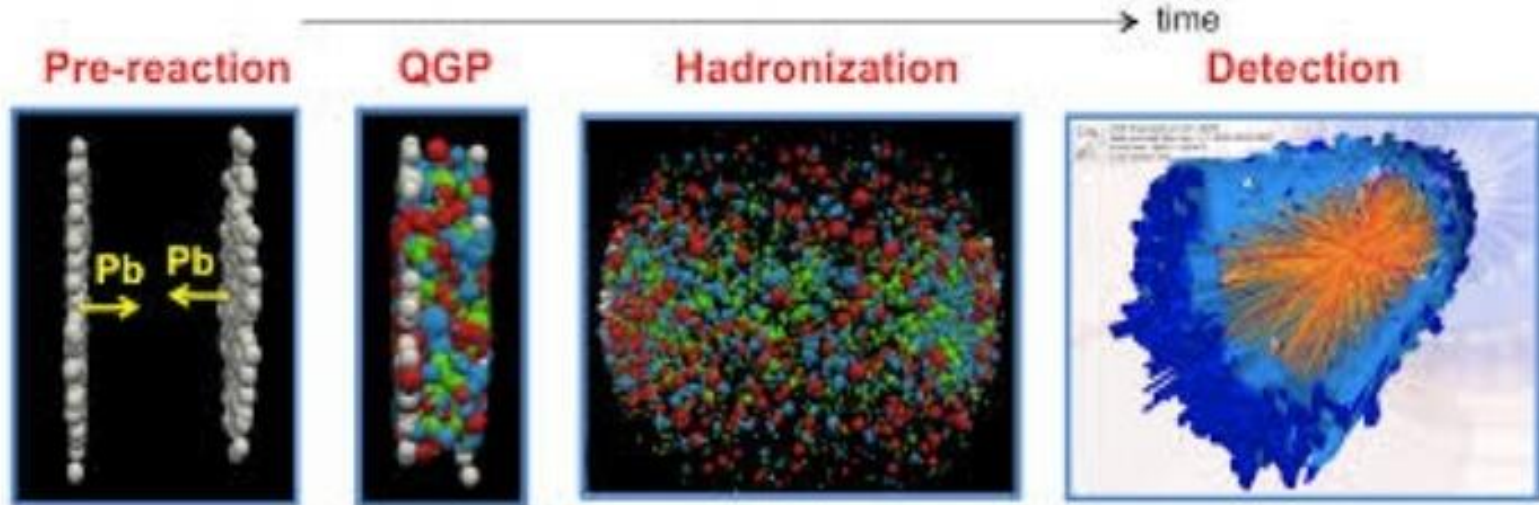


# Heavy quark dynamics in QCD matter



**Santosh Kumar Das**

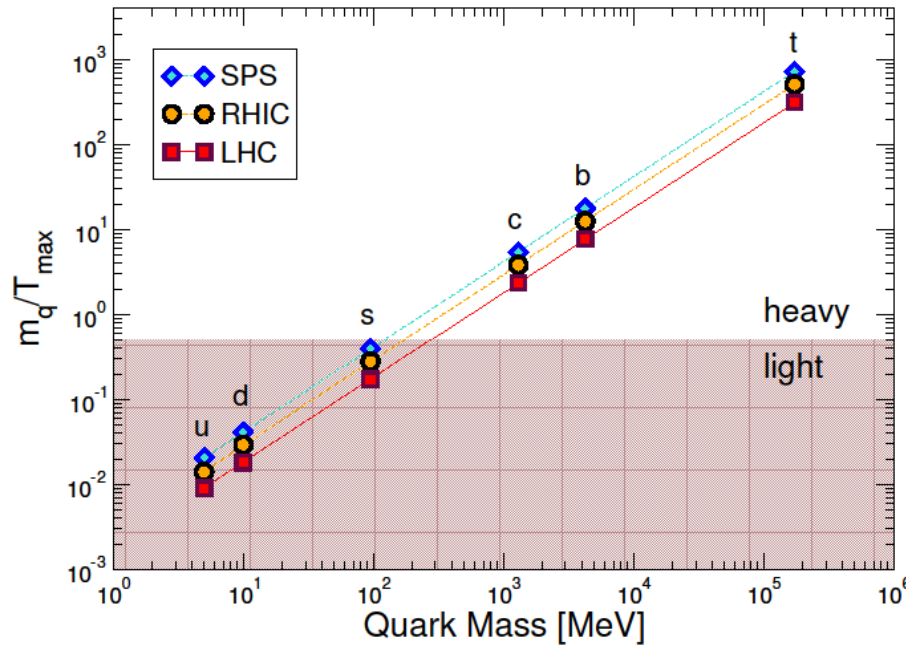
**School of Physical Science  
Indian Institute of Technology Goa  
Goa, India**



# **OUTLINE .....**

- ☐ Introduction
- ☐ Quark Gluon Plasma - the primordial fluid
- ☐ Heavy quark transport coefficients –  $R_{AA}$  and  $v_2$
- ☐ Heavy quark momentum evolution and hadronization
- ☐ Initial stage effects – EM field, Angular momentum, Glasma
- ☐ Summary and outlook

# Heavy Quark & QGP



**SPS to LHC**

$\sqrt{s} = 17.3 \text{ GeV to } 2.76 \text{ TeV} \sim 100 \text{ times}$

$T_i = 200 \text{ MeV to } 600 \text{ MeV} \sim 3 \text{ times}$

$\tau$  relaxation time

$$M_{c,b} \gg \Lambda_{QCD}$$

Produced by pQCD process (before equilibrium)  
(Early production)

$$\tau_{c,b} \gg \tau_{QGP}$$

They go through all the QGP life time

$$M_{c,b} \gg T_0$$

No thermal production

## Boltzmann Kinetic equation

$$\left( \frac{\partial}{\partial t} + \frac{\mathbf{p}}{E} \frac{\partial}{\partial \mathbf{x}} + \mathbf{F} \cdot \frac{\partial}{\partial \mathbf{p}} \right) f(\mathbf{x}, \mathbf{p}, t) = \left( \frac{\partial f}{\partial t} \right)_{col}$$

➤ The plasma is uniform ,i.e., the distribution function is independent of  $\mathbf{x}$ .

➤ In the absence of any external force,  $\mathbf{F}=0$

$$R(\mathbf{p}, t) = \left( \frac{\partial f}{\partial t} \right)_{col} = \int d^3k [\omega(\mathbf{p} + \mathbf{k}, k) f(\mathbf{p} + \mathbf{k}) - \omega(\mathbf{p}, k) f(\mathbf{p})]$$

$$\omega(\mathbf{p}, k) = g \int \frac{d^3q}{(2\pi)^3} f'(q) v_{q,p} \sigma_{p,q \rightarrow p-k, q+k} \longrightarrow \text{is rate of collisions which change the momentum of the charmed quark from } p \text{ to } p-k$$

$$\omega(\mathbf{p} + \mathbf{k}, k) f(\mathbf{p} + \mathbf{k}) \approx \omega(\mathbf{p}, k) f(\mathbf{p}) + \mathbf{k} \cdot \frac{\partial}{\partial \mathbf{p}} (\omega f) + \frac{1}{2} k_i k_j \frac{\partial^2}{\partial p_i \partial p_j} (\omega f)$$

$$\frac{\partial \mathbf{f}}{\partial t} = \frac{\partial}{\partial \mathbf{p}_i} \left[ \mathbf{A}_i(\mathbf{p}) \mathbf{f} + \frac{\partial}{\partial \mathbf{p}_j} [\mathbf{B}_{ij}(\mathbf{p}) \mathbf{f}] \right]$$

B. Svetitsky PRD 37(1987)2484

where we have defined the kernels

$$\mathbf{A}_i = \int d^3k \omega(\mathbf{p}, k) \mathbf{k}_i \rightarrow \text{Drag Coefficient}$$

$$\mathbf{B}_{ij} = \int d^3k \omega(\mathbf{p}, k) \mathbf{k}_i \mathbf{k}_j \rightarrow \text{Diffusion Coefficient}$$

# Langevin Equation

The Fokker-Planck equation can be recast to Langevin equation:

$$d\mathbf{r} = \frac{\mathbf{p}}{E} dt$$

$$\frac{dp}{dt} = -\gamma(p)p + \zeta \quad \text{with} \quad \langle \zeta_i(t) \zeta_k(t') \rangle = D \delta(t-t') \delta_{jk}$$

where  $\gamma$  is the deterministic friction (drag) force

$\zeta$  is stochastic force

For the bulk evolution: Hydrodynamics/Transport

Transport coefficients are connected by Fluctuation Dissipation Theorem.

Moore, Teaney, PRC 71 (2005) 064904

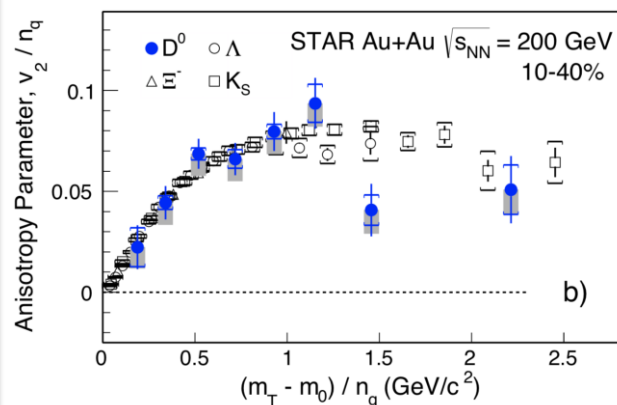
Van Hees, Greco, Rapp, PRC 73 (2006) 034913

## Heavy quark initialization

- ✧ r-space: N\_coll (Glauber mode)
- ✧ p-space: NLO (pQCD)

# Heavy quark physics at different scales

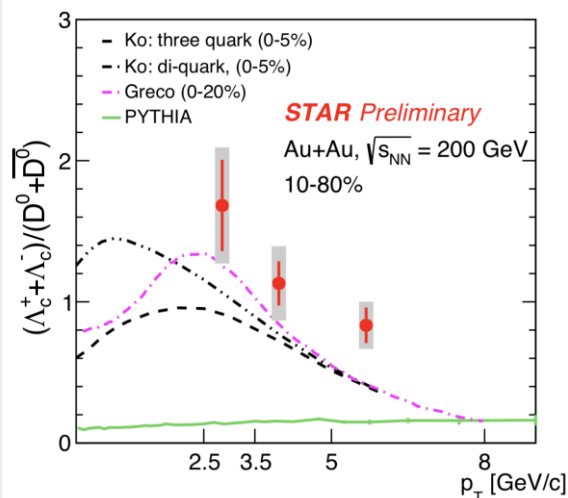
**low  $p_T$**



**Study thermalization  
process of HQ**

**Constrain diffusion  
coefficient  $D_s$**

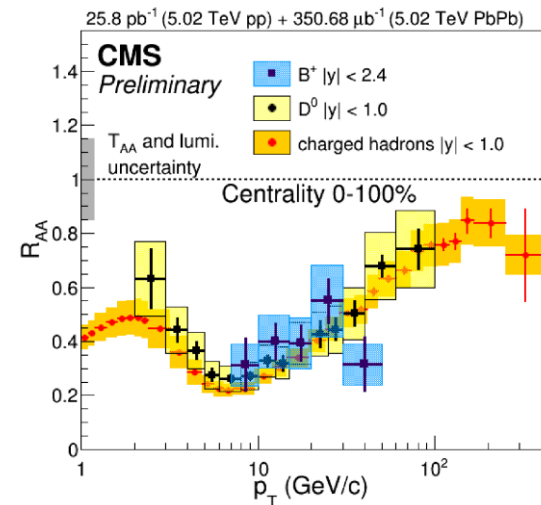
**medium  $p_T$**



**Study hadronization  
process of HQ**

**Constrain hadron  
wave-function**

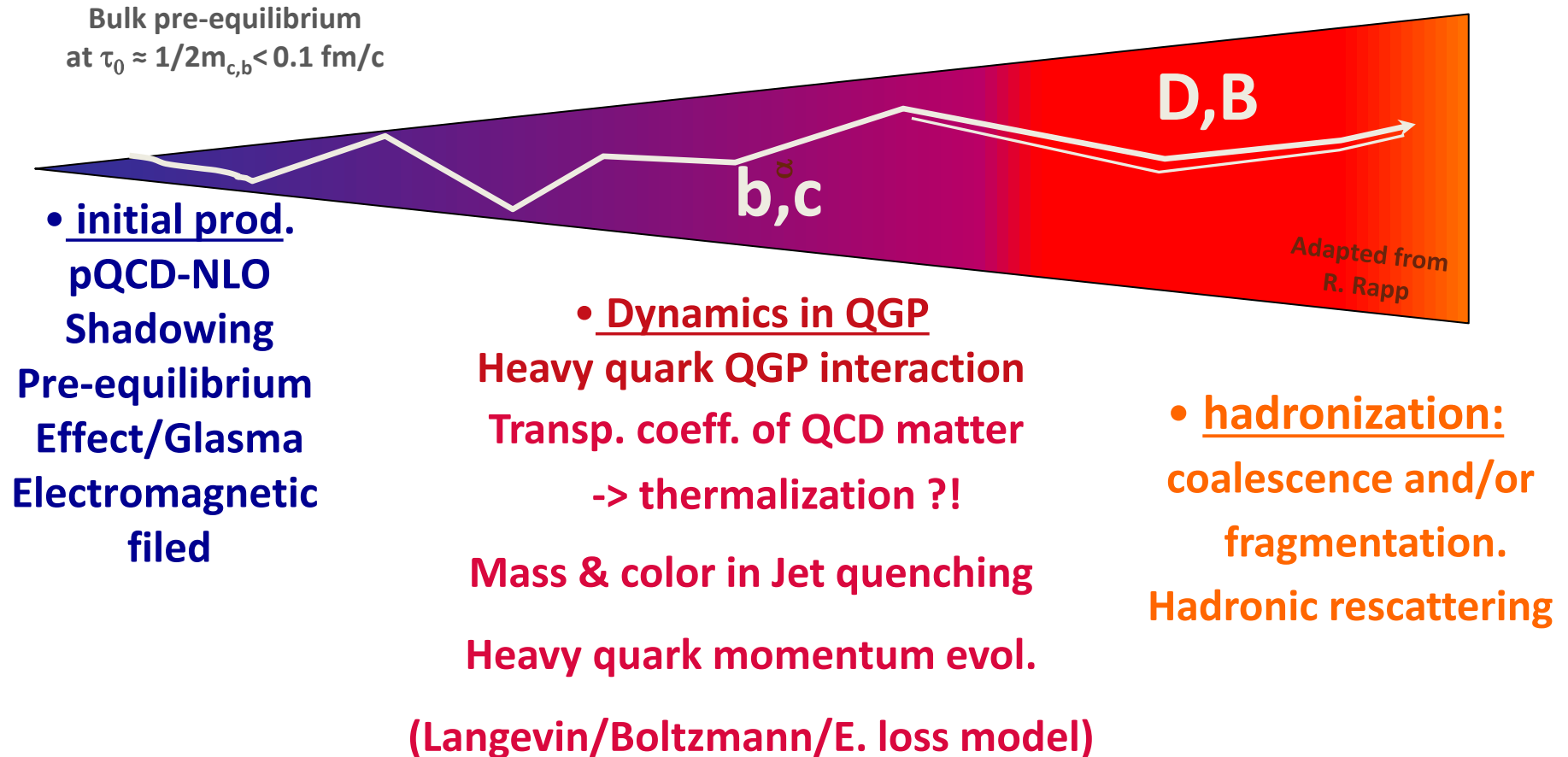
**high  $p_T$**



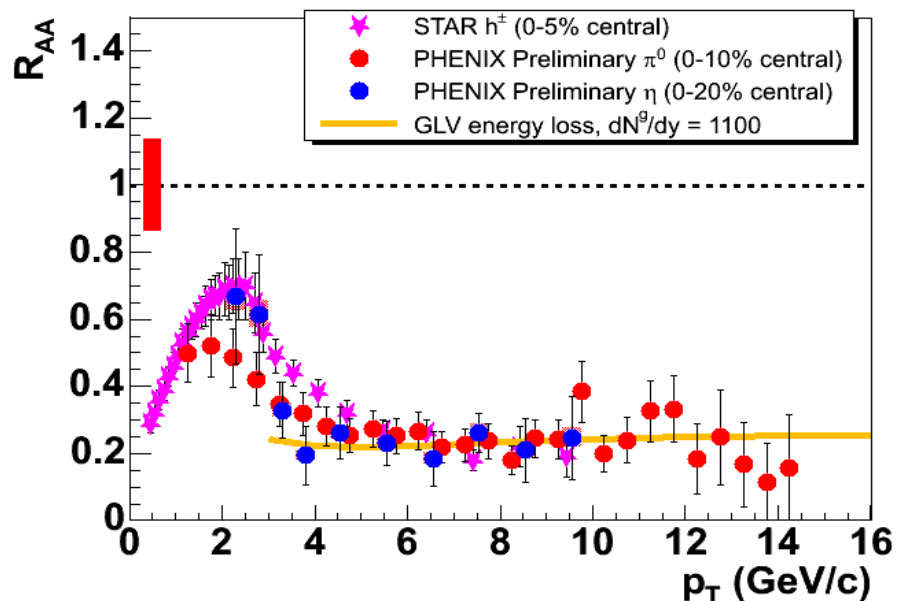
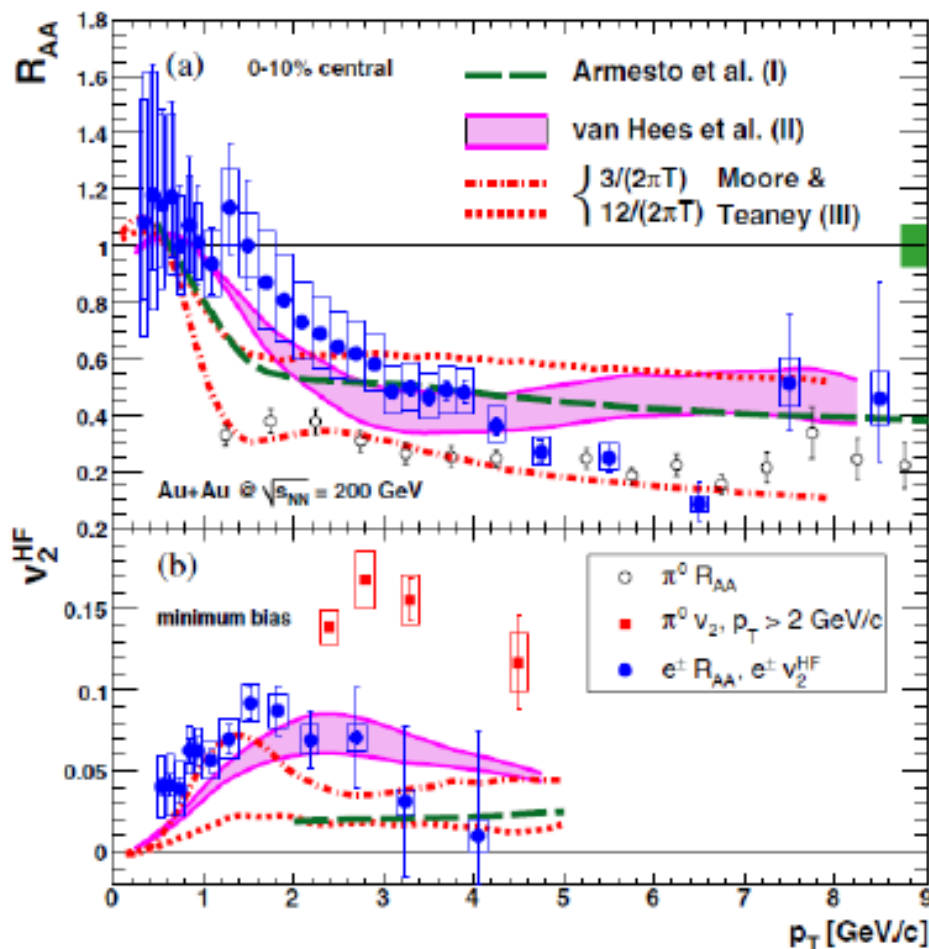
**Study parton energy  
loss and mass effect**

