AXIONS IN THE EARLY UNIVERSE

Selected Reading

DJEM, "Axion Cosmology", Ch. 3.

Kolb & Turner, Ch. 3, 10

Sikivie, astro-ph/0610440

Order of Service

1. PQ symmetry breaking $T \sim f_a$
2. Shift symmetry breaking $T \sim \mu$
3. Axion field evolution $H \sim m$

Q1: When and how are initial conditions set?

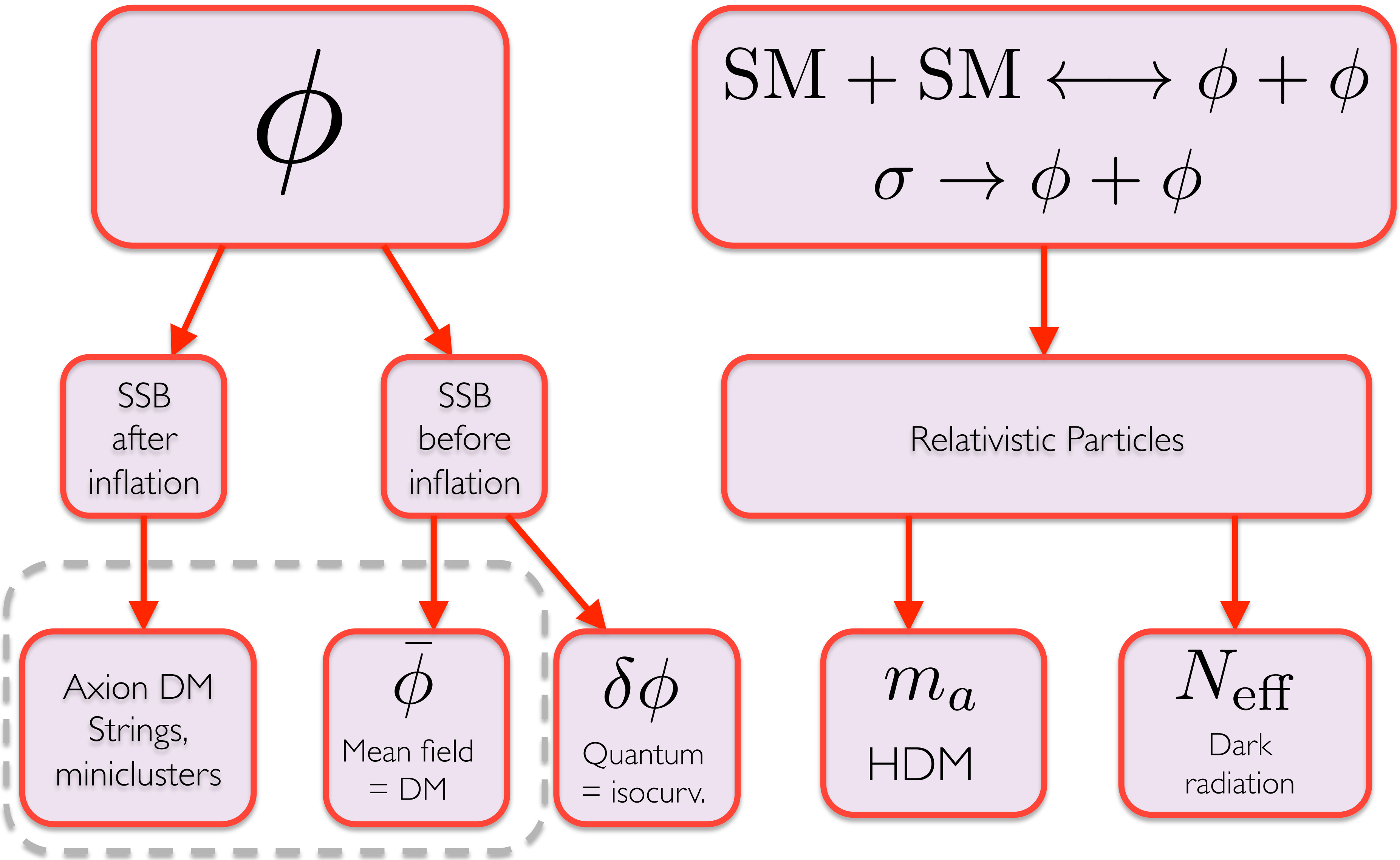
Q2: What produces relic axions?

Sources of Cosmic Axions

Classical Axion Condensate

Thermal Production
Non-thermal decays

Sources of Cosmic Axions



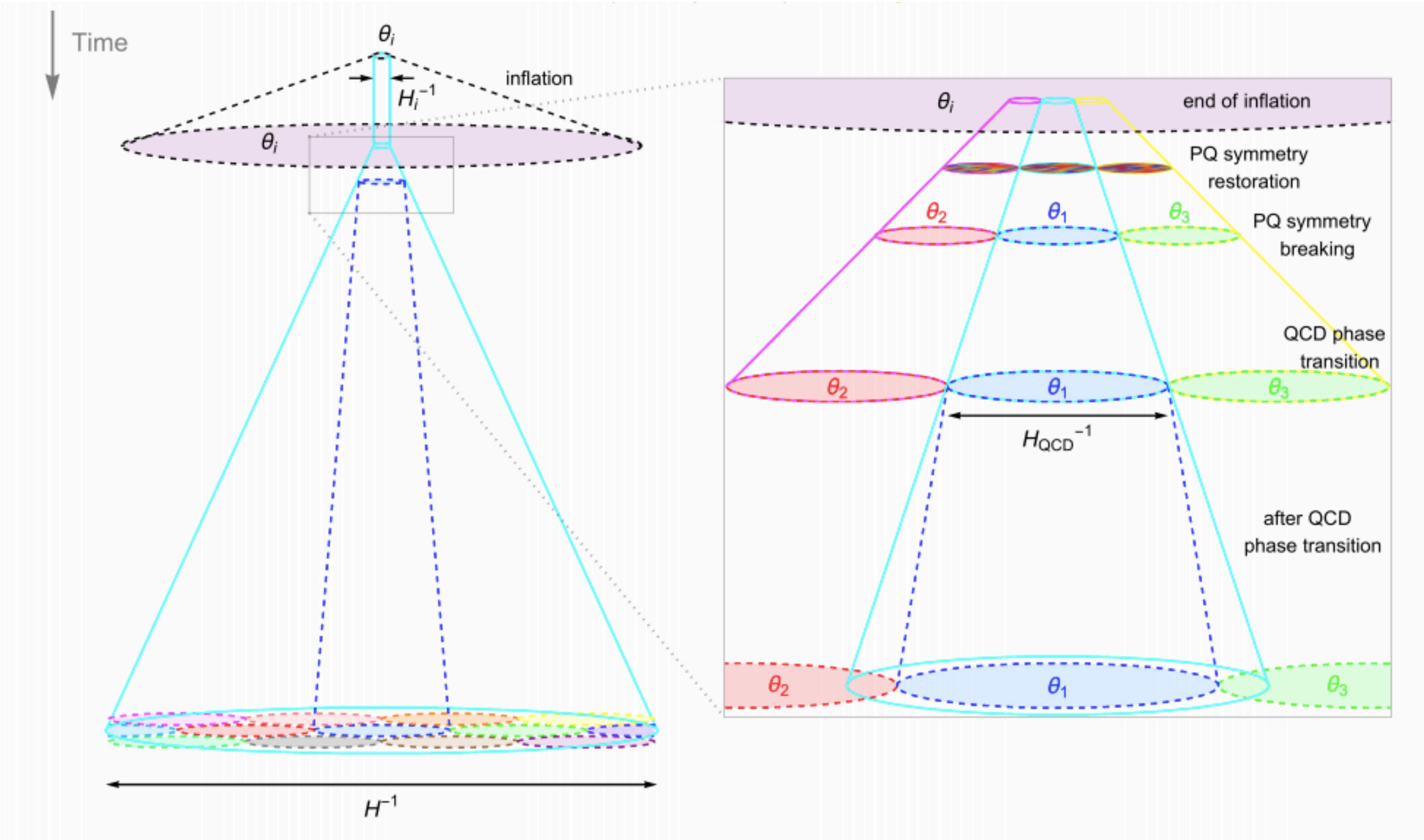
$\delta\phi$

Quantum fluctuations

See Vivian Poulin's lecture

"Scenario A"	"Scenario B"
Strings, domain walls, miniclusters	$\bar{\phi}$ Mean field
PQ Broken After Inflation	PQ Broken During Inflation

Scenario A



Time
θ_i
inflation
H_i^{-1}
θ_i
H^{-1}
θ_i
end of inflation
PQ symmetry restoration
θ_2
θ_1
θ_3
PQ symmetry breaking
QCD phase transition
θ_2
θ_1
θ_3
H_{QCD}^{-1}
after QCD phase transition
θ_2
θ_1
θ_3

Scenario B

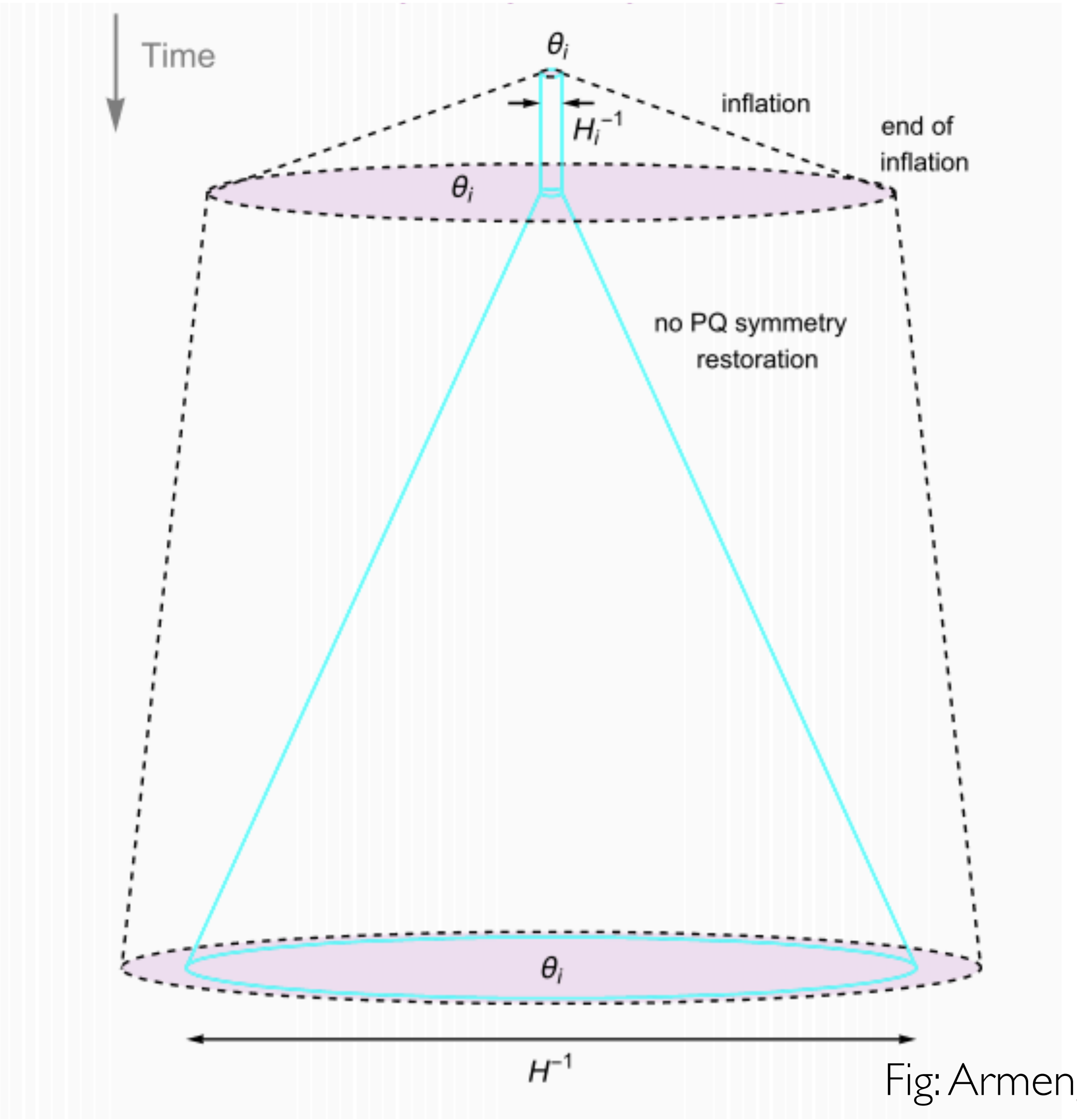


Fig: Armengaud et al (2019)

