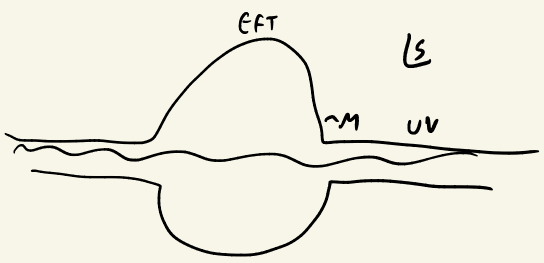


Tolley + Wang + Zhou 2011.02400
SCH + van Duong 2011.02957

Scalar EFT $\quad M_4 \supset g_2(s^2+t^2+u^2) + g_3\, stu + \cdots$

$$g_2 = \left\langle \frac{1}{m^4}\right\rangle, \quad g_3 = \left\langle \frac{3-2\mathcal{J}(\mathcal{J}+1)}{m^6}\right\rangle, \quad 0 = \left\langle \frac{\mathcal{J}(\mathcal{J}+1)(\mathcal{J}(\mathcal{J}+1)-8)}{m^8}\right\rangle$$

$$\langle\cdots\rangle = \underbrace{\sum_{\substack{J\geq 0\\ \text{even}}} \int_{M^2}^{\infty} \frac{dm^2}{m^2}\, \mathrm{Im}\, a_J(m)}_{\geq 0} (\cdots)$$

Q: given g_2, what is $\min(g_3)$?

Anatomy of a bootstrap problem:

- positive unknowns
- some obs. to max/min
- crossing equation

(CFT) $f^2_{\mathcal{O}}$ ... f_0^2 ... $\sum(\ldots - \ldots) = 0$

Ad-hoc Cauchy-Schwartz $\Rightarrow \left\langle \frac{\mathcal{J}(\mathcal{J}+1)}{m^6}\right\rangle \leq \frac{8}{M^2} g_2 \Rightarrow M^2 g_3 \geq -16\, g_2$

Systematic strategy:

take linear combination:

$$F[J,m] = a\frac{1}{m^4} + M^2\frac{3-2\mathcal{J}(\mathcal{J}+1)}{m^6} + \beta\frac{\mathcal{J}(\mathcal{J}+1)(\mathcal{J}(\mathcal{J}+1)-8)}{m^8}M^4$$

s.t. $F[J,m]\geq 0$ for all possible heavy states ($J\geq 0$ even, $m\geq M$)

$\langle F[J,m]\rangle \geq 0$

$\Rightarrow a\, g_2 + g_3 + \beta\cdot 0 \geq 0$

$M^2 g_3 \geq -a\, g_2 \Rightarrow$ maximize $-a$ over all such positive functionals.

good news: efficient algorithms (numerically) solve such problems!

i) simplest strategy: sample a fine grid of (J,m) (N points on)

$$N\updownarrow \begin{pmatrix} \vdots & \vdots & \vdots \end{pmatrix}\cdot\begin{pmatrix}a\\1\\g_3\end{pmatrix} \geq \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix} \; N\updownarrow.$$

Mathematica LinearOptimization[] can solve this.

ii) impose $\forall$ mass at once, for some finite list of spins.

thm: a poly(x) is positive for $x\in\mathbb{R}$ iff poly$(x) = \sum(\text{poly})^2$

$x\geq 0$ iff poly$(x) = \sum()^2 + x\sum()^2$.

$\frac{1}{m^2}\to\frac{x}{x+1}$ $\quad m\geq 1 \Rightarrow x\geq 0$

a poly of degree $2q$ is a sum of squares iff

iff poly$(x) = (1\; x \cdots x^q)\cdot M\cdot\begin{pmatrix}1\\x\\\vdots\\x^q\end{pmatrix}$ where $M\succeq 0$

$\Rightarrow$ by finding positive matrices M, we solve ∞ constraints ($\forall m\geq M$)

"Semi-definite programming" SDPB.

once a "numerically positive" functional is found, positivity is easy to verify

EFT coefficient	Lower bound	Upper bound
$\tilde{g}_3$	-10.346	3
$\tilde{g}_4$	0	0.5
$\tilde{g}_5$	-4.096	2.5
$\tilde{g}_6$	0	0.25
$\tilde{g}'_6$	-12.83	3
$\tilde{g}_7$	-1.548	1.75
$\tilde{g}_8$	0	0.125
$\tilde{g}'_8$	-10.03	4
$\tilde{g}_9$	-0.524	1.125
$\tilde{g}'_9$	-13.60	3
$\tilde{g}_{10}$	0	0.0625
$\tilde{g}'_{10}$	-6.32	3.75

