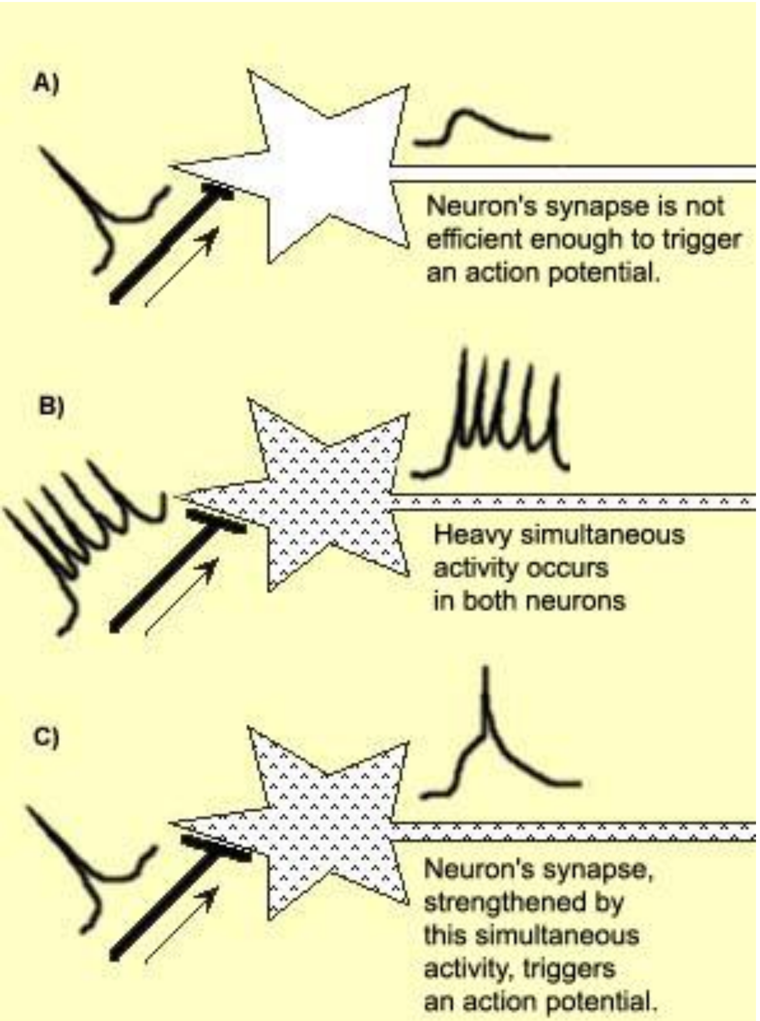
- Can other kinds of networks, in particular those that occur in biology, exhibit balance ?

- And if so, what is the mechanism of evolution to balance ?

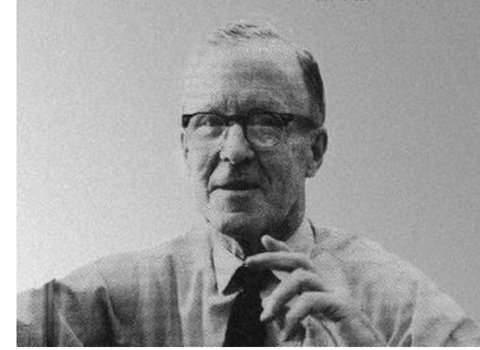
In particular, can balance be achieved as an outcome of link adaptation dynamics that depends on the state dynamics of the nodes of the network

“Learning” to Balance

Modifying the synaptic weights by Hebb rule



http://thebrain.mcgill.ca



Donald O Hebb (1904-85)

Hebb’s hypothesis (1949)

Neurons that fire together, wire together

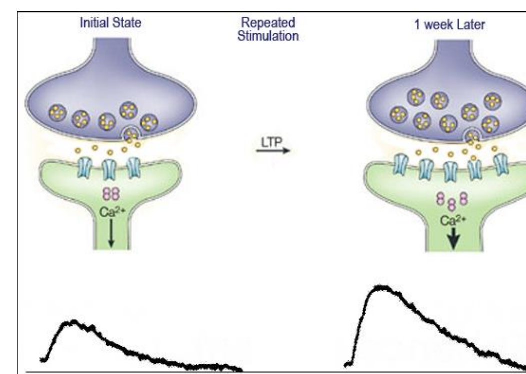
Intuitive interpretation

Agents behaving alike have their ties strengthened, while those behaving differently gradually develop antagonistic relations.