The Cosmic Axion Field

Scalar field action (canonical kinetic term) in curved space:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

Apply the Euler-Lagrange equations:

$$\partial_\mu \frac{\delta S}{\delta \partial_\mu \phi} - \frac{\delta S}{\delta \phi} = 0$$

The Cosmic Axion Field

Scalar field action (canonical kinetic term) in curved space:

$$S = \int \mathrm{d}^4x\sqrt{-g}\left[-\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - V(\phi)\right]$$

Apply the Euler-Lagrange equations:

$$\Rightarrow \Box\phi - \partial_\phi V = 0 \qquad \Box = \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu)$$

In the FRW spacetime:

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2\mathrm{d}\vec{x}^2$$

$$\Box = -\partial_t^2 - 3H\partial_t + \nabla^2$$

For FRW we should strictly ignore this. We include it approximately when curvature is small (Scenario A)

Energy-Momentum Tensor

Einstein-Hilbert action:

$$S = \int \mathrm{d}^4x \sqrt{-g} \left[\frac{M_{pl}^2}{2} R + \mathcal{L}_M \right]$$

$$R = g^{\mu\nu} R_{\mu\nu}$$

Ricci scalar

Euler-Lagrange equations → Einstein field equations:

$$G_{\mu\nu} = M_{pl}^{-2} T_{\mu\nu}$$

$$G_{\mu\nu} := R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R \qquad T_{\mu\nu} := \frac{\delta S_M}{\delta g^{\mu\nu}}$$

Components define the energy density, pressure and velocity:

$$T^0{}_0 := -\rho \quad T^i{}_i := \delta^i{}_j P + \Sigma^i{}_j \quad T^0{}_i := (\rho + P) v_i$$

Energy-Momentum Tensor

This defines the energy momentum tensor for the scalar field:

$$T^{\mu}{}_{\nu} = g^{\mu\alpha}\partial_\alpha\phi\partial_\nu\phi - \frac{\delta^{\mu}{}_{\nu}}{2}\left[g^{\alpha\beta}\partial_\alpha\phi\partial_\beta\phi + 2V\right]$$

In FRW only the energy density and pressure are non-vanishing:

$$\bar{\rho} = \frac{1}{2}\dot{\phi}^2 + V \quad \bar{P} = \frac{1}{2}\dot{\phi}^2 - V$$

"Relic Density"

Compute the final value of the energy density given the initial conditions for the Klein Gordon equation:

$$\rho(t_i) \to \rho(t_0)$$

The cosmic density parameter is:

$$\Omega_a h^2 := \frac{\rho(t_0) h^2}{3 H_0^2 M_{pl}^2}$$ For DM: $\Omega_a h^2 = 0.120 \pm 0.001$

$$H_0 = 100h \text{ km s}^{-1} \text{ Mpc} = h M_H = 2.13h \times 10^{-33} \text{ eV}$$

Our task is to compute the function:

$$\rho(t_0) = f(\phi_i, \dot{\phi}_i, m_a, \cdots)$$

VACUUM REALIGNMENT: SCENARIO B

Selected Reading

DJEM, "Axion Cosmology", Ch. 4, App. C, D

Wantz & Shellard, arXiv:0910.1066

Fox, Pierce, & Thomas, hep-th/0409059

Borsanyi et al, arXiv:1606.07494

Klein-Gordon Equation

Inflation smoothes the background field → neglect spatial derivatives

$$\ddot{\phi} + 3H\dot{\phi} + \frac{\chi(T)}{f_a}\partial_\theta U(\theta) = 0$$

(Hubble) Damped

T-dependent mass

Anharmonic oscillator

Key feature: Hubble friction dominates early until H~m and "freezes" field → Initial conditions show attractor behaviour:

$$\dot{\phi}(t_i) \rightarrow 0$$

Aside: for the cosine potential this is the e.o.m. of a circular pendulum on a table with time-dependent angle of inclination.

An Exact Solution

Start with the simplest version of this equation, which still displays all the key features:

1. T-independent mass. Relevant if H~m after T~μ. QCD axion with fa >10^{16} GeV. String axions, since $\mu \sim M_{string} >> \Lambda$.
2. No "back reaction" in the Friedmann equation. Relevant for axion DM, when H~m should occur before matter radiation equality.
3. Harmonic potential. Relevant for $\theta_i < 1$.

2 → can assume expansion is dominated by a single fluid:

$$a(t) \propto t^p \Rightarrow H = \frac{p}{t}$$

Standard hot big bang → p=1/2. Matter dom → p=2/3

An Exact Solution

$$\phi(t) = a^{-3/2}(t/t_i)^{1/2}\left[C_1 J_n(mt) + C_2 Y_n(mt)\right]$$

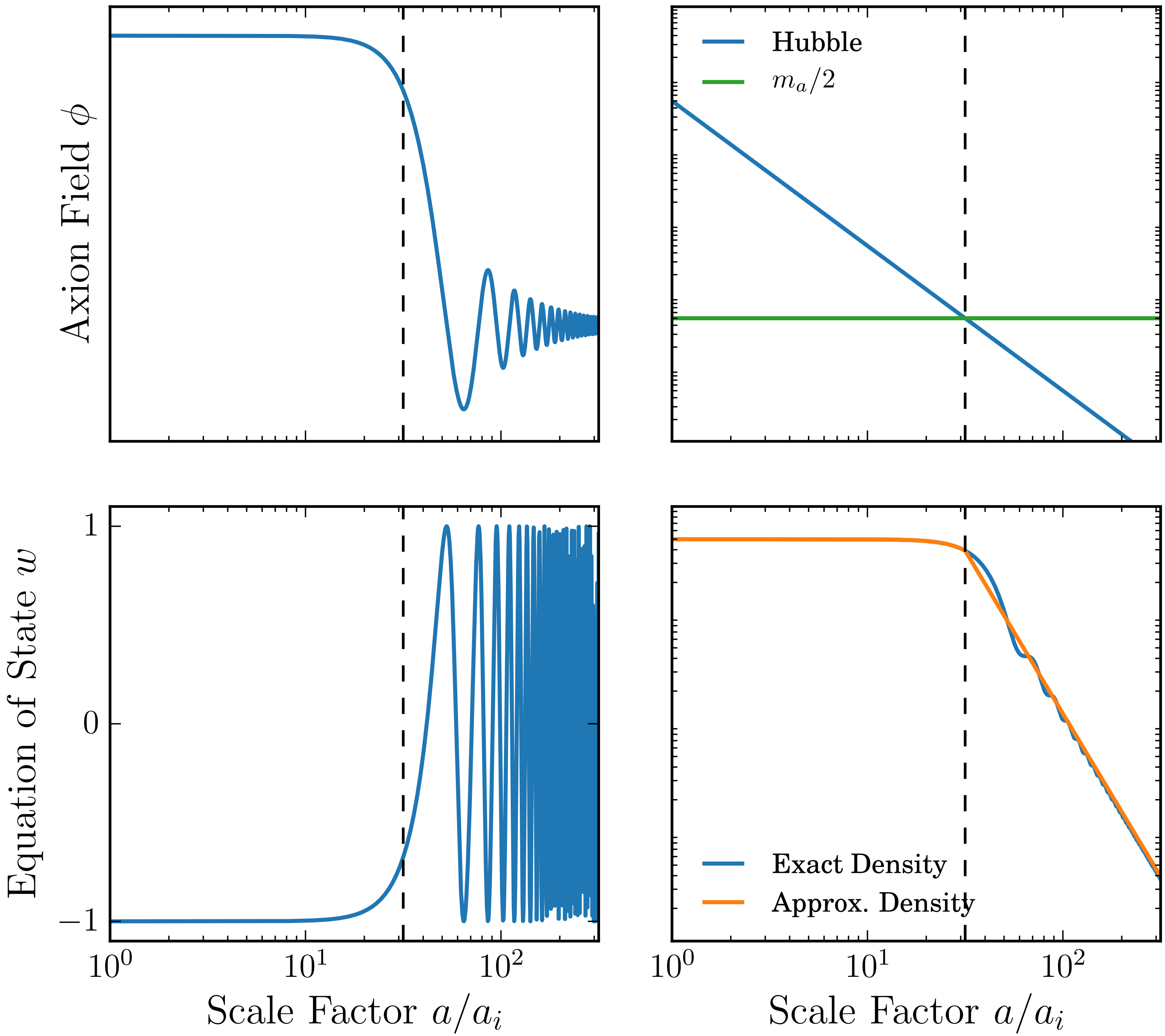
$$n = (3p - 1/2)$$

Constants C1 and C2 set by i.c.'s such that initial field velocity =0
Harmonic equation is linear in ϕ, so the initial condition scales out.

Asymptotic behaviour:

$$t \ll t_i \qquad \phi = \phi(t_i)$$

$$t \gg t_i \qquad \phi \propto a^{-3/2}\cos(mt)$$



Axion Field ϕ
Hubble
$m_a/2$
Equation of State w
1
0
−1
Exact Density
Approx. Density
10^0
10^1
10^2
Scale Factor a/a_i
10^0
10^1
10^2
Scale Factor a/a_i

A Useful Approximation

Initial value $$\rho(a_{\rm osc}) \approx \rho(t_i) = \frac{1}{2} m_a^2 \phi_i^2$$

