

Semi-abelian gauge theories, non-invertible symmetry & string tension beyond N-ality.

Mithat Ünsal, NCSU.

with Mendel Nguyen, NCSU

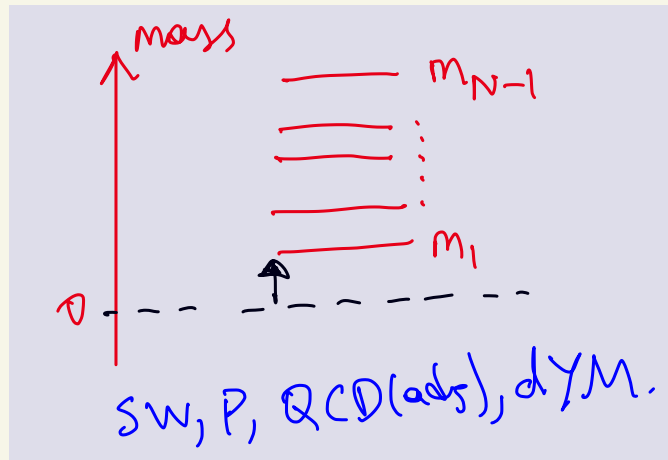
Yuya Tanizaki, Yukawa Institute, Kyoto.

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MOTIVATION

- There are a handful of QFTs in 3d & 4d where confinement and mass gap generation can be understood analytically.
- Polyakov model on $\mathbb{R}^3$ (75)
Seiberg-Witten on $\mathbb{R}^4$ (94)
QCD(adj) on $\mathbb{R}^3 \times S^1$ (07)
deformed Yang-Mills on $\mathbb{R}^3 \times S^1$ (08).
- In all of the above; $SU(N) \longrightarrow U(1)^{N-1}$ due to either an adjoint Higgs vev (algebra valued) or gauge holonomy (group valued).
"Dynamical abelianization"
- In all; $S_N =$ Weyl group of $SU(N)$ is Higgsed.
 $\hookrightarrow$ permutation group.
- Higgsing of S_N pervades the physics of these systems & renders them quite distinct from pure YM.

- Spectrum of fundamental string tensions. $N-1$ types in SW & Polyakov. (see Douglas, Shenker 95).
- Spectrum of "glueball" masses. $N-1$ distinct dual photon masses in all abelianizing theories.



while in YM.

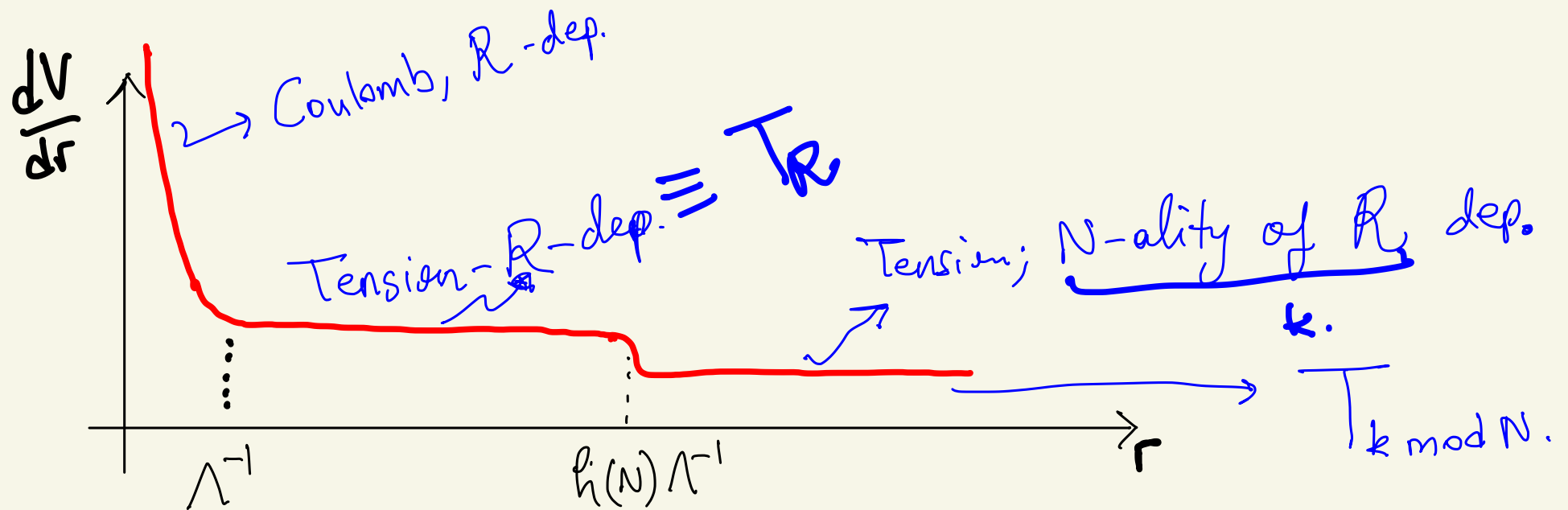


- Splitting to $N-1 \Rightarrow$ Imprint of broken S_N .
- Tensions dictated by charges $w \in \Gamma_w$ & not N -ality! (sometimes presented as deficiency. Not so, it is a feature.)
(weight lattice)

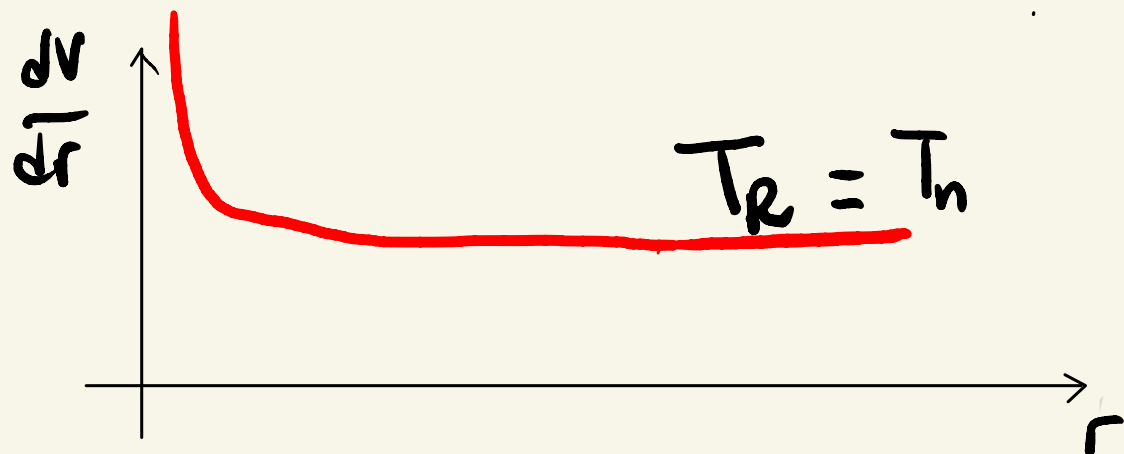
CURIOUS FACT ABOUT STRING TENSION IN YM.

