International School and Workshop on Probing Hadron Structure at the Electron-Ion Collider

School (January 29-February 3, 2024)

Topics of lectures:

(i) Pedagogical lectures on QCD and physics of EIC

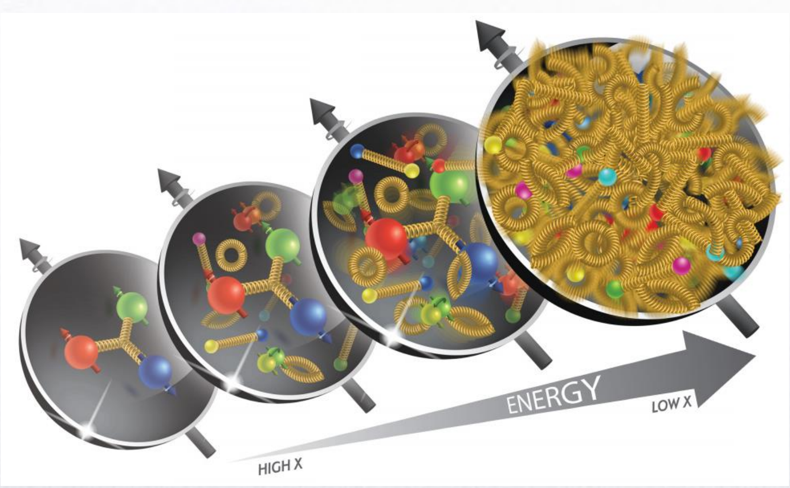
(ii) Elastic and deep inelastic scattering (iii) Exclusive processes

(iv) Single spin asymmetries and TMDs (v) Hadron structure in experiments (vi) Heavy ion physics

Date and time	Mon, 29 Jan	Tue, 30 Jan	Wed, 31 Jan	Thu, 1 Feb	Fri, 2 Feb	Sat, 3 Feb
09:15 - 09:30	Opening remarks					
09:30 - 10:30	PEIC	PEIC	EP	HSE	EP	Informal Discussion
10:30 - 11:00	Tea/Coffee					
11:00 - 12:00	EDIS	EDIS	HSE	SSA	HIP	Informal Discussion
12:00 - 13:00	PEIC	PEIC	EP	HIP	EP	
13:00 - 14:30	Lunch					
14:30 - 15:30	EDIS	EDIS	SSA	SSA	HIP	
15:30 - 16:00	Tea/Coffee					
16:00 - 17:00	EDIS	EDIS	SSA	HIP	Interactive session on data analysis	
17:00 - 18:00	Discussion	SSA	HIP	HSE	Interactive session on data analysis	

ICTS INTERNATIONAL CENTRE for THEORETICAL SCIENCES
TATA INSTITUTE OF FUNDAMENTAL RESEARCH

Center for Frontiers in Nuclear Science



? "Pedagogical QCD" : main features and open problems

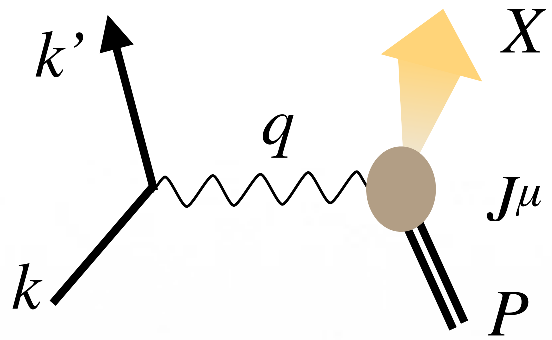
Marco Radici
INFN - Pavia
INFN Istituto Nazionale di Fisica Nucleare

Operator Product Expansion

Factorization from another point of view: the OPE

1. Justification

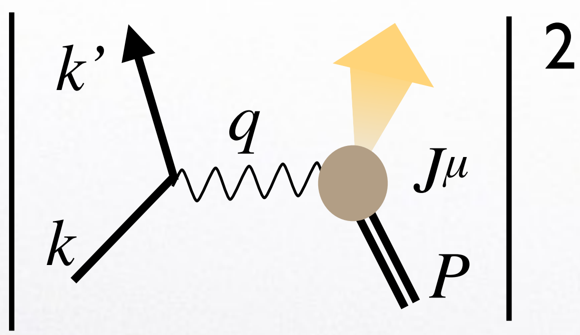
consider the inclusive DIS



scattering amplitude $\mathscr{M} = \bar{u}(k')\,\gamma_\mu\,u(k)\,\dfrac{e^2}{Q^2}\,\langle P_X\,|\,J^\mu(0)\,|\,P\rangle$

phase space $dR = (2\pi)^4\,\delta(P+q-P_X)\,d^4P_X\,d^4k'$

cross section $d\sigma \,\propto\, \displaystyle\int_X |\,\mathscr{M}\,|^2\,dR$



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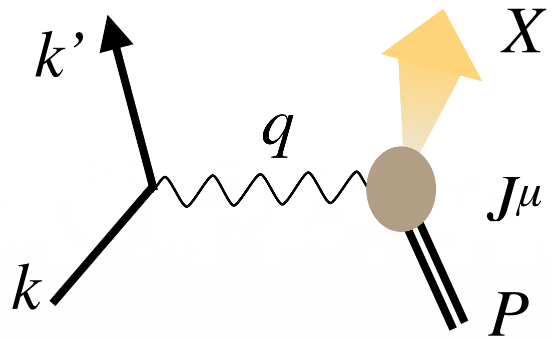

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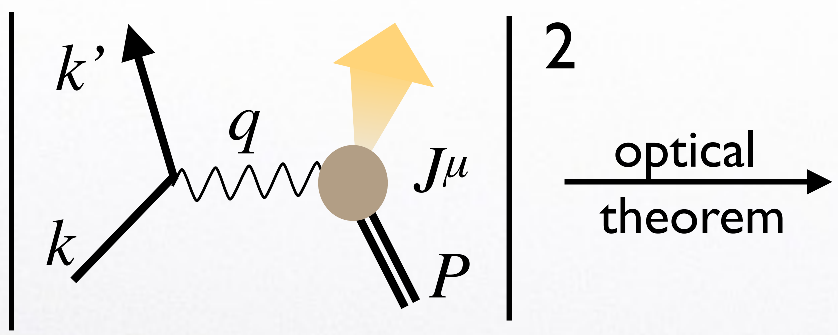

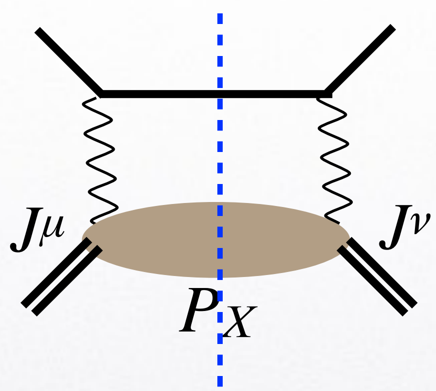

$$L_{\mu\nu} = 2k_\mu k'_\nu + 2k_\nu k'_\mu - Q^2\,g_{\mu\nu}$$ leptonic tensor

$$W^{\mu\nu} = \int d^4P_X\,(2\pi)^4\,\delta(P_X - P - q) \times \langle P\,|\,\hat{J}^\mu(0)\,|\,P_X\rangle\,\langle P_X\,|\,J^\nu(0)\,|\,P\rangle$$ hadronic tensor

cut-diagram notation:
cross section = product of (amplitude) x (amplitude)*
particles entering cut are on-shell

Operator Product Expansion

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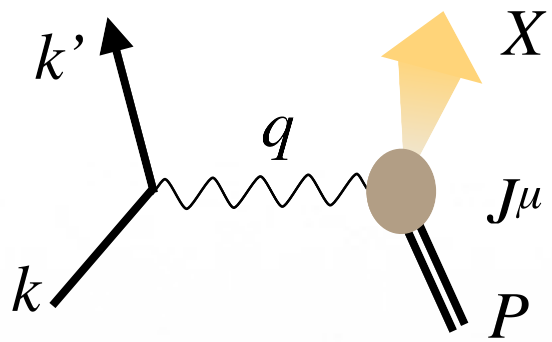

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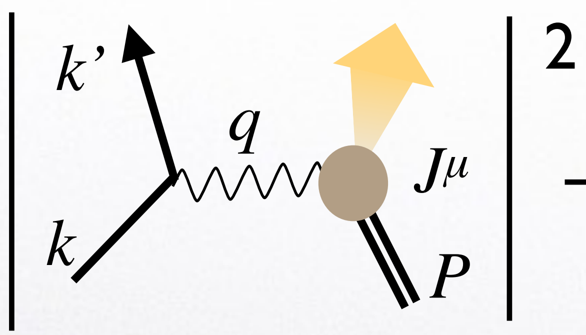
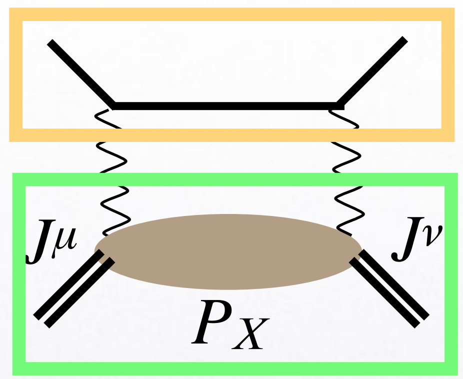


optical theorem $\longrightarrow$



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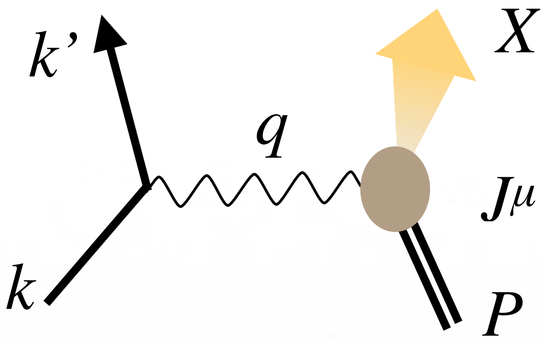

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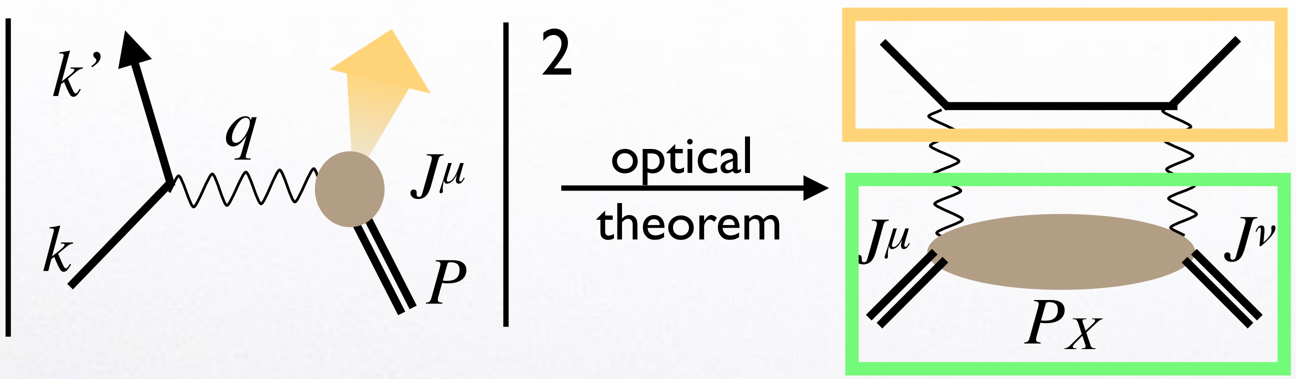

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hadronic tensor

matrix element of bilocal operator $= \int d\xi\, e^{iq\cdot\xi}\, \langle P \,|\, \left[\hat{J}^\mu(\xi)\,,\, \hat{J}^\nu(0) \right] \,|\, P \rangle$

$\hat{J}^\mu = \bar{\psi}\, \gamma^\mu\, \psi$

parton e.m. current

check! $\hat{J}^{\mu\dagger} = \hat{J}^\mu$

Operator Product Expansion

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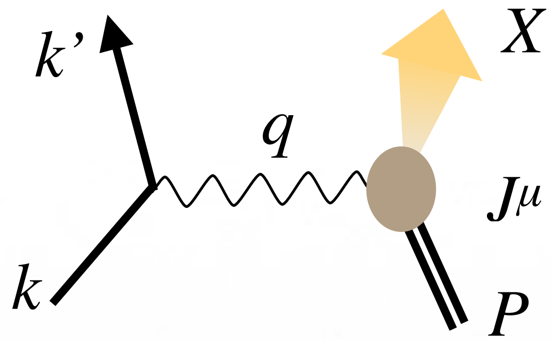

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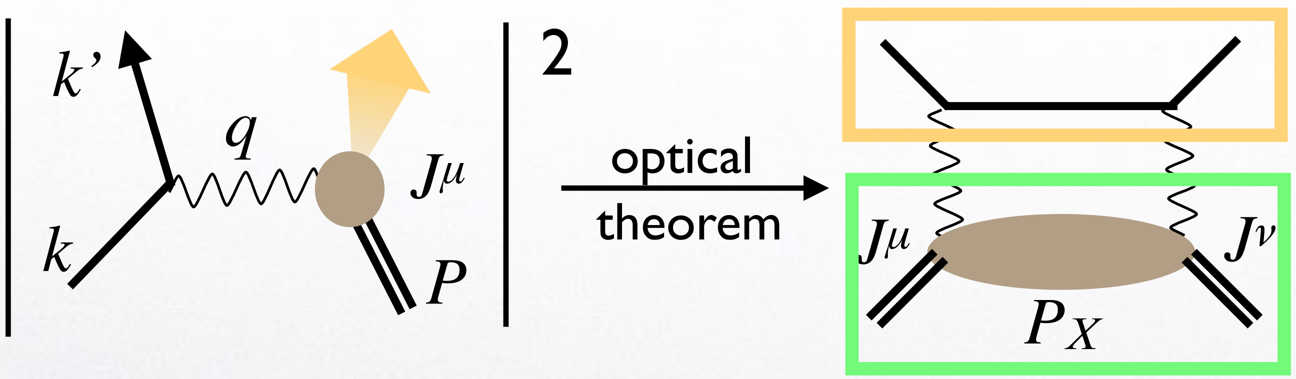

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parton e.m. current

dominated by time-like short distances $\xi^2 \to 0$, but ill defined !

check! $\hat{J}^{\mu\dagger} = \hat{J}^\mu$

Operator Product Expansion

DIS regime:

$Q^2 \to \infty$

$x = \frac{Q^2}{2P \cdot q}\Big|_{\mathrm{TRF}} = \frac{Q^2}{2M\nu}$ fixed

Target Rest Frame $\Rightarrow \ \nu \to \infty$

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Riemann - Lebesgue theorem:

for $|q\cdot\xi| \to \infty$, large oscillations and cancelations;

integral is dominated by terms with $|q\cdot\xi| \leq K$ constant

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Then, $(q\cdot\xi)|_{\mathrm{cm}} = \nu\,\xi^0 \leq K \quad \Rightarrow \quad \xi^0 \leq \dfrac{K}{\nu} \xrightarrow{\nu\to\infty} 0$

space-like distances $\xi^2 < 0$ are forbidden by causality;

for time-like distances $\xi^2 \geq 0$, $\xi^2 = (\xi^0)^2 - \vec{\xi}^2 \geq 0 \Rightarrow (\xi^0)^2 \geq \vec{\xi}^2 \xrightarrow{\nu\to\infty} 0$

The integral is dominated by short time-like distances $\xi^2 \to 0$, but in this limit the bilocal operator is ill defined. Example: free neutron scalar field $\phi(x)$ with propagator $\Delta(x$-$y)$

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$$\langle 0 | \mathcal{T}[\phi(x)\,\phi(y)] | 0 \rangle = -\,i\,\Delta(x-y) = i \int \frac{d^4p}{(2\pi)^4} \frac{e^{-ip\cdot(x-y)}}{p^2 - m^2 + i\varepsilon}$$

for $x \to y$, the integral is divergent :

$$= \frac{m}{4\pi^2} \frac{K_1\left(m\sqrt{-(x-y)^2 + i\varepsilon}\right)}{\sqrt{-(x-y)^2 + i\varepsilon}} - \frac{i}{4\pi}\,\delta\left((x-y)^2\right) \xrightarrow{x\to y} \infty$$

K_1 modified Bessel funct. of 2^o kind

2. Definition

$$\hat{A}(x)\,\hat{B}(y) = \sum_{i=0}^{\infty} C_i(x-y)\,\hat{O}_i\left(\frac{x+y}{2}\right)$$

local operators, regular for $x \to y$, typically $\hat{O}_0 = \mathbf{I}$

Wilson coefficients, singular for $x \to y$, ordered in decreasing singularity

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Example: the Wick theorem

$$\lim_{x\to y} \mathcal{T}\left[\phi(x)\,\phi(y)\right] = :\phi(x)\,\phi(y): + \langle 0|\,\mathcal{T}\left[\phi(x)\,\phi(y)\right]|0\rangle$$

$$= 1\cdot\hat{O}_1 + C_0(x-y)\,\mathbf{I}$$

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3. Application to inclusive DIS

$$W^{\mu\nu} = \int d\xi\, e^{iq\cdot\xi}\,\langle P|\left[\hat{J}^\mu(\xi)\,,\hat{J}^\nu(0)\right]|P\rangle \;=\sum_{\{\alpha\}} C^{\mu\nu}_{\{\alpha\}}\left(\frac{M}{Q}\right)^{t-2}$$

twist t = canonical dimension - spin of operator $\hat{O}_i$

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twist expansion \ perturbative expansion		leading order LO	NLO	NNLO	…
		1	α_s	α_s^2	…
leading twist 2	1	parton model	perturbative QCD →		
subleading twist 3	$1/Q$	↓ OPE			
twist 4	$1/Q^2$				
…	…				

OPE rigorous proof only for inclusive processes. It can be effectively extended to all semi-inclusive hard processes.