Long-term potentiation

First empirical observation (Lomo, 1966) supporting Hebb’s hypothesis

Persistent increase in synaptic strength after high-freq stimulation



Coevolution of coupling strength & spin dynamics

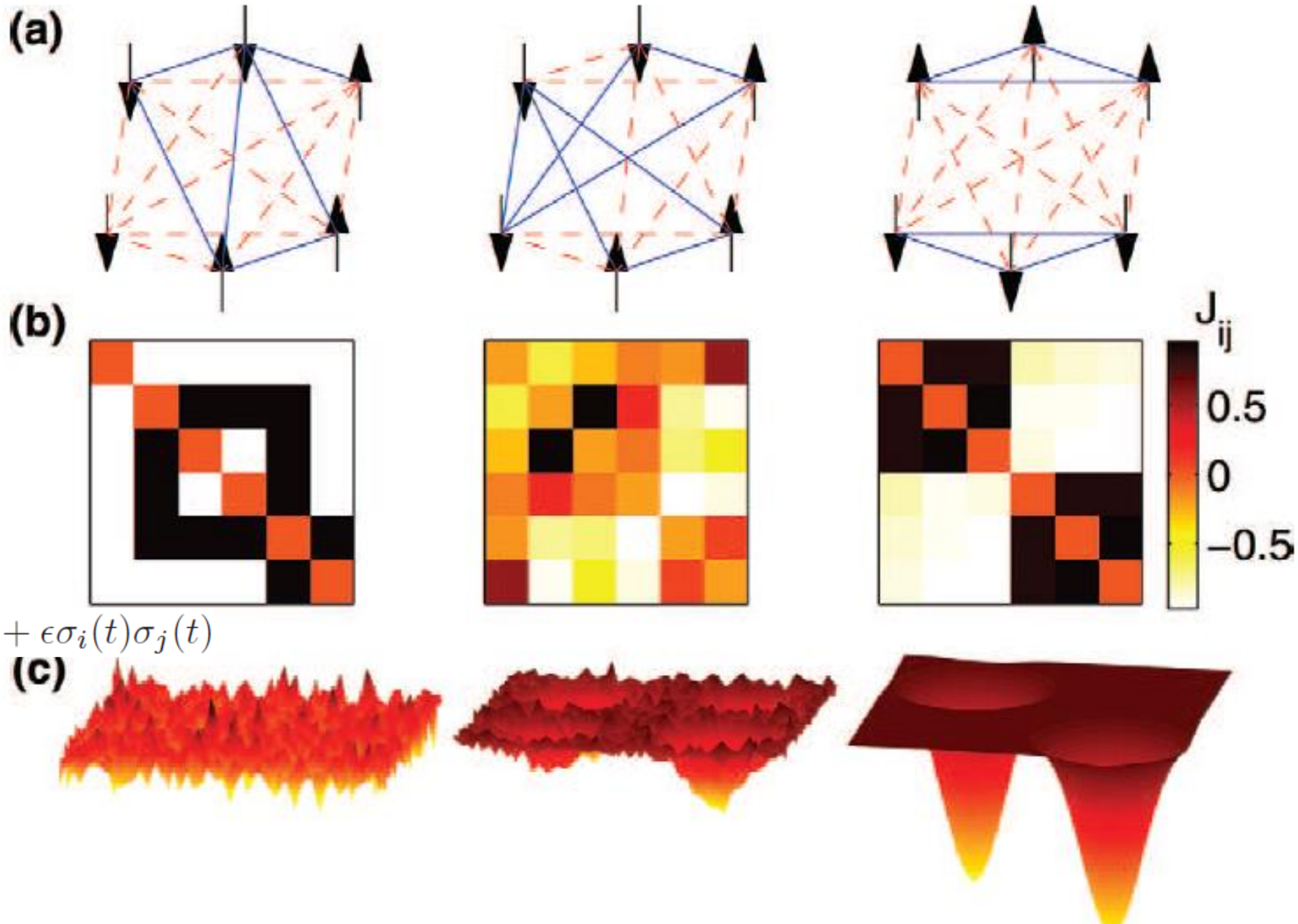
Spin orientations

$$\mathcal{E} = -\sum_{i \neq j} J_{ij}\sigma_i\sigma_j$$

Interaction matrix

$$J_{ij}(t+1) = (1-\epsilon)J_{ij}(t) + \epsilon\sigma_i(t)\sigma_j(t)$$

Energy landscape (schematic)



Initial (Frustrated)

Intermediate

Final (Balanced)

Image: R Singh, S Dasgupta & SS, EPL (2014)

Structural balance: A tool for seeing some order behind the chaos of financial markets ?

Image: Wikipedia

Robust signatures of systemic crises

Financial markets undergo fluctuations at all times – but only occasionally these spill over into the real economy resulting in economic collapse

E.g.,1929 Great Recession and the 2008 Great Recession

Can we distinguish such crises events from day-to-day ups and downs of the market ?

For this purpose look at the long-term evolution of a market over almost a century:
the daily closing prices of all stocks in New York Stock Exchange between 31 Dec 1925 and 1 Feb 2012

[CRSP (Center for Research in Security Prices) database]

Identifying the network of relations between different stocks in terms of how similar their prices movements are over time

By spectral analysis of cross-correlation matrix

Correlation Analysis

1. Construct the correlation matrix C composed of correlation values between every pair of stocks

Correlation between returns for stocks i and j:

$$C_{ij} = \langle r_i \, r_j \rangle \quad \text{where} \quad r_i = [R_i - \langle R_i \rangle]/\sigma_i$$

Data set:

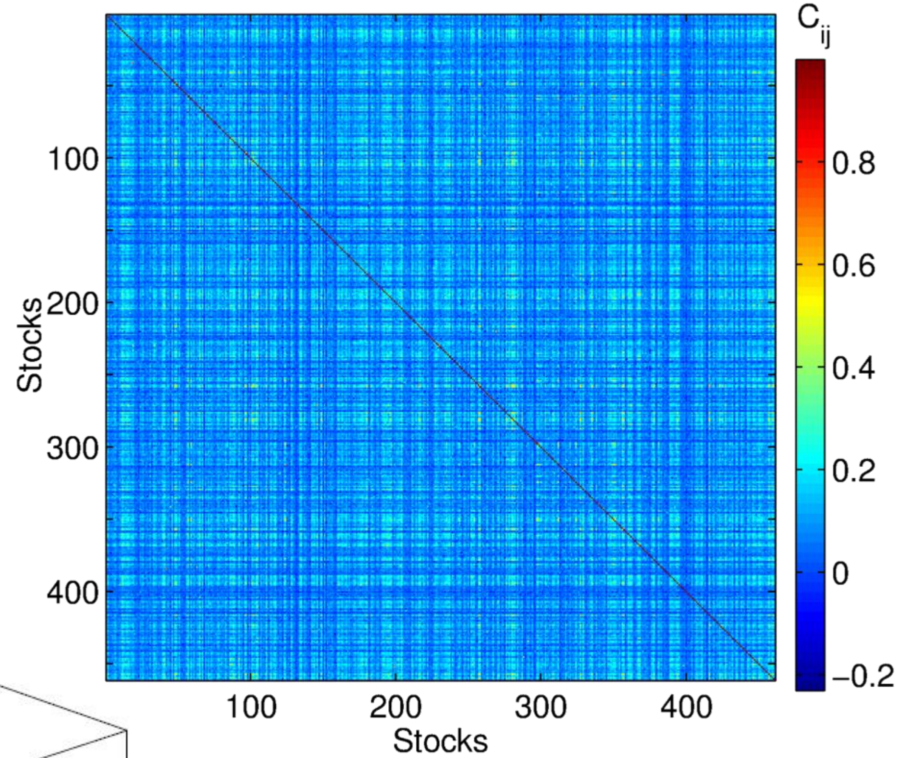
Data split into 85 overlapping periods of 1000 days (labeled Period 0 to Period 84), the temporal window being shifted by 260 days

Stocks with > 50 missing days not considered

To compare different periods, we consider 300 or 500 stocks in each

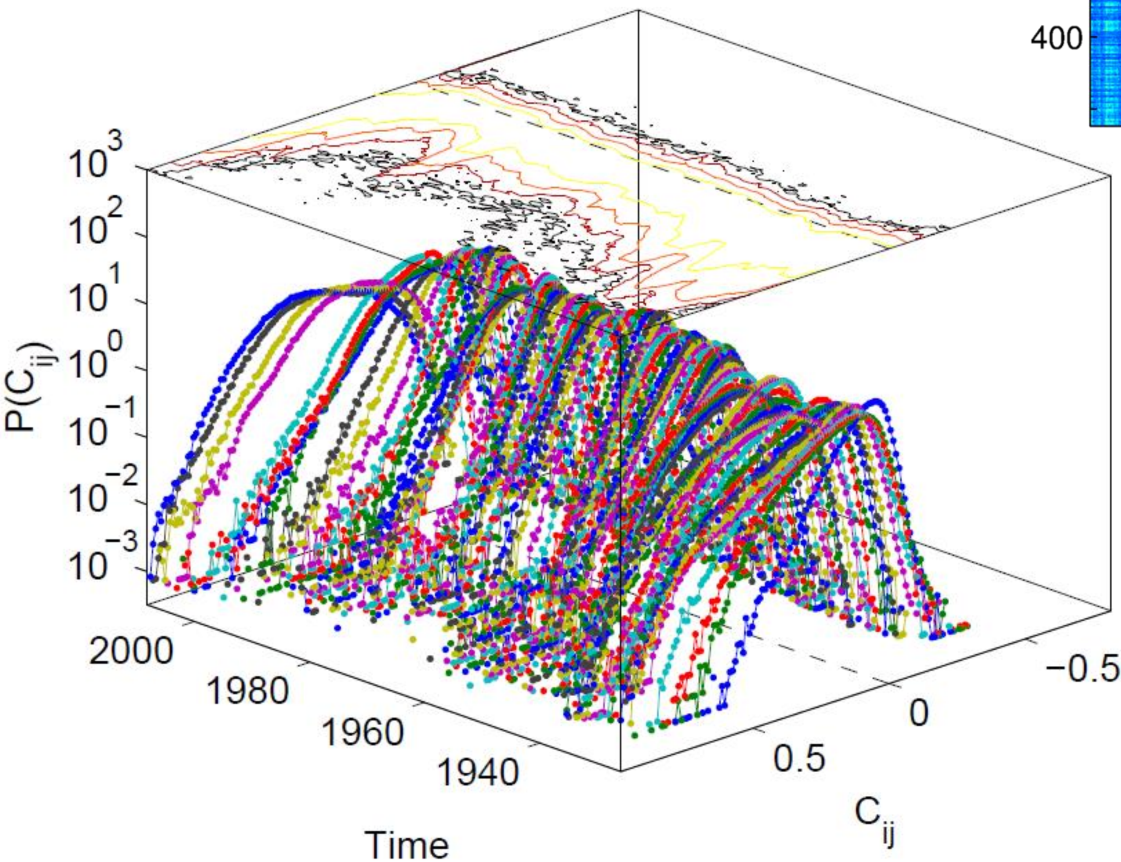
Sector to which a stock belongs identified by SIC (Standard Industrial Classification) code

Correlation Matrix



Distribution of magnitudes of cross-correlations varies with time over the period considered.