- In YM or QCD with adjoint matter, we are usually told that tensions are dictated by N -ality.
- However, the real story is actually more interesting.

$$\psi_R \bullet \xleftarrow{r} \bullet \overline{\psi}_R \rightarrow \text{representation}$$

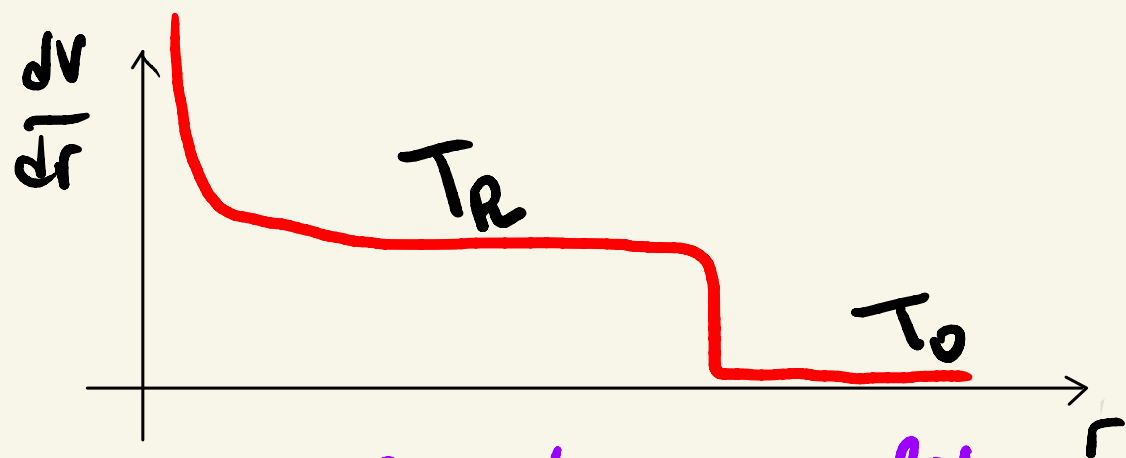


based on Bali (93), de Forcrand, Philipsen (99), Wipf et al. (10), Greensite (03),
(not widely appreciated)

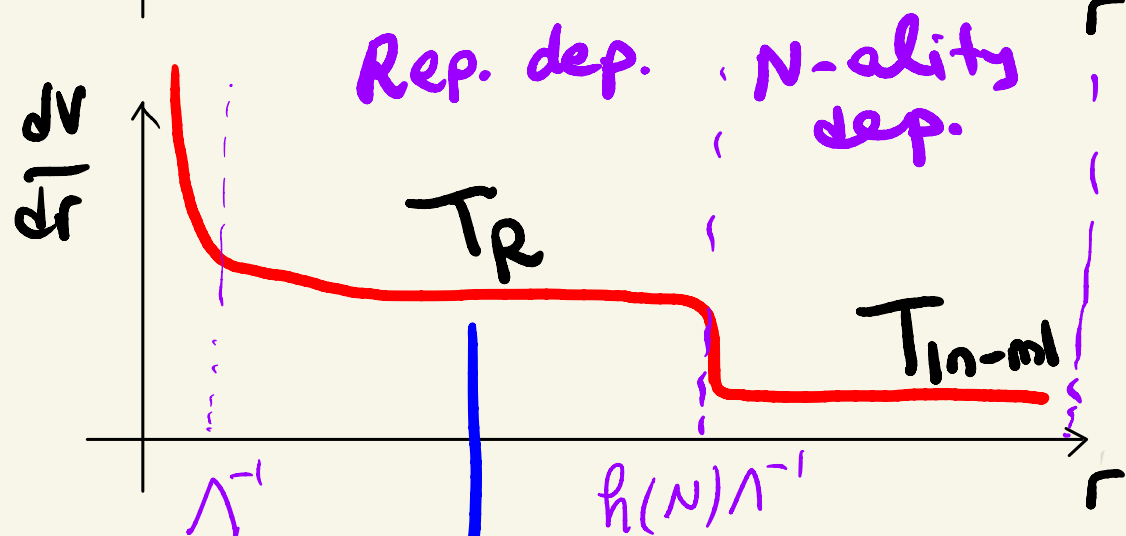


$$R = F, F \otimes F, \dots, \underbrace{F \otimes \dots \otimes F}_n$$

$$n \leq \lfloor \frac{N}{2} \rfloor$$



$$R = \text{ad}_J, \text{ad}_J \otimes \text{ad}_J, \dots$$



$$R = \underbrace{F \otimes \dots \otimes F}_n \otimes \underbrace{\bar{F} \otimes \dots \otimes \bar{F}}_m$$

WHY?

DEFINITION OF SEMI-ABELIAN THEORY.

- We construct two interesting gauge theories.
 - $U(1)^{N-1}$ theory with global non-abelian discrete sym. S_N
 - $U(1)^{N-1} \rtimes S_N$ semi-abelian g.t. obtained by gauging S_N . (coupling to S_N TQFT)
- Semi-direct product.

Abelianizing theories
 $SU(N) \rightarrow U(1)^{N-1}$
 S_N : Higgsed

Semi-abelian
 $U(1)^{N-1} \rtimes S_N$
 S_N : not Higgsed

Non-abelian
 $SU(N)$ theories.

S_N : not Higgsed

- Many properties of semi-abelian theory are much closer to pure YM than abelianizing theories.

3d LGT

- Wilson formulation
 - Villain formulation
- } of 3d $U(1)^{N-1}$ Lattice g.t. (LGT)
- Equivalent at weak coupling.
 - Villain more convenient for exact dualities.
 - Wilson better for gauging S_N .
- We work with 3d LGT; very concrete and simple illustration of some general ideas.

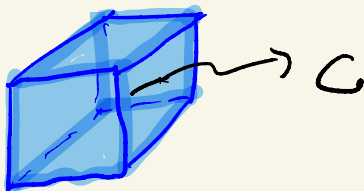
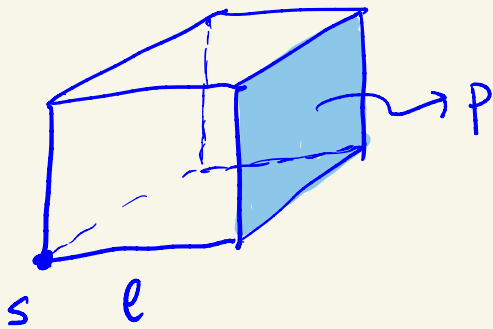
Differential forms and dual lattice.

Λ_3 : Original lattice

$C(r)$: r -cells, $r=0,1,2,3$

site, link, plaquette, cube.

$\{s, l, p, c\}$
 $\parallel$
 $\times$



$\tilde{\Lambda}_3$: Dual lattice.

$\tilde{c}(r)$

$$\star C^{(r)} = \tilde{c}^{(d-r)}$$

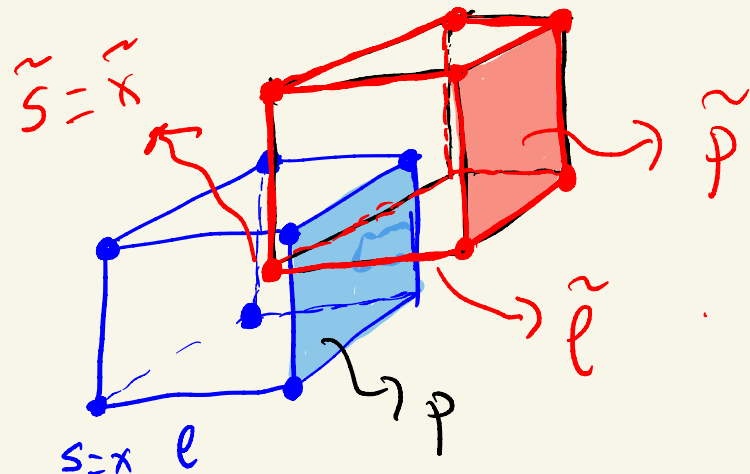
$$\star s = \tilde{c}$$

$$\star l = \tilde{p}$$

$$\star p = \tilde{l}$$

$$\star c = \tilde{s} = \tilde{x} = x + \frac{1}{2}(\hat{1} + \hat{2} + \hat{3})$$

shifted by half.



Wilson formulation

$$S_W = \beta \sum_P \sum_{i=1}^N (1 - \cos f_P^i) - i \sum_e v_e \sum_{i=1}^N a_e^i ;$$

$$f_P^i = (da^i)_P ; \quad a_e^i \in [0, 2\pi)$$

Lagrange multiplier.
produce constraint $\sum_{i=1}^N a_e^i = 0$.

- Constraint tells us that only $N-1$ photons are physical.

- Global S_N symmetry manifest.

$$(a_e^1 \dots a_e^N) \xrightarrow{P \in S_N} (a_e^{P(1)}, \dots, a_e^{P(N)})$$

- Pure gauge theories studied to date are not equipped with non-abelian global symmetries.
- This is the first example of this new class, and opens a new window. Wait & see, especially when we gauge S_N .

Villain formulation.

$$S = \frac{1}{4\pi e^2} \sum_p (F_p + 2\pi n_p)^2, \quad F_p = (dA)_p$$

A_e valued in $\mathbb{R}^{N-1}$

n_p valued in $\Gamma_r \subset \mathbb{R}^{N-1}$

Γ_r : Root lattice of $SU(N)$.