4. Factorization

By applying the same technique of Wick theorem, it can be shown that the dominant contribution to the hadronic tensor of inclusive DIS comes from the so-called "handbag" diagram:

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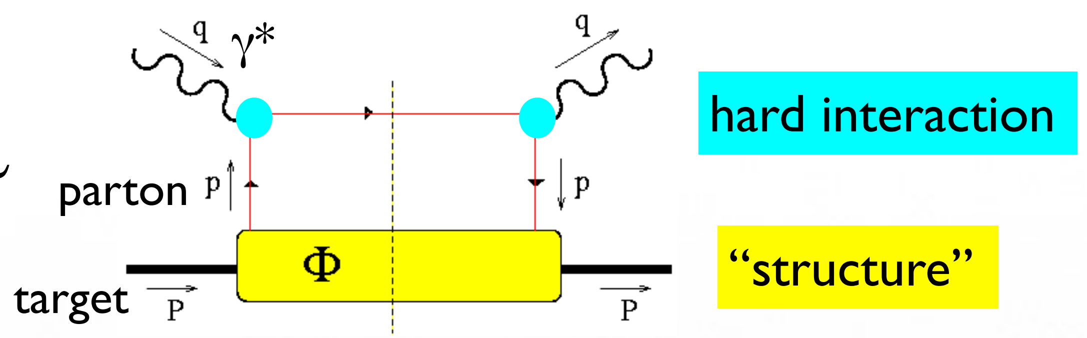

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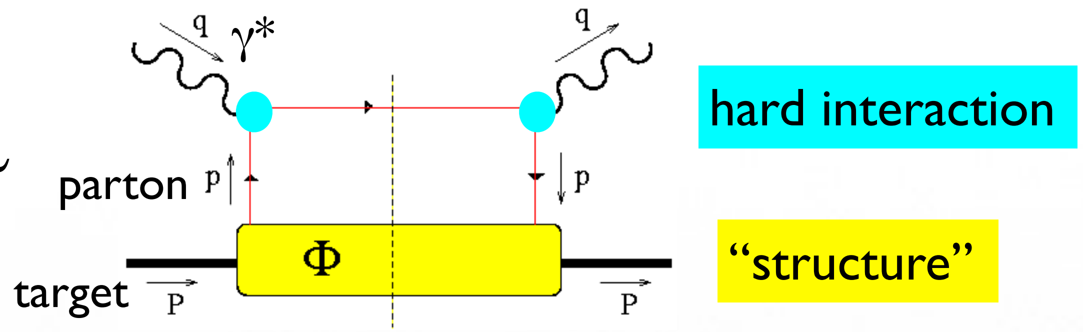

Φ bilocal quark-quark correlator:
$$\Phi_f(p,P) = \int d^4 P_X \, \delta(P - p - P_x) \, \langle P | \bar{\psi}_f(0) | P_X \rangle \, \langle P_X | \psi_f(0) | P \rangle$$
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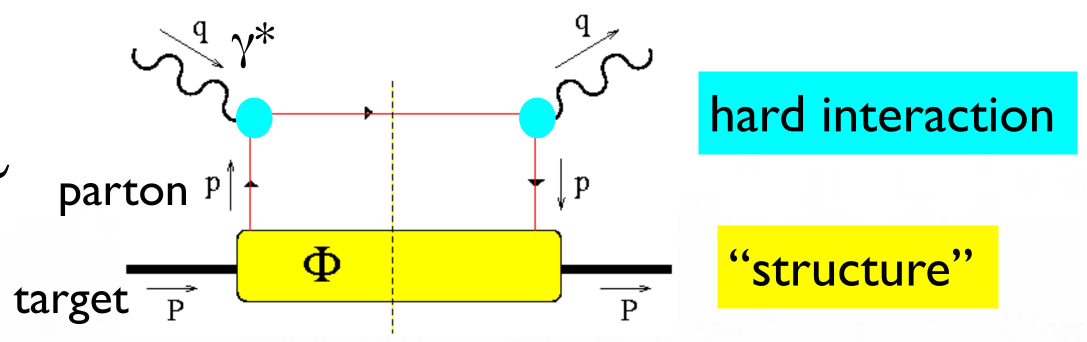

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By taking suitable projections, one can extract from the "structure" the leading-twist part, the subleading part at twist 3, at twist 4, etc..

5. Extract PDF

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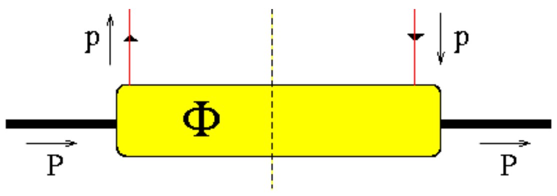

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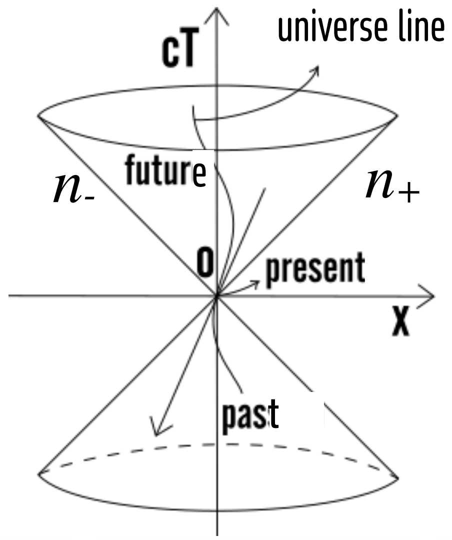
Light-cone variables

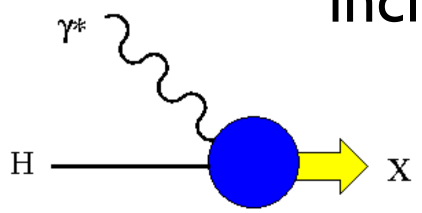
4-vector a^μ $\quad a^\mu = (a_0, a_1, a_2, a_3) = (a_+, a_-, \mathbf{a}_\perp)$

light-cone coordinates $\quad a_\pm = \dfrac{a_0 \pm a_3}{\sqrt{2}} \quad \mathbf{a}_\perp = (a_1, a_2)$

light-cone basis $\quad n_\pm^2 = 0 \qquad a_\pm = a \cdot n_\mp$

$n_+ \cdot n_- = 1$





inclusive DIS: target H absorbs momentum from γ^*

initially, target at rest: $P^\mu = 0$

if $\mathbf{q} \parallel \hat{z}$, $P_z = 0 \;\rightarrow\; P'_z = |\mathbf{q}| \gg M$

$$P = (M,0,0,0)$$

$$\downarrow \quad q = (\nu, 0, 0, |\mathbf{q}|)$$

$$P' = (\sqrt{M^2 + P_z'^2}, 0, 0, P'_z)$$

$$P' \approx (|\mathbf{q}|, 0, 0, |\mathbf{q}|)$$

DIS regime → component "+" dominant
component "-" suppressed

$$P' \approx (P'_+ = \sqrt{2}\,|\mathbf{q}|, P'_- \approx 0, \mathbf{0}_\perp)$$

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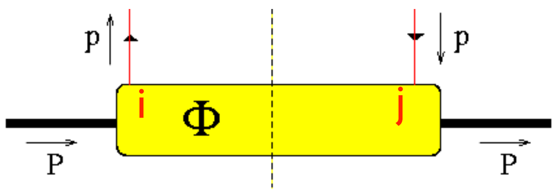

By taking suitable projections, one can extract from the "structure" the leading-twist part, the subleading part at twist 3, at twist 4, etc..

DIS regime $\rightarrow P^+ = \frac{1}{\sqrt{2}}(P^0 + P^3)$ dominant component; take partons collinear, $p^+ = xP^+$, and integrate other components. Let's define

$$\Phi_f^{[\Gamma]}(x) = \frac{1}{P^+} \int \frac{dp^-\, d\vec{p}}{(2\pi)^4}\, \mathrm{Tr}\left[\Big(\Phi_f(p,P) \Big)_{ji} (\Gamma)_{ij} \right] \Bigg|_{p^+ = xP^+}$$

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$$\underset{i\,j}{\Phi_f(p,P)} = \int d^4P_X\, \delta(P-p-P_x)\, \langle P|\bar\psi_f(0)|P_X\rangle\, \langle P_X|\psi_f(0)|P\rangle$$
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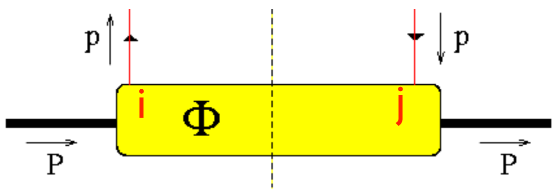


By taking suitable projections, one can extract from the "structure" the leading-twist part, the subleading part at twist 3, at twist 4, etc..

DIS regime $\rightarrow P^+ = \frac{1}{\sqrt{2}}(P^0+P^3)$ dominant component; take partons collinear, $p^+ = xP^+$, and integrate other components. Let's define

$$\Phi_f^{[\Gamma]}(x) = \frac{1}{P^+}\int \frac{dp^-\, d\vec{p}}{(2\pi)^4}\, \mathrm{Tr}\left[\left(\Phi_f(p,P)\right)_{ji}\,(\Gamma)_{ij}\right]\Big|_{p^+=xP^+}$$

project with $\Gamma \equiv \gamma^+ = \frac{1}{\sqrt{2}}(\gamma^0+\gamma^3)$: leading twist $\Phi_q^{[\gamma^+]} = q(x)$ unpolarized PDF

similarly for gluons

$\Gamma \equiv \mathbf{I}$ subleading twist 3 $\Phi_q^{[I]} = \frac{M}{P^+}\, e(x)$ connected to N scalar form factor

… … …

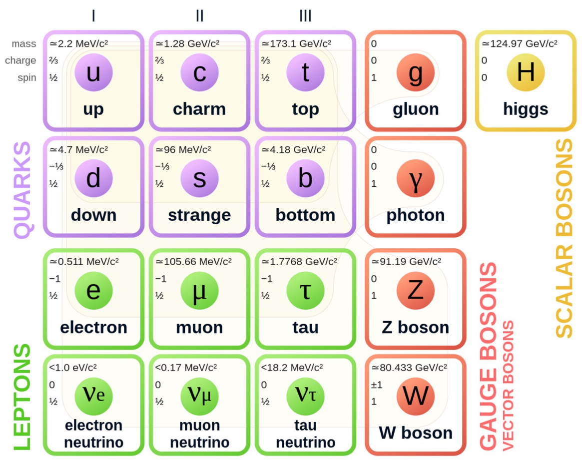
similarly for polarized PDFs

Outline

❸ open problems

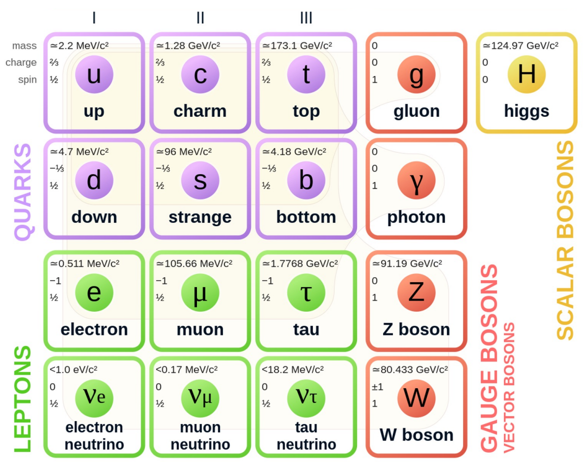
The Nucleon mass

origin of Nucleon mass



$$\mathscr{L}_{\mathrm{QCD}} = \bar{\psi}(x)\,\left[i\,\gamma^\mu\,D_\mu - m\right]\,\psi(x) - \frac{1}{4}\,\left(F^a_{\mu\nu}\right)^2$$

Standard Model: gluons in $F_{\mu\nu}$ are massless; quark mass m comes from interaction with Higgs field; no other mass scale



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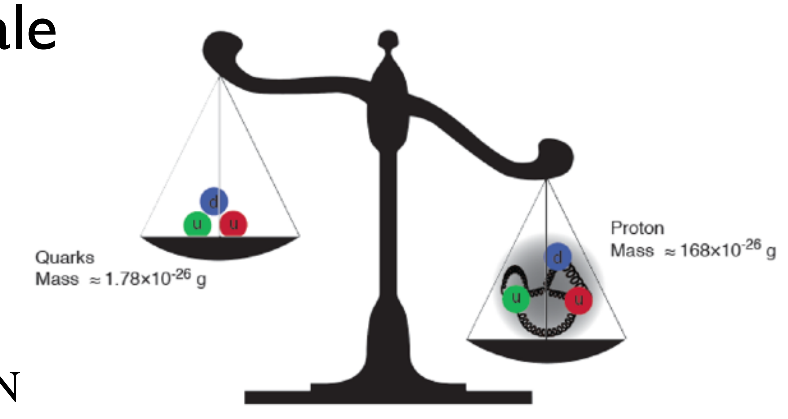