Distributions are highly asymmetric, skewed towards +ve values.



Degree of asymmetry varies with time, with the distributions in the 1930s and early 1940s and the 1990s onwards era having the largest degree of **skewness** – coincides with periods of marked upheaval in economy.

The problem with analyzing raw correlations

Cross-correlations of infinitely long time series of different variables will reflect actual inter-relations between them

But in reality, we only get data over a finite time !

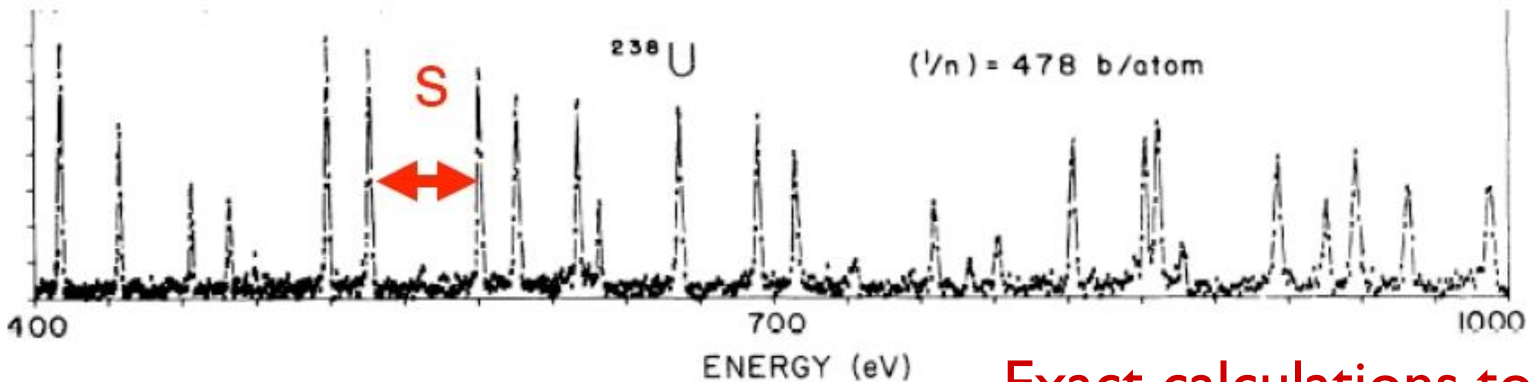
Due to stochastic fluctuations, even the output of uncorrelated random processes may exhibit spurious correlations, if we calculate C over finite-length time-series !

Question: Can we identify true interactions between variables by filtering out the effect of cross-correlations generated because of such finite time effects ?

Solution: Look at the spectral properties of C and compare with that of an ensemble of random cross-correlation matrices

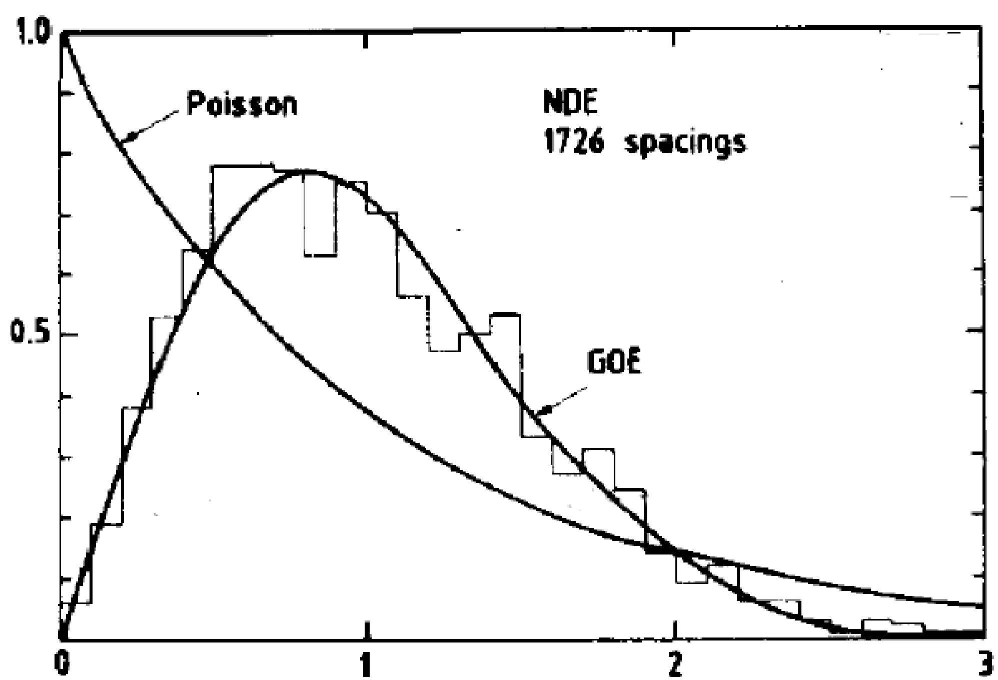
The spectrum of "Wigner-ium"

Nuclear physics: What is the energy spectrum of a complex nucleus ?



Exact calculations too difficult!

Image: O. Bohigas, R.U. Haq and A. Pandey, in *Nuclear Data for Science and Technology* (1983)



Wigner: Instead of focusing on specific energy levels, look at the spectral properties (eigenvalues & eigenvectors) of an ensemble of random (Hamiltonian) matrices

