Dynamic mass generation in d=2 Dirac fermions

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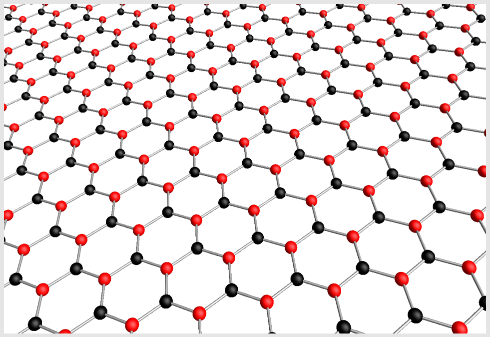
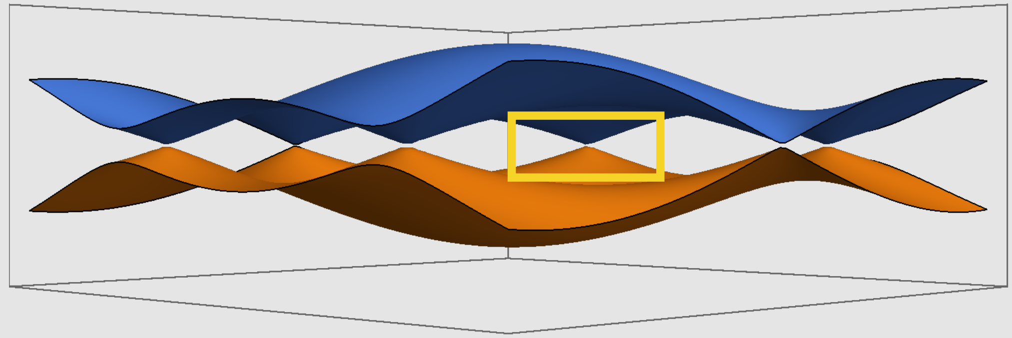
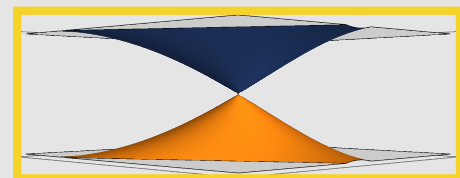
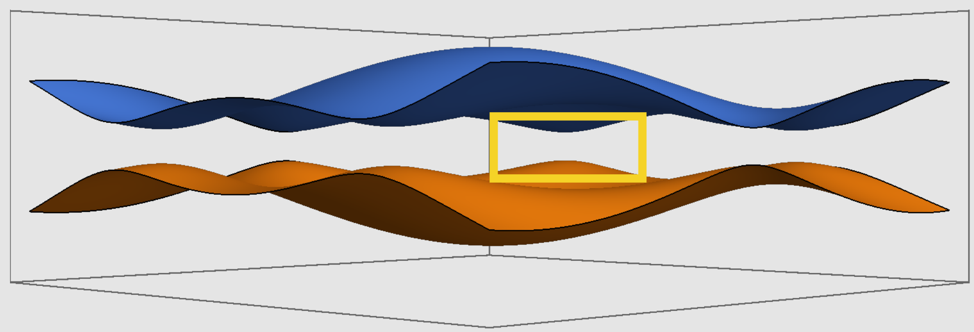
Graphene Dispersion

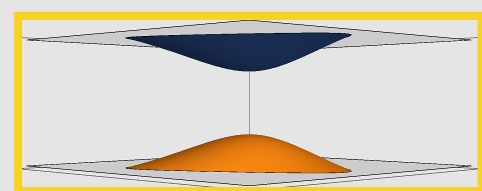






$$H = -t \sum_{\langle ij \rangle} \left(c_i^\dagger c_j + \text{h.c.} \right)$$



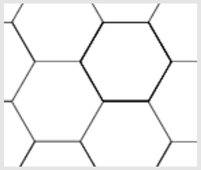


Is this an accident?

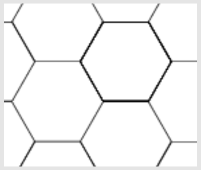
How stable is this touching?

Interactions create mass

$$H_{tV} = -t\sum_{\langle ij\rangle}\left(c_i^\dagger c_j + \text{h.c.}\right) + V\sum_{\langle ij\rangle}(n_i - \frac{1}{2})(n_j - \frac{1}{2})$$



$$H_{\text{Hubb}} = -t\sum_{\langle ij\rangle}\left(c_{i\sigma}^\dagger c_{j\sigma} + \text{h.c.}\right) + U\sum_{i}(n_{i\uparrow} - \frac{1}{2})(n_{i\downarrow} - \frac{1}{2})$$



Internal Symmetries

$$H_0 = -\sum_{<ij>\sigma} t_{ij}(c^\dagger_{i\sigma}c_{j\sigma} + c^\dagger_{j\sigma}c_{i\sigma})$$

$\mathrm{SU}(2)_s$ $$\vec{\mathcal{S}}_i = c^\dagger_{i\alpha}\vec{\sigma}_{\alpha\beta}c_{i\beta}$$

$\mathrm{SU}(2)_c$ $$\vec{\mathcal{C}}_i = (\zeta_i(c^\dagger_{i\uparrow}c^\dagger_{i\downarrow} + c_{i\downarrow}c_{i\uparrow}), -\imath\zeta_i(c^\dagger_{i\uparrow}c^\dagger_{i\downarrow} - c_{i\downarrow}c_{i\uparrow}), c^\dagger_{i\uparrow}c_{i\uparrow} + c^\dagger_{i\downarrow}c_{i\downarrow} - 1)$$

$\mathbb{Z}_2^{sc}$ $$Fc_{i\downarrow}F^\dagger = \zeta_i c^\dagger_{i\downarrow} \quad Fc_{i\uparrow}F^\dagger = c_{i\uparrow}$$

$$H_0 = \sum_{<i,j>a} \frac{i}{2} t_{ij}\gamma^a_i\gamma^a_j$$

Hubbard interaction

$$H_U = U \sum_i \left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right)$$

$$\left(n_{i\uparrow} - \frac{1}{2} \right) \left(n_{i\downarrow} - \frac{1}{2} \right) \propto \vec{\mathcal{S}}_i \cdot \vec{\mathcal{S}}_i - \vec{\mathcal{C}}_i \cdot \vec{\mathcal{C}}_i$$

Invariant under $\mathrm{SU}(2)_s$ & $\mathrm{SU}(2)_c$

Odd under $\mathbb{Z}_2^{sc}$

(U is like Ising field!)