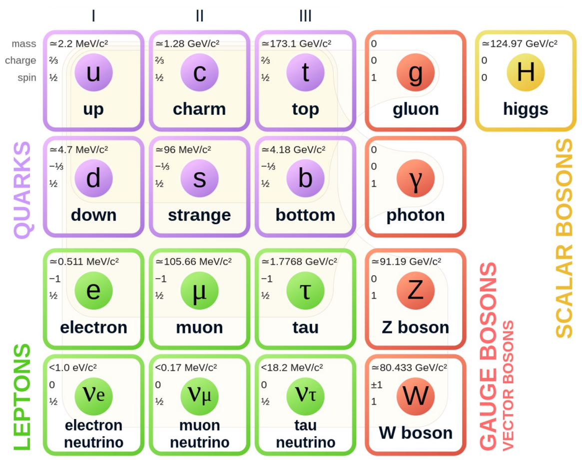
Nucleon = ensemble of quarks + gluons

$m_{\rm N} \sim 1$ GeV gluons massless

$\sim 168\ 10^{-26}$ g $m_{\rm q} \sim 10$ MeV $\sim 1.78\ 10^{-26}$ g $\sim 1\%\ m_{\rm N}$

Where does the N mass come from ?

origin of Nucleon mass



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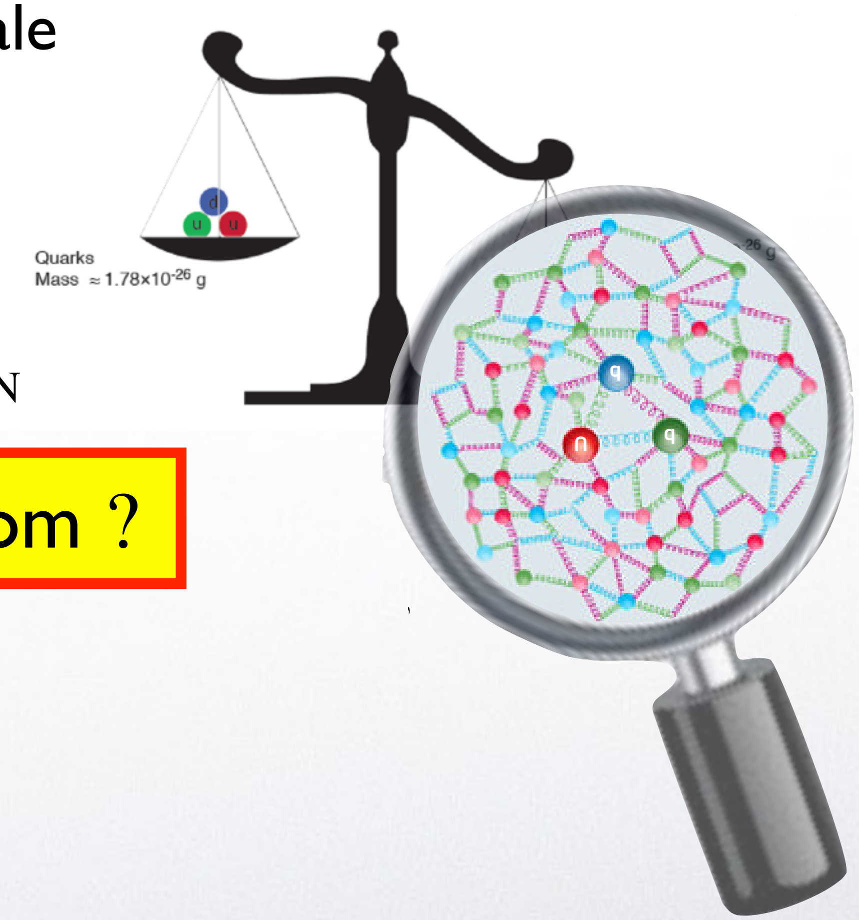

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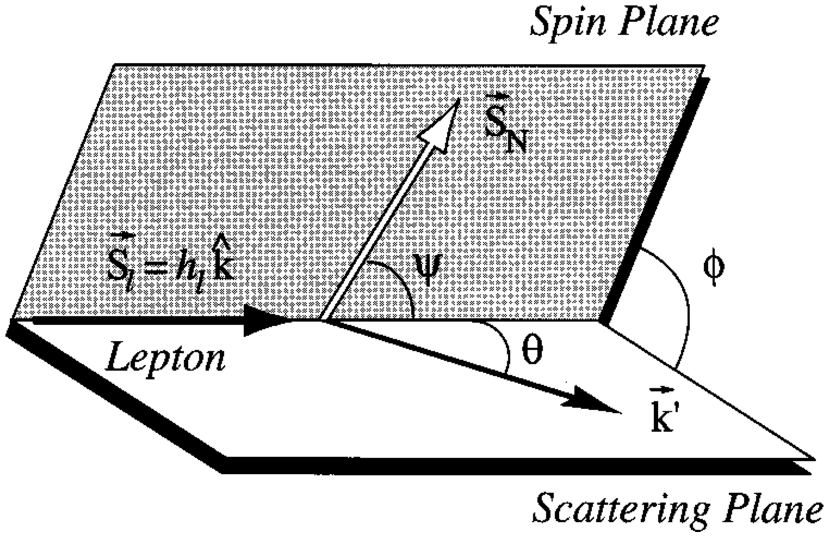
Where does the N mass come from ?

99% of N mass comes from the energy of quark-gluon interactions that bind them inside the N

We don’t understand this highly nonlinear dynamics

The Nucleon spin

origin of Nucleon spin



switching on polarization: lepton helicity $|\vec{S}_\ell| = h_\ell = \pm 1$

N spin such that $P \cdot S_N = 0$ and normalized.
Since $P^2 = M^2$ time-like $\Rightarrow$ space-like $S_N^2 = -\vec{S}_N^2 = -1$

$$S_N^{\parallel} = |\vec{S}_N| \cos\psi = \cos\psi$$

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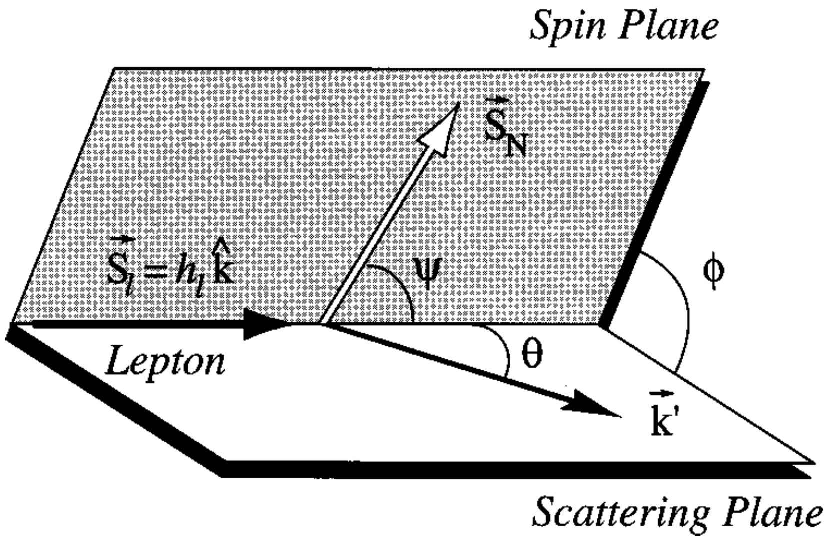

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$$\tilde{\gamma} = \frac{\sqrt{Q^2}}{\nu} \xrightarrow{\text{DIS}} 0$$

inclusive DIS: unpolarized + polarized parallel (ψ=0) or transverse (ψ=π/2)

$$\frac{d\sigma}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4} \left[A(y)\, F_2(x,Q^2) - y^2\, F_L(x,Q^2)\right]$$

$$F_L = F_2 - 2x\, F_1$$

$$\frac{d\Delta\sigma_{\parallel}}{dx\, dQ^2} = h_\ell \frac{8\pi\alpha^2 y}{Q^4} \left(1 - \frac{y}{2}\right) \underline{g_1(x,Q^2)}$$

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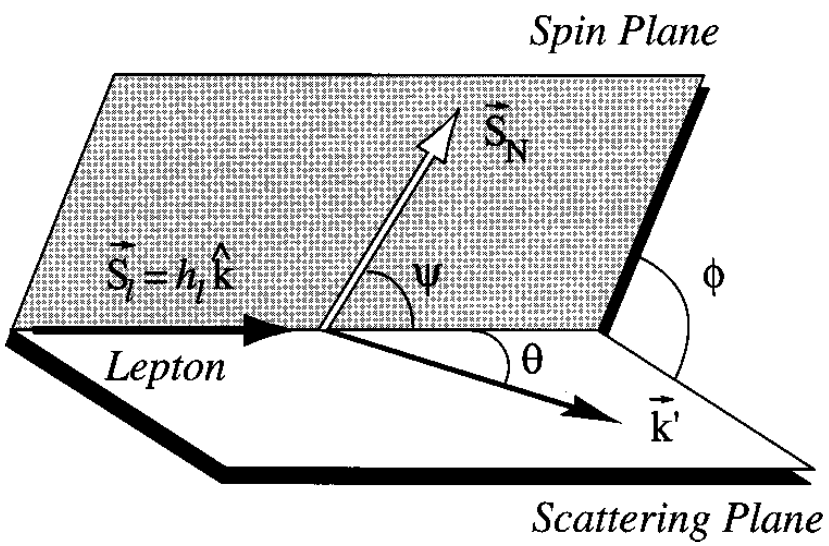

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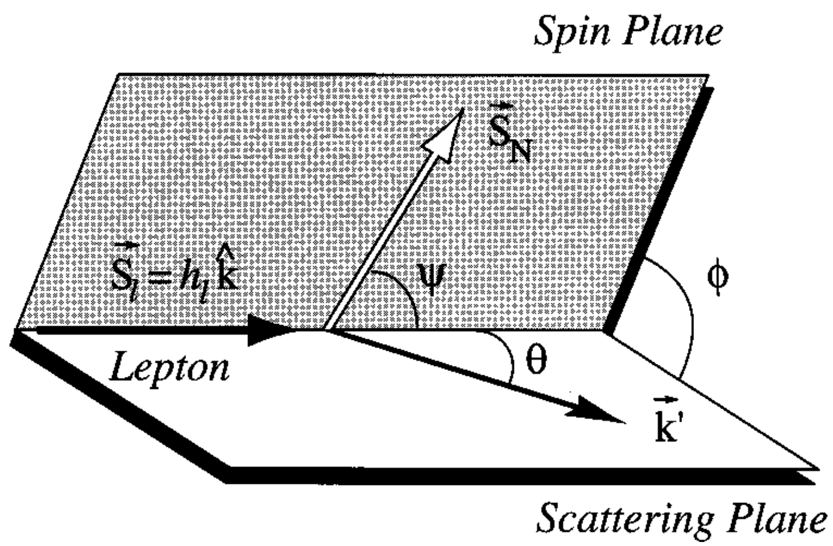
γ^* - N cross section: $\sigma^{\lambda_{\gamma^*}}_{J_z}$ ← γ^* polarization ← γ^*-N total J

γ^* - N asymmetries:
$$\begin{cases} A_1 = \dfrac{\sigma^T_{1/2} - \sigma^T_{3/2}}{\sigma^T_{1/2} + \sigma^T_{3/2}} \approx \dfrac{g_1}{F_1} \\[2ex] A_2 = \dfrac{2\,\sigma^{LT}}{\sigma^T_{1/2} + \sigma^T_{3/2}} = \tilde{\gamma}\, \dfrac{g_1 + g_2}{F_1} \ll A_1 \end{cases}$$

origin of Nucleon spin



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helicity asymmetries:
$$\begin{cases} A_\parallel = \dfrac{d\Delta\sigma_\parallel}{2\, d\sigma} \\[2ex] A_\perp = \dfrac{d\Delta\sigma_\perp}{2\, d\sigma} \end{cases}$$
combinations of A_1 and A_2 : $\boxed{\text{by measuring } A_\parallel, A_\perp, \text{ get } g_1(x,Q^2)}$

the "Spin crisis"

Define 1^{st} Mellin moment: $\Gamma_1(Q^2) = \int_0^1 dx\, g_1(x, Q^2)$

in parton model
$$= \frac{1}{2} \sum_f e_f^2 \int_0^1 dx \left[\phi_f^{\uparrow}(x) - \phi_f^{\downarrow}(x)\right] \equiv \frac{1}{2} \sum_f e_f^2 \, \Delta f \longleftarrow$$

% of N spin carried by flavor f

$$= \frac{1}{2} \left[\frac{4}{9}\Delta u + \frac{1}{9}\Delta d + \frac{1}{9}\Delta s\right]$$

$$= \frac{1}{12}\left(\Delta u - \Delta d\right) + \frac{1}{36}\left(\Delta u + \Delta d - 2\,\Delta s\right) + \frac{1}{9}\left(\Delta u + \Delta d + \Delta s\right)$$

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using info from weak decays of baryon octet (Cabibbo theory).

"reduced Wigner-Eckart theorem": all matrix elements depend on two parameters F & D, fitted to data.

$F = 0.42 \pm 0.004\,,\, D = 0.73 \pm 0.003$

Assuming perfect $\mathrm{SU}(3)_f$ symmetry and $\Delta s=0$

isovector A_3 from β-decay

octet A_8 from hyperon decay

singlet $A_0 = \Delta\Sigma$

$A_3 = F+D$

$A_8 = 3F\text{-}D$

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Assuming perfect SU(3)$_f$ symmetry and $\Delta s=0$

Theory: Ellis-Jaffe sum rule $\quad \Gamma_1 = \frac{1}{12} A_3 + \frac{1}{36} A_8 + \frac{1}{9} A_0 = \frac{1}{2} \left(F - \frac{1}{9} D\right) = 0.170 \pm 0.004$

$\Delta\Sigma = A_0 = 0.60 \pm 0.12$

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Ashmann et al. (EMC), P.L. ***B206*** *(88) 364*

the “Spin crisis”

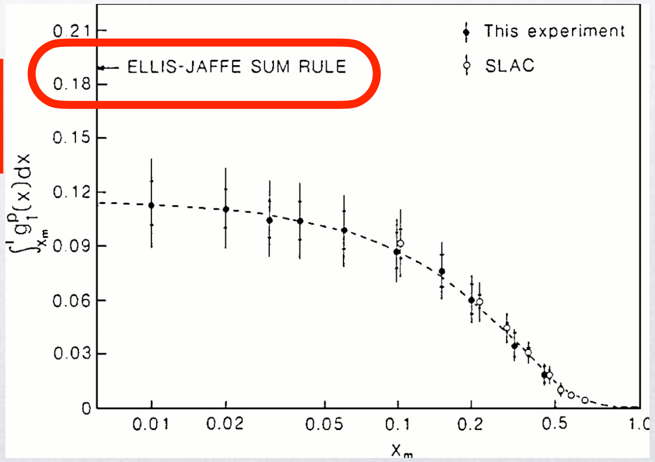
Experiment: EMC (’88)

