Medium modifications to jet angularities using SCET with Glauber gluons

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Based on *[arXiv: 2305.10237 [hep-ph]]* with Ankita Budhraja and Balbeer Singh

Jet era at the LHC

- The EIC will provide new information about the structure of nucleons as well as nucleons present in a nuclear environment
- But another aspect of dynamics in nuclear collisions is collectivity, and has been explored extensively at RHIC and LHC
- Jets are useful hard-probes of this collectivity, and we have a large collection of data on fully reconstructed jets in heavy ion collisions at the LHC (at all three experiments ATLAS, CMS and ALICE) and interesting jet results from STAR
- In particular, there is data on jet yields (R_{AA}), jet substructure observables eg. jet Fragmentation distributions $D(z)$ (longitudinal structure), jet shapes, $\gamma - /Z - /jet-$ tagged jet spectra and substructures ...
- Here we study the modification of jet angularities in the QGP

Jet angularities

- Consider a jet (start with *pp* collisions) formed by a collection of particles of momenta $\{p_i\}$
- Define and observable,

$$\tau_a = \sum_{i \in jet} z_i \Delta R_{i,jet}^{2-a}$$

- $$z_i = \frac{p_{T,i}}{p_{T,jet}}, \quad \Delta R_{i,jet} = \sqrt{\Delta\phi_{i,jet}^2 + \Delta\eta_{i,jet}^2}$$

  [Berger et. al. (2003); Hornig et. al (2009); Thaler et. al (2014)]
- For infrared safety need $a < 2$. To have a virtuality separation between the soft and collinear modes, we further only take $a < 1$.
- Wider jets have larger τ_a. Can change the relative distribution by changing a
- Eventually, hope to learn about the QGP by looking at jets in *AA* collisions

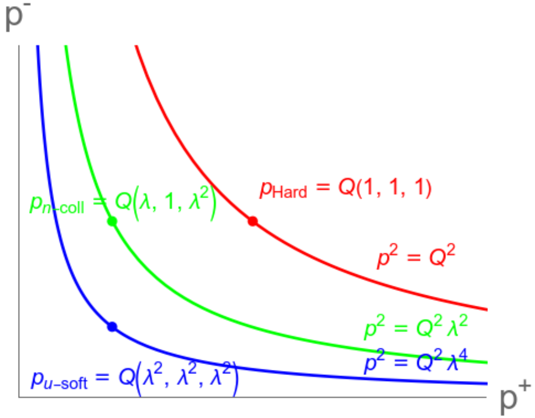
Illustrative example: Thrust

- $a = 0$ well known as the thrust *[Farhi (1977)]*

$$\tau_0 = \sum_{i \in jet} z_i \Delta R^2_{i,jet}$$

- Let the jet direction be $\hat{n}$. We are interested in small angle deviations, i.e. $\tau \ll 1$
- Substantial contribution comes from energetic partons ($z_i \sim 1$) that are collinear with the jet
- These have momenta $p^\mu \sim p_{T,jet}(1, \hat{n}, 0_\perp)$. In light cone coordinates $p_{n-coll} \sim p_{T,jet}(0, 1, 0_\perp)$. More typically, $p_{n-coll} = p_{T,jet}(\lambda^2, 1, \lambda)$ where $\lambda^2 \sim \tau_0$ ($\lambda \sim \Delta R_{i,jet}$)
- Ultra-soft partons have much smaller energy ($z_i \ll 1$). But they also give a similar contribution to τ as ΔR is larger $p_{u-soft} = p_{T,jet}(\lambda^2, \lambda^2, \lambda^2)$

Relevant modes

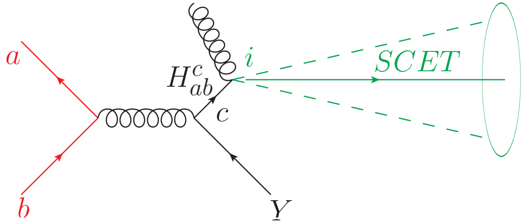


- $Q \sim p_{T,jet}$
- *[Manohar, Stewart, Pirjol, Bauer,.. .. Beneke ..]*
- For general a, $p_{u-soft} = p_{T,jet}(\lambda^{2-a}, \lambda^{2-a}, \lambda^{2-a})$ and $\tau \sim \lambda^{2-a}$

