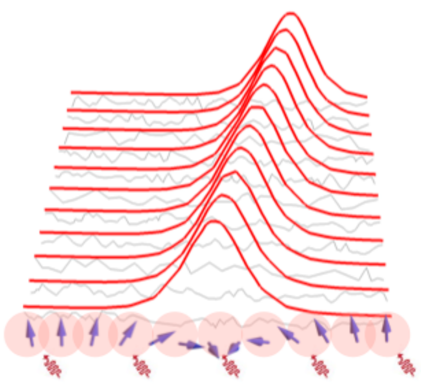
Superdiffusive hydrodynamics in isotropic spin chains

Romain Vasseur

(UMass Amherst)

ICTS workshop



Gopalakrishnan & RV, PRL ‘19

Gopalakrishnan, RV & Ware, PNAS ‘19

De Nardis, Gopalakrishnan, Ilievski & RV, PRL ‘20

Ilievski, De Nardis, Gopalakrishnan, RV, Ware, PRX ‘21

De Nardis, Gopalakrishnan, RV, Ware, PRL ‘21

UMass Amherst

Acknowledgements

B. Ware
(UMass—>
NIST/JQI)

S. Gopalakrishnan
(Penn State)

J. De Nardis
(Cergy-Pontoise)

E. Ilievski
(Ljubljana)

Also based on earlier works with:
U. Agrawal, V. Bulchandani, A. Friedman, D. Huse, C. Karrasch,
V. Khemani, J. Lopez, J. Moore…

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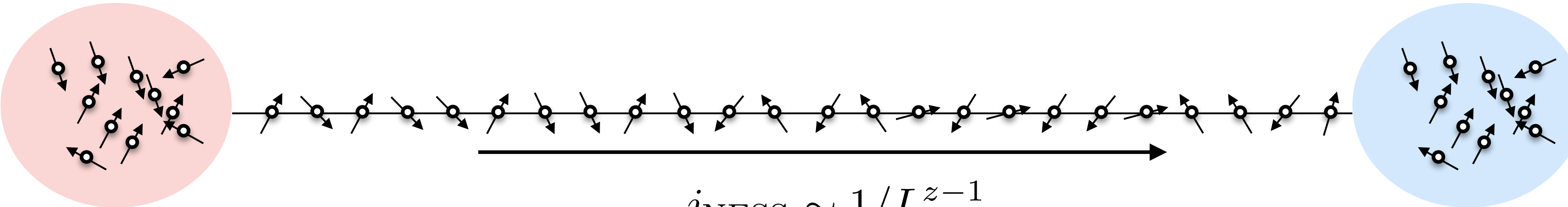
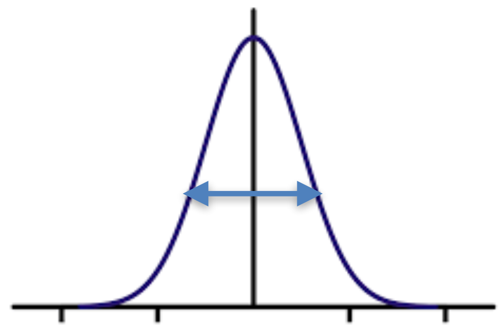
Spin transport



$$j_{\mathrm{NESS}} \sim 1/L^{z-1}$$

- Different pictures/setups: open (reservoirs), linear response to local perturbation, Kubo formula, …

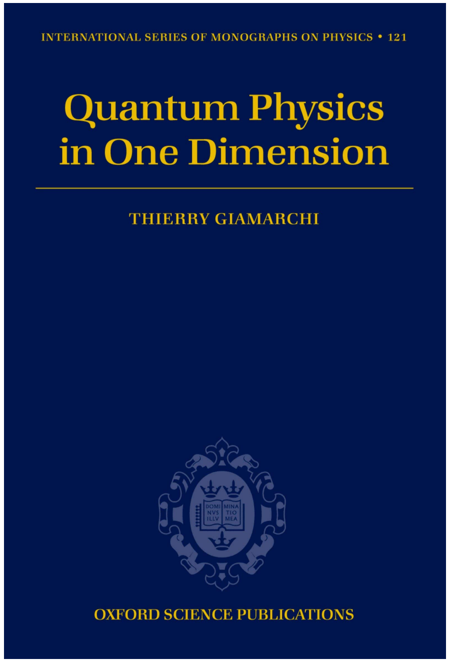


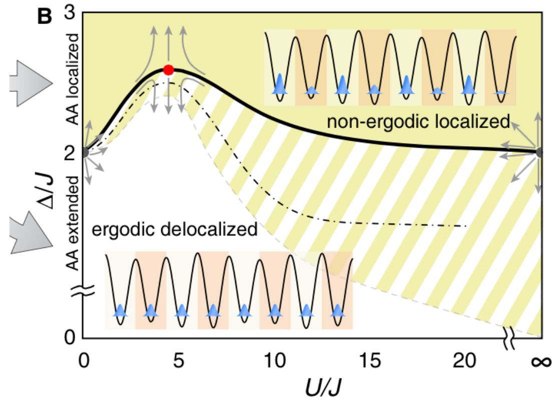
$$x \sim t^{1/z}$$

- Expected behavior:
  - **Diffusive**: $j = -D\nabla m$, finite d.c. conductivity, $z = 2$
  - **Ballistic**: $\sigma(\omega) = \pi D \delta(\omega) + \ldots$, finite Drude weight, $z = 1$
  - **Anomalous diffusion**

