International School and Workshop on Probing Hadron Structure at the Electron-Ion Collider

School (January 29-February 3, 2024)

Topics of lectures:

(i) Pedagogical lectures on QCD and physics of EIC

(ii) Elastic and deep inelastic scattering (iii) Exclusive processes

(iv) Single spin asymmetries and TMDs (v) Hadron structure in experiments (vi) Heavy ion physics

Date and time	Mon, 29 Jan	Tue, 30 Jan	Wed, 31 Jan	Thu, 1 Feb	Fri, 2 Feb	Sat, 3 Feb
09:15 - 09:30	Opening remarks					
09:30 - 10:30	PEIC	PEIC	EP	HSE	EP	Informal Discussion
10:30 - 11:00	Tea/Coffee					
11:00 - 12:00	EDIS	EDIS	HSE	SSA	HIP	Informal Discussion
12:00 - 13:00	PEIC	PEIC	EP	HIP	EP	
13:00 - 14:30	Lunch					
14:30 - 15:30	EDIS	EDIS	SSA	SSA	HIP	
15:30 - 16:00	Tea/Coffee					
16:00 - 17:00	EDIS	EDIS	SSA	HIP	Interactive session on data analysis	
17:00 - 18:00	Discussion	SSA	HIP	HSE	Interactive session on data analysis	

ICTS International Centre for Theoretical Sciences
Tata Institute of Fundamental Research

Center for Frontiers in Nuclear Science

"Pedagogical QCD" :

Marco Radici
INFN - Pavia

INFN Istituto Nazionale di Fisica Nucleare

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International Centre *for* Theoretical Sciences — ICTS — Tata Institute of Fundamental Research

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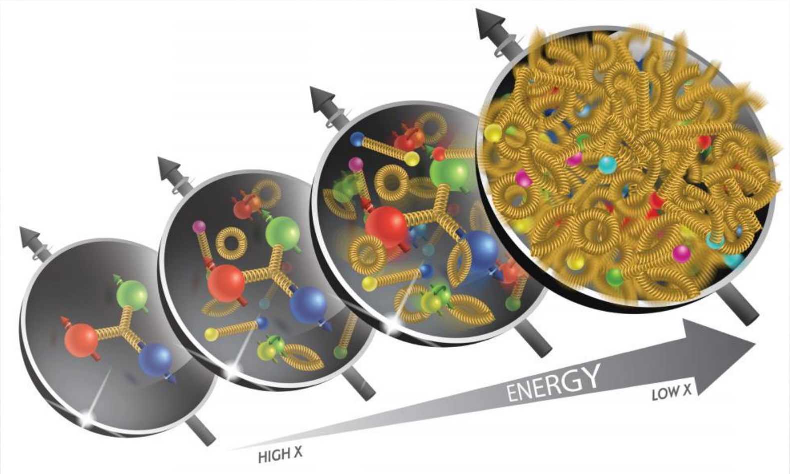


?

"Pedagogical QCD" : main features and open problems

Marco Radici
INFN - Pavia

INFN Istituto Nazionale di Fisica Nucleare

Useful references

- **Books**
  - Peskin-Schroeder - *Quantum Field Theory*
  - Muta - *Foundations of Quantum Chromodynamics*
  - Collins - *Foundations of perturbative QCD*
  - Ellis-Stirling-Webber - *QCD and Collider Physics*
  - Devenish-CooperSarkar - *Deep Inelastic Scattering*
  - Field - *Applications of Perturbative QCD*
  - Aitchison-Hey - *Gauge Theories in Particle Physics*
  - Roberts - *The structure of the proton*

- **Lecture notes & Handbooks**
  - R. Jaffe - Erice School https://arxiv.org/pdf/hep-ph/9602236.pdf
  - CTEQ Handbook of perturbative QCD https://www.physics.smu.edu/scalise/cteq/handbook/v1.1/handbook.pdf

- **Papers**
  - references added to slide when needed

Outline

❶ the Standard Model: why is Quantum ChromoDynamics so "exotic" ?
(QCD)

