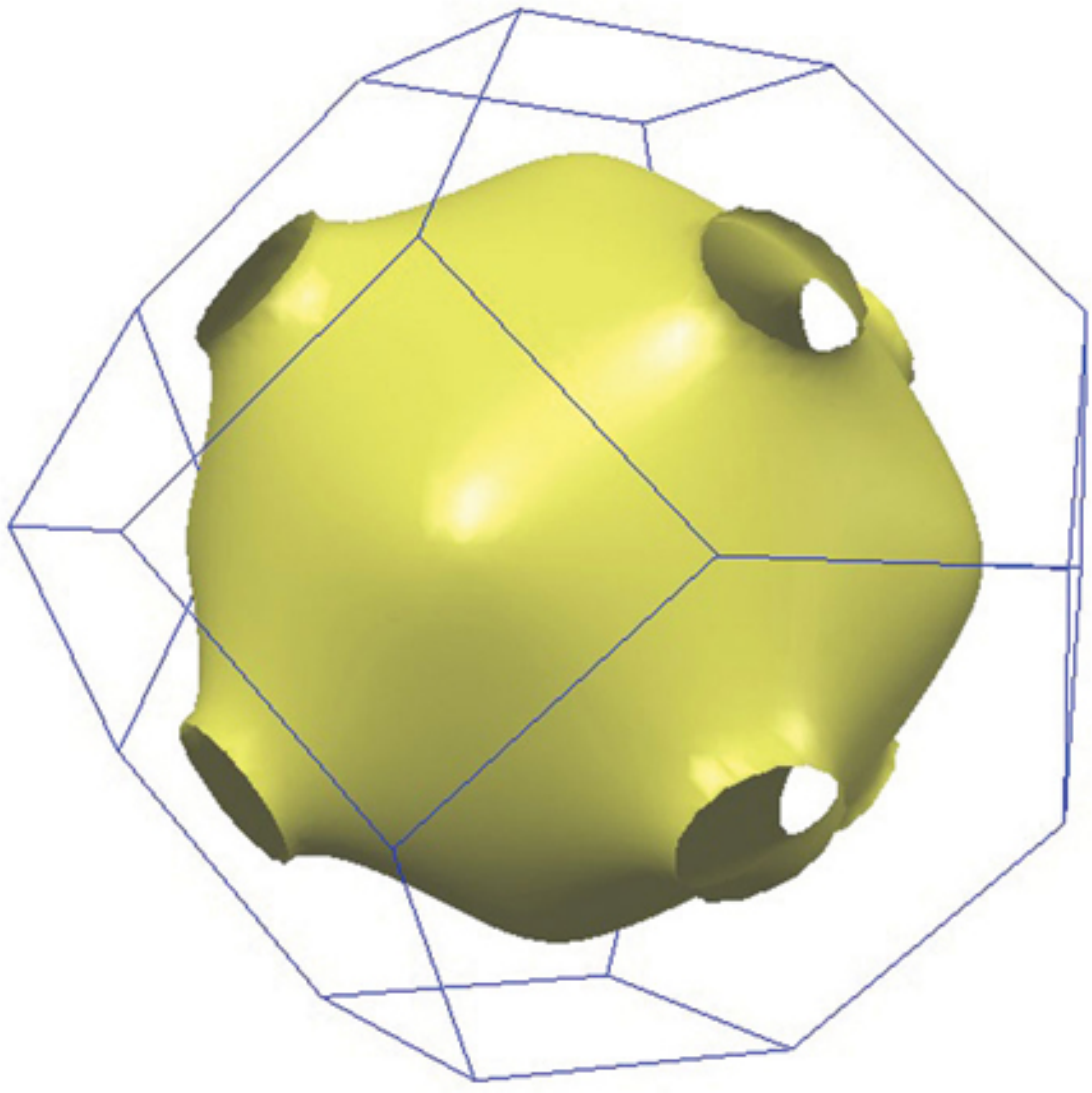

Emergent symmetries, Luttinger's theorem and 't Hooft anomalies in metals

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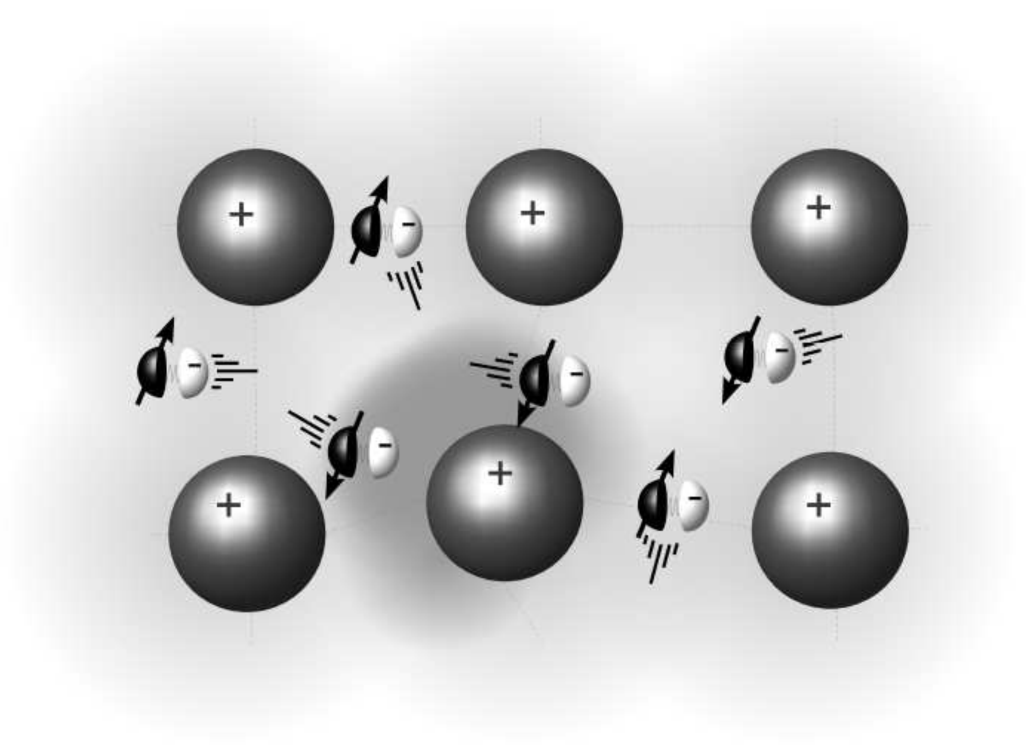
What is a metal?

A state of matter that conducts electricity.

In solids, this is due to mobile electrons that are liberated from the individual atoms.



Electrons in metals: a degenerate fermi fluid

Even room temperature is cold (T $\ll$ degeneracy temperature): electron motion is quantum rather than thermal.

Ingredients of the physics of metals:

A degenerate fluid of electrons that is affected by