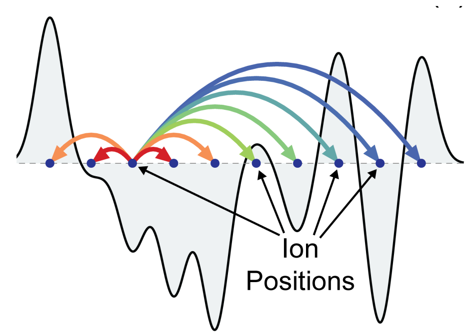
Picture credit: Jacopo De Nardis

1D is special

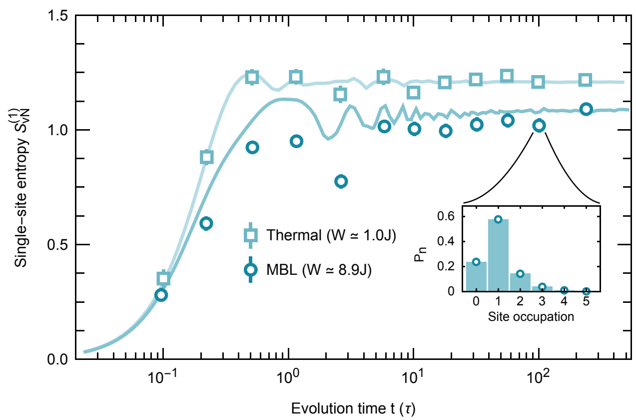




Schrieber et al, Bloch group '15



Smith et al, Monroe group '15



Lukin et al, Greiner group '18

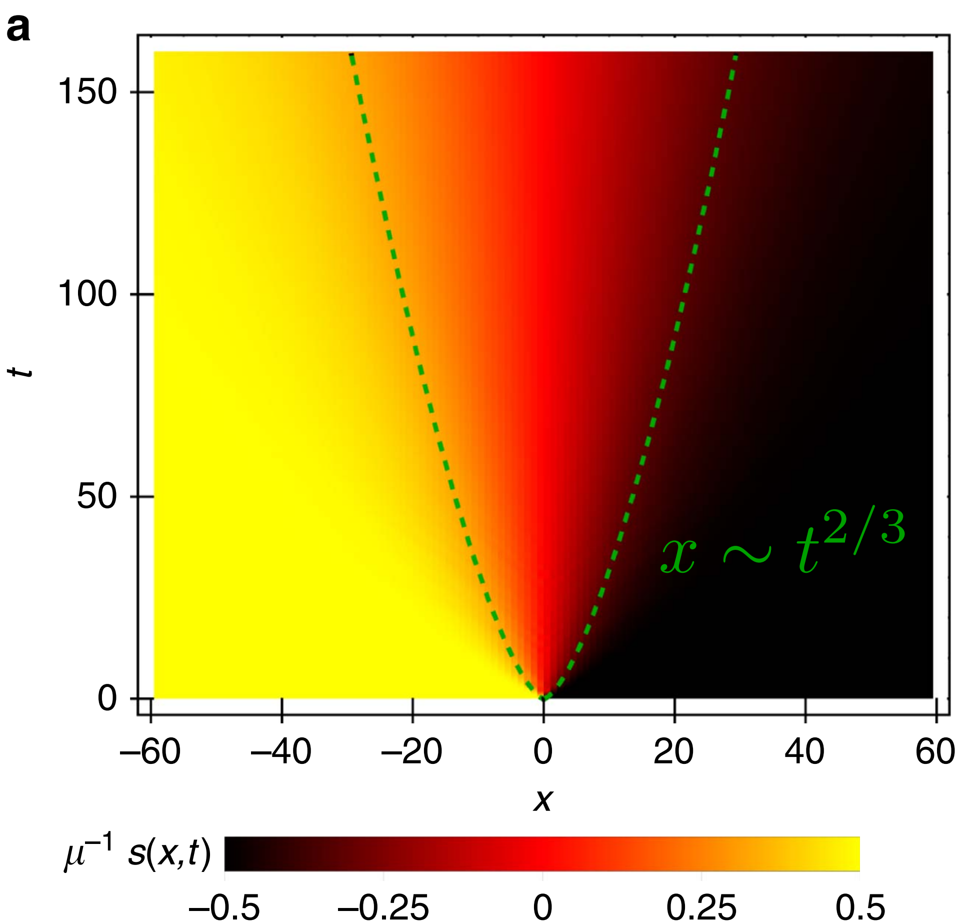
Luttinger Liquids

Many-body Localization

Conventional Hydrodynamics breaks down

(Long time tails in Galilean-invariant gases)

Superdiffusion



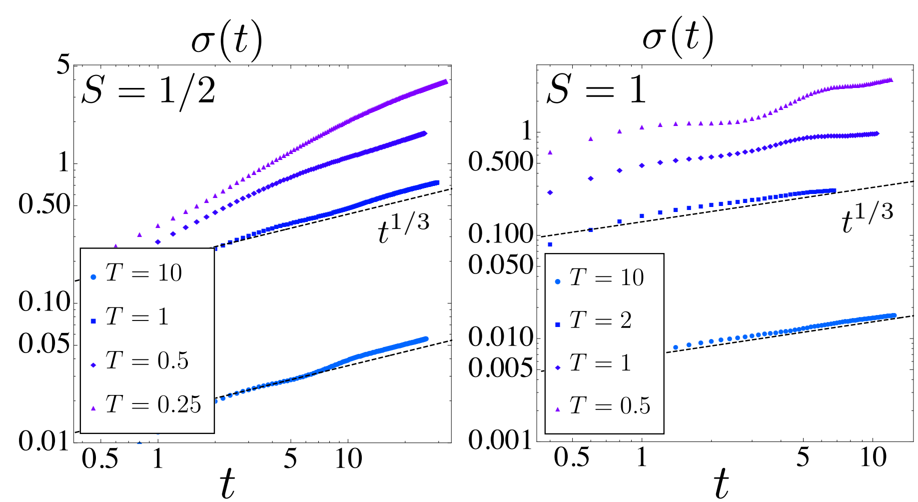
Ljubotina, Znidaric & Prosen '17
Nat Comm

Heisenberg spin-1/2 antiferromagnet:

$$H = \sum_i \vec{S}_i . \vec{S}_{i+1}$$

Spin transport is superdiffusive!

$$x \sim t^{1/z} \qquad z = 3/2$$

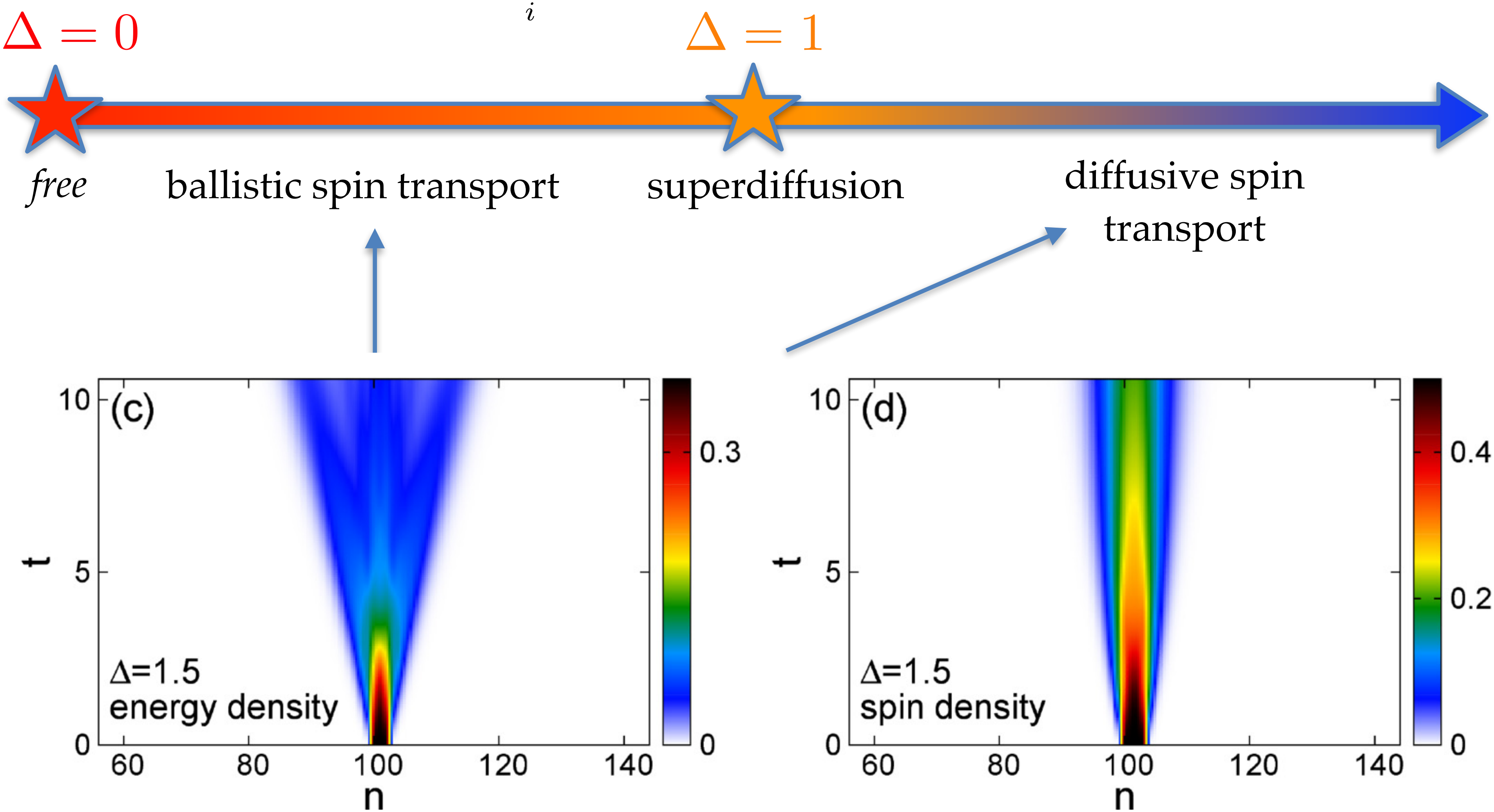


Integrable   Non-Integrable!

