Time Crystals

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Jagiellonian University in Krakow, Poland

People involved in the research on time crystals:

- Egidijus Anisimovas, Vilnius
- Jakov Braver, Vilnius
- Bryan Dalton, Melbourne
- Alexandre Dauphin, Barcelona
- Dominique Delande, Paris
- Karthik Eswaran, Kraków
- Chu-hui Fan, Changchun
- Krzysztof Giergiel, Melbourne
- Weronika Golletz, Kraków
- Peter Hannaford, Melbourne
- Tobias Herr, Hamburg
- Ali Emami Kopaei, Kraków
- Arkadiusz Kosior, Innsbruck
- Arkadiusz Kuroś, Kielce
- Maciej Lewenstein, Barcelona
- Lute Maleki, Pasadena
- Andrey B. Matsko, Pasadena
- Pawel Matus, Dresden
- Marcin Mierzejewski, Wroclaw
- Florian Mintert, London
- Artur Miroszewski, Warsaw
- Rick Mukherjee, London
- Luis Morales-Molina, Santiago
- Frederic Sauvage, London
- Andrzej Syrwid, Stockholm
- Xuedong Tian, Guilin
- Hossein Taheri, Riverside
- Jia Wang, Melbourne
- Damian Włodzyński, Kraków
- Majid Yazdani-Kachoei, Kraków
- Jakub Zakrzewski, Kraków
- Giedrius Zlabys, Vilnius

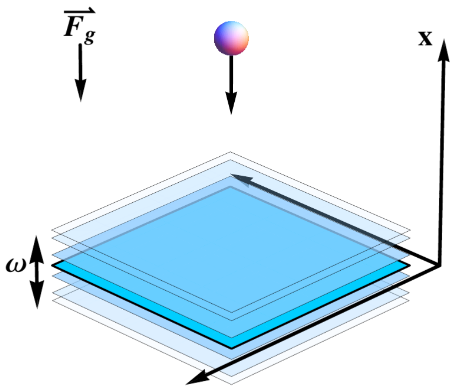
Plan:

- **Creating crystalline structures in time**
- **Spontaneous formation of time crystals**
- **Towards time-tronics**

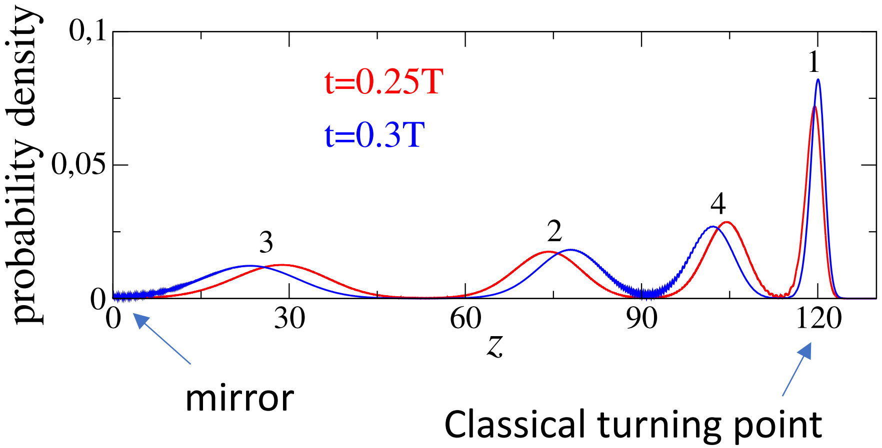
Creating crystalline structures in time

Crystalline structures in time

s: 1 resonance



4:1 resonance



A. Buchleitner, D. Delande, J. Zakrzewski, Phys. Rep. 368, 409 (2002).

Crystalline structures in time

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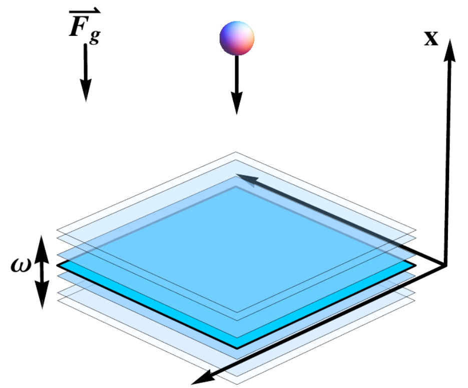

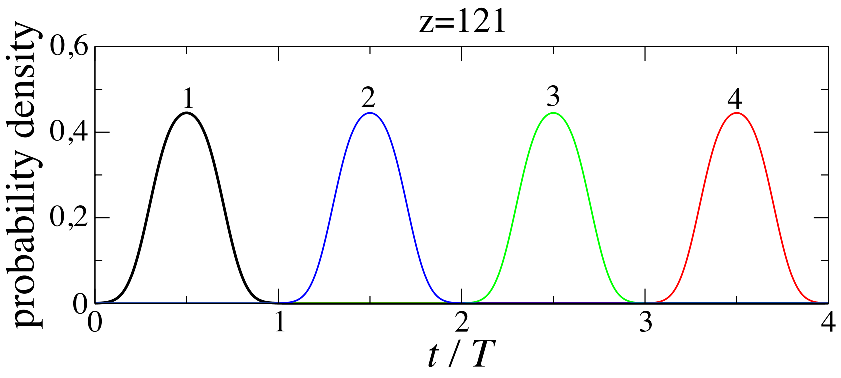

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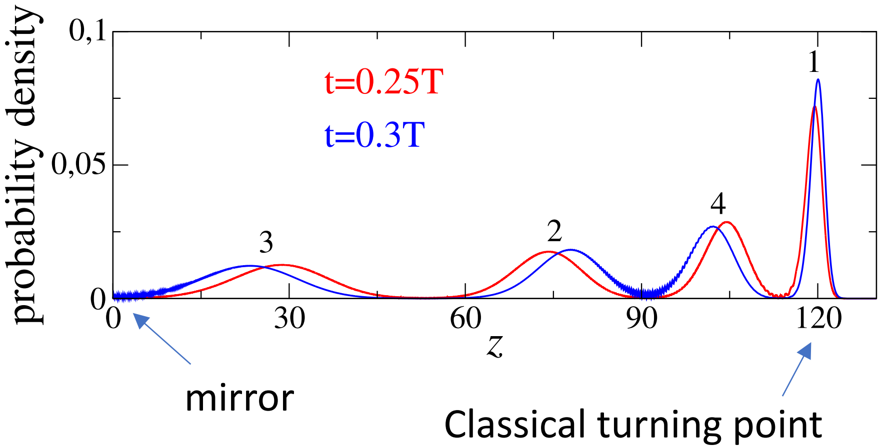


A. Buchleitner, D. Delande, J. Zakrzewski, Phys. Rep. 368, 409 (2002).

$$E = -\frac{J}{2}\sum_{i=1}^{s} a_{i+1}^{*}\, a_i + c.c.$$

KS, Sci. Rep. 5, 10787 (2015).

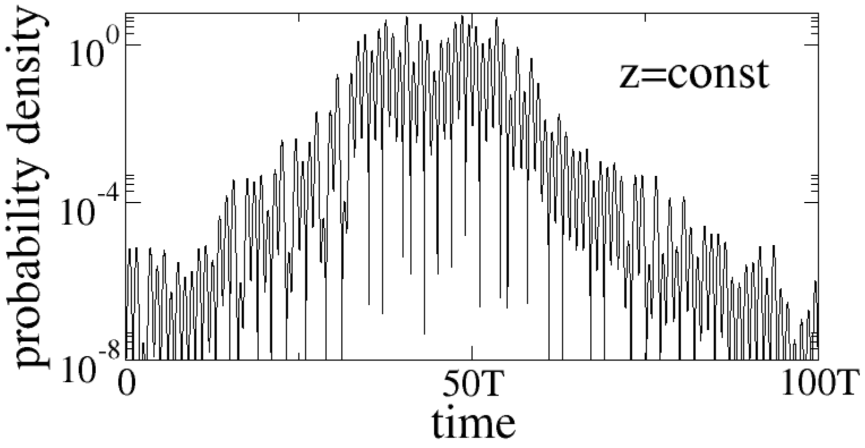
Phase space crystals: L. Guo, M. Marthaler, G. Schon, Phys. Rev. Lett. 111, 205303 (2013).

Anderson localization in the time domain

$H'(t)$ is a perturbation that fluctuates in time but $H'(t+sT)=H'(t)$

$$E=-\frac{J}{2}\sum_{i=1}^{s}(a_{i+1}^{*}a_i+c.c)+\sum_{i}^{s}\epsilon_i|a_i|^2$$

Example for $s=100$



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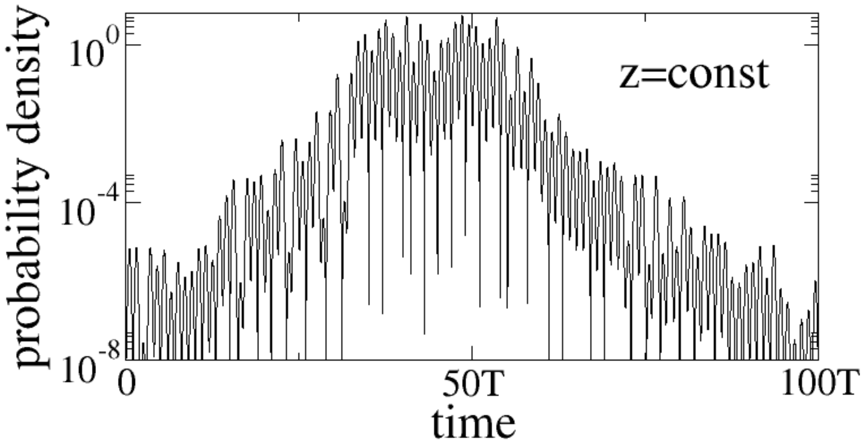

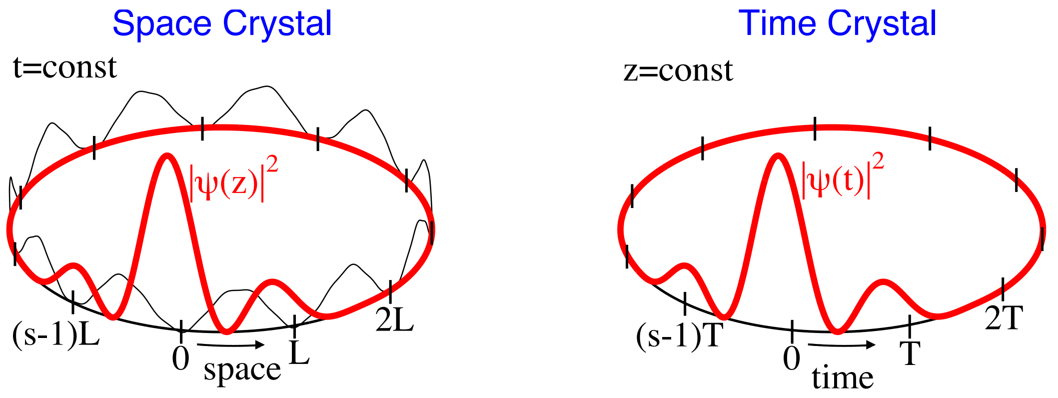

KS, Sci. Rep. 5, 10787 (2015).

