Pseudoscalars From 'Dark' Extra Dimensions: The 'Dark matter' Problem

Sandipan Sengupta

Dept of Physics
IIT Kharagpur
INDIA

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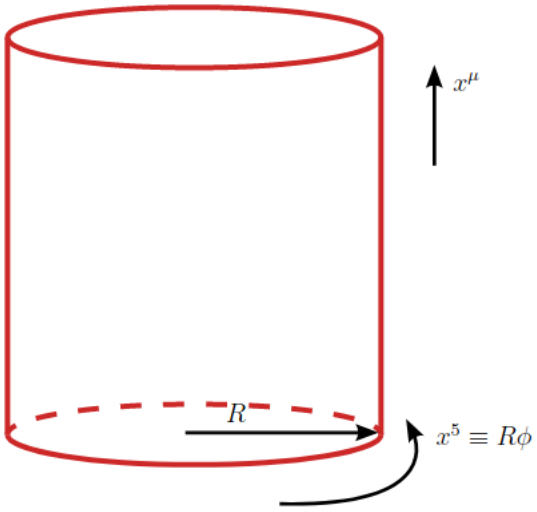
The lure of extra dimensions

• In physics, there has been a few remarkable instances where an idea itself has gone farther than the original purpose behind its very inception

• Perhaps, the idea of extra dimensions is one such example

• One of the earliest applications was due to Kaluza in his bold attempt to unify Einstein gravity and Maxwell theory in one higher dimension; Klein proposed the justification as to why we won't see this 5th dim (compactification)
(Kaluza '21, Klein '26;
Nordstrom '14, Weyl '18)

The Kaluza-Klein idea



- Other scenarios:

Large (Arkani-Hamed, Dimopoulos, Dvali, '98; Antoniadis+ADD '98),
Warped (Randall-Sundrum '99),
..
and so on

- However, in general, any such formulation predicts an infinite tower of massive KK excitations which should be seen at the colliders at some energy: proof of extra dimensions! No trace till now though

Alternative ideas on extra dimension and the 'Dark matter' problem

• Here, we discuss a new formulation based on extra dimension of vanishing proper length (in vacuum gravity), and show why such a framework could be relevant to the 'dark matter' physics

• Further, it exhibits a pseudoscalar ('axion-like'??) structure, with potential implications

• Simpler, in the sense that:
a) the 5th dim is not detectable in principle,
b) one does not have to worry about whether it should be small or large or warped (and the associated nontrivial phenomenology, e.g. KK towers etc)

Extra dimension of 'zero (proper) length': The proposed dynamical theory

• 5D action in vacuum (coordinates t, x, y, z, v):

$$\mathcal{S} = \frac{1}{L^3}\int d^5x\ \epsilon^{\mu\nu\alpha\beta\rho}\epsilon_{IJKLM}\ \hat{e}^I_\mu \hat{e}^J_\nu \hat{e}^K_\alpha \hat{R}_{\beta\rho}{}^{LM}(\hat{w})$$
$$\equiv \frac{1}{L^3}\int d^5x\ \epsilon_{IJKLM}\hat{e}^I \wedge \hat{e}^J \wedge \hat{e}^K \wedge \hat{R}^{LM}(\hat{w})$$

• 5D EOM:
$\delta\hat{w}:\ \epsilon_{IJKLM}\ \hat{e}^I \wedge \hat{e}^J \wedge D\hat{e}^K = 0$
$\delta\hat{e}:\ \epsilon_{IJKLM}\ \hat{e}^I \wedge \hat{e}^J \wedge \hat{R}^{KL}(\hat{w}) = 0$

• To explore the consequences of an extra dimension of zero metrical length, we need to find the most general soln to these EOM where $\hat{e}^I_\mu$ has a zero eigenvalue
$\det \hat{g}_{\mu\nu} = \det[\hat{e}^I_\mu \hat{e}_{\nu I}] = 0$

• The zero may be chosen to lie along the 5th direction (v): $\hat{e}^I_v = 0$, implying: $\hat{e}^I_\mu = \begin{bmatrix} e^i_a & 0 \\ 0 & 0 \end{bmatrix}$
5-inverse $\hat{e}^\mu_I$ does not exist ($\det \hat{e} = 0$), but 4-inverse e^a_i of the tetrads does ($\det e \neq 0$)

Emergent Einstein theory in 4D

- A purely geometric stress-tensor emerges:

$$\bar{R}_{ab} - \frac{1}{2} g_{ab} \bar{R} = t_{ab} + [L^2]_{ab} + [S^2]_{ab} - \chi^2 g_{ab}$$

$$\left[L^2\right]_{ab} = 2 L_a L_b + L_c L^c g_{ab},$$

$$\left[S^2\right]_{ab} = -[2 S_a{}^c S_{cb} - S_{cd} S^{cd} g_{ab}]$$

- The last 3 pieces come from the nonvanishing torsion in the 5-geometry defined by the noninvertible pentad $\widehat{e}$