De Nardis, Medenjak, Karrasch & Ilievski, PRL '19

Dynamical phase transition

$$H = \sum_i J \left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z \right)$$



DMRG data for the XXZ spin chain, from Karrasch, Moore & Heidrich-Meisner '14

Spin diffusion coexisting with ballistic energy transport?

KPZ hydrodynamics

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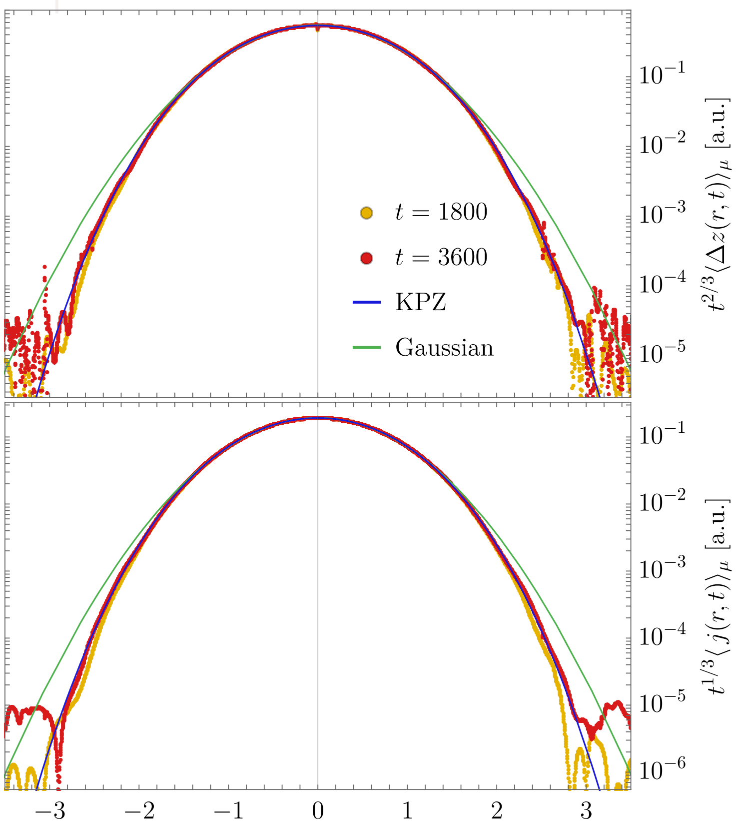
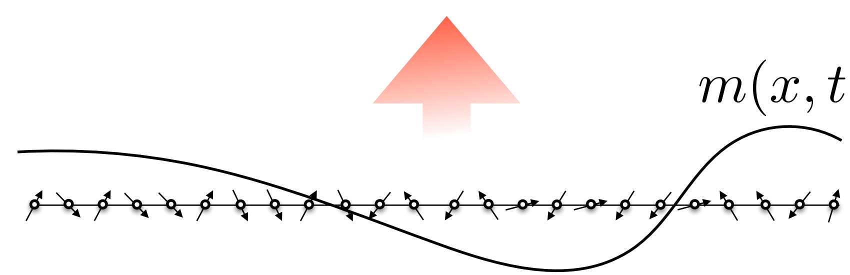
Kardar-Parisi-Zhang Physics in the Quantum Heisenberg Magnet

Marko Ljubotina, Marko Žnidarič, and Tomaž Prosen
Phys. Rev. Lett. **122**, 210602 – Published 31 May 2019



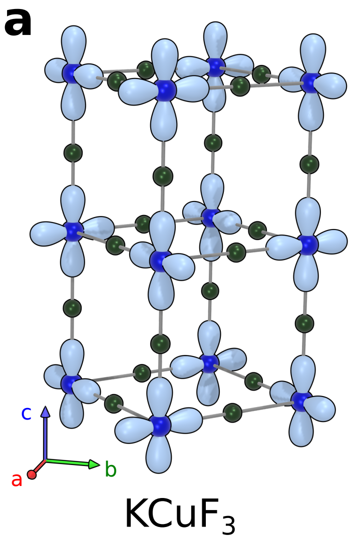
$$\langle \hat{S}^z_n(t)\hat{S}^z_0(0)\rangle \simeq \frac{\chi}{(\lambda_{\mathrm{KPZ}}\, t)^{2/3}} f_{\mathrm{KPZ}}\left(\frac{n}{(\lambda_{\mathrm{KPZ}}\, t)^{2/3}}\right)$$

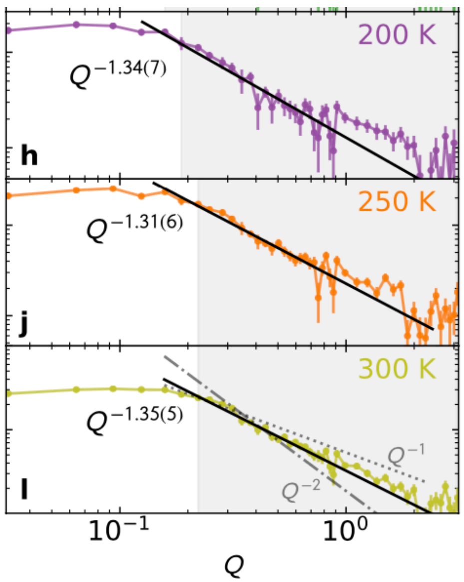
$$\partial_t m + \partial_x(\lambda m^2 + \dots) = D_{\mathrm{reg}}\partial_x^2 m + \text{noise}$$



Picture credit: Jacopo De Nardis

Experiments





nature physics

LETTERS

https://doi.org/10.1038/s41567-021-01191-6

Check for updates

Detection of Kardar–Parisi–Zhang hydrodynamics in a quantum Heisenberg spin-1/2 chain

A. Scheie[1,7], N. E. Sherman[2,3,7], M. Dupont[2,3], S. E. Nagler[1], M. B. Stone[1], G. E. Granroth[1], J. E. Moore[2,3] and D. A. Tennant[4,5,6]

Neutron scattering

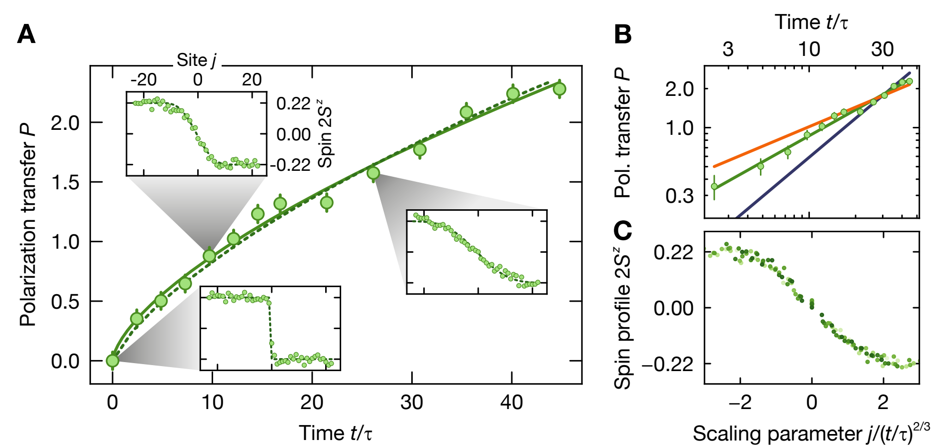
$$\mathcal{S}(Q, \omega \to 0) \sim Q^{-z}$$

See Joel's talk

Quantum gas microscopy of Kardar-Parisi-Zhang superdiffusion

David Wei,[1,2] Antonio Rubio-Abadal,[1,2,*] Bingtian Ye,[3] Francisco Machado,[3,4] Jack Kemp,[3] Kritsana Srakaew,[1,2] Simon Hollerith,[1,2] Jun Rui,[1,2,†] Sarang Gopalakrishnan,[5,6] Norman Y. Yao,[3,4] Immanuel Bloch,[1,2,7] and Johannes Zeiher[1,2]

Cold atoms



Superuniversality: see Enej's talk

See also Dupont & Moore, PRB '20, Krajnik, Ilievski, Prosen, Scipost '20

- Superdiffusive transport observed in a large class of quantum chains
- Key ingredients:
  - Spin rotation SU(2) symmetry (non-Abelian)
  - Integrability seems to play a key role (though superdiffusion has been observed in non-integrable models!)
- Question: How do we understand this? Start with integrable limit (Heisenberg chain), then add perturbations

$$\hat{H} = -t \sum_{\langle i,j\rangle,\sigma} \left(\hat{c}^\dagger_{\sigma,i}\hat{c}_{\sigma,j} + h.c.\right) + U\sum_i \hat{n}_{\uparrow,i}\hat{n}_{\downarrow,i}$$

Fava, Ware, Gopalakrishnan, Vasseur, Parameswaran PRB '20

$\Delta x \propto t^{2/3}$ and $C(0,t) \propto t^{-2/3}$

Ilievski, De Nardis, Gopalakrishnan, RV, Ware, PRX '21
See Enej's talk!