- the periodic potential of ion cores (and their vibrations)
- Electron-electron coulomb repulsion
- impurities/other defects in the solid

Conventional metals: Fermi liquid theory

An ordinary metal, eg, Copper.

Effects of inter-electron Coulomb repulsion weakened
due to Pauli exclusion.

Low energy theory: ``Elementary particles''(a.k.a ``quasiparticles'')

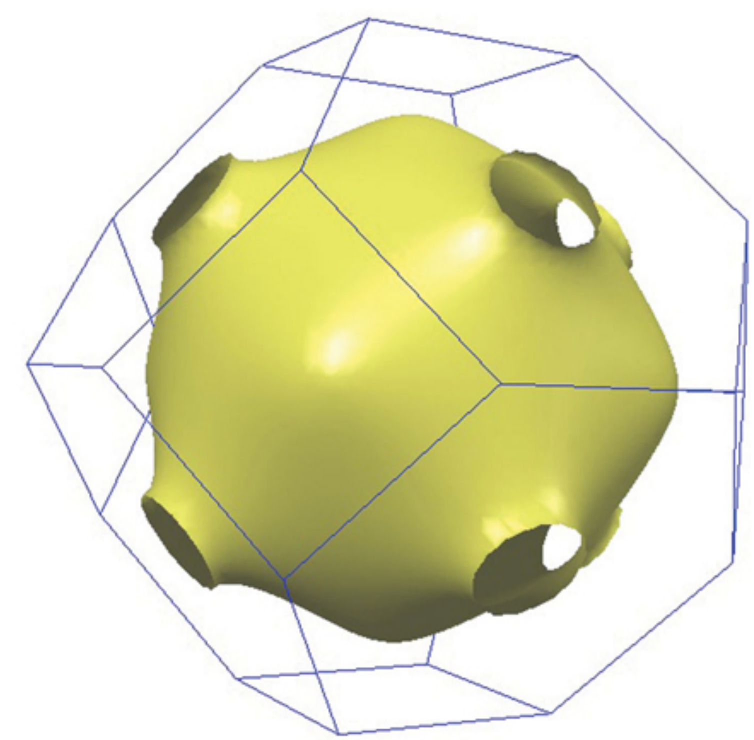
Fermions with electric charge e, spin-1/2 that fill up a Fermi sea in momentum space.

Highest occupied momenta form a Fermi surface.

Quasiparticles long lived near Fermi surface, and have well-defined energy-momentum relation.

Shape of Fermi surface: not a sphere.