❷ Factorization Theorems, evolution equations and all that

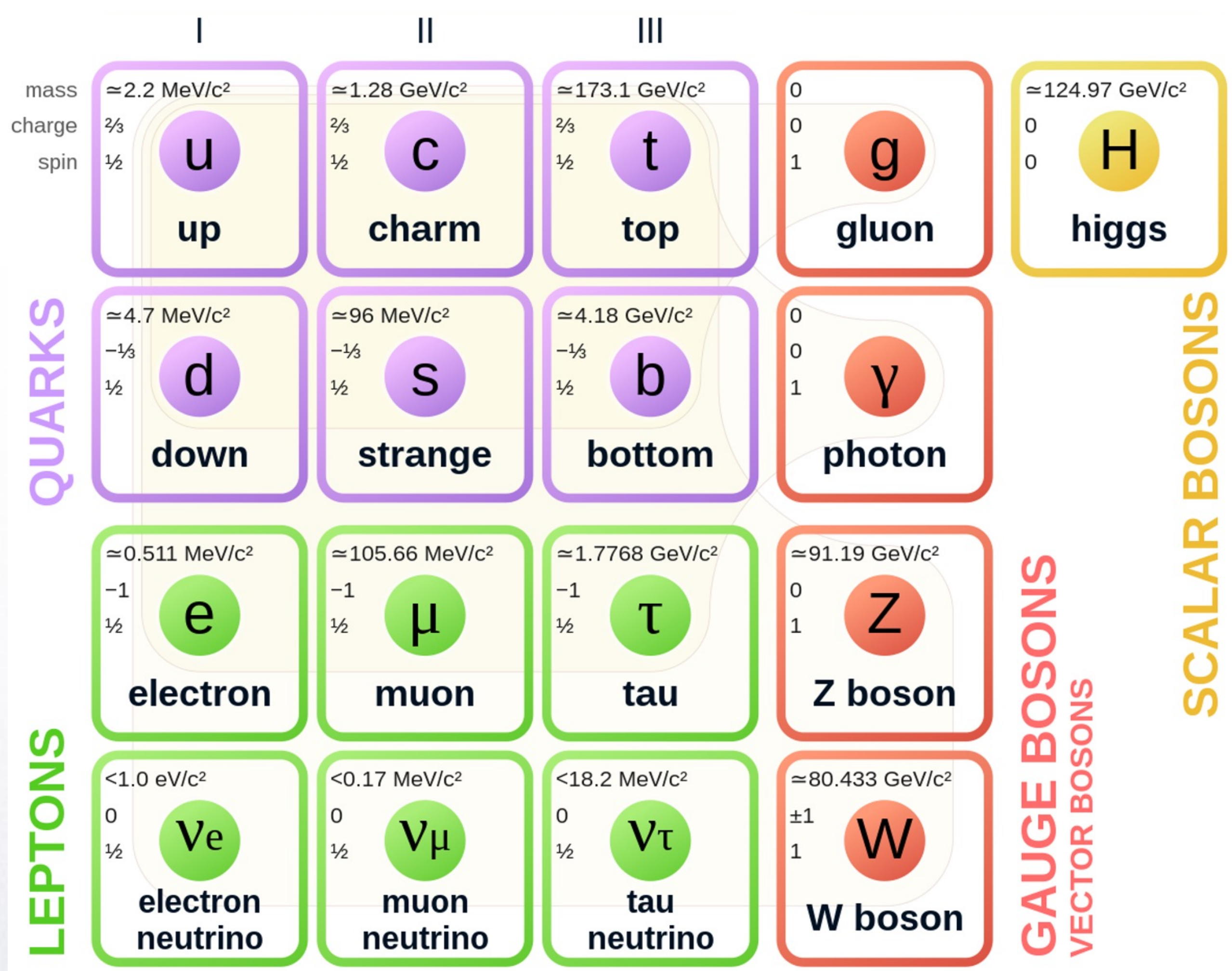
❸ open problems
- where do the Nucleon mass and spin come from?
- beyond the collinear approximation
- chiral-odd structures
- nuclear matter effects
- saturation: a new state of matter ?

Outline

❶ the Standard Model: why is QCD so "exotic" ?