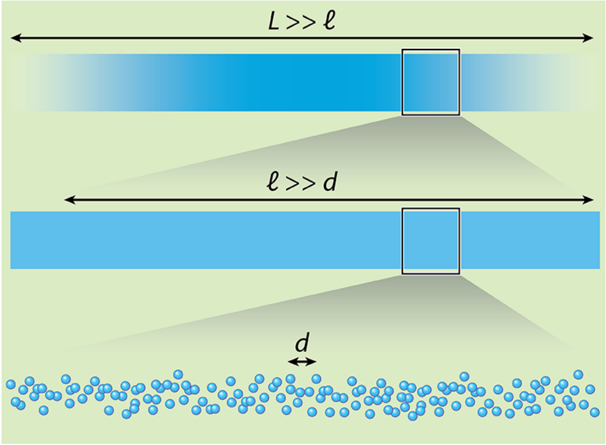
Also survives integrability breaking up to long times, more on this later...

Theory: Generalized hydrodynamics

- New hydrodynamics framework:

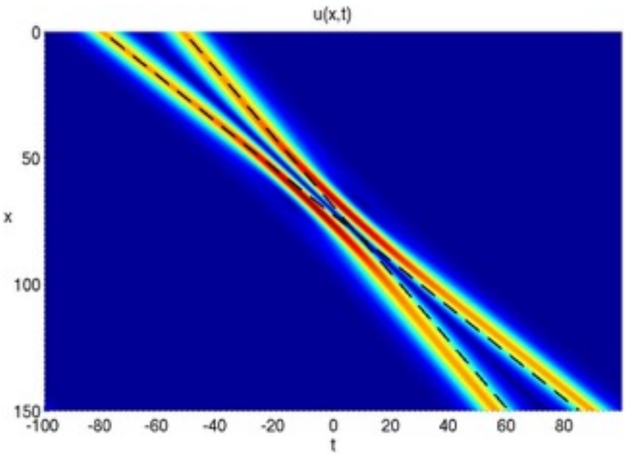
Castro-Alvaredo, Doyon & Yoshimura PRX '16
Bertini, Collura, De Nardis & Fagotti PRL '16
…

Picture credit: J. Dubail, Physics 9, 153 (2016)



- Two complementary pictures

VS



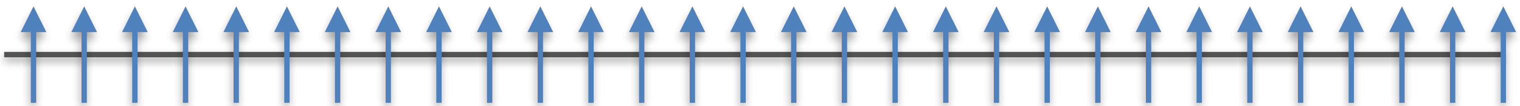
Hydrodynamic approach that takes all conserved quantities into account

Kinetic theory approach for the stable quasiparticles (=solitons) of such models

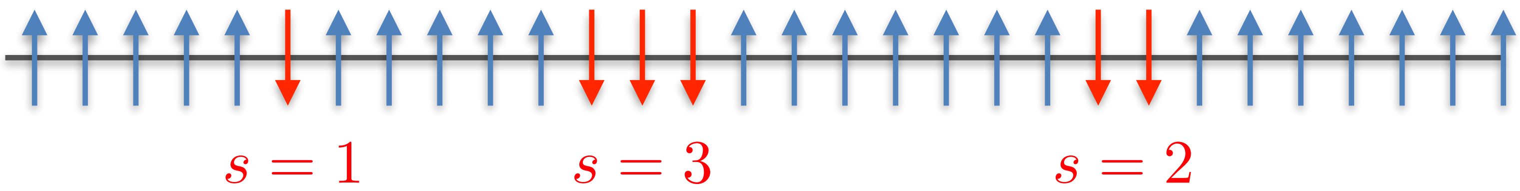
Doyon, Yoshimura & Caux, PRL'17
V. Bulchandani, RV, C. Karrasch & J.E. Moore, PRB '17

Quasiparticles: magnons and strings

Start from reference ("vacuum") state:



Magnons and bound states of magnons ("string"), stable quasiparticles at finite T



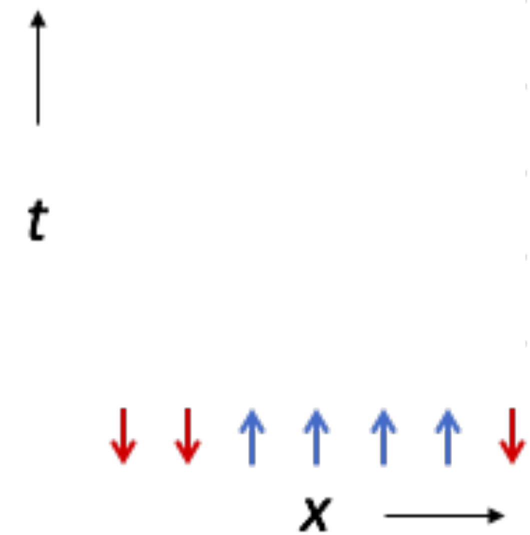
Behave like semi-classical particles:

Density: $\rho_s \sim \frac{1}{s^3}$ Velocity: $v_s \sim \frac{1}{s}$ Charge: $m_s = s$

ballistically moving charged quasiparticles —> ballistic transport?

Screening

- **Magnon: spin down moving in a majority of spins up, becomes a spin up in a majority of spins down when scattering with a bigger "string" (cartoon in FM limit)**



- Spends half its time being up or down

``Quantum Bowling", Ganahl, Haque & Evertz PRB '14

- Quasiparticle "s" gets screened when it collides with a QP s' bigger than itself:

Density: $$\sum_{s'>s} \rho_{s'} \sim 1/s^2$$

- Using velocity $v_s \sim 1/s$, this gives a screening time scale: $\tau_s \sim s^3$

Anomalous diffusion

At time t, only magnon bound states with $s \sim t^{1/3}$ contribute to transport!