**Constrain jet transport  
parameter  $\hat{q}$**

# Studying the HF at RHIC and LHC



# Heavy flavor at RHIC (2007)

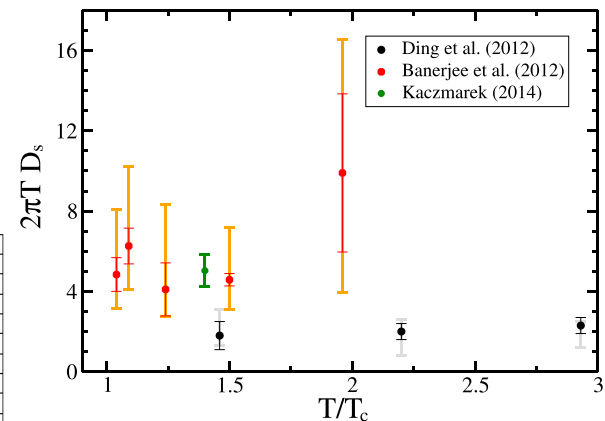
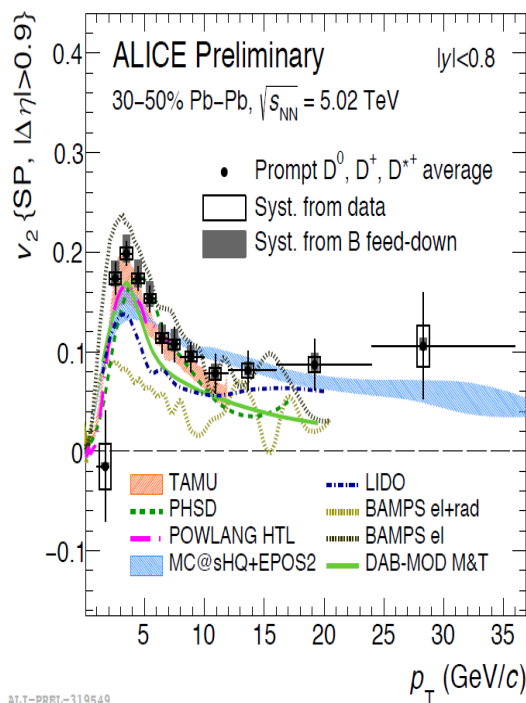
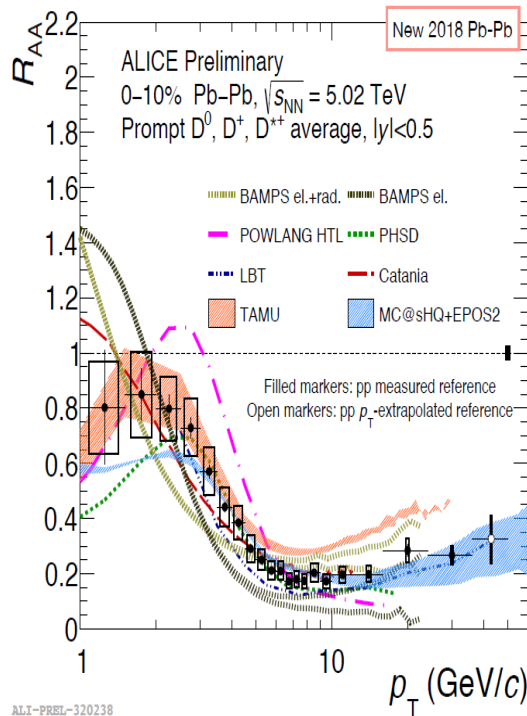


At RHIC energy heavy flavor suppression is similar to light flavor

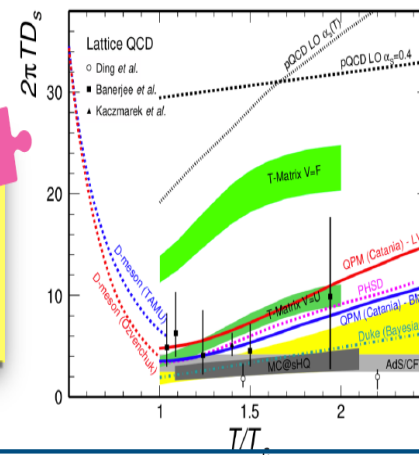
Simultaneous description of  $R_{AA}$  and  $v_2$  is a tough challenge for all the models.



# $R_{AA}$ and $v_2$ Comparison with models

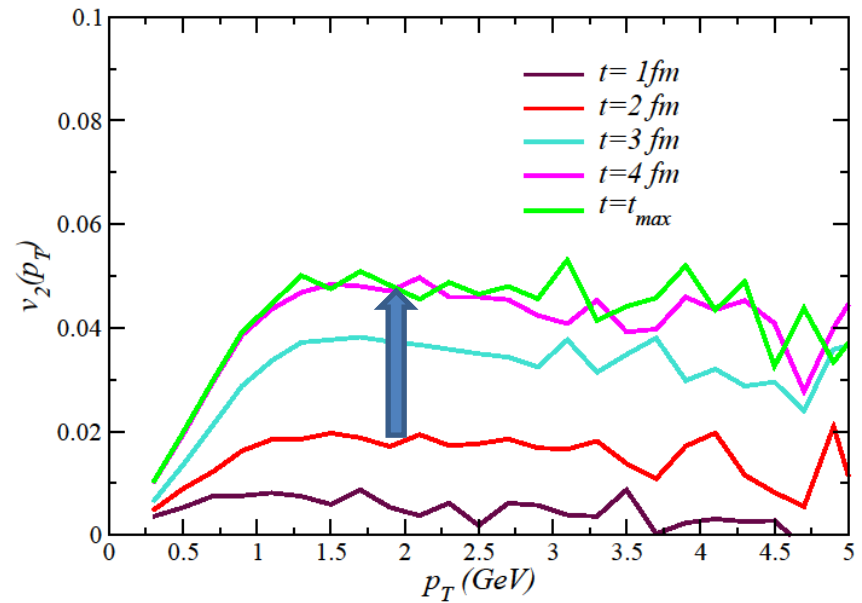
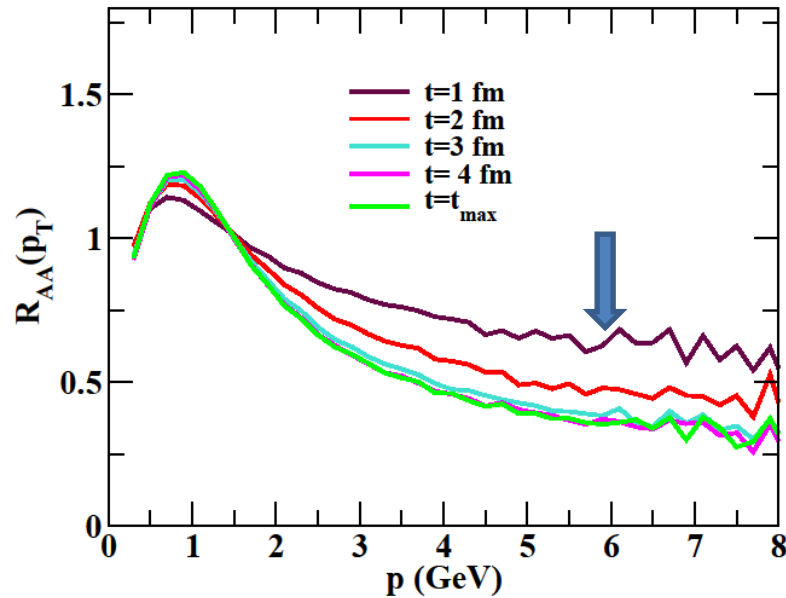


TAMU: PLB 735,445-450(2014),  
arXiv:1905.09216  
PHSD: PRC 92, 014910 (2015)  
POWLANG: EPJC 75,121(2015)  
MC@sHQ+EPOS2: PRC 89 014905 (2014)  
LBT: PLB 777 (2018) 255-259  
LIDO: arXiv:1810.08177  
BAMPS: JPG 42, 115106 (2016)  
Djordjevic: PRC 92, 024918 (2015)  
CUJET3.0: JHEP 02 (2016) 169  
SCET: JHEP 03 (2017) 146  
DAB-MOD: PRC 96 (2017) 064903  
Catania: Eur. Phys. J. C (2018) 78: 348

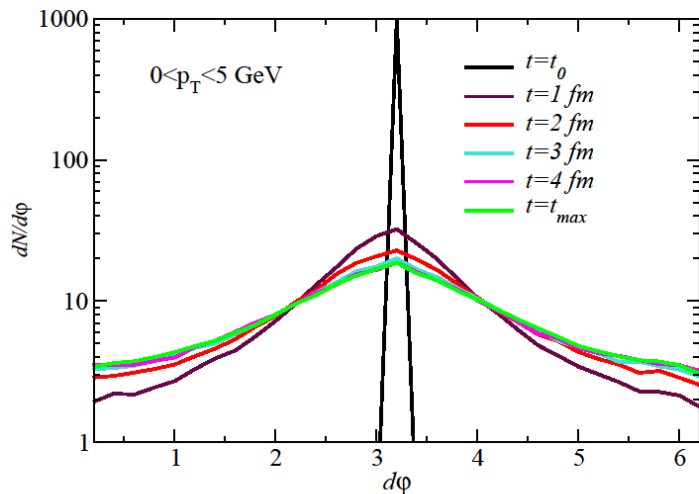


- Simultaneous description of  $R_{AA}$  and  $v_2$  is **challenging** in the whole measured  $p_T$  range!
- Experimental measurements start to **provide constraint to the models** for the characterization of the charm and beauty interaction with the medium
  - constraints on plasma transport parameters**, such as the heavy-quark diffusion coefficient

# Time evolution of Heavy quarks observables



Das, Scardina, Plumari, Greco  
*J. Phys. Conf. Ser.* 668 (2016) 012051

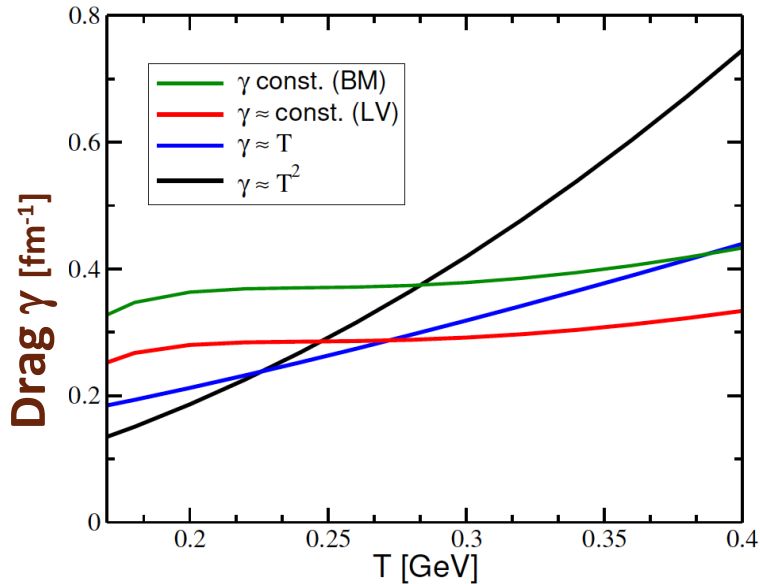


**$R_{AA}$  and  $dN/d\phi$  developed during the early stage of the evolution  $\rightarrow T_i$**

**$V_2$  developed during the later stage of the evolution  $\rightarrow T_c$**

**T dependence of the interaction i.e the transport Coefficients are the essential ingredient for the simultaneous description of HQ observables**

# Impact of T dep. interaction on $R_{AA} - v_2$



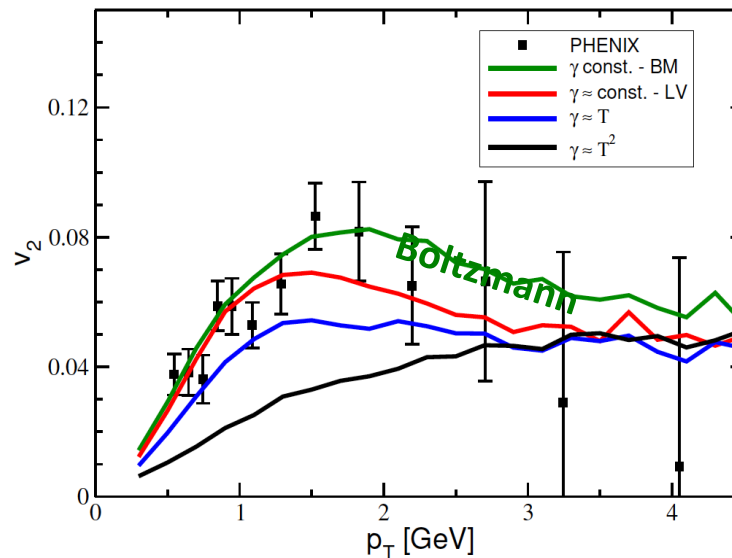
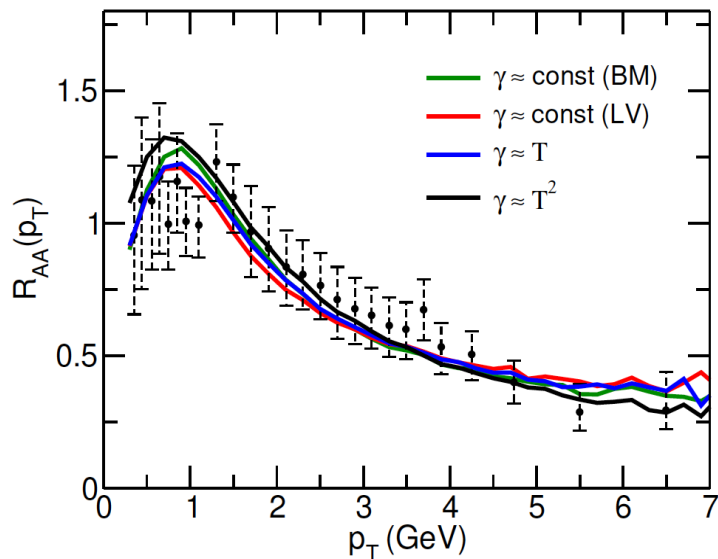
Looking at it beyond the specific modelings

➤  $\gamma \approx T^2$  [AdS/CFT, pQCD  $\alpha_s = \text{const}$ ]

➤  $\gamma \approx T$  [pQCD strong  $\alpha_s$  running]

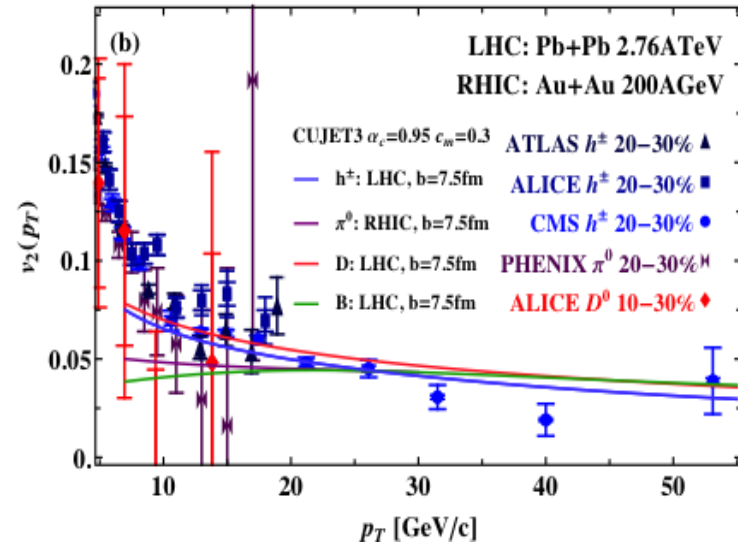
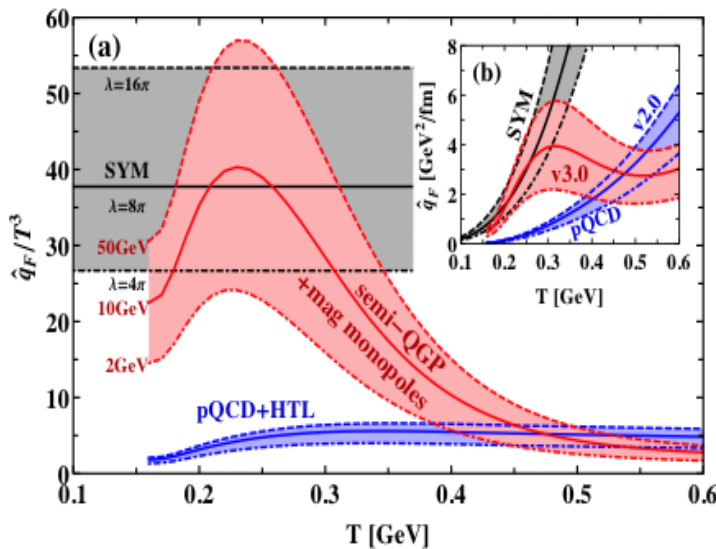
➤  $\gamma \approx \text{const.}$  [QPM, PHSD, T-matrix]

$\gamma$  rescaled to fit  $R_{AA}(p_T)$ , D from FDT



# $R_{AA}$ vs. $v_2$ puzzle

Different temperature dependence of the interaction strength may lead to different  $v_2$  for the same  $R_{AA}$ .



Semi-quark-gluon monopole plasma model increases  $\hat{q}$  around  $T_c$  and enhances hard probes'  $v_2$ .