DIS $\vec{\mu}\,\vec{p} \to \mu p$

at Q^2=10.7 GeV^2

$\Gamma_1(Q^2) = \int_{x_{\min}}^1 dx\, g_1(x,Q^2) = 0.126 \pm 0.010 \pm 0.015$

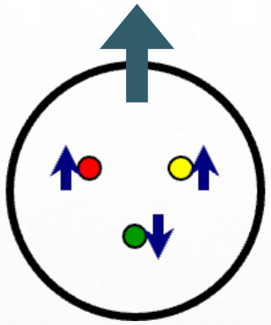
$$\Delta\Sigma = 0.12 \pm 0.17$$



origin of Nucleon spin

Parton model $\Delta\Sigma = 1$ (quarks give the N spin)

$$\frac{1}{2} = \frac{1}{2}\,\Delta\Sigma$$



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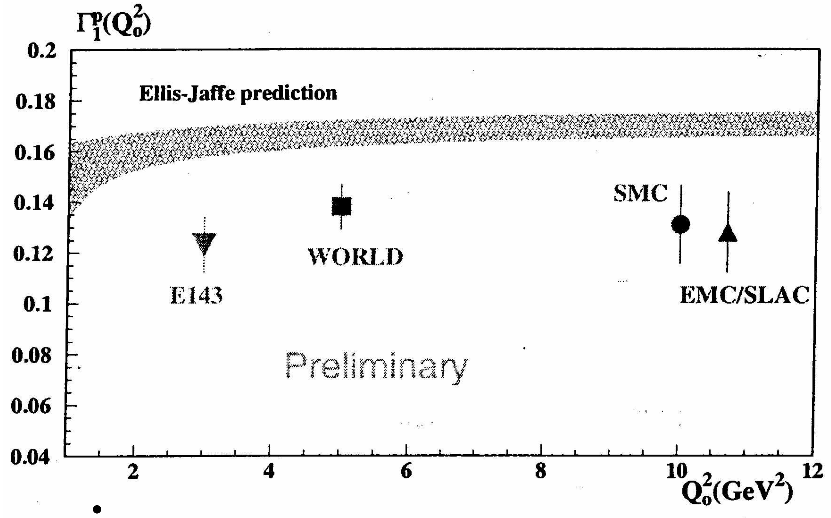
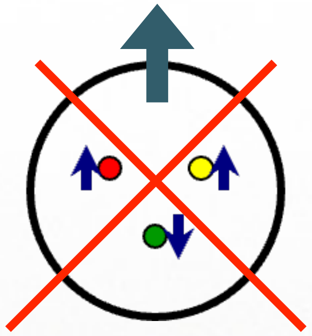
result confirmed by other exp's



Therefore, quarks carry only a small fraction of N spin

$$\frac{1}{2} \neq \frac{1}{2}\Delta\Sigma$$



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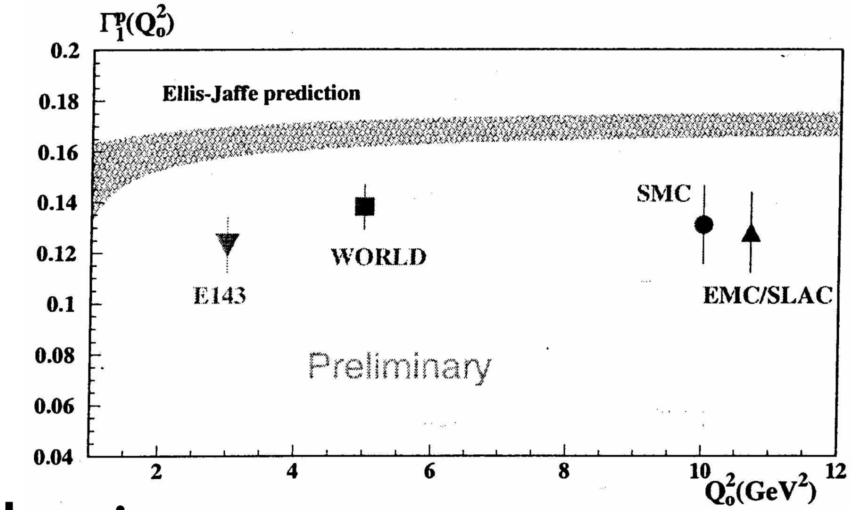

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At RHIC (BNL), measured spin asymmetries

$\vec{p}\,\vec{p} \to \mathrm{jet(s)} + X$ STAR

P.R.L. **115** ('15)
P.R. D**95** ('17); D**98** ('18)

$\vec{p}\,\vec{p} \to \pi + X$ PHENIX

P.R. D**90** ('14)

access to gluon helicity $\Delta g(Q^2) = \int_0^1 dx\, \Delta g(x, Q^2)$ at tree level

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g \ ?$$

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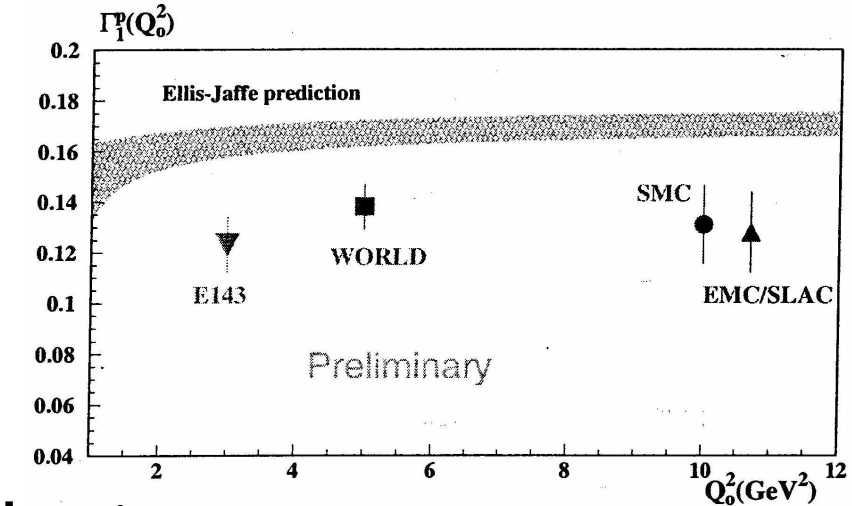

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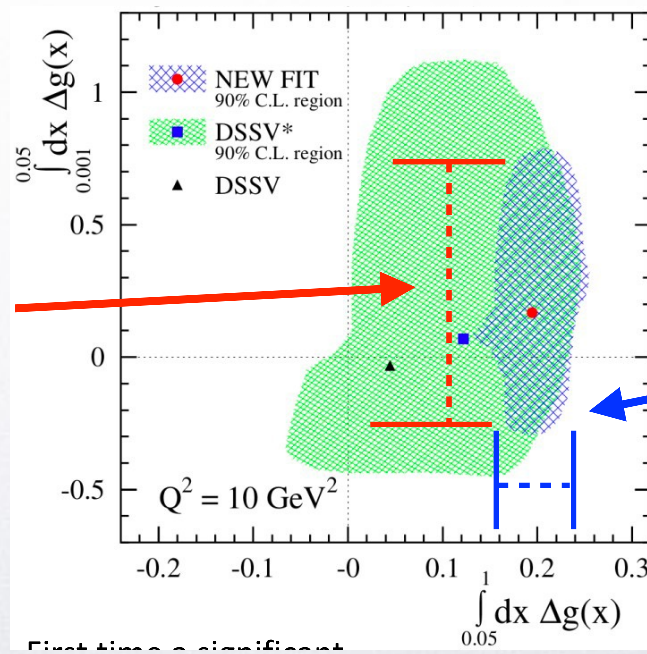
but large uncertainties at x < 0.05



$$\int_{0.05}^{1} dx\, \Delta g(x, Q^2 = 10) = 0.218 \pm 0.027$$

first clear evidence of $\Delta g > 0$ for $x > 0.05$ at $Q^2 = 10$ GeV^2

DSSV14 fit P.R.L. **113** ('14); arXiv:1902.10548

NNPDFpol1.1 fit N.P. B**887** ('14); arXiv:1702.05077

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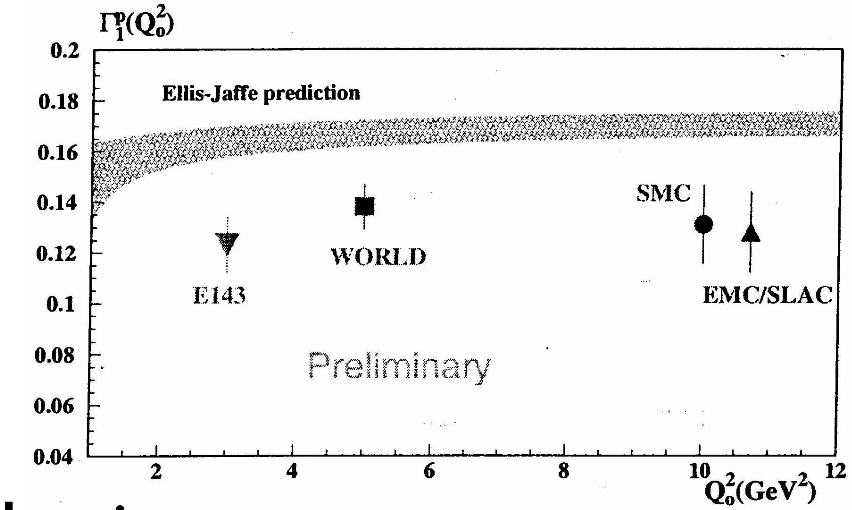

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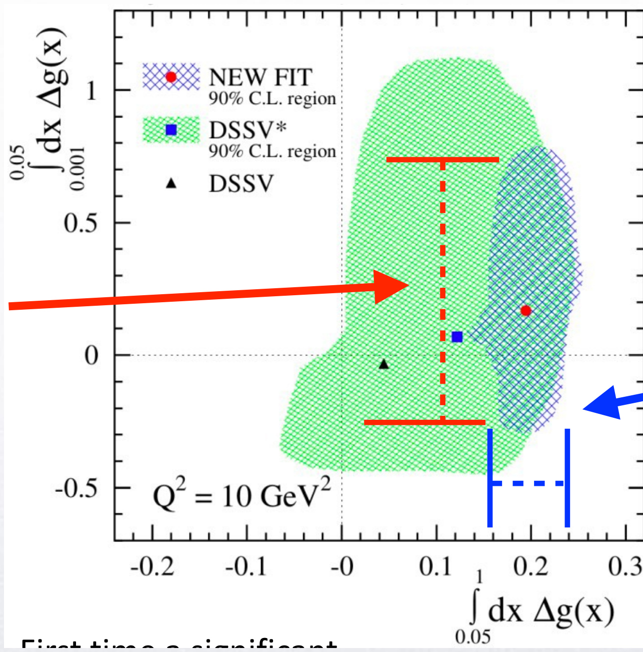
P.R. D**90** ('14)

$$\boxed{\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta g + L_q + L_g \;?}$$

orbital motion ?

but large uncertainties at x < 0.05



$$\int_{0.05}^{1} dx\, \Delta g(x, Q^2 = 10)$$
$$= 0.218 \pm 0.027$$

first clear evidence of $\Delta g > 0$ for x > 0.05 at Q^2 = 10 GeV^2

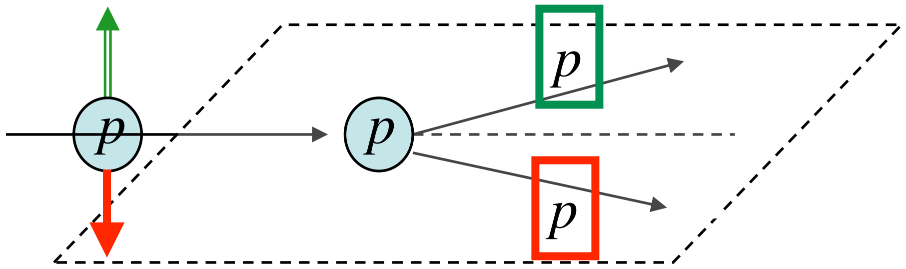
DSSV14 fit P.R.L. **113** ('14); arXiv:1902.10548

NNPDFpol1.1 fit N.P. B**887** ('14); arXiv:1702.05077

Outline

Going beyond collinear

Example #2: elastic p-p scattering



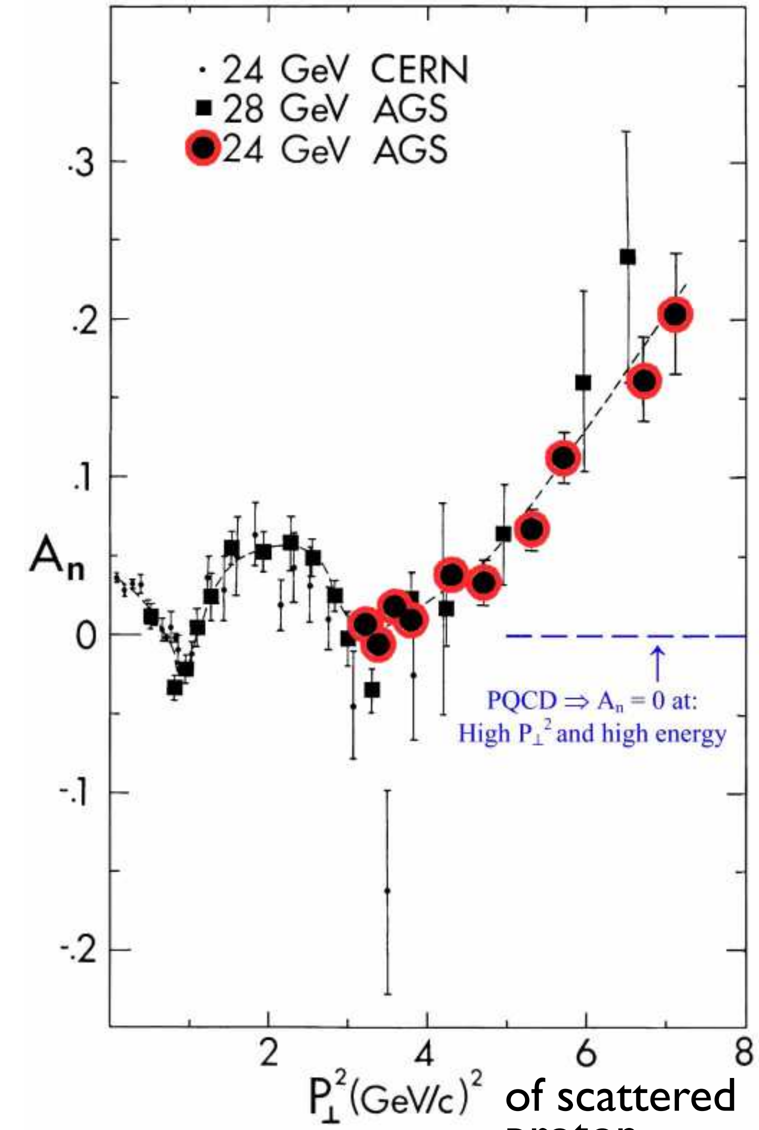
$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$

$p^{\uparrow} p \to p\, p$ versus $p^{\downarrow} p \to p\, p$

correlation between spin of the proton and k_T of partons

$\leftrightarrow$ orbital motion

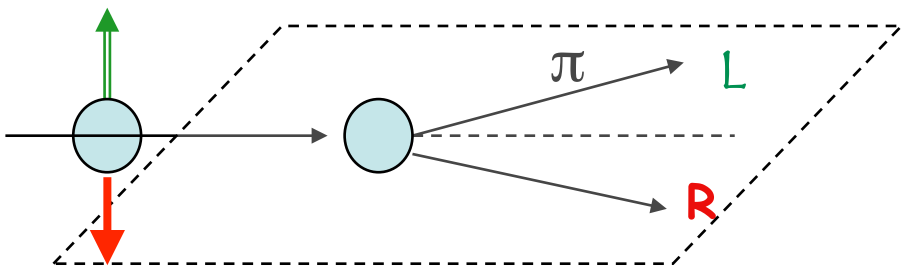
for a review, see
Krisch, E.P.J. **A31** (07) 417



of scattered proton

Example #3: semi-inclusive p-p collisions

$$p^{\uparrow}\, p \to \pi\, X$$



$$A_N = \frac{d\sigma^{\uparrow} - d\sigma^{\downarrow}}{d\sigma^{\uparrow} + d\sigma^{\downarrow}}$$ single-spin asymmetry

perturbative QCD $\propto \frac{m_q}{p_T}\, \alpha_s \sim \mathcal{O}(10^{-3})$

Kane, Pumplin, Repko, P.R.L. **41** ('78) 1689

Example #3: semi-inclusive p-p collisions

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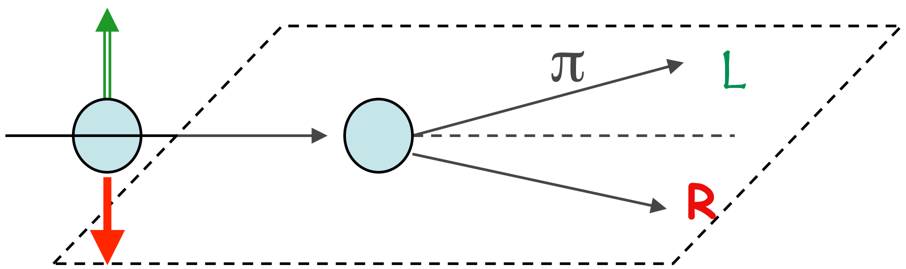