- Basic fields appear in algebraic combinations and not through derivatives: not dynamical

- Only gravitational intn, since cannot couple to matter in 5D

- $\omega_t = \frac{1}{3},\ \omega_L = -\frac{1}{3},\ \omega_S = 1,\ \omega_\chi = -1$

- Demanding the individual energy densities to be nonnegative, the equation of state of the (t, L, S) fluid is bounded: $-\frac{1}{3} \leq \omega \leq 1$

New physics: Not emergent 'matter'

$\bar{R}_{ab} - \frac{1}{2} g_{ab} \bar{R} = t_{ab} - \chi^2 g_{ab} + [L^2]_{ab} + [S^2]_{ab} = 8\pi G T_{eff}$

• χ^2 is to be identified with the cosmological constant Λ, and sets the length scale of the multiplet (L, S) which have a common origin

• Hence, these fields are characterized by a large length scale (act over distances $\sim H_0^{-1}$)

• $\rho_{eff} \sim \frac{1}{Gl^2} \sim \frac{M_p}{l_p l^2}$ unlike ordinary matter scaling $\sim \frac{m}{l^3}$; No inherent mass-scale of their own (non-particulate)

• In other words, these fields cannot exchange energy through collisions with matter particles (inert)

• Even though one may invoke some 'effective theory' in 4D with a coupling to four dimensional propagating matter, these are always Planck-suppressed
(e.g. $L \sim l_P \sqrt{-g} S^{ab} \partial_a \phi \partial_b \phi$, $L \sim l_P^2 \sqrt{-g} L^a L^b \partial_a \phi \partial_b \phi$)

A potential resolution to the 'dark matter' problem

• In view of these unmistakably suggestive features, we propose:
The non-propagating geometric fields (t, L, S) could supercede the (notion of) 'dark matter' in vacuum gravity theory with an extra dim of vanishing proper length

• Even if this theoretical framework is provocative enough, it must face up to the challenge as to how much of the observational data it could explain; Needs a long and elaborate effort

• Also, does it suggest anything beyond the standard scenarios?

Pseudoscalar structure: Axionlike??

$\bar{R}_{ab} - \frac{1}{2} g_{ab} \bar{R} = t_{ab} + [L^2]_{ab} + [S^2]_{ab} - \chi^2 g_{ab}$

- There exists a unique pseudoscalar mode ϕ within the axial vector L_a; Contributes to $T^{(eff)}_{ab}$ as:

$L_a L_b + \frac{1}{2} L_c L^c g_{ab} \rightarrow l_P^2 \left[\partial_a \phi \partial_b \phi + \frac{1}{2} (\partial_c \phi \partial^c \phi) g_{ab} \right]$

- To be contrasted with the typical axionic contribution (sign, Planck suppression, eqn of state, ..)

- EOS is negative ($\omega_\phi = -\frac{1}{3}$)

Let us now apply our framework directly to the case of rotation curves in spiral galaxies, one of the main motivations behind the 'Dark Matter' hypothesis

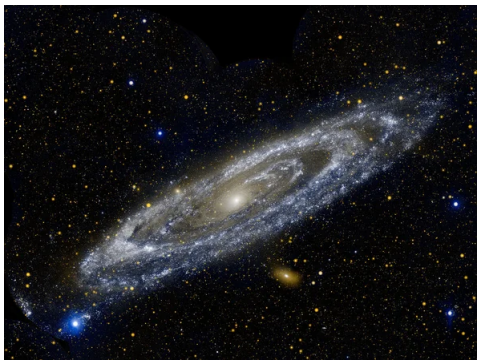
Galactic rotation curves

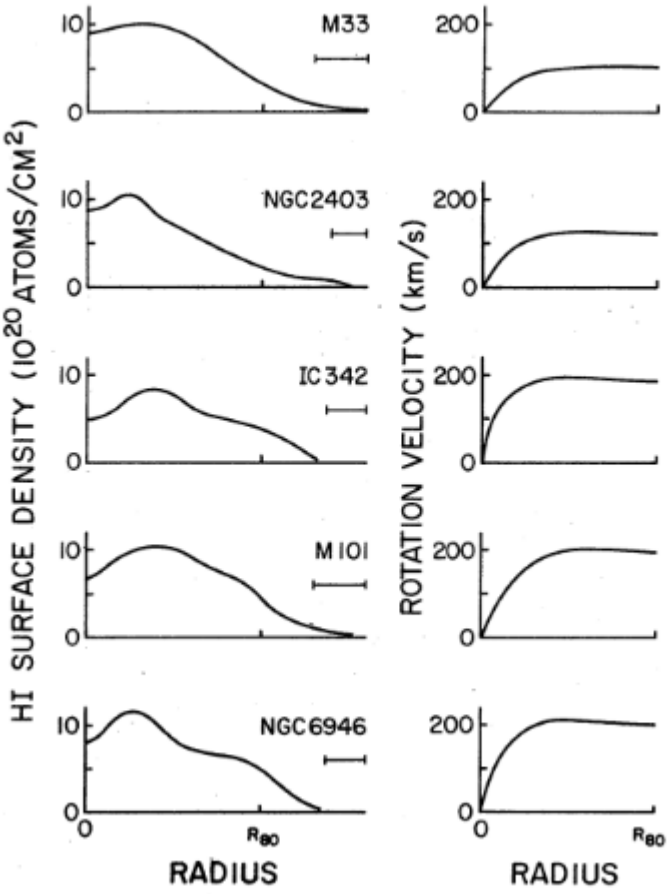
• While Newtonian gravity predicts a falling $v^2(r) = \frac{GM}{r}$ for test particles at the outer regions, observations confirm a flat profile: $v^2(r) \to const.$ (Rubin & Ford '70, ...)