``Landau Fermi Liquid Theory''



Filled states unavailable for scattering

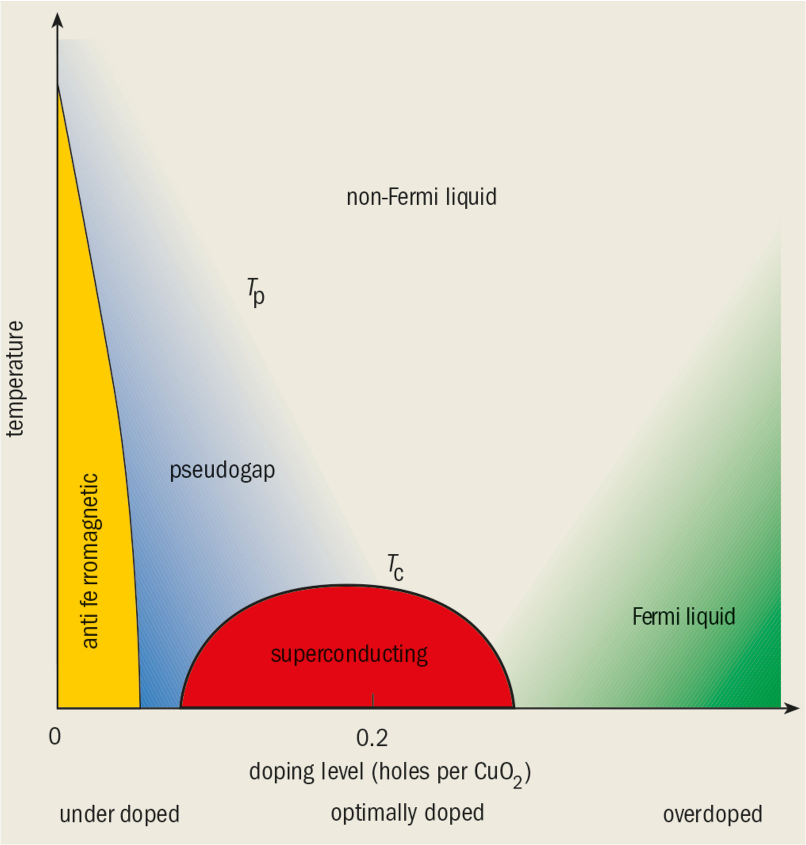
Strange non-fermi liquid metals

Last 30+ years: Many metals that violate Fermi liquid theory, some down to very low temperature

Prominent examples:

1. Parent metal of many high temperature superconductors

2. Several metals near the onset of magnetism



The mystery of strange metals

Grand challenge in contemporary physics:

How should we think about metals where the `quasiparticle concept' has broken down?

Nature of a `coarse-grained' low energy effective theory?

Expect such a theory will capture universal aspects of several strange metals irrespective of microscopic origins.

Strategy of these lectures

Discuss some very general (model-independent) properties of ***clean*** metals i.e focus on the idealized situation where there are no impurities.

Whether or not impurities are central to the essential physics of prominent strange metals, particularly their transport, is not a fully settled question.

Nevertheless it is interesting to see how much we can learn by ignoring the disorder.

We will see that any low energy theory of a metal in such a clean system obeys some severe constraints.

Plan

Lecture 1: Emergent symmetries, Luttinger's theorem and 't Hooft anomalies in metals

Lecture 2: Electrical transport in clean metals: general constraints and some routes to non-fermi liquid transport

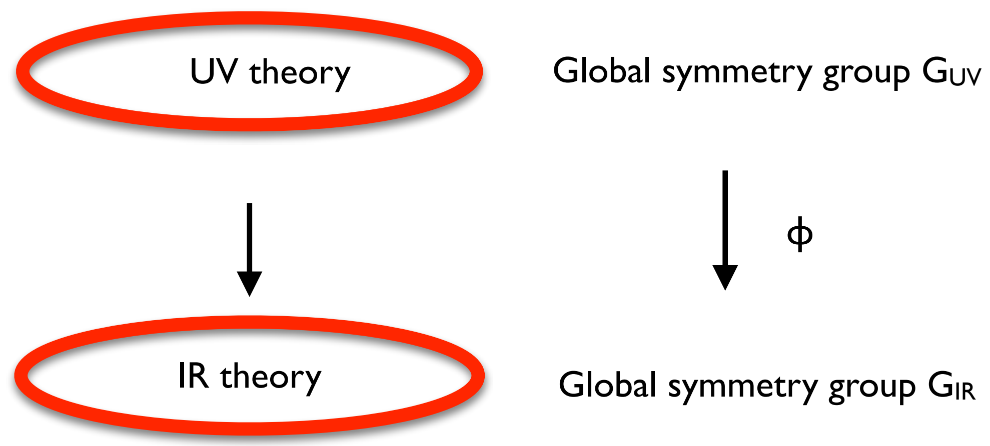
Global symmetry in quantum many body physics

I will be interested in situations where G_{UV} is not spontaneously broken in the IR.

Global symmetry in quantum many body physics

G_{IR} may be `bigger' than G_{UV} (the IR theory may have emergent symmetry).

G_{IR} may have a property known as 't Hooft anomaly (eg, chiral anomaly of massless fermions) which will be constrained by the UV theory.



't Hooft anomalies are `topological' properties of how symmetry is realized - they are robust to deformations within the same phase of matter.

Study of topological phases thus informs study of other non-topological phases of matter (such as non-fermi liquids)

The UV Global symmetry

I will consider UV systems with a global *internal U(1) symmetry* and *(lattice) translation symmetries* on a d-dimensional lattice.