Transition scale $$3H(a_{\rm osc}) = m_a$$

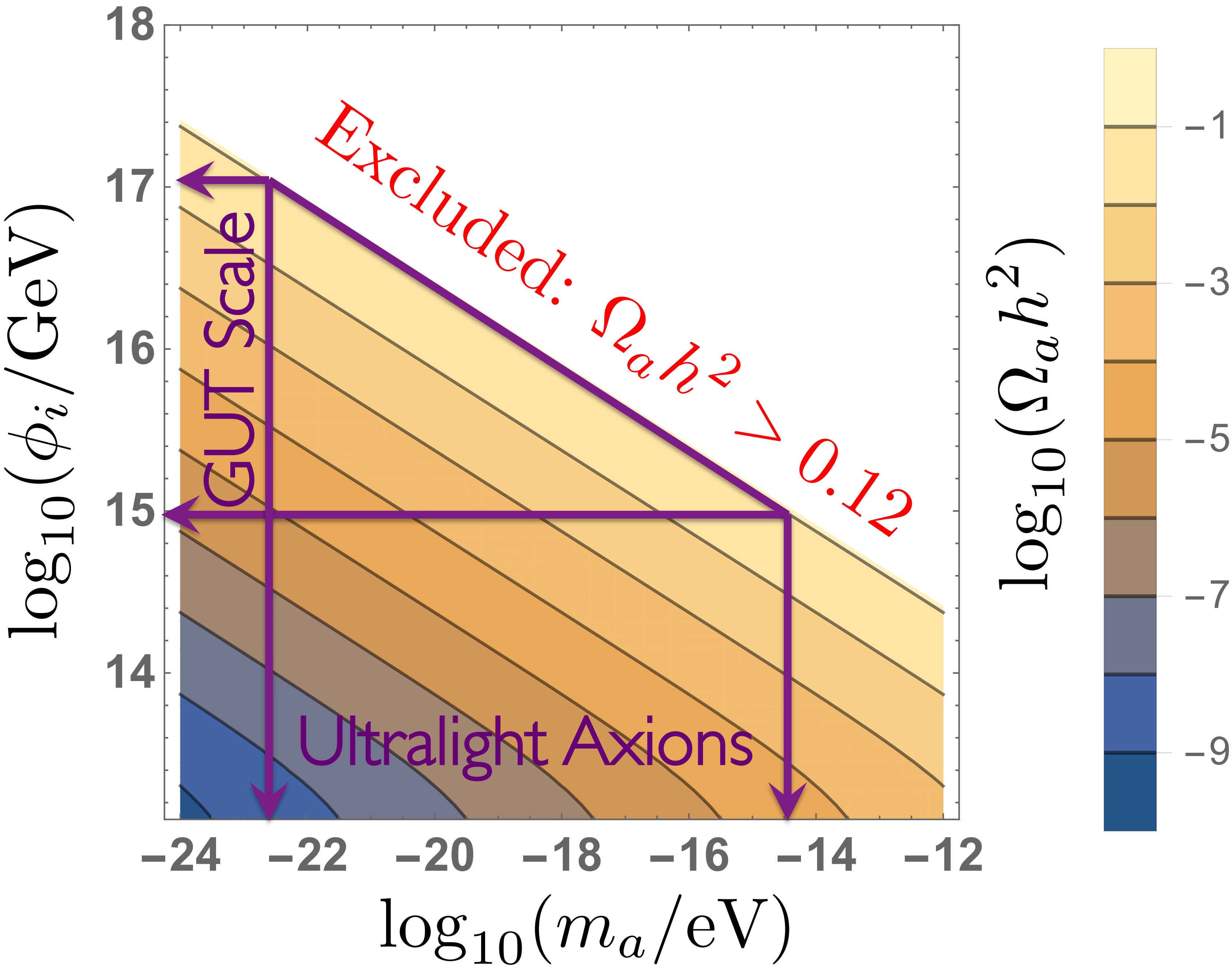
Early time (DE-like) $$\rho(a) \approx \rho(a_{\rm osc}) \qquad (a < a_{\rm osc})$$

Late time (DM-like) $$\rho(a) \approx \rho(a_{\rm osc}) \left(\frac{a_{\rm osc}}{a}\right)^3 \quad (a > a_{\rm osc})$$

A Useful Approximation

Closed form expression for relic density:

$$\Omega_a \approx \begin{cases} \frac{1}{6}(9\Omega_r)^{3/4}\left(\frac{m_a}{H_0}\right)^{1/2}\left\langle\left(\frac{\phi_i}{M_{pl}}\right)^2\right\rangle \text{ if } a_{\rm osc} < a_{\rm eq} \\ \frac{9}{6}\Omega_m\left\langle\left(\frac{\phi_i}{M_{pl}}\right)^2\right\rangle \text{ if } a_{\rm eq} < a_{\rm osc} \lesssim 1 \\ \frac{1}{6}\left(\frac{m_a}{H_0}\right)^2\left\langle\left(\frac{\phi_i}{M_{pl}}\right)^2\right\rangle \text{ if } a_{\rm osc} \gtrsim 1 \end{cases}$$



Excluded: $\Omega_a h^2 > 0.12$
GUT Scale
Ultralight Axions
$\log_{10}(\phi_i/\mathrm{GeV})$
$\log_{10}(m_a/\mathrm{eV})$
$\log_{10}(\Omega_a h^2)$
18
17
16
15
14
–24
–22
–20
–18
–16
–14
–12
–1
–3
–5
–7
–9

Initial Field Value: Caution!

Normally we simply take this as a free parameter.
But don't forget the inflationary fluctuations!

$$\phi(t_i) = \sqrt{\langle \phi_i^2 \rangle} = \sqrt{\theta_i^2 f_a^2 + H_I^2/4\pi^2} \approx \theta_i f_a$$

Only valid if $\theta_i f_a \gg H_I$

→ Cannot set $\phi=0$ by tuning θ even if $f_a>10^{14}$ GeV
→ There is a minimum contribution to the relic density
→ If $f_a<10^{14}$ GeV the inflation contribution can dominate for θ if tensor to scalar ratio is large.

Important Lessons

Axions behave like DM (density ~a^{-3}) whenever H<m.
Axions behave like DE (negative pressure) whenever H>m.

Axion DE (or axion inflation), V needs to dominate H:

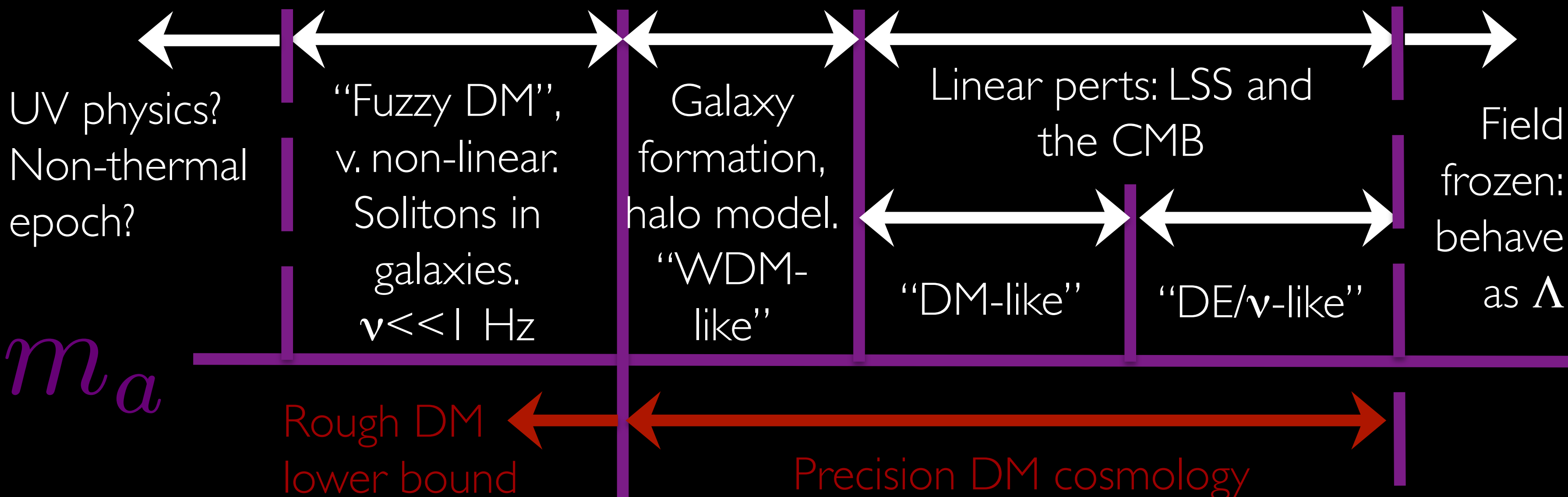
$$3H^2 M_{pl}^2 \approx \frac{1}{2} m_a^2 \theta_i^2 f_a^2$$

H>m → f must be of order the Planck scale.
For a single axion, this runs into the "Weak Gravity Conjecture", and is a subject of intense study.
Inflation needs long, flat potentials (60 e-folds). DE easier since we only see ~ few e-folds.

What about axion Dark Matter?

Axion DM Scales

Consider the Hubble scale at different epochs:



Physics:	BBN	Size of dSph	Non-linear	Equality	Today
Hubble: [eV]	10^{-15}	10^{-22}	10^{-24}	10^{-28}	10^{-33}

The QCD Axion

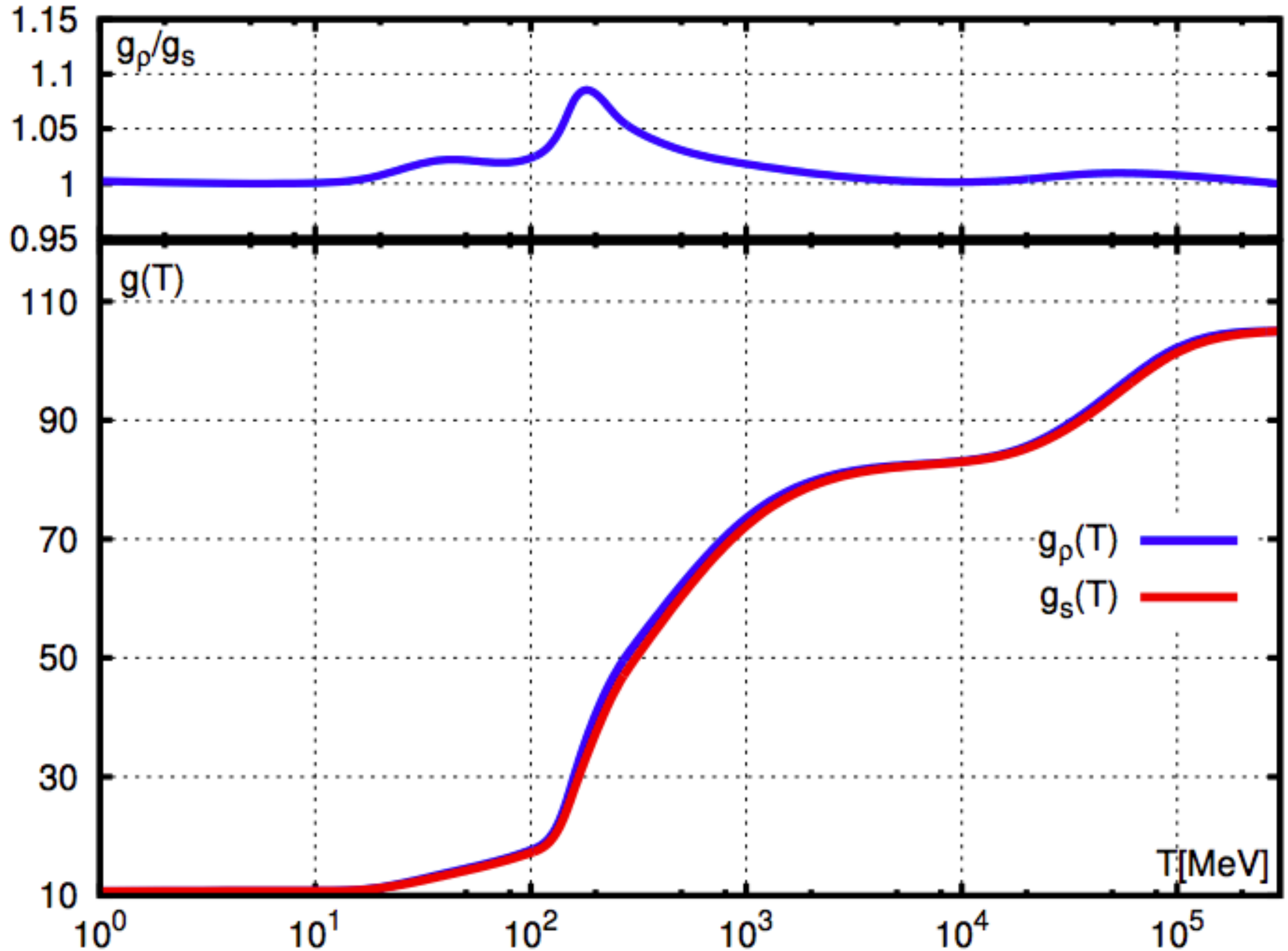
We must revoke all of our simplifying assumptions:

The QCD Axion

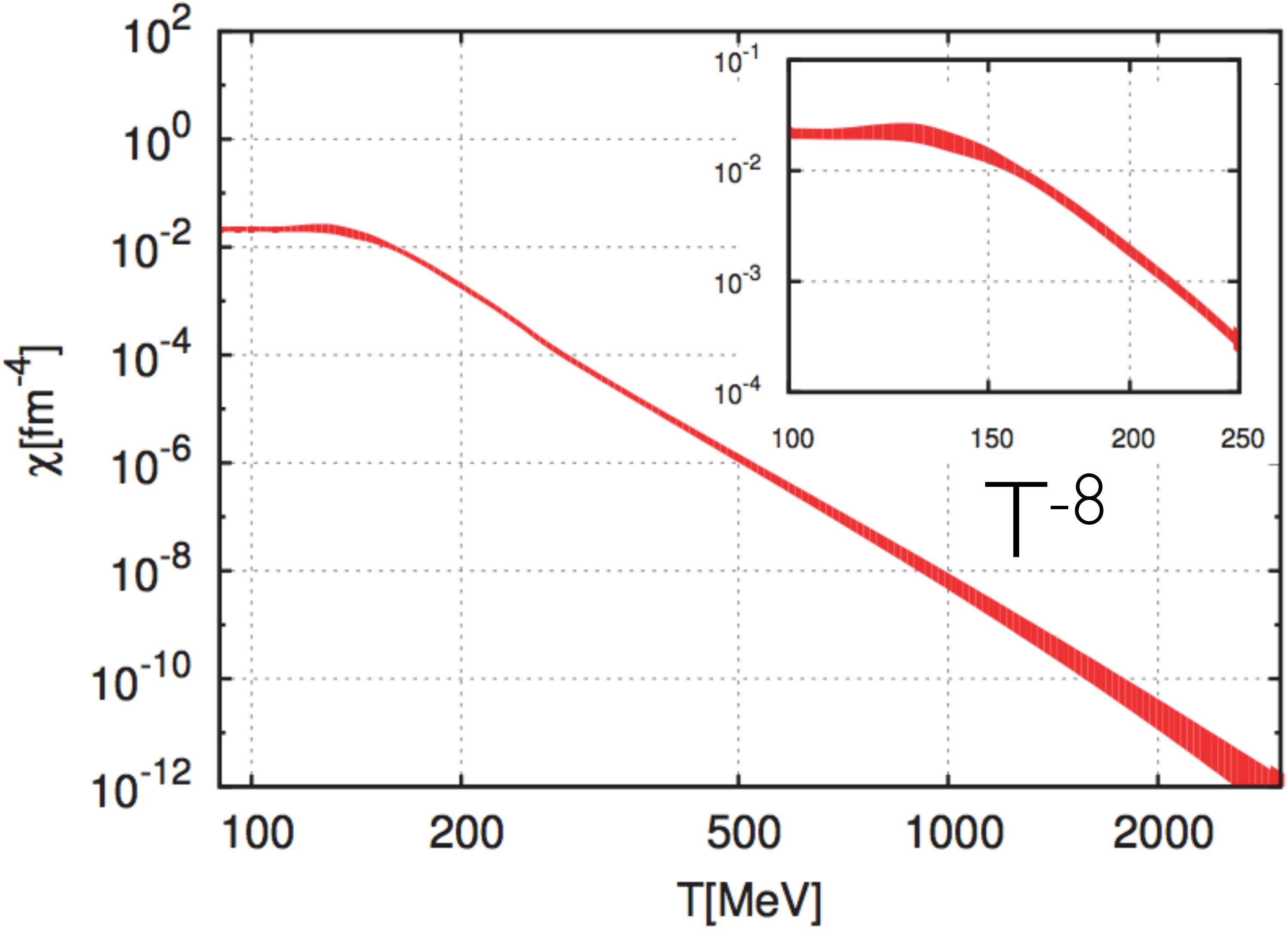
We must revoke all of our simplifying assumptions:

- Hubble not simple fluid, $g^*(T)$ needed during QCD phase transition.
- Topological susceptibility important, since $H \sim m(T)$ occurs while $m(T)$ still growing.
- Anharmonicities are known, so we may as well use the full potential.
- Relic density can be achieved for $f < 10^{14}$ GeV → HI contribution can dominate.

We covered all these points last lecture…

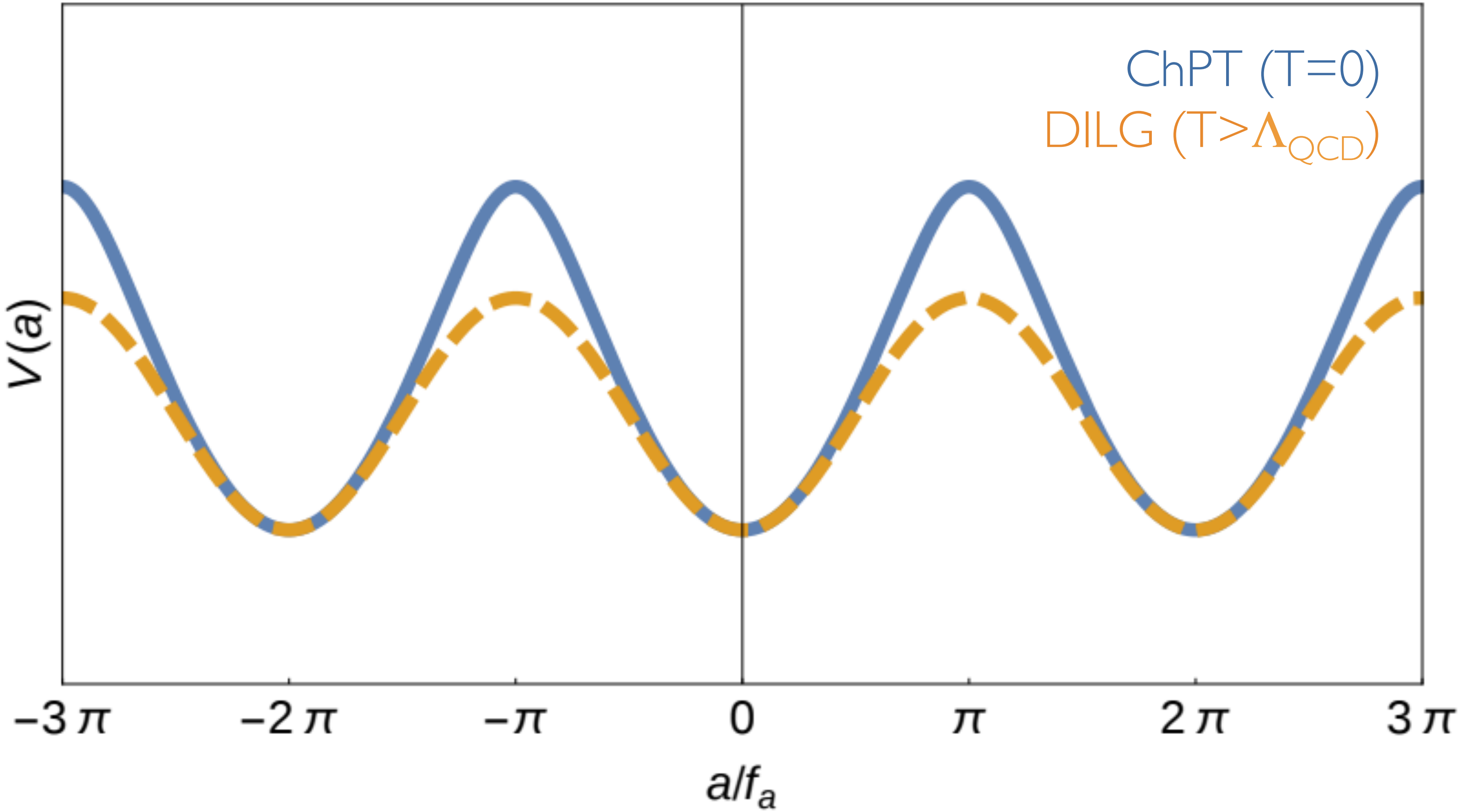


Lattice QCD g*(T) from Borsanyi et al (2016) → table fits.
See also Wantz & Shellard (2009) for analytic fits and calculation.



$$\chi(0) = (75.6\ \mathrm{MeV})^4$$

Borsanyi et al (2016)
See tabulated fitting functions



di Cortona et al (2015)

Oscillation Temp. and f Scaling

Define:

$$3H(T_{\rm osc}) = m_a(T_{\rm osc})$$

Recall:

$$3H^2 M_{pl}^2 = \frac{\pi^2}{30} g_{\star,\rm R}(T) T^4$$

$$m_a(T) = m_{a,0} \left(\frac{T}{\Lambda_{\rm QCD}} \right)^{-n}$$

$$m_{a,0} \approx 5.9 \times 10^{-6}\ {\rm eV} \left(\frac{10^{12}\ {\rm GeV}}{f_a} \right)$$

Approximating g* as a series of step functions:

$$T_{\rm osc} \propto f_a^{-1/(n+2)}$$

Adiabatic Invariant

Due to the mass evolution we cannot use the approximation:

Early time (DE-like) $$\rho(a) \approx \rho(a_{\rm osc}) \qquad (a < a_{\rm osc})$$

Late time (DM-like) $$\rho(a) \approx \rho(a_{\rm osc}) \left(\frac{a_{\rm osc}}{a}\right)^3 \qquad (a > a_{\rm osc})$$

However, since m(T) evolution (T evolves over H) is slow compared to the frequency of oscillation (m>H), there is an invariant:

$$\mathcal{A} = \frac{1}{2\pi} \oint p\,\mathrm{dq} \qquad q = \theta \qquad p = a^3 \dot{\theta}$$

Adiabatic Invariant

This implies that the axion comoving number density is conserved while the mass evolves:

$$\rho(T) = m_a(T) n_a(T) = m_a(T) \left(\frac{a(T_{\rm osc})}{a(T)} \right)^3 n_a(T_{\rm osc})$$

$$n_a(T_{\rm osc}) = \frac{1}{2} m_a(T_{\rm osc}) \langle \theta^2 \rangle_{\rm cycle} f_a^2$$

Using entropy conservation: $a(T) \propto g_{\star,\rm S}(T)^{-1/3} T^{-1}$

We find the final scaling for the relic density in terms of f:

$$\Omega_a h^2 \sim f_a^{(n+3)/(n+2)}$$

[Exercise: show this]

The adiabatic invariant only applies strictly when the potential is in the harmonic regime, which brings us on to the last point…

Anharmonic Corrections

In general these must be computed numerically. Solve the evolution in the full potential. However, they can be fit and then applied to the existing approximations:

$$\Omega_a h^2 = \mathcal{F}_{\rm an}(\theta)\Omega_a h^2|_{\rm harm.}$$

We know the asymptotic form of F:

$$\mathcal{F}_{\rm an}(\theta) \approx 1; \quad (\theta \lesssim 1)$$

$$\mathcal{F}_{\rm an}(\theta) \to \infty; \quad (\theta \to \pi)$$

The divergence can be shown to take the form:

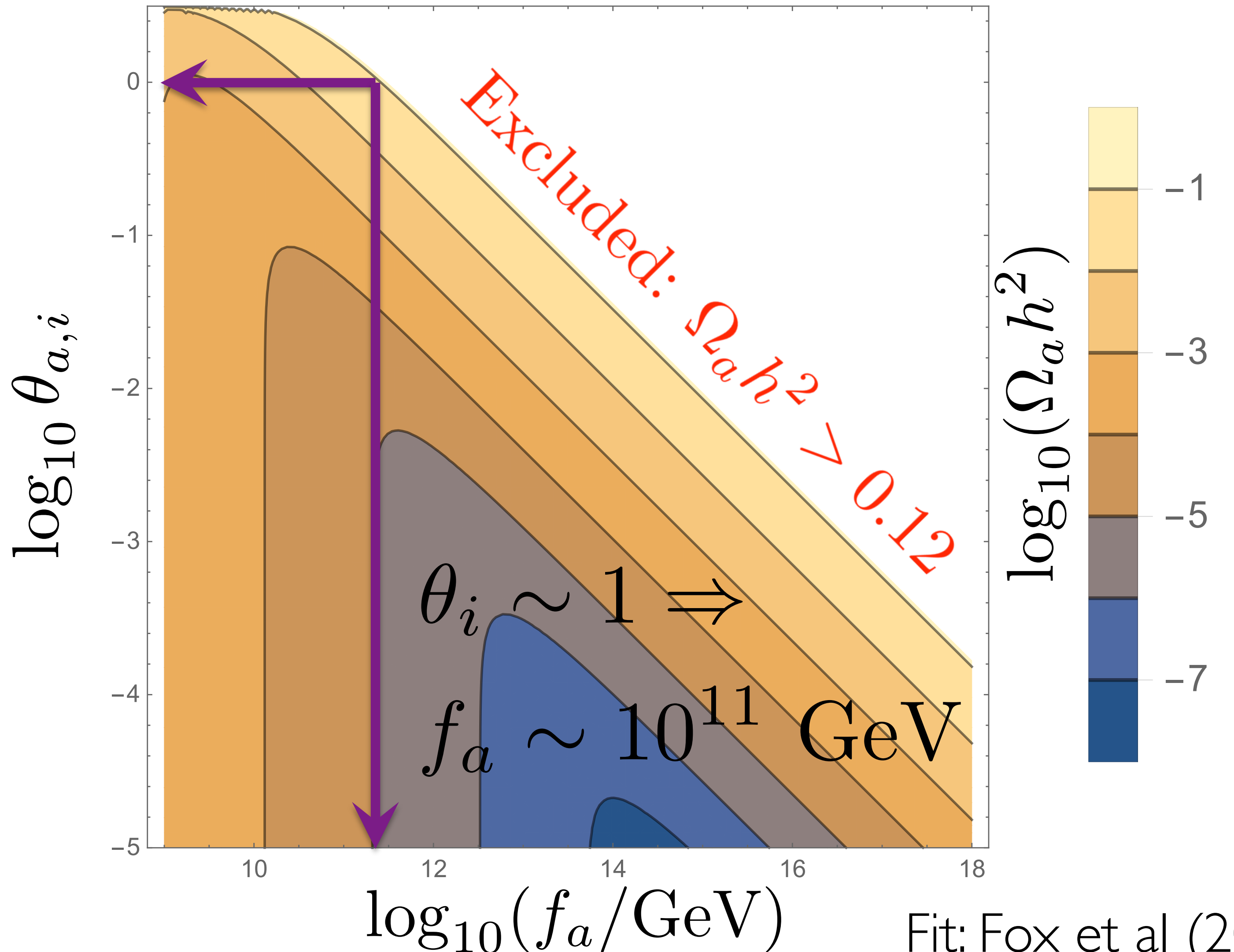
Lyth (1992)

$$m_a(t_{\rm osc})t_{\rm osc} = \ln[e/q(\theta_i/\pi)]$$ For q(x) polynomial

For a series of fits, see Diez-Tejedor & Marsh (2017)

Code: AxionRelic (ask me), GAMBIT module soon (Seb Hoof)

Fits to the QCD Relic Density



Fit: Fox et al (2004)

STRING DECAY: SCENARIO A

Selected Reading

DJEM, "Axion Cosmology", Ch. 3.

Vaquero et al, arXiv:1809.09241

Hiramatsu et al, arXiv:1202.5851

Gorghetto et al arXiv:1806.04677

Formulation of the Problem

The PQ symmetry breaks after inflation → solve SSB

$$S = \int \mathrm{d}^4x\sqrt{-g}\,[-(\partial_\mu\varphi)(\partial^\mu\varphi*) - V(\varphi)]$$

$$V(\varphi) = \lambda(f_a^2/2 - \varphi^2)^2 + \chi(T)\left(1 - \frac{\mathrm{Re}(\varphi)}{f_a}\right)$$

Field begins at the origin at high T with small perturbations, eom:

$$\ddot{\varphi} + 3H\dot{\varphi} - \nabla^2\varphi + \partial_\varphi V = 0$$

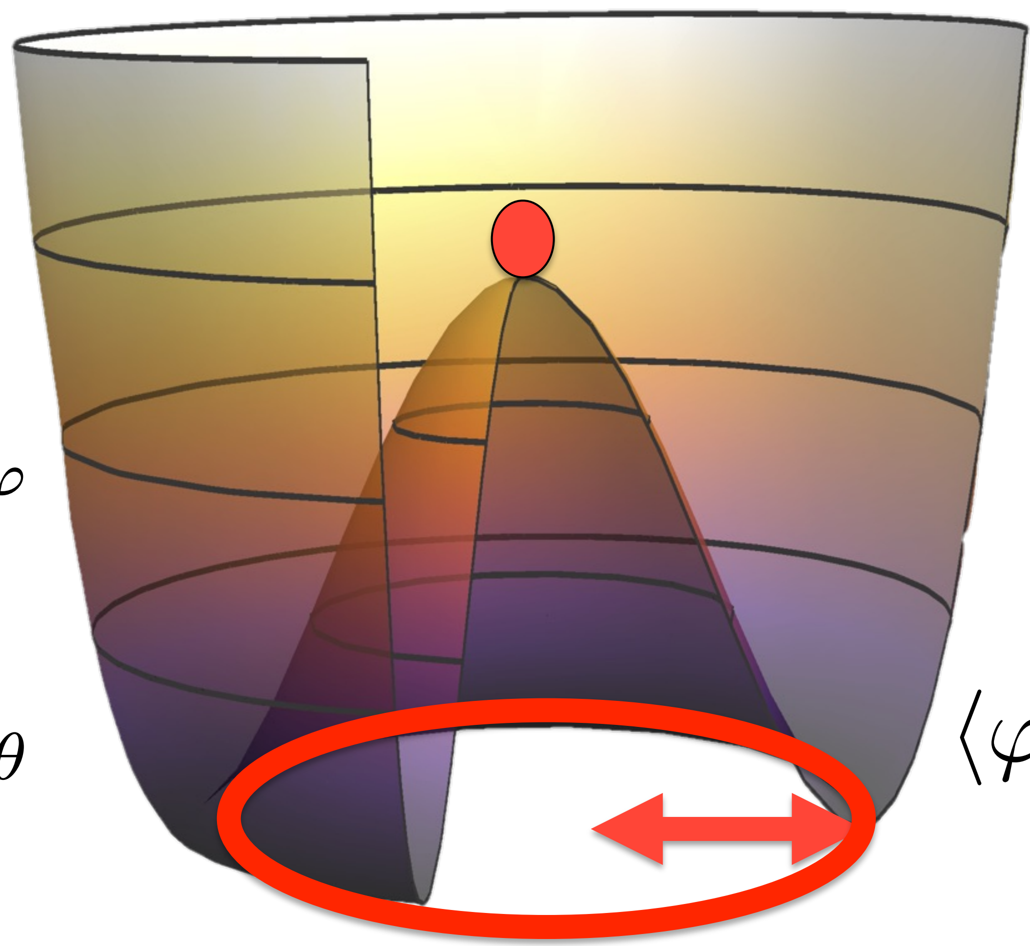
Gradient energy cannot be neglected → this is a PDE problem requiring lattice field theory.

Spontaneous Symmetry Breaking

$H \gg m_\varphi$

$\langle \varphi \rangle = 0$

$\varphi = \chi e^{i\theta}$



$H \ll m_\varphi$

$\langle \varphi \rangle = f_a/\sqrt{2}$

$\theta = \phi/f_a$

Kibble Mechanism

These dynamics happen locally. Due to the shift symmetry, θ is only defined within a horizon volume.

$$\ddot{\theta} + 3H\dot{\theta} - \nabla^2\theta = 0$$

Fourier transform, and we see that gradients act like a mass:

$$\theta = \int \frac{\mathrm{d}^3 k}{(2\pi)^3} e^{i\vec{k}\cdot\vec{x}}\theta_k \qquad \ddot{\theta} + 3H\dot{\theta} + k^2\theta = 0$$

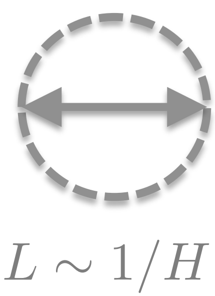
Thus the field is driven to "zero" by the "gradient potential" within each patch where $H<k$.

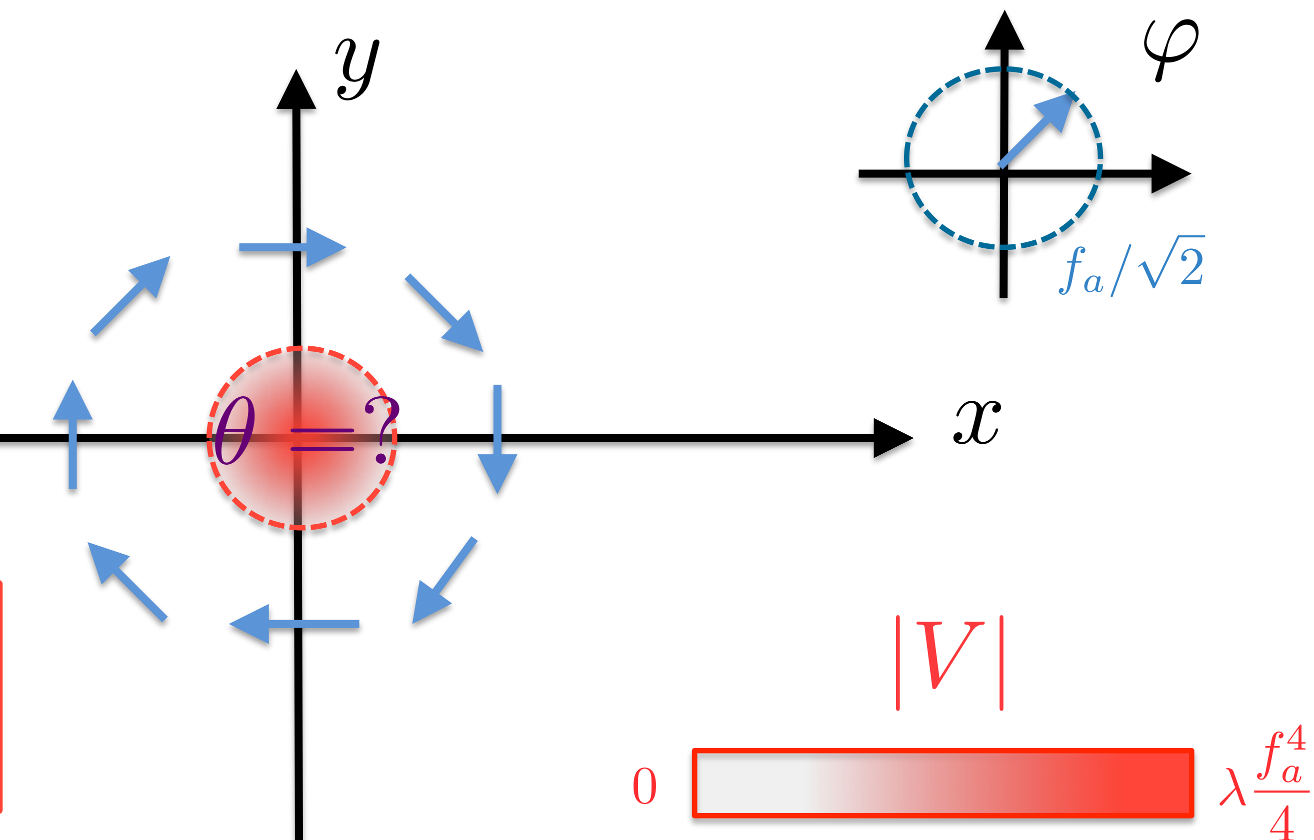
BUT, these patches are only locally defined due to shift symmetry → different θ in different horizon volumes.

As H shrinks, this process continues, s.t. patches $L \sim 1/H$.