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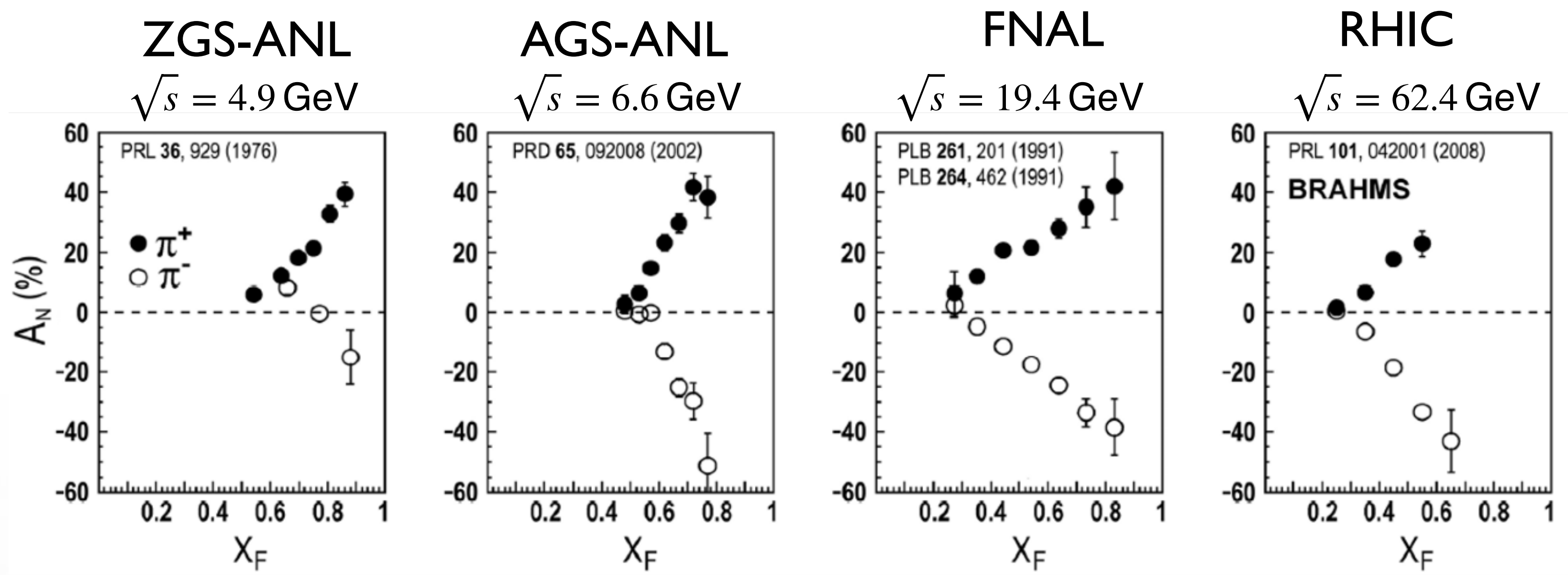
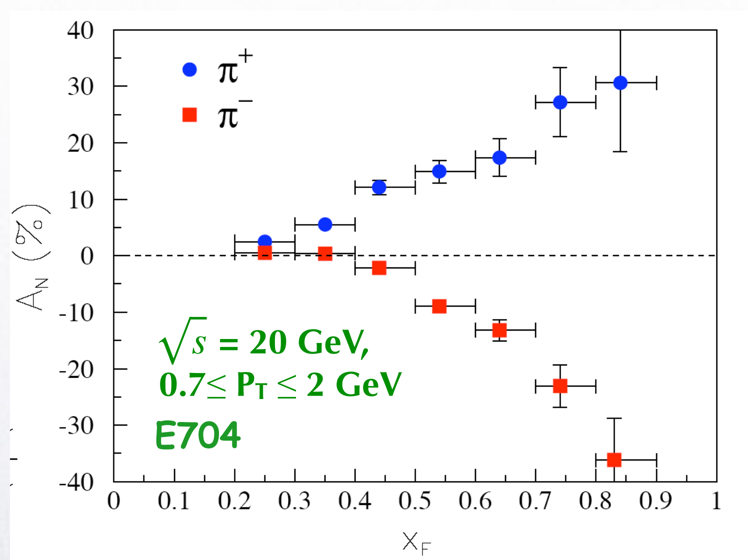
perturbative QCD $\propto \frac{m_q}{p_T}\, \alpha_s \sim \mathcal{O}(10^{-3})$

Kane, Pumplin, Repko, P.R.L. **41** ('78) 1689

Instead, large asymmetries observed.
Evidence of correlation between spin of the proton and k_T and flavor of partons





Persisting also up to $\sqrt{s} = 200$ GeV

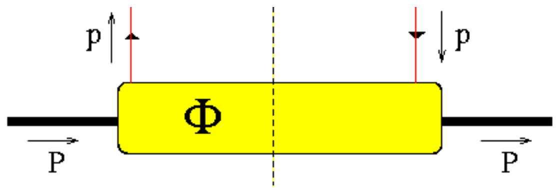
STAR

Adams *et al.* (STAR), PRL **92** (04) 171801

Chiral-odd structures

chiral-odd structures

quark-quark correlator



$$= \int \frac{d^4\xi}{(2\pi)^4}\, e^{-i p\cdot\xi}\, \langle P\,|\,\bar{\psi}_f(\xi)\,\psi_f(0)\,|\,P\rangle$$

leading-twist projections

$$\Phi_f^{[\Gamma]}(x) = \frac{1}{P^+}\int \frac{dp^-\, d\vec{p}}{(2\pi)^4}\,\mathrm{Tr}\left[\left(\Phi_f(p,P)\right)_{ji}\,(\Gamma)_{ij}\right]\Big|_{p^+=xP^+}$$

spin-1/2 hadron (Nucleon)

$\Gamma = \gamma^+ \rightarrow$

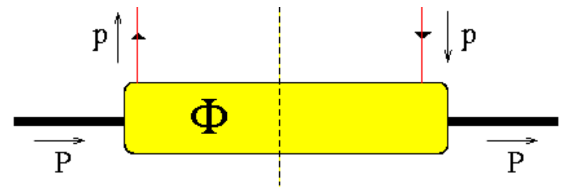
quark polarization

N polarization	U •	L →	T ↑
U ○	$\mathbf{f_1}$		$h_1^\perp$
L →		g_{1L}	$h_{1L}^\perp$
T ↑	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

unpolarized PDF

chiral-odd structures

quark-quark correlator



$$= \int \frac{d^4\xi}{(2\pi)^4} \, e^{-ip\cdot\xi} \, \langle P \,|\, \bar{\psi}_f(\xi) \, \psi_f(0) \,|\, P \rangle$$

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spin-1/2 hadron (Nucleon)

quark polarization

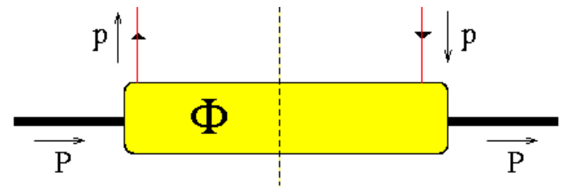
N polarization

	U •	L →	T ↑	
U	f_1		$h_1^{\perp}$	unpolarized PDF
L →		g_{1L}	$h_{1L}^{\perp}$	helicity PDF
T ↑	$f_{1T}^{\perp}$	g_{1T}	h_1 $h_{1T}^{\perp}$	transversity PDF

$\Gamma = \gamma^+ \to$ unpolarized PDF

$\Gamma = \gamma^+ \gamma_5 \to$ helicity PDF

$\Gamma = i\,\sigma^{i+} \gamma_5 \to$ transversity PDF quark-quark correlator



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spin-1/2 hadron (Nucleon)

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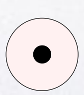
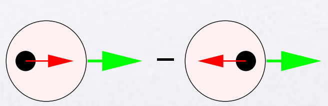
N polarization	U •	L →	T ↑
$\Gamma = \gamma^+ \rightarrow$ U	f_1		$h_1^\perp$
$\Gamma = \gamma^+\gamma_5 \rightarrow$ L →		g_{1L}	$h_{1L}^\perp$
$\Gamma = i\,\sigma^{i+}\gamma_5 \rightarrow$ T ↑	$f_{1T}^\perp$	g_{1T}	h_1 $h_{1T}^\perp$

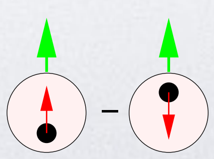
unpolarized PDF

helicity PDF

transversity PDF

$\Gamma = \gamma^+ \rightarrow \bar{\psi}\, \gamma^+ \psi \rightarrow R^\dagger R + L^\dagger L$ sum of right-handed / left-handed quark densities $f_1 =$ 

$\Gamma = \gamma^+ \gamma_5 \rightarrow \bar{\psi}\, \gamma^+ \gamma_5 \psi \rightarrow R^\dagger R - L^\dagger L$ difference " " " $g_1 =$ 

$\Gamma = i\,\sigma^{i+}\gamma_5 \rightarrow \bar{\psi}\, i\,\sigma^{i+}\gamma_5\, \psi \rightarrow L^\dagger \gamma_i R - R^\dagger \gamma_i L$ not diagonal on helicity basis $h_1 =$ 

"chiral-odd" structure

chiral-odd structures

define $\gamma^{\pm} = \frac{1}{\sqrt{2}}\left(\gamma^0 \pm \gamma^3\right)$ $\qquad \mathscr{P}^{\pm} = \frac{1}{2}\gamma^{\mp}\gamma^{\pm}$

projector: $\left(\mathscr{P}^{\pm}\right)^2 = \mathscr{P}^{\pm}$ $\qquad \left[\mathscr{P}^{+}, \mathscr{P}^{-}\right] = 0$

$\mathscr{P}^{+} + \mathscr{P}^{-} = I$

given Dirac vector $|\psi\rangle = \left|\begin{matrix}\phi \\ \chi\end{matrix}\right|$

"good" light-cone component (dominant)

"bad" component

$\mathscr{P}^{+}\,|\psi\rangle = \phi \qquad \mathscr{P}^{-}\,|\psi\rangle = \chi$

define helicity projectors: $\mathscr{P}_{R,L} = \frac{1 \pm \gamma_5}{2}$ $\qquad \left[\mathscr{P}_{R,L}, \mathscr{P}^{\pm}\right] = 0$

show that

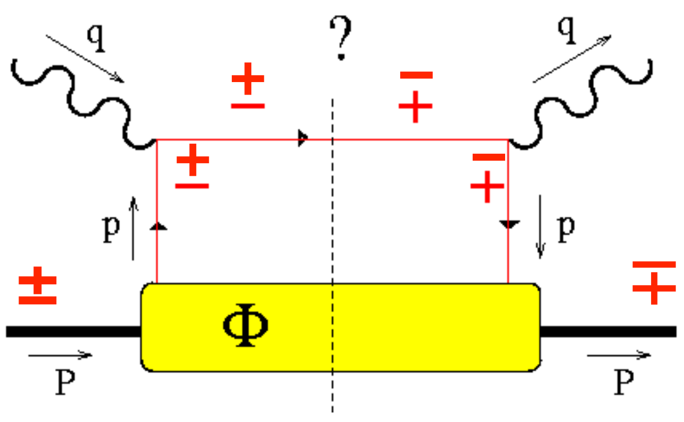
$\bar{\psi}\,\gamma^{+}\psi \to R^{\dagger}R + L^{\dagger}L$ with $R, L \equiv \mathscr{P}_{R,L}\,\phi$

$\bar{\psi}\,\gamma^{+}\gamma_5\,\psi \to R^{\dagger}R - L^{\dagger}L$

$\bar{\psi}\,i\,\sigma^{i+}\gamma_5\,\psi \to L^{\dagger}\gamma_i R - R^{\dagger}\gamma_i L$

QCD conserves helicity at leading order,
transversity does not

→ transversity suppressed in inclusive DIS
it has no corresponding structure function
PDF $f_1 \leftrightarrow$ structure function F_1
PDF $g_1 \leftrightarrow$ structure function g_1
PDF $h_1 \leftrightarrow$?



Need a process with another chiral-odd partner

chiral-odd structures

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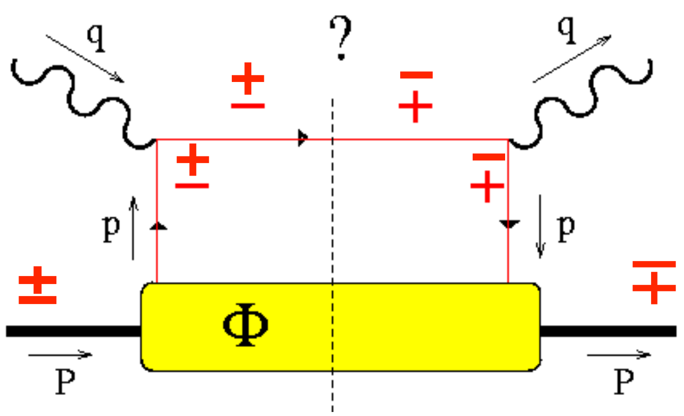
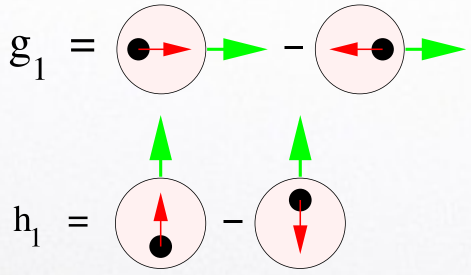


Need a process with another chiral-odd partner

Nevertheless, it's a leading-twist PDF with very interesting properties:



in a nonrelativistic theory, $g_1 = h_1$ because would differ just by a rotation
$\rightarrow$ transversity contains info on relativistic motion of quarks

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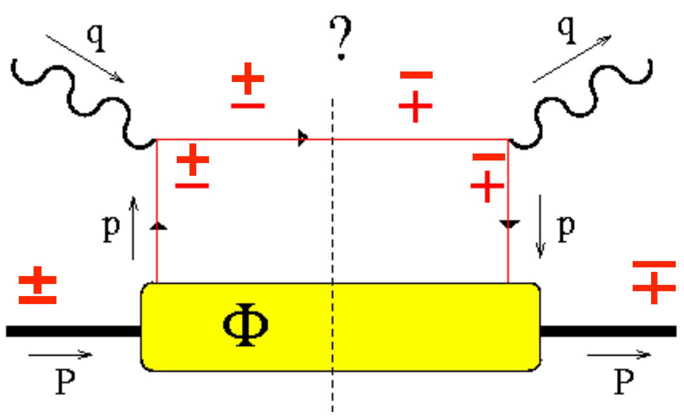

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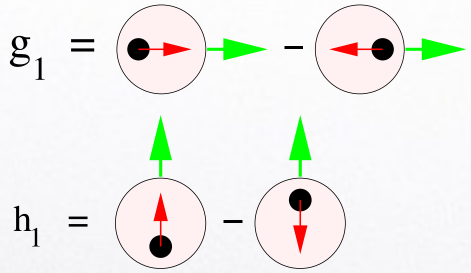