Factorization of the hard process I

- The separation of scales allows us to write an EFT of collinear and soft modes only: SCET. These are the modes that contribute to the observable, τ
- The hard modes, $\sim Q^2$, can be integrated out
- In the EFT their effect will show up as short distance coefficients of higher dimensional operators that can be found from matching
- Schematically, $\mathcal{L} = \mathcal{L}_{SCET}^{(0)} + \sum_{n,a} \frac{1}{Q^n} C_{n,a} \mathcal{O}^{n,a}$
- Analogy with $1/M$ power counting in NRQCD

Factorization of the hard process II



- The matching of the hard process to SCET to obtain the Wilson coefficient computed in QCD
- It involves the inclusive (don't care about Y) production of a hard parton c (H^c_{ab}). The hard process was computed NLO *[Aversa et. al. (1989)]*
- This hard parton "fragments" to i with a virtuality much smaller than p_T. The resulting parton i is collinear and describable in SCET

Factorization of the hard process III

- Schematically,

$$\frac{d\sigma^{AA\to\tau^a Y}}{d\tau_a dp_T d\eta} = \sum_{abc}\sum_i \int dx^a dx^b f_a(x_a,\mu)\otimes f_b(x_b,\mu)$$
$$\otimes H^c_{ab}(x_a,x_b,\eta,p_T/z,\mu)\times \mathcal{M}^{SCET}_{c\to i}(z,\tau_a,p_T,\mu)$$

- Slightly more complicated because first select jets of radius $\mathcal{R}\sim 0.2-0.5$ and then measure τ_a *[ALICE (2021)]*. Two hierarchies of collinear theories, one at $p_T\mathcal{R}$ and one at $p_t\tau_a$ (assuming $\tau_a \ll \mathcal{R}$) *[Kang, Lee, Ringer (2018)]*

Factorization of the hard process IV

- Factorization

$$\frac{d\sigma^{AA\to\tau^a Y}}{d\tau_a dp_T d\eta} = \sum_{abc}\sum_i \int dx^a dx^b f_a(x_a,\mu)\otimes f_b(x_b,\mu)$$
$$\otimes H^c_{ab}(x_a,x_b,\eta,p_T/z,\mu)\times \mathcal{H}_{c\to i}(z,p_T\mathcal{R},\mu)$$
$$\times \mathcal{J}(\tau^c_a,p_T,\mu)\otimes \mathcal{S}(\tau^s_a,p_T,R,\mu)$$

- $\mathcal{J}_i(\tau^c_a,p_T,\mu)$ gives the collinear contribution to τ_a
- $\mathcal{S}_i(\tau^s_a,p_T,R,\mu)$ gives the soft contribution to τ_a
- Now we can focus on the *SCET* dynamics since these will be the key differences in *pp* and heavy ion collisions

SCET: basic structure

- A lagrangian of soft and collinear fields. Eg. $A^\mu = A_s^\mu + A_n^\mu$
- The fermion spinor has large components in the helicity bases and the small components can be integrated out
- The lagrangian for the fermionic field collinear with direction n has the following form

$$\mathcal{L}_{n\xi} = e^{-ix\cdot\mathcal{P}}\bar{\xi}_n[in\cdot D + i\not{D}_{n,\perp}\frac{1}{i\bar{n}\cdot D_n}\not{D}_{n,\perp}]\frac{\not{\bar{n}}}{2}\xi_n$$