$g_2\geq 0$

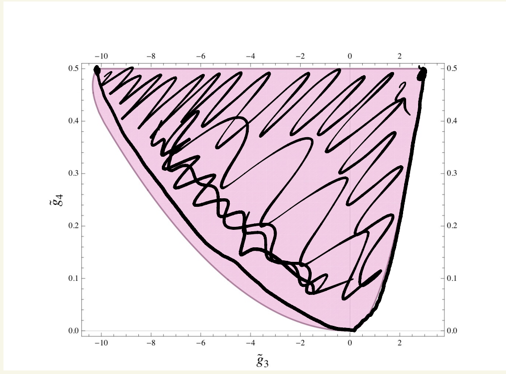
$\tilde{g}_3 = g_3\frac{M^2}{g_2}$

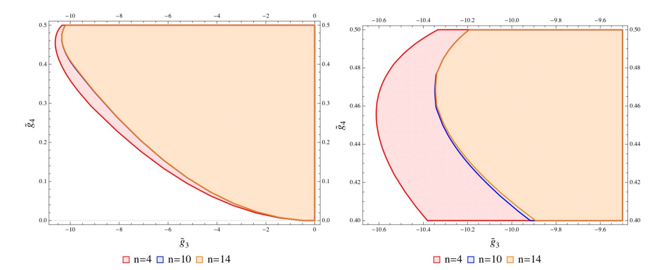
"Higher-derivative couplings don't grow faster than geometric"

$$\frac{1}{M^2-s}\to\frac{1}{M^2}+\frac{s}{M^4}+\frac{s^2}{M^6}+\cdots$$

even if UV theory is strongly coupled, and has states of all spins.

causality $\to$ EFT-like power counting.





(a) Convergence is rapidly achieved by adding null constraints. (b) Close-up near the left kink at (−10.19, 0.5).

Figure 10: Convergence of the $(\tilde{g}_3, \tilde{g}_4)$ allowed region (shaded) with the Mandelstam degree n of crossing equations. The positive section of the horizontal axis is omitted since that region converges immediately.

Two dual approaches:

i) Rule out

ii) Rule in (exhibit some 2→2 S-matrix that satisfies our axioms)

Ruling 'in':

$$\mathcal{M}_{\text{spin-0}} = \frac{1}{m^2-s}+\frac{1}{m^2-t}+\frac{1}{m^2-u},$$
$$\mathcal{M}_{stu\text{-pole}} = \frac{m^4}{(m^2-s)(m^2-t)(m^2-u)} - \gamma(d)\mathcal{M}_{\text{spin-0}}$$

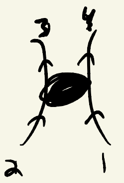
satisfies all our axioms!

Caveats:

- Higher-dim ops in SM $\to$ $(HL)^2$ $\Rightarrow \nu$ masses dim5.

$\to g_1(H^\dagger D_\mu H)^2$, $(\bar\psi\psi)^2$, $\cdots$ dim6

these have spin 1, and are not probed by spin>1 dispersion.



$M\sim s$ $\quad$ ~~not s^2~~

(even if we assumed convergence for J=1 sum rules, they are not "positive" $\oint\frac{ds}{s^2}M \Rightarrow g_1 = \left\langle \frac{|c^{\phi\phi_1}|^2 - |c^{\phi\bar\phi}|^2}{m^2}\right\rangle$

- unitarity $\Rightarrow 0\leq \mathrm{Im}\, a_\ell(s)\leq 2$

$$|1+ia_\ell|\leq 1 \Leftrightarrow \begin{pmatrix}\mathrm{Im}\, a_\ell & \mathrm{Re}\, a_\ell\\ \mathrm{Re}\, a_\ell & 2-\mathrm{Im}\, a_\ell\end{pmatrix}\succeq 0$$

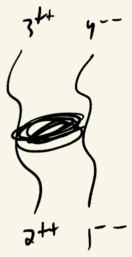
Guerrieri + Sever 2106.10257

Chiang + Huang + Li + Rodina + Weng 2105.02862, 2201.07177

EFTs of gravity

massless spin 2 particles $d=4$ $M \ll M_{pl}$

Lorentz, causality



$$\mathcal{M} \sim \langle 14\rangle^4 [23]^4 \Big(\underbrace{\frac{8\pi G}{stu} + 8\pi G \frac{g_3^2\, su}{t} + g_4 + \cdots}_{F(s,t)} \Big)$$

$g_4 \Downarrow \sim \frac{8\pi G}{M^6}$

$$0 = \oint F(s, p^2 - s)\, ds\, s \qquad B_2.$$

$$-B_2^{(p)}\big|_{low} = B_2\big|_{high}$$

contact terms don't contribute!

$$\frac{8\pi G}{p^2} + 8\pi G\, g_3^2\, p^6 = \sum_J \int \frac{dm^2}{m^6} \left[|c^{++}|^2 d_J \left(1 - \frac{2p^2}{m^2}\right) + |c^{+-}|^2 \right]$$

Task: find a positive functional $\int_0^M dp\, \psi^{magic}(p)\, B_2(p^2)$ s.t. RHS is pos,

and LHS is $8\pi G \cdot$ const.