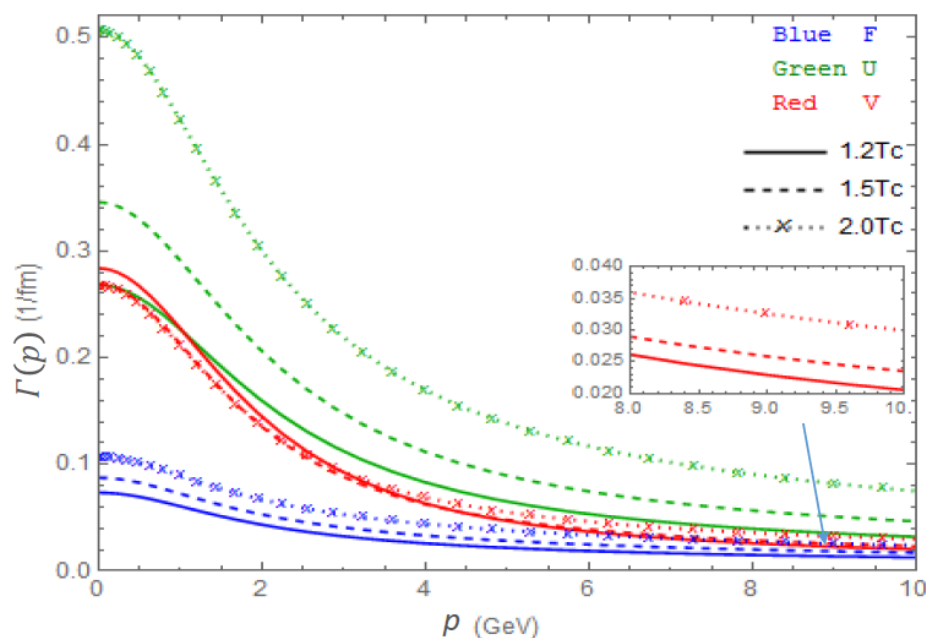
J. Xu, J. Liao and M. Guyllassi  
Chin. Phys. Lett. 32, 092501 (2015)

# T-matrix developments [TAMU]:

## ❖ Relaxation rate (drag coefficient)

$$\Gamma(p) = \frac{1}{2\omega_Q(p)} \sum \int d^3\tilde{q} d^3\tilde{q}' d^3\tilde{p}' n_i(\omega_q) \cdot \frac{(2\pi)^4}{d_c} C_f |T(E_{cm}|\mathbf{p}_{cm}, \mathbf{p}'_{cm})|^2 \delta^4(p + q - p' - q') \left(1 - \frac{p p'}{p^2}\right)$$

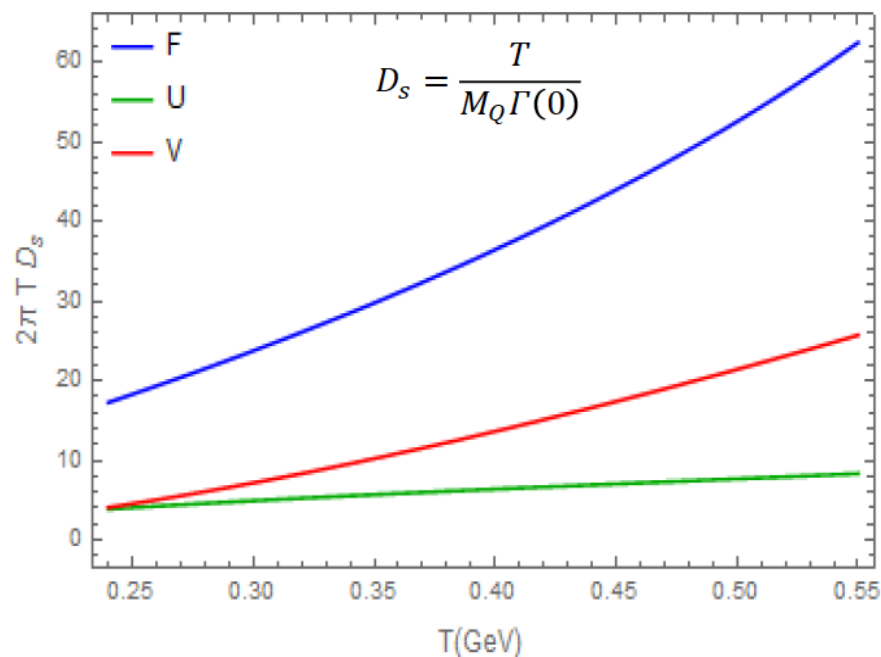
Relaxation rate for **FUV**



## ❖ For V

- Infrared enhancement due to long range force
- Different (slightly reversed)  $T$  dependence at low  $p$
- Recover usual  $T$  dependence at high  $p$

Spatial Diffusion Coefficient

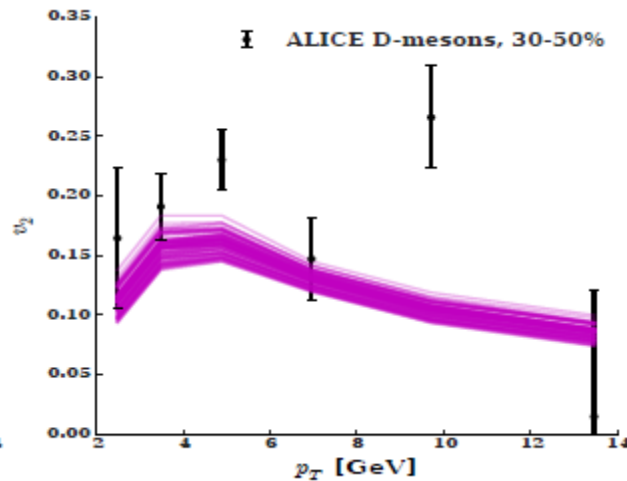
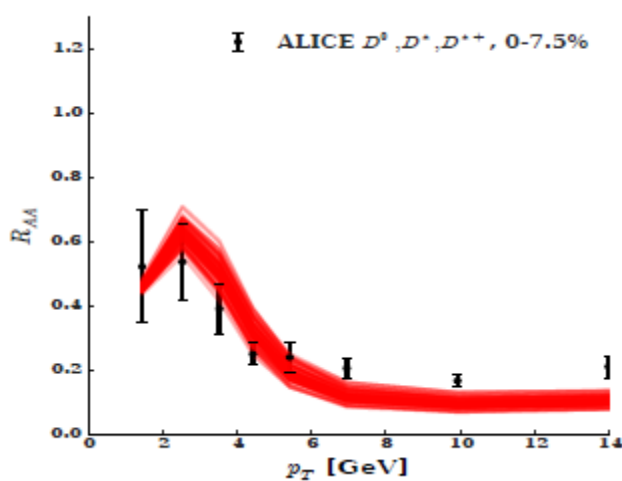


**Radiative loss in T-matrix approach**  
**Large suppression due to thermal mass**  
**Impact on observables yet to see**

# Bayesian model to data analysis [Duke]:

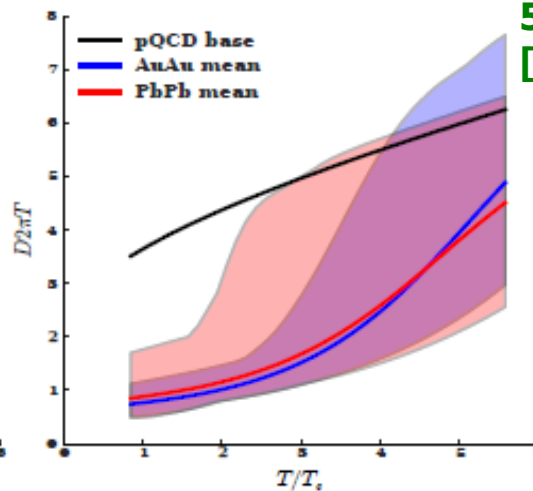
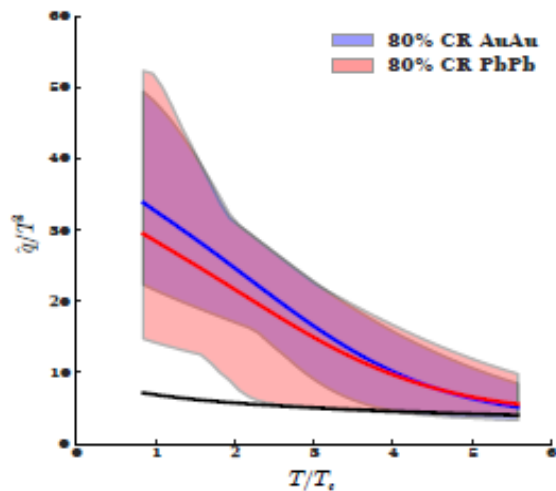
Simultaneously calibrate all model parameters through model-to-data comparison.

Extract the probability distribution of all parameters which best describe the data.



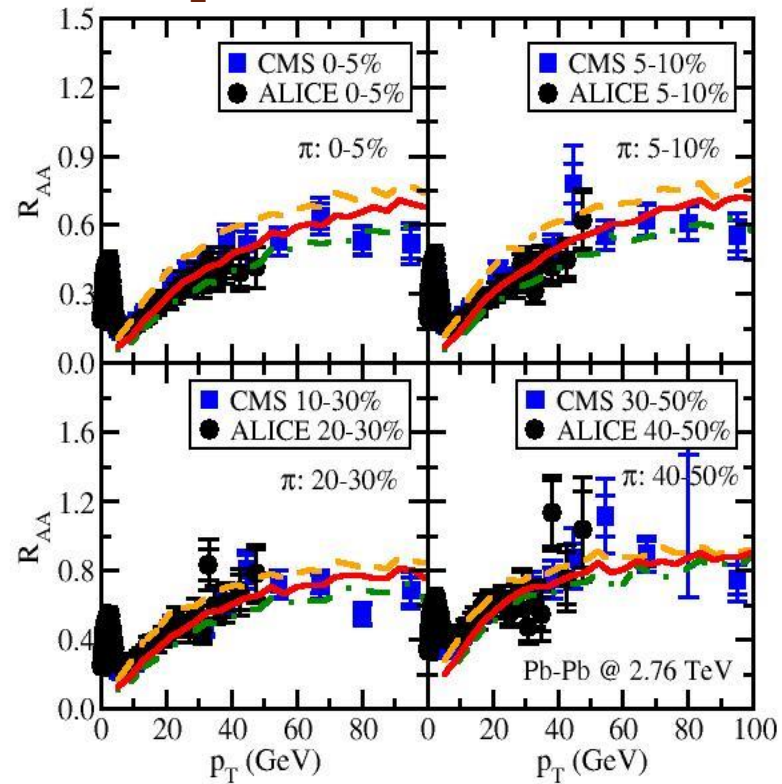
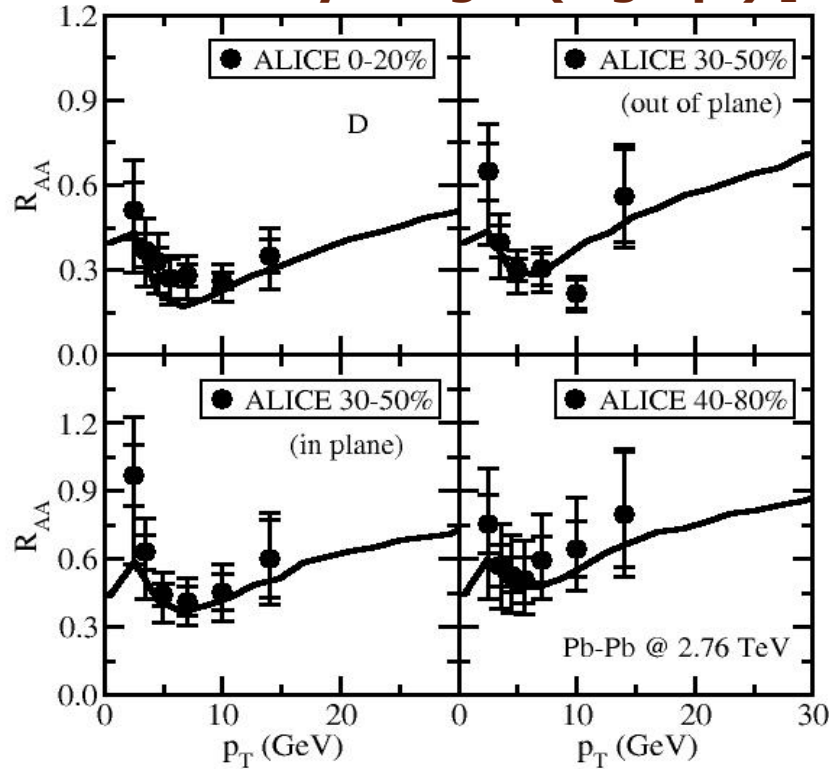
One can extract both the temperature and momentum dependence of heavy quark transport coefficients .

$$T, p\text{-dependence: } \hat{q} = \hat{q}_{pQCD} * preK * \left(1 + K_p e^{-\frac{|p|^2}{2\sigma_p^2}}\right) * \left(1 + K_T e^{-\frac{(T-T_c)^2}{2\sigma_T^2}}\right) \quad D_s = T^2/\hat{q}$$



5 dimensional Parameter space:  $[preK, K_p, \sigma_p, K_T, \sigma_T]$

## Approach from low to high pt from heavy to light (high-pt) [LBL-CCNU]



Spectrum of medium-induced gluon (higher-twist formalism):

$$\frac{dN_g}{dx dk_{\perp}^2 dt} = \frac{2\alpha_s C_A P(x)}{\pi k_{\perp}^4} \hat{q} \left( \frac{k_{\perp}^2}{k_{\perp}^2 + x^2 M^2} \right)^4 \sin^2 \left( \frac{t - t_i}{2\tau_f} \right)$$

**Linearized Boltzmann transport model**  
**Both collisional and radiative loss**

$$K_p = 1 + A_p e^{-|\vec{p}|^2 / 2\sigma_p^2},$$

**(T-matrix like features)**

$$K_T = 1 + A_T e^{-(T - T_c)^2 / 2\sigma_T^2}$$

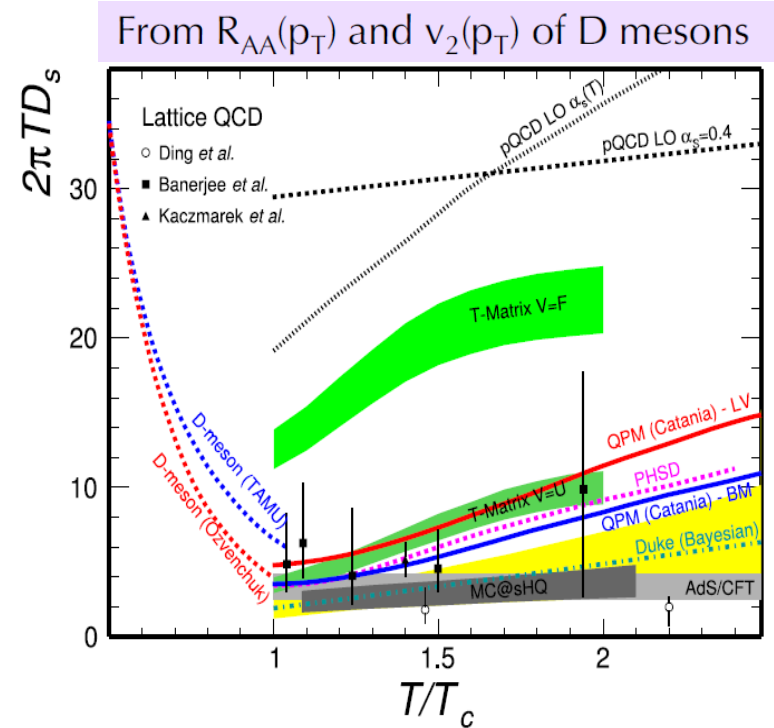
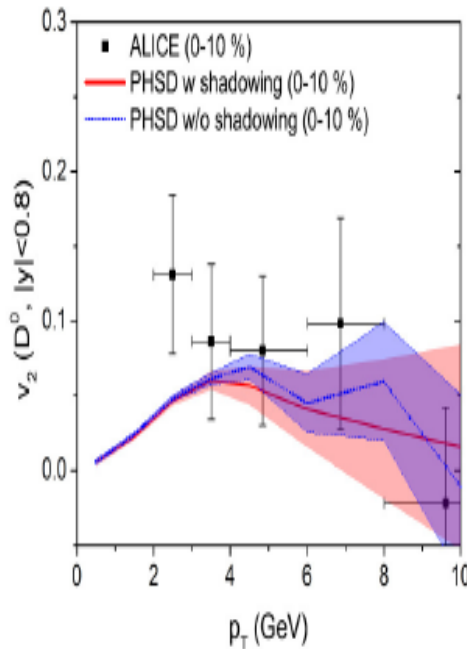
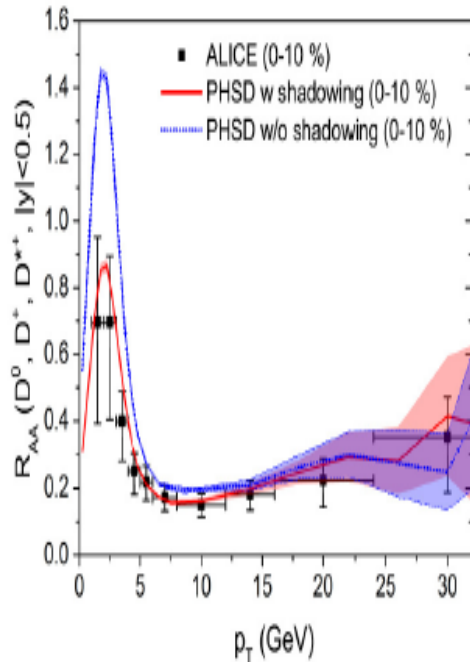
**(Catania-QPM, PHSD, T-matrix)**

**T and p dependent K factor needed starting from pQCD**

Cao, Luo, Qin and Wang, PRC, 94, 014909 2016



# RAA and v2 @PHSD



**Dong, Greco, PPNP 104 (2019) 97-141**

**T. Song et al. PRC 93, 034906 (2016)**

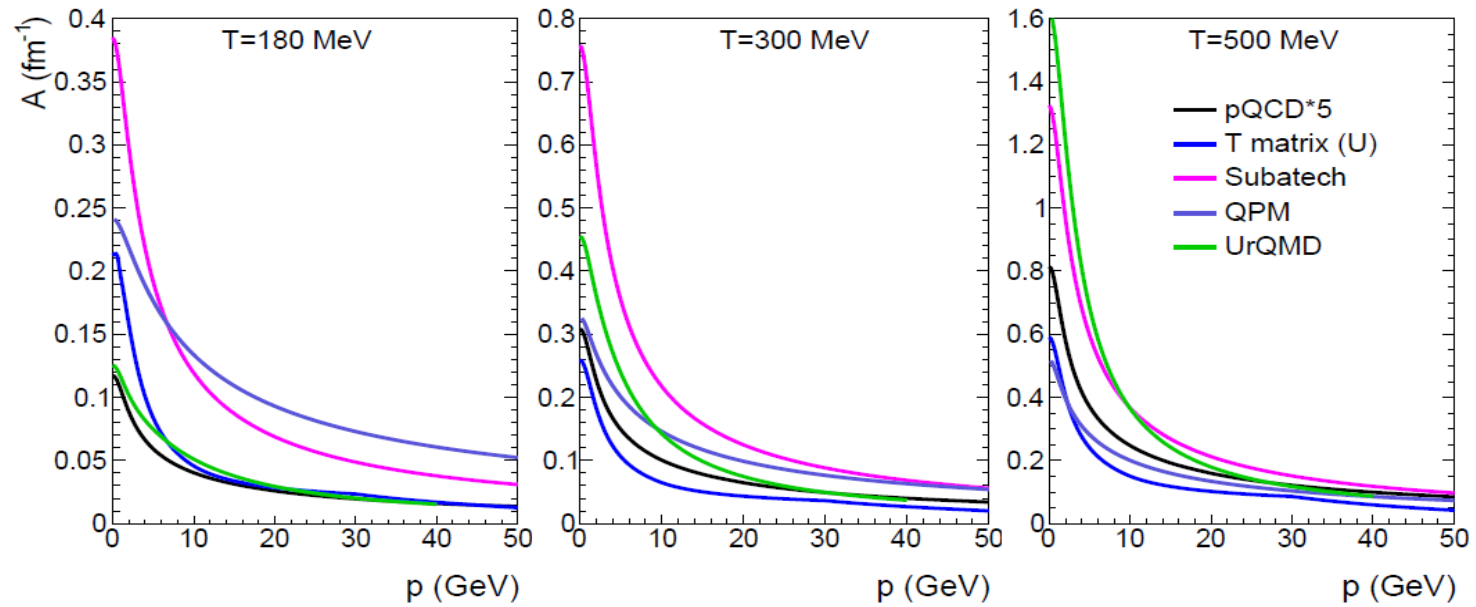
**Brambilla et. al 2007.10078 [hep-lat]  
Altenkort et al. 2009.13553 [hep-lat]**