- $\mathcal{P}$ is a momentum projection operator that selects the large component of the momentum $[Q(0,1,\lambda)]$. The collinear derivatives are
  - $iD^\mu_{n\perp} = \mathcal{P}^\mu_\perp + gA^\mu_{n\perp}$
  - $i\bar{n}\cdot D_n = \bar{\mathcal{P}} + g\bar{n}\cdot A_n$
- Only coupling between collinear quarks and soft gluons is from the $in\cdot D$ term. This can be removed by gauge transformations (soft Wilson lines appear in currents). This is why the collinear ($\mathcal{J}_i$) and the soft factors ($\mathcal{S}_i$) appear separately
- Similar steps in the gauge sector

SCET: Ingredients

- Gauge invariant collinear quark and gluon fields are built out of the SCET collinear fields (ξ_n,A_n) and the collinear Wilson line (U_n)

$$\chi_n(x) = U_n^\dagger(x)\xi_n(x)$$
$$\mathcal{B}^\mu_{n,\perp}(x) = \frac{1}{g} U_n^\dagger(x) iD^\mu_{n,\perp} U_n(x)\,.$$

- $$U_n(x) = \mathcal{P}\exp\left(-ig\int_0^\infty dy\,\bar{n}\cdot A_n(x+\bar{n}y)\right)\,,$$

- Y_n, $Y_{\bar{n}}$ defined analogously with collinear fields A_n replaced by soft fields A_s

Angularity operators

- Collinear function for angularity

$$\mathcal{J}_q(\tau_a^c, p_T, \mu) = \frac{1}{2N_c} Tr\left[\frac{\bar{\not{n}}}{2}\langle 0|\delta(\omega - \bar{n}\cdot\mathcal{P})\delta(\tau_a^c - \hat{\tau}_a^c)\chi_n(0)|JX\rangle \right.$$
$$\left. \langle JX|\bar{\chi}_n(0)|0\rangle \right],$$
$$\mathcal{J}_g(\tau_a^c, p_T, \mu) = -\frac{\omega}{2(N_c^2-1)}\langle 0|\delta(\omega - \bar{n}\cdot\mathcal{P})\delta(\tau_a^c - \hat{\tau}_a^c)\mathcal{B}_{n\perp\mu}(0)|JX\rangle$$
$$\langle JX|\mathcal{B}_{n\perp}^{\mu}(0)|0\rangle .$$

$\omega \sim p_T$ is the large component of the initiating parton i.

- Soft function for angularity

$$\mathcal{S}_q(\tau_a^s, p_T, R, \mu) = \frac{1}{N_c}\langle 0|\bar{Y}_n\delta(\tau_a^s - \hat{\tau}_a^s)Y_{\bar{n}}|X\rangle\langle X|\bar{Y}_{\bar{n}}Y_n|0\rangle ,$$
$$\mathcal{S}_g(\tau_a^s, p_T, R, \mu) = \frac{1}{N_c^2-1}\langle 0|\bar{Y}_n\delta(\tau_a^s - \hat{\tau}_a^s)Y_{\bar{n}}|X\rangle\langle X|\bar{Y}_{\bar{n}}Y_n|0\rangle ,$$

Resummation

- H, $\mathcal{H}$, $\mathcal{J}$, $\mathcal{S}$ have to be computed at the same scale μ. But NLO expressions have large logs of μ/p_T, $\mu/(p_T\mathcal{R})$, μ/τ_a respectively. Resum using RG
- $$\mu\frac{d}{d\mu}\mathcal{J}_i^{\rm vac}(\tau_a, p_T, R, \mu) = \int d\tau_a'\,\gamma_{\mathcal{J}_i}(\tau_a-\tau_a', p_T, R, \mu)\,\mathcal{J}_i^{\rm vac}(\tau_a', p_T, R, \mu)\,.$$