0-form gauge invariance: $A_e \rightarrow A_e + (d\lambda)_e$ $\lambda_s \in \mathbb{R}^{N-1}$
1-form gauge inv. $A_e \rightarrow A_e + 2\pi\beta_e$; $n_p \rightarrow n_p - (d\beta)_p$, $\beta_e \in \Gamma_r$.

• Since $\mathbb{R}^{N-1} / 2\pi\Gamma_r \simeq U(1)^{N-1}$; this also defines
a $U(1)^{N-1}$ LGT.

$$\mathbb{Z} = \sum_{\{n_p \in \Gamma_r\}} \int_{\mathbb{R}^{N-1}} [dA_e] e^{-S}$$

• Action in Villain form; can be exactly dualized.

Global symmetries

- 1-form symmetry: $A_e \rightarrow A_e + \Theta_e$; $\Theta_e \in \mathbb{R}^{N-1}$ & $(d\Theta)_p = 0$.
- Discrete non-abelian 0-form symmetry:

$$A_e \rightarrow \pi A_e \quad ; \quad n_p \rightarrow \pi n_p$$

II: $O(N-1)$ transformations that preserve root lattice Γ .

Weyl group of $SU(N)$, S_N & $\mathbb{Z}_2$ charge conj.

$$A_e \rightarrow A_e - \alpha(\alpha \cdot A_e)$$

$$A_e \rightarrow -A_e$$

$$n_p \rightarrow n_p - \alpha(\alpha \cdot n_p)$$

$$n_p \rightarrow -n_p$$

$$G^{[0]} = \begin{cases} S_N \times \mathbb{Z}_2 & N \geq 3 \\ \mathbb{Z}_2 & N=2 \end{cases}$$

$$G^{[1]} = \{U(1)^{N-1}\}$$

Weyl reflections: $S_\alpha(v) = v - 2(\alpha \cdot v)\alpha$

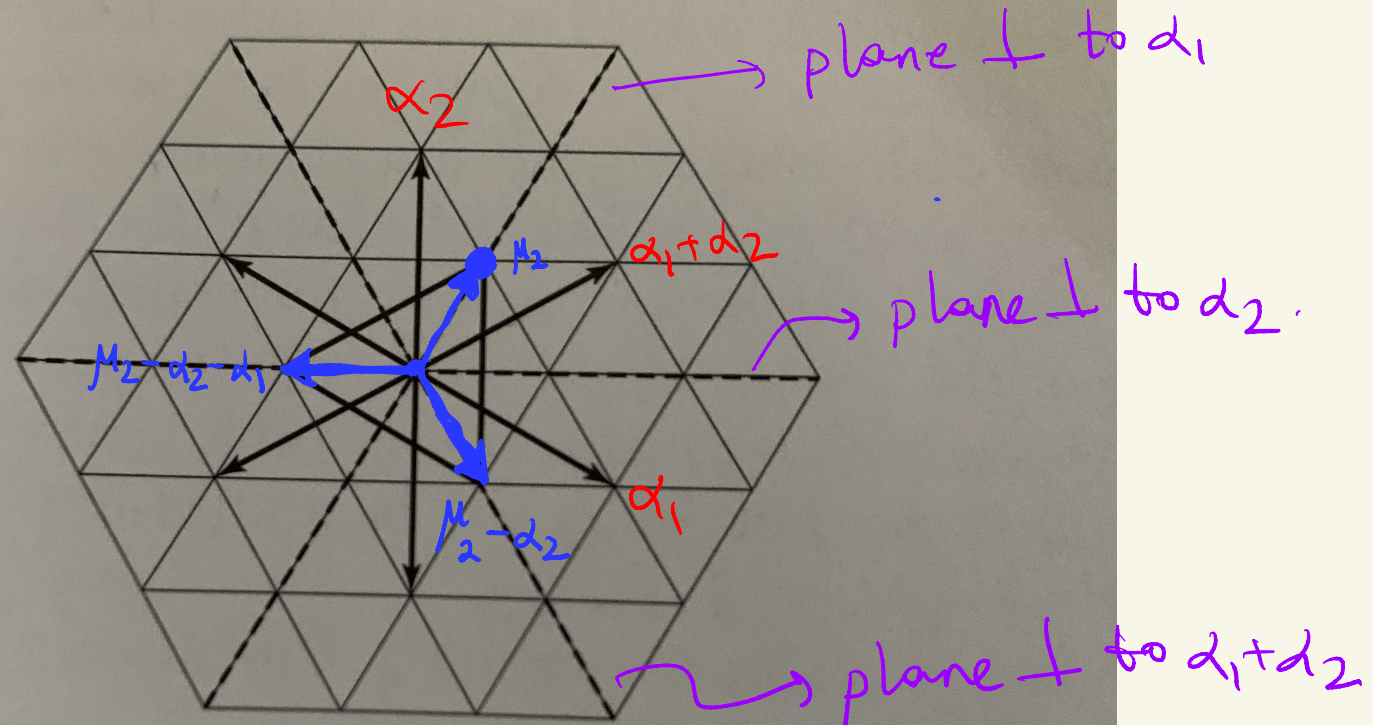
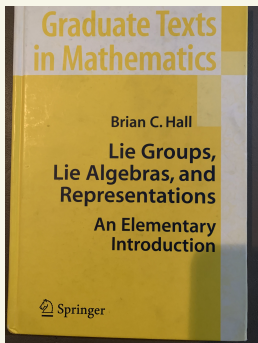