Spin-charge lattice model

Liu, Huffman, Chandrasekharan, Kaul (2021)

$$H_{\rm SC} = -\sum_{\langle ij\rangle} \exp\left(\kappa\eta_{ij}\sum_\sigma (c^\dagger_{i\sigma}c_{j\sigma} + c^\dagger_{j\sigma}c_{i\sigma})\right).$$

- Model has full O(4) symmetry.
- Terms with 2,4,6,8 operators.
- completely local.
- “designer” model.
- $t_{ij} = \sinh(\kappa)\eta_{ij}$
- $\kappa \ll 1$ “semi-metal”

$$\mathbf{g} = 2\tanh(\kappa/2)$$

QMC algorithm

$$H_{SC} = -\sum_b H_b$$

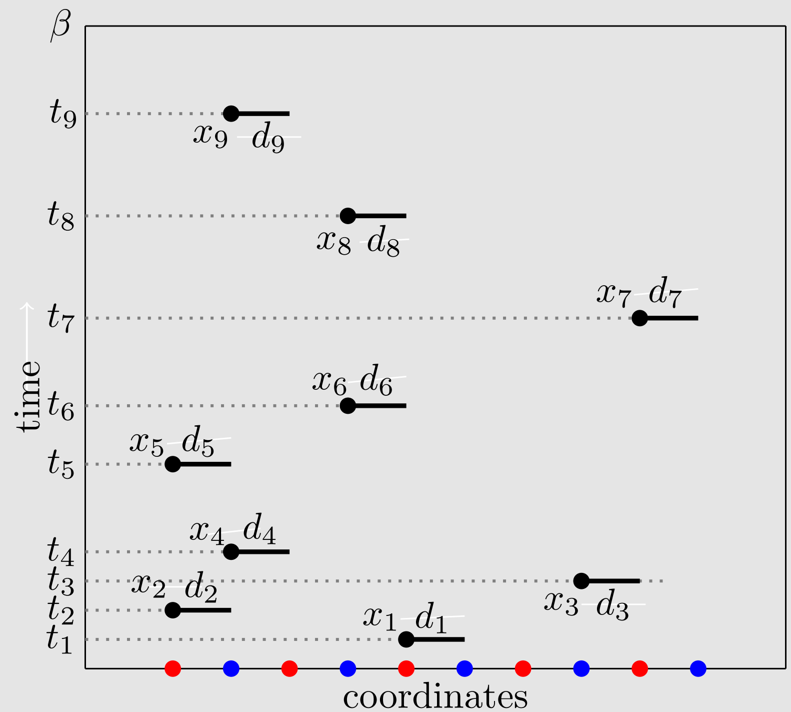
$$H_b = e^{\kappa\eta_b \sum_\sigma (c^\dagger_{i\sigma} c_{j\sigma} + c^\dagger_{j\sigma} c_{i\sigma})}$$

$$\mathrm{Tr}[H_{b_N} \ldots H_{b_2} H_{b_1}]$$

$$\Big\downarrow$$

$$\mathrm{Det}[1 + M_{b_N} \ldots M_{b_2} M_{b_1}] > 0$$

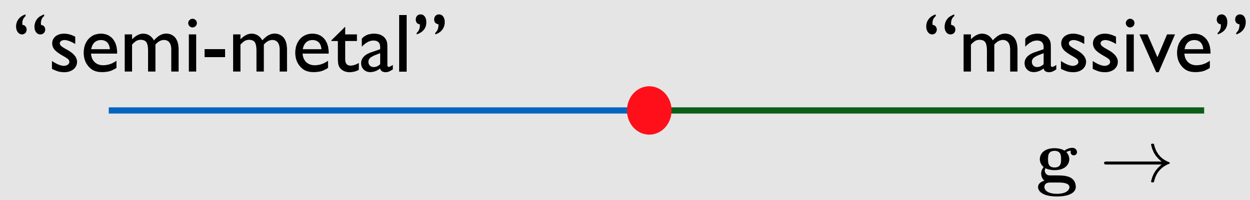
(BSS formula)



[b] configuration

Huffman, Chandrasekharan (2019)

QMC data

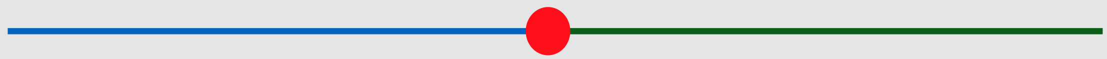


$$C_S = \langle S^z_{\frac{L}{2}\hat{\mathbf{e}}_\mathbf{x}} S^z_\mathbf{0} \rangle$$

$$C_U = \langle (n_\uparrow - \frac{1}{2})(n_\downarrow - \frac{1}{2})_{\frac{L}{2}\hat{\mathbf{e}}_\mathbf{x}} (n_\uparrow - \frac{1}{2})(n_\downarrow - \frac{1}{2})_\mathbf{0} \rangle$$

QMC data

“semi-metal” “massive”