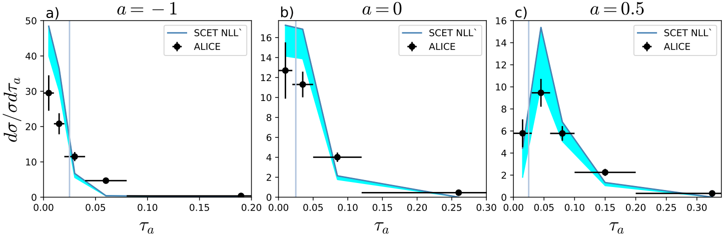
  The collinear anomalous dimensions are given by

  $$\gamma_{\mathcal{J}_i}(\tau_a, p_T, R, \mu) = \frac{\alpha_s(\mu)}{\pi}\left\{\delta(\tau_a)\left(2b_i+\frac{(2-a)}{(1-a)}\ln\frac{\mu^2}{p_T^2}\,C_i\right)-\frac{2}{1-a}C_i\left[\frac{1}{\tau_a}\right]_+\right\}.$$
- $$\mu\frac{d}{d\mu}\mathcal{S}_i^{\rm vac}(\tau_a, p_T, R, \mu) = \int d\tau_a'\,\gamma_{\mathcal{S}_i}(\tau_a-\tau_a', p_T, R, \mu)\,\mathcal{S}_i^{\rm vac}(\tau_a', p_T, R, \mu)\,,$$

  The soft anomalous dimensions are given as

  $$\gamma_{\mathcal{S}_i}(\tau_a, p_T, R, \mu) = \frac{2\alpha_s(\mu)\,C_i}{\pi\,(1-a)}\left\{\left[\frac{1}{\tau_a}\right]_+-\ln\frac{\mu\,R^{1-a}}{p_T}\,\delta(\tau_a)\right\}.$$
- NLL formalism and computation *[Kang, Lee, Ringer (2018)]*

Comparison with *pp*



- Ungroomed jet angularity distribution τ_a in *pp* collisions from*[ALICE (2021)]* for $R = 0.4$
- $80 < p_T < 100$
- Compared with *[Budhraja, RS, Singh (2023)]*. $\mu = [1 - 2]\, p_T$

Heavy ion collisions

Jet function using splitting

- Jet functions in SCET in some cases can be directly computed from the spin-averaged QCD splitting functions. For eg. see *[Ritzmann et. al. (2014), Cal et. al (2019)]*
- For angularity jet functions,

$$\mathcal{J}^{vac}_{i\to jk}(\tau_a, p_T, R, \mu) = \frac{\alpha_s(\mu)}{\pi}\frac{e^{\epsilon\gamma_E}\mu^{2\epsilon}}{\Gamma(1-\epsilon)}\sum_{j,k}\int dx \frac{dk_\perp}{k_\perp^{2\epsilon-1}}\mathcal{P}_{i\to jk}(x, k_\perp)\delta(\tau_a-\hat{\tau}_a)$$

  x is the momentum fraction carried by the final parton, $k_\perp$ is its transverse momentum.
- We checked that for *pp*, this approach gives the same jet functions and running as *[Kang, Lee, Ringer (2018)]*
- Useful because medium affects the splitting functions

Medium effect on jet functions

- $\mathcal{J} = \mathcal{J}^{vac} + \mathcal{J}^{med}$
- $$\mathcal{J}^{med}_{i\to jk}(..) \sim \sum_{j,k} \int dx \frac{d^2 k_\perp}{(2\pi)^2} P^{med}_{i\to jk}(k_\perp, x)\delta(\tau_a - \hat{\tau}_a)$$
- The physics of $P^{med}_{i\to jk}$ is medium induced bremmstrahlung. *BDMPS-Z, GLV, Wiedemann, Gyulassy, Wang, Majumder...*

Coupling between Glauber and collinear modes

- Medium particles (η) interact with the collinear modes in the jet function via Glauber modes (off-shell modes)
- This coupling induces radiation
- Eg. interaction between collinear quarks and the medium has the form,

$$\mathcal{L}_G(\chi_n, \mathcal{B}_n, \eta) = \sum_{q,p,p'} e^{-i(q+p-p')\cdot x} \left(\frac{1}{2} \bar{\chi}_{n,p'} \Gamma^{\nu,a}_{qqG} \bar{\not{n}} \chi_{n,p} \right) \bar{\eta} \Gamma^{\delta,a}_{s} \eta \Delta_{\nu\delta}(q),$$