String Formation

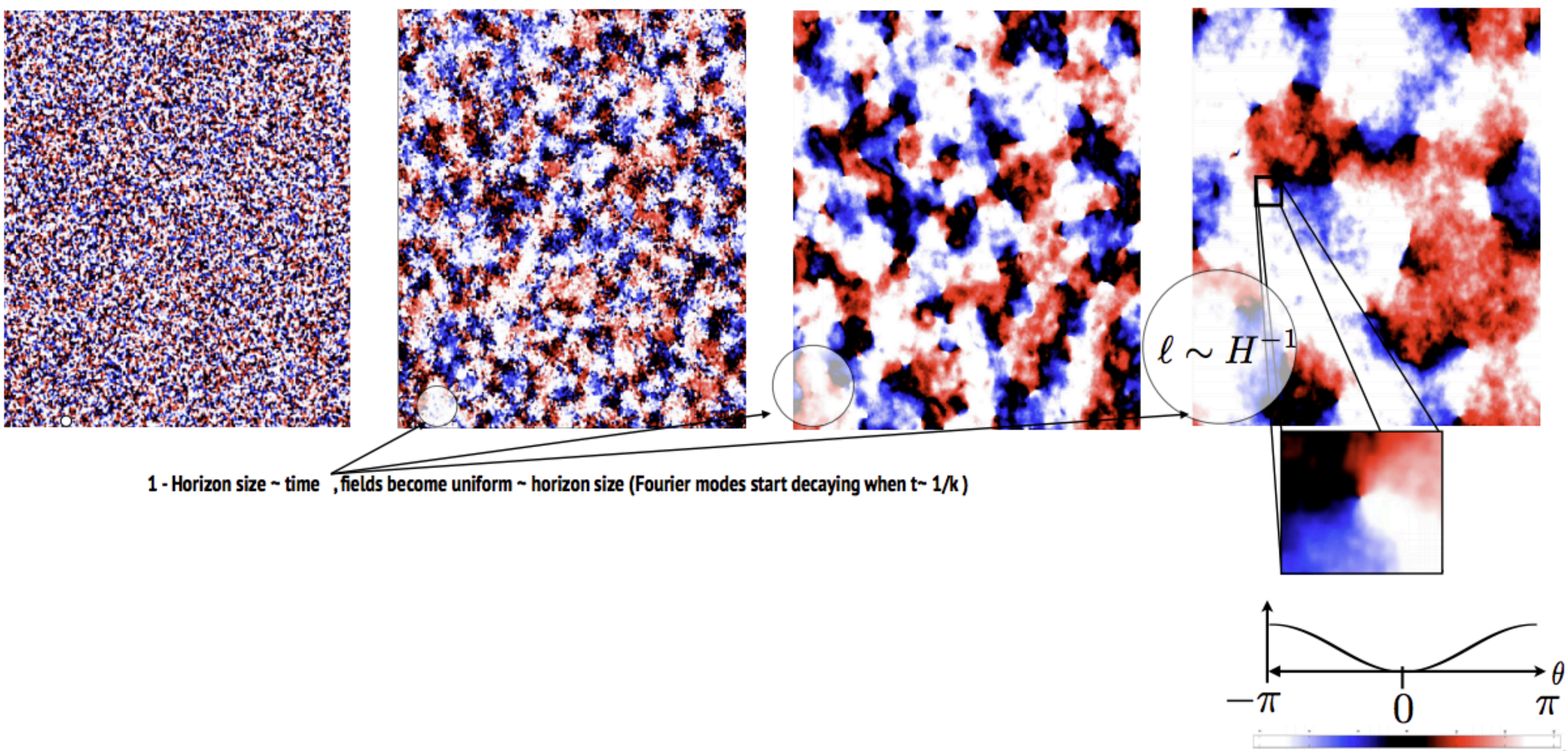
This process leads to a mapping from the complex plane of the field [which has symmetry group U(1)] to the physical space, which has symmetry group R3 → formation of "topological defects":





Due to the U(1)→R3 mapping, there can be points where the field undergoes winding

- Axion strings form by Kibble mechanism
- Energy logarithmically distributed around, tension $\mu \simeq \pi f_A^2 \log\left(\frac{f_A}{H}\right)$



Javier Redondo, slides from "Patras" (2018)

String Profile and Dynamics

Consider the complex field profile in cylindrical polars (R,ψ,z):

$$\varphi = \frac{f_a}{\sqrt{2}} g(m_\varphi R) e^{i\psi}$$

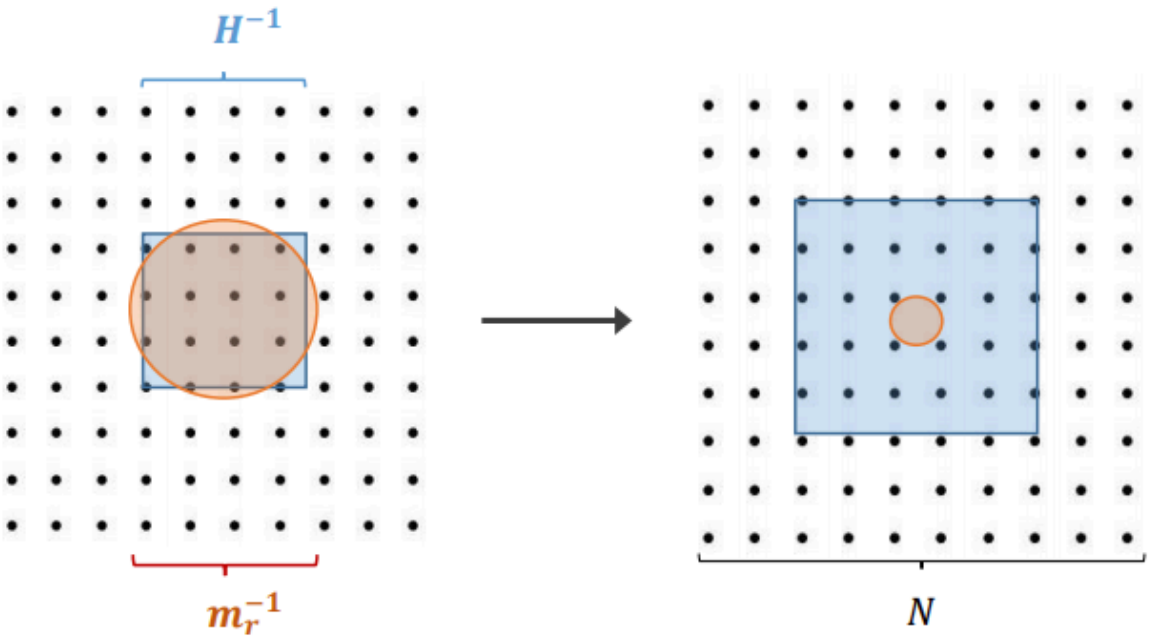
The function g(x) has the asymptotic properties:

$g(x) \to 0$	as	$x \to 0$	i.e. unbroken PQ	$V(\varphi) = \lambda \frac{f_a^4}{4}$
$g(x) \to 1$	as	$x \to \infty$	i.e. broken PQ	$V(\varphi) = 0$

These strings can be simulated approximately using the Nambu-Goto action, but then the properties like reconnection must be put in by hand. Modern approach: brute force Klein-Gordon eqn.

Gorghetto et al (2018)
https://www.youtube.com/watch?v=DbvM7emtodo

Hierarchies Restrict Simulation



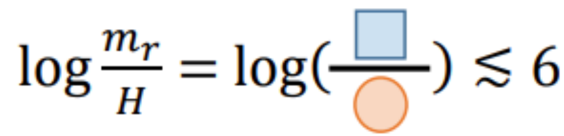


Fig: Gorghetto et al (2018)

1. Small log range and extrapolate. — Hiramatsu et al, Gorghetto et al
2. "PRS" or "fat string" approx. — Vaquero et al, Gorghetto et al
3. Multi-field trick. — Moore et al

Axions From String Decay

Really just a variation on misalignment mechanism…

We say "strings decay into axions" but it is all just field oscillations.

Compute $\rho_{\rm string} \approx \rho_a$ at the end of the simulation.

$$\rho_{\rm string} = \xi \frac{\mu}{t^2}$$

tension

$f_a^2 \ln(f_a d_a)$

$$d_a = \frac{\sqrt{\xi}}{H}$$

per horizon

"Scaling solution": $\xi \to \mathrm{const.}$

"Attractor solution": $\xi \to \xi(t)$ logarithmic scaling violation.

Axions From String Decay

Integrate the string spectrum:

$$n_a(t) = \int \frac{\mathrm{d}k}{k} \frac{\partial \rho}{\partial k}$$

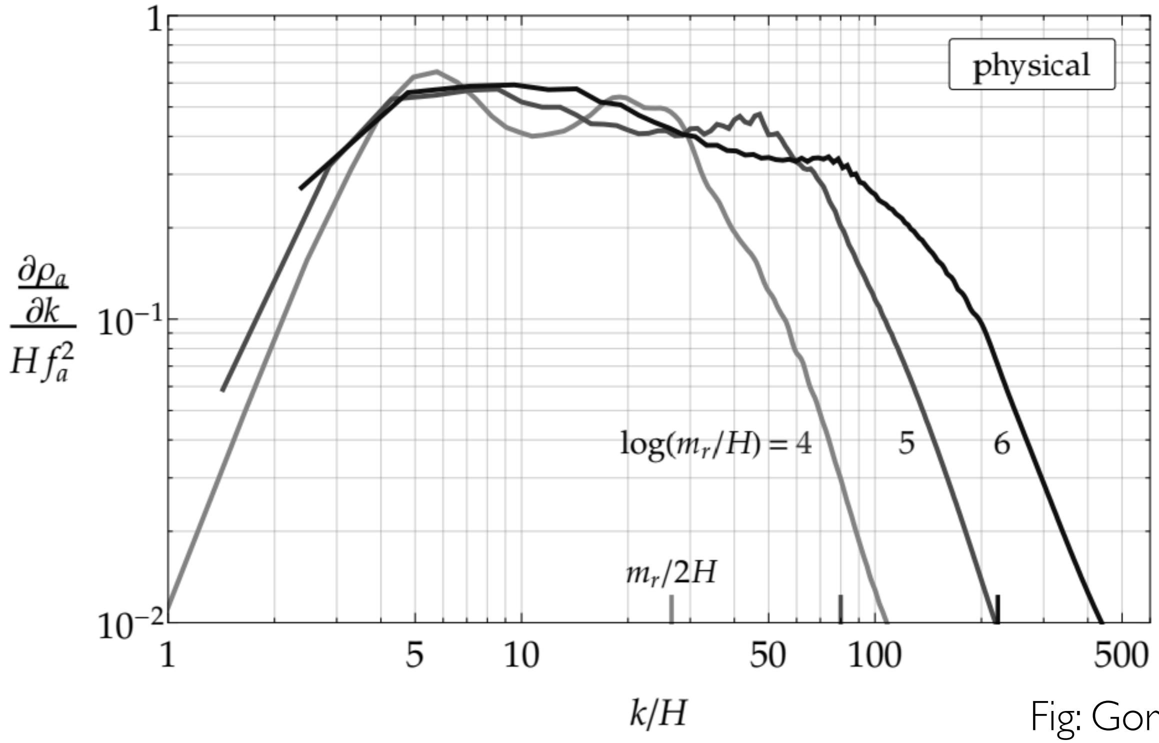


Fig: Gorghetto et al (2018)

Axions From String Decay

Parameterise your ignorance:

1. Compute the "naïve" misalignment result with the average value (many horizon volumes):

$$\langle\theta^2\mathcal{F}_{\rm an}(\theta)\rangle = \frac{1}{2\pi}\int_{-\pi}^{\pi}{\rm d}\theta\ \theta^2\mathcal{F}_{\rm an}(\theta) = c_{\rm an.}\frac{\pi^2}{3}$$