(In condensed matter physics the global U(1) symmetry corresponds to electric charge conservation.)

I will not specify the Hamiltonian other than to require that it is `local' (i.e is a sum of operators that each act on local regions of space).

This includes almost all models of interest in standard discussions of strongly interacting electrons (eg, the Hubbard model and variants)

Compressible quantum matter

Let n (= electrical charge) be the generator of the global U(1) symmetry, and μ the corresponding chemical potential.

The compressibility $\kappa = \frac{d\langle n \rangle}{d\mu}$.

I will be interested in phases of matter where κ is non-zero.

Within such a phase the charge density can be tuned continuously.

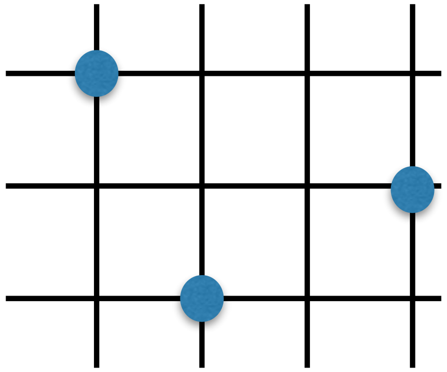
The classic example is a free fermi gas at a non-zero density.

The non-fermi liquid metals we eventually wish to understand are all compressible.

Lattice filling

With a global U(1) and lattice translation symmetries, we can define the lattice filling ν = average charge per unit cell.

In a compressible phase we can tune ν continuously.

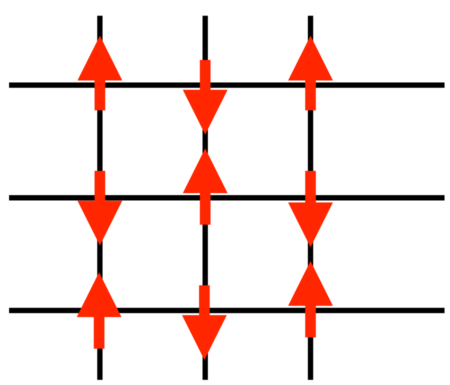


Lattice translations in the IR theory

Unit lattice translation in UV theory ~ infinitesimal translations in the IR theory

More precisely we should allow for action by an internal symmetry of the IR theory.

Example: Ising antiferromagnet



Tune to criticality →

IR - continuum ϕ^4 theory;
Unit lattice translation: $\phi \to -\phi$

There may be some exceptions to this if the IR does not involve spatial coarse-graining but we will set this subtlety aside as a future worry.

Constraints from the UV on the IR theory: a simple example

Assume IR theory is fully gapped, and is smoothly connected to a band insulator

Only possible if UV theory has lattice filling with ν even.

Note that this statement is independent of any Hamiltonian.

Constraints from the UV on the IR theory: A famous example

Luttinger's theorem in Fermi Liquids

Volume of Fermi surface fixed by electron filling: $$\frac{2V_F}{(2\pi)^d} = \nu \; mod \; Z$$

Luttinger (1960s): perturbative proof; Oshikawa (2000): nonperturbative argument

Also a ***Hamiltonian-independent statement*** so long as ground state is a Fermi liquid.

Revisiting Luttinger's theorem

In the next few slides I will revisit, from a modern viewpoint Luttinger's theorem:

Cast as a statement about the emergent symmetry and the property known as the 't Hooft anomaly of the low energy Fermi liquid theory

This viewpoint will allow us to generalize Luttinger's theorem to more general compressible phases, including non-fermi liquid metals.

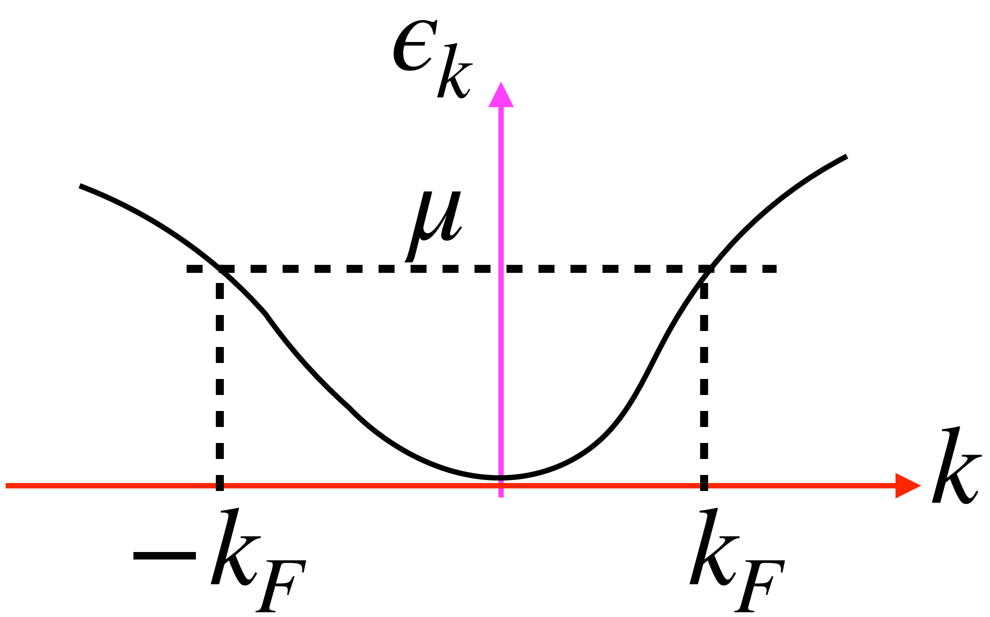
1d compressible matter:

emergent symmetry, chiral anomaly, and Luttinger's theorem

1d compressible matter

Free fermions at non-zero density in1d:

IR theory - massless Dirac fermion



Global symmetry U(1) x U(1)

Add interactions: marginal perturbation leading to a fixed line
(condensed matter physics: a.k.a Luttinger Liquid)

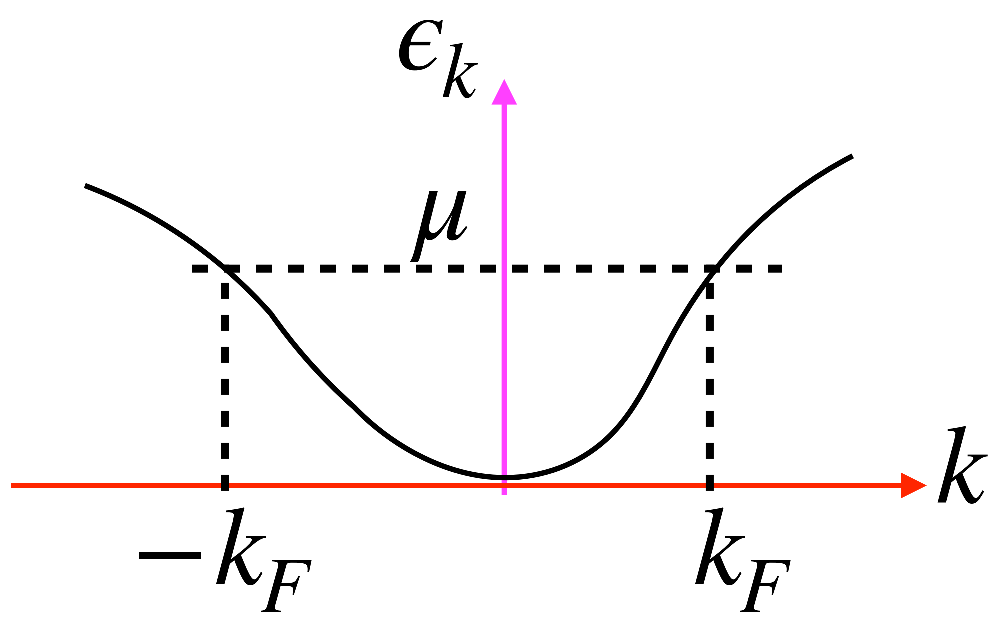
Preserves U(1) x U(1) symmetry.

1d compressible matter (cont'd)

Total charge $Q \sim n_L + n_R$

Total momentum(*) $P \sim k_F(n_R - n_L)$

(Embedding the G_{UV} into G_{IR})



IR global symmetry U(1) x U(1) is broken by external gauge fields, eg, turn on electric field E coupling to total charge.

$$\partial_\mu j_L^\mu = -E/2\pi$$

Chiral anomaly (example of t' Hooft anomaly)