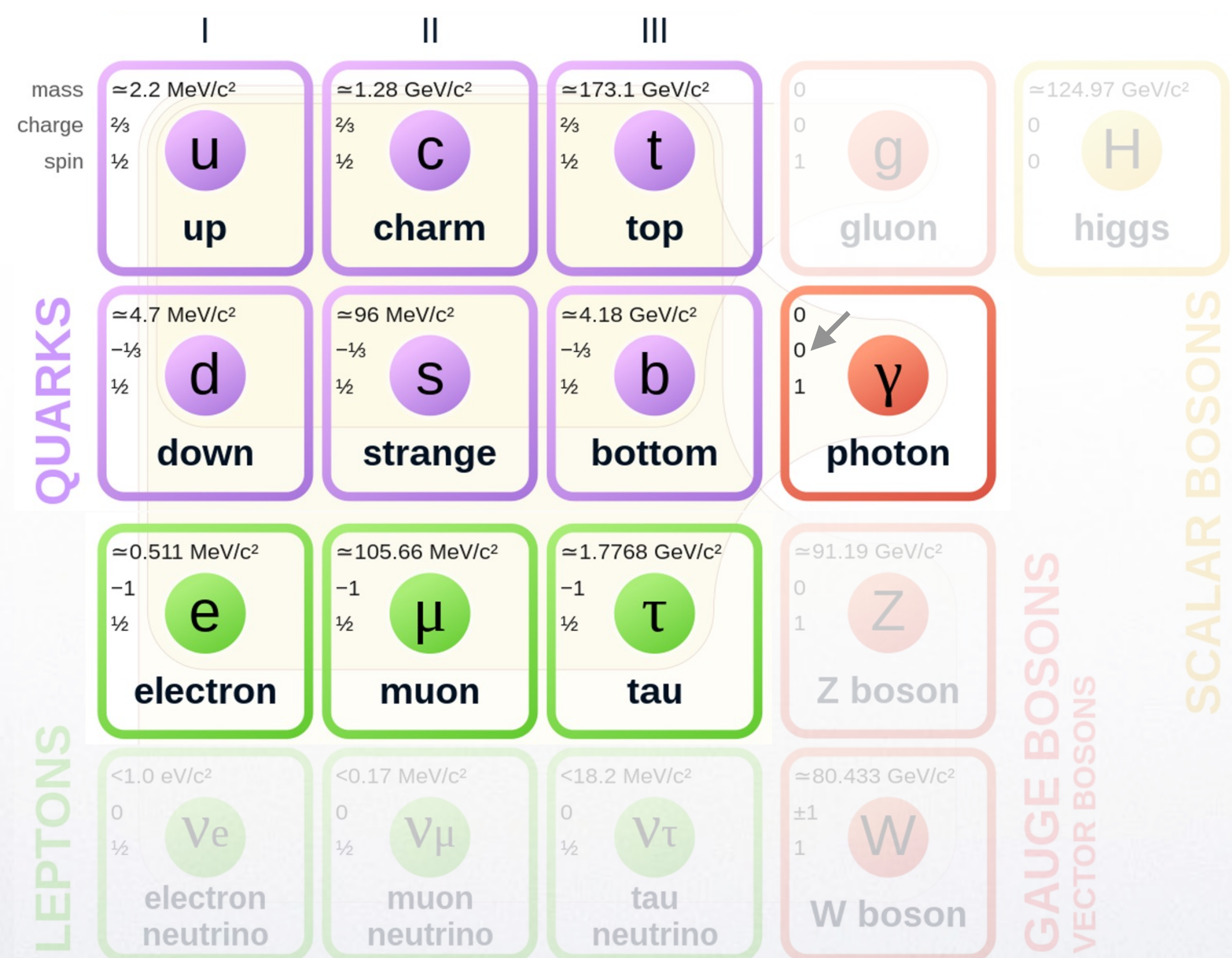
elementary particles   force carriers

Fermions   Bosons



elementary particles force carriers

Fermions Bosons

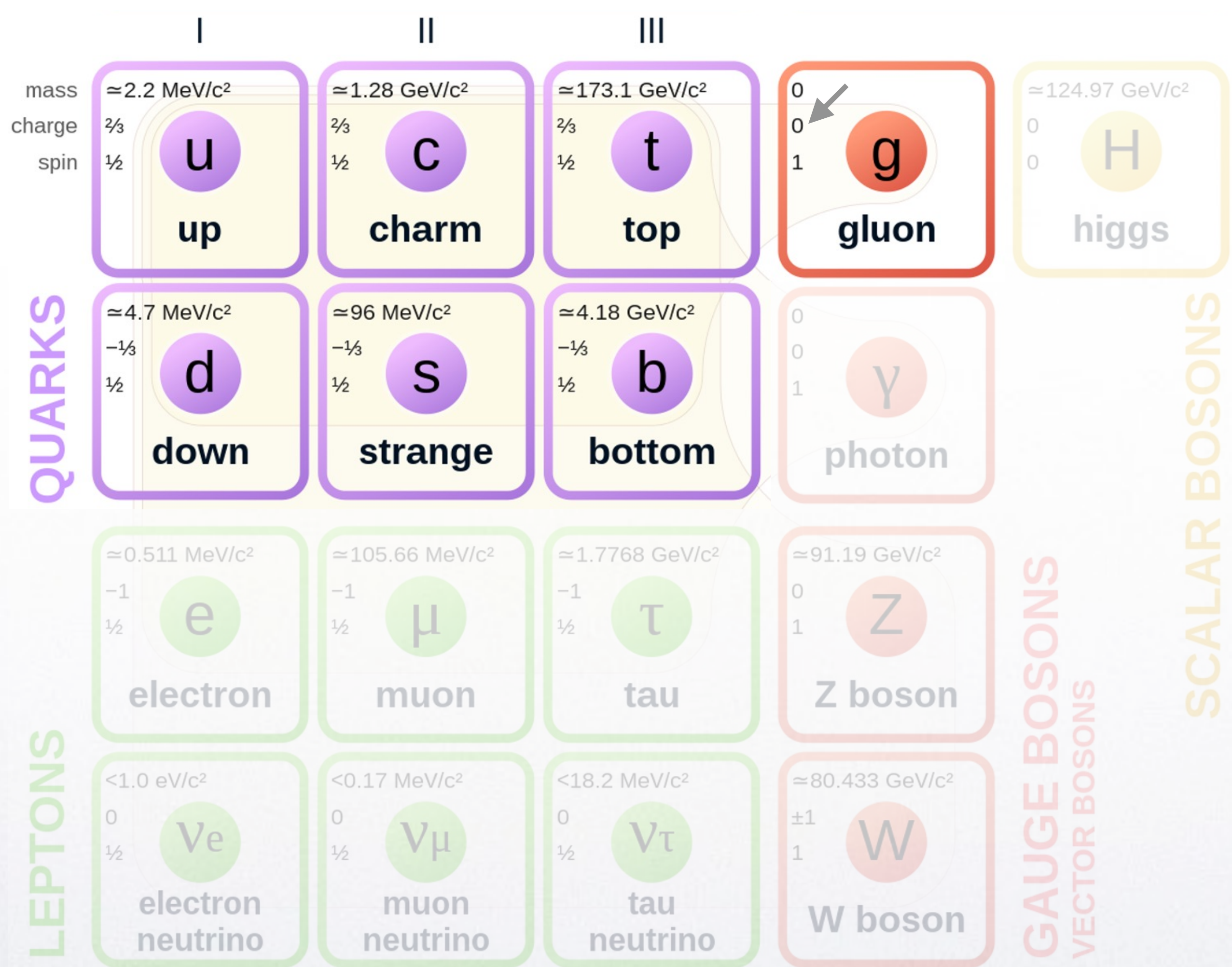


electromagnetic interactions

massless, neutral photon