Distribution of spacing between neighboring energy levels

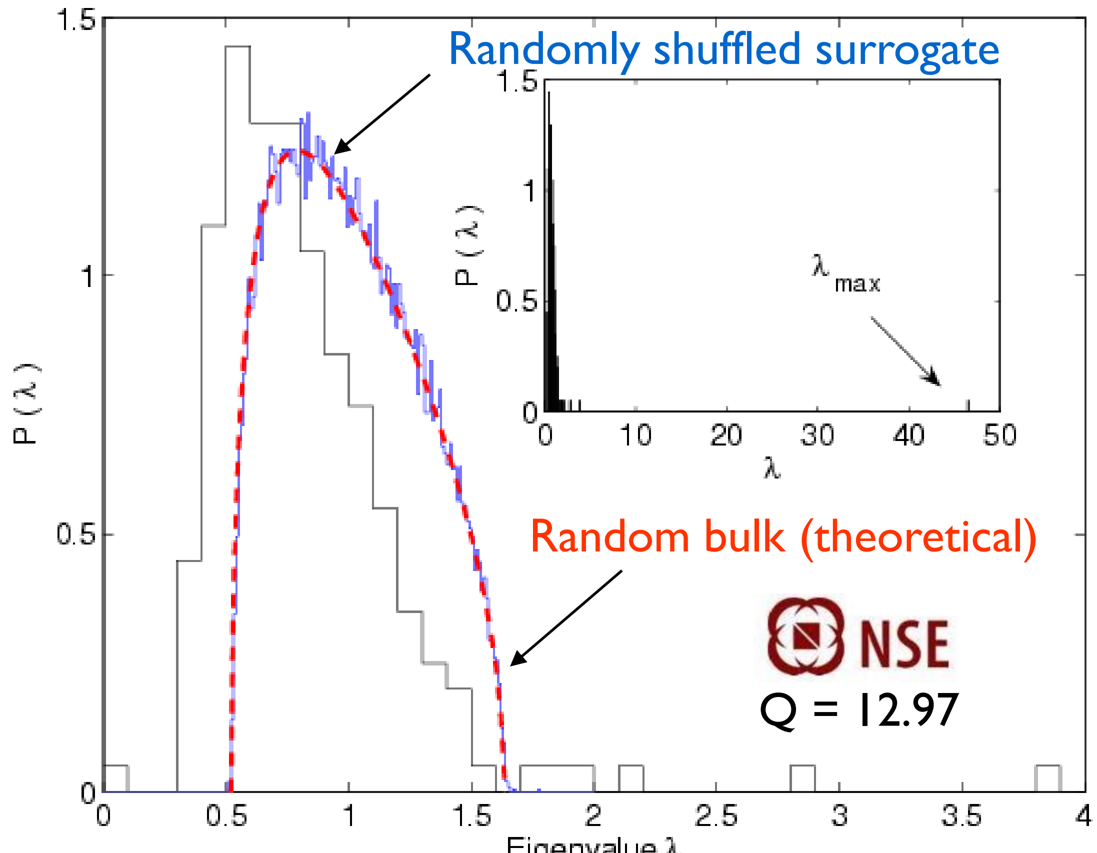
$$P(s) = s\exp(-[\pi/4]s^2)$$

II. Obtain eigenvalues of the correlation matrix

If all stocks are uncorrelated, C will be a random (Wishart) matrix with Marchenko-Pastur distribution (1967)

$$P(\lambda) = \frac{[Q/2\pi]\sqrt{[(\lambda_{max} - \lambda)(\lambda - \lambda_{min})]}}{\lambda} \quad \text{where } Q = T/N$$

Bounds of random distrn : $\lambda_{max} = [1+(1/\sqrt{Q})]^2$ and $\lambda_{min} = [1-(1/\sqrt{Q})]^2$

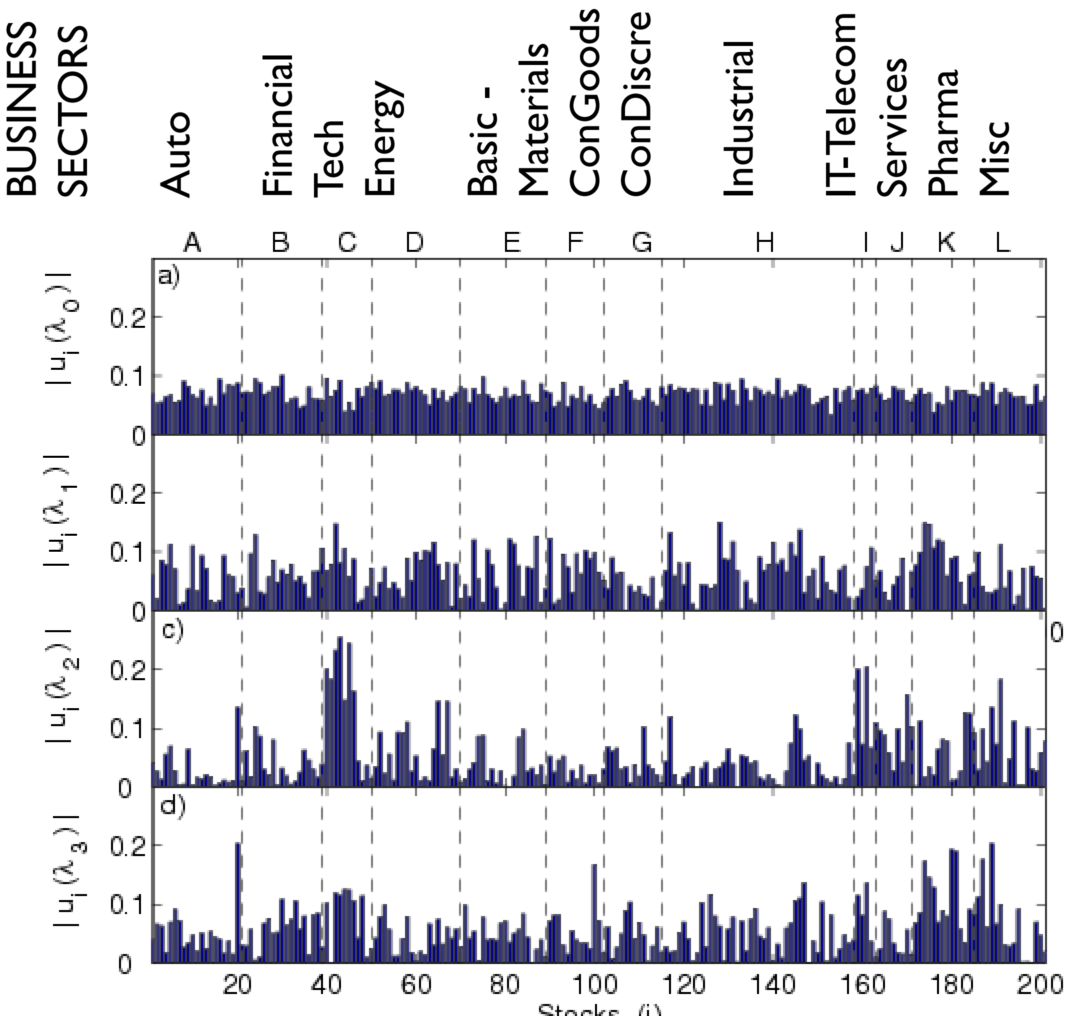


A small fraction of eigenvalues (~ 3%) deviate from random behavior

The largest eigenvalue is more than 28 times larger than the predicted max. random bound
+ a few "intermediate" eigenvalues

Deviating eigenvalues ⇨ Information about interaction structure of the market

Look at eigenvectors *u* of the largest few eigenvalues



All stocks contribute (almost) uniformly to *largest* eigenvalue

⇨ Market mode

Common component affecting all stocks with same bias

Intermediate eigenvalues should reflect group structure in market if eigenvectors are *localized*

However …

largest eigenmode (*market*) dominates all intra-group correlations (if existing).

$\Rightarrow$ no straightforward detection of significantly related groups of stocks.