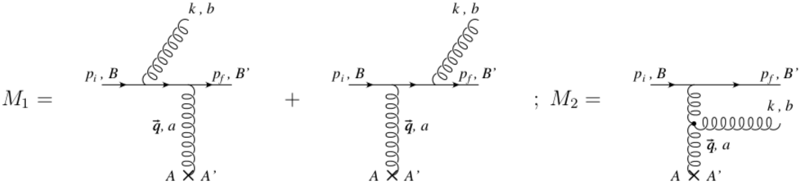
where Γ_{qqG} are vertices, η and $\bar{\eta}$ are source fields and $\Delta_{\nu\delta}$ is the Glauber gluon propagator.

Medium effect on jet functions

- $\mathcal{J} = \mathcal{J}^{vac} + \mathcal{J}^{med}$
- $$\mathcal{J}^{med}_{i\to jk}(..) \sim \sum_{j,k} \int dx \frac{d^2 k_\perp}{(2\pi)^2} P^{med}_{i\to jk}(k_\perp, x)\delta(\tau_a - \hat{\tau}_a)$$
- The physics of $P^{med}_{i\to jk}$ is medium induced bremmstrahlung. *BDMPS-Z, GLV, Wiedemann, Gyulassy, Wang, Majumder...*

Single gluon emission

- Single (transverse) scattering from the medium and resulting induced gluon emission from an energetic parton



- Typical scattering momentum m_D ($\ll p_T$). Mean free path between scatterings λ_{mf}
- These exchange gluons (emitted by the medium "scattering centres" A above) are called Glauber modes. In many approximations, taken to be static
- The emitted gluons (k, b) affect the distribution of partons

Medium effect on angularities

- Medium splitting kernels derived using $SCET_G$ in *[Vitev, Ovanesyan (2011, 2012, 2013)]*. For eg, in the small x limit,

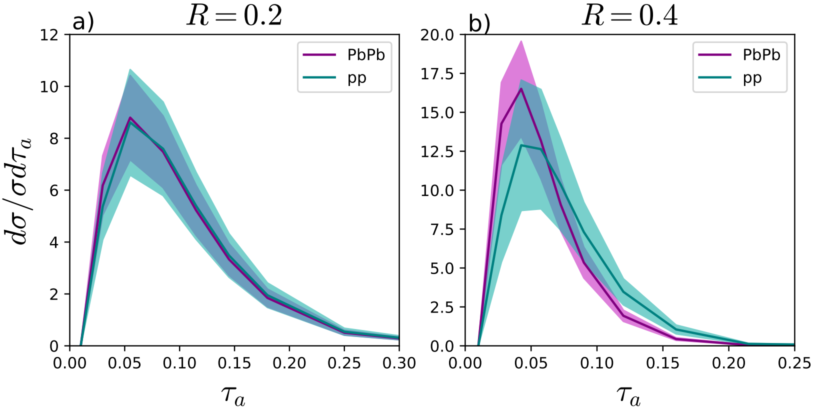
$$x\frac{dN^{med}_{q\to qg}}{dxd^2k_\perp} = \alpha_s \int_0^L d\Delta z d^2q_\perp \frac{1}{\sigma}\frac{d^2\sigma}{dq_\perp^2}\frac{2k_\perp q_\perp}{k_\perp^2(q_\perp - k_\perp)^2} \times \left[1 - \cos\left(\frac{(q_\perp - k_\perp)^2\Delta z}{x\omega}\right)\right]$$

- *[Gyulassy, Wang (1994)]*

$$\frac{1}{\sigma}\frac{d^2\sigma}{dq_\perp^2} = \frac{m_D^2}{\pi(q_\perp^2 + m_D^2)^2}$$