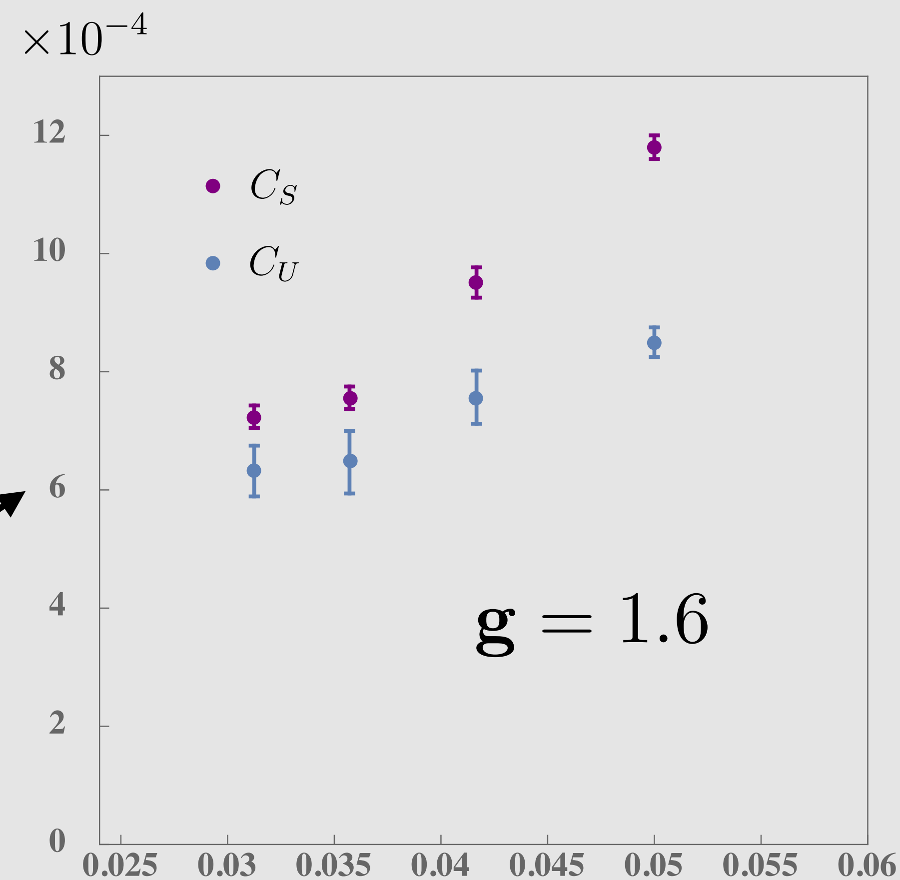


$\mathbf{g} \rightarrow$

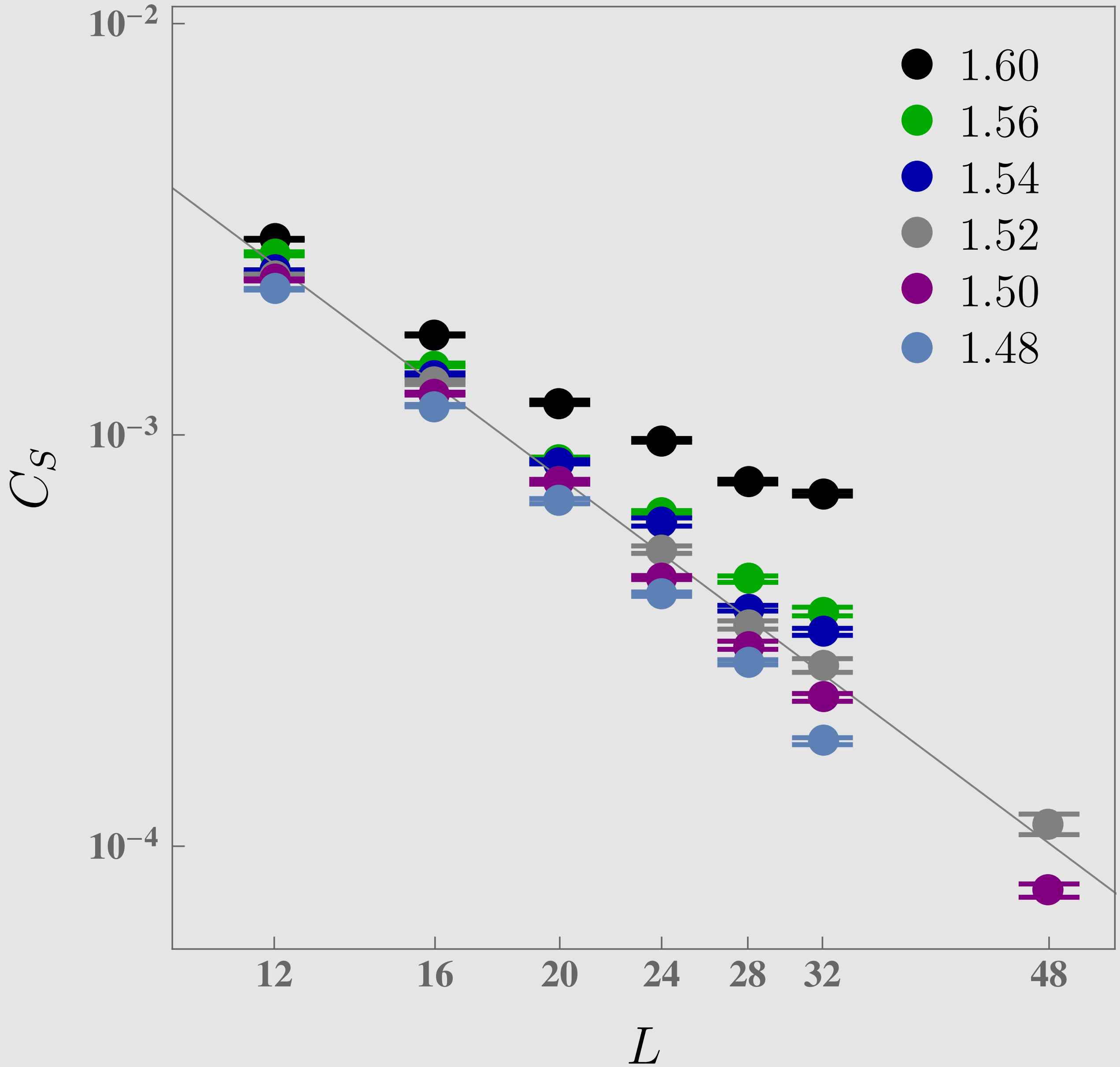
$$C_S = \langle S^z_{\frac{L}{2}\hat{\mathbf{e}}_\mathbf{x}} S^z_\mathbf{0} \rangle$$

$$C_U = \langle (n_\uparrow - \frac{1}{2})(n_\downarrow - \frac{1}{2})_{\frac{L}{2}\hat{\mathbf{e}}_\mathbf{x}} (n_\uparrow - \frac{1}{2})(n_\downarrow - \frac{1}{2})_\mathbf{0} \rangle$$