For this purpose, use

Matrix Decomposition Technique

Aim: removing the effect of (i) market mode & (ii) random noise

Expanding correlation matrix as $C = \Sigma_i \lambda_i u_i^T u_i$

Allows decomposition of C into contributions due to

- *market*, common for all stocks
- *groups* of co-moving stocks (identified with various business sectors)
- *random*, idiosyncratic effects for each stock

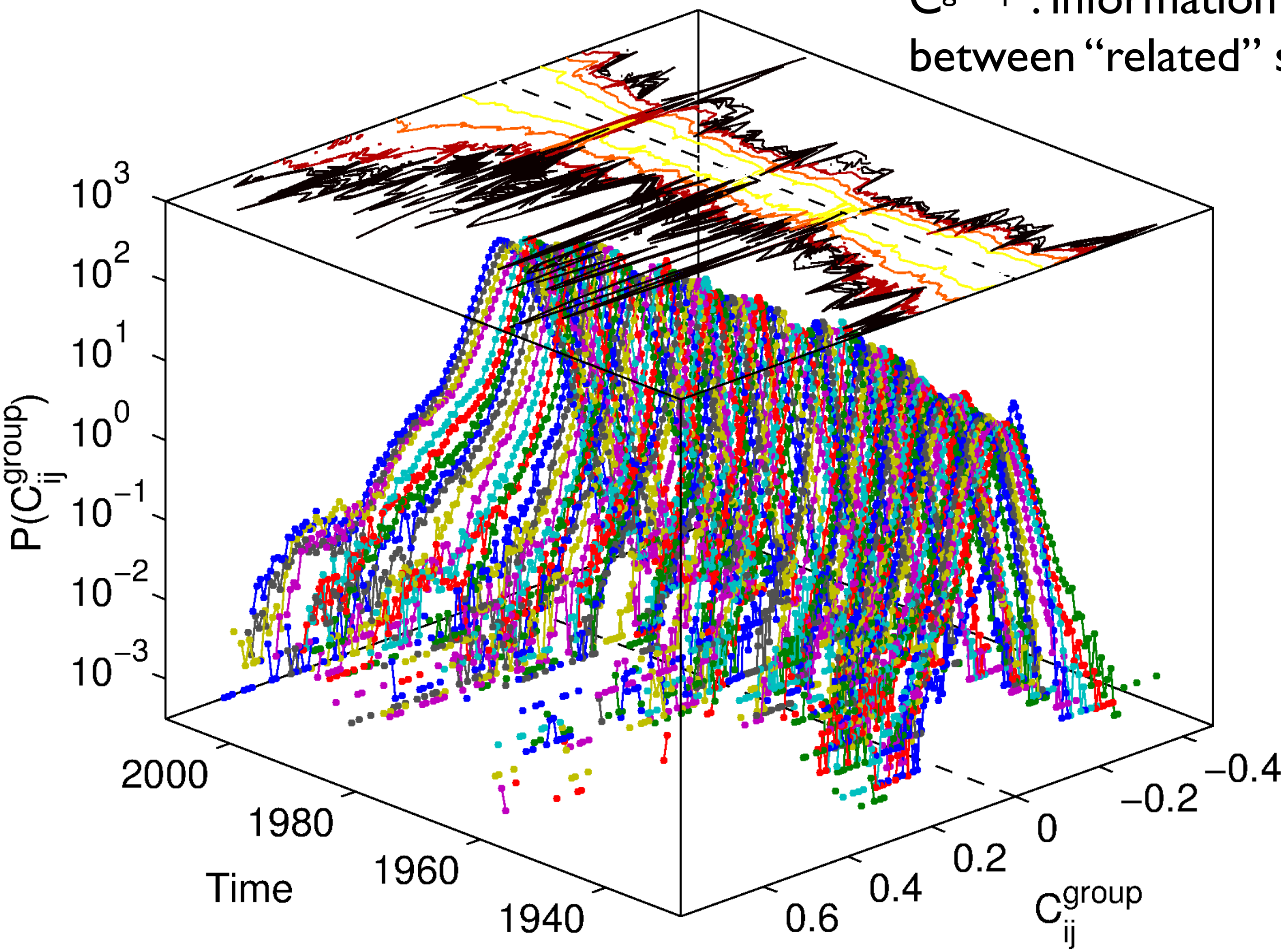
$$C = C_{market} + C_{sector} + C_{random}$$

$$= \lambda_0 u_0^T u_0 + \Sigma_{i=1,\ldots,N_{group}} \lambda_i u_i^T u_i + \Sigma_{i=N_{group}+1,\ldots,N-1} \lambda_i u_i^T u_i$$

Largest eigenvalue | Intermediate eigenvalues | Random bulk eigenvalues

Focus on group modes

C^{group} : information about interaction between “related” stocks



Higher incidence of larger values of positive correlations in certain periods is representative of higher correlation between groups of stocks.

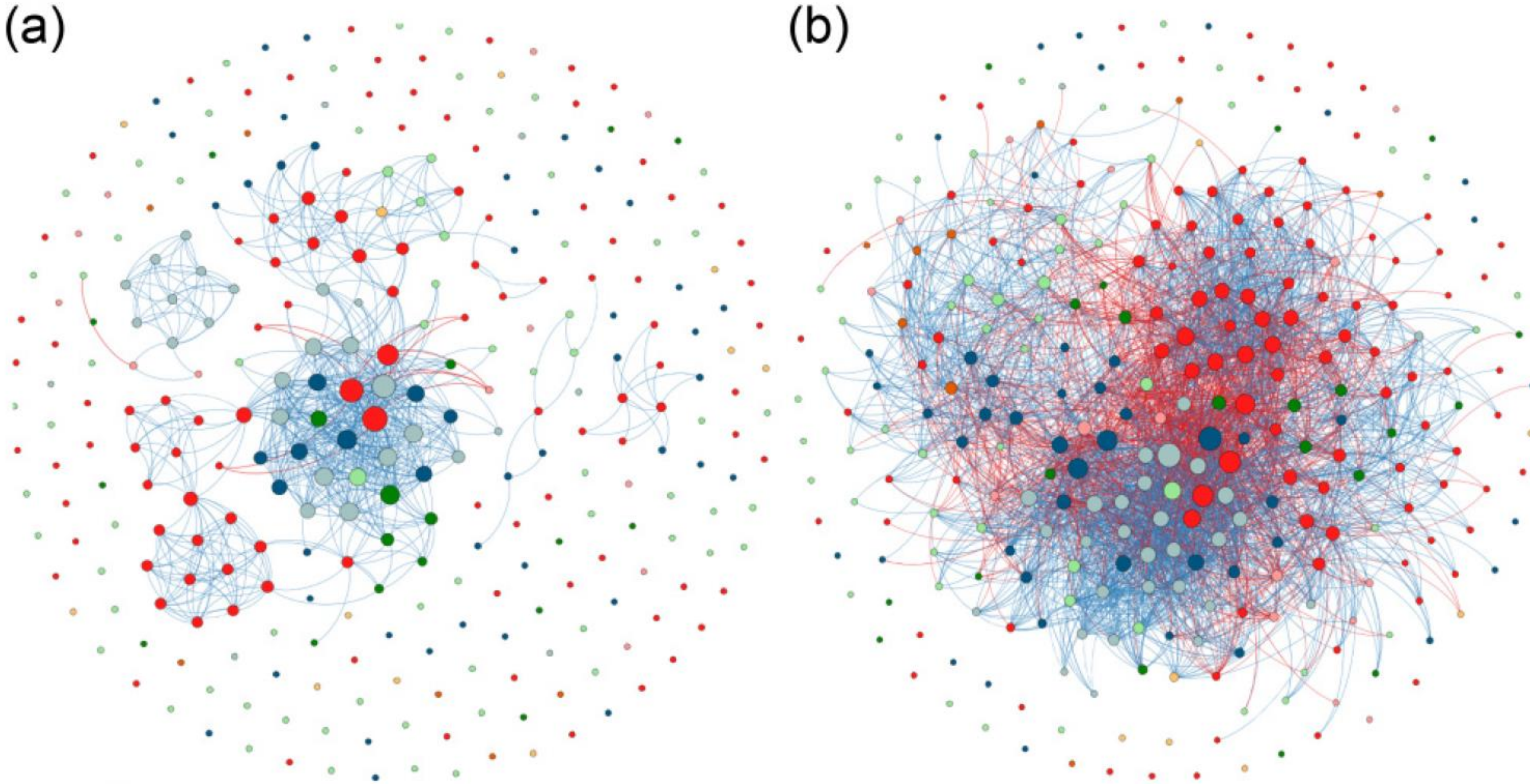
Reconstructing the interaction network

Method: Use C^{group} to generate an adjacency matrix A, such that

- $A_{ij} = 1$ if $|C^{group}_{ij}| > 3 \times$ std dev of C^{random} distribution
- $A_{ij} = 0$ otherwise