Rubin and Kent Ford (white hat) check on their equipment at Lowell Observatory in 1965 during one of their first observing runs together.

Carnegie Institution, Department of Terrestrial Magnetism

[Courtesy: Astronomy, June 2016]





The hydrogen surface density profile (left) and the rotation curves (right) of 5 galaxies as obtained by Rogstad and Shostak in 1972 (Courtesy: G. Bertone & D. Hooper, Rev. Mod. Phys. 90, 45002 (2018))

A few fundamental questions to chase ..

a) Could flat rotation curves emerge as dynamical solutions of the EOM here within the general framework just elucidated?

b) Is there a natural explanation of the stability of the halo?

c) Is there a way to predict the constituents of the halo 'fluid' and also the eqn of state?

d) Any possible signature of the emergent pseudoscalar structure?

Model of the halo

- The spacetime outside the galactic disc:

$ds^2 = -e^{\mu(r)}dt^2 + e^{\lambda(r)}dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)$

Circular velocity: $v^2(r) = \frac{1}{2}r\mu'(r)$

- Insert this metric into the EOM for the theory here; The resulting set of equations indeed admit solutions with flat rotation curves

- Ignore the radiation piece t_{ab} in the (t, L, S) field multiplet for simplicity

Solutions

- Newtonian limit:
$e^{\mu(r)} = Ar^{2v^2},\ e^{\lambda(r)} = B$
(Nucamendi et. al PRD '01)
$w \approx 0,\ \frac{\rho_S}{\rho_L} \approx \frac{1}{3}$

- Non-Newtonian limit:
$e^{\mu(r)} = Ar^{2v^2},\ e^{\lambda(r)} = -Cr^{\frac{2(1+2v^2)}{1+v^2}}$
(Rahaman et. al PLB 2010)
$\omega \approx -\frac{1}{3},\ \frac{\rho_S}{\rho_L} \approx 0$

- Gen soln:
$\rho_{L,S} \approx \frac{2v^2}{r^2} + 3Cr^{2v^2},\ P_{L,S} \approx \frac{v^4}{r^2} - Cr^{2v^2}$
(Consistent with $\rho \sim \frac{1}{l^2}$ anticipated earlier)

- Any possible explanation of the non-New. soln? look at the pseudoscalar, same EOS!

- Note that this phase cannot be explained using a dust-only 'dark matter' hypothesis

New Physics: Unification of length scales

• The scale of the nonluminous field multiplet (L, S) is fixed by the emergent Λ with $\frac{1}{\sqrt{\chi}} \sim 10^{26}\ m$; In our framework, this must generate any typical length scale of the galactic physics

• Indeed so. Empirically, such a scale is precisely defined by typical galactic accelerations: $a_0 \sim 10^{-10}\ ms^{-2} \sim 10^{-26}\ m^{-1}\ (c = 1)$

• Thus, we have:
$l_{L,S} \sim l_{DE} \sim l_{Galaxy}$, the fundamental origin of all these scales is that of the 5D connection (may be contrasted with, e.g. the 'MOND' programme, which assumes the existence of a new fundamental length scale a_0 whose origin has no explanation within that framework itself)

To summarize..

• We have explored the dynamical implications of a gravity theory with an extra dimension of vanishing proper length in the context of the 'dark matter' problem

• The emergent 4D theory exhibits a field composite which is proposed to supercede the 'dark matter'

• Imports:
A dynamical (and geometric) basis to understand
the stability of halo, its constituents (t, L, S), its equation of state throughout the cosmic history ($[1, -\frac{1}{3}]$), rotation curves as solutions;
A pseudoscalar structure with possible relevance

• Can predict the relative contribution of the constituents (L, S) to total density (given ω)

Summary

• At this stage it would be premature to assert that one could indeed obtain new testable constraints; However, alongwith cosmological observations, that possibility seems to be open

• Explore: Cosmic history, CMB physics, Non-spirals, Relation to baryons, ...
(some in progress)

Thank you!