- **Kubo formula:**

$1/s^3$ Screening

$$\sigma(\omega) = \int_0^\infty dt e^{i\omega t} \langle J(t) j(0) \rangle \sim \int_0^\infty dt e^{i\omega t} \sum_s \rho_s (v_s m_s)^2 e^{-t/\tau_s}$$

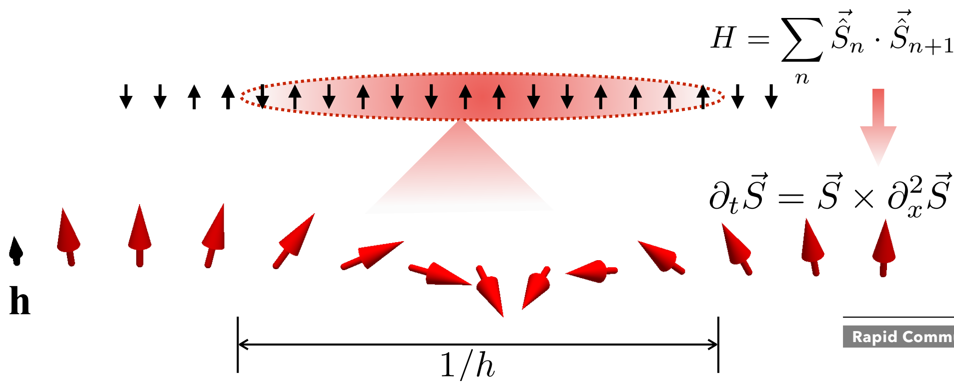
$1/s \times s$

$$\sigma(\omega) \sim \omega^{-1/3}$$

Superdiffusion!

Gopalakrishnan & RV, PRL ‘19
De Nardis, Gopalakrishnan, RV, Ware, PRL ‘21

Where does KPZ come from?



Quasiparticles constructed above reference vacuum state. SU(2) “gauge” Degree of freedom.

PHYSICAL REVIEW B **101**, 041411(R) (2020)

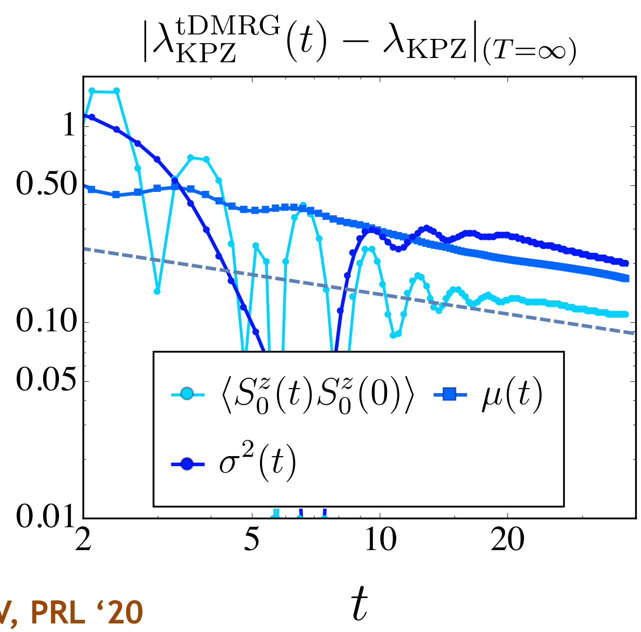
Rapid Communications

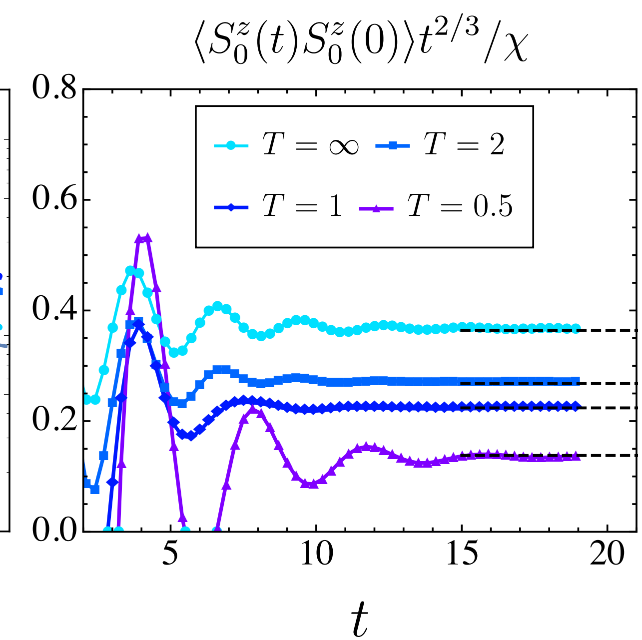
Kardar-Parisi-Zhang universality from soft gauge modes

Vir B. Bulchandani

Department of Physics, University of California, Berkeley, Berkeley, California 94720, USA

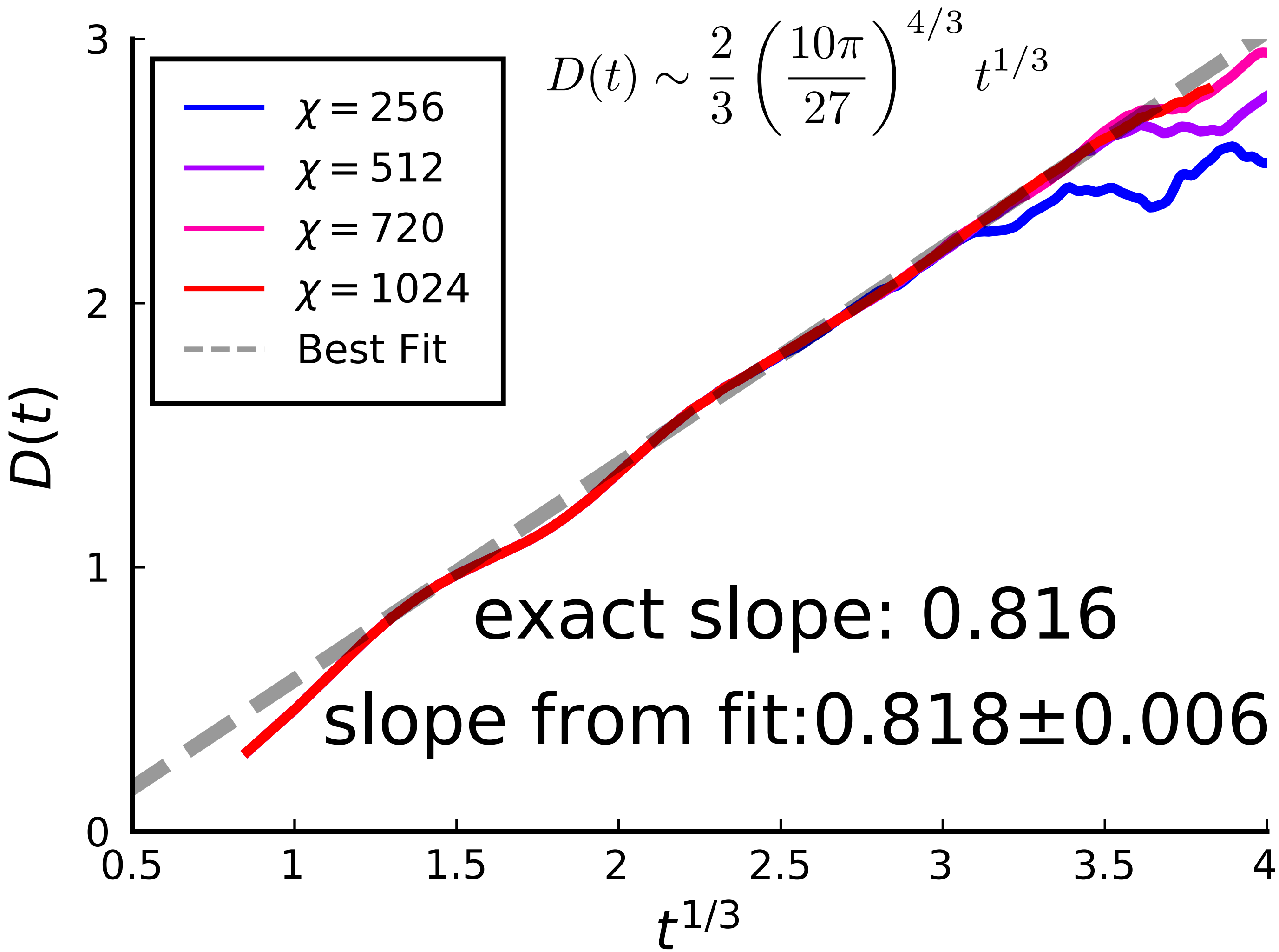
- Mapping between soft goldstone modes And giant quasiparticles
- Can even compute the KPZ coupling (“superdiffusion constant”)