2. Introduce an additional fudge factor fit to simulations:

$$\Omega_a h^2 = \Omega_a h^2|_{\rm mis.}(1+\alpha_{\rm dec.})$$

Domain Walls

Fig: Armengaud et al (2019)

Complex phase identifies:

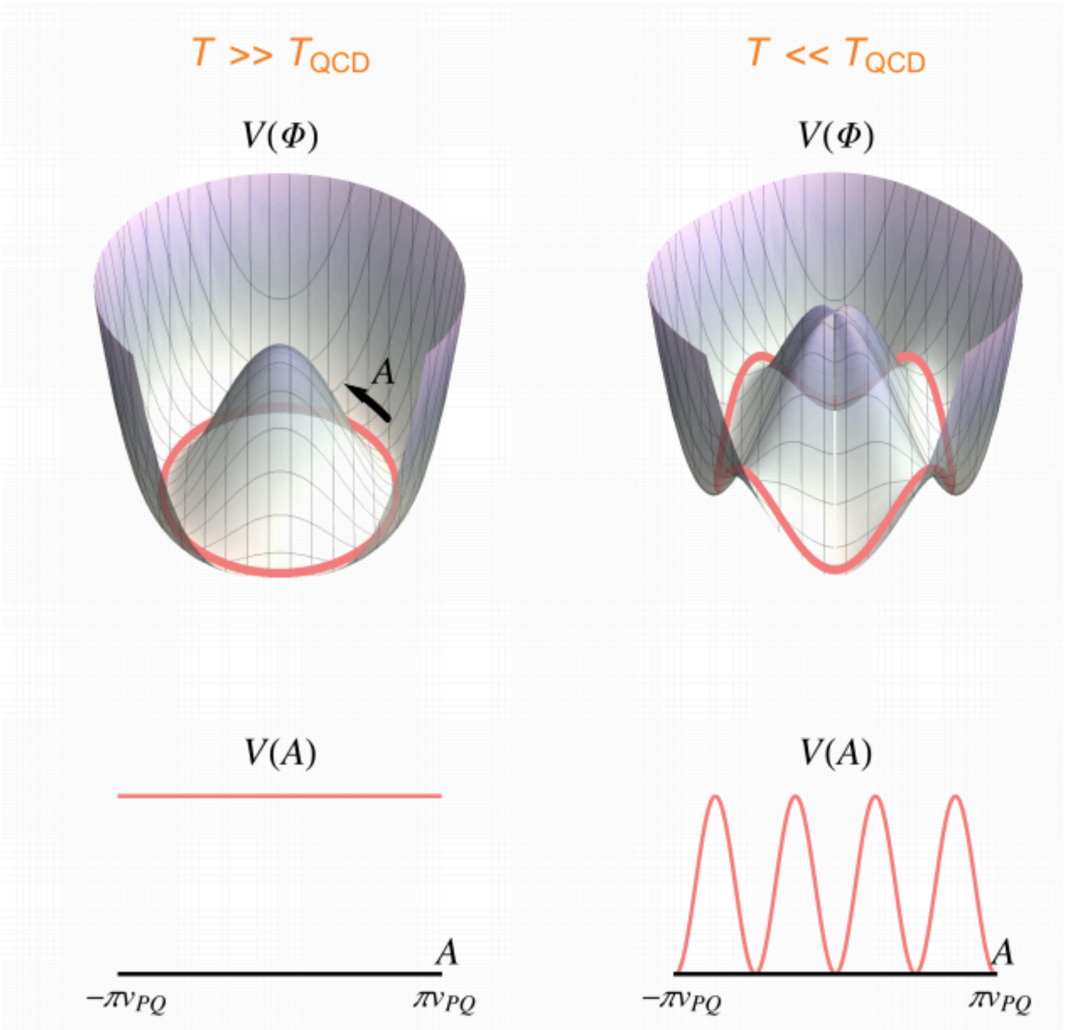
$$\theta_{\mathrm{PQ}} \to \theta_{\mathrm{PQ}} + 2\pi$$

If the PQ C>1 then:

$$\theta_{\mathrm{QCD}} = \mathcal{C}\theta_{\mathrm{PQ}}$$

$$V \sim \cos\theta_{\mathrm{QCD}}$$

→ potential has multiple minima for one PQ rotation
→ different vacua possible
→ "domain walls".



Example with C=3

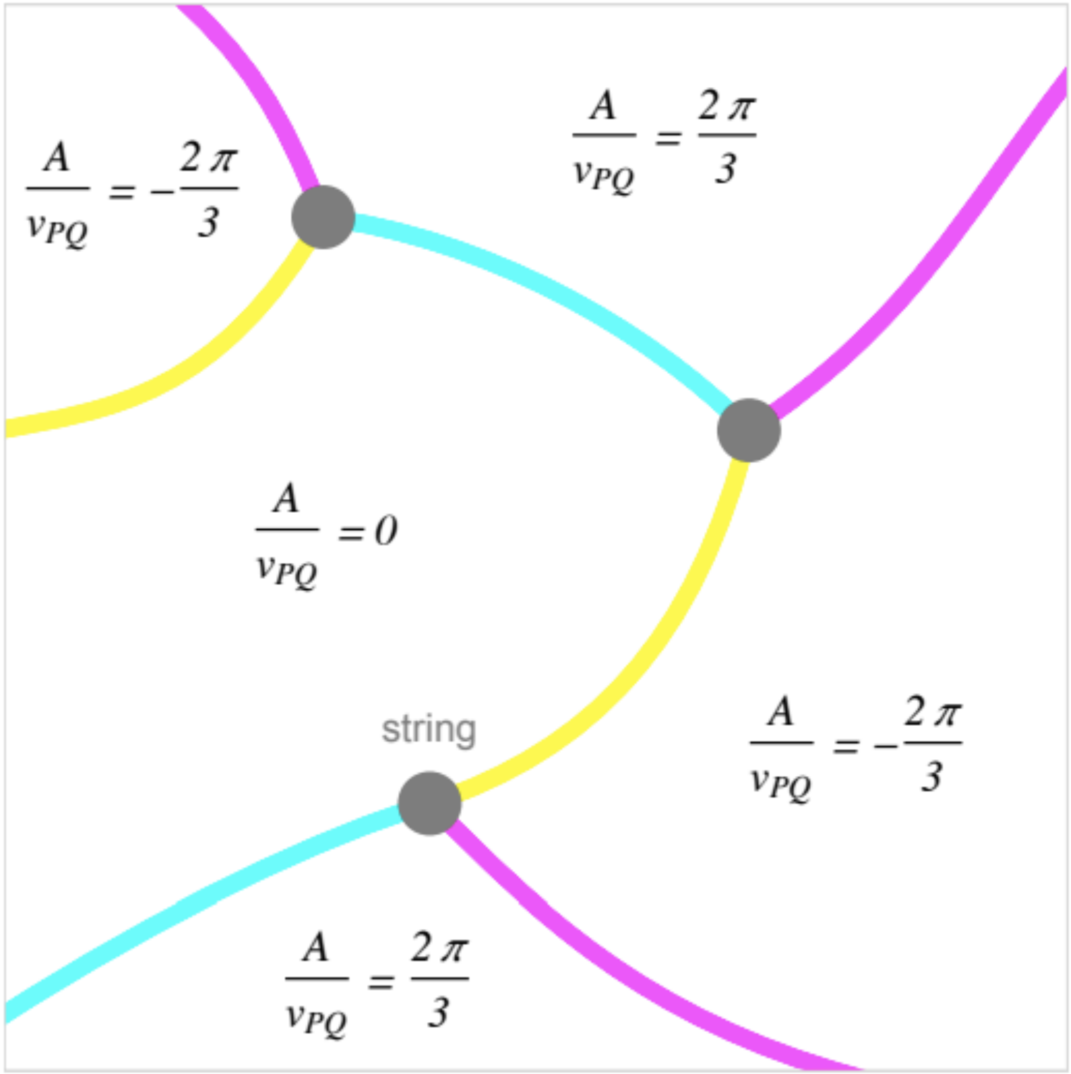
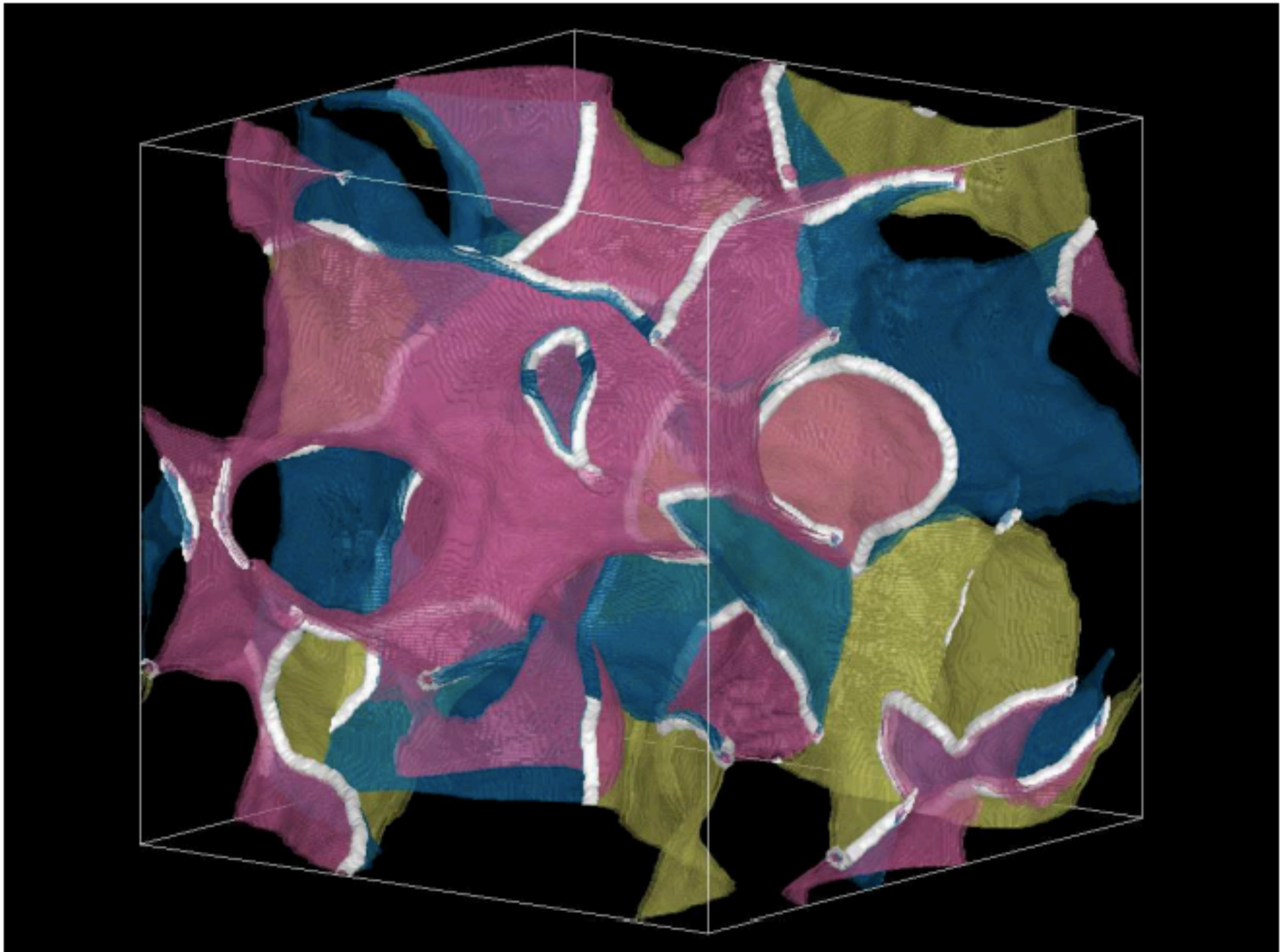