elementary particles force carriers

Fermions Bosons

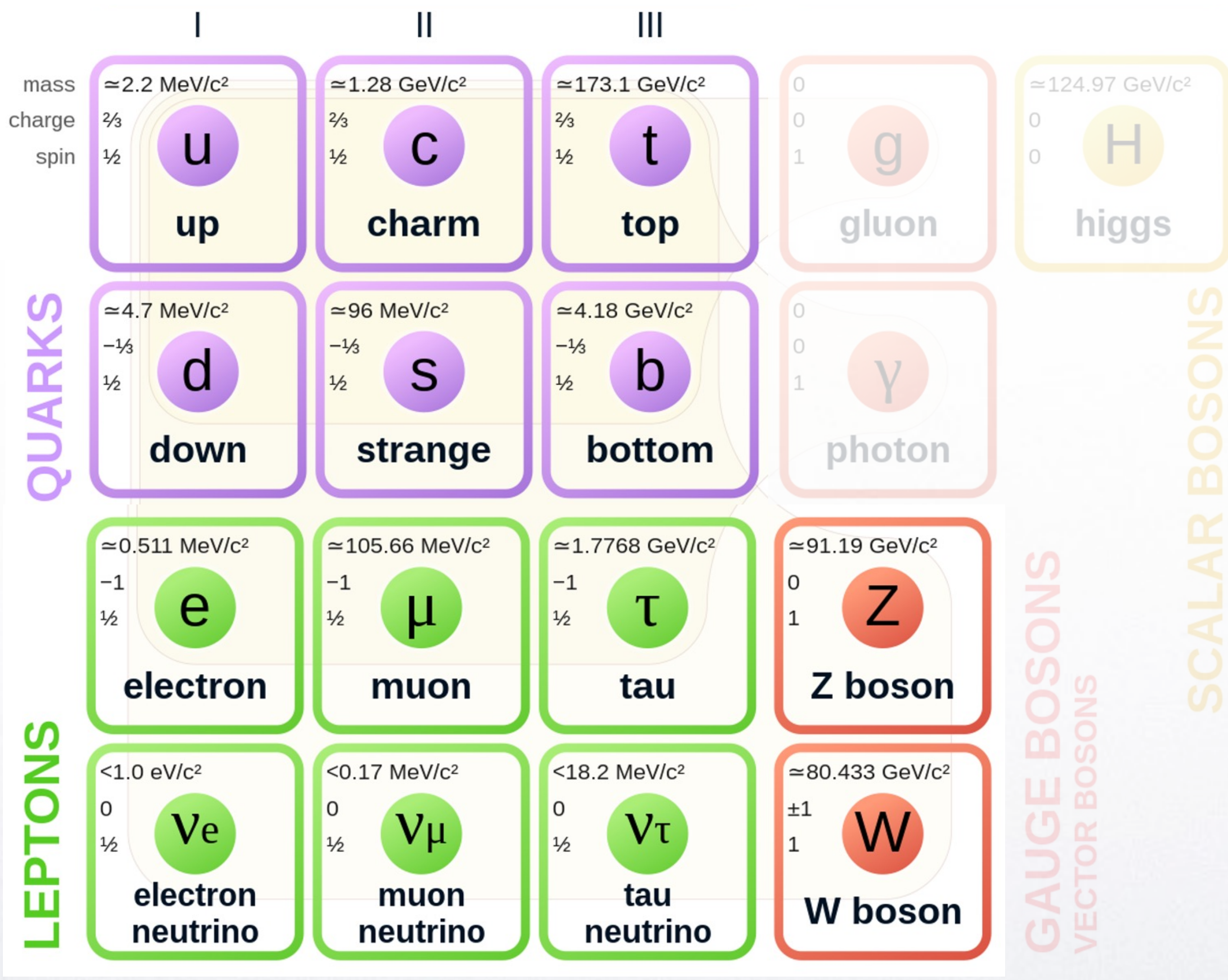


strong interactions

massless, neutral
but colored gluon

elementary particles force carriers

Fermions Bosons



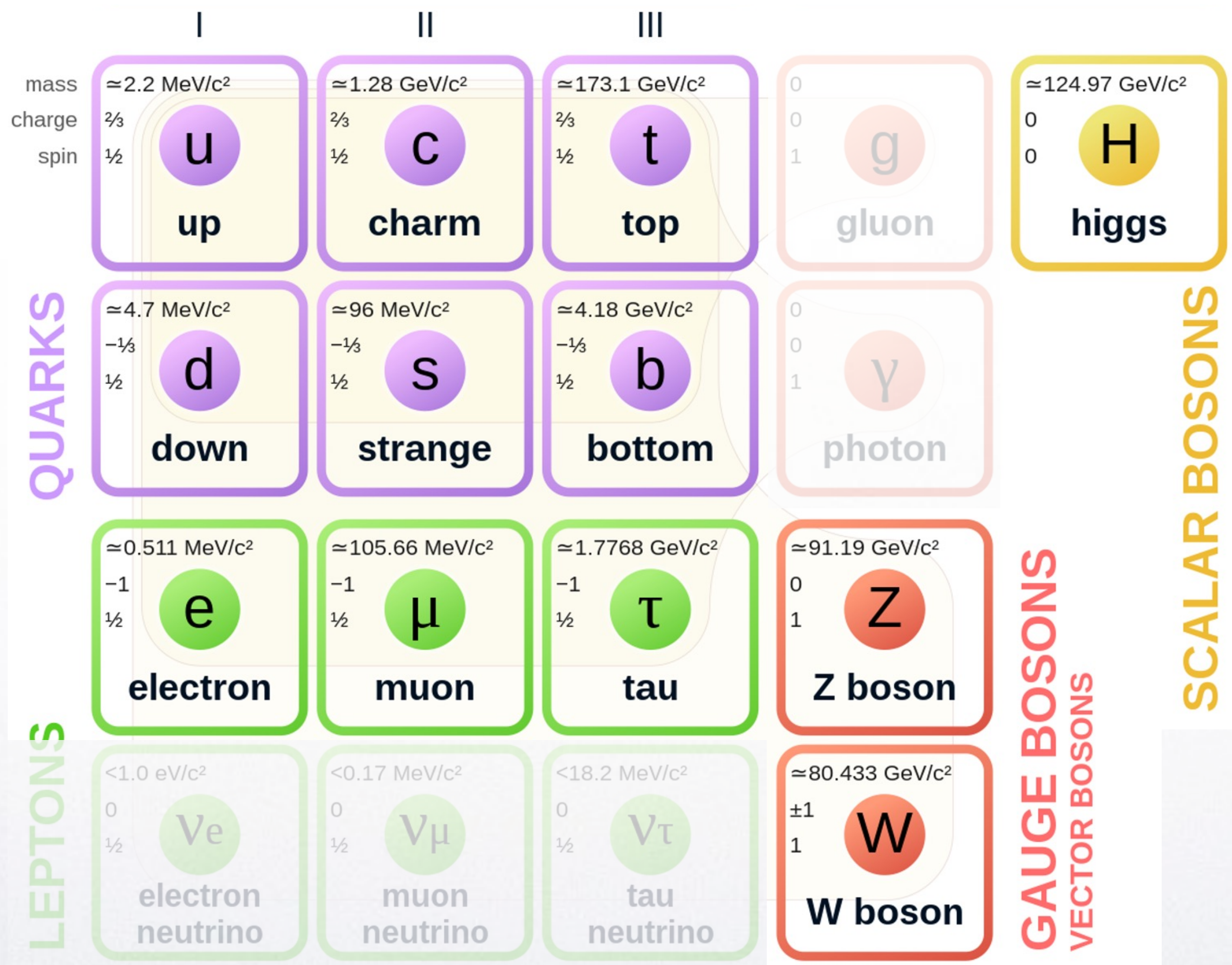