Fig. 5.3. Weyl group for $\mathfrak{sl}(3; \mathbb{C})$

$$S_{\alpha_2}(\mu_2) = \mu_2 - \alpha_2$$

$$S_{\alpha_1}(\mu_2 - \alpha_2) = \mu_2 - \alpha_2 - \alpha_1$$

Weyl reflections: $S_{\alpha}(v) = v - \alpha(\alpha \cdot v)\alpha$

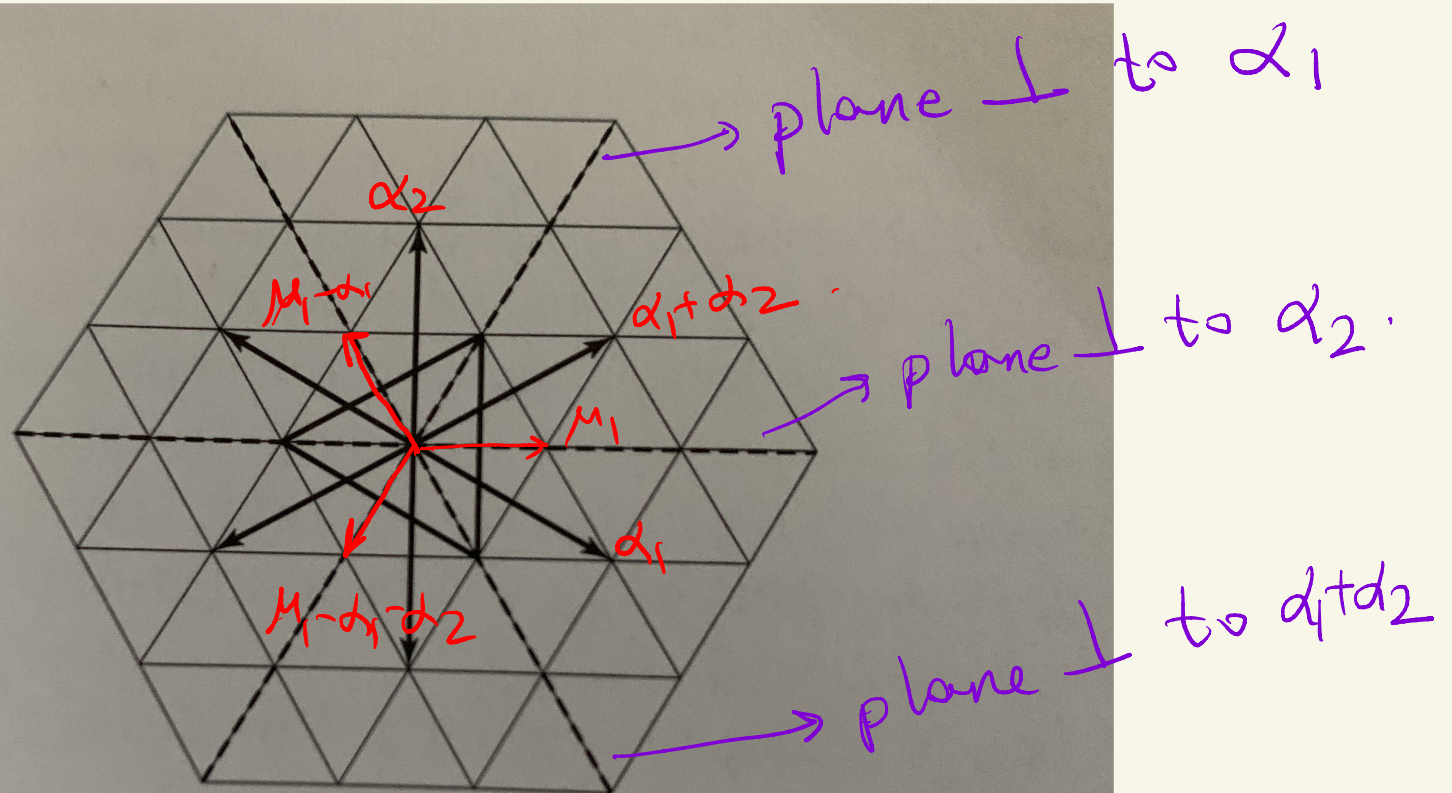


Fig. 5.3. Weyl group for $\mathfrak{sl}(3; \mathbb{C})$

μ_1

$$S_{\alpha_1}(\mu_1) = \mu_1 - \alpha_1$$

$$S_{\alpha_2}(\mu_1 - \alpha_1) = \mu_1 - \alpha_1 - \alpha_2$$

Observables.

- Wilson lines. $W_w(c) = e^{i \int_c w \cdot A}$

Invariance under 1-form gauge transformation $\Rightarrow w \in \underline{\Gamma_w}$
weight lattice.

- Transformation under global symmetries.

$$G^{[0]}: W_w(c) \mapsto W_{\pi^{-1}w}(c).$$

$$G^{[1]}: W_w(c) \mapsto W_w(c) e^{i \int_c w \cdot \theta}$$

hence contractible loops are invariant under $G^{[1]}$ and non-contractible loops transform.

Villain $\Rightarrow \Gamma_w$ -ferromagnet $\Rightarrow$ Magnetic Coulomb gas.

- Poisson resummation.

$$\sum_{n_p \in \Gamma_r} e^{-\frac{1}{4\pi e^2} (F_p + 2\pi n_p)^2} = \sum_{k_p \in \Gamma_w} e^{-\pi e^2 k_p^2 + i k_p \cdot F_p}$$

- Integrate out A_e exactly. $\Rightarrow (d^\dagger k)_e = 0. \Rightarrow$
 $(\star k)_e = (dm)_e$ where $m_{\tilde{x}}$ is Γ_w -valued
 scalar on the dual lattice $\tilde{\Lambda}_3$.

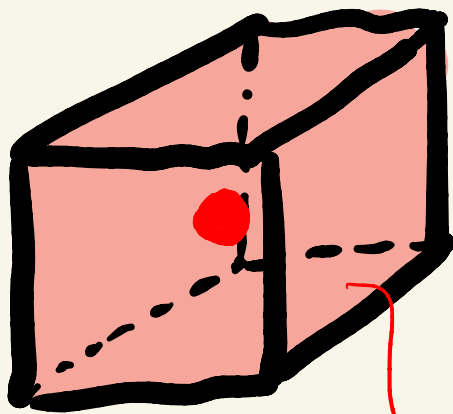
$$\mathcal{Z} = \sum_{\{m_{\tilde{x}} \in \Gamma_w\}} e^{-\pi e^2 \sum_{\tilde{e}} (dm)_{\tilde{e}}^2}$$

Γ_w -ferro.
 Exact dual
 of Villain.

- Let us replace $m(\tilde{x})$ with a continuous field. Using another Poisson resummation;

$$\sum_{m(\tilde{x}) \in \Gamma_w} \delta(\sigma(\tilde{x}) - 2\pi m(\tilde{x})) = \sum_{q(\tilde{x}) \in \Gamma_r} e^{i q(\tilde{x}) \cdot \sigma(\tilde{x})}$$

$$\mathbb{Z} = \int [d\sigma(\tilde{x})] \sum_{\{q(\tilde{x}) \in \Gamma_r\}} e^{-\frac{e^2}{4\pi} \sum_{\tilde{x}} (\partial_\mu \sigma(\tilde{x}))^2 + i \sum_{\tilde{x}} q(\tilde{x}) \sigma(\tilde{x})}$$



$$q(\tilde{x}) = \oint n_p \in \Gamma_r$$

faces, center $\tilde{x}$

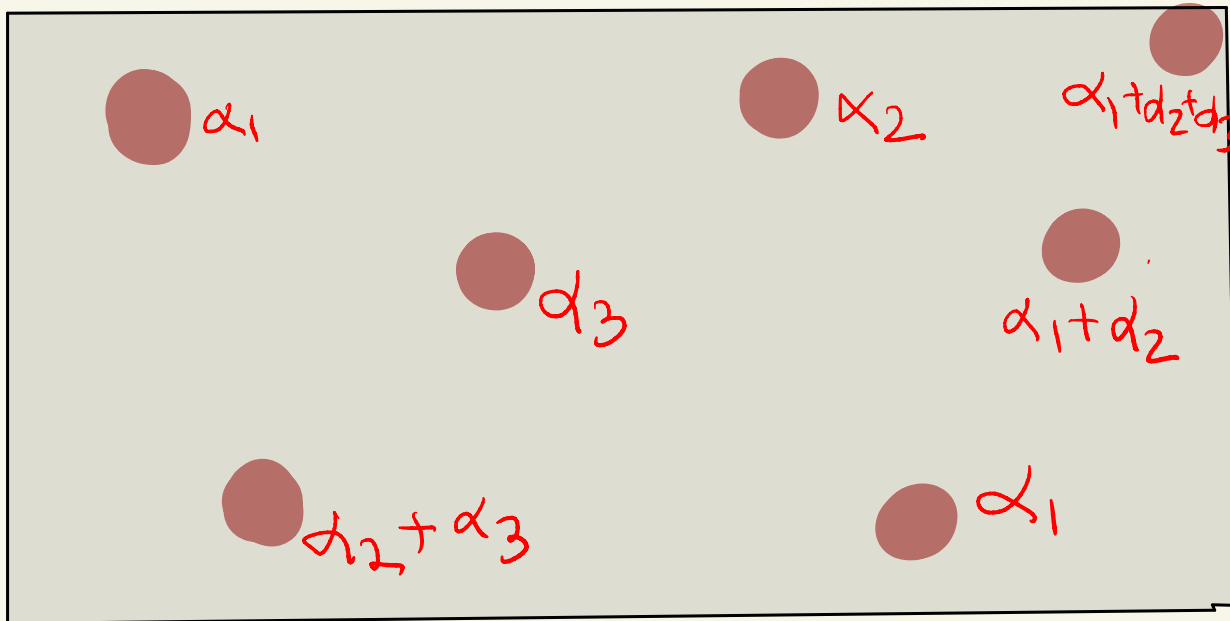
→ Magnetic charge.

→ Gauss surface.

- Integrating out $\sigma(\tilde{x})$ exactly; we reach to Coulomb gas representation.

$$Z = \sum_{\{q(\tilde{x}) \in \Gamma_r\}} e^{-\frac{\pi}{e^2} \sum_{\tilde{x}, \tilde{x}'} v(\tilde{x} - \tilde{x}') q(\tilde{x}) \cdot q(\tilde{x}')}$$

- Euclidean Vacuum: Proliferation of monopoles, but very different from Polyakov model & deformed YM due to unbroken S_N .



EFT.