Node colors represent sector

Image: C Kuyyamudi, A S Chakrabarti & S Sinha, PRE (2019)



Dec, 2002 - Dec, 2006

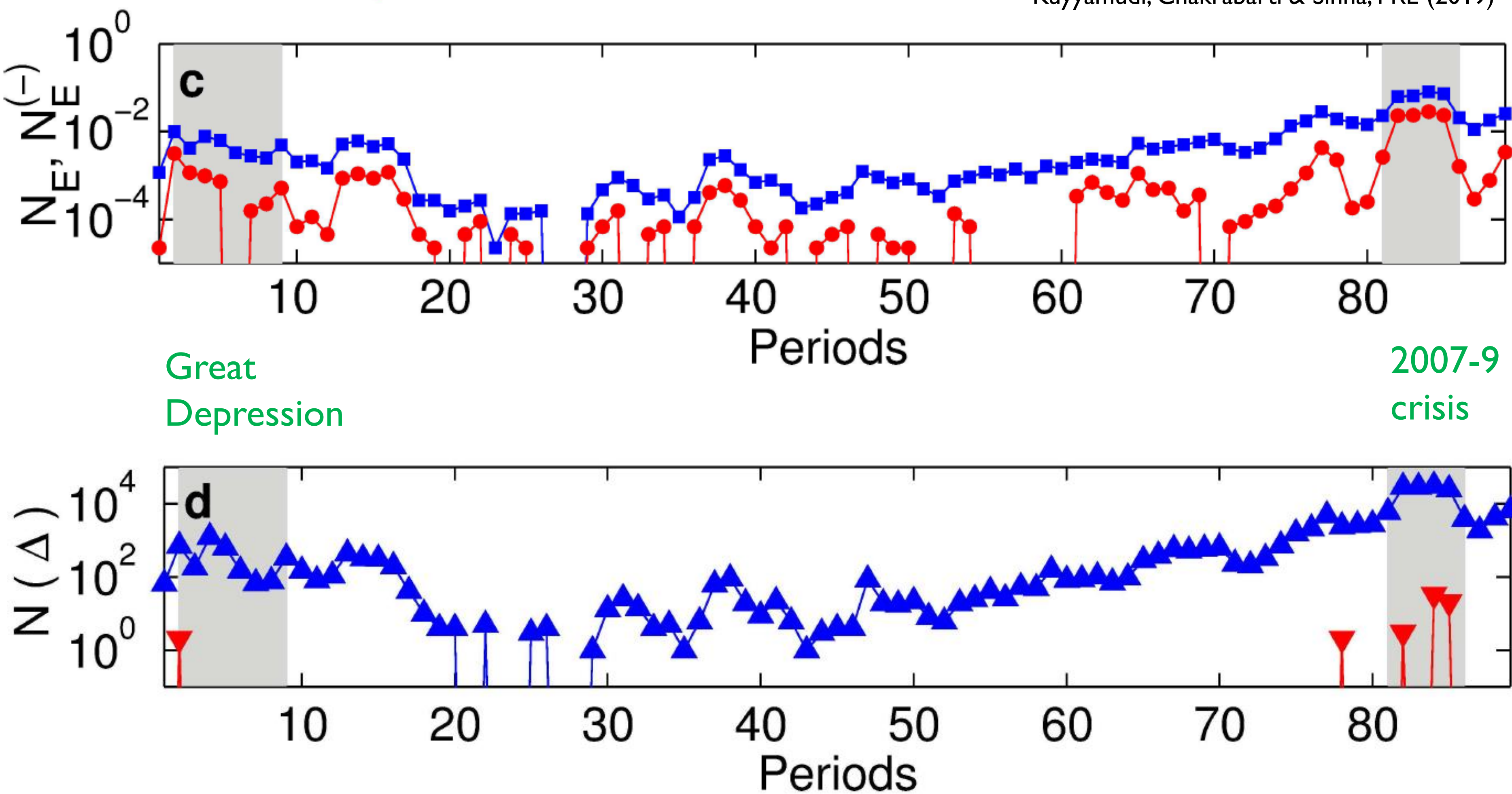
Great recession Feb, 2008 - Feb, 2012

Blue links: +ve interactions, **Red** links: – ve interactions

During a major crisis, network structure shows increased

- Connection density,
- Number of negative edges, and
- Clustering

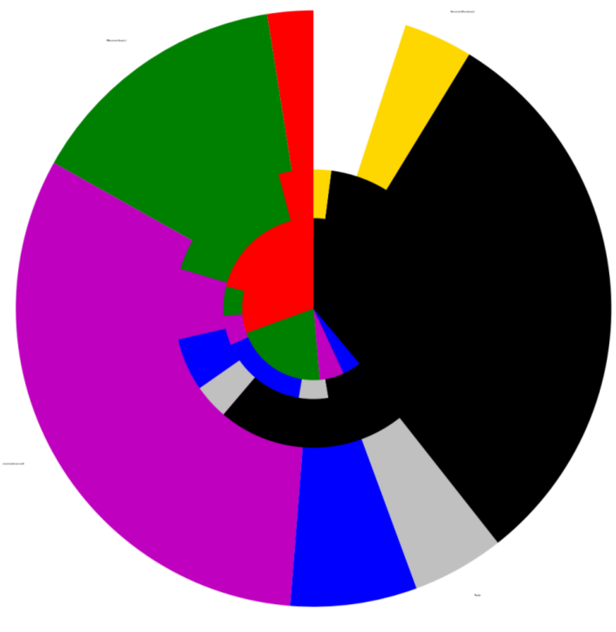
Kuyyamudi, Chakrabarti & Sinha, PRE (2019)



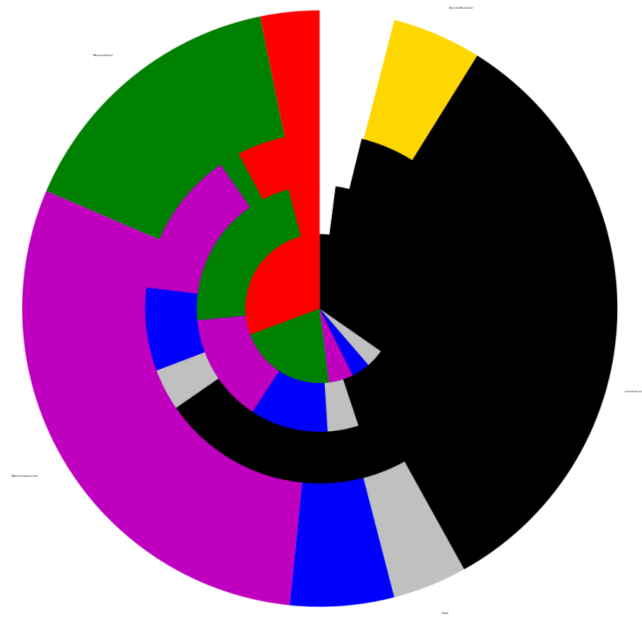
Degree and size of inner cores increase during crisis