weak interactions

massive bosons
neutral Z^0
charged $W^{\pm}$

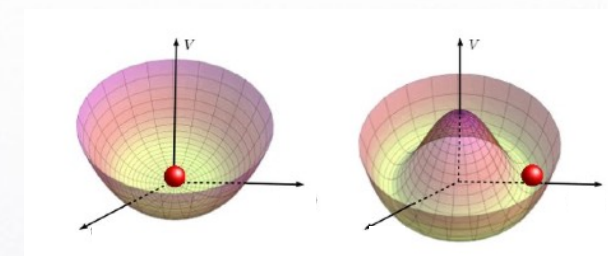
the Standard Model

elementary particles force carriers

Fermions Bosons



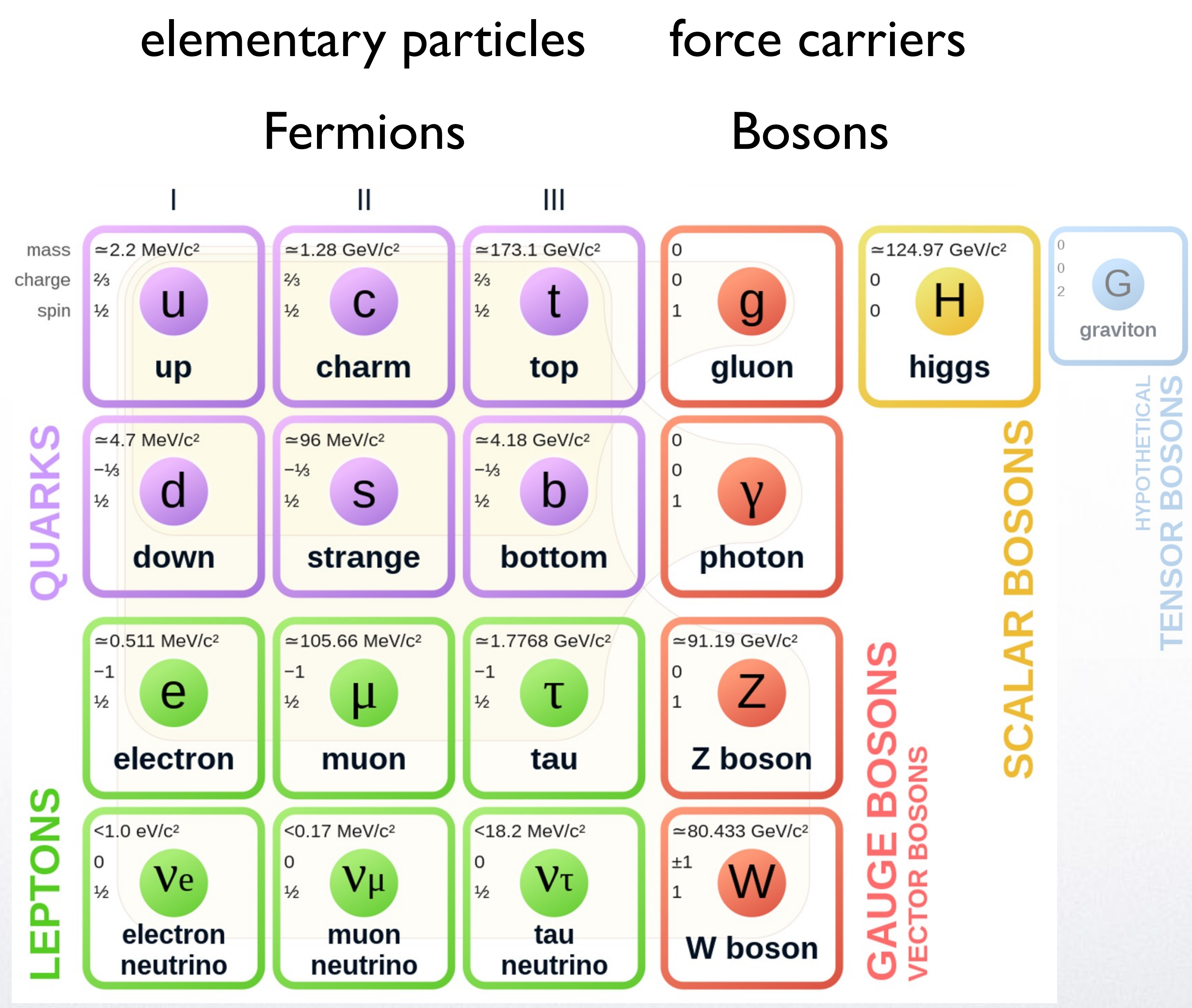
all particles (except ν) take their mass from interaction with Higgs boson through spontaneous breaking of a local symmetry



$\mu > 0$
$\phi=0$ stable
unbroken symmetry

$\mu < 0$
$\phi=0$ instable
broken symmetry

incomplete theory:
- quantum gravity?