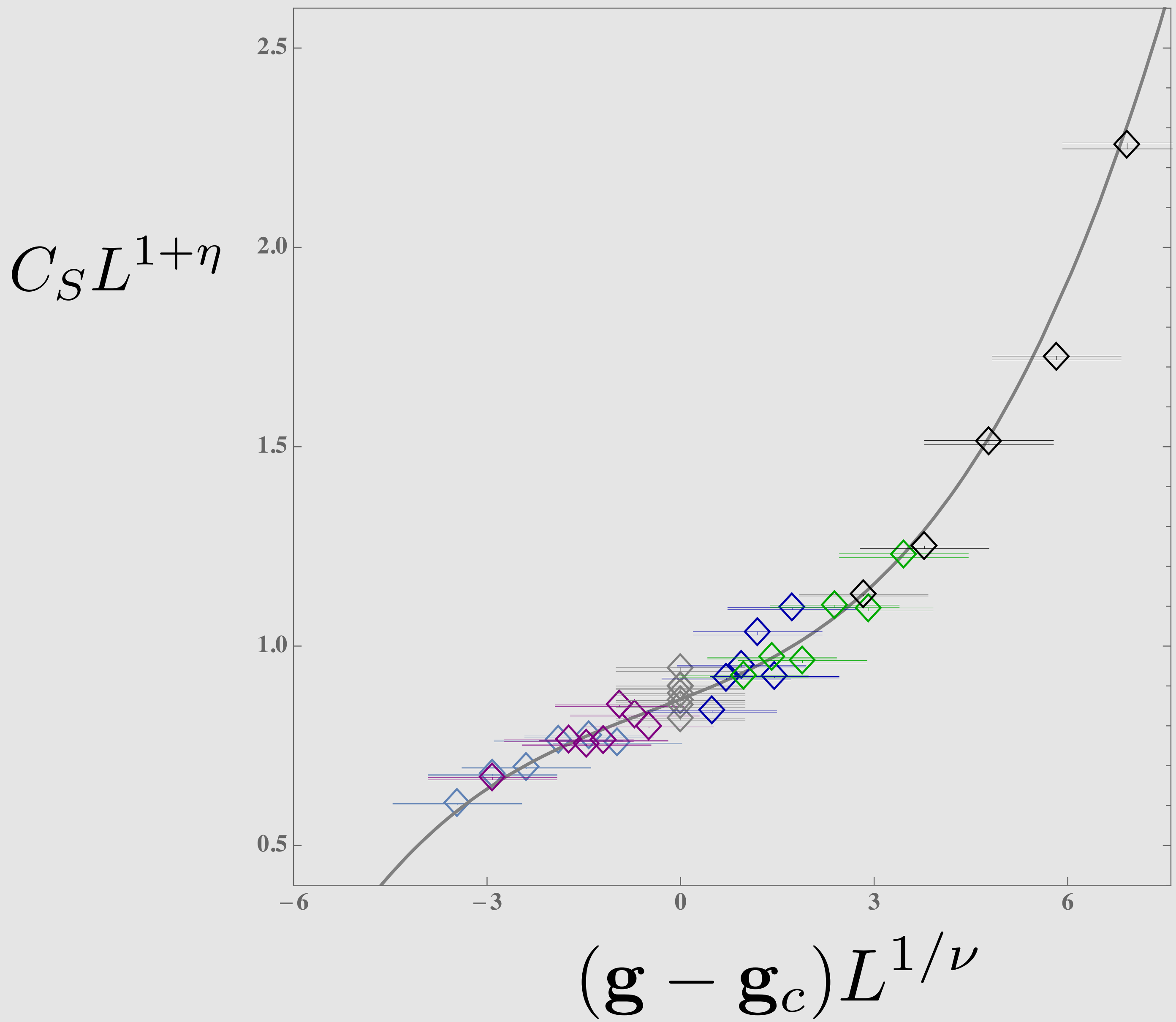
Massive phase breaks both spin-charge-Ising and SU(2)



Quantum Critical Scaling



Scaling Collapse



Mean Field

$$H_{0+\mathrm{MF}} = -\sum_{\langle ij\rangle\sigma} t_{ij}\left(c^\dagger_{i\sigma}c_{j\sigma} + c^\dagger_{j\sigma}c_{i\sigma}\right) + \sum_i \zeta_i\left(\vec{\phi}^s_i \cdot \vec{\mathcal{S}}_i + \vec{\phi}^c_i \cdot \vec{\mathcal{C}}_i\right)$$

$$\zeta_i = \pm 1$$

$$\vec{\mathcal{S}}_i = c^\dagger_{i\alpha}\vec{\sigma}_{\alpha\beta}c_{i\beta}$$

$$\vec{\mathcal{C}}_i = (\zeta_i(c^\dagger_{i\uparrow}c^\dagger_{i\downarrow} + c_{i\downarrow}c_{i\uparrow}), -\imath\zeta_i(c^\dagger_{i\uparrow}c^\dagger_{i\downarrow} - c_{i\downarrow}c_{i\uparrow}), c^\dagger_{i\uparrow}c_{i\uparrow} + c^\dagger_{i\downarrow}c_{i\downarrow} - 1)$$

$$Fc_{i\downarrow}F^\dagger = \zeta_i c^\dagger_{i\downarrow} \quad Fc_{i\uparrow}F^\dagger = c_{i\uparrow}$$