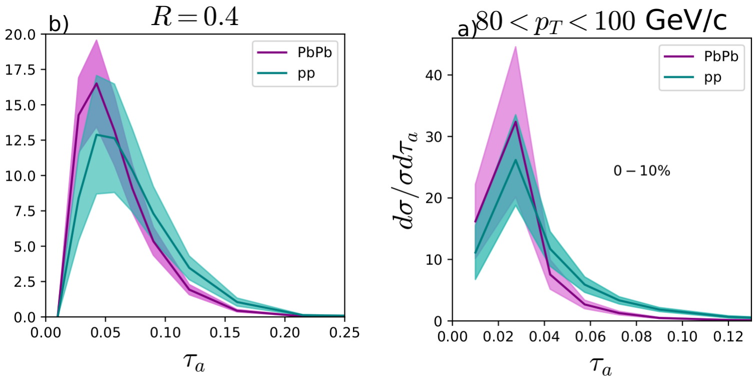
- We model the medium as a Bjorken expanding medium and use an average L and T for every centrality bin

Angularities for different R



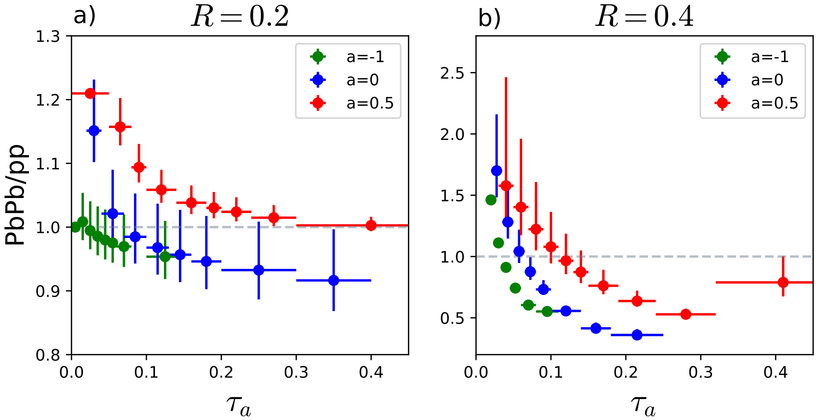
- Normalized τ_a distributions for $p_T \in [40, 60]$GeV, $a = 0$, $0 - 10\%$ centrality. $R = 0.2$ (left) $R = 0.4$ (right) Interpretation: (a) additional medium induced emissions at intermediately small angle (b) the jets in *PbPb* came from "would be" higher energy, narrower jets in *pp*, and have smaller τ_a

Angularities for different p_T



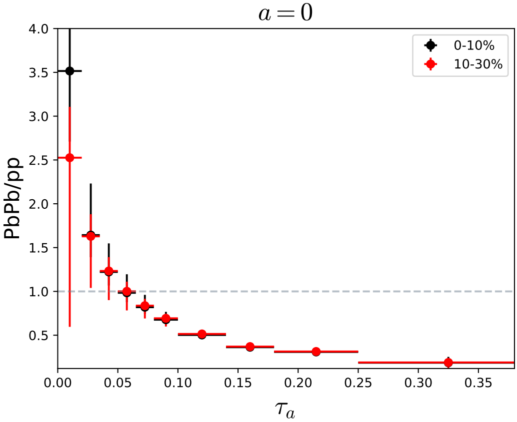
- $p_T \in [40, 60]$GeV (left), $p_T \in [80, 100]$GeV (right), $a = 0$, $R = 0.4$, $0 - 10\%$ centrality

Ratios for different a



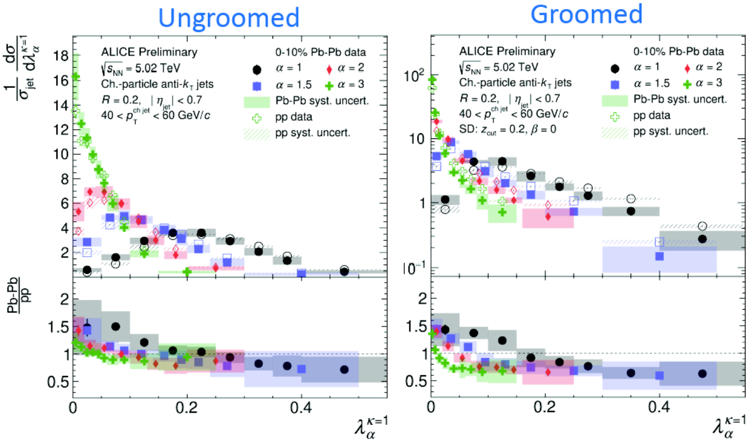
- Ratios of normalized distributions for different a