- $$\mathcal{L} = -\frac{e^2}{4\pi} \sum_{\tilde{x}} \sigma(\tilde{x}) \Delta \sigma(\tilde{x}) + 2e^{-I} \sum_{\tilde{x}} \sum_{\alpha \in \Phi} \cos(\alpha \cdot \sigma(\tilde{x}))$$

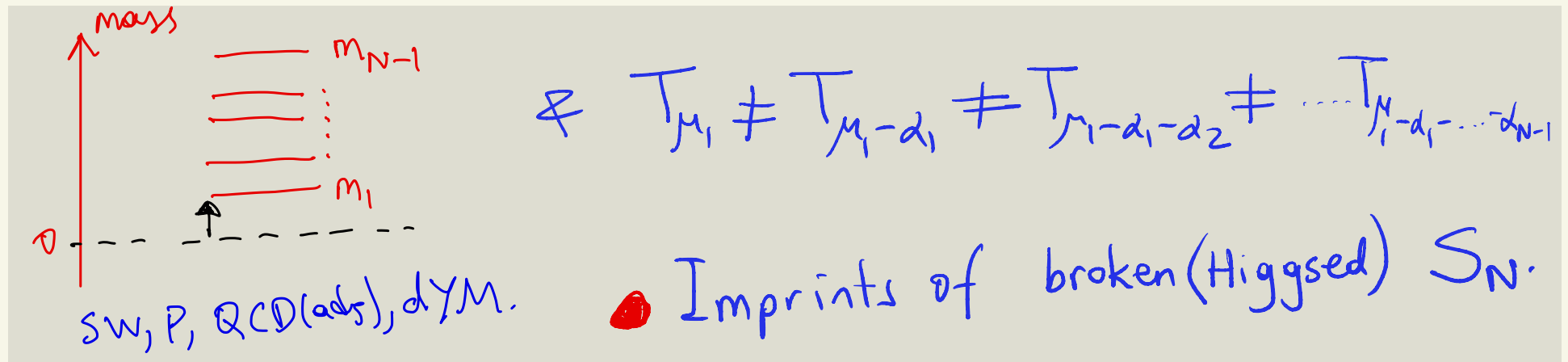
Φ : all $N^2 - N$ roots of $SU(N)$ algebra. $\alpha \in \text{Adj}(SU(N))$
 All on the same footing due to unbroken S_N .

- In Polyakov model; the action are hierarchical.
 Our construction is more egalitarian.

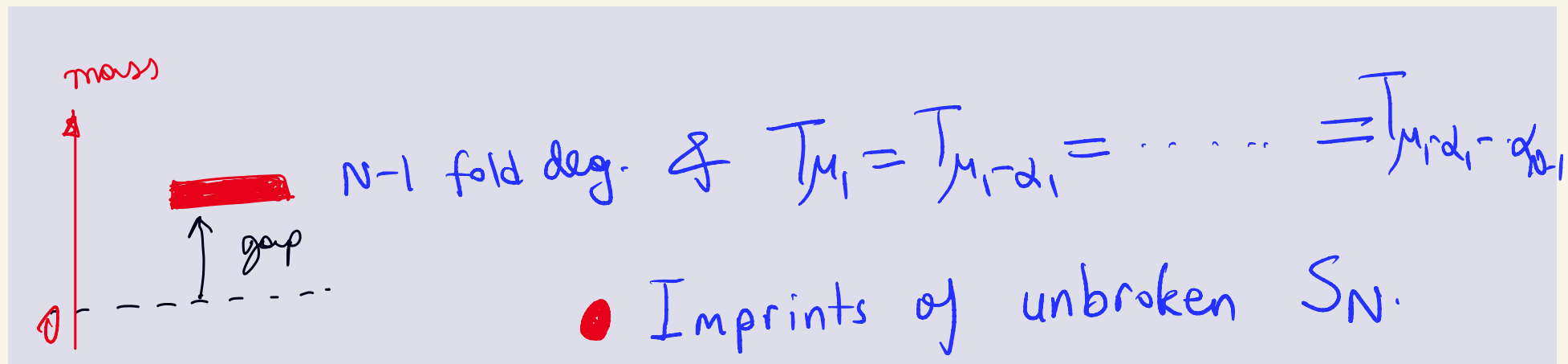
$SU(N) \rightarrow U(1)^{N-1}$ (the best we can do is: Simple roots in Δ_0 .

$$\begin{array}{ll}
 \alpha_1, \alpha_2, \dots, \alpha_{N-1} & : e^{-S_0} \\
 \alpha_1 + \alpha_2, \alpha_2 + \alpha_3, \dots, \alpha_{N-1} + \alpha_N & : e^{-2S_0} \\
 & : e^{-3S_0} \\
 \alpha_1 + \alpha_2 + \alpha_3, \dots & : \\
 \vdots & \\
 \alpha_1 + \dots + \alpha_{N-1} & : e^{-(N-1)S_0}
 \end{array}$$

- In Polyakov; there are $N-1$ dual photon mass & $N-1$ fundamental string tension. (same in SW $N=2$)



- In $U(1)^{N-1}$ LGT with S_N .



Wilson loops & string tensions.

$w \in \Gamma_w$ charge.

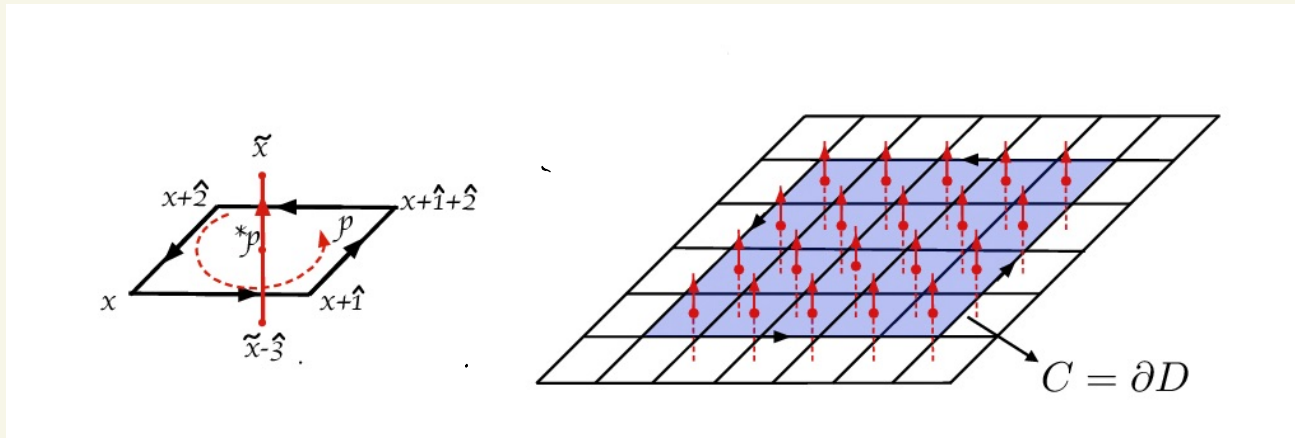
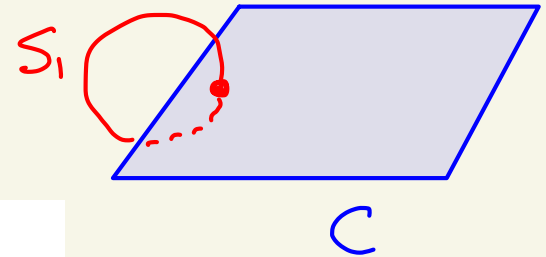
$C = \partial D$ contractible loop.