Field Theory

Liu, Huffman, Chandrasekharan, Kaul (2021)

$$\mathcal{L}_{\mathrm{Y}} = -\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + g_s\vec{\phi}_s \cdot \vec{M}_s + g_c\vec{\phi}_c \cdot \vec{M}_c$$

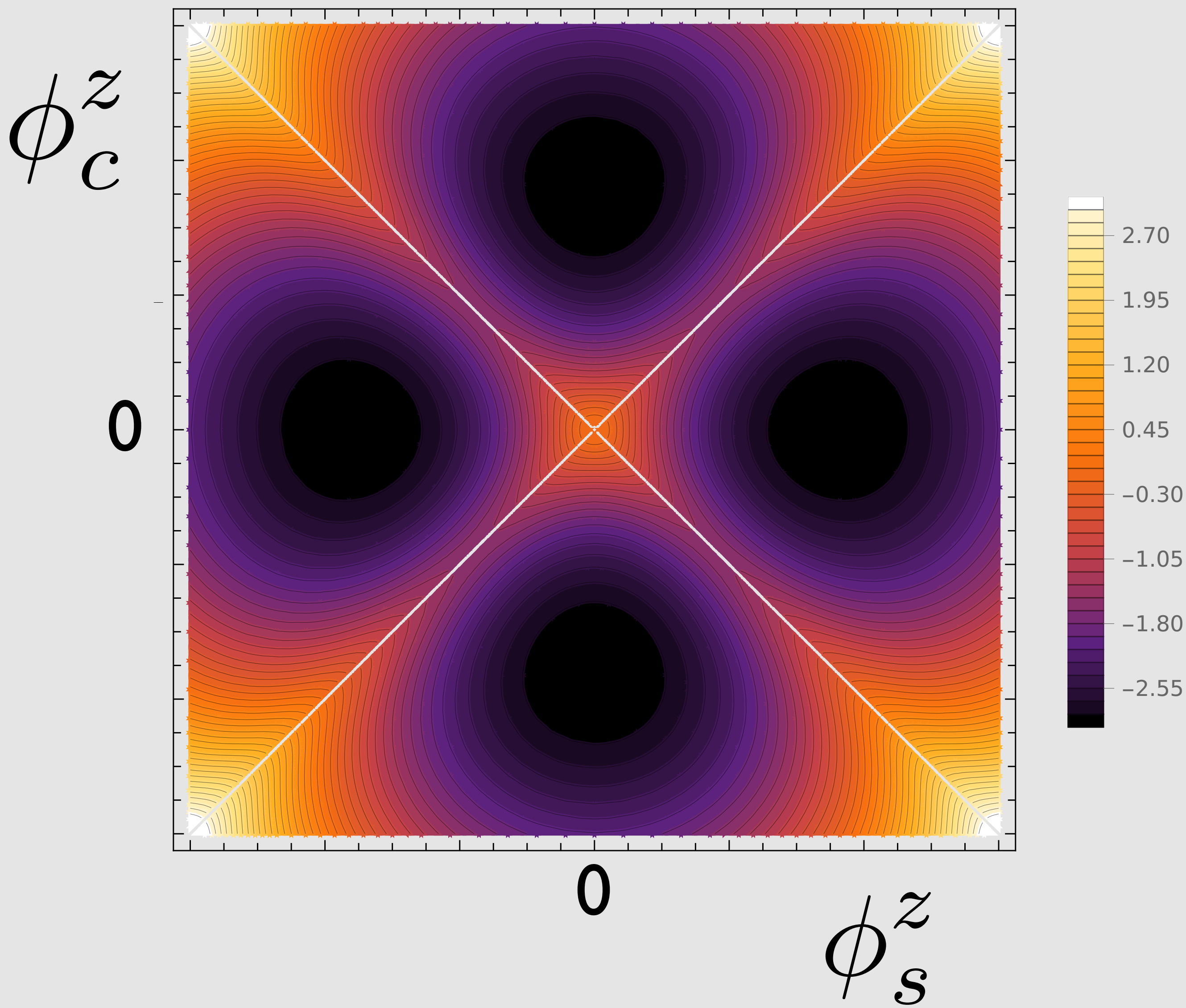
$$\mathcal{L}_{\mathrm{B}} = \sum_{a=s,c}\left(\frac{1}{2}\partial_{\mu}\vec{\phi}_a \cdot \partial^{\mu}\vec{\phi}_a + \frac{1}{2}m_a^2\vec{\phi}_a \cdot \vec{\phi}_a + \frac{1}{4!}\lambda_a(\vec{\phi}_a \cdot \vec{\phi}_a)^2\right) + \frac{1}{12}\lambda_{sc}(\vec{\phi}_s \cdot \vec{\phi}_s)(\vec{\phi}_c \cdot \vec{\phi}_c)$$