KS, D. Delande, PRA 94, 023633 (2016).
K. Giergiel, KS, PRA 95, 063402 (2017).
D. Delande, L. Morales-Molina, KS, PRL 119, 230404 (2017).

Mott insulator phase in the time domain

$$H = -\frac{J}{2}\sum_{i}^{s}\left(\hat{a}_{i+1}^{\dagger}\,\hat{a}_i + h.c.\right) + \frac{1}{2}\sum_{i,j=1}^{s} U_{ij}\,\hat{n}_i\,\hat{n}_j$$

where $U_{ij} = \frac{g_0}{sT}\int_0^{sT} dt\ \int_0^{\infty} dz\ |w_i|^2\ \left|w_j\right|^2$

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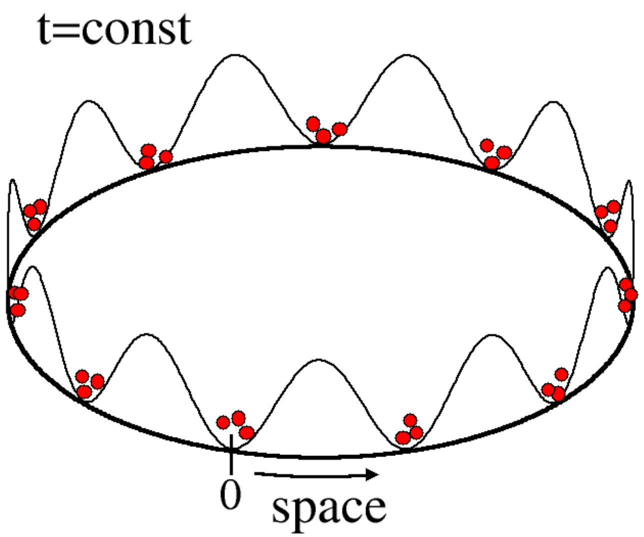

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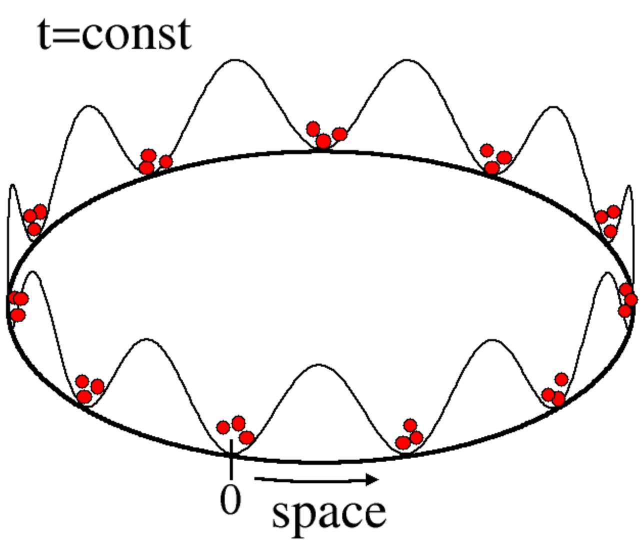
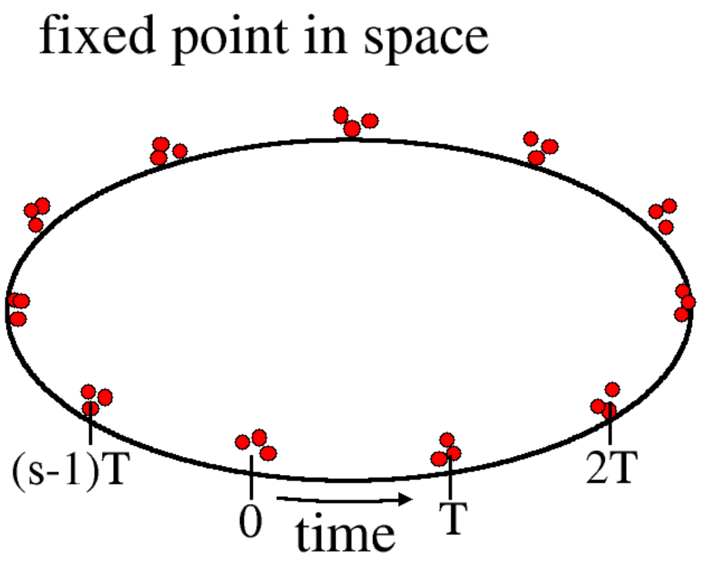
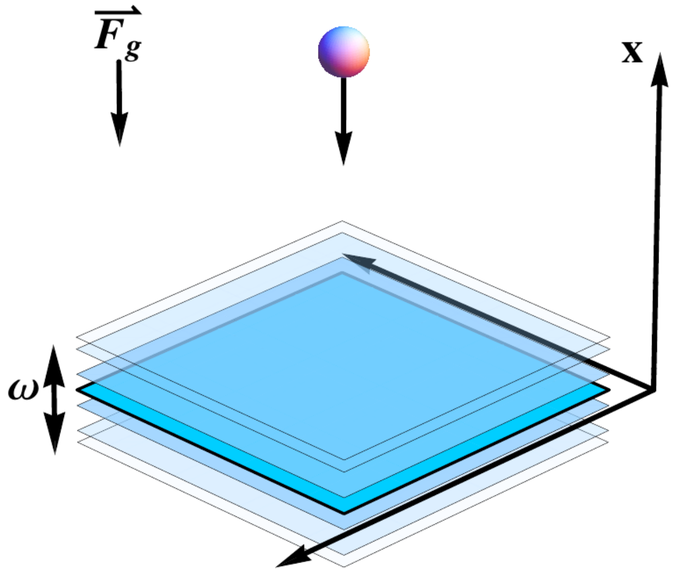
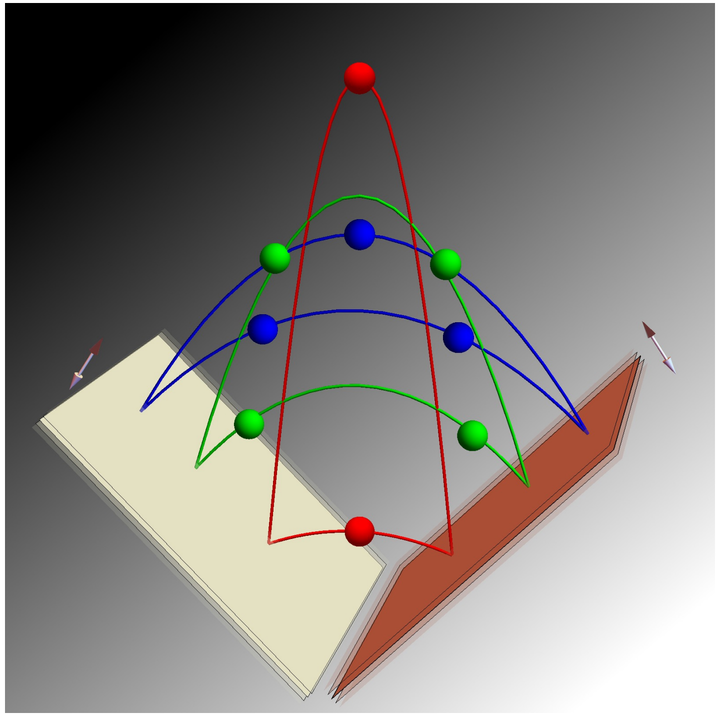




KS, Sci. Rep. 5, 10787 (2015).

Multi-dimensional time lattices





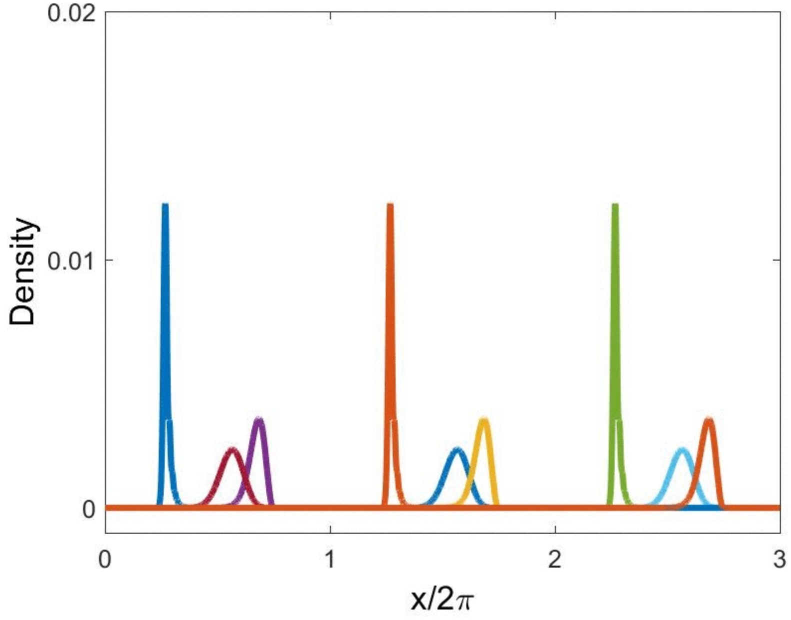
$$H = -\frac{J}{2}\sum_{\langle i,j\rangle}(\hat{a}_i^\dagger \hat{a}_j + h.c.) + \frac{1}{2}\sum_{i,j} U_{ij}\,\hat{n}_i\,\hat{n}_j$$

K. Giergiel, A. Miroszewski, KS, PRL 120, 140401 (2018).

Time-space lattices

$$H = \frac{p_x^2}{2} + V_0 \sin^2(x) \quad \Longrightarrow \quad H = \frac{p_x^2}{2} + V_0 \sin^2(x - \lambda \cos \omega t)$$