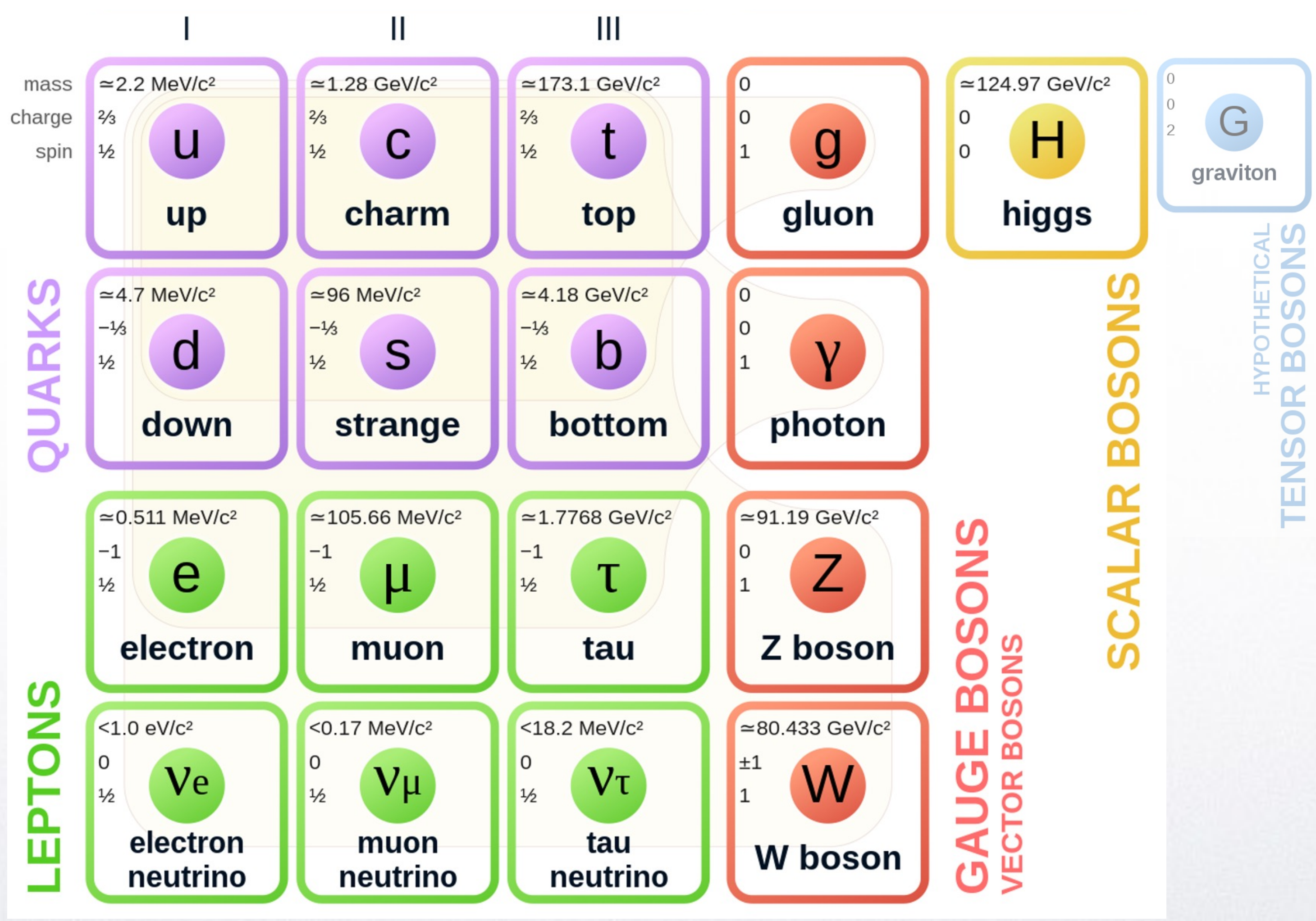
the Standard Model

elementary particles force carriers

Fermions Bosons

incomplete theory:

- quantum gravity?
- why 3 generations?