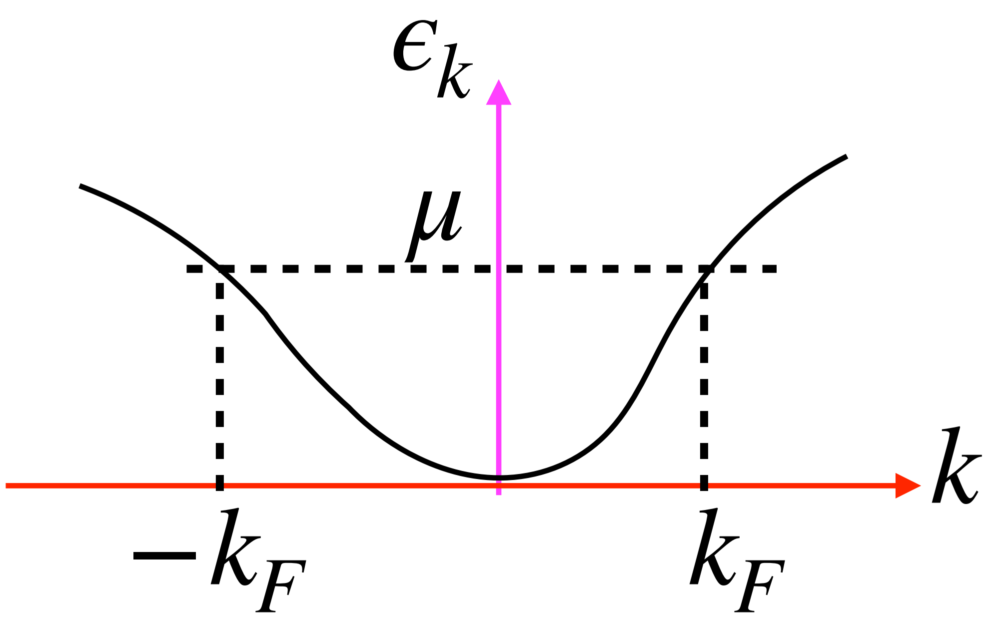
$$\partial_\mu j_R^\mu = E/2\pi$$

(*) For simplicity, assume continues translation symmetry in UV; argument can be extended if there is a lattice.

Chiral anomaly and Luttinger's theorem

Total charge $Q \sim n_L + n_R$

Total momentum $P \sim k_F(n_R - n_L)$



In original UV theory: $dP/dt = nE$

In IR theory: (from anomaly) $dP/dt = k_F d(n_R - n_L)/dt = k_F EL/\pi$
(L = length of system)

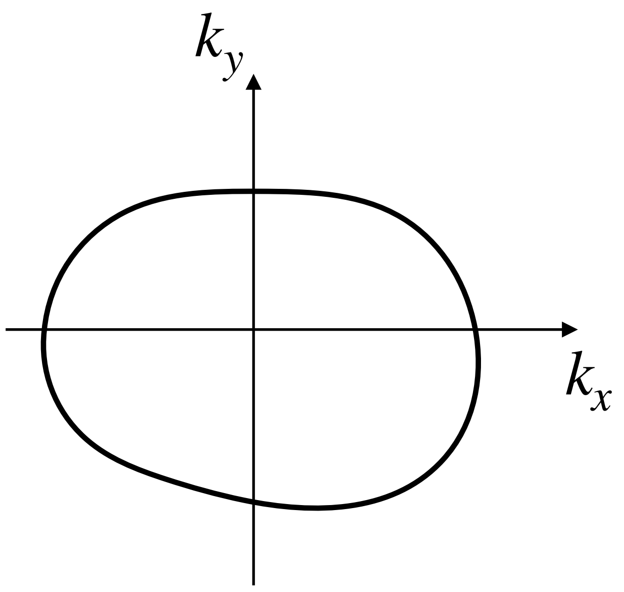
Comparing gives $k_F = \pi n/L$ which is Luttinger's theorem

2d Landau Fermi liquids

The Landau Fermi liquid in d = 2

G_{UV} = U(1) x lattice translations (= Z^2)

IR theory: Quasiparticles near a sharp Fermi surface



IR Hamiltonian:

$$H = \sum_k \epsilon_k n_k + 1/2 \sum_{k,k'} F_{kk'} n_k n_{k'}$$

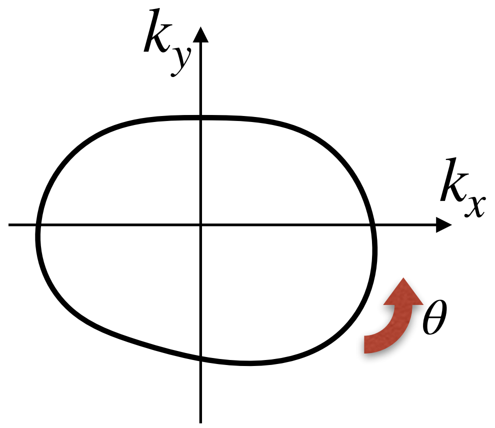
n_k = quasiparticle number at point k near Fermi surface

Emergent symmetry of the Fermi Liquid

``Quasiparticles are separately conserved for each Fermi surface point"

For each point on Fermi surface, there is a conserved charge density n_θ

$n_\theta d\theta$ is the number of quasiparticles between θ and $\theta + d\theta$



General IR symmetry element: $e^{i \int d\theta \, f(\theta) \, n_\theta}$ for smooth functions $f(\theta)$.

These define smooth maps from a circle to U(1) which form a group known as the `loop group' $\equiv LU(1)$ (identify as G_{IR})

Embedding microscopic symmetries

Total charge $n \sim \int d\theta \; n_\theta$

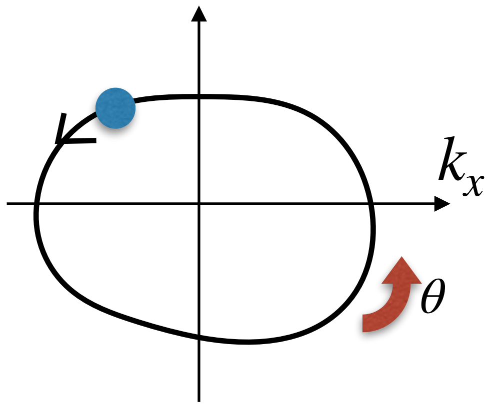
Unit lattice translations along $\alpha = (x, y)$ direction : $T_\alpha \sim e^{-i\int d\theta \; k_{F\alpha} \; n_\theta}$

(setting lattice constants to be 1).