De Nardis, Gopalakrishnan, Ilievski & RV, PRL ‘20

KPZ coupling and superdiffusion constant



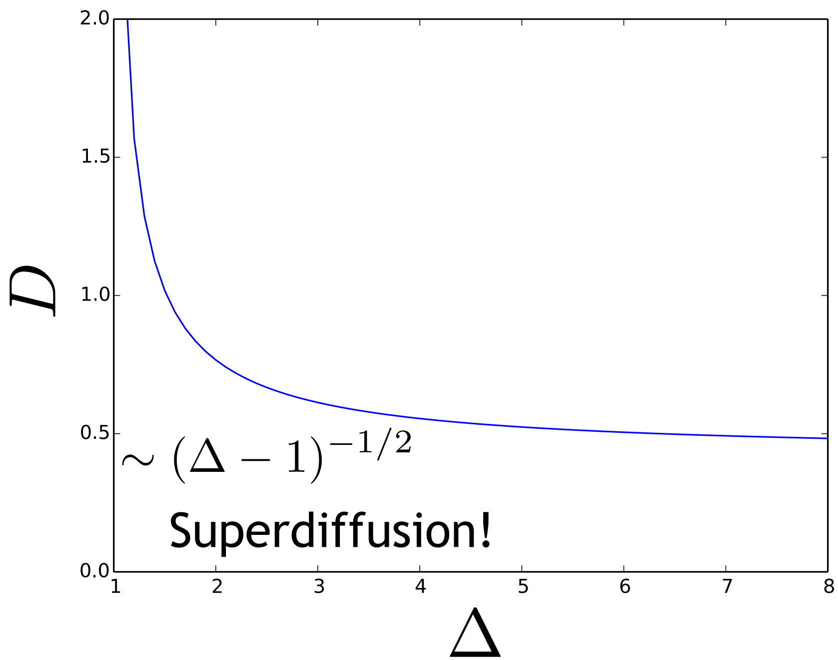
De Nardis, Gopalakrishnan, Ilievski & RV, PRL '20
De Nardis, Gopalakrishnan, RV, Ware PRL '21

Breaking the SU(2) symmetry

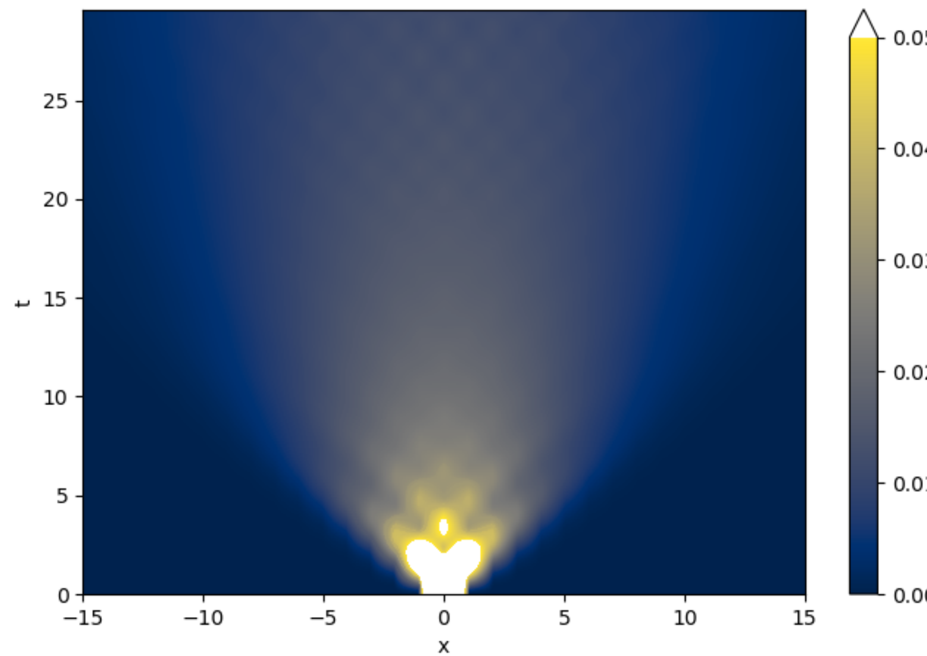
$$H = \sum_i J\left(S_i^x S_{i+1}^x + S_i^y S_{i+1}^y + \Delta S_i^z S_{i+1}^z\right)$$

Giant QP are exponentially suppressed,
Finite d.c. conductivity
(= diffusion)

De Nardis, Bernard, Doyon, Scipost '19
S. Gopalakrishnan & RV, PRL '19
S. Gopalakrishnan, RV & B. Ware, PNAS '19



$$\langle S_z(x,t) S_z(0,0)\rangle$$

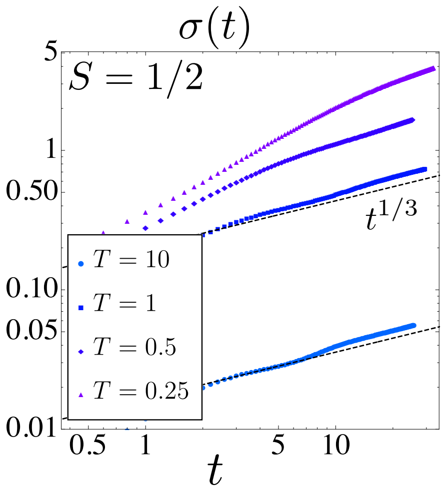


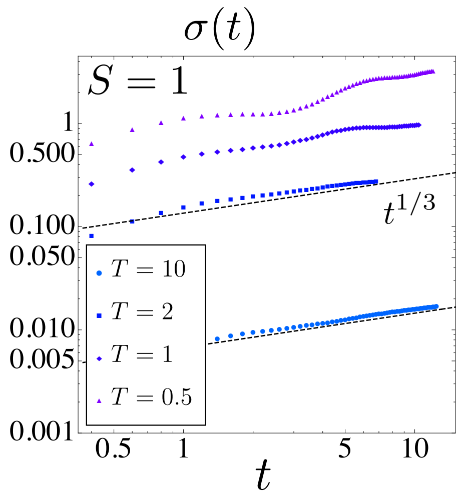
- **Diffusion constant (large T):**

$$D = \frac{2\sinh\eta}{9\pi}\sum_{s=1}^{\infty}(1+s)\left[\frac{s+2}{\sinh\eta s} - \frac{s}{\sinh\eta(s+2)}\right]$$

$$\eta \;=\; \cosh^{-1}(\Delta)$$

Integrability breaking





Integrable **Non-Integrable!**

De Nardis, Medenjak, Karrasch & Ilievski, PRL '19

Superdiffusion appears to be robust to breaking integrability?

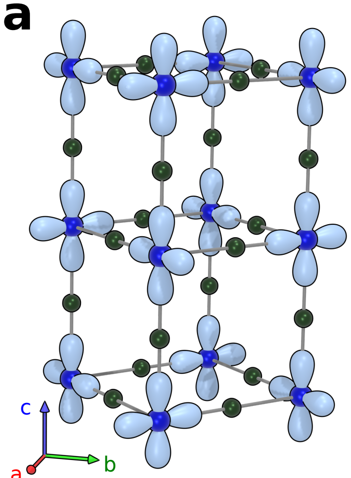
Why?

SU(2) hydro: vanilla diffusion?