$$\vec{M}_s = \bar{\psi}_{\alpha}\vec{\sigma}_{\alpha\beta}\psi_{\beta}$$

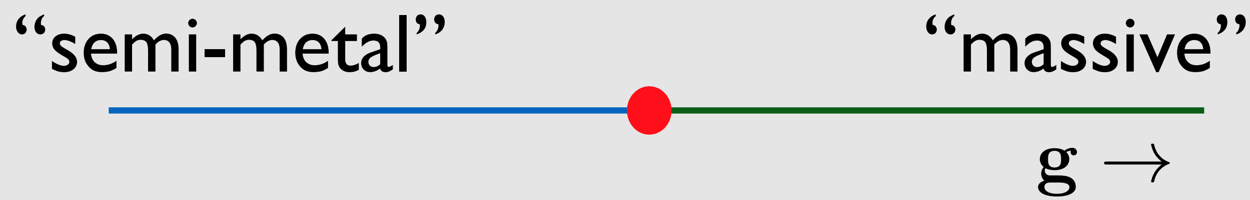
$$\vec{M}_c = (\psi_{\downarrow}^T\gamma^0\psi_{\uparrow} + \bar{\psi}_{\uparrow}\gamma^0\bar{\psi}_{\downarrow}^T, i(\psi_{\downarrow}^T\gamma^0\psi_{\uparrow} - \bar{\psi}_{\uparrow}\gamma^0\bar{\psi}_{\downarrow}^T), \bar{\psi}_{\uparrow}\psi_{\uparrow} + \bar{\psi}_{\downarrow}\psi_{\downarrow})$$

condensate energetics

Effective Potential

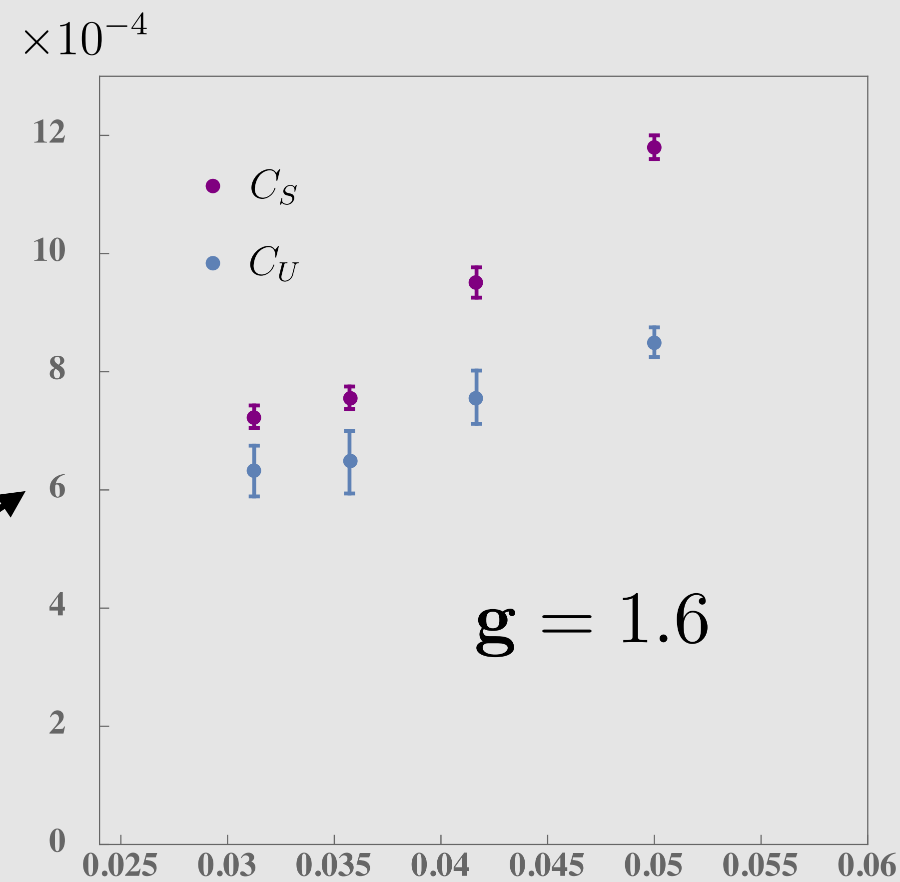


QMC data



$$C_S = \langle S^z_{\frac{L}{2}\hat{\mathbf{e}}_\mathbf{x}} S^z_{\mathbf{0}} \rangle$$

$$C_U = \langle (n_\uparrow - \frac{1}{2})(n_\downarrow - \frac{1}{2})_{\frac{L}{2}\hat{\mathbf{e}}_\mathbf{x}} (n_\uparrow - \frac{1}{2})(n_\downarrow - \frac{1}{2})_{\mathbf{0}} \rangle$$



Massive phase breaks
both spin-charge-Ising
and SU(2)

RG: eps-expansion

Liu, Huffman, Chandrasekharan, Kaul (2021)