Both U(1) and lattice translations of the UV map to elements of the LU(1) IR symmetry.

In fact we can take the action of translations to define the `Fermi momentum' in the IR theory.

The anomaly: a physical manifestation



Turn on external electromagnetic field

Separate conservation of n_θ destroyed - only total charge is conserved.

Example: External uniform magnetic field

$$\frac{d\vec{k}}{dt} = -\frac{d\epsilon}{d\vec{k}} \times \vec{B}$$

Semiclassical: quasiparticle moves around Fermi surface.

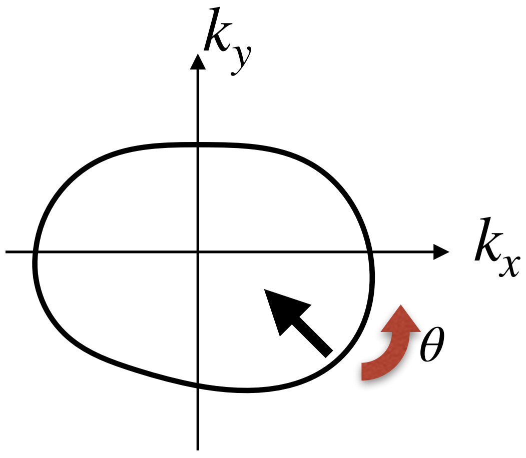
Fully quantum treatment?

A useful physical picture

The Fermi surface is the boundary of the rigid occupied Fermi sea.

In a magnetic field, the interior of the Fermi sea is rigid but there is a chiral `edge' state in momentum space.

Can in fact show that in a magnetic field the Fermi sea can be thought of as showing an integer quantum Hall state in momentum space.



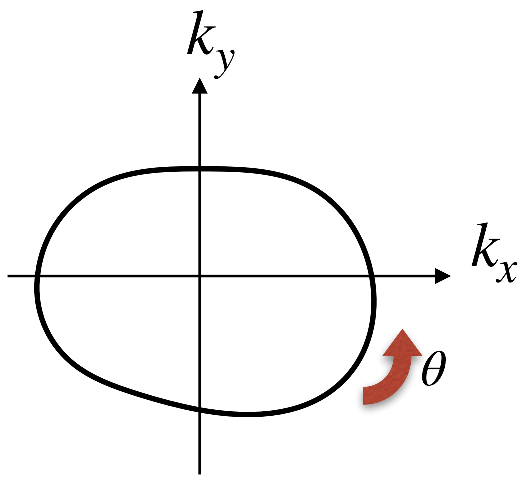
Formal manifestation of the anomaly

Else, Thorngren, TS, 2020

Turn on 2π flux of the electromagnetic field (A_x, A_y)

For the `chiral' k-space edge state at the Fermi surface the n_θ satisfy an algebra (familiar for integer quantum Hall edge states)

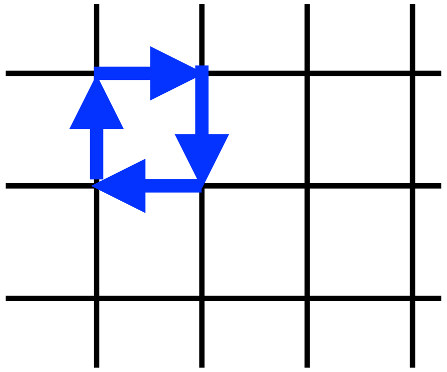
$$[n_\theta, n_{\theta'}] = -\frac{i}{2\pi}\delta'(\theta - \theta')$$



See also Nguyen and Son 2018, Barci, Fradkin, Ribeiro 2018

Luttinger's theorem from the anomaly

UV theory: With 2π flux, the discrete unit translations do not commute:



$$T_x T_y T_x^{-1} T_y^{-1} = e^{2\pi i \nu}$$

IR theory: Use $T_\alpha = e^{-i a_\alpha \int d\theta \; k_{F\alpha}(\theta) \; n_\theta}$

and the commutation algebra $[n_\theta, n_{\theta'}] = -\frac{i}{2\pi} \delta'(\theta - \theta')$

=> $T_x T_y T_x^{-1} T_y^{-1} = e^{i V_F a_x a_y / 2\pi}$

Matching these exactly gives Luttinger's theorem.

Comments

Apart from Luttinger's theorem, several (but not all) universal properties of the Fermi liquid follow just from knowing its emergent symmetry and anomaly.

Eg: response to electric fields, quantum oscillations,….

These `kinematic' properties must be distinguished from `dynamical' properties that require knowledge of details of the IR Hamiltonian, eg, the Fermi velocity.

Some ``advanced'' comments

A powerful way to think about 't Hooft anomalies:

Couple background gauge fields to G_{IR} .

Theory in D spacetime dimensions with t' Hooft anomaly:
Gauge invariance obtained by extending gauge field action to D+1 dimensions with a topological action related to a Symmetry Protected Topological (SPT) phase.

t'Hooft anomaly in D spacetime dimensions ↔ SPT phases in D+1 dimensions.