3: 1 resonance



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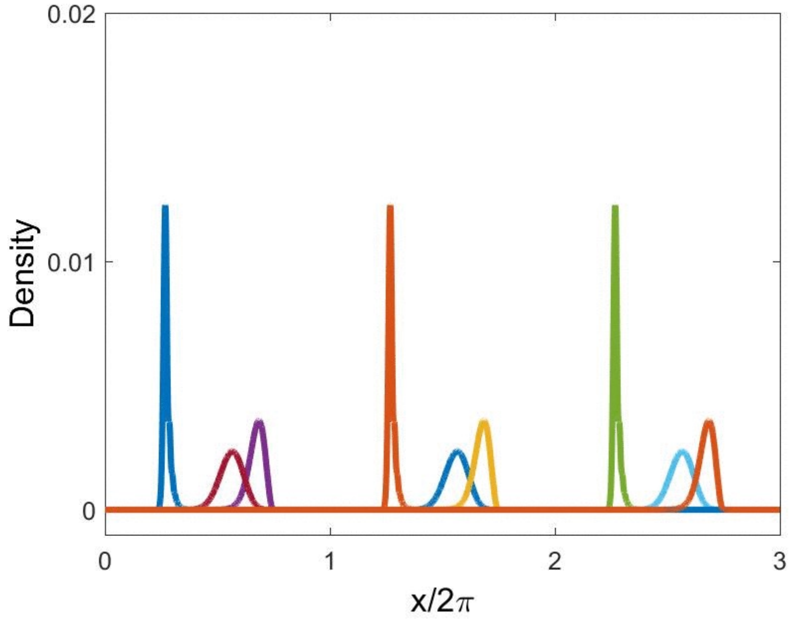

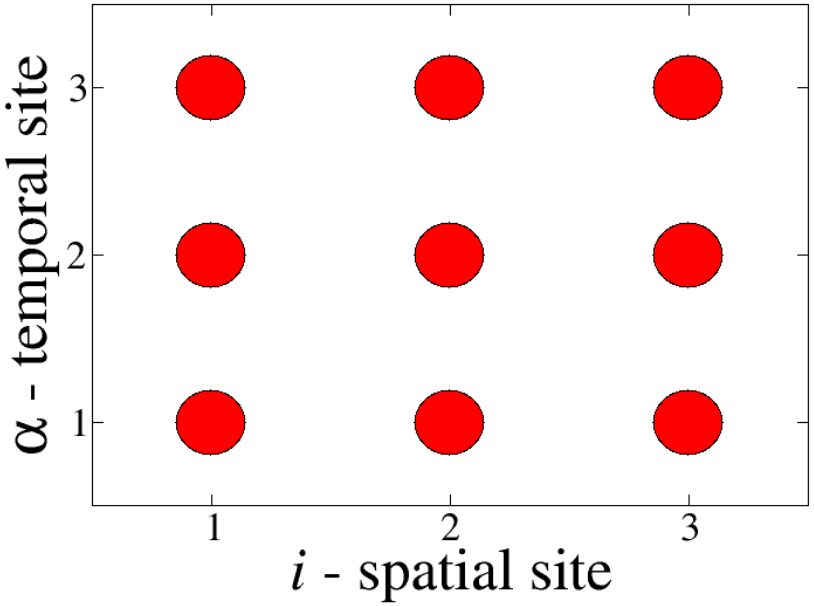

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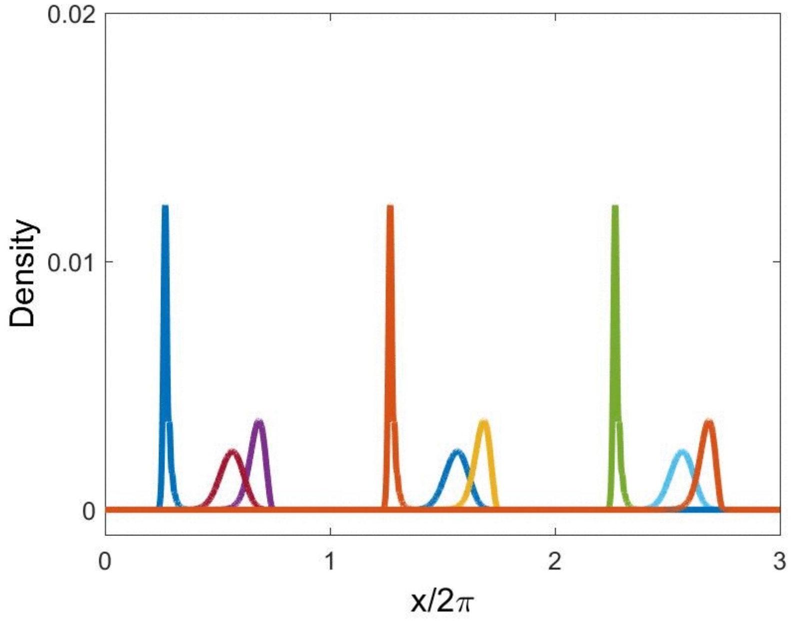
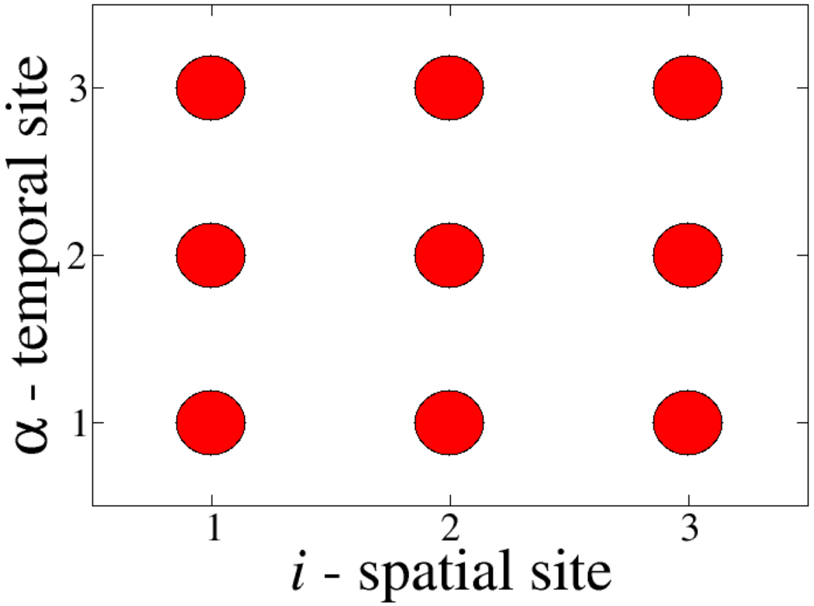




Shaking of 3D optical lattice

6D time-space lattice

G. Zlabys, Ch-hui Fan, E. Anisimovas, KS, PRB 103, L100301 (2021);
Braver, Anisomovas, KS, PRB 108, L020303 (2023).

Time crystalline structures

- **Anderson localization in the time domain,**

  KS, Sci. Rep. 5, 10787 (2015).

  KS, D. Delande, PRA 94, 023633 (2016).

  K. Giergiel, KS, PRA 95, 063402 (2017).

  D. Delande, L. Morales-Molina, KS, PRL 119, 230404 (2017).

- **Mott insulator in the time domain,**

  KS, Sci. Rep. 5, 10787 (2015).

- **Many-body localization in time crystalline structures,**

  M. Mierzejewski, K. Giergiel, KS, PRB 96, 140201 (2017).

- **Topological time crystals,**

  K. Giergiel, A. Dauphin, M. Lewenstein, J. Zakrzewski, KS, NJP 21, 052003 (2019).

  A. Emami Kopaei, X. Tian, K. Giergiel, KS, PRA 106, L031301 (2022).

- **Multi-dimensional time lattices,**

  K. Giergiel, A. Miroszewski, KS, PRL 120, 140401 (2018).

  K. Giergiel, A. Kuroś, A. Kosior, KS, PRL 127, 263003 (2021).

  W. Golletz, A. Czarnecki, KS, A. Kuroś, NJP, 24, 093002 (2022).

- **6D time-space crystalline structures,**

  G. Zlabys, C.-h. Fan, E. Anisimovas, KS, PRB 103, L100301 (2021).

  Y. Braver, C.-h. Fan, G. Zlabys, E. Anisimovas, KS, PRB 106, 144301 (2022).

  Y. Braver, E. Anisimovas, KS, PRB 108, L020303 (2023).

- **Exotic long-range interaction in time lattices,**

  K. Giergiel, A. Miroszewski, KS, PRL 120, 140401 (2018).

  K. Giergiel, A. Kuroś, A. Kosior, KS, PRL 127, 263003 (2021).

- **Anderson and topological molecules,**

  K. Giergiel, A. Miroszewski, KS, PRL 120, 140401 (2018).

  P. Matus, K. Giergiel, KS, PRA 103, 023320 (2021).

  A. Kopaei, X. Tian, K. Giergiel, KS, PRA 106, L031301 (2022).

Spontaneous formation of time crystals

Brief history

- F. Wilczek, PRL 109, 160401 (2012).
  Idea of time crystals.

- K. Sacha, PRA 91, 033617 (2015).
  Discrete time crystal in a driven atomic system.

- V. Khemani et al., PRL 116, 250401 (2016).
  D. V. Else et al., PRL 117, 090402 (2016).
  Discrete time crystals in driven spin systems.

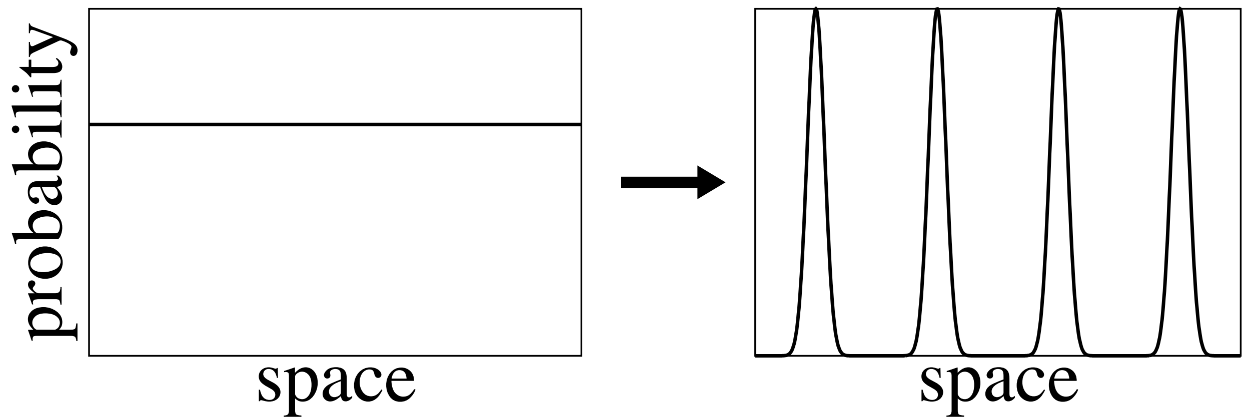
- J. Zhang et al., Nature 543, 217 (2017).
  S. Choi et al., Nature 543, 221 (2017).
  First experimental realizations.