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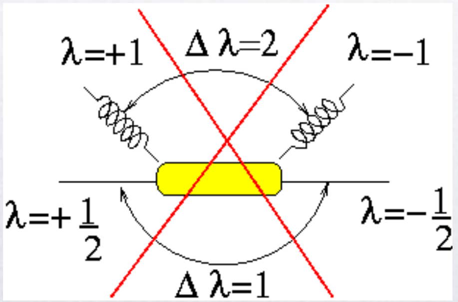


in a spin-$1/2$ hadron, there is no gluon transversity
→ evolution of transversity of quarks is decoupled from gluons
very different from helicity

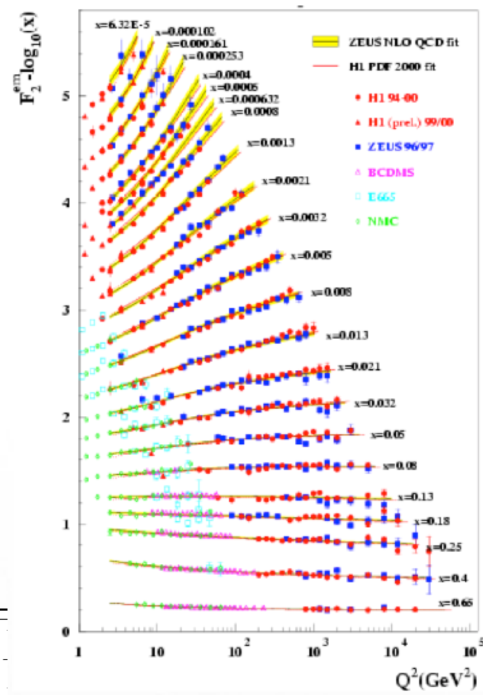
chiral-odd structures

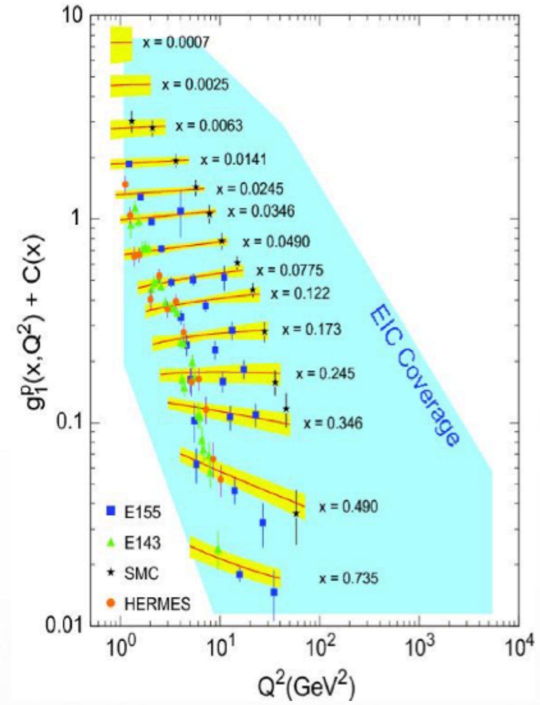
world data for proton F_2 proton g_1 h_1

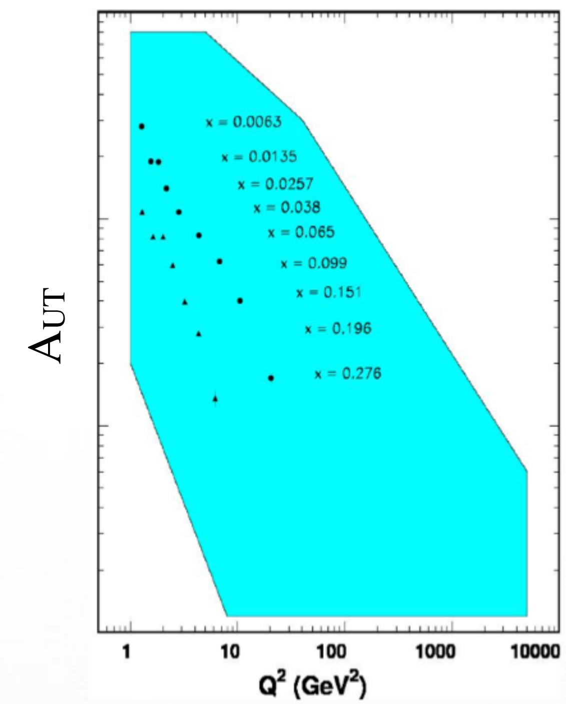
thousands of data

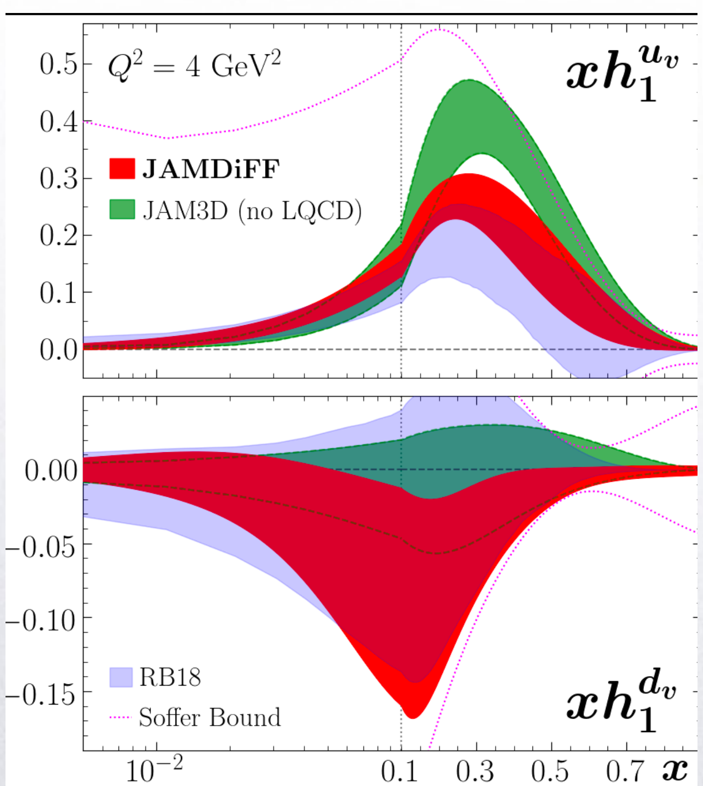
hundreds of data

tens of data









some phenomenological extractions using different mechanisms

JAM3D → Collins effect

Gamberg et al., P.R.D **106** ('22) 034014

JAMDiFF, RB18
→ di-hadron production

Cocuzza et al., arXiv:2308.14857

Radici & Bacchetta, P.R.L. **120** ('18) 192001

- agreement on valence up
- large uncertainty on valence down

1^{st} Mellin moment of transversity: the tensor "charge"

(not really a charge, since it scales → no associated conserved current in $\mathscr{L}_{\mathrm{QCD}}$)

$$\delta q(Q^2) = \int_0^1 dx \, \left[h_1^q(x, Q^2) - h_1^{\bar{q}}(x, Q^2) \right]$$

1^{st} Mellin moment of transversity: the tensor "charge"

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$$\langle PS|\,\bar{q}\,\sigma^{\mu\nu}\,q\,|PS\rangle = (P^\mu S^\nu - P^\nu S^\mu)\,\delta q(Q^2) = \int_0^1 dx\,\left[h_1^q(x,Q^2) - h_1^{\bar{q}}(x,Q^2)\right]\,(P^\mu S^\nu - P^\nu S^\mu)$$

matrix element of tensor operator

- calculable on lattice with high precision

the tensor charge

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$$\langle PS \,|\, \bar q\, \sigma^{\mu\nu}\, q \,|\, PS \rangle$$

extraction from data in range $[x_{\mathrm{min}}, x_{\mathrm{max}}]$, problems with extrapolation outside it

matrix element of tensor operator

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tension ? discussion in progress…

the tensor charge

$1^{\rm st}$ Mellin moment of transversity: the tensor "charge"

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$$(P^\mu S^\nu - P^\nu S^\mu)\, \delta q(Q^2) = \int_0^1 dx \left[h_1^q(x,Q^2) - h_1^{\bar q}(x,Q^2) \right] (P^\mu S^\nu - P^\nu S^\mu)$$

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matrix element of tensor operator

extraction from data in range $[x_{\rm min}, x_{\rm max}]$, problems with extrapolation outside it

- calculable on lattice with high precision

tension ? discussion in progress…

$\boxed{\text{no tensor operators in } \mathscr{L}_{\rm SM} \text{ at tree level !}}$

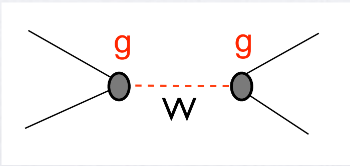
Is transversity a low-energy footprint of new physics (BSM) at higher scale ?

direct access to BSM at $M_{\rm BSM}$

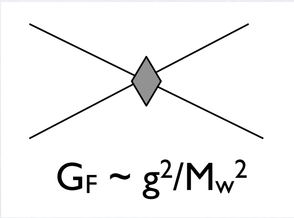
footprint of BSM operators at low energy

energy

Example: weak CC interaction

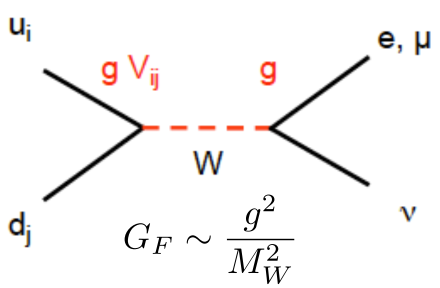


$\xrightarrow{Q^2 \ll M_W^2}$



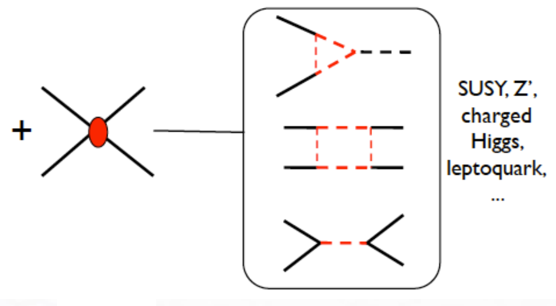
Fermi contact term

$\boxed{1.\ n\ \beta\text{-decay:}\ n \to p\, e^-\, \bar{\nu}_e}$



tree-level SM Lagrangian

$$\mathscr{L}_{\rm SM} \sim G_F\, V_{ud}\, \bar{e}\, \gamma_\mu\, (1-\gamma_5)\, \nu_e\, \langle p\,|\,\bar{u}\,\gamma^\mu\,(1-\gamma_5)\, d\,|\,n\rangle$$



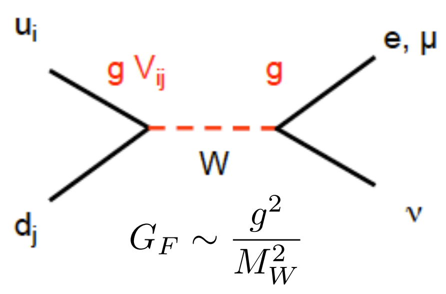
effective BSM Lagrangian
with tensor coupling

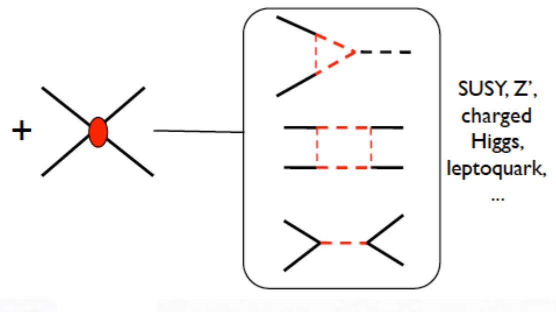
$$+\mathscr{L}_{\rm eff} \sim G_F\, V_{ud}\, \varepsilon_T\, \bar{e}\, \sigma_{\mu\nu}\, \nu_e\, g_T\, \langle p\,|\,\bar{u}\,\sigma^{\mu\nu}\, d\,|\,n\rangle$$

BSM tensor coupling

the tensor charge

$1.\ n\ \beta$-decay: $n \to p\, e^-\, \bar{\nu}_e$





tree-level SM Lagrangian

$$\mathscr{L}_{\mathrm{SM}} \sim G_F\, V_{ud}\, \bar{e}\, \gamma_\mu\, (1-\gamma_5)\, \nu_e\, \langle p\,|\,\bar{u}\,\gamma^\mu\,(1-\gamma_5)\,d\,|\,n\rangle$$

isospin symmetry implies

$$\langle p, S_p\,|\,\bar{u}\,\sigma^{\mu\nu}\,u - \bar{d}\,\sigma^{\mu\nu}\,d\,|\,p, S_p\rangle$$

$$\propto\ \delta u(Q^2) - \delta d(Q^2) \equiv g_T(Q^2)$$

effective BSM Lagrangian
with tensor coupling

$$+\mathscr{L}_{\mathrm{eff}} \sim G_F\, V_{ud}\, \varepsilon_T\, \bar{e}\, \sigma_{\mu\nu}\, \nu_e\ g_T\, \langle p\,|\,\bar{u}\,\sigma^{\mu\nu}\,d\,|\,n\rangle$$

BSM tensor coupling

isovector tensor charge

$$\varepsilon_T\, g_T \approx \frac{M_W^2}{M_{\mathrm{BSM}}^2}$$

$\boxed{\text{precision on couplings} \to \text{bound on BSM scale}}$

(0.1% → [3-5] TeV)

2. CP violation (CPV):

effective BSM Lagrangian CPV couplings related to fermion Electric Dipole Moment (EDM)

$$\mathscr{L}_{\rm EDM} \sim e \sum_{f=u,d,s,c} \tilde{d}_f \, \bar{\psi}_f \, \sigma_{\mu\nu} \gamma_5 \psi_f \, F^{\mu\nu} \qquad F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu$$

quark EDM

quark tensor charge

Neutron EDM

$$\tilde{d}_N = \tilde{d}_u \, \delta u(Q^2) + \tilde{d}_d \, \delta d(Q^2) + \tilde{d}_s \, \delta s(Q^2) + \tilde{d}_c \, \delta c(Q^2)$$

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effective BSM Lagrangian CPV couplings related to fermion Electric Dipole Moment (EDM)

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quark EDM

quark tensor charge

Neutron EDM

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experimental bounds + improved knowledge on quark tensor charges $\rightarrow$ constraints on CP violation encoded in quark EDM