Anomaly of boundary theory canceled by `anomaly inflow' from higher dimension.

In these terms, how should we think about the 't Hooft anomaly of the 2d Fermi liquid?

Coupling a background gauge field to G_{IR}

t' Hooft anomaly signaled by breakdown of gauge invariance which is cured by extending the gauge fields to one higher dimension with a topological action.

For G_{IR} = loop group LU(1), the gauge field $A_\mu = (A_0, A_x, A_y, A_\theta)$ which are

all functions of of (t, x, y, θ).

Thus we have a four-dimensional gauge field.

(A_θ can be interpreted as a Berry connection on the Fermi surface).

Any anomaly will be related to a 5D topological action of a U(1) gauge field.

t’ Hooft anomaly of the Fermi liquid

Corresponding topological action: 5D Chern-Simons theory

$$S[A] = \frac{m}{24\pi^2} \int A \wedge dA \wedge dA \qquad \text{with } m \in Z$$

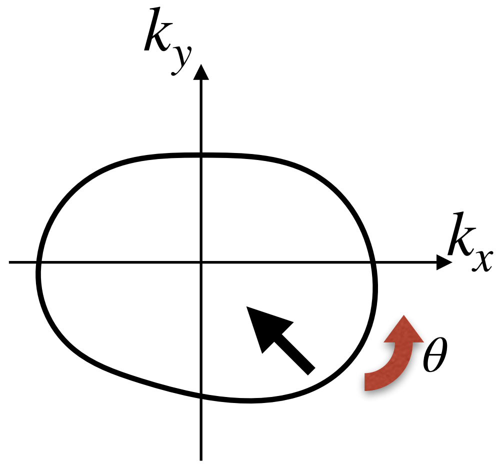
Claim: $m = \pm 1$ correctly captures physics of the Fermi liquid.

A useful physical picture

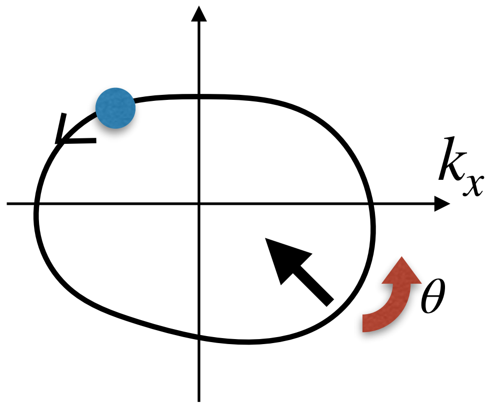
Interpret the 5th dimension as going into the interior of the Fermi surface.

We can think of the 5D U(1) gauge field as living in 2 space + 1 time + 2 momentum directions.

The Fermi surface is a boundary of the rigid occupied Fermi sea.



An interesting point of view



Turn on 2π flux of the electromagnetic field (A_x, A_y).

5D Chern-Simons term reduces to a 3D Chern-Simons for the remaining 3 components.

This can be thought of as an `integer quantum Hall effect' in momentum space in the interior of the Fermi surface.

The Fermi surface is the boundary of this momentum space integer

quantum Hall state, and hence has a chiral fermion.

Constraints on more general compressible phases
(not necessarily Fermi liquids)

Generalized Luttinger Theorem

Else, Thorngren, TS, 2020

Theorem: For any irrational ν in d > 1 with G_{UV} = U(1) x lattice translations, the IR theory must have a large emergent internal symmetry G_{IR} (precise statement: cannot be a compact Lie group).

Further G_{IR} must have a precise anomaly that is related to the lattice filling.

Fermi liquids satisfy this by way of

(1) infinite dimensional emergent symmetry

(2) anomaly of this emergent symmetry <=> Luttinger's theorem

Beyond Fermi liquids: a simple possibility

Emergent ***internal symmetry*** of a class of non-fermi liquid metals (with G_{UV} = U(1) x lattice translations):

- Infinite dimensional emergent continuous symmetry - same as Fermi liquid or some variant thereof

``Ersatz Fermi Liquids"

Many examples in literature

Other more exotic structure of emergent symmetries may be possible but the class above is already very rich and poorly understood.

Ersatz Fermi liquids

Even if G_{IR} and the anomaly are the same as in the fermi liquid, detailed (universal) dynamical properties can be very different.

Only `kinematic’ properties will be the same.

Infinite dimensional continuous symmetry => infinite number of emergent conserved quantities.

Very strong implications for transport and other dynamical properties in such non-fermi liquid metals.

Conclusions/plan for next lecture

Luttinger's theorem of Fermi liquids: view through the lens of the emergent IR symmetry and its anomaly.

Generalization to other compressible phases: emergence of a large continuous IR symmetry with a specific anomaly to match filling constraint.

Next talk: Role of symmetry in transport in a clean metal

``Electrical transport in clean metals: general constraints and some routes to non-fermi liquid transport"