Formation of space crystals

$$[\hat{H}, \hat{T}] = 0$$

$\hat{T}$ – translation operator of all particles by the same **arbitrary** vector

t = const

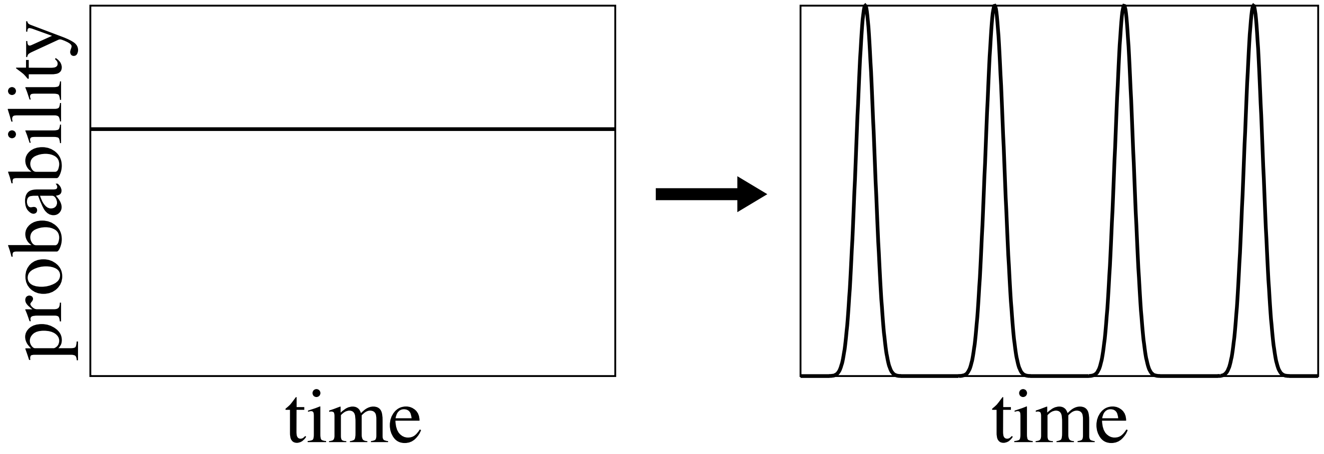


$$\langle\, \hat{\psi}^\dagger(x)\, \hat{\psi}^\dagger(x')\, \hat{\psi}(x')\, \hat{\psi}(x)\, \rangle$$

Formation of time crystals?

Eigenstates of a time-independent Hamiltonian H are also eigenstates of the time translation operator e^{-iHt}

$\vec{r}$ is fixed



$$\langle\, \hat{\psi}^{\dagger}(t)\, \hat{\psi}^{\dagger}(t')\, \hat{\psi}(t')\, \hat{\psi}(t)\, \rangle$$

F. Wilczek, PRL 109, 160401 (2012).
P. Bruno, PRL 111, 070402 (2013).
H. Watanabe and M. Oshikawa, PRL 114, 251603 (2015).
A. Syrwid, J. Zakrzewski, KS, PRL 119, 250602 (2017).
V. K. Kozin and O. Kyriienko, ”Quantum Time Crystals from Hamiltonians with Long-Range Interactions”, PRL 123, 210602 (2019).

Periodically driven closed systems

$$\left(H(t) - i\hbar\frac{\partial}{\partial t}\right)|\psi_n(t)\rangle = E_n|\psi_n(t)\rangle$$

$$|\psi_n(t+T)\rangle = |\psi_n(t)\rangle$$

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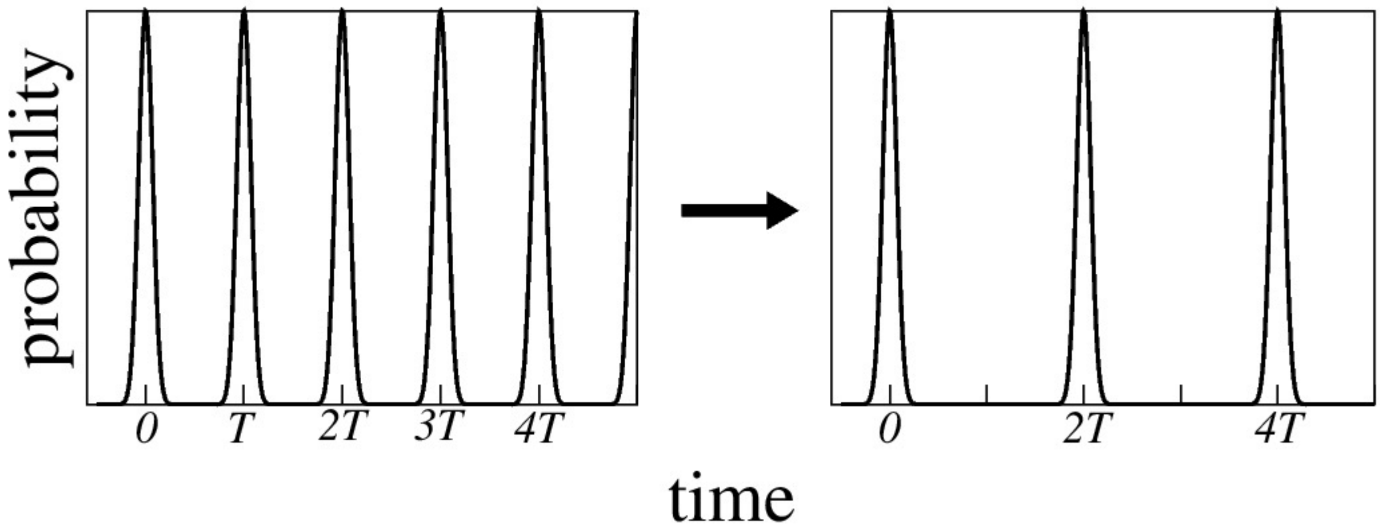
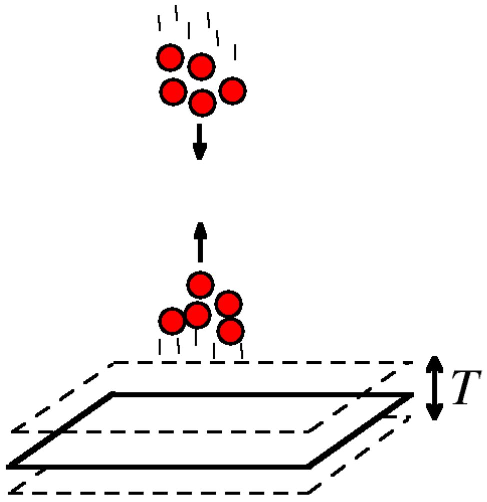
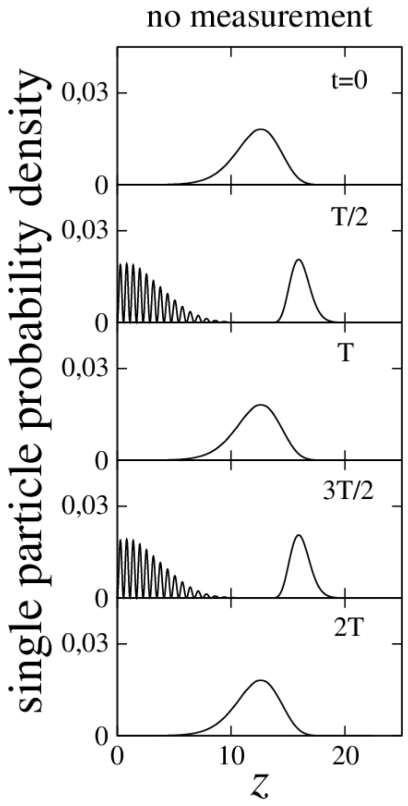
Discrete time crystals:



Discrete time crystals

Bosons with attractive interactions (2: 1 resonance)





$N = 10^4$

$$|\psi\rangle \approx \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}}$$

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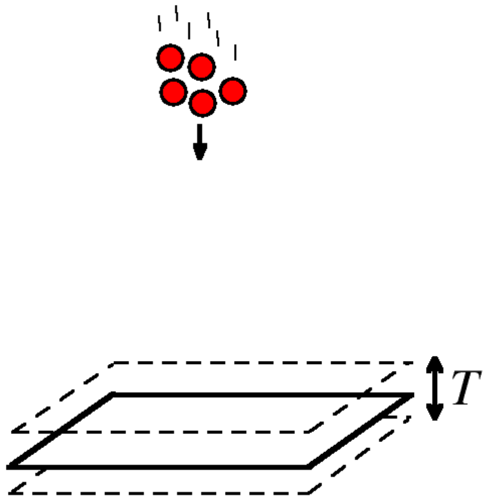

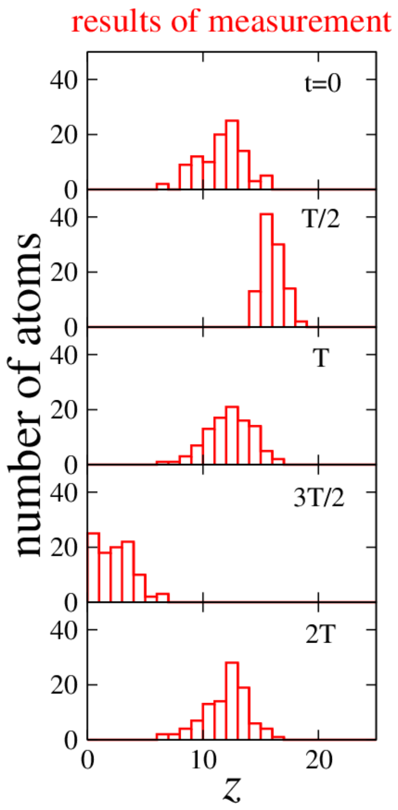

$N = 10^4$

$$|\psi\rangle \approx \frac{|N,0\rangle + |0,N\rangle}{\sqrt{2}} \quad \Longrightarrow \quad |N-1,0\rangle \ \text{ or } \ |0,N-1\rangle$$

KS, Phys. Rev. A 91, 033617 (2015).

Discrete time crystals

Many-body localized spin systems

V. Khemani, A. Lazarides, R. Moessner, L. S. Sondhi, Phys. Rev. Lett. 116, 250401 (2016).
D. V. Else, B. Bauer, C. Nayak, Phys. Rev. Lett. 117, 090402 (2016).