**Heavy quark transport coefficients computed within Polyakov loop plasma showing similar T and p-dependence of heavy quark transport coefficients like T-matrix, QPM and PHSD.**

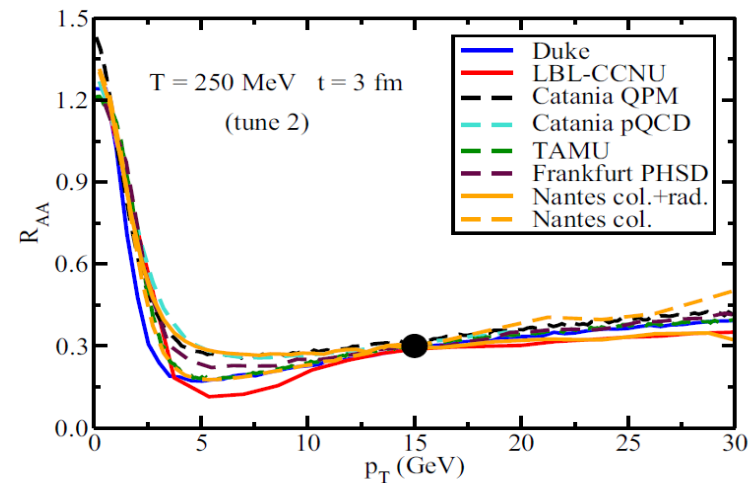
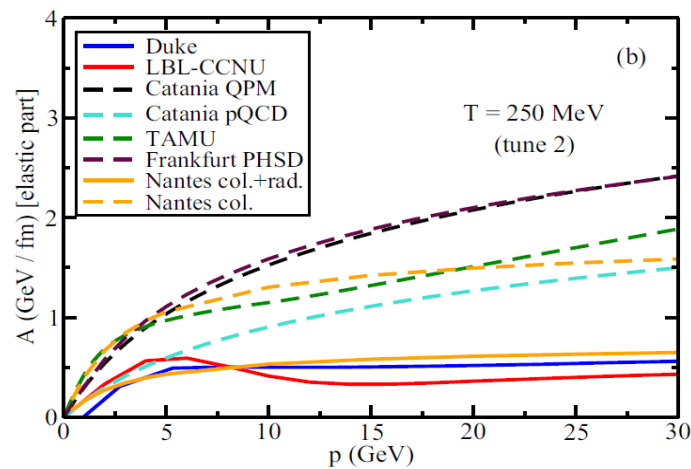
**Singh et al. PRD 100 (2019)114019**



**A systematic attempts are going on within the EMMI-RRTF and "JET-HQ" working groups to find a common agreement between different groups:**



**R. Rapp et.al NPA 979 (2018) (EMMI-RRTF)**

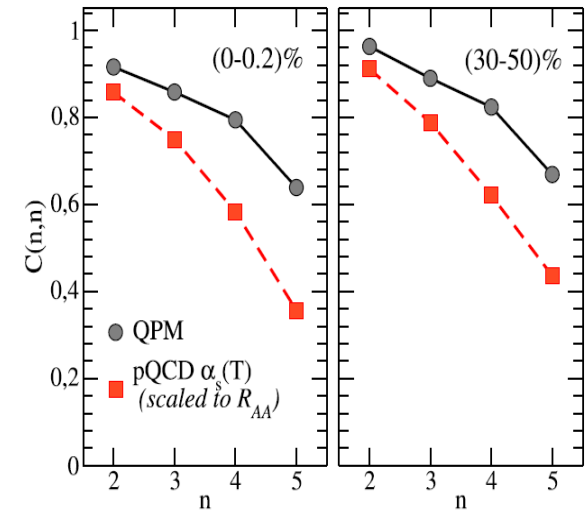
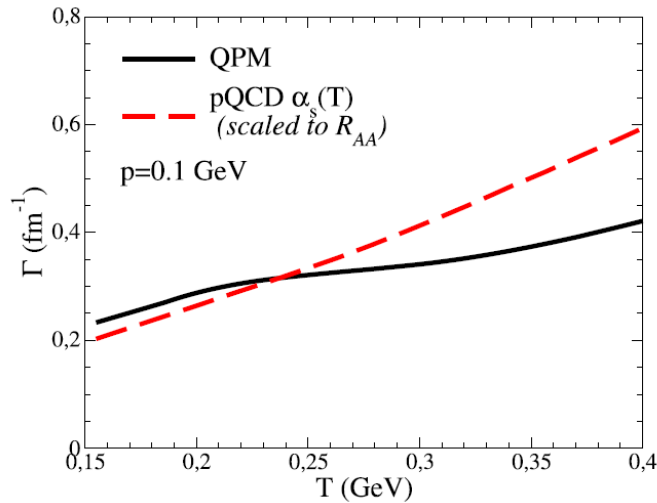
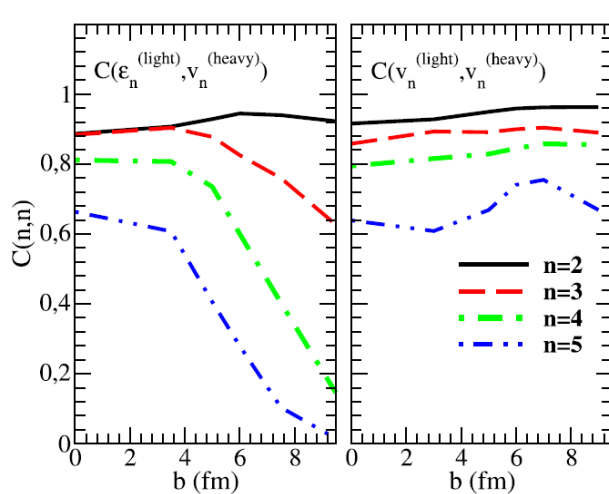


**S. Cao et. al PRC 99, 054907 (2019) (JET-HQ)**

# New observables:

## Heavy-light event-by-event correlation

$$C(n, m) = \frac{\sum_i (v_n^{L,i} - \langle v_n^L \rangle)(v_m^{H,i} - \langle v_m^H \rangle)}{\sqrt{\sum_i (v_n^{L,i} - \langle v_n^L \rangle)^2 \sum_i (v_m^{H,i} - \langle v_m^H \rangle)^2}}$$



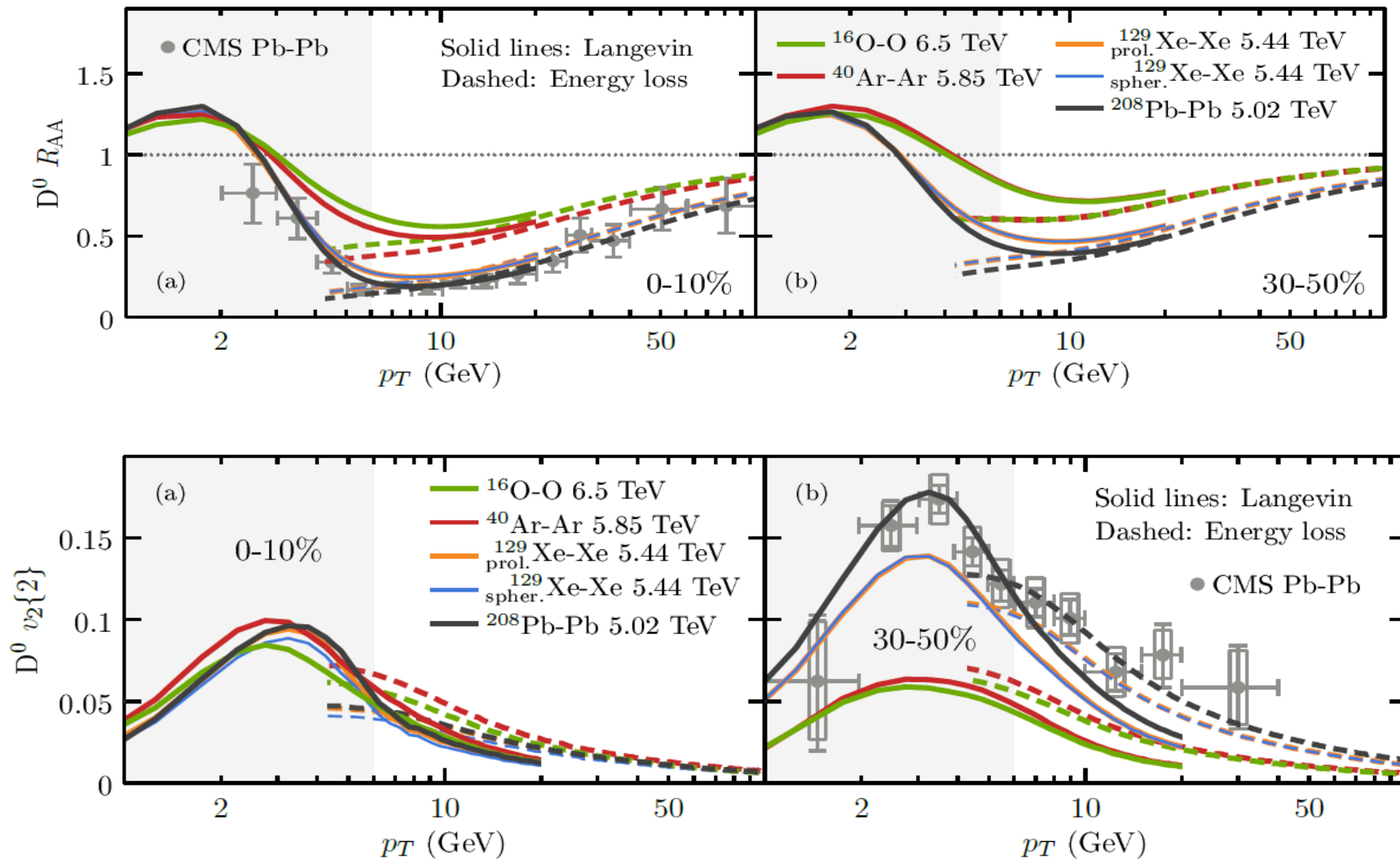
$V_n(D)$  more correlated to  $v_n(N_{ch})$  than  $\varepsilon_n$

Very large sensitivity to T dep. of Ds

This can put further constrain on heavy quark transport coefficients

Plumari Coci, Minissale, Das, Sun, Greco  
PLB 805 (2020) 135460

# System size scan of D meson $R_{AA}$ and $v_2$



**In central collisions the  $v_2$  is independent of system size.  
System size vs Eccentricity**

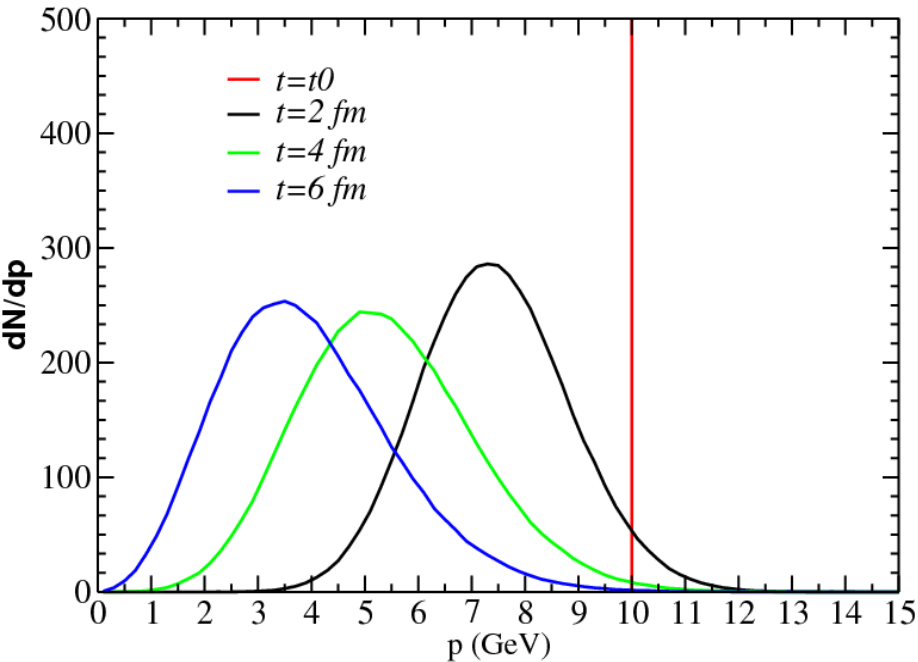
**R. Katz et. al, arxiv:1907.03308**

# Evolution: Boltzmann vs Langevin (Charm)

Momentum evolution starting from a  $\delta$  (Charm) in a Box

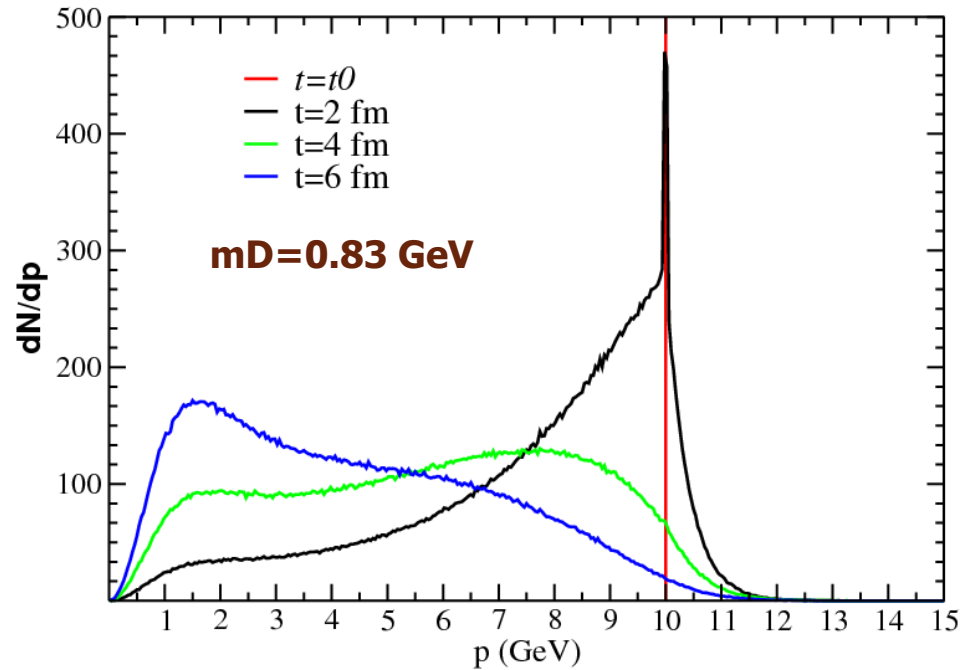
$$\frac{dN}{d^3 p_{initial}} = \delta(p - 10 \text{ GeV})$$

Langevin



In case of Langevin the distributions are Gaussian as expected by construction

Boltzmann



In case of Boltzmann the charm quarks does not follow the Brownian motion

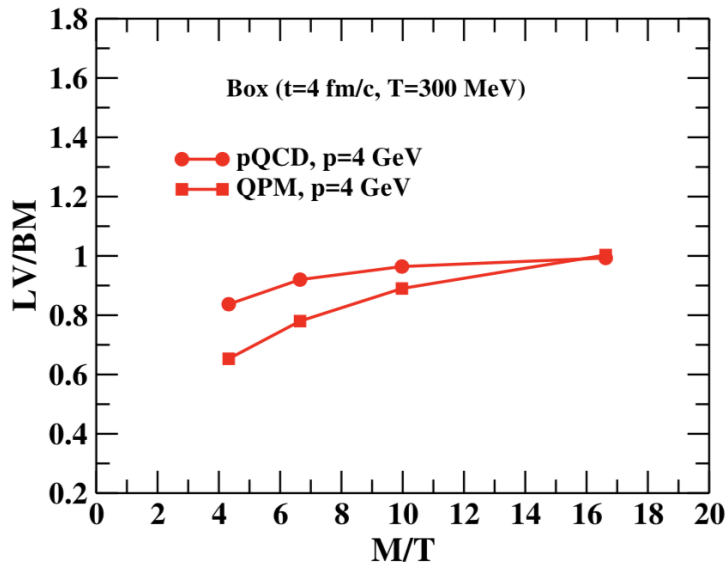
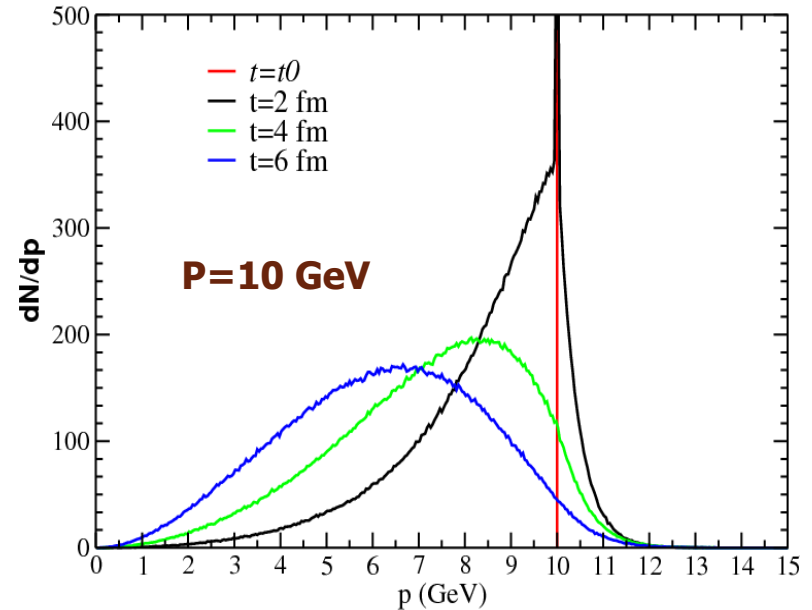
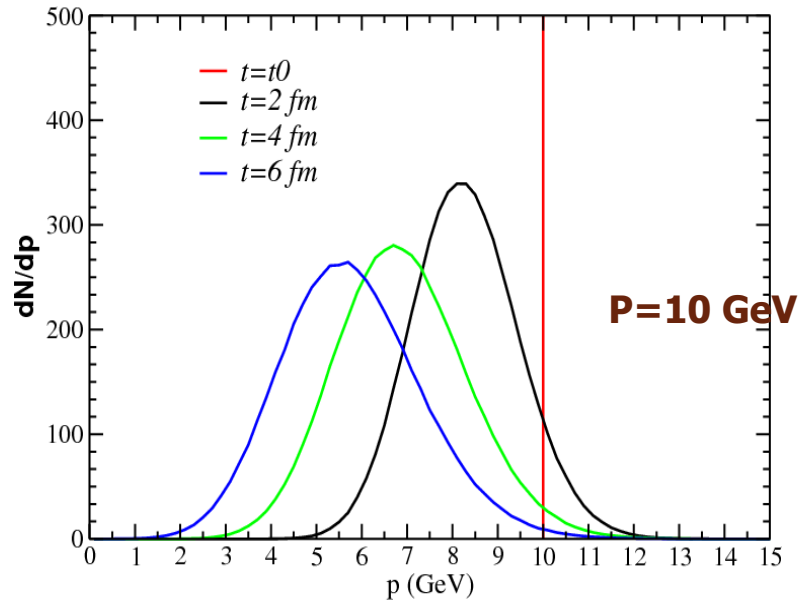
Das, Scardina, Plumari and Greco  
PRC,90,044901(2014)

# Momentum evolution starting from a $\delta$ (Bottom)

Langevin

In a Box

Boltzmann



Das, Scardina, Plumari and Greco  
PRC,90,044901(2014)

Langevin dynamics overestimate the interaction  
Boltzmann generate more  $v_2$  for the same RAA.