$$W_w(C) = e^{i \oint_C w \cdot A}$$

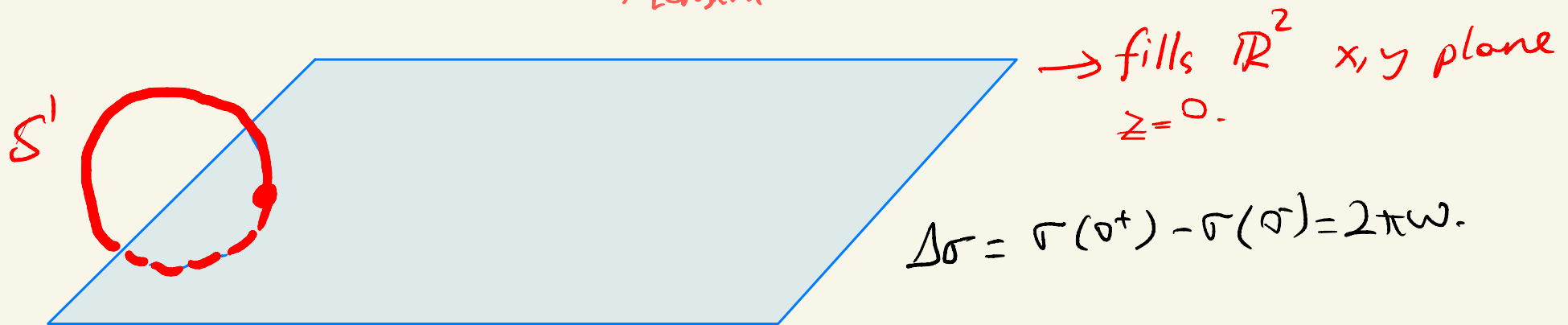
$$W_w(C) = e^{i \int_D w \cdot F} = e^{i \sum_{p \in \Lambda_2} [D]_p (w F_p)}$$

$$W_w(C) = \int \mathcal{D}\sigma \quad e^{-\frac{e^2}{2\pi} \int d^3x \left\{ \frac{1}{2} |d\sigma - 2\pi w [D]|^2 + M^2 \sum_{\alpha \in \phi^\dagger} (1 - w \alpha \sigma) \right\}}$$

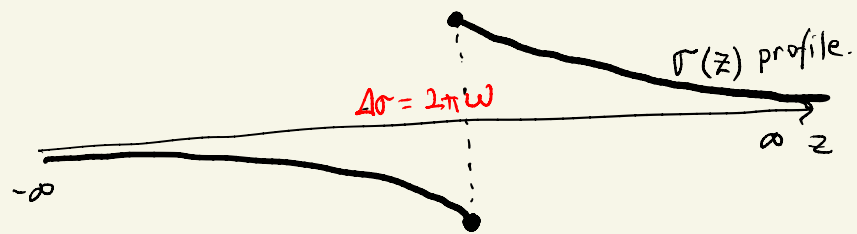
$W_w(C)$: Defect operator in dual formulation.
Delete C ; restrict path integral to config.
 $\oint_S d\sigma = 2\pi w$



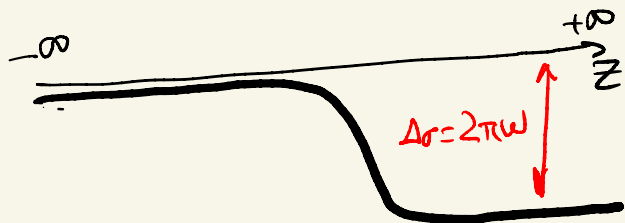
- $W_w(c) \sim e^{-T_w \text{Area}[D]}$.
 $\hookrightarrow$ tension $\hookrightarrow$ Area law.



$$\Delta\sigma = \sigma(\sigma^+) - \sigma(\sigma^-) = 2\pi w.$$



- Action of $\sigma(z) =$
 $S = \text{Area}[D] \times T_w$
 where T_w is instanton action
 in reduced QM system.
- $A[D]$ is area law.



$$T_w = \min_{\sigma(z)} \frac{e^2}{2\pi} \int_{-\infty}^{+\infty} dz \left(\frac{1}{2} \left(\frac{d\sigma}{dz} \right)^2 \right) + M^2 \sum_{\alpha \in \Phi^+} (1 - \cos \alpha \cdot r)$$

• One can calculate tensions with a plausible ansatz, $\omega\sigma(z)$.
The main point is if $\omega_1 \neq \omega_2$; $\omega \in \Gamma_w$; the tensions are typically different.

• Let $\mu_1, 2\mu_1, \mu_2, \alpha \in \Gamma_w$ denote highest weight of F, S, AS, adj rep $su(N)$ respectively.

T_{μ_1} : minimal tension

$$T_{2\mu_1} = 2 T_1$$

$$T_{\mu_2} = \frac{2(N-2)}{N-1} T_{\mu_1}$$

$$T_{\alpha} = 2 T_1 \quad \text{etc.}$$

• Fairly obviously; there is no $\omega \in \Gamma_w$ for which tension vanishes. This simple fact will be important later.

Gauging S_N is good.

- Clearly, properties of our abelian theory with S_N global symmetry resembles to YM more than dynamically abelianizing theories.
- But global symmetries are different from YM.

$$\begin{array}{l} \hline U(1)^{N-1} \text{ with global } S_N \\ \hline G^{[0]} : S^N \rtimes \mathbb{Z}_2 \rightarrow \text{charge conj.} \\ G^{[1]} : U(1)^{N-1} \end{array}$$

$$\begin{array}{l} \hline SU(N) \text{ YM} \\ \hline G^{[0]} : \mathbb{Z}_2 \rightarrow \text{charge conj.} \\ G^{[1]} = \mathbb{Z}_N^{[1]} \end{array}$$

- Let us gauge S_N and construct $U(1)^{N-1} \rtimes S_N$ theory. Then, we will show that

$$\begin{array}{l} \hline U(1)^{N-1} \rtimes S_N \\ \hline G^{[0]} : \mathbb{Z}_2 \\ G^{[1]} : \mathbb{Z}_N \end{array}$$

Semi-abelian gauge theory.

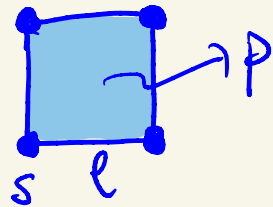
- Start with Wilson formulation. and gauge S_N .
- Embed elements of $U(1)^{N-1} \rtimes S_N$ inside $SU(N)$ as

$$P \cdot C \in SU(N) ; \quad C = (e^{ia_1}, \dots, e^{ia_N})$$

$P \in S_N$: $N \times N$ matrix rep. of Weyl reflection.

$$(P_1 \cdot C_1) (P_2 \cdot C_2) = \underbrace{P_1 P_2}_{\in S_N} \cdot \underbrace{(P_2^{-1} C_1 P_2 C_2)}_{\in U(1)^{N-1}}$$

- Let $(P_e \cdot C_e) \in SU(N)$ denote link variable.



$$S = \sum_P \beta_1 \left(\text{tr} \left(1_N - \prod_{(e \in \partial P)} (P_e \cdot C_e) \right) \right) + \underbrace{\sum_P \beta_2 \text{tr} \left(1_N - \prod_{e \in \partial P} P_e \right)}_{S_N}$$

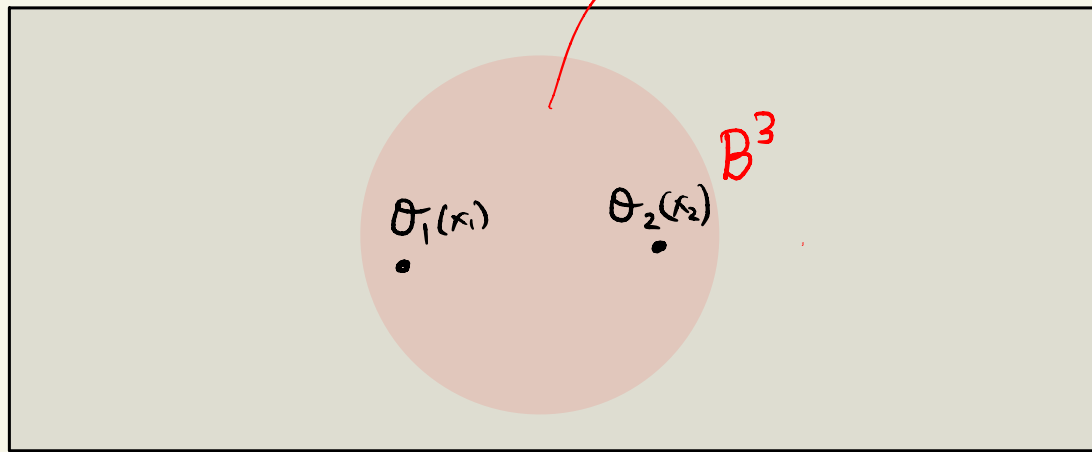
Coupling to S_N TQFT.