Chain of spins,

$$H = \sum_i (J_i \sigma_i^z \sigma_{i+1}^z + h_i^z \sigma_i^z + h_i^x \sigma_i^x)$$

But every period *T*, there is a spin flip,

$$U = e^{i\frac{\pi}{2}(1-\epsilon)\sum_i \sigma_i^x}.$$

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$$|\psi\rangle \approx \frac{|\uparrow\uparrow\uparrow \cdots \uparrow\rangle_z + |\downarrow\downarrow\downarrow \cdots \downarrow\rangle_z}{\sqrt{2}} \Longrightarrow |\uparrow\uparrow\uparrow \cdots \uparrow\rangle_z \text{ or } |\downarrow\downarrow\downarrow \cdots \downarrow\rangle_z$$

Bosons on a ring

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2} + g_0 \sum_{i<j}^{N} \delta(x_i - x_j) + \lambda \sum_{i=1}^{N} \cos(2x_i - \omega t)$$

Discrete time and space translation symmetries:

$t \to t + \frac{2\pi}{\omega}$ or $x_i \to x_i + \pi$

K. Giergiel, J. Wang, B. Dalton, P. Hannaford, KS, PRB 108, L180201 (2023).

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Discrete time and space translation symmetries:

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In the frame moving with the frequency $\omega/2$:

$$H = \sum_{i=1}^{N} \left[\frac{p_i^2}{2} + \lambda \cos(2x_i) \right] + g_0 \sum_{i<j}^{N} \delta(x_i - x_j)$$

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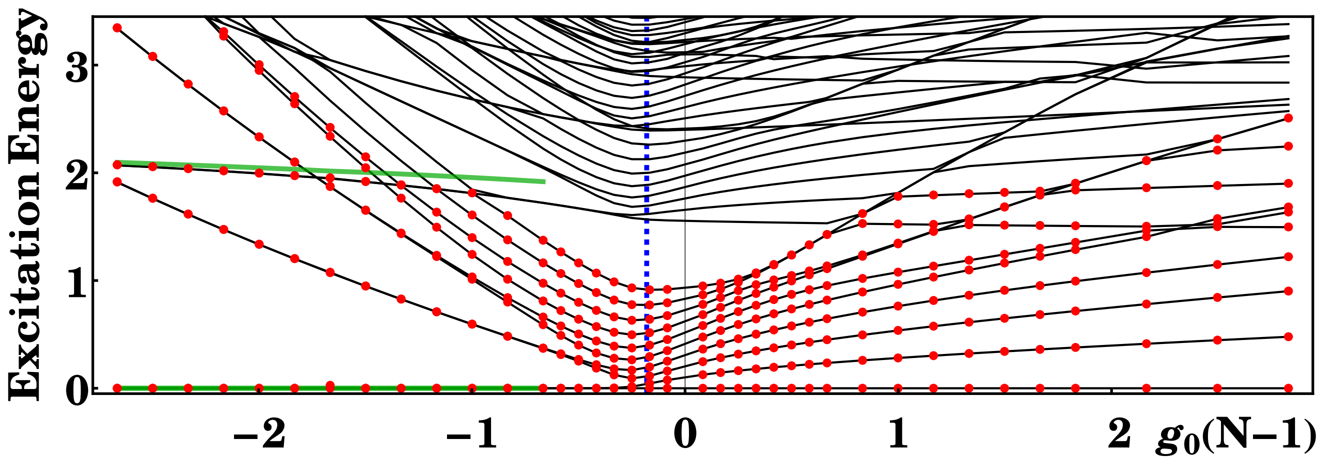
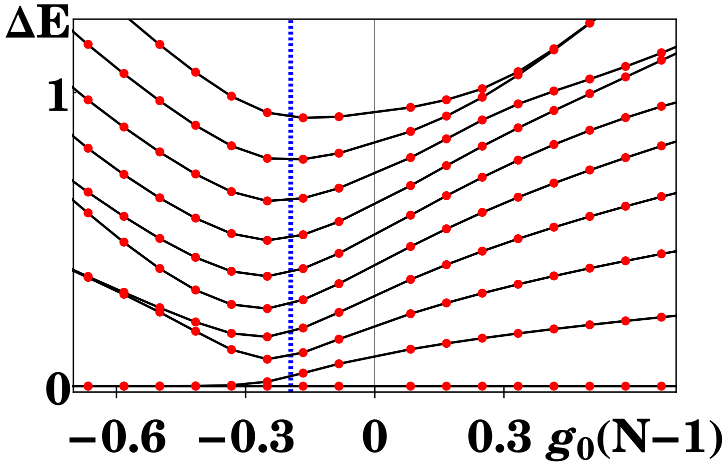
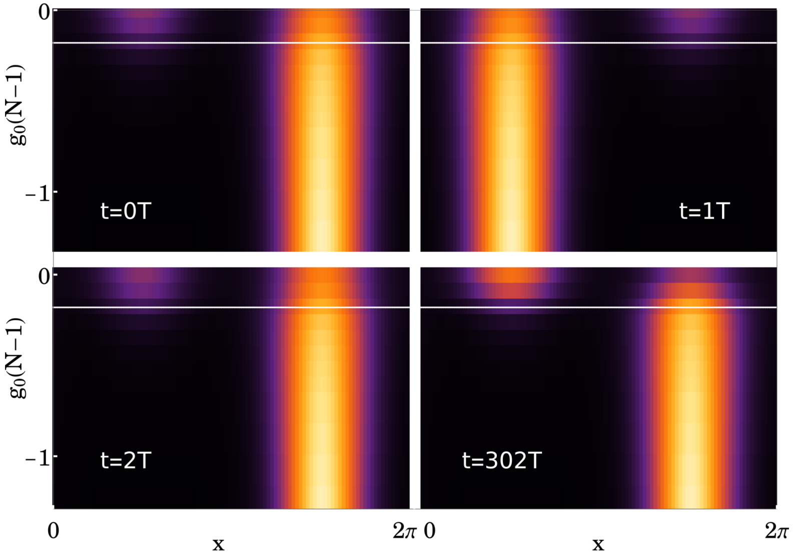
Illustration for $N = 9$







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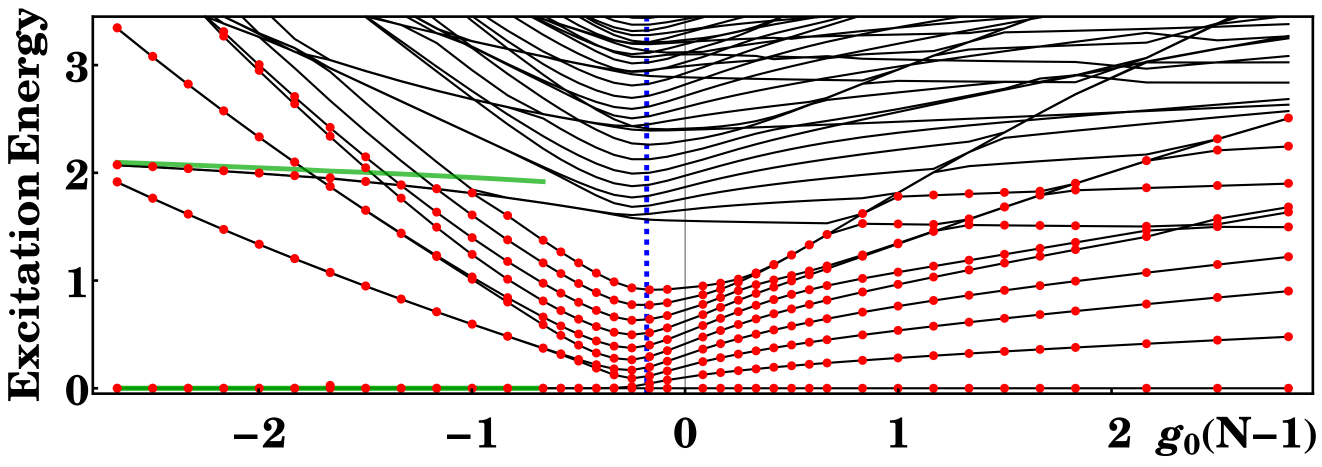

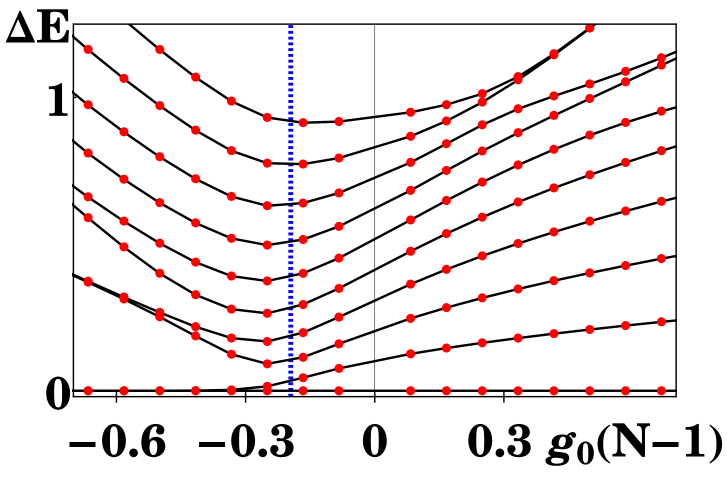

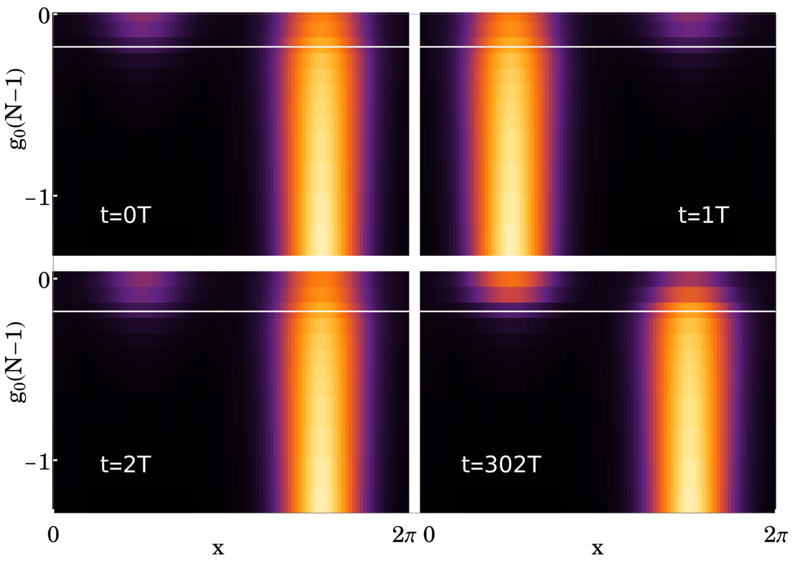

Periodically kicked bosons on a ring:

$$H = \sum_{i=1}^{N} \frac{p_i^2}{2} + g_0 \sum_{i<j}^{N} \delta(x_i - x_j) + \lambda T \sum_{i}^{N} \cos(2x_i) \sum_{m} \delta(t - mT)$$