Ratios for different centralities



- Ratios of normalized distributions for different centralities

Angularities, preliminary



- Preliminary data from *[ALICE AA (QM 2022).]*

Summary

- Jet substructure is sensitive to medium properties and can help us understand finer properties of jet dynamics in the medium
- Medium effects on angularity can be incorporated using the medium modified splitting functions
- Angularity distributions show a relative increase at low τ_a and a decrease at high τ_a
- Caveats: Simple medium, non-perturbative effects like hadronization,...

Looking ahead to EIC

- At the EIC, collective dynamics in the final state are not expected to be present
- However, it is a cleaner environment compared to heavy ion collisions, and quantitative calculations of substructure observables and comparison with data can be useful tools for providing additional constraints on the initial state (for eg. on the modified parton distribution functions)

Backup slides

Non-perturbative effects in the shape function

- Non-perturbative dynamics can be appropriately included in a shape function, $\mathcal{S}_{\rm np}$, which can be convolved with the resummed perturbative distribution

$$\frac{d\sigma}{d\eta dp_T d\tau_a} = \int dk \, \frac{d\sigma^{\rm pert}}{d\eta dp_T d\tau_a}\left(\tau_a - \frac{k}{p_T R}\right) \mathcal{S}_{\rm np}(k)\,.$$

- Single parameter parameterization, Ω_a*[Aschenauer et. al (2019)]*

$$\mathcal{S}_{\rm np}(k) = \frac{4k}{\Omega_a^2}\exp\left(-\frac{2k}{\Omega_a}\right),$$

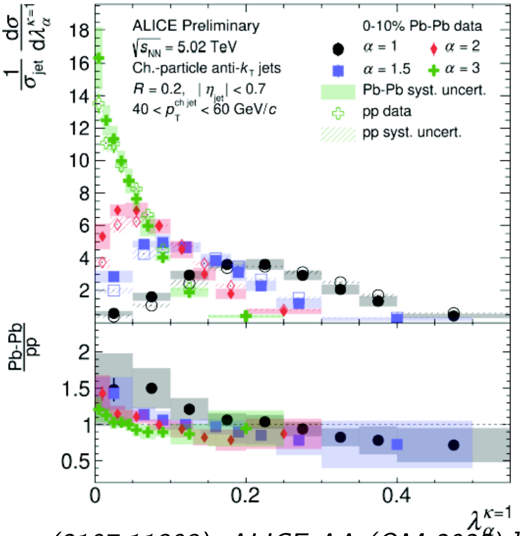
- The 'a' dependence factors out as *[Lee, Sterman (2007)]*

$$\Omega_a = \frac{\Omega_{a=0}}{1-a}\,, \qquad (1)$$

- Can be estimated from a global fit to the jet angularity data for different choices of 'a'
- $\Omega_0 \approx 0.35$ for a jet with $80 < pT < 100$ and $R = 0.4$., $\Omega_0 \approx 0.8$ for $40 < pT < 60$.
- Not modified between *pp* and *PbPb*

Angularities

Ungroomed



- *[ALICE pp (2107.11303); ALICE AA (QM 2022).]* Calculations from Monte-Carlo studies (JEWEL, JETSCAPE, H-T, Hybrid)