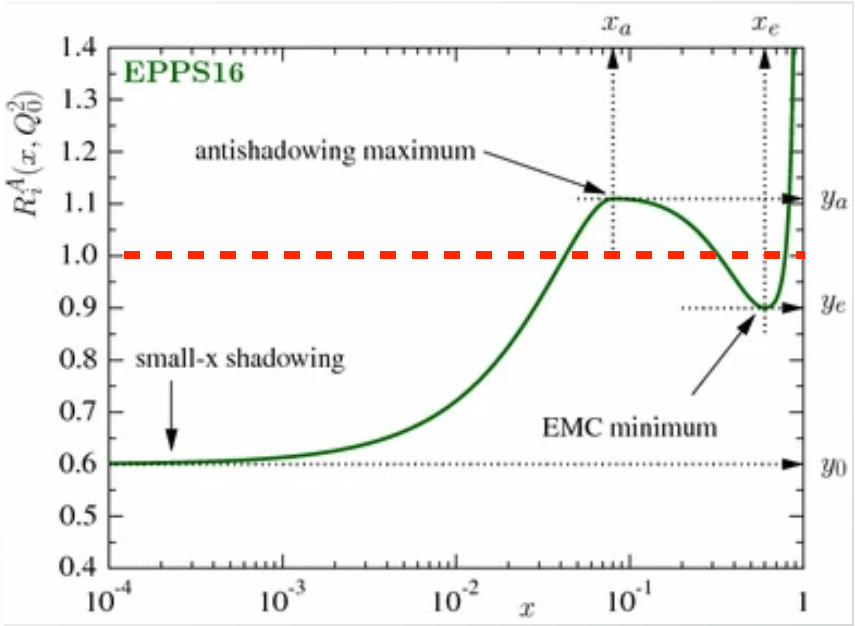
Nuclear matter effects

- nuclear PDFs are different from free-Nucleon PDFs :

$$\phi_A^f(x, Q^2) = \frac{Z}{A}\,\phi_p^f(x, Q^2) + \frac{N}{A}\,\phi_n^f(x, Q^2)$$

$$R_A^f(x; Q^2) = \frac{\phi_A^f(x; Q^2)}{\phi_p^f(x; Q^2)} \neq 1$$

Eskola et al., E.P.J. C77 (16) 163



the EMC effect

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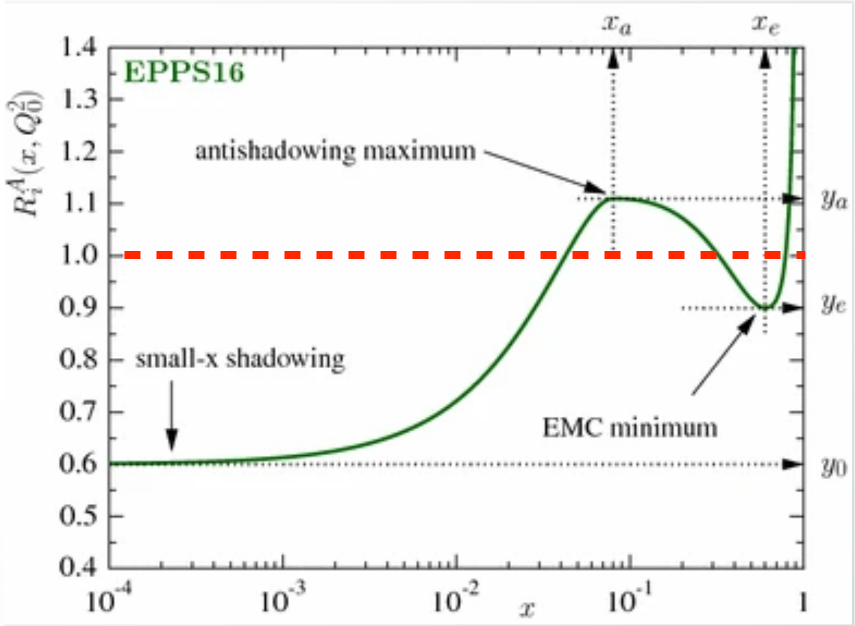
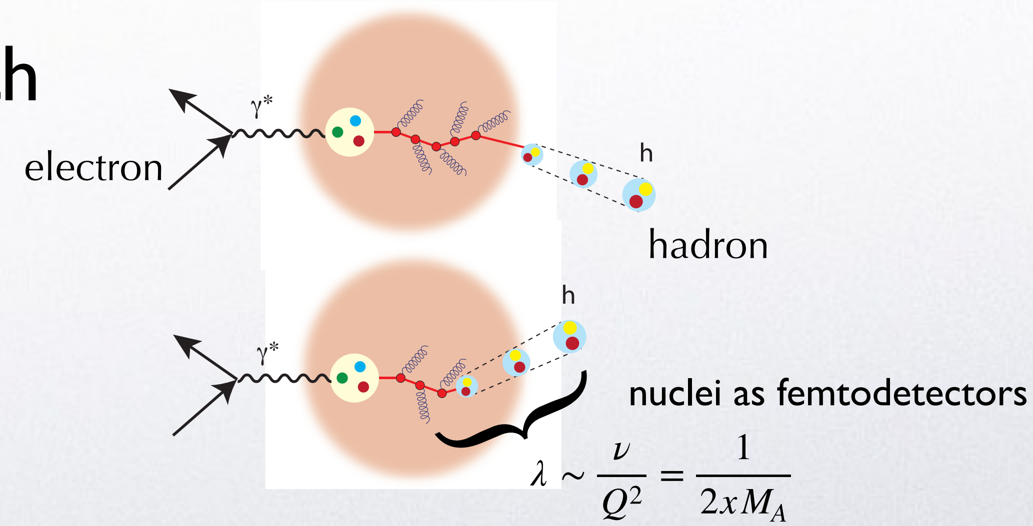


Eskola et al., E.P.J. C77 (16) 163

- how do colored quarks and gluons interact with nuclear medium and propagate through it?

- which mechanism creates nuclear binding from quark-gluon interactions?



two pillars of the EIC science case

nuclear PDF

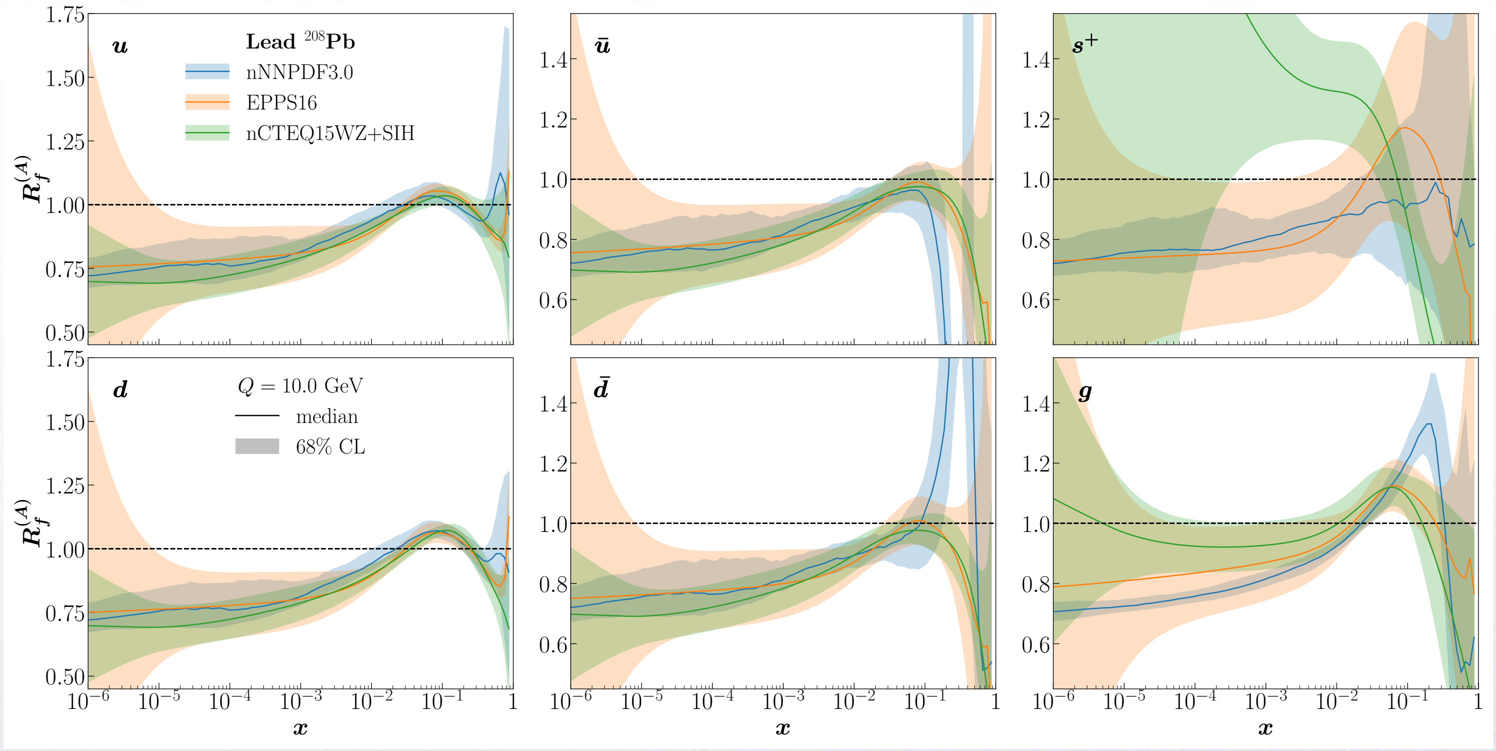
- inclusive DIS on nuclei A $$\frac{d\sigma^{eA\to e'+X}}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left[A(y)\,F_2^A(x,Q^2) - y^2\,F_L^A(x,Q^2)\right]$$

- fitting F_2^A, F_L^A on a large range in $x \to$ extract nuclear PDF (nPDF)

- **nNNPDF3.0** *Abdul Khalek et al., arXiv:2201.12363*
2151 pts (NC&CC DIS + DY + LHC + di-jet + prompt γ & D^0)
$\chi^2/N_{pts} = 1.09$

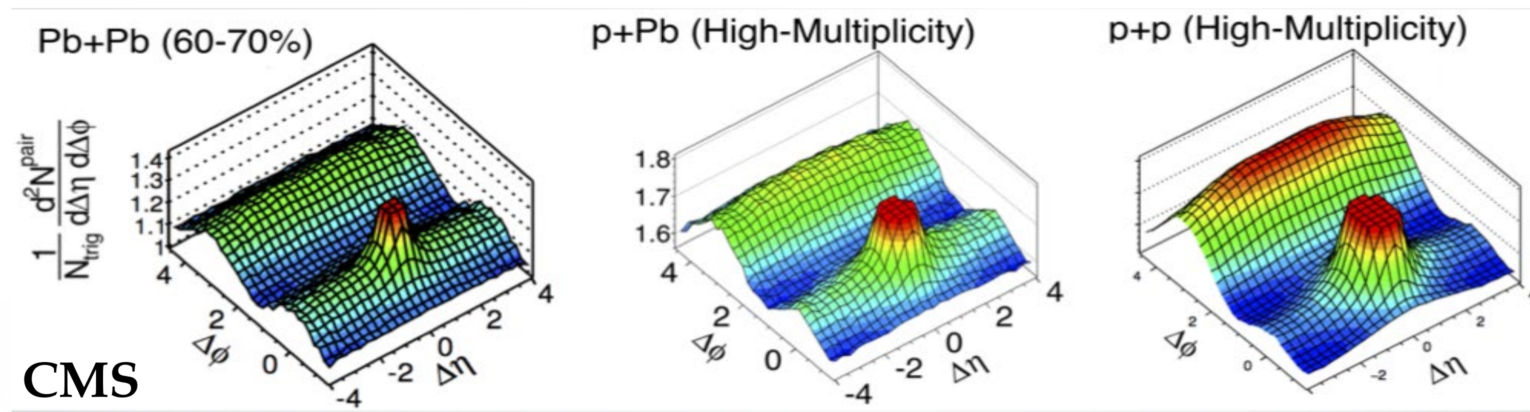
- **EPPS16nlo** *Eskola et al., E.P.J. C77 (16) 163*
1905 pts (NC&CC DIS + LHC)
$\chi^2/N_{pts} = 0.896$

- **nCTEQ15** *Kovarik et al., arXiv:1509.00792*
740 pts (NC&CC DIS+DY+π RHIC)
$\chi^2/N_{pts} = 0.81$

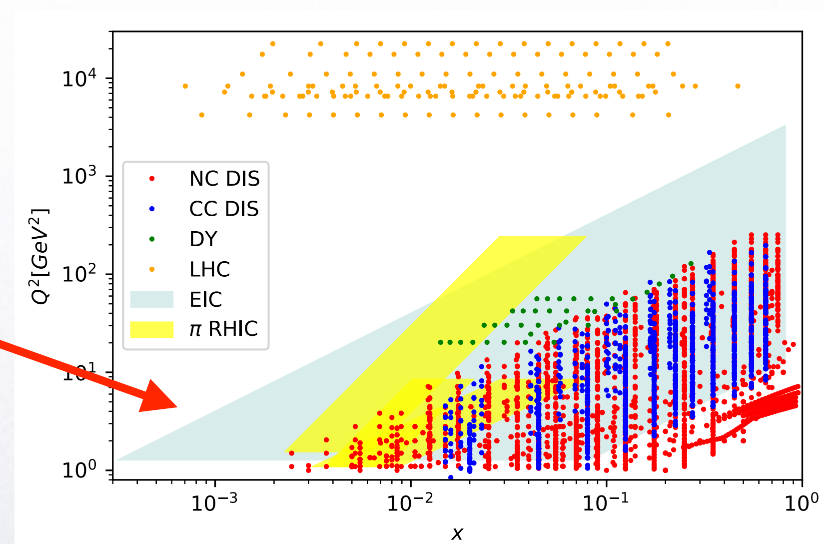


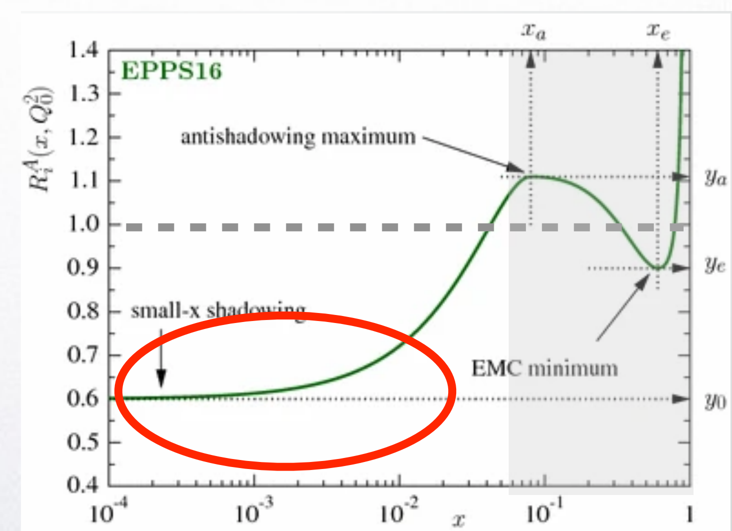
nPDF determine the initial state of ion collisions

e.g., useful to determine initial configurations that lead to understand collective phenomena



currently, significant coverage of phase space.
But still missing very low x





terra incognita: does R_A saturate? or keep falling?

existing data

arXiv:2103.05419, N.P.**A1026** (22) 122447

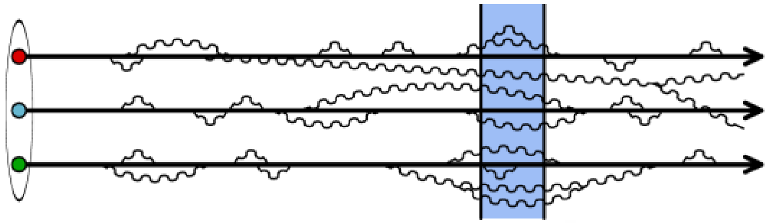
Saturation: a new
universal state of matter?

