Quadrature rules on manifolds: References

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For the Koksma-Hlawka inequality and lower bounds for the discrepancy, see [1, 4, 10, 32, 37, 42, 43].

For quadrature rule errors in Sobolev spaces on the sphere, see [12, 13, 14, 26, 27, 28, 29, 30, 33]. For general compact manifolds (and beyond), see [8, 9].

For the Marcinkiewicz-Zygmund inequality for the moments of independent random variables, see [16, 36, 35].

For area regular partitions for the sphere, see [1, 6, 7, 21, 34, 41, 45], and for Ahlfors regular metric measure spaces, see [25].

For the existence of almost positive summability kernels on manifolds, see [24].

For the proof of the Korevaar-Meyers conjecture on spherical designs, see [5, 6]: For manifolds, see [19, 20, 23].

One can find the Brouwer degree theory theorem used to prove the above conjecture in [40, Theorem 1.2.9].

The Marcinkiewicz-Zygmund inequality for diffusion polynomials can be found in [38].

Hadamard's parametrix for the wave equation: see [2, 31, 44].

For the existence of dyadic cube partitions, see [17, 18].

For Sierpinski's measure theoretic result, see [22, page 38].

For Cassels inequality, see [3, 11, 15, 39].

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