- $\beta_2 \rightarrow \infty$ imposes flatness condition on S_N gauge field.

$$\prod_{e \in \partial p} P_e = 1_N$$

- As long as correlators do not involve non-contractible cycles on M_3 ; local dynamics must be the same as abelian model

T^3



- P_e can be fixed to 1 inside B^3 . Local dynamics same as abelian model.

- Globally different theories. e.g. correlators involving non-contractible cycles know about S_N TQFT.

- In the $\beta_2 \rightarrow \infty$ limit; S_N monopoles are energetically forbidden.

$$\left\langle \sigma_1(x_1) W_{(x_1, x_2)}^{S_N} \sigma_2(x_2) \right\rangle_{U(1)^{N-1} \rtimes S_N} = \left\langle \sigma_1(x_1) \sigma_2(x_2) \right\rangle_{U(1)^{N-1}}$$

- Local dynamics same as abelian theory.
- But spectrum of line operators & global symmetries are distinct.

Center symmetry & a puzzle

- Center sym. is a 1-form symmetry (acting on lines) generated by co-dim 2 defect.

- E.g. $U(1)$ LGT. (easiest to describe on lattice)

Let $U_\ell \equiv U_\mu(x) = e^{i\theta_\mu(x)}$. $S \sim \prod_{\ell \in \partial P} U_\ell = U_\mu(x) U_\nu(x+\mu) U_\mu^\dagger(x+\nu) U_\nu^\dagger(x)$

$$Z(U(1)) : U_\mu(x) \rightarrow \underbrace{e^{i a \mu}}_{\text{global.}} U_\mu(x)$$

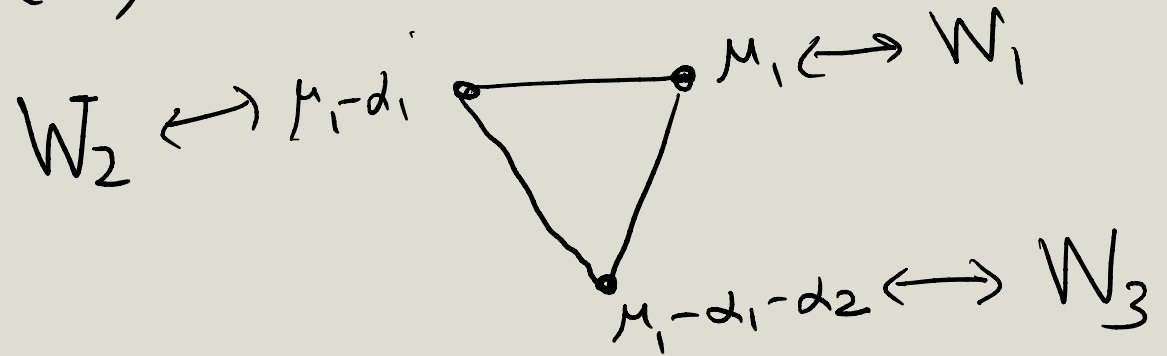

- $Z(U(1)^{N-1}) = U(1)^{N-1}$

$$Z(S^N) = 1$$

$$Z(U(1)^{N-1} \rtimes S^N) = \mathbb{Z}_N \text{ just like pure } SU(N) \text{ YM.}$$

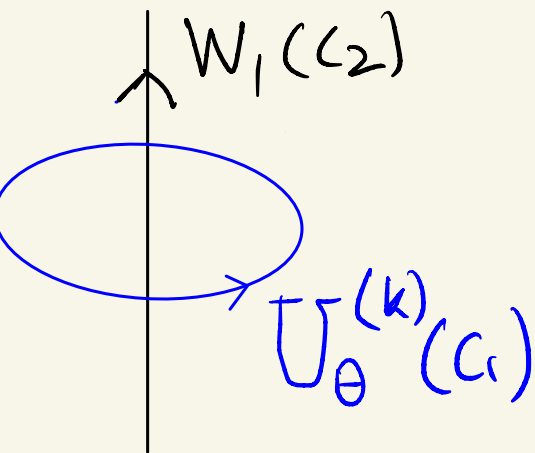
Wilson Loops & symmetry generators in abelian $U(1)^{N-1}$ model.

Wilson loops : $W_k(C) = e^{i \left(\mu_1 - \sum_{j=1}^{k-1} \alpha_j \right) \cdot \int_C A}$ $k=1, \dots, N-1$

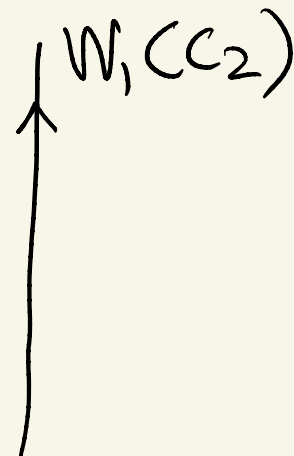


$[U(1)^{(1)}]^{N-1}$

generators: $U_\theta^{(k)}(C) = e^{i \frac{\theta}{2\pi} \int_C \alpha_k \cdot d\sigma}$ $k=1, \dots, N-1$
 $\sigma \in \mathbb{R}^{N-1} / 2\pi \Gamma_w$



$= e^{i\theta \delta_{k1}}$



Eigenvalue eq.
 $W_1(C_2)$: Eigen-operator.
 $U_\theta(C_1)$: Topological line operator.

Gauging S_N

- After gauging S_N , generators & Wilson operators are no longer gauge invariant.
- We must symmetrize Wilson loops & topological operators to build gauge invariant objects.

- Wilson loops in $U(1)^{N-1} \rtimes S_N$ theory.

$$W_{fd}(C) = W_1(C) + W_2(C) + \dots + W_N(C)$$

- $\mathbb{Z}_N$ generators.

$$U_n(C) = \prod_{k=1}^{N-1} U_{\frac{2\pi}{N}k_n}^{(k)}(C) = e^{i \frac{n}{N} \int_C (\alpha_1 + 2\alpha_2 + \dots + (N-1)\alpha_{N-1}) \cdot d\tau}$$

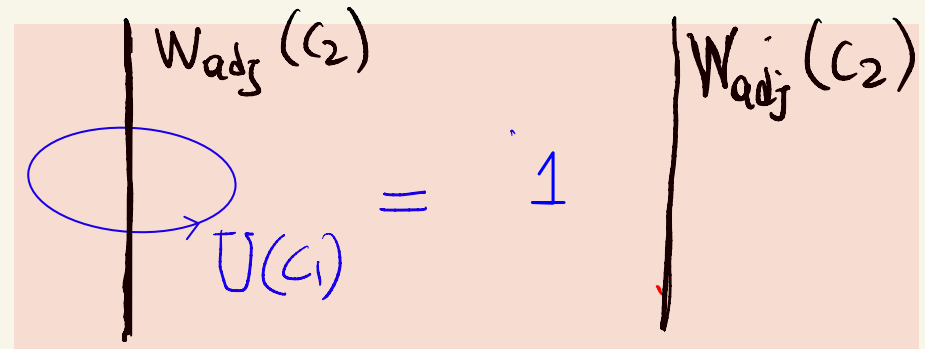
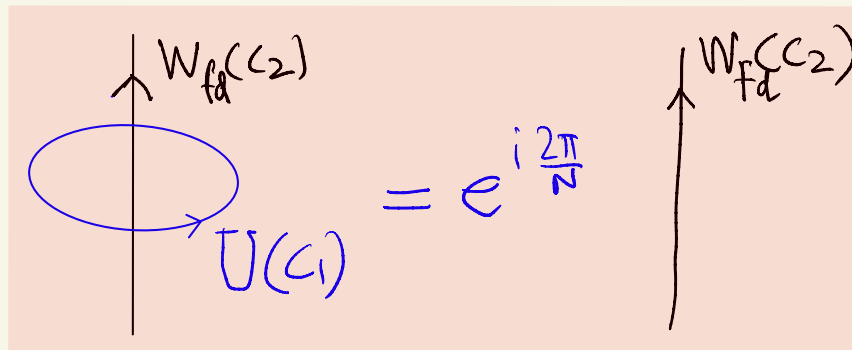
$$U_n(C) U_m(C) = U_{n+m \bmod N}(C)$$

Fusion obeys group multiplication laws.