2007-9 crisis

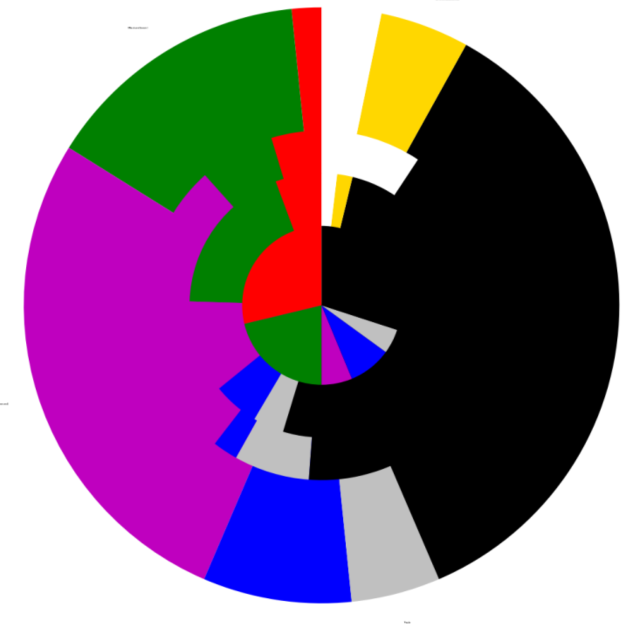
Jan 30, 2006 – Jan 13, 2010

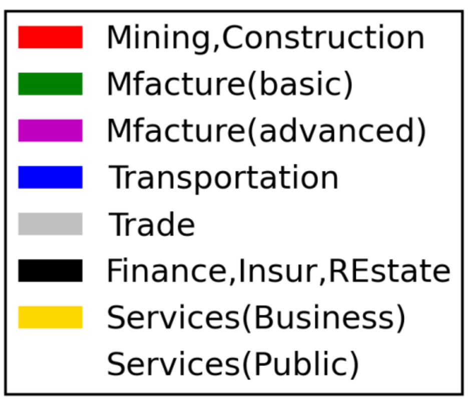


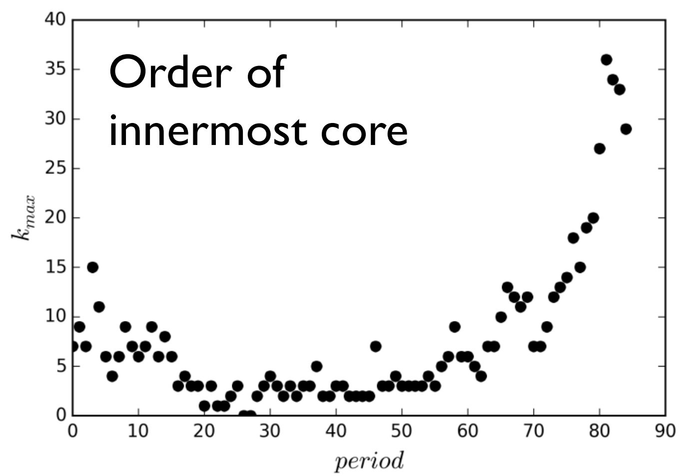
Feb 8, 2007 – Jan 24, 2011

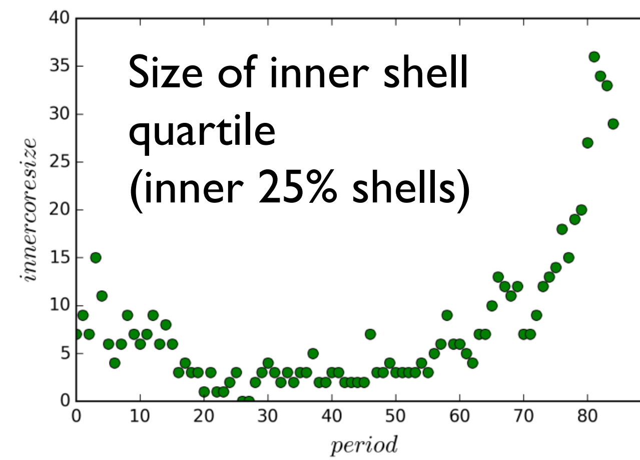


Feb 21, 2008 - Feb 2, 2012









An even more robust indicator of economic crisis is the loss of

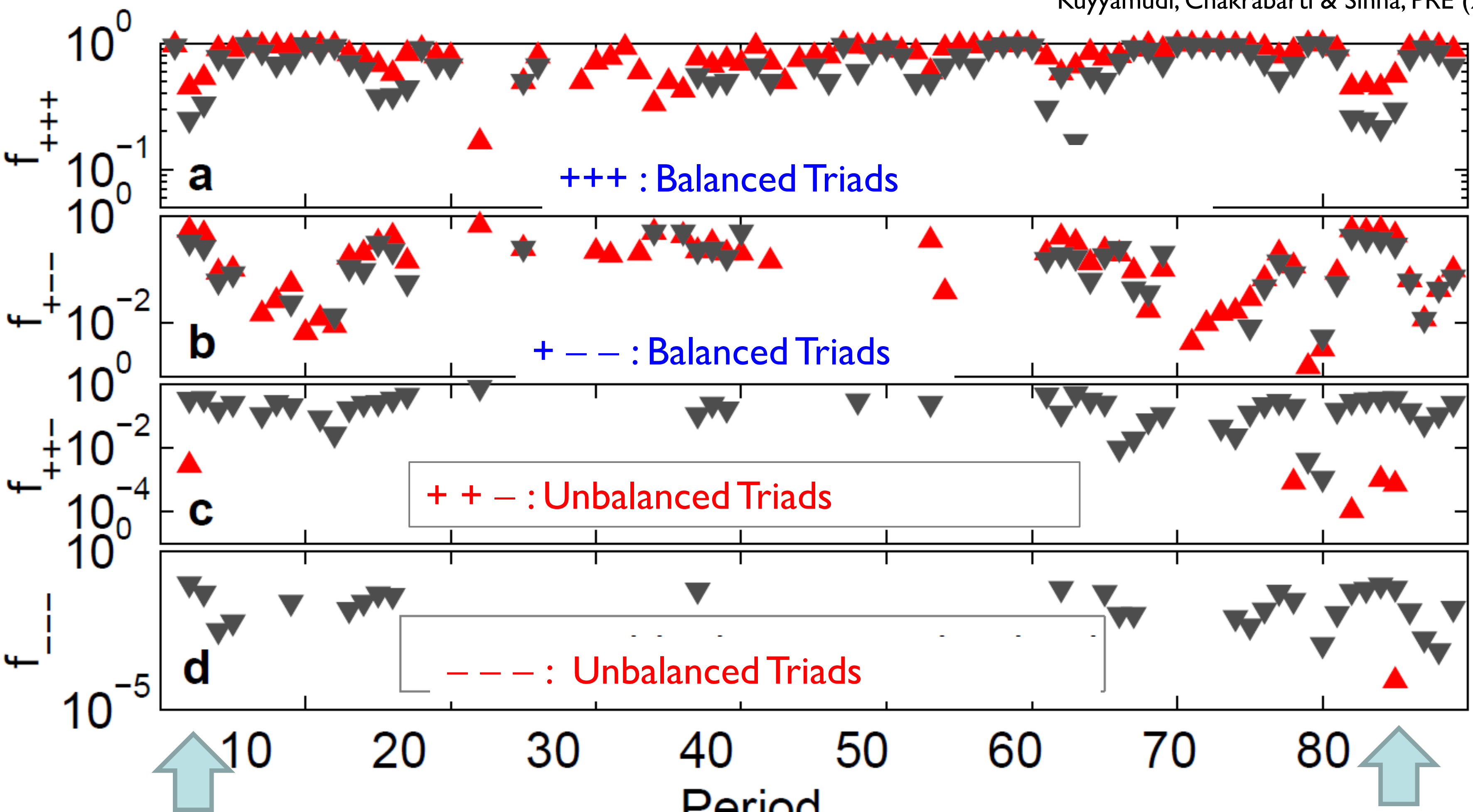
structural balance

i.e., the emergence of frustrated triads in the interaction network

A closer look at triangles in the NYSE stock interaction networks

Red: empirical networks, **Gray**: degree-preserved randomized networks

Kuyyamudi, Chakrabarti & Sinha, PRE (2019)