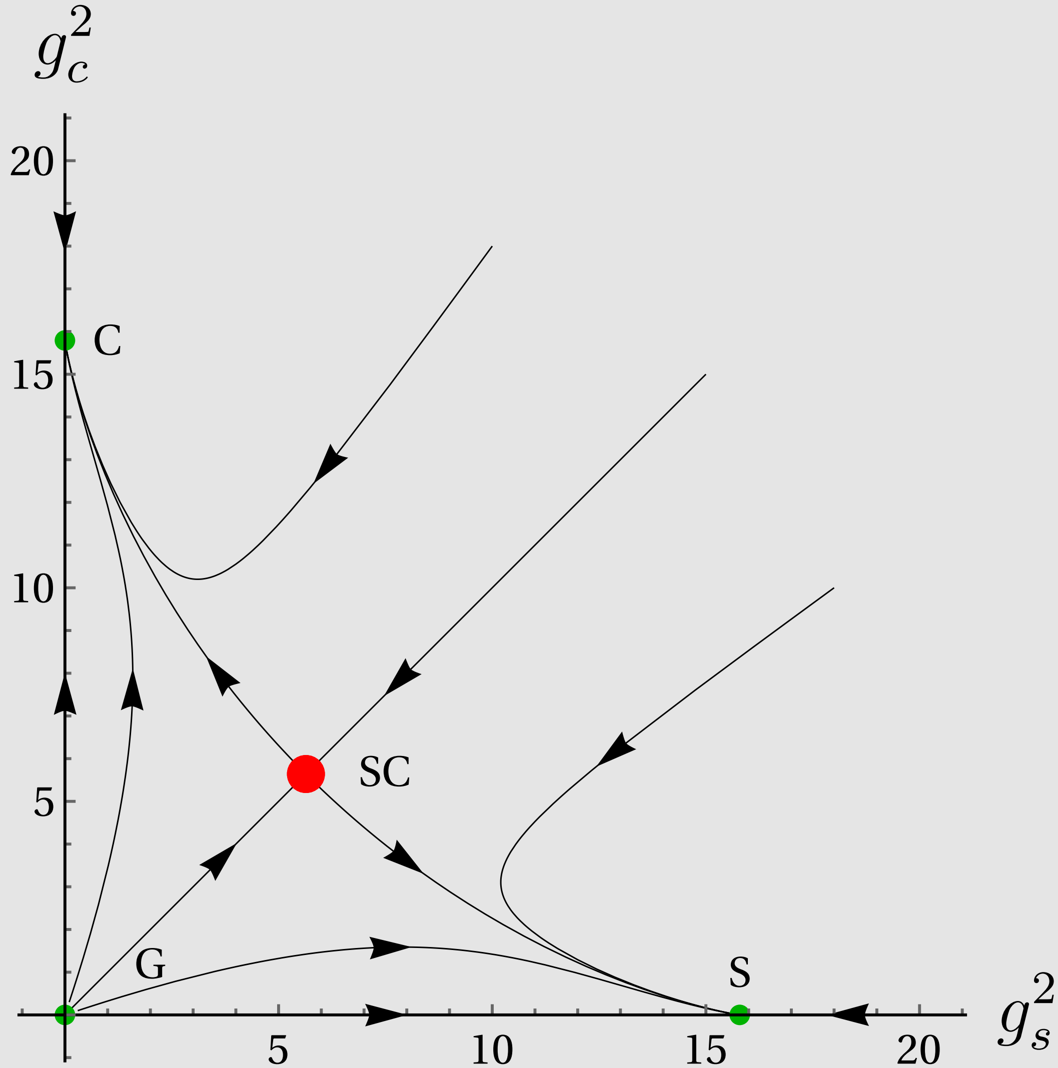
$$\mathcal{L}_{\mathrm{Y}} = -\bar{\psi}\gamma^{\mu}\partial_{\mu}\psi + g_s\vec{\phi}_s \cdot \vec{M}_s + g_c\vec{\phi}_c \cdot \vec{M}_c$$

$$\mathcal{L}_{\mathrm{B}} = \sum_{a=s,c}\left(\frac{1}{2}\partial_{\mu}\vec{\phi}_a \cdot \partial^{\mu}\vec{\phi}_a + \frac{1}{2}m_a^2\vec{\phi}_a \cdot \vec{\phi}_a + \frac{1}{4!}\lambda_a(\vec{\phi}_a \cdot \vec{\phi}_a)^2\right) + \frac{1}{12}\lambda_{sc}(\vec{\phi}_s \cdot \vec{\phi}_s)(\vec{\phi}_c \cdot \vec{\phi}_c)$$

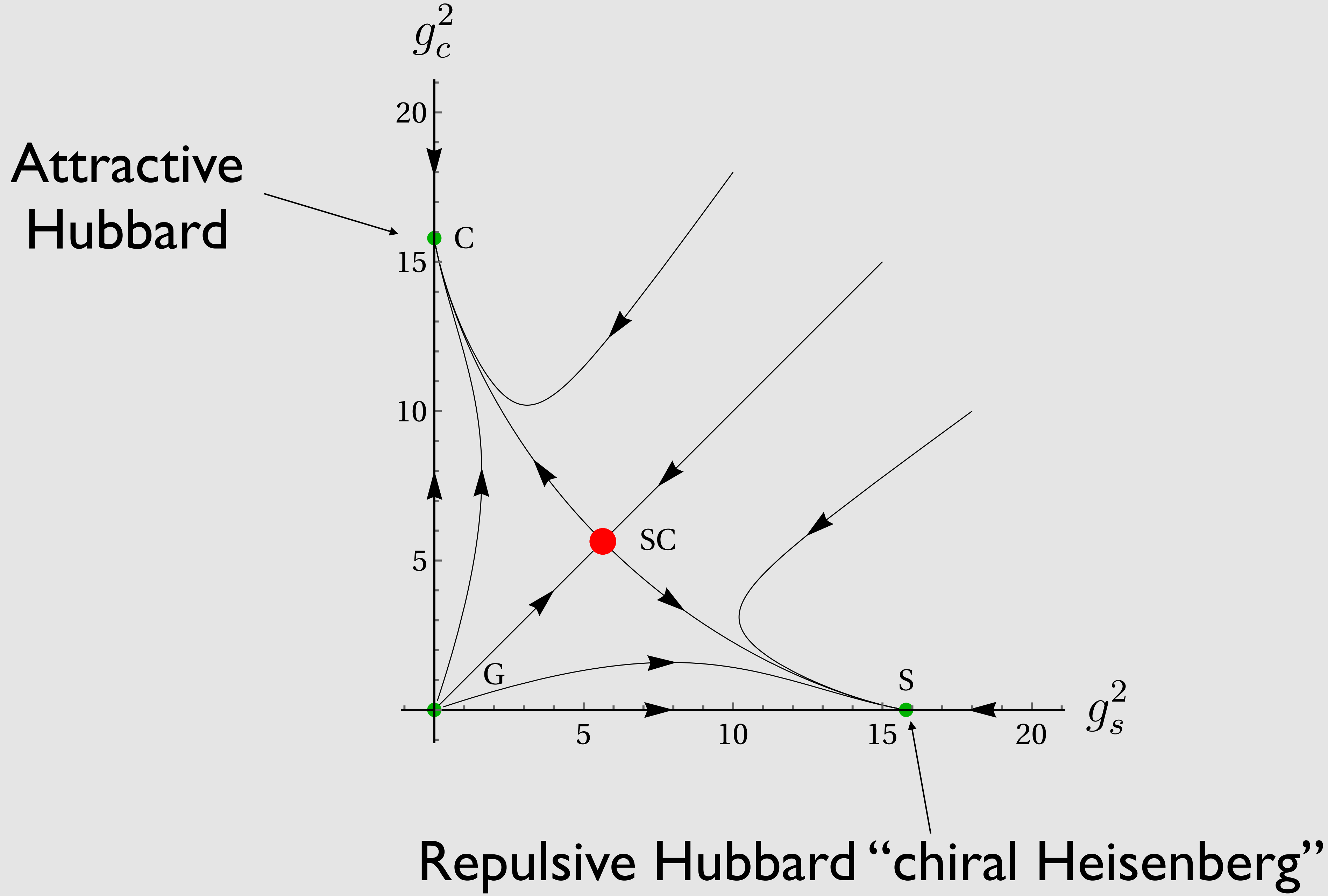
$$g_s,\, g_c,\, \lambda_s,\, \lambda_c,\, \lambda_{sc}$$