Rapp et. al. EMMI-RRTF, NPA 979 (2018)

# Hadronization: Coalescence plus Fragmentation

Fragmentation function gives the probability to get a hadron from a parton:

$$f_H(p_T) = \sum_p f_p(p_T / z) \otimes D_{p \rightarrow H}(z)$$

$\langle z \rangle \sim 0.9$  for charm quark and  $\langle z \rangle \sim 0.5$  for light quark

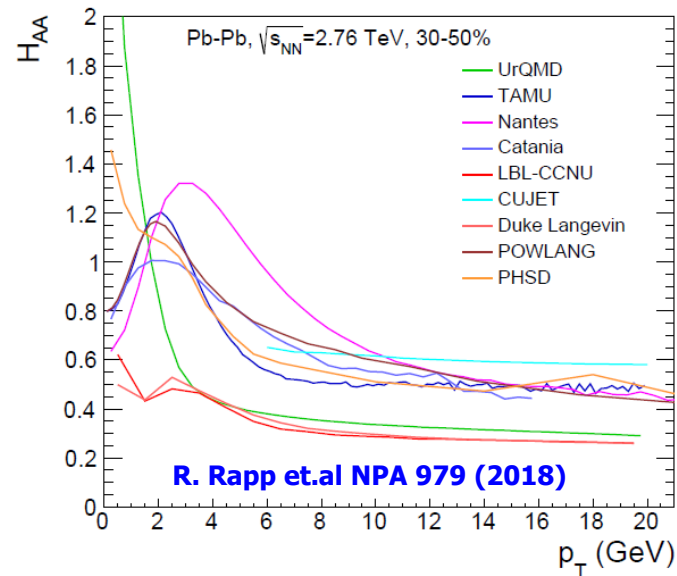
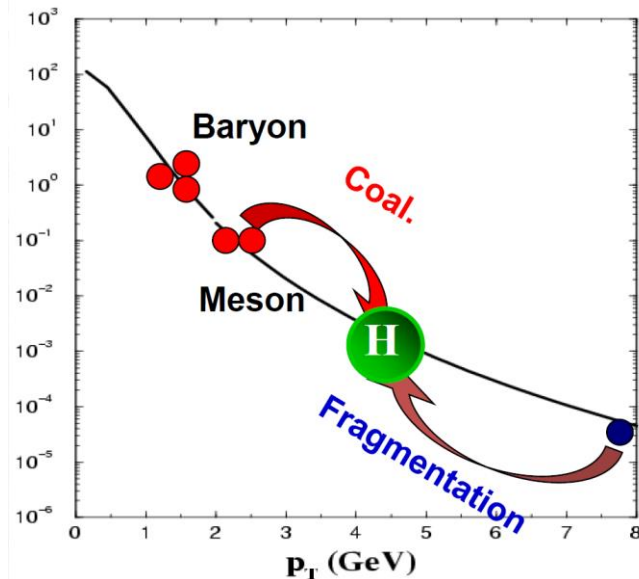
Coalescence is the convolution of two /three parton distribution folded by a wave function:

$$\frac{dN_{Meson}}{d^2 p_T} = g_M \sum_{i,j} P_q(i) P_q(j) \delta^{(2)}(p_T - p_{iT} - p_{jT}) f_M(x_i, x_j; p_i, p_j)$$

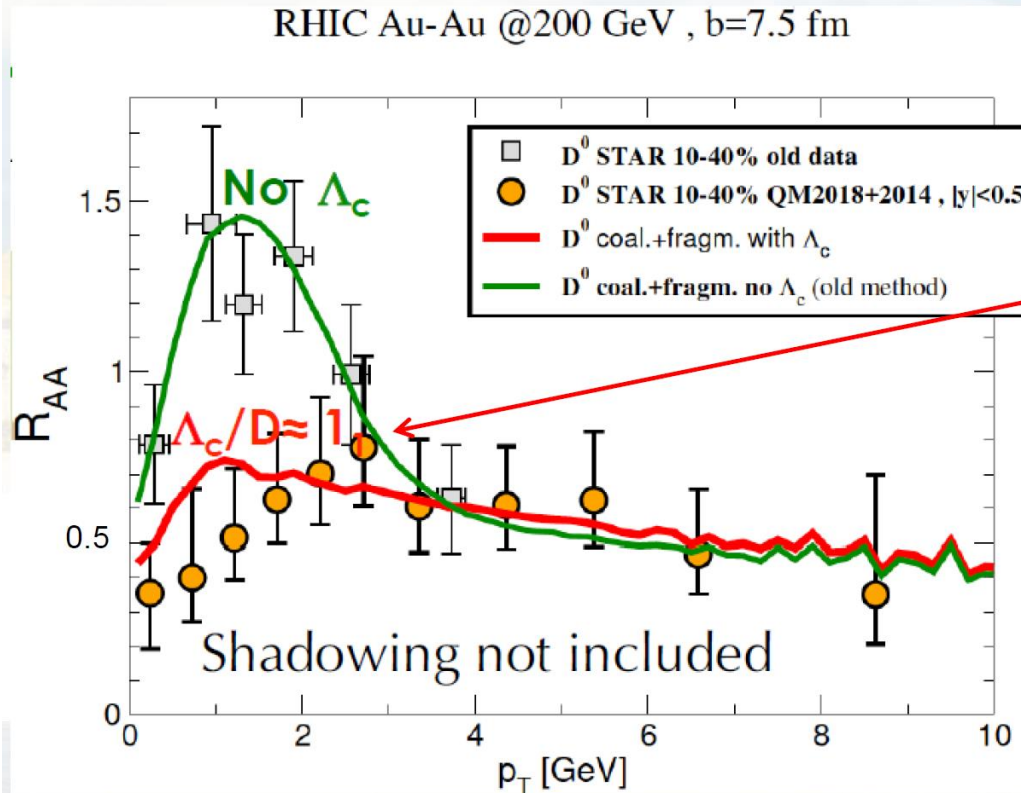
V. Greco, C.M. Ko, and P. L'evai  
PRL 90, 202302 (2003)

Hadron wave function

$$\frac{dN_{Baryon}}{d^2 p_T} = g_B \sum_{i,j,k} P_q(i) P_q(j) P_q(k) \delta^{(2)}(p_T - p_{iT} - p_{jT} - p_{kT}) f_B(x_i, x_j, x_k; p_i, p_j, p_k)$$



# Impact of heavy baryon to meson ratio on heavy quark suppressions



## New STAR data in QM2018

- Big effect at RHIC where coalescence dominates
- Smaller but still significant also at LHC

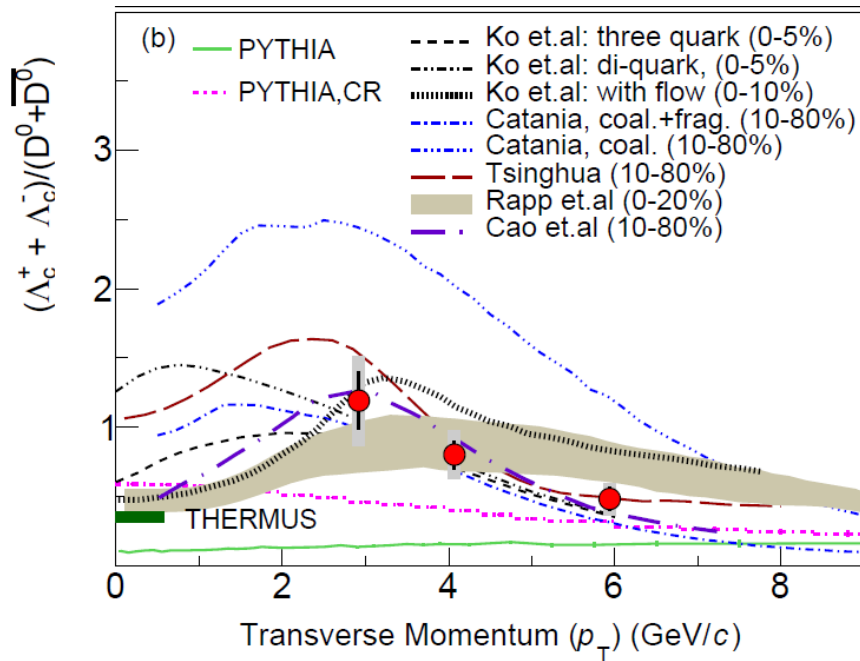
$R_{AA}$  of  $D^0$  decreases because part of charm quark makes coalescence in charmed Lambdas, while in pp charm quarks fragment mainly in D mesons

Minissale et.al (SQM-2019)

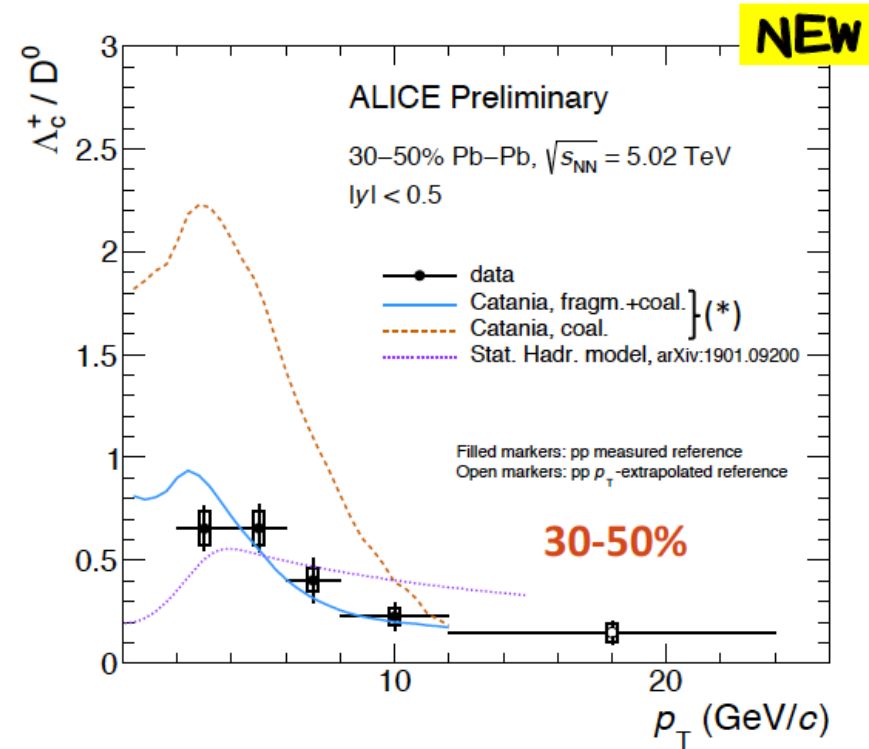


# Heavy Baryon to meson ratio

(Serve as a tool to disentangle different hadronization mechanisms)



STAR, Phys. Rev. Lett. 124, 172301 (2020)



$$P_{coal}=1$$

to all hadron at  $p \rightarrow 0$

Baryon in resonance recombination model

Plumari et al. EPJC 78 (2018) 4, 348

He, Rapp, PRL 124 (2020) 042301



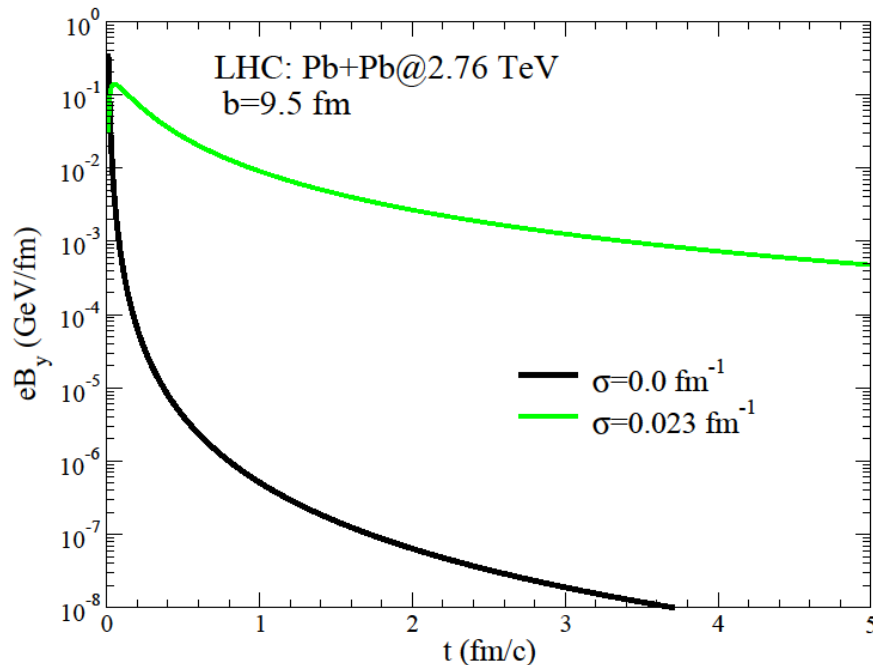
# Impact of EM field on heavy quark dynamics at LHC

$$dp_j = -\Gamma p_j dt + \sqrt{dt} C_{jk}(t, p + \xi dp) \rho_k + F_{ext} dt$$

$$F_{ext} = q(E' + v \times B')$$

$$E' = \gamma(E + v \times B) - (\gamma - 1)(E \cdot \hat{v}) \hat{v}$$

$$B' = \gamma(B - v \times E) - (\gamma - 1)(B \cdot \hat{v}) \hat{v}$$



**Electromagnetic field has been included in the Langevin equation as a external force.**

**We consider both E and B.**

$$B_x=B_z=0$$

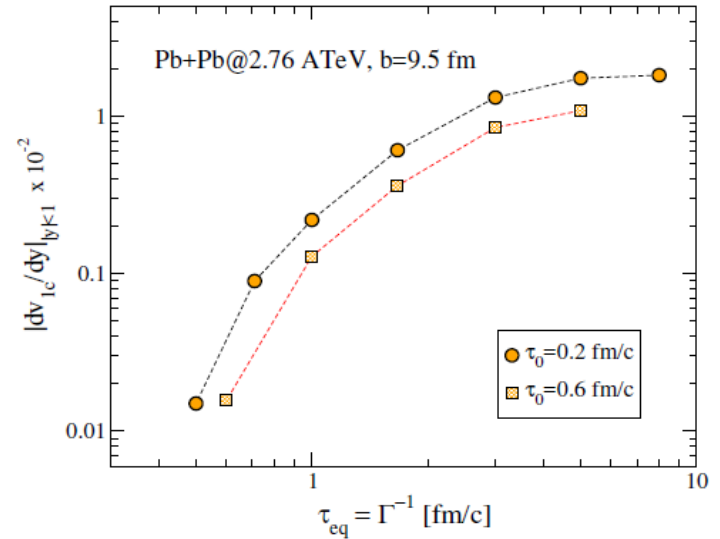
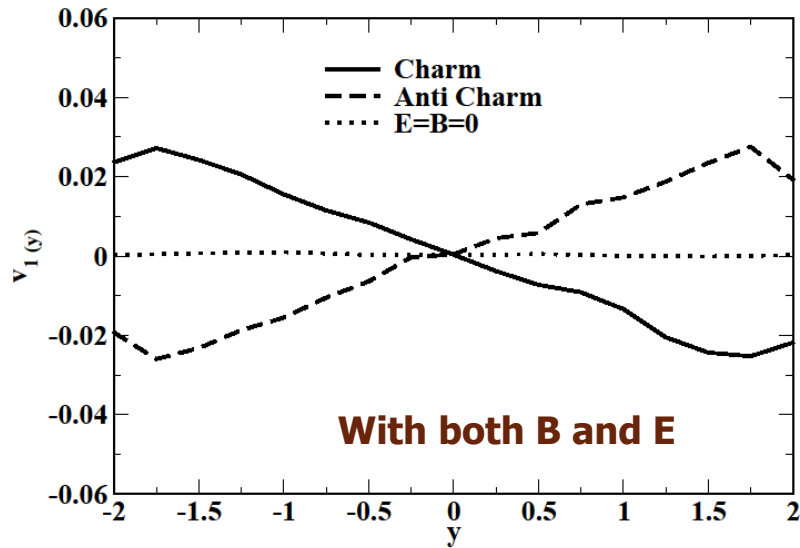
$$\text{And } E_y=E_z=0$$

$$v_1 = \langle \frac{p_x}{p_T} \rangle$$

**Gursoy, Kharzeev and Rajagopal  
PRC 89, 054905 (2014)**

**Das, Plumari, Chartarjee, Scardina, Greco, Alam  
Phys. Lett. B, 768 (2017) 260**

# Heavy quark $v_1$ @LHC

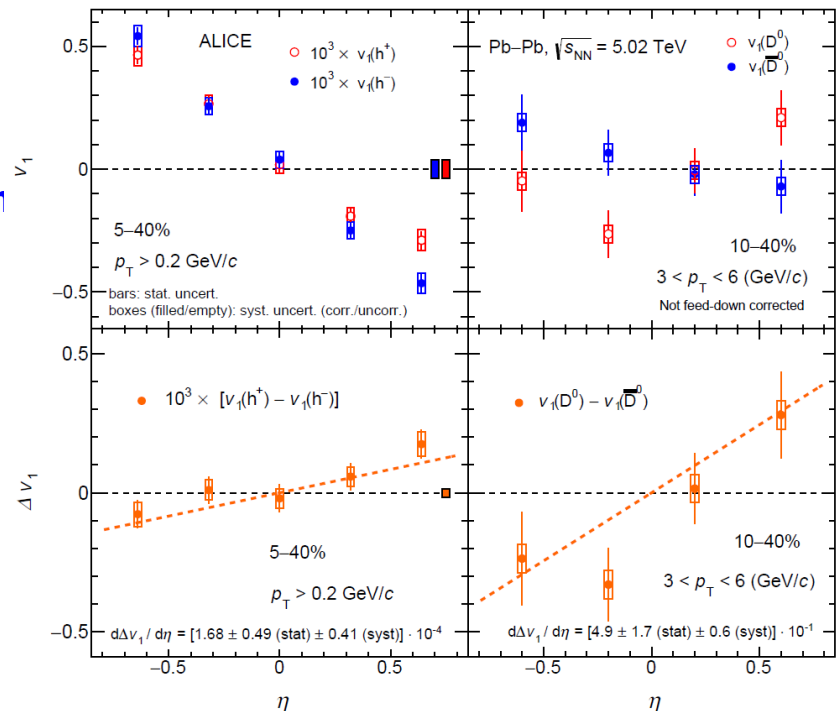


**Das, Plumari, Chartarjee, Scardina, Greco, Alam**  
**Phys. Lett. B, 768 (2017) 260**

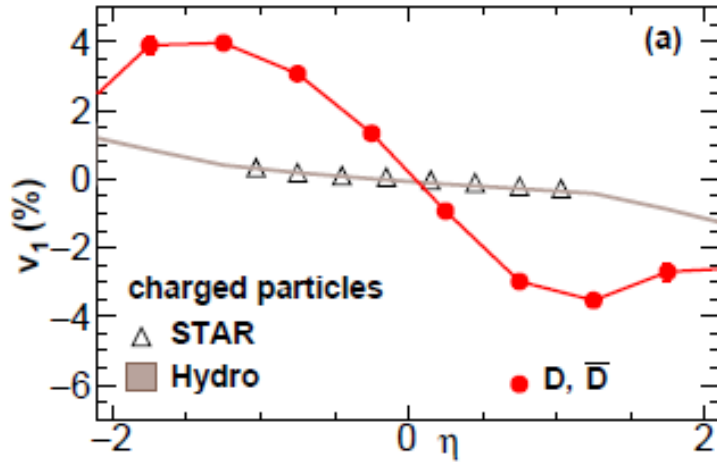
**Heavy quark  $v_1$  is larger than light quark  $v_1$ .**