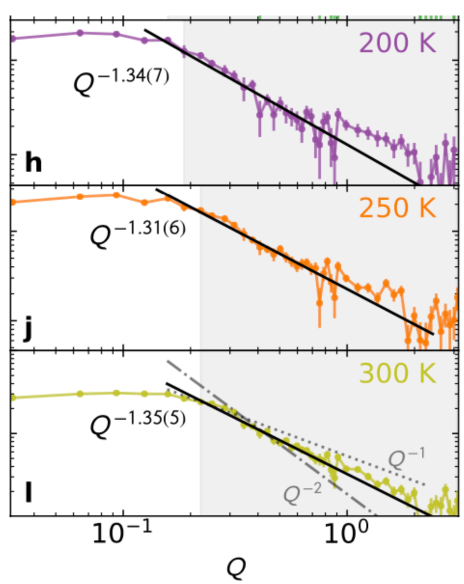
P. Glorioso, L. Delacretaz, X. Chen, R. Nandkishore, A. Lucas

Scipost '21

a



$KCuF_3$



nature physics

LETTERS

https://doi.org/10.1038/s41567-021-01191-6

Check for updates

Detection of Kardar–Parisi–Zhang hydrodynamics in a quantum Heisenberg spin-1/2 chain

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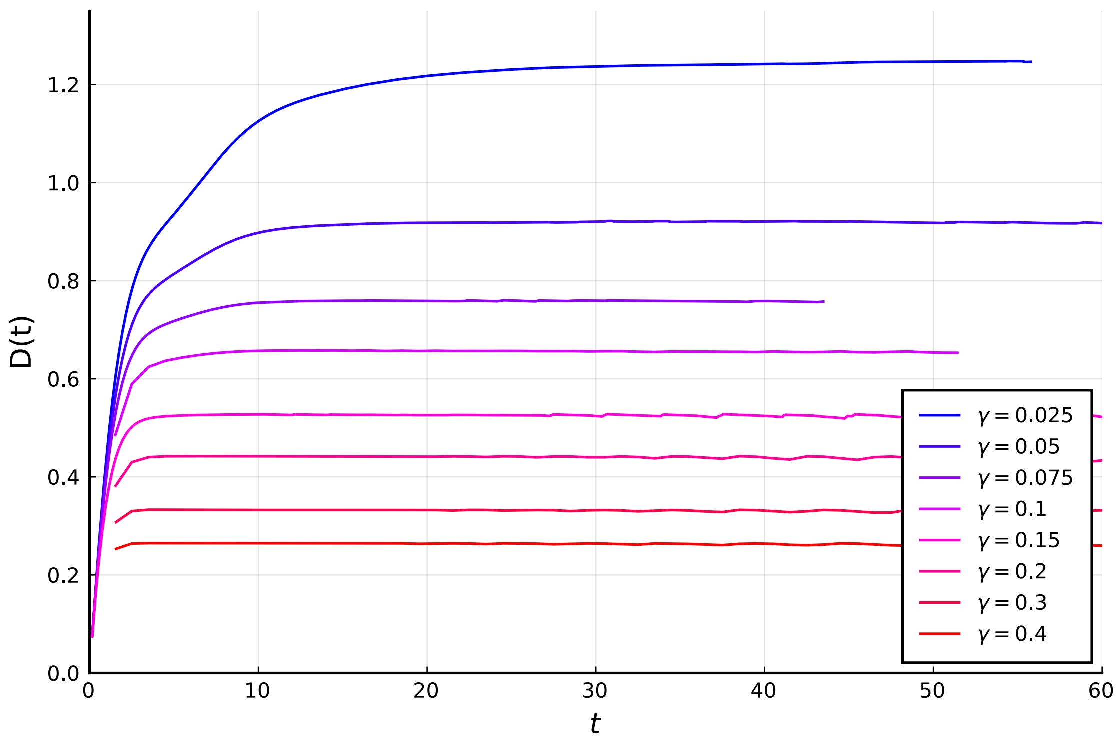
Integrability breaking: lifetimes

- **Quasiparticles can decay, have a finite lifetime**
- **Fermi Golden Rule estimate:** $\Gamma_s \sim \mathrm{d}.o.s. \times |V|^2$

For a generic perturbation breaking SU(2) (say noise coupling to Sz), rate Γ_s increases with s. Giant quasiparticles decay quickly…

Vanilla diffusive transport



De Nardis, Gopalakrishnan, RV, Ware PRL '21

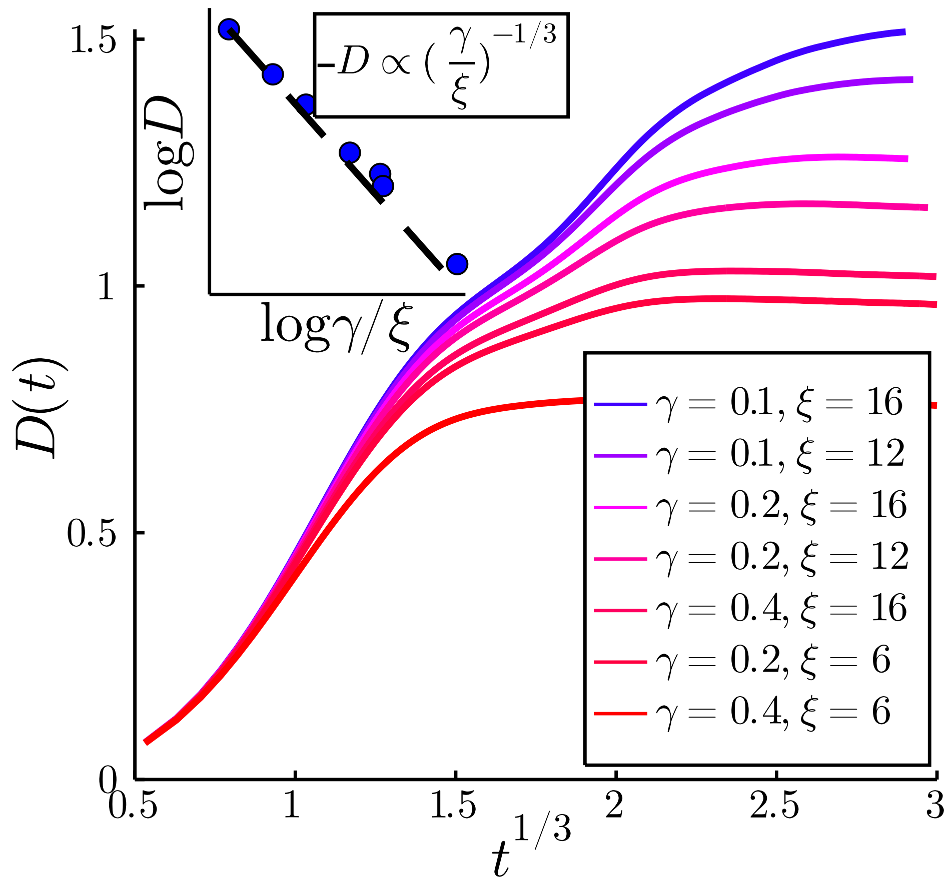
Non-analytic diffusion constant

$$H_\eta = H_0 + \sum_j \eta_j(t) O_j$$
$$\langle \eta_j(t)\eta_{j'}(t')\rangle = \gamma f(j-j')\delta(t-t')$$

Non-analytic dependence of D on the perturbation strength!