$\Rightarrow$ ψ^{magic} proves that gravity is attractive.

guarantees bounds: $G - \# g_4 = \psi^{magic} - \psi_{g_4} \geq 0$

$$\frac{g_4}{8\pi G} \leq \# \frac{1}{M^6} \log(M/m_{IR})$$

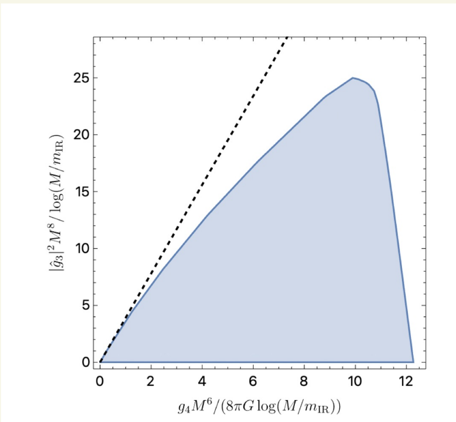
(in $d=4$: if positive at all b $\Rightarrow$ can't vanish at $p \to 0$

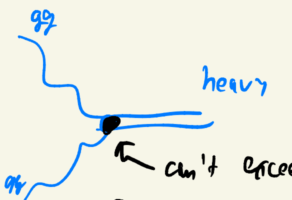
$\Rightarrow$ log-IR divergent on graviton pole)

interpretation:

$$\mathcal{L} = \frac{1}{16\pi G} \int d^4x \left(R + g_3 \frac{\mathrm{Riem}^3}{3!} + \frac{g_4}{8\pi G} \mathrm{Riem}^4 + \cdots \right)$$

$\mathrm{Riem}^3 \cancel{R}$ EOM





$M \ll M_{pl}$

can't exceed gravitational strength

$\sqrt{8\pi G} \cdot O(1)$

if $g_3 \sim r_0^4$ were measured,

new higher-spin states must exist at $M \lesssim \frac{1}{r_0}$

(Ex: $\sim (10\,km)^4$ $\Rightarrow$ $M \lesssim 10^{-11}\, eV$)

$$f(R) = R + (\partial\phi)^2 + V(\phi)$$

unconstrained by us.

with gravity:

- how far from GR can it be? $\max(g_4)$ given mass M of higher-spin states
- how close to GR can it be? need Planck-scale effects?

10D SUSY   Guerrieri + Penedones + Vieira 2102.02847

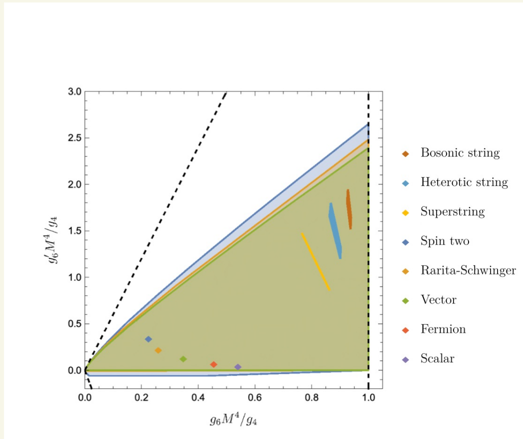
theory that minimizes Riem^4 is a string theory!

SCH + Mazac + Rastelli + Simmons-Duffin 2102.08951

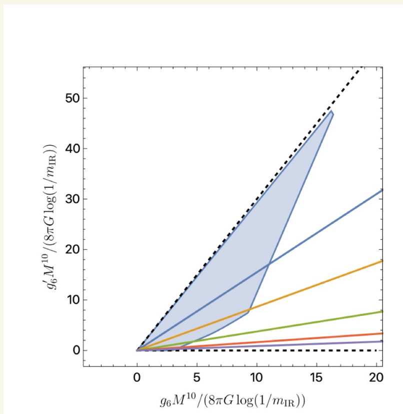
SCH + Li + Parra-Martinez + Simmons-Duffin 2201.06602

$$|\widehat{g}_3|^2 M^8 \leq 24.9 \log(M/m_{\rm IR}) - 27.6\,,$$

$$\frac{g_4 M^6}{8\pi G} \leq 12.3 \log(M/m_{\rm IR}) - 13.5\,.$$



(a)



(a)