**Recent data from ALICE indicates splitting  
in D and Dbar  $v_1$ .**

**ALICE Collaboration, PRL 125 (2020) 2, 022301**

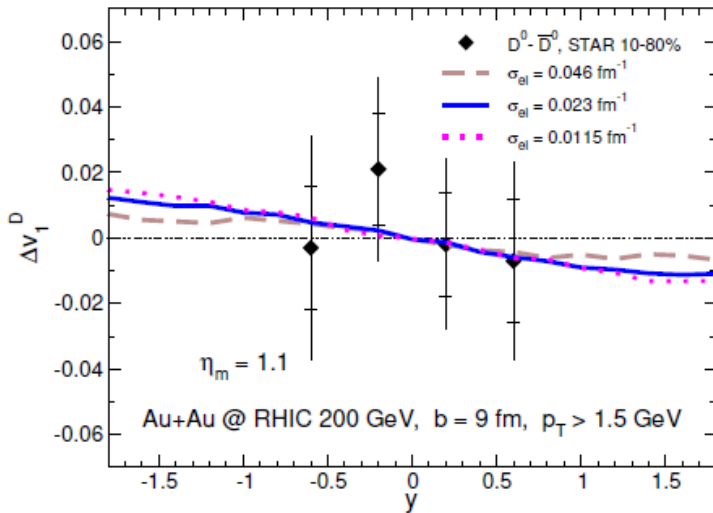


# Initial vorticities and electromagnetic field

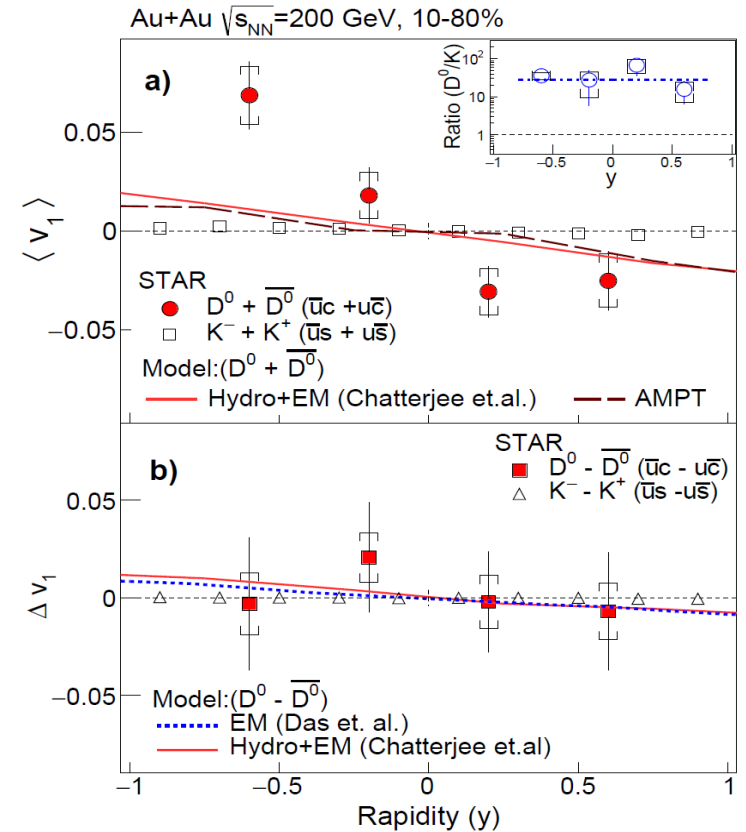


**Impact of tilt bulk: Forward backward asymmetry**

*Chatterjee and Bozek, PRL 120 (2018) 192301*



*Oliva, Plumari, Greco, arxiv:2009.11077 [hep-ph]*



*STAR, PRL 123 (2019) 16, 162301*

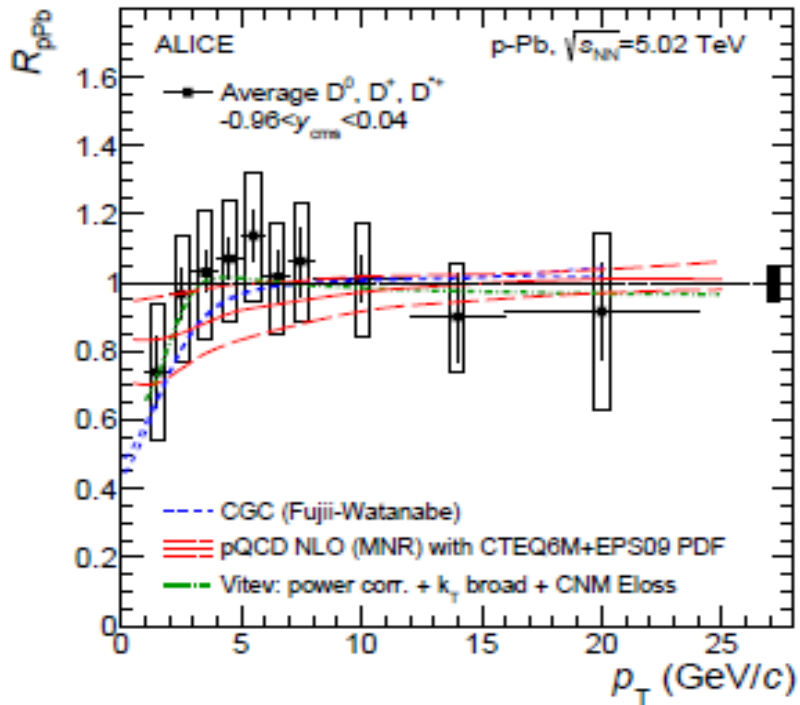
**Heavy quark Transport coefficient in Magnetic field:**

*K. Fukushima et al. PRD, 93 (2016) 074028*

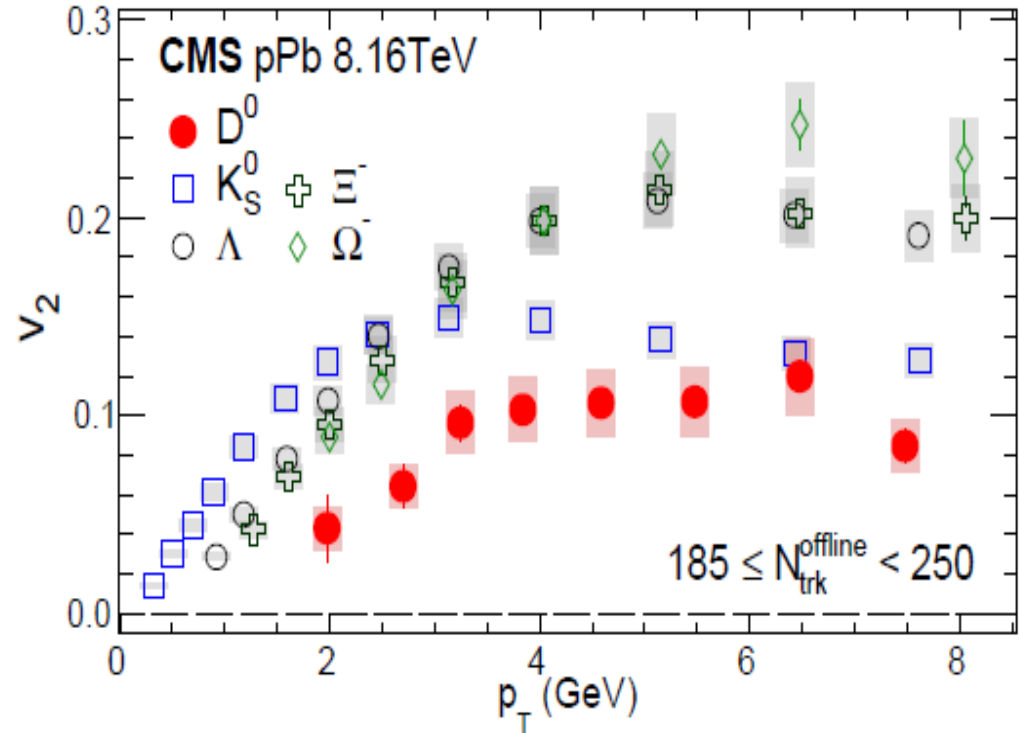
*M. Kurin et al. PRD, 101 (2019) 074003*

*B. Singh et al. arXiv:2004.11092*

# Heavy quark in small system (p-nucleus)



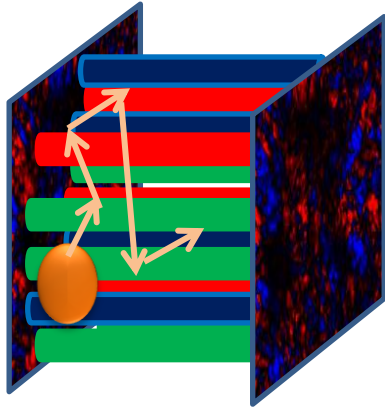
ALICE Collaboration  
 Phys. Rev. Lett. 113 (2014) 232301



CMS Collaboration  
 arXiv:1804.09767v2

What mechanism could build up  $v_2$  without energy loss?

# Heavy quarks as probes of the evolving Glasma



(Adapted from M. Ruggieri)

$$t_{\text{formation}} \approx \frac{1}{2m_c} \approx 0.06 \text{ fm}/c$$

[ Talk by Mueller on Monday ]

[Talk by Boguslavski on Tuesday]



***HQs can probe the very early evolution of the Glasma fields***

**Hamilton equations of motion of  $c$ -quarks:**

$$\frac{dx_i}{dt} = \frac{p_i}{E} \quad E = \sqrt{p^2 + m^2}$$

$$v \equiv \frac{p}{E} \quad \text{(Relativistic) Velocity}$$

$$E \frac{dp_i}{dt} = gQ_a F_{i\nu}^a p^\nu,$$

$$\frac{dp}{dt} = qE + q(v \times B) \quad \text{Lorentz force}$$

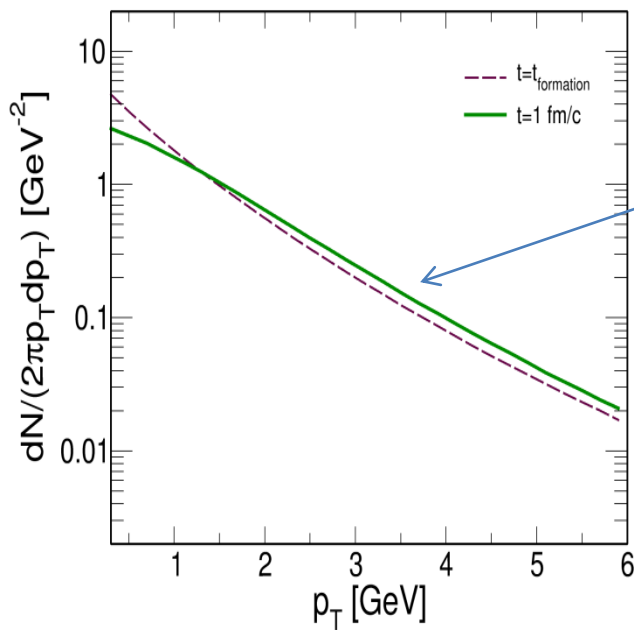
$$E \frac{dQ_a}{dt} = -gQ_c \varepsilon^{cba} A_b \cdot p$$

Wong (1979)

$$D_\mu J_a^\mu = 0 \quad \text{Gauge-invariant conservation of the color Current carried by charm quarks + gluons}$$

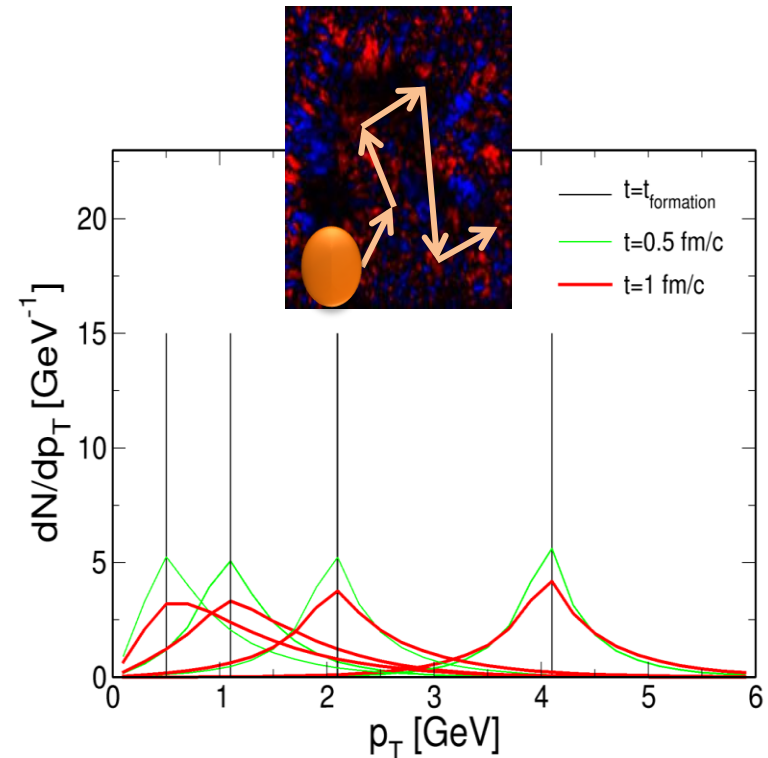
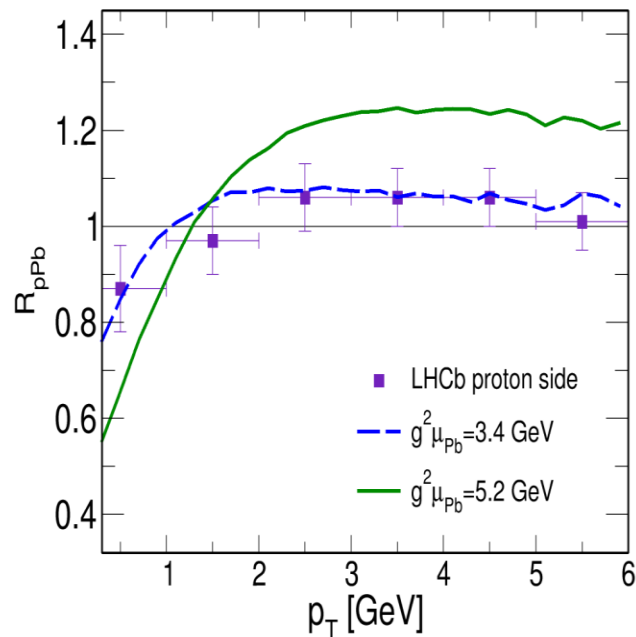
$$J_a^\mu = \bar{c} \gamma^\mu T_a c$$