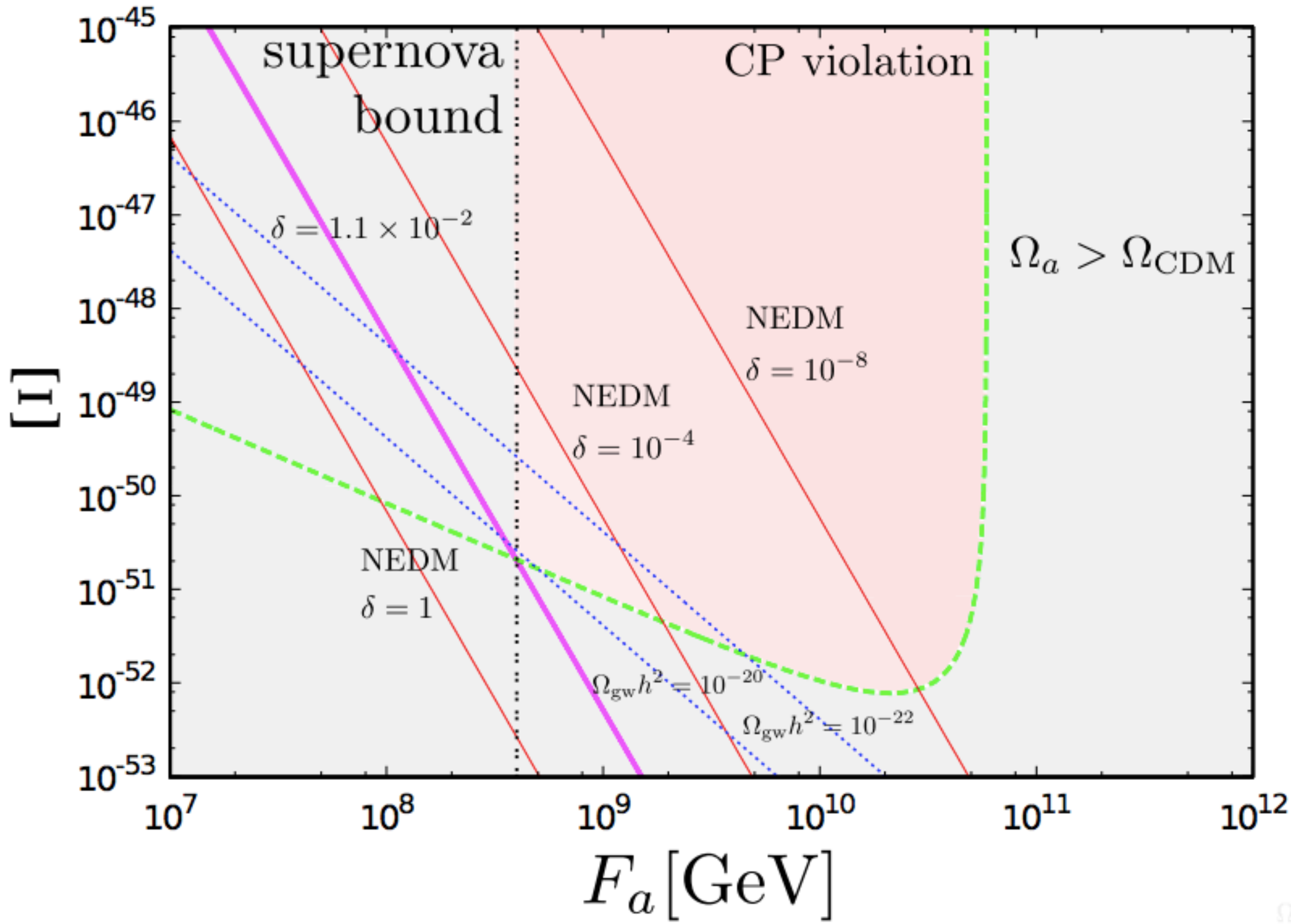
Fig: Armengaud et al (2019)

Fig: Hiramatsu et al (2012)

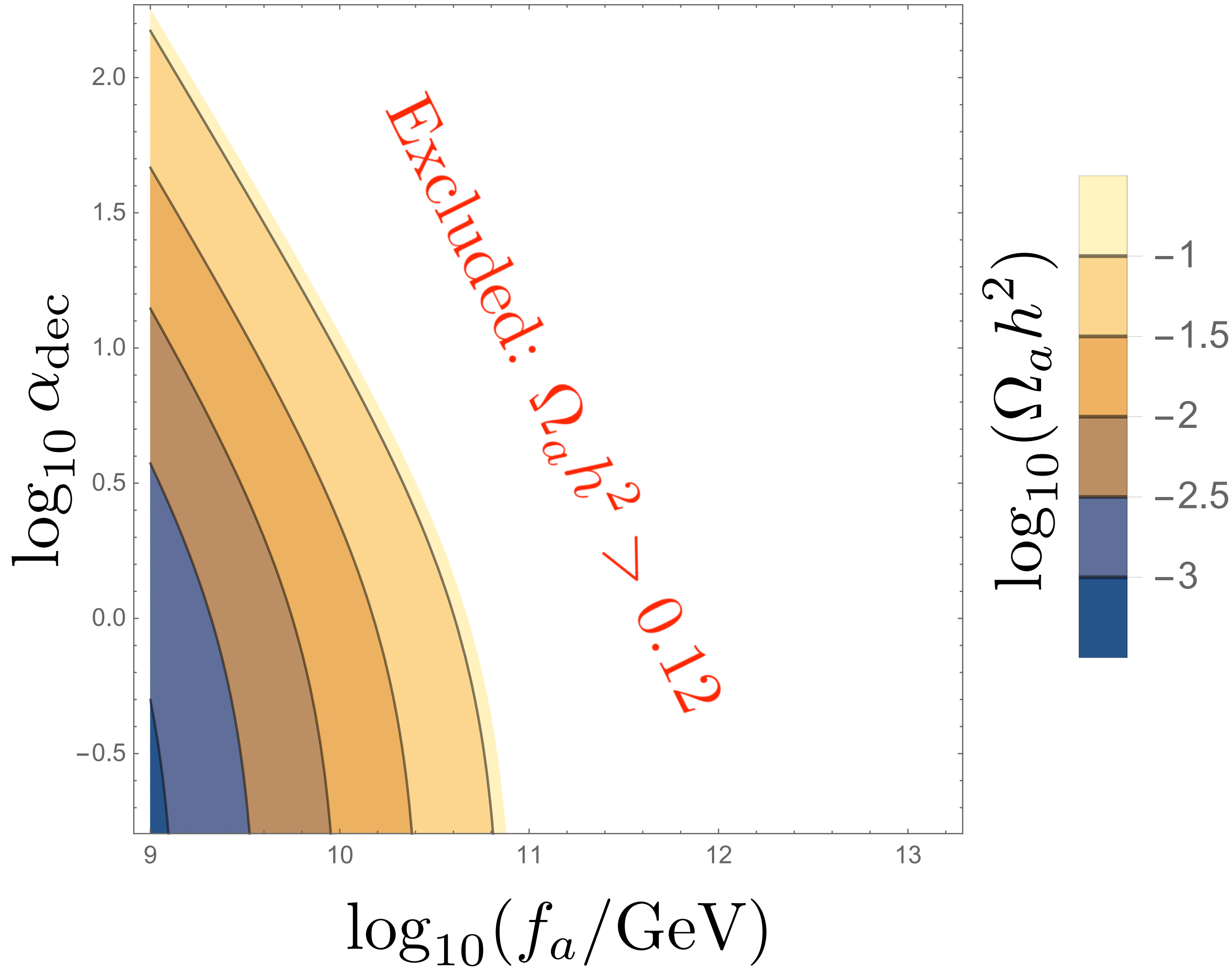


Walls require a "biasing potential" Ξ to decay → explicitly break PQ → spoil solution to strong CP problem by tuning factor δ.

Small biasing → delayed decay → increased relic density → lower f_a.

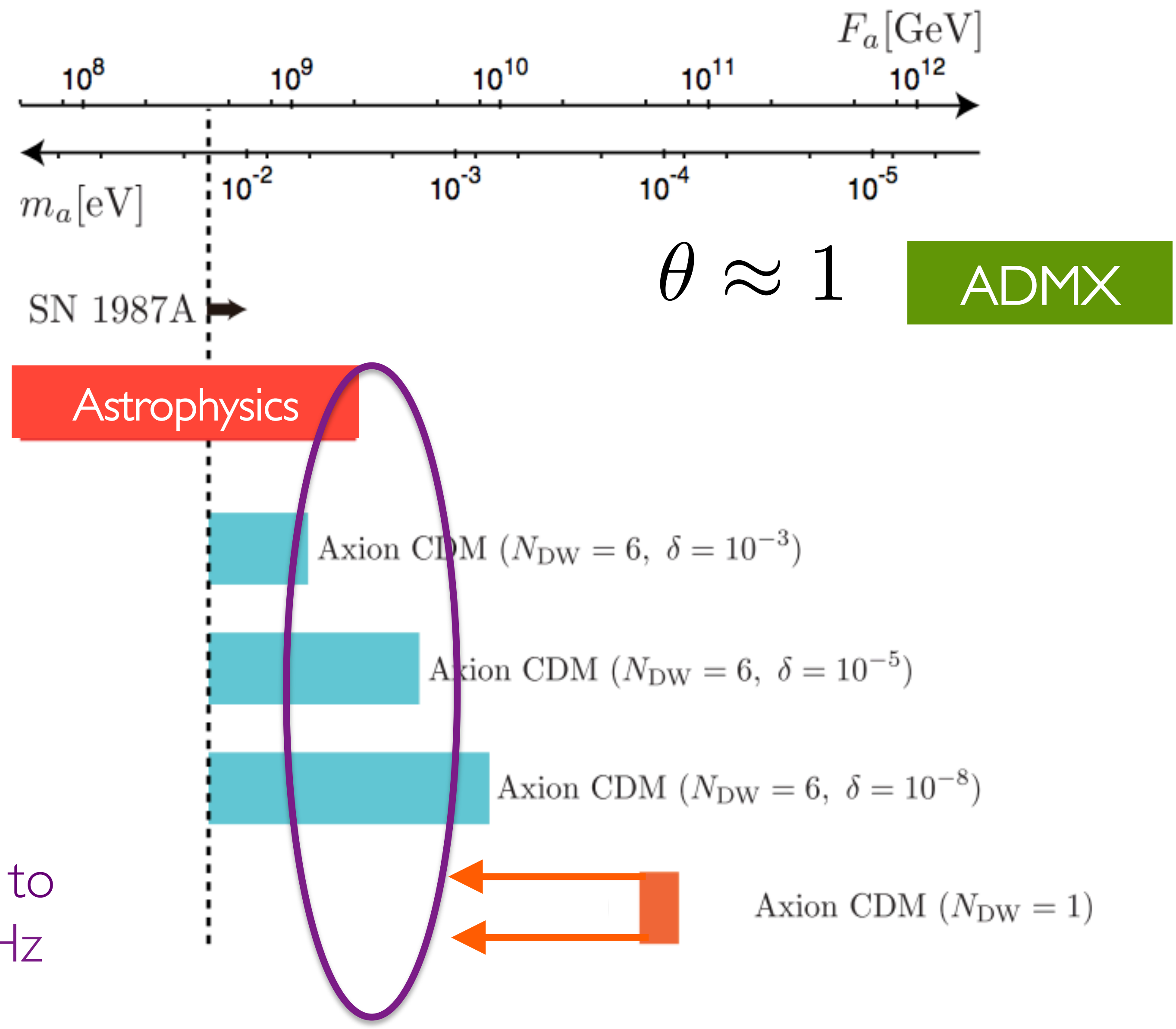


Relic Abundance with Defects



Adapted from: Kawasaki et al (2014)

Scenario B allows lighter axions.



Scenario A seems to favour meV → THz