At low x, very surprising phenomena…

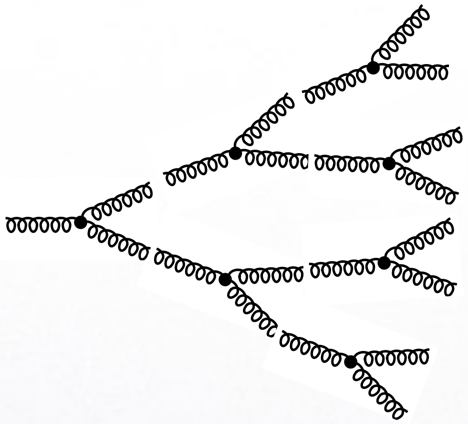
$$\mathscr{L}_{\rm QCD} = \bar{\psi}(x)\,\left[i\,\gamma^\mu\, D_\mu - m \right]\,\psi(x) - \frac{1}{4}\left(F^a_{\mu\nu}\right)^2$$

gluons carry color charge, can self-interact

high energy, low x : parton fluctuations time-dilated; long-lived gluons radiate smaller-x gluons, which in turn radiate more…



boosted hadron



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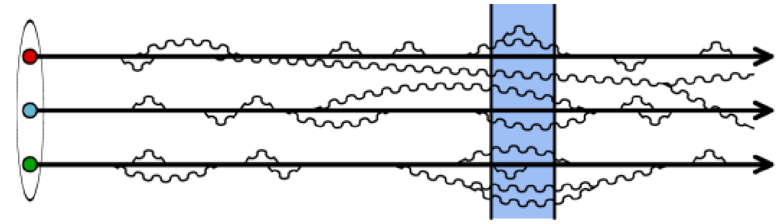

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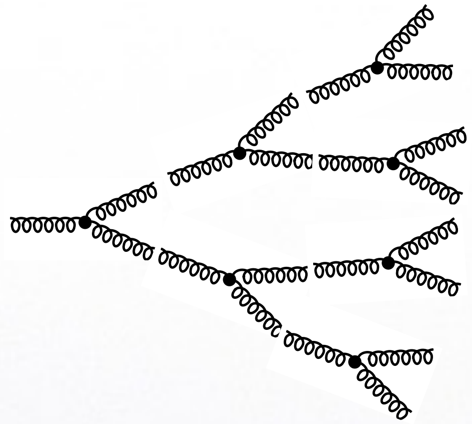
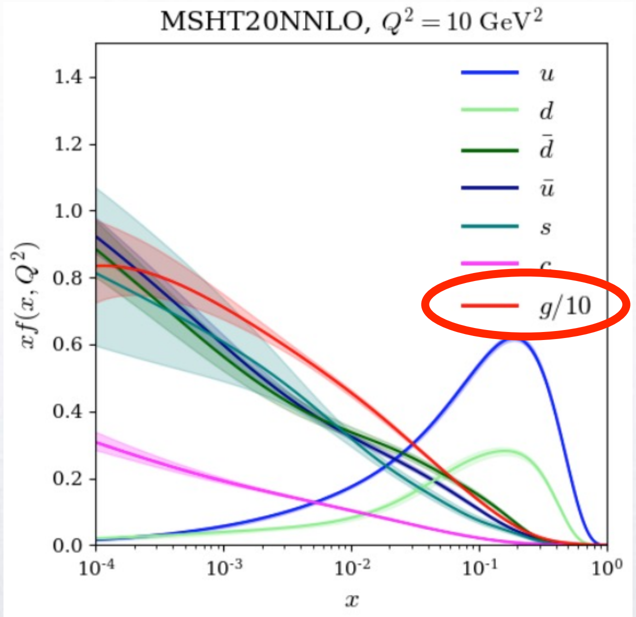
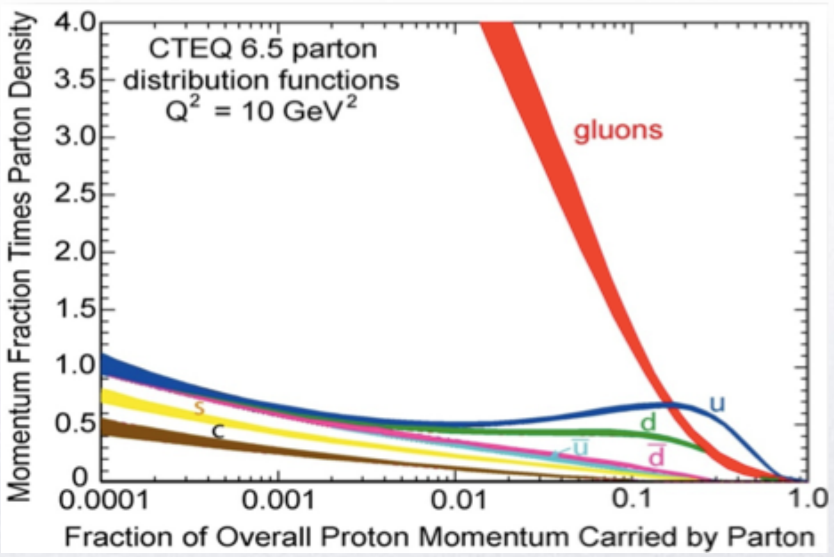


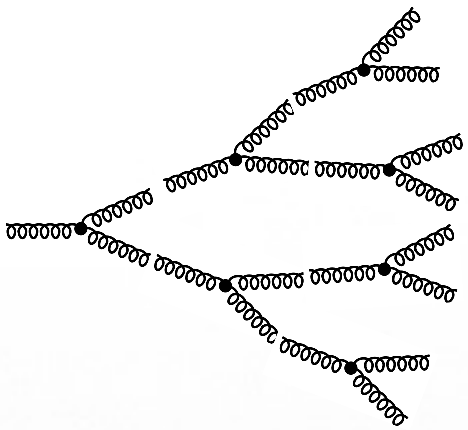
$$\frac{d\,g(x,Q^2)}{d\,\log(1/x)} = \alpha_s\,K_{\rm BFKL} \otimes g(x,Q^2)$$

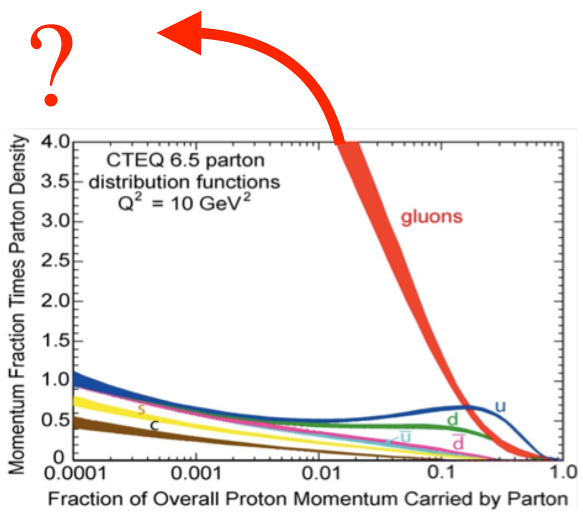
… rapid growth of gluon density at lower x





The rapid growth of gluon density at lower x must stop to preserve unitarity! Froissart bound: the total cross section should grow as $\sigma \sim \log^2 s$.
Gluons must recombine to balance the splitting:





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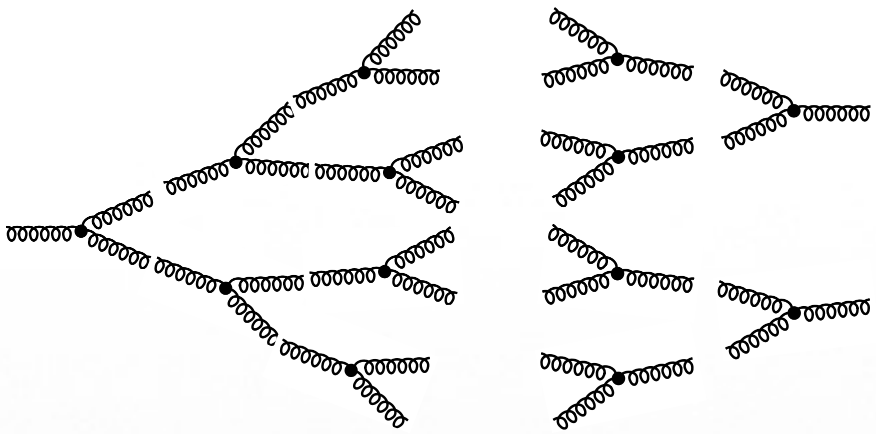

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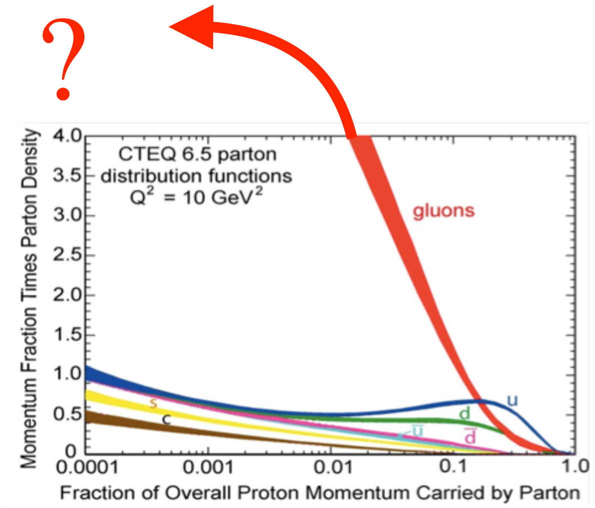
saturation



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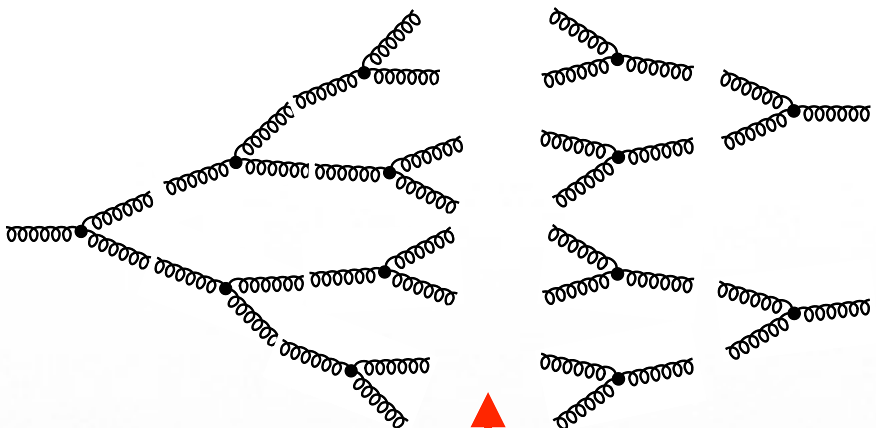

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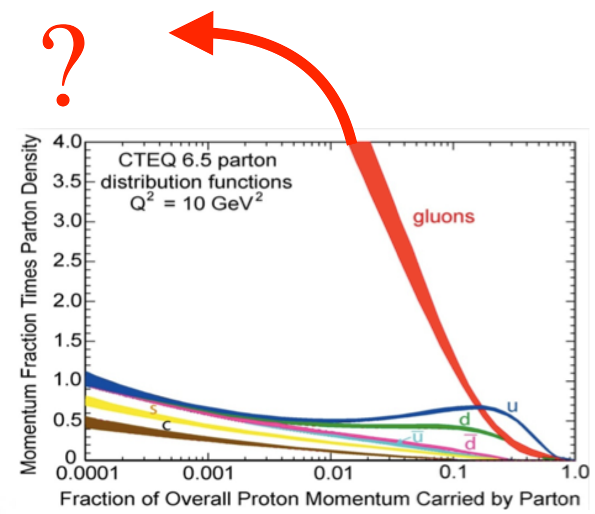
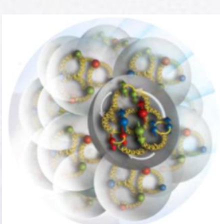
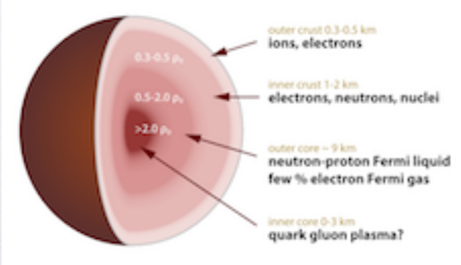


Where (= at which x and scale) does saturation set in?



New universal state of gluonic matter at large densities?

another pillar of EIC science case



implications for astrophysics of neutron stars

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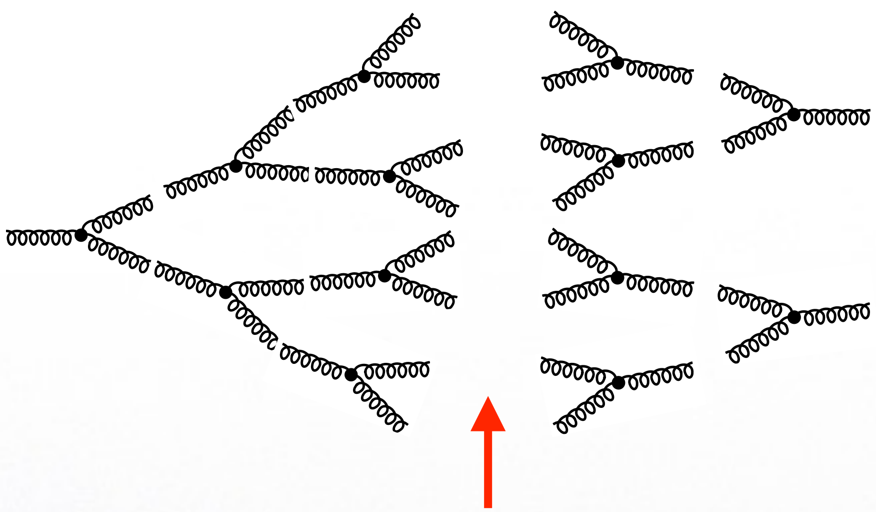

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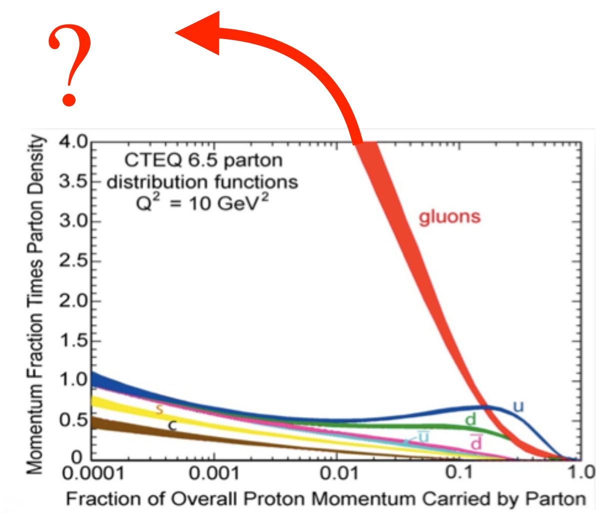

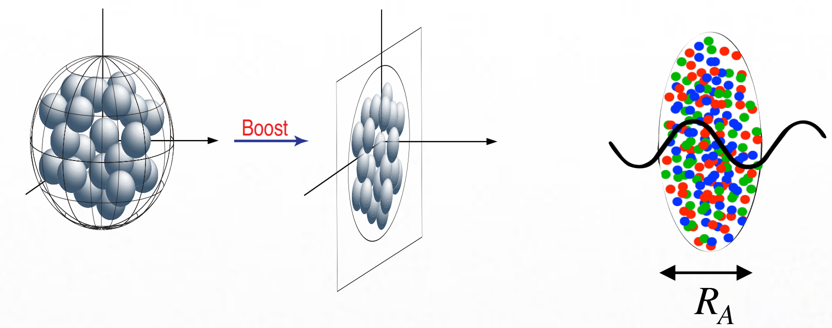

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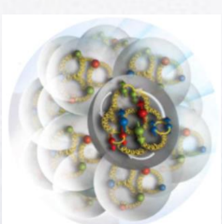
Color-Glass Condensate model *Iancu et al., P.L.* ***B510*** *(01) 133*

$$L \sim \frac{1}{2xM} \sim 2R_A \sim A^{1/3}$$

saturation scale $\left(Q_s^A(x)\right)^2 \sim \left(\frac{A}{x}\right)^{1/3}$

not clearly seen at HERA



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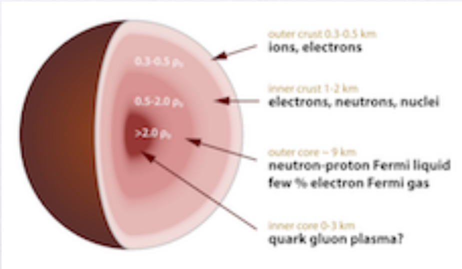

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THANK YOU
for your
ATTENTION!