***Equations of motion of heavy quarks are solved in the background given by the evolving Glasma fields***

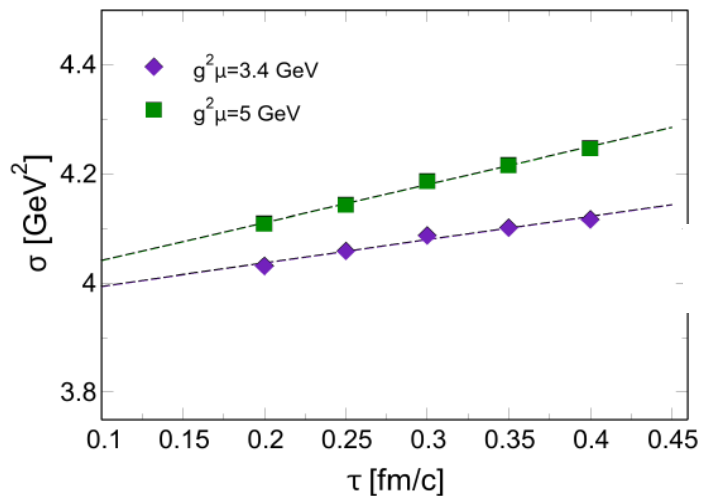
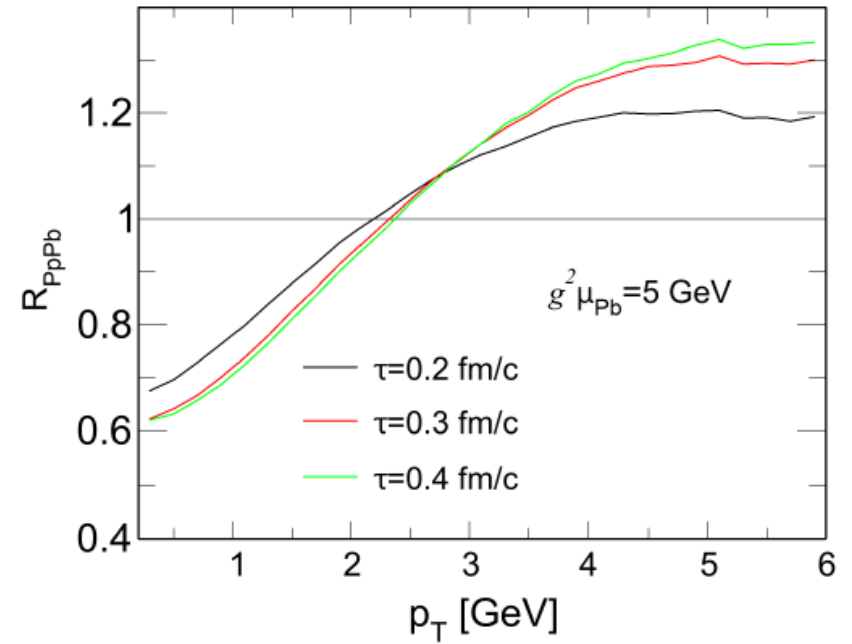
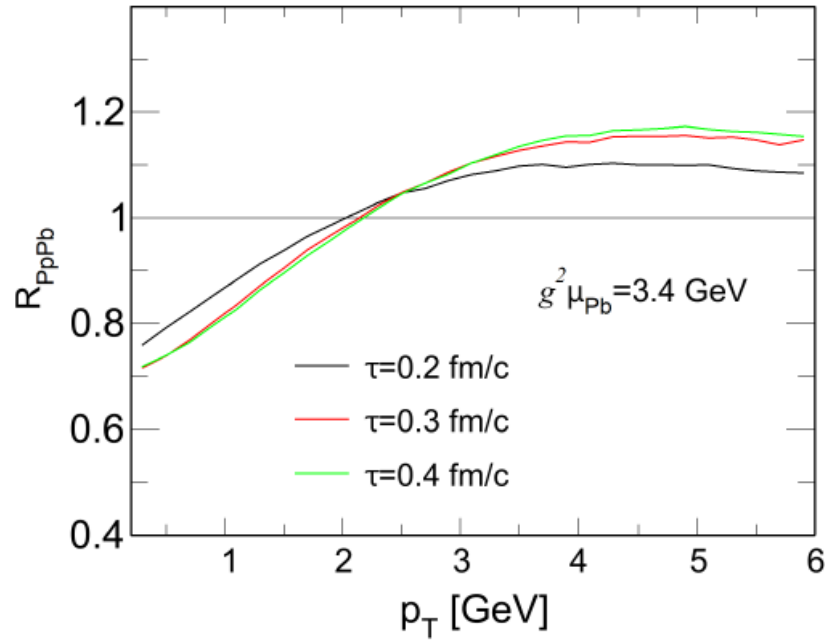


**Initial distribution: from perturbative QCD**  
**Evolution: interaction with the Glasma**

**$R_{pPb} \neq 1$**   
**Interaction with the fields created by the collision**



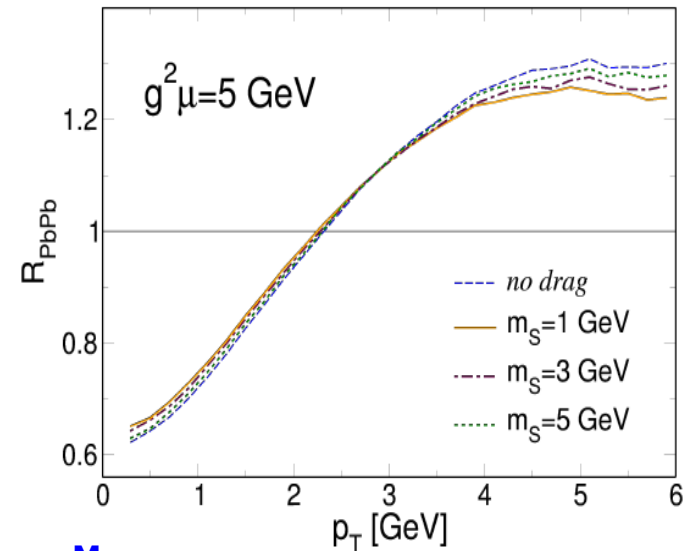
# Heavy quark dynamics in Expanding Glasma



$$E \frac{dp_i}{dt} = Q_a F_{i\nu}^a p^\nu - E \Gamma p_i,$$

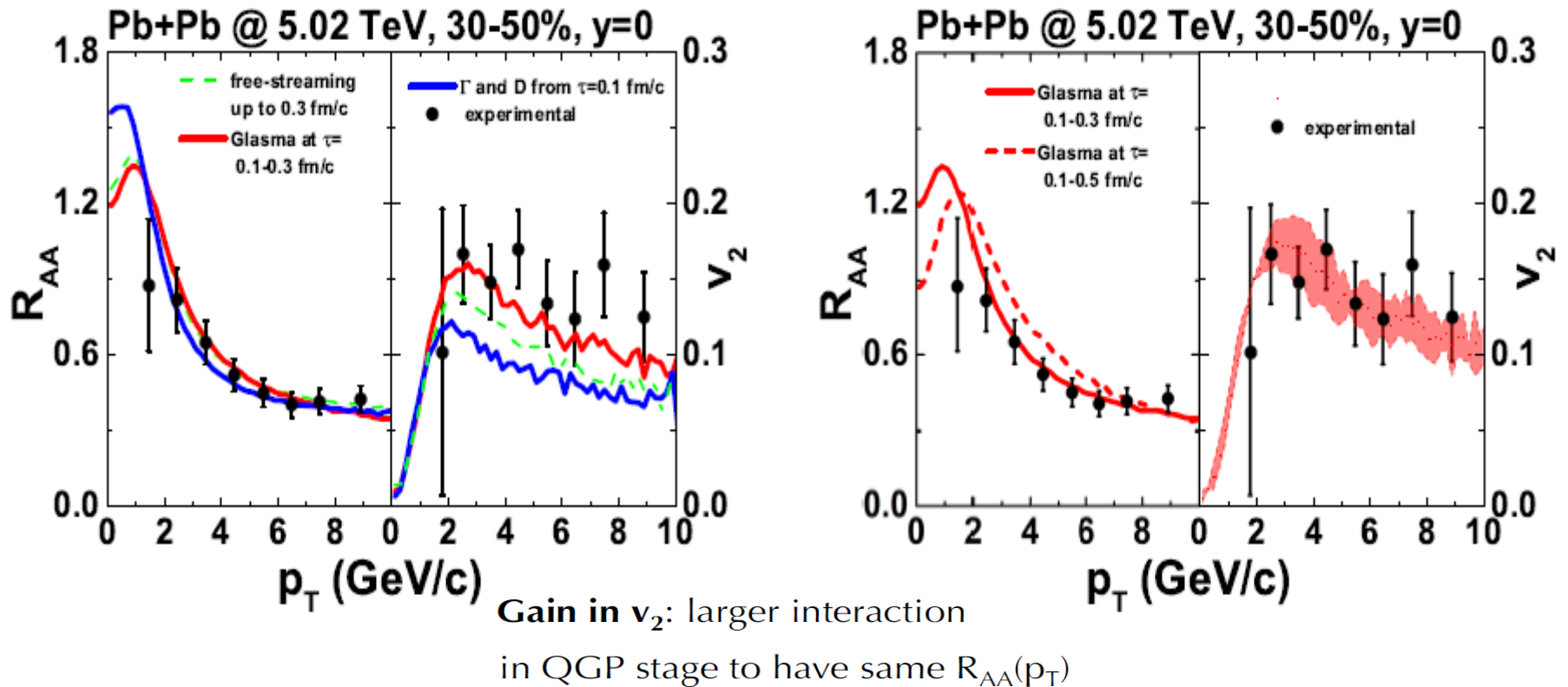
$$D = \Gamma E \tau,$$

$$\sigma \equiv \langle (p_T - \langle p_T \rangle)^2 \rangle = 2D\tau + \text{constant}$$



Liu, Plumari, Das, Greco, Marco  
PRC, in press

# Impact of Glasma on a heavy quark observables at LHC (Heavy quark dynamics in Glasma plus Plasma)



**This indicates an initial pre-thermal stage is unlikely to be described in terms of a standard drag and diffusion dynamics, because even if one tune such coefficients to reproduce the same  $R_{AA}(p_T)$  this would imply a significantly smaller  $v_2$ .**



# Few recent works on heavy quark diffusion in pre-equilibrium phase

*S. Mrowczynski, EPJA 54 (2018) 3, 4*

*M. Carrington et al. NPA 1001 (2020) 121914*

*K. Boguslavski et al. JHEP 09 (2020) 077*

Talk by K. Boguslavski on Tuesday

A. IPP et al. 2009.14206 [hep-ph]

[ Talk by D. Mueller on Monday ]

## Within Kinetic Theory:

S. K. Das et al. JPG, 44 (2017) 095112

T. Song et al. PRC, 101 (2020), 044901

Impact of pre-equilibrium phase is significant

# Conclusions and Perspectives:

- ❖ **Open Heavy Flavor Physics at RHIC and LHC**

$R_{AA}$  and  $v_2 \rightarrow D_s(T) \rightarrow IQCD$

- ❖ **More precision data and New Observables**

$V_n(HQ) - V_n(LQ)$ ,  $\Lambda_c$ ,  $dN/d\Phi$ , System size scan and bottom quark observables, will allow significant advantage to understand the hot QCD matter

- ❖ **Heavy quark  $V_1 \rightarrow$  EM field and Angular momentum**

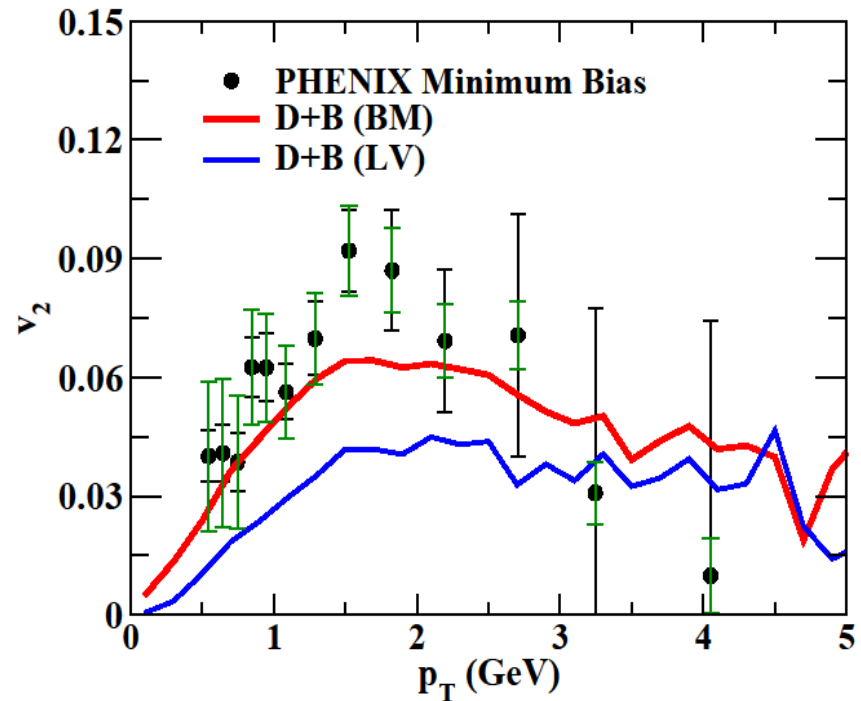
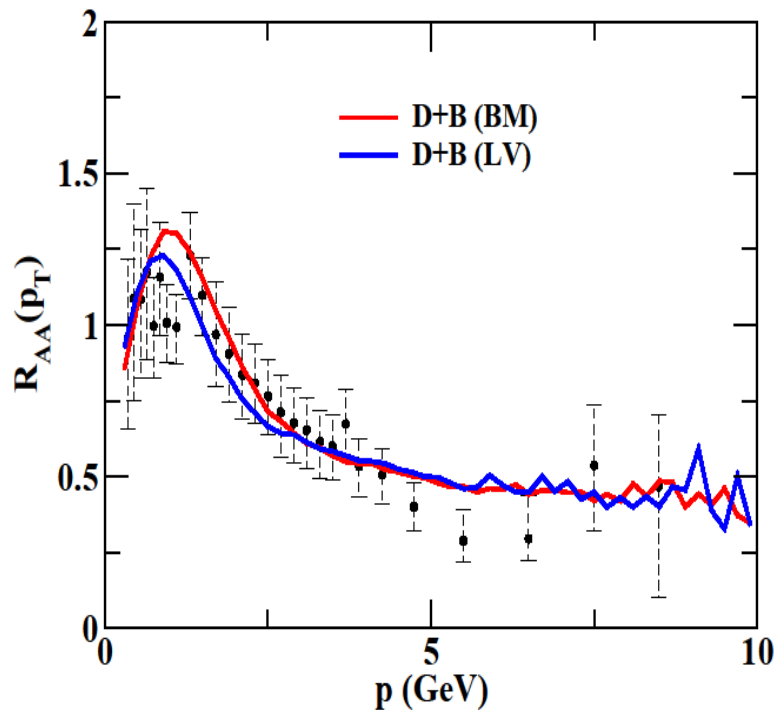
- ❖ **Heavy quark as a probe of the pre-equilibrium phase  $\rightarrow$  Glasma**

Thank You



# $R_{AA}$ and $v_2$ at RHIC

(With near isotropic cross-section)

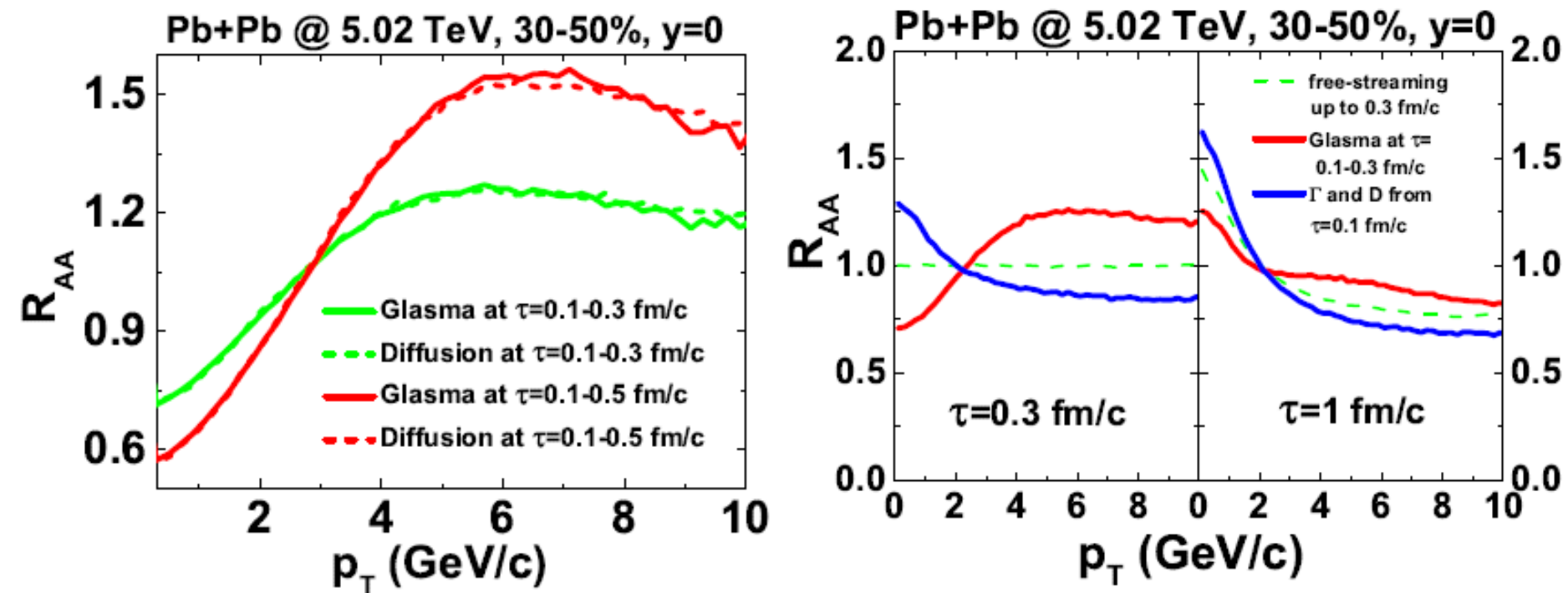


Das, Scardina, Plumari and Greco  
PRC,90,044901(2014)

At fixed  $R_{AA}$  Boltzmann approach generate larger  $v_2$  .  
(depending on  $m_D$  and  $M/T$ )

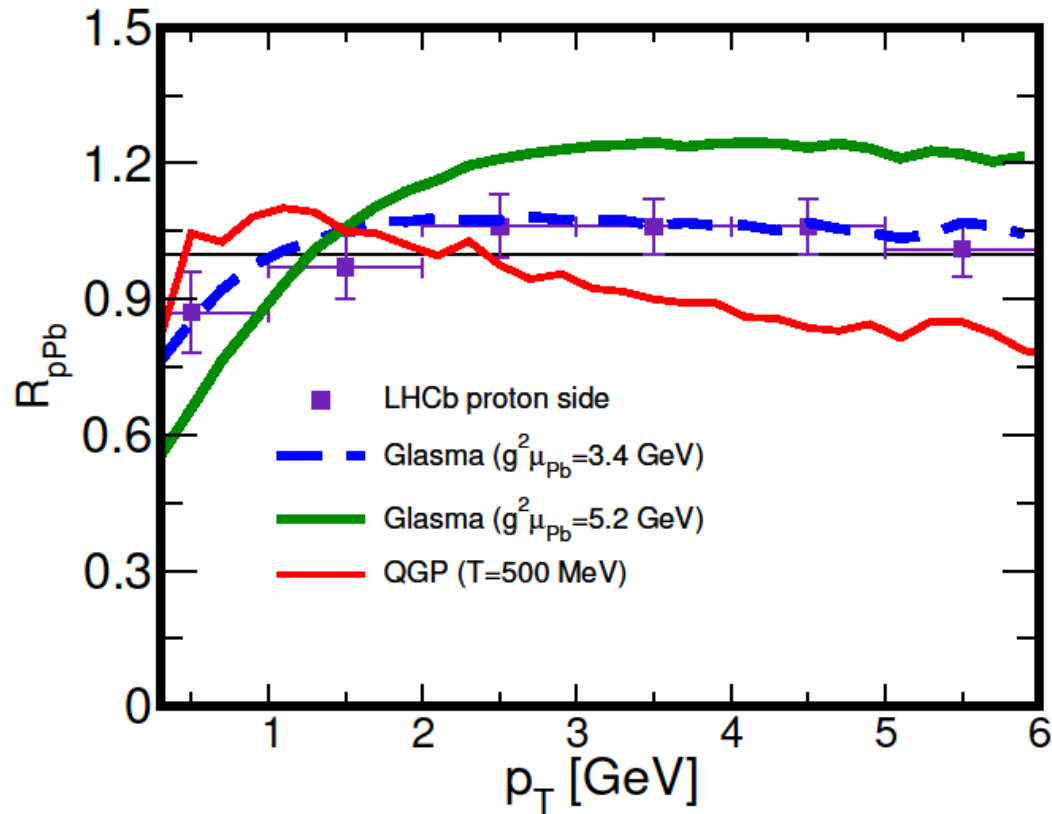
With isotropic cross section one can describe both  $R_{AA}$  and  $V_2$   
simultaneously within the Boltzmann approach