In the frame moving with the frequency $\omega/2$:

$$H \approx \sum_{i=1}^{N} \left[\frac{p_i^2}{2} + \lambda \cos(2x_i) \right] + g_0 \sum_{i<j}^{N} \delta(x_i - x_j)$$

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Spontaneous formation of time quasi-crystals

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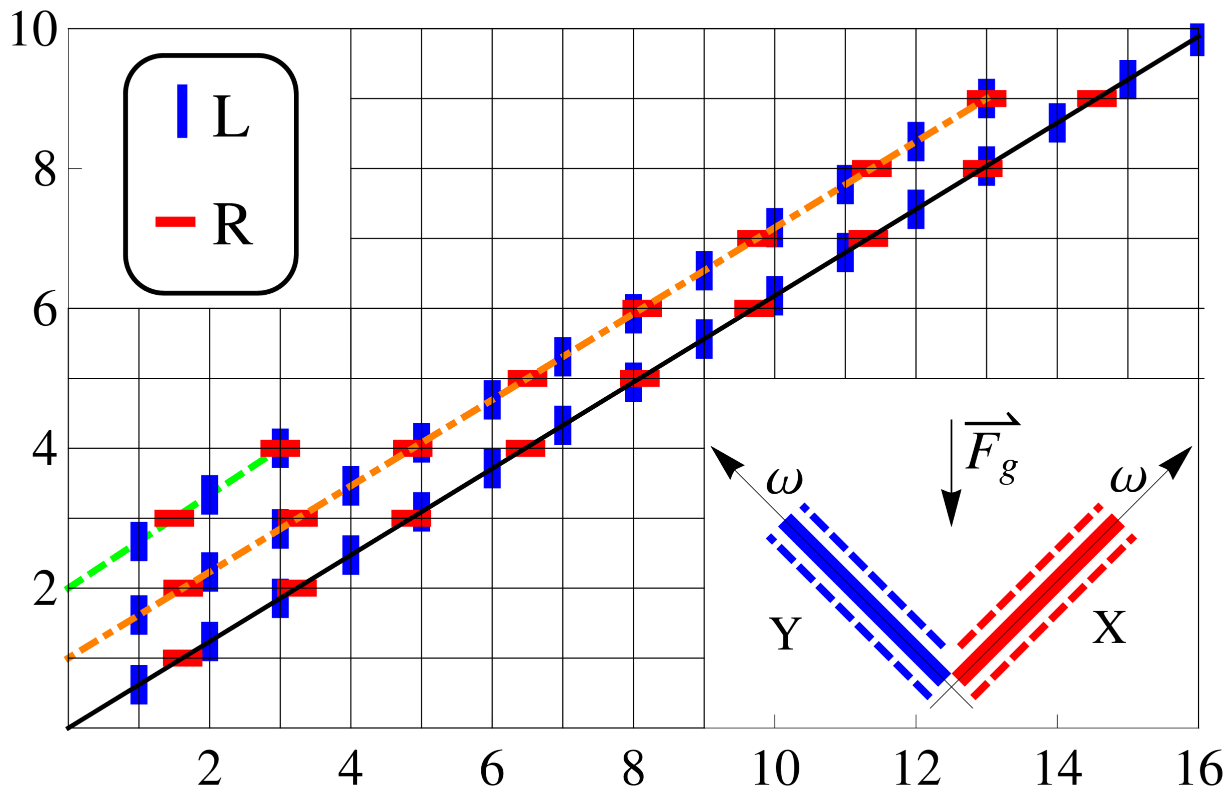
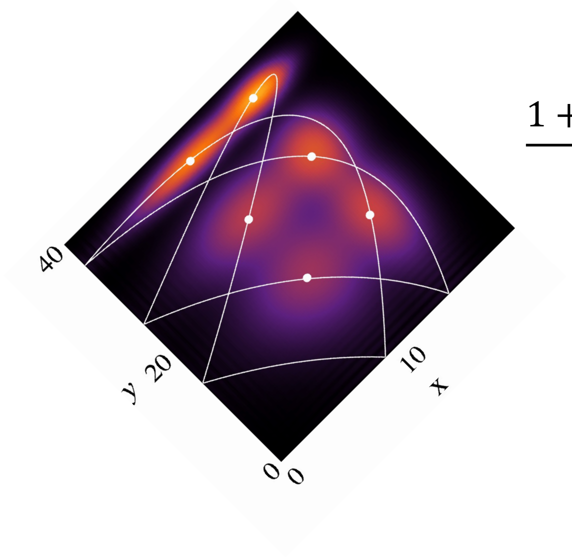
Fibonacci quasi-crystal: *LRLLRLRLLR* ...



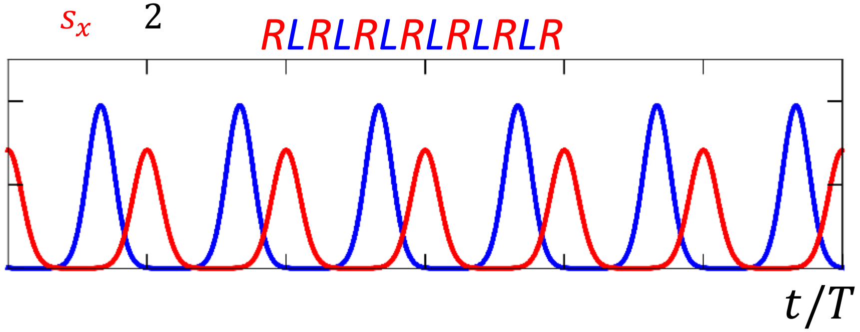
Spontaneous formation of time quasi-crystals

$$H(t+T) = H(t)$$

$s_x : 1$ and $s_y : 1$ resonances



$$\frac{1+\sqrt{5}}{2} \approx \frac{s_y}{s_x} = \frac{3}{2}$$



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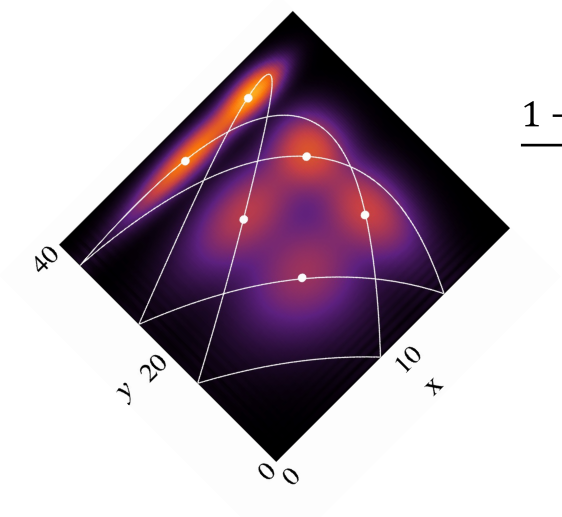
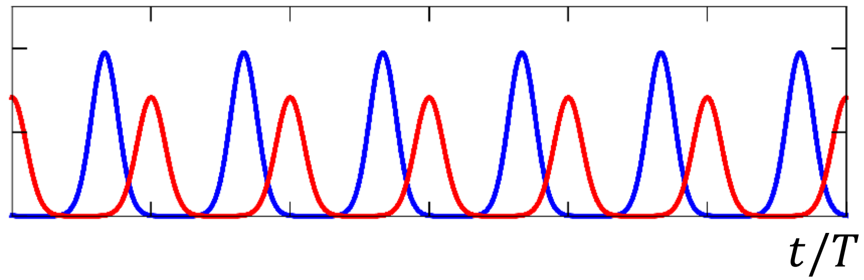
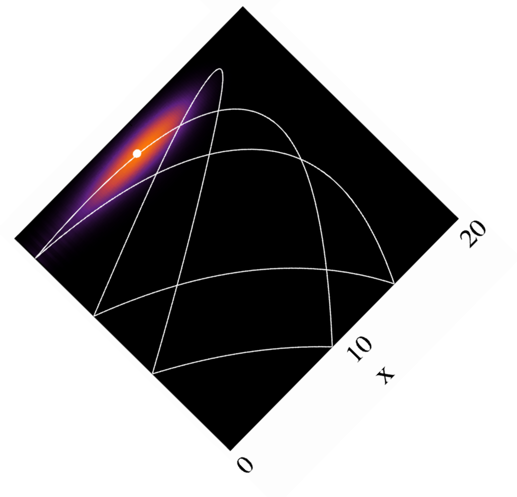
$s_x : 1$ and $s_y : 1$ resonances



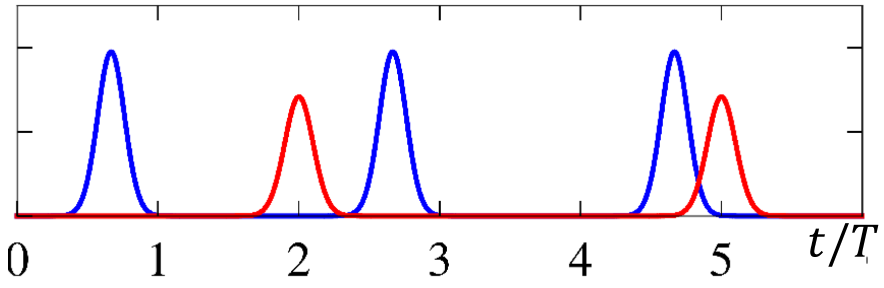
$$\frac{1+\sqrt{5}}{2} \approx \frac{s_y}{s_x} = \frac{3}{2}$$