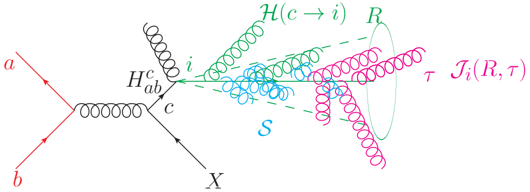
Factorization formula

- For $a < 1$ angularities and $\tau_a^{\frac{1}{2-a}} \ll R$,

$$\frac{d\sigma^{AA\to(jet[R,\tau_a])X}}{d\tau_a dp_T d\eta} = \sum_{abc}\sum_i f_a(x_a,\mu)\otimes f_b(x_b,\mu)$$

$$\otimes H_{ab}^c(x_a,x_b,\eta,p_T/z,\mu)\times\mathcal{H}_{c\to i}(z,p_T R,\mu)$$

$$\otimes \mathcal{J}(\tau_a^c,p_T,R,\mu)\otimes\mathcal{S}(\tau_a^s,p_T,R,\mu)$$



Resumming the jet function using splitting

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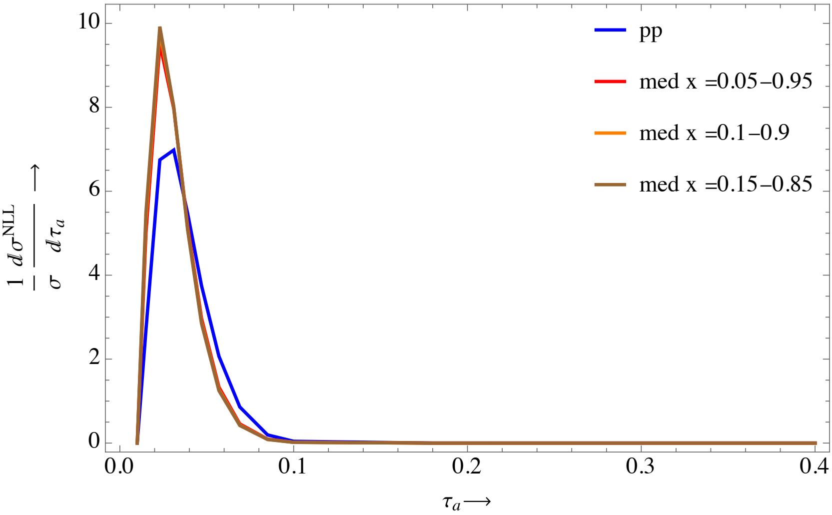
  x is the momentum fraction carried by the final parton, $k_\perp$ is its transverse momentum.

- $$\hat{\tau}_a = p_T^{a-2}k_\perp^{2-a}(x^{a-1}+(1-x)^{a-1})\,.$$
- In vacuum,

$$\mathcal{P}_{i\to jk}(x,k_\perp) = \frac{1}{k_\perp^2}\mathcal{P}_{i\to jk}(x)\,, \tag{2}$$

  with $\mathcal{P}_{i\to jk}(x)$ being the usual Altarelli-Parisi QCD splitting functions

Cutoff dependence



- Cutoff dependence *[Budharaja, RS, Singh [2305.10237]]*

Coherent emission

- For independent scattering and (Bethe-Heitler)

$$\omega \frac{dI}{d\omega} \sim \frac{\alpha_s}{\pi} N_c$$

- For independent gluon emission (Bethe-Heitler)

$$\omega \frac{dI}{dzd\omega} \sim \frac{\alpha_s}{\pi} N_c (\frac{1}{\lambda})$$

- Formation time of the emitted gluon $l_{coh} = t_f \sim \frac{\omega}{k_\perp^2}$
- When $L > t_f > \lambda$ emission contributions from multiple scatterings add coherently. Effectively only one emission for a coherence length
- The net transverse momentum transferred during this period $k_\perp^2 \sim l_{coh} m_D^2 / \lambda$. Thus $l_{coh} = \sqrt{\frac{m_D^2}{\lambda\ \omega}}$
- Only a single emission per coherence length

$$\omega \frac{dI}{dzd\omega} \sim \frac{\alpha_s}{\pi} N_c (\frac{1}{l_{coh}}) = \frac{\alpha_s}{\pi} N_c \sqrt{\frac{m_D^2}{\lambda\ \omega}}$$

Emission suppressed by $\frac{1}{\sqrt{\omega}}$ (LPM)