## Impact of Glasma on a heavy quark observables at LHC (Glasma vs Plasma)



**Glasma induce a diffusion of charm quarks in momentum space resulting in a tilt of their spectrum without a significant drag.**

# Heavy quark suppression in pPb: Glasma vs Plasma

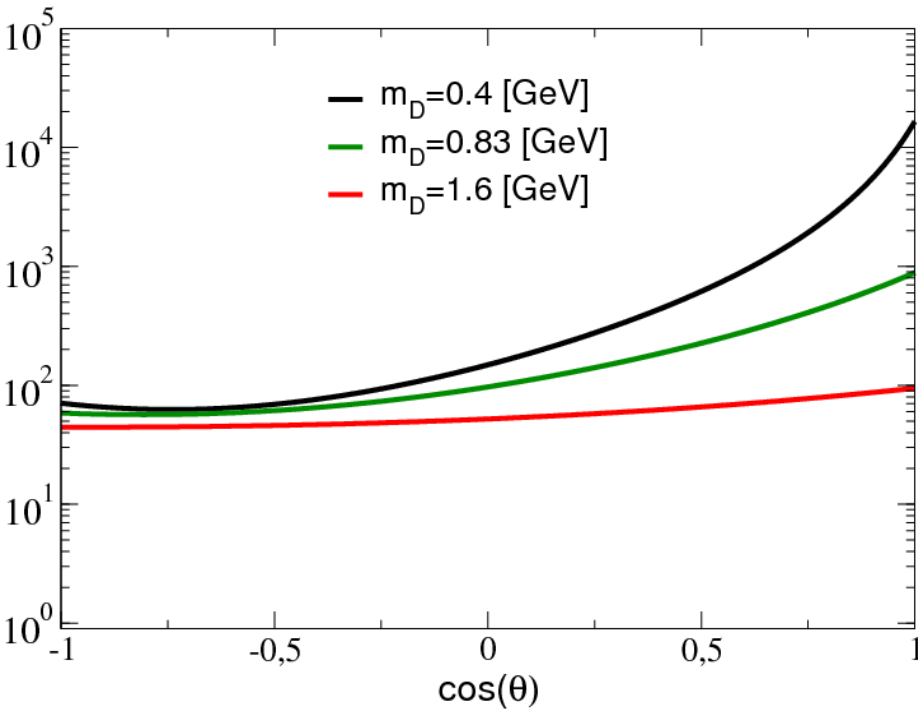


In Plasma: high momentum particle loose energy shifted to low momentum domain.  
In Glasma: low momentum particle get accelerated and shifted to high momentum domain

# Boltzmann vs Langevin (Charm)

$T=400$  MeV

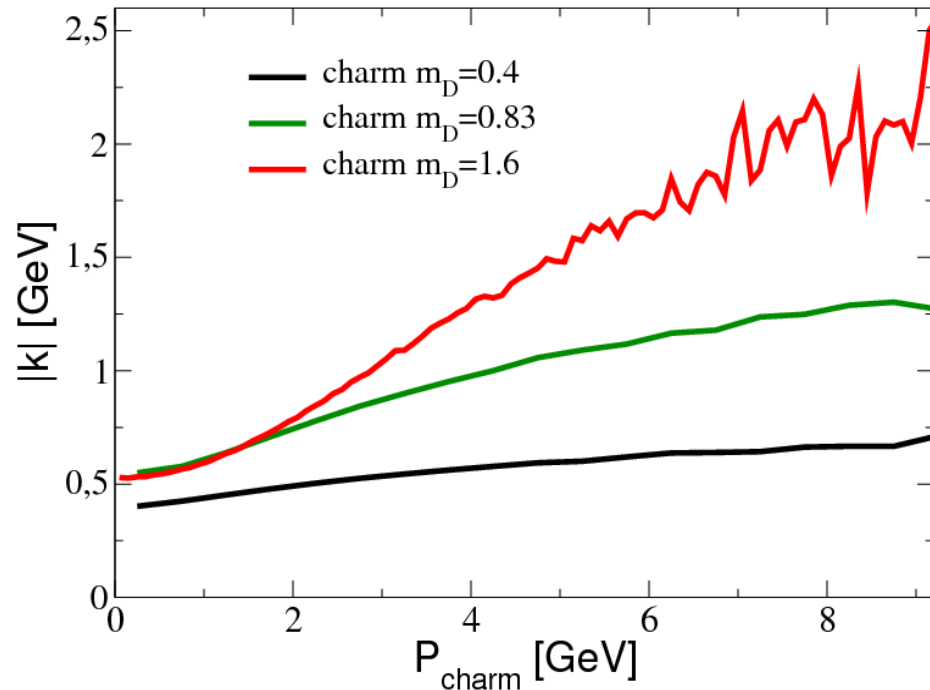
## Angular dependence of $\sigma$



Decreasing  $m_D$  makes the  $\sigma$  more anisotropic

Hees, Greco, Rapp, PRC, 73, 034913 (2006)

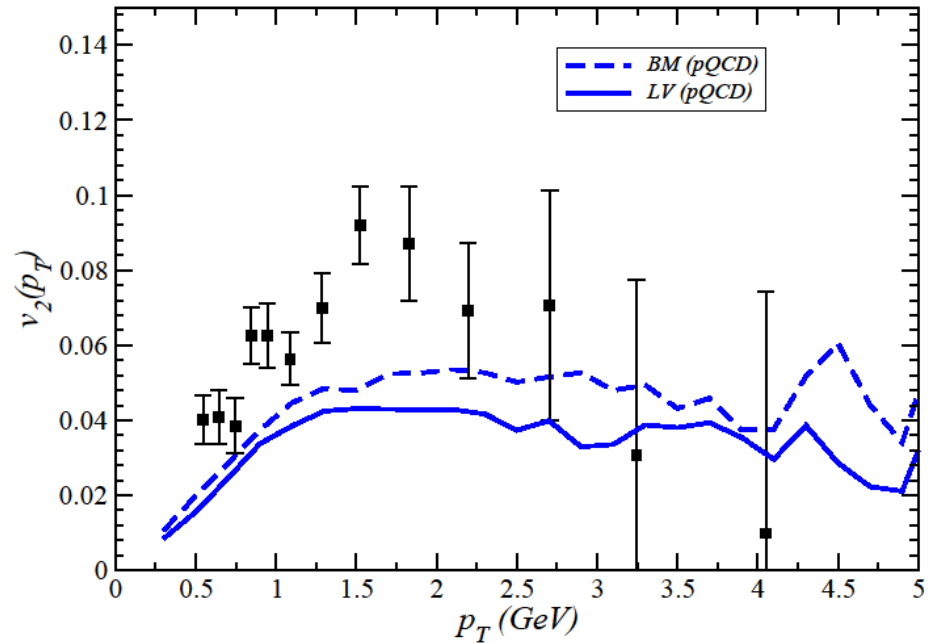
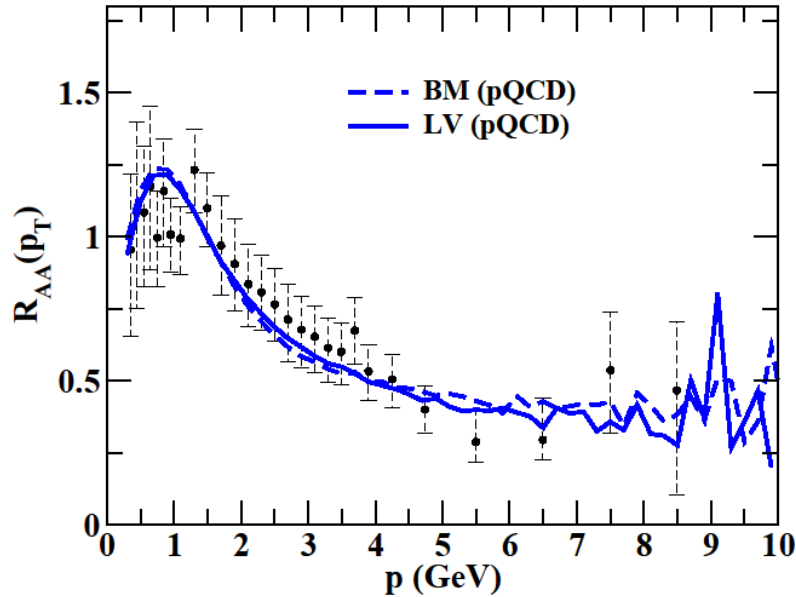
## Momentum transfer vs P



→ Smaller average momentum transfer

Das, Scardina, Plumari and Greco  
PRC, 90, 044901 (2014)

# $R_{AA}$ and $v_2$ at RHIC at $mD=gT$

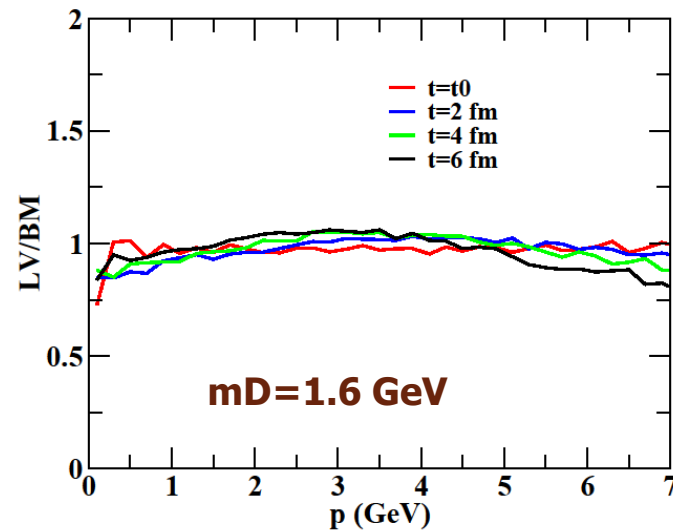
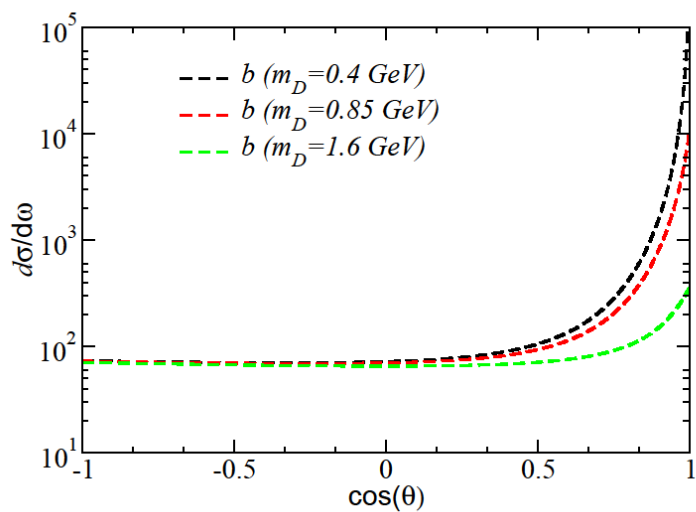
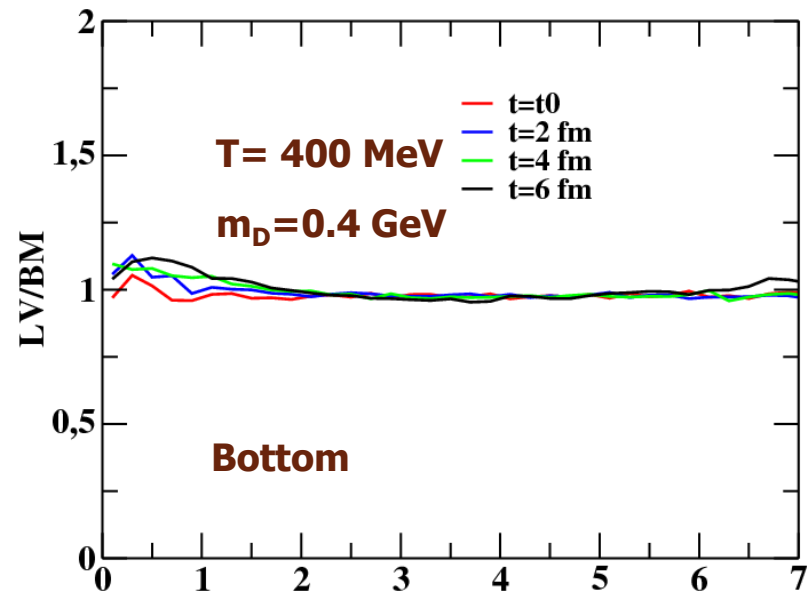
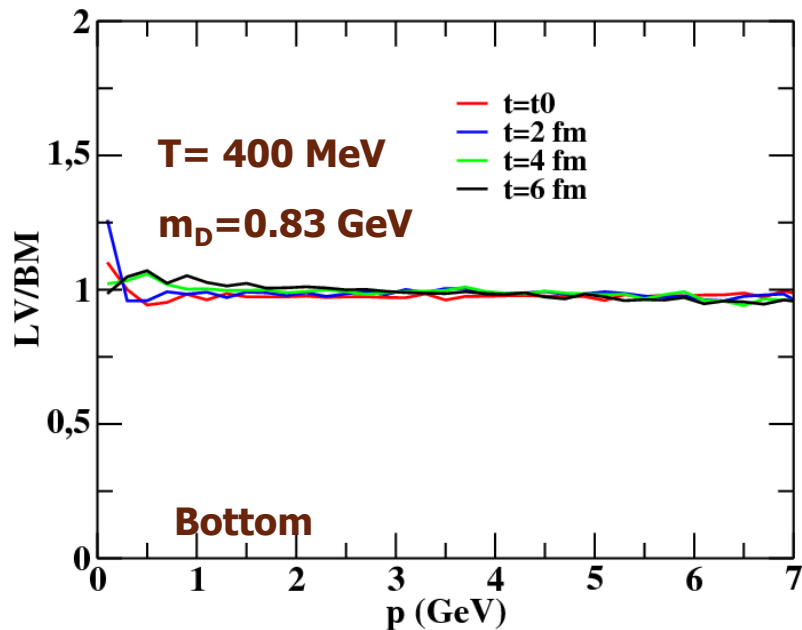


Das, Scardina, Plumari and Greco  
PRC,90,044901(2014)

**At fixed RAA Boltzmann approach generate larger  $v_2$  .  
(depending on  $mD$  and  $M/T$ )**

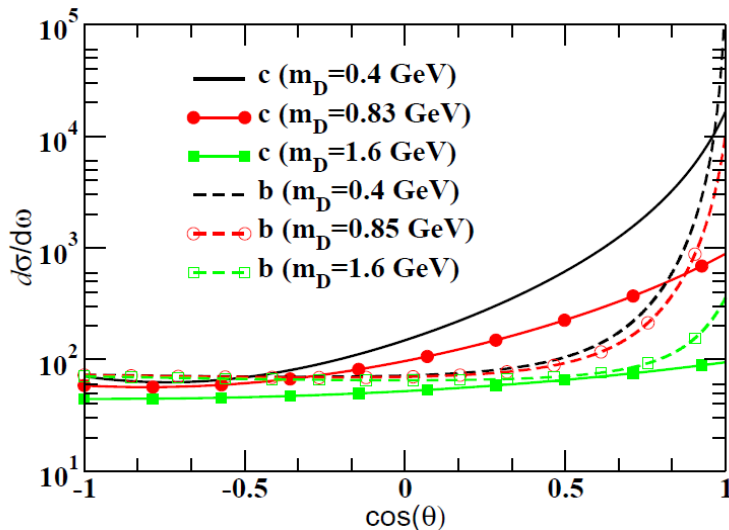
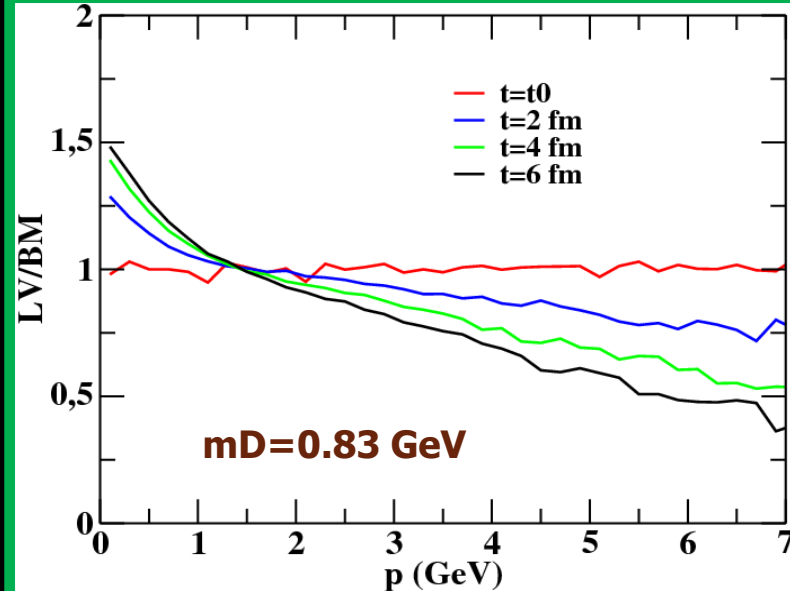
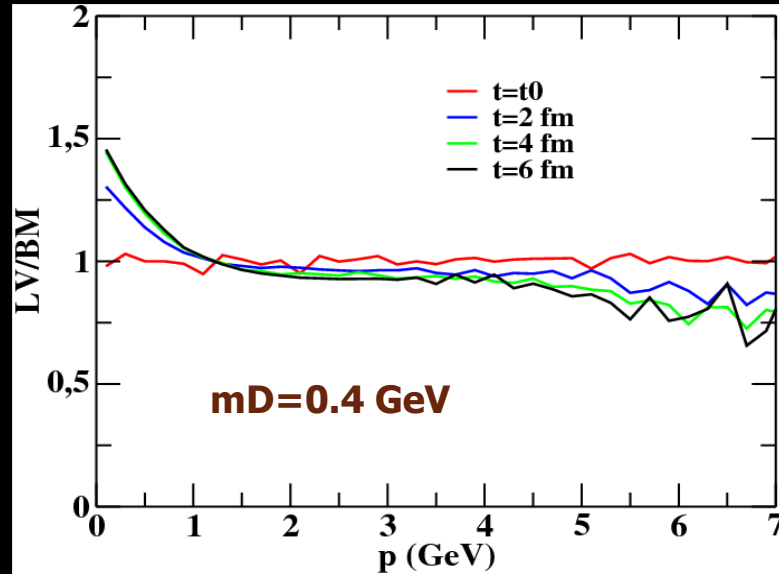


# Bottom: Boltzmann = Langevin



But Larger  $M_b/T$  ( $\approx 10$ ) the better Langevin approximation works

# Boltzmann vs Langevin (Charm)



Das, Scardina, Plumari and Greco  
PRC,90,044901(2014)