RLRLRLRLRLRLR





LRLLR

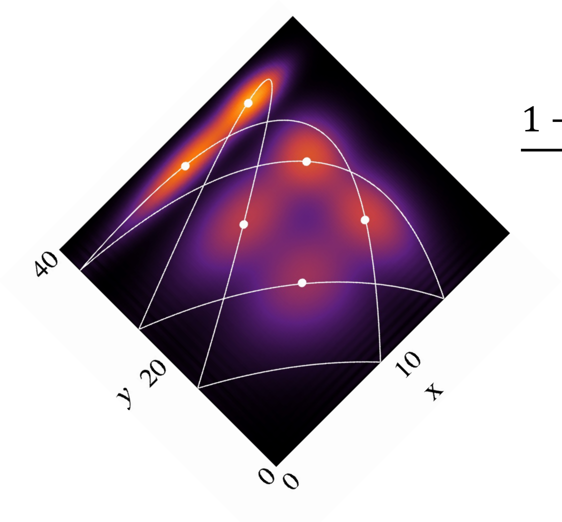


K. Giergiel, A. Kuroś, KS, "Discrete Time Quasi-Crystals", PRB 99, 220303(R) (2019)

Spontaneous formation of time quasi-crystals

$$H(t+T) = H(t)$$

$s_x : 1$ and $s_y : 1$ resonances



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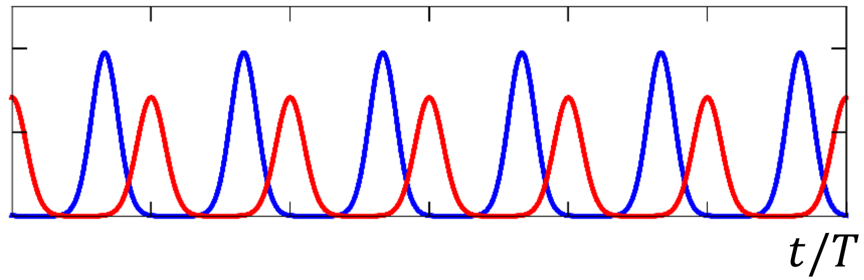
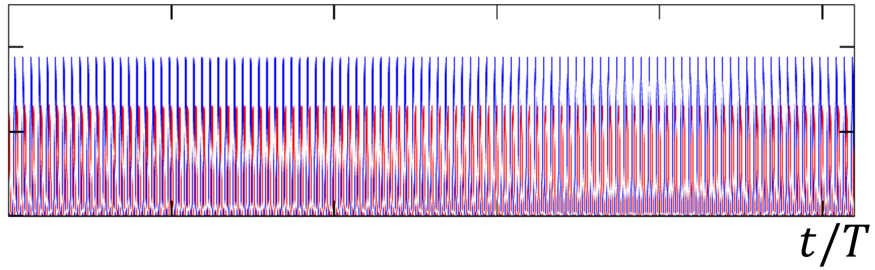
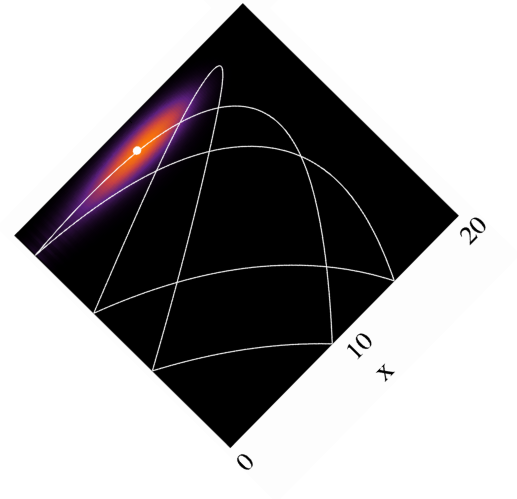
RLRLRLRLRLRLR



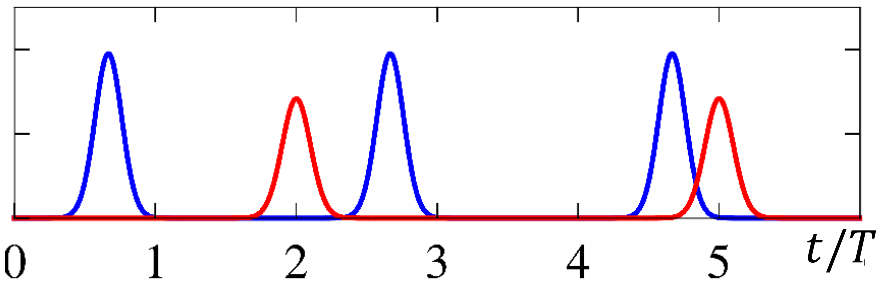
$\frac{s_y}{s_x} = \frac{13}{8}$

RLRLRLRLRLRLRLRLRL . . .

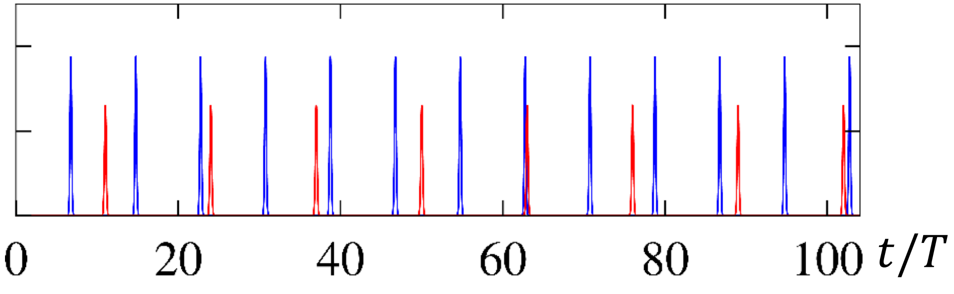




LRLLR



LRLLRLRLLR . . .



K. Giergiel, A. Kuroś, KS, "Discrete Time Quasi-Crystals", PRB 99, 220303(R) (2019)

Towards time-tronics

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Periodically driven systems

Condensed matter in time lattices

- Arbitrary 1D time lattices.
- Temporal disorder.
- Arbitrary effective long-range interactions.
- 2D or 3D time lattices.
- Even 6D time-space lattices.

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- Big discrete time crystals evolving with periods even 100 times longer than the driving period — discrete time crystals with many temporal sites.
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Time-tronics

- Useful devices where crystalline structures in time play a crucial role.
- Experiments: P. Hannaford (Melbourne), H. Taheri (UC Riverside).

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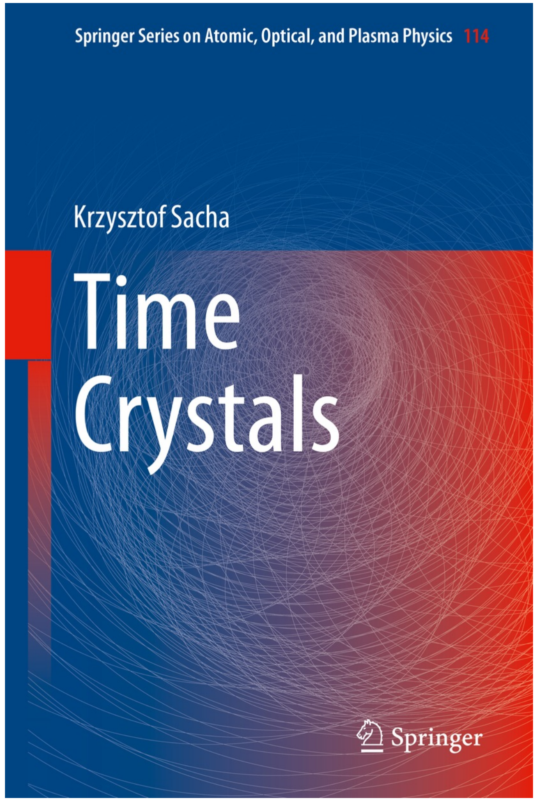

Many-body localization induced by temporal disorder

For example for bosons:

$$H = -\frac{J}{2}\sum_{i}^{s}\left(\hat{a}_{i+1}^{\dagger}\,\hat{a}_i + h.c.\right) + \sum_{i}^{s}\epsilon_i\,\hat{a}_i^{\dagger}\,\hat{a}_i + \frac{1}{2}\sum_{i,j=1}^{s}U_{ij}\,\hat{n}_i\,\hat{n}_j$$

Many-body localization (MBL):

- absence of thermalization,
- logarithmic growth of the entanglement entropy,

M. Mierzejewski, K. Giergiel, KS, PRB 96, 140201(R) (2017).

Plaform for time crystal research

A particle which can perfom periodic motion in 1D:

$$H_0(x,p) \rightarrow H_0(I) \quad \Longrightarrow \quad I = const, \qquad \theta = \Omega(I)t + \theta_0$$

Time-periodic perturbation, $H = H_0 + H_1$, where:

$$H_1 = f(t)\, h(x) \quad \rightarrow \quad H_1 = \left(\sum_k f_k e^{ik\omega t}\right)\left(\sum_n h_n e^{in\theta}\right)$$

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Assume s:1 resonance, $\omega = s\,\Omega(I_s)$. In the moving frame, $\Theta = \theta - \frac{\omega}{s}t$,

$$H \approx \frac{P^2}{2m_{eff}} + \sum_k f_{-k}\, h_{ks}\, e^{iks\Theta}, \qquad P = I - I_s$$

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For example for $f(t) = \lambda\cos(\omega t)$, we get $H \approx \frac{P^2}{2m_{eff}} + V_0\cos(s\,\Theta)$.

Topological time crystals

Mirror oscillations $\propto \lambda \cos(\omega t) + \lambda_1 \cos\left(\frac{\omega t}{2}\right)$

SSH model:

$$H \approx -\sum_{i=1}^{s/2} \left(J\, b_i^* a_i + J'\, a_{i+1}^* b_i \right)$$

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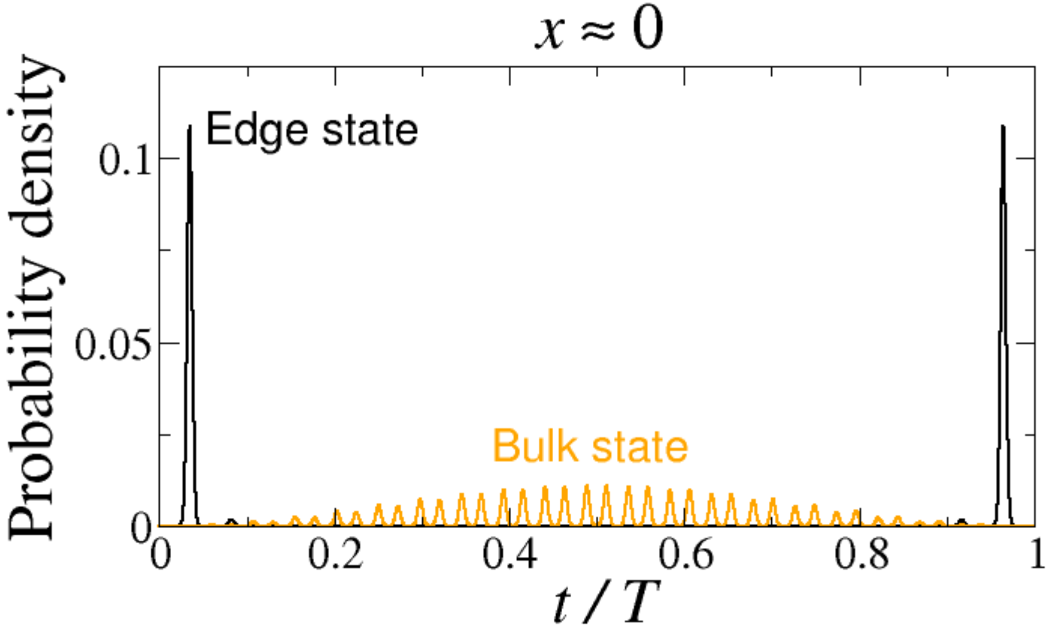
Mirror oscillations $\propto\ \lambda\cos(\omega t)+\lambda_1\cos\left(\frac{\omega t}{2}\right)\ +\ f(t)$

$f(t)$ creates an Edge in time.