$$\begin{aligned}
\frac{dg_s^2}{d\log\ell} &= \varepsilon g_s^2 - \frac{1}{8\pi^2}(5g_s^4 + 9g_s^2 g_c^2), \\
\frac{d\lambda_s}{d\log\ell} &= \varepsilon\lambda_s - \frac{1}{8\pi^2}\left(\frac{11}{6}\lambda_s^2 + \frac{1}{2}\lambda_{sc}^2 + 8g_s^2\lambda_s - 48g_s^4\right), \\
\frac{d\lambda_{sc}}{d\log\ell} &= \varepsilon\lambda_{sc} - \frac{1}{8\pi^2}\left(\frac{5}{6}(\lambda_s + \lambda_c)\lambda_{sc} + \frac{2}{3}\lambda_{sc}^2 + 4(g_s^2 + g_c^2)\lambda_{sc} - 48g_s^2 g_c^2\right).
\end{aligned}$$

Yukawa flow

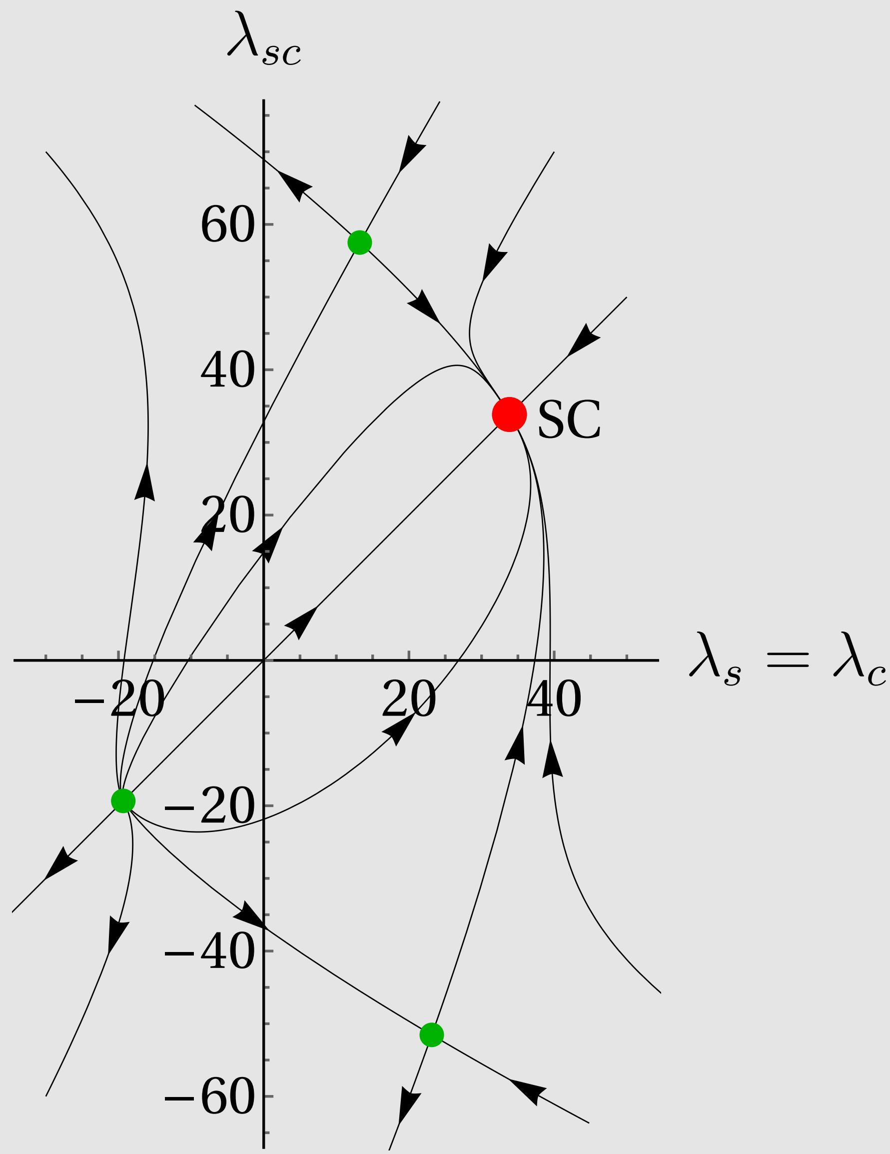


Yukawa flow



Attractive Hubbard
g_c^2
20
15
10
5
C
SC
G
S
5
10
15
20
g_s^2
Repulsive Hubbard "chiral Heisenberg"

Boson flow



- stable with sc symmetry

Conclusions

Liu, Huffman, Chandrasekharan, Kaul (2021)

Spin-charge flip symmetry

New phase with spontaneously broke SC flip

New critical point:

- superconductivity & anti-ferromagnetism quantum critical
- systematic study with epsilon-expansion
- higher order epsilon, bootstrap
- other phase transition affected by this symmetry?