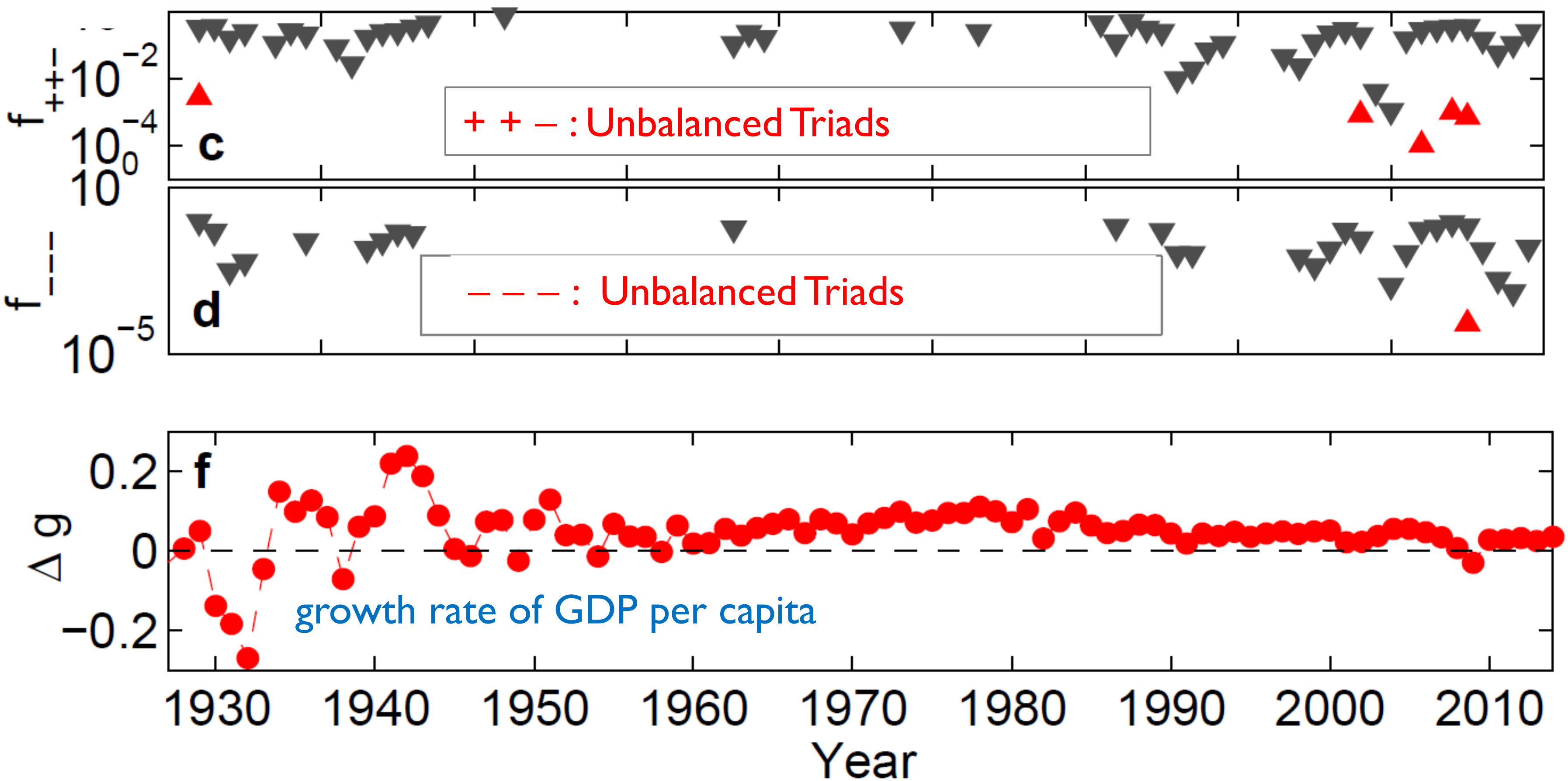


1929 Great Crash

2007-08 Crisis

Frustration ⇒ Systemic Risk

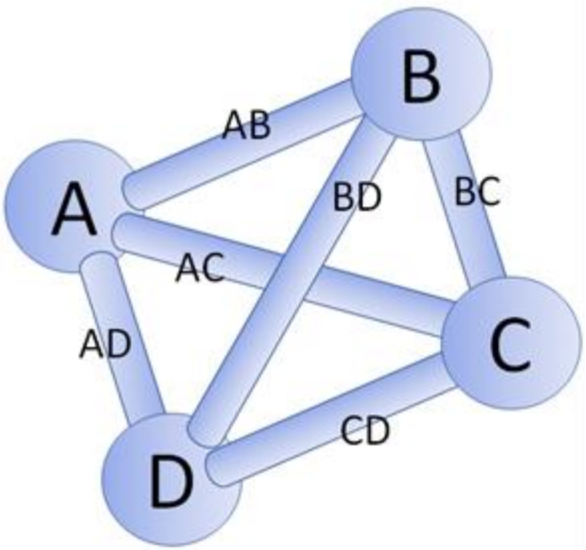
Frustrated triads appear in individual eigenmodes in many periods, but in the network only prior to/during systemic crisis in the economy (e.g., negative growth rate of GDP per capita)



Qualitatively similar behaviour shown by NASDAQ and the FOREX markets

Assignment

Consider a fully connected network of 4 nodes shown on the right, the nodes being labelled A,B,C,D and the links designated AB,BC,CD, etc. (i.e., indicating the pair of nodes that each link connects).



(a) If each link can be either +ve or –ve, what is the total number of possible configurations that can be obtained (where the configurations can be represented as {+,+,+,+,+,+}, {+,+,+,+,+,–},{+,+,+,+,–,+}, {+,+,+,–,+,+}, etc.)?

[Note that each of the links are distinct, i.e., AB being negative in a network where all other links are positive is a distinct configuration compared to one in which AD (for instance) is the only link which is negative.]

(b) How many distinct configurations will have 3 links positive and 3 links negative?

(c) Find how many of the total number of configurations with distinct assignment of link signs that you calculated in (a) are balanced. Note that a network is balanced if every closed loop or cycle is balanced, i.e., product of the signs of the links in the loop is + ve. However, instead of having to look at all 4-cycles as well as 3-cycles (triads), you can use the Cartwright-Harary theorem, according to which a fully connected network is balanced if each of the triads (ABC, ABD, etc.) are each individually balanced.

[Hint: find how many distinct triads are there in the network. If for a given configuration, even one of these triads is <u>not</u> balanced (i.e., has an odd number of negative links) the configuration will be <u>not</u> balanced.]