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6D time-space lattice

$$H = \frac{p_x^2 + p_y^2 + p_z^2}{2} + V_0[\sin^2(x - \lambda\cos\omega t) + \sin^2(y - \lambda\cos\omega t) + \sin^2(z - \lambda\cos\omega t)]$$

$$W_{\vec{i},\vec{\alpha}}(r,t) = w_{i_x,\alpha_x}(x,t)\ w_{i_y,\alpha_y}(y,t)\ w_{i_z,\alpha_z}(z,t)$$

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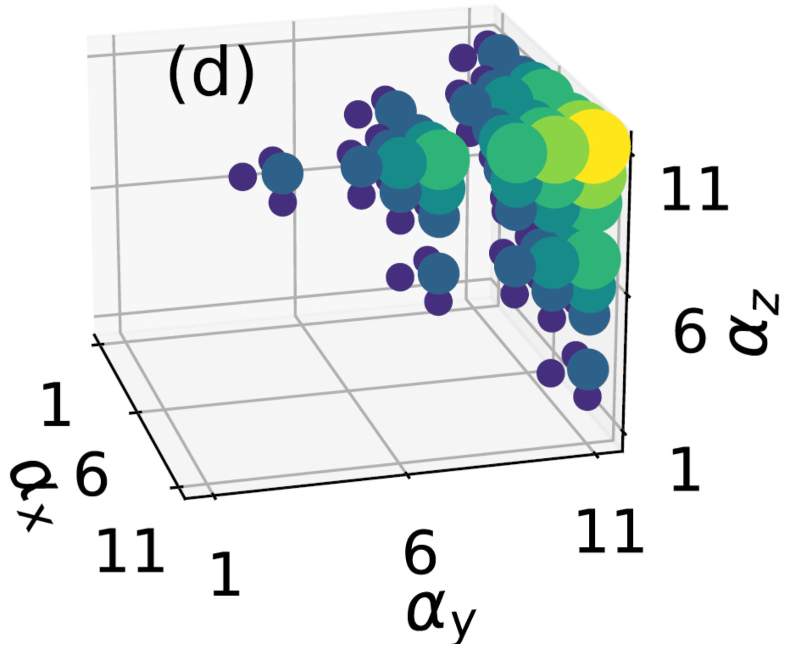

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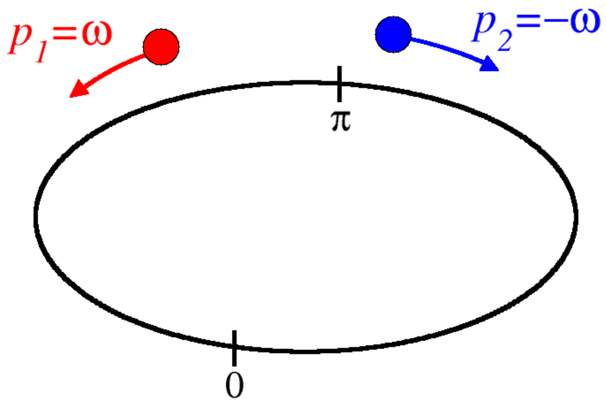
Time engineering: Anderson molecules

Two atoms bound together not due to attractive interaction but due to destructive interference

$$H = \frac{p_1^2 + p_2^2}{2} + \delta(x_1 - x_2)\, f(t) \quad \Rightarrow \quad H_{eff} = \frac{P_1^2 + P_2^2}{2} + V_d(X_1 - X_2) = \frac{P_{CM}^2}{4} + p^2 + V_d(x)$$



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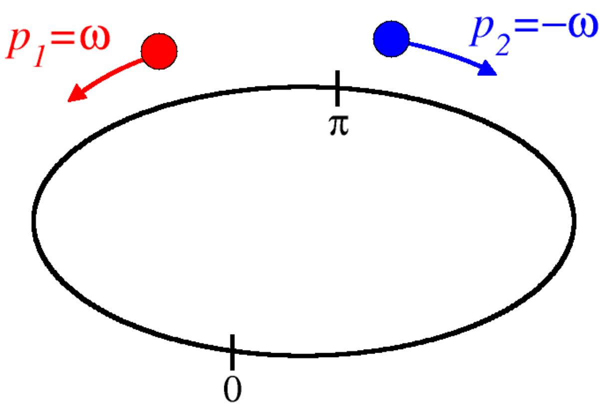

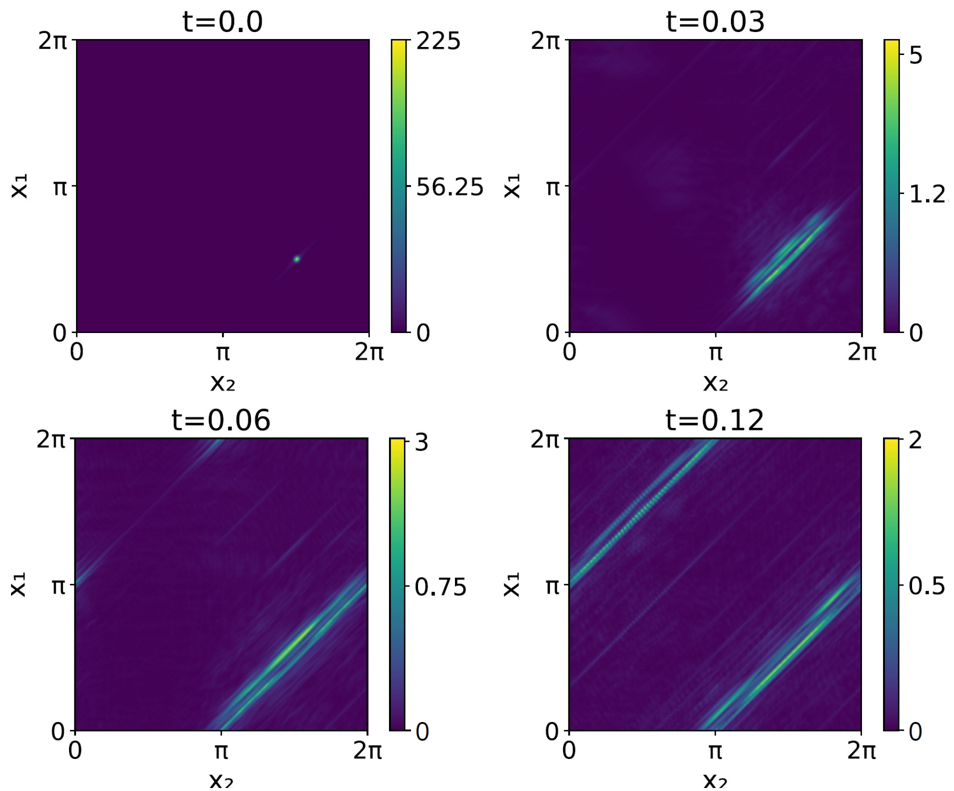

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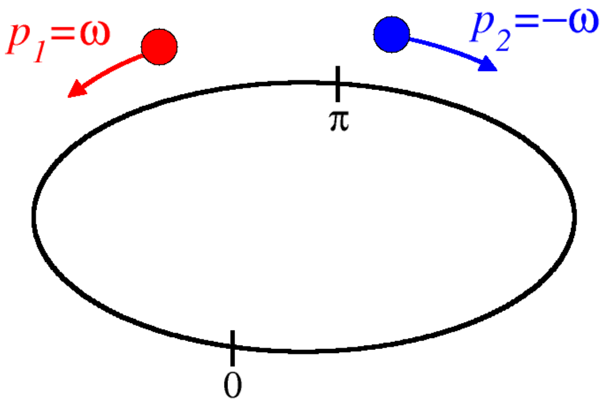

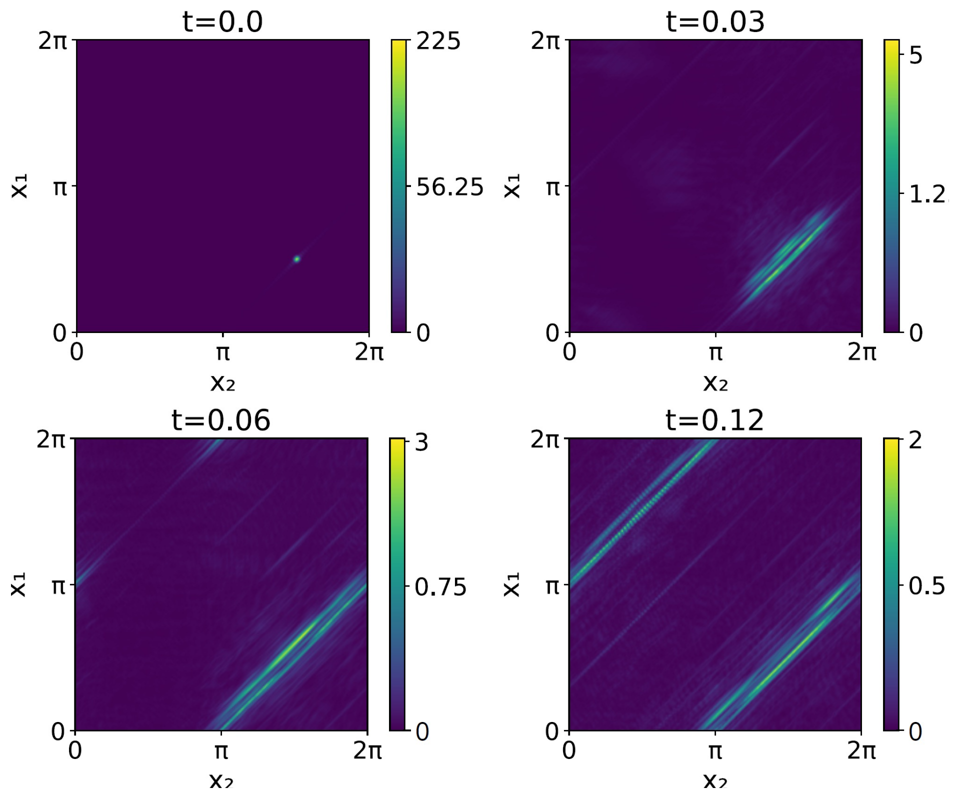

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