$$D \sim \gamma^{-1/3}$$

See also Znidaric PRL '20

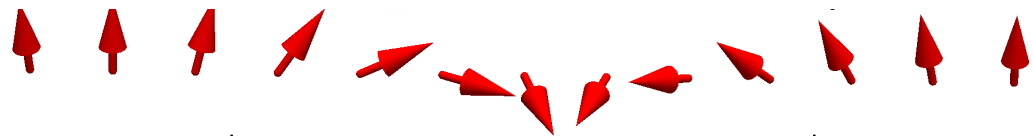


De Nardis, Gopalakrishnan, RV, Ware PRL '21

Integrability breaking: anomalous diffusion

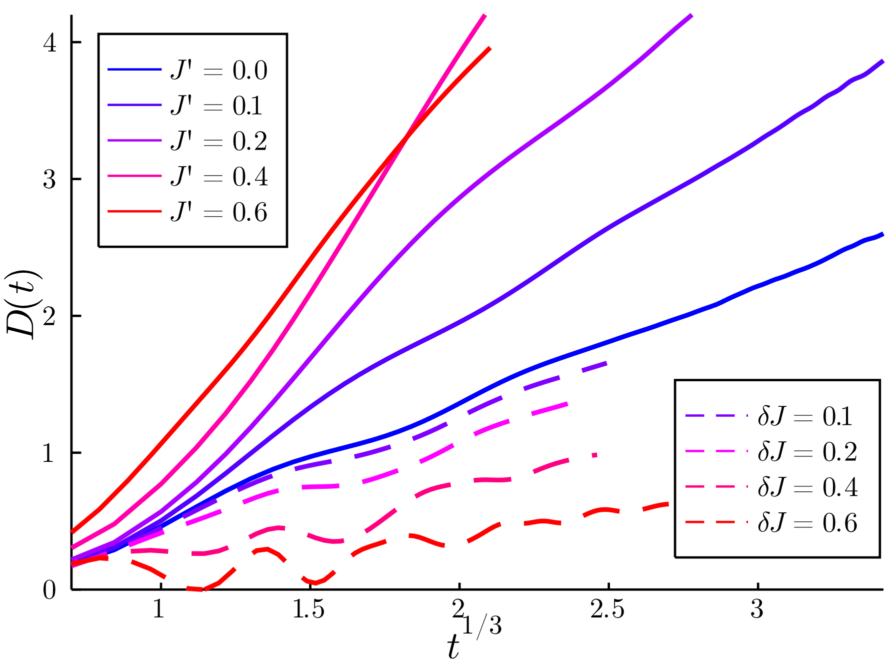
De Nardis, Gopalakrishnan, RV, Ware PRL '21

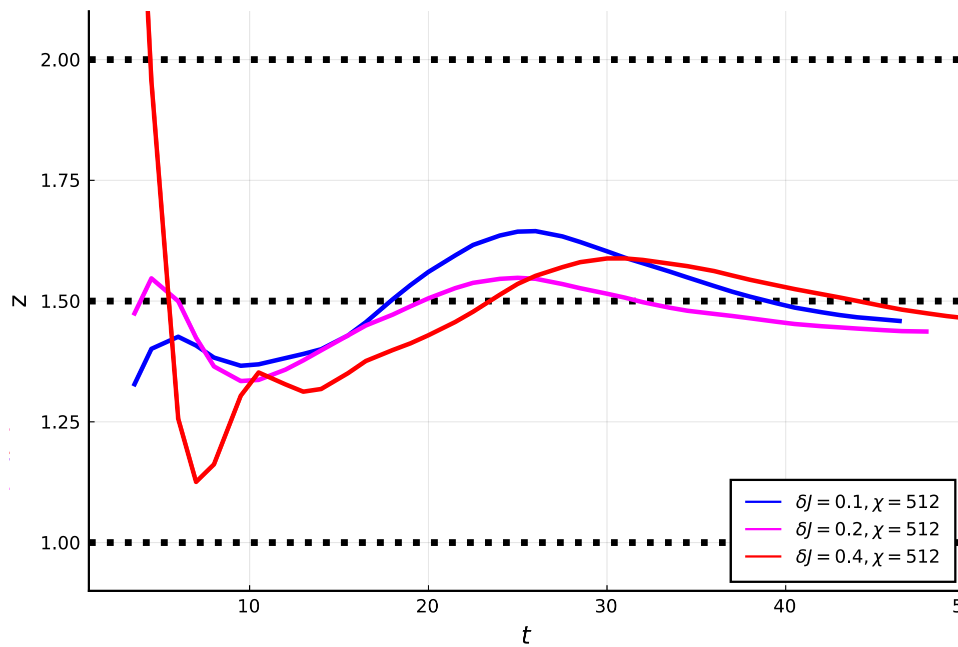
BUT if perturbation preserves SU(2), matrix element suppressed as 1/s (Goldstone physics), and density of states factors can only suppress this further



z=3/2 for all accessible time scales!!

$$H = J\sum_i \vec{S}_i.\vec{S}_{i+1} + \delta J \sum_i \vec{S}_i.\vec{S}_{i+2}$$





Integrability breaking: anomalous diffusion

See also: De Nardis, Medenjak, Karrasch & Ilievski PRL ’20 for logs in a different regime

Noisy SU(2)-symmetric perturbation: energy is not conserved

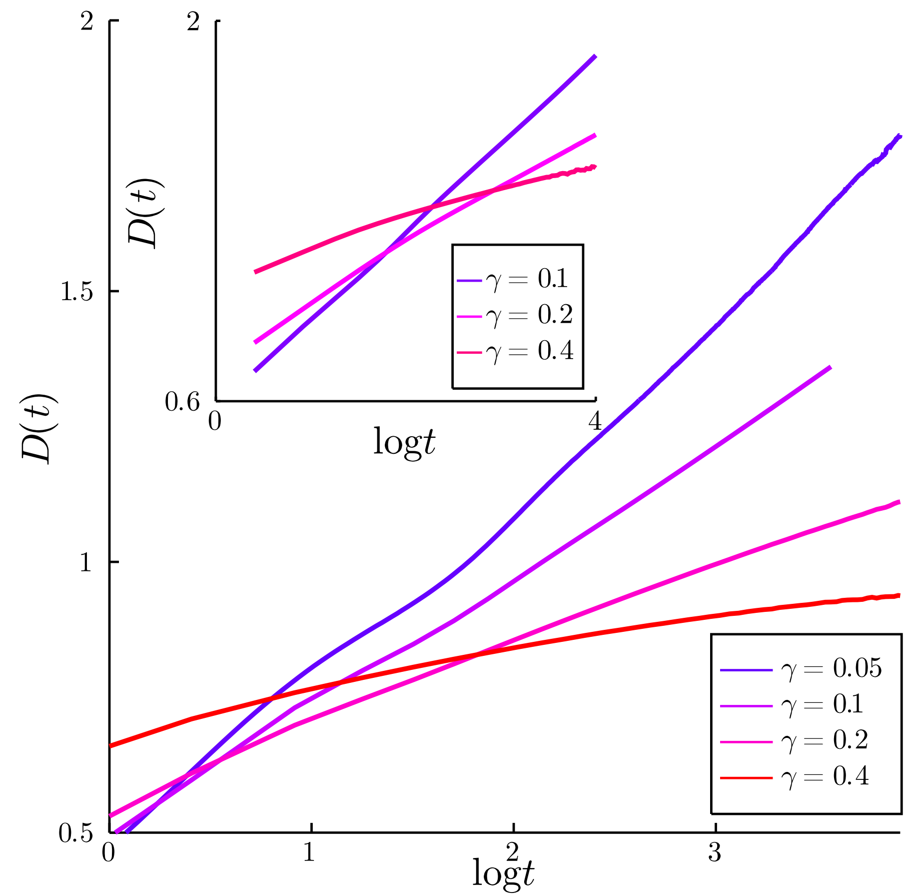
Giant quasiparticles are still long lived!

$$\Gamma_s \lesssim 1/s^2$$

$$\sigma(\omega) \sim \int dt e^{i\omega t} \sum_s \frac{1}{s^3} \mathrm{e}^{-t/s^2}$$

$$\sigma(\omega) \sim |\log \omega|$$

Log (super)diffusion!



De Nardis, Gopalakrishnan, RV, Ware PRL ‘21

Conclusion

- Anomalous (superdiffusive) transport in integrable isotropic spin chains

- Relatively simple mechanism, integrability needed only to stabilize quasiparticle excitations at high temperature

- Remarkably stable to integrability-breaking perturbations

- KPZ? Non-perturbative effects? Higher-order processes?