$\mathbb{Z}_N^{[1]}$ center & failure of N-ality rule.

$$\mathbb{Z}_N^{[1]} : W_{fd} \mapsto e^{2\pi i/N} W_{fd}. \quad \text{or}$$

$$\langle U(C_1) W_{fd}(C_2) \rangle = e^{\frac{2\pi i}{N} \text{Link}(C_1, C_2)} \langle W_{fd}(C_2) \rangle$$



- Q:** Does 1-form $\mathbb{Z}_N^{[1]}$ center sym. control string tensions?
 No. $T_{\mu_1} \neq 0$, but $T_{\alpha} \neq 0$ as well. (zero N-ality). $T_{2\mu_1} \neq T_{\mu_2}$ (both N-ality 2).
- Infinitely many string tensions, instead of just N.

Puzzle: How is the presence of infinitely many string tensions compatible with finite center symmetry?

Non-invertible topological lines & symmetry.

- Need to explain the failure of the N-lity rule.
- $\Rightarrow$ Non-invertible top. symmetry. Generalization of 1-form $\mathbb{Z}_N^{[1]}$ center symmetry.
- Symmetrizing center generators $U_\theta^{(1)}(C)$ in abelian theory:

$$\begin{aligned} T_\theta(C) &= \frac{1}{N!} \sum_{P \in S_N} P U_\theta^{(1)}(C) P^{-1} \\ &= \frac{1}{N(N-1)} \sum_{\alpha \in \Phi} e^{i \frac{\theta}{2\pi} \int_C \alpha \cdot d\sigma} \end{aligned}$$

- Satisfies all features of 1-form symmetry, **except group multiplication law.**

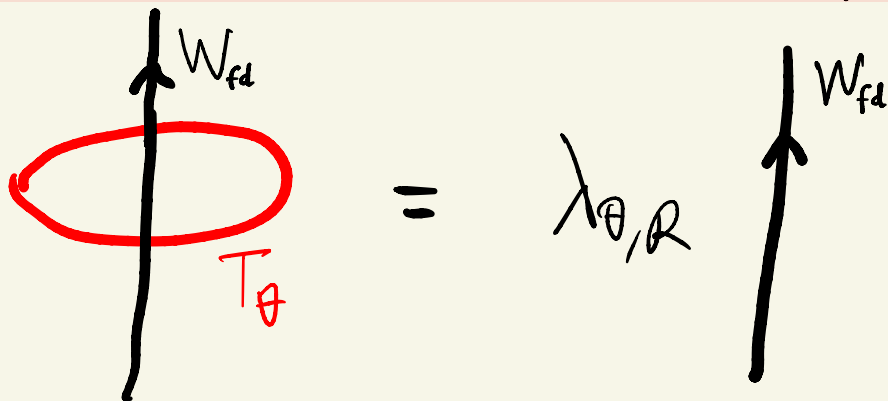
$$T_\theta(C) T_{\theta'}(C) \neq T_{\theta+\theta'}(C).$$

Symmetry, but not a group.

- Consider Wilson loop that corresponds to $\omega \in \Gamma_w$.
electric charge in weight lattice.
 e.g. $W_R = \frac{1}{N!} \sum_{P \in S_N} P W_\omega P^{-1}$ etc.

$$T_\theta(c) W_R(c') = \lambda_{\theta,R} W_R(c')$$

$$\lambda_{\theta,R} = \frac{1}{N(N-1)} \sum_{\alpha \in \Phi} e^{i\theta \alpha \cdot \omega}$$



$$T_\theta W_{fd} = \lambda_{\theta,R} W_{fd}$$

$$T_\theta(c) W_{fd}(c') = \frac{1}{N} (N-2+2\cos\theta) W_{fd}(c')$$

Tension for N-ality zero rep.

- Here is the main point concerning non-invertible sym. Despite the fact that W_{adj} is trivial under $Z_N^{(1)}$; $U(C_1) W_{\text{adj}}(C_2) = 1 W_{\text{adj}}(C_2)$; it obeys

$$T_\theta(C_1) W_{\text{adj}}(C_2) = \underbrace{\frac{(N-2)(N-3) + 4(N-2) \cos \theta + 2 \cos(2\theta)}{N(N-1)}}_{\text{non-trivial, eigenvalue.}} W_{\text{adj}}(C_2)$$

- String tension beyond N-ality is characterized by non-invertible topological line operators T_θ .

- T_θ is a 1-form sym. But it does not have an inverse. $T_{-\theta} = T_\theta$. Fusion rule does not conform with group law. Fusion category???

Distinguishing two N-ality two reps.

- Consider two N-ality 2-reps, W_{sym} & W_{asym} .
- Eigen-operators of non-invertible generators $T_\theta(C_1)$ are W_{asym} & $(W_{\text{sym}} - W_{\text{asym}})$.

$$T_\theta(C_1) (W_{\text{sym}} - W_{\text{asym}})(C_2) = \frac{N-2 + 2\cos 2\theta}{N} (W_{\text{sym}} - W_{\text{asym}})$$

$$T_\theta(C_1) W_{\text{asym}}(C_2) = \frac{(N-2)(N-3) + 2 + 4(N-2)\cos\theta}{N(N-1)} W_{\text{asym}}(C_2)$$

- consistent with $T_{\mu_2} \neq T_{2\mu_1}$.

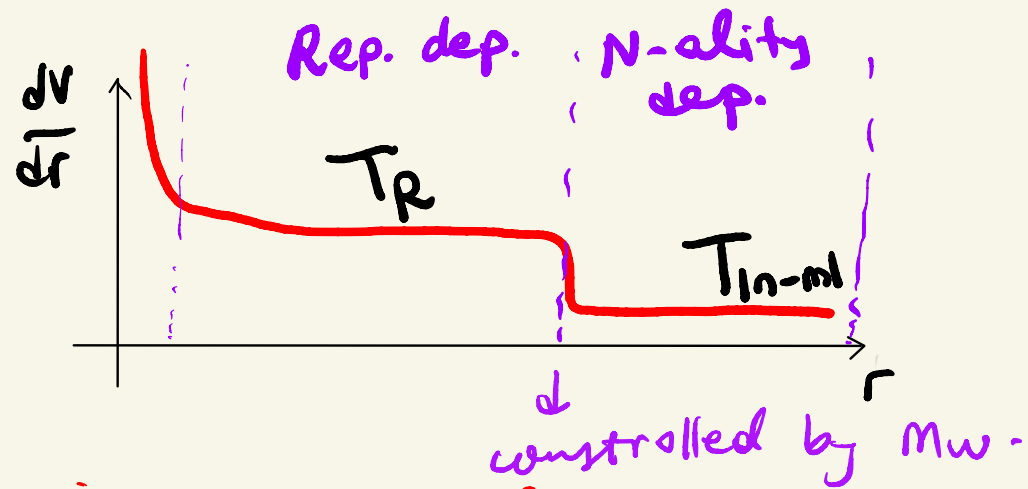
Dynamical electric charges & breaking non-invertible sym.

- Add W-bosons. Since eigenvalue of W_{adj} must be one, (perimeter law or adjoint string breaking);

$$T_\theta(C_1) W_{adj}(C_2) = \lambda_{\theta, adj} W_{adj}(C_2)$$

the only solution is for $\theta=0$, and we lose non-inv. symmetry.

- If W is heavy; then;



- suspiciously curious, isn't it?

Conclusions- speculations

- Abelian and semi-abelian dynamics are quite different from other solvable, abelianizing theories: $\mathcal{N}=2$ SW, $\text{QCD}(\text{adj})$, deformed YM. Provides new insights.
- Non-invertible sym. is very likely also present in pure YM as an approximate sym. We believe that it becomes exact at $N=\infty$ limit and string tensions are not controlled by N -ality.
- Non-inv. sym can very likely provide a meaning to confinement in theories such as $\text{QCD}(F)$ and YM with G_2 group etc. It also gives a meaning to confinement of adjoint probes in $\text{SU}(N)$ YM.