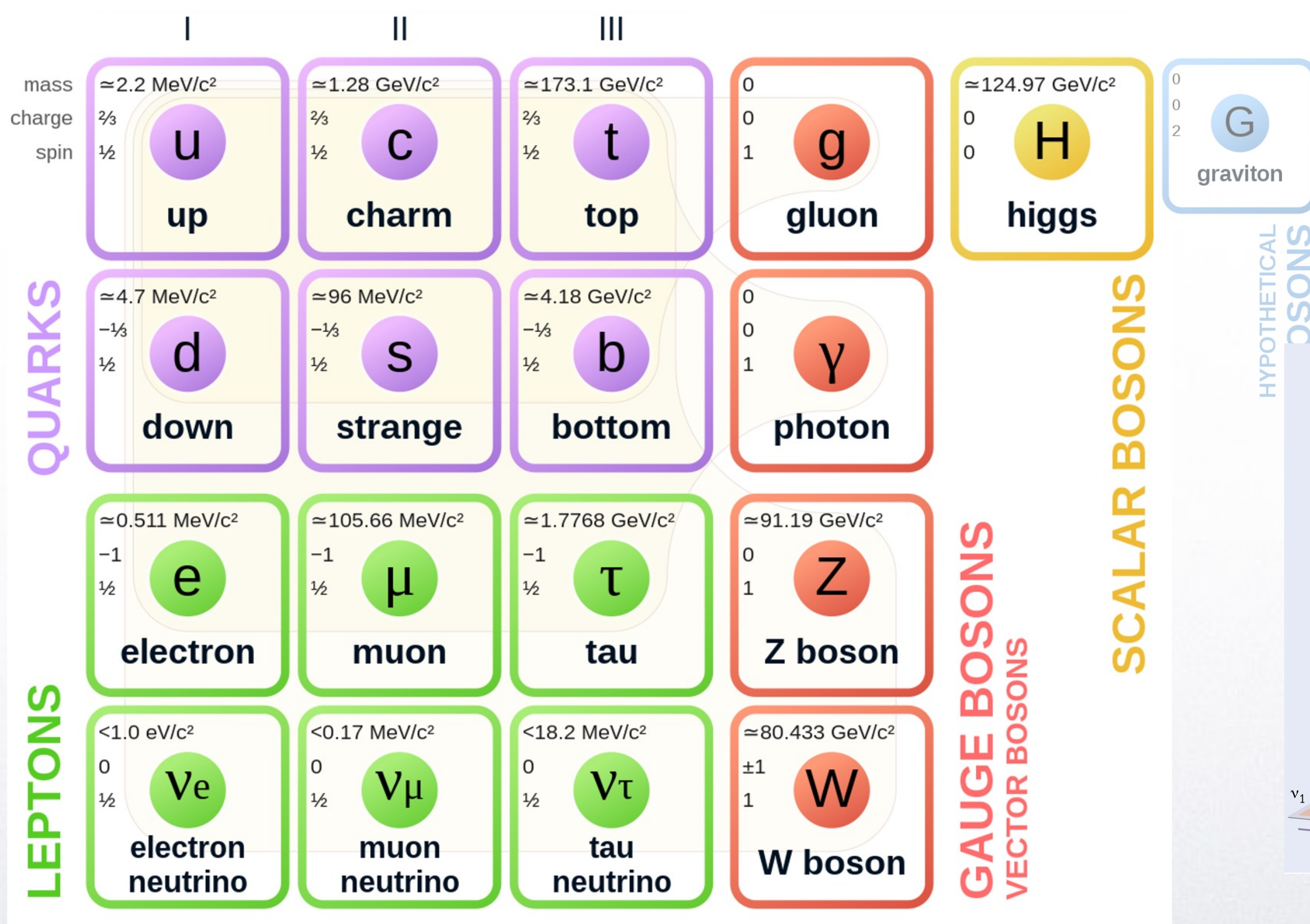


elementary particles force carriers

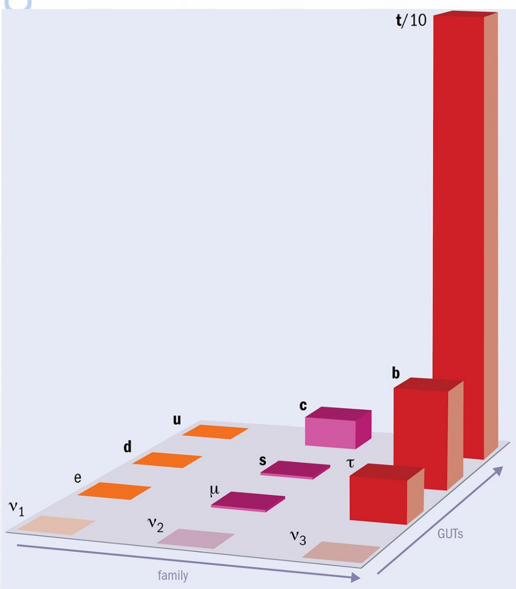
Fermions Bosons

incompleteincomplete theory:

- quantum gravity?
- why 3 generations?
- hierarchy?
- origin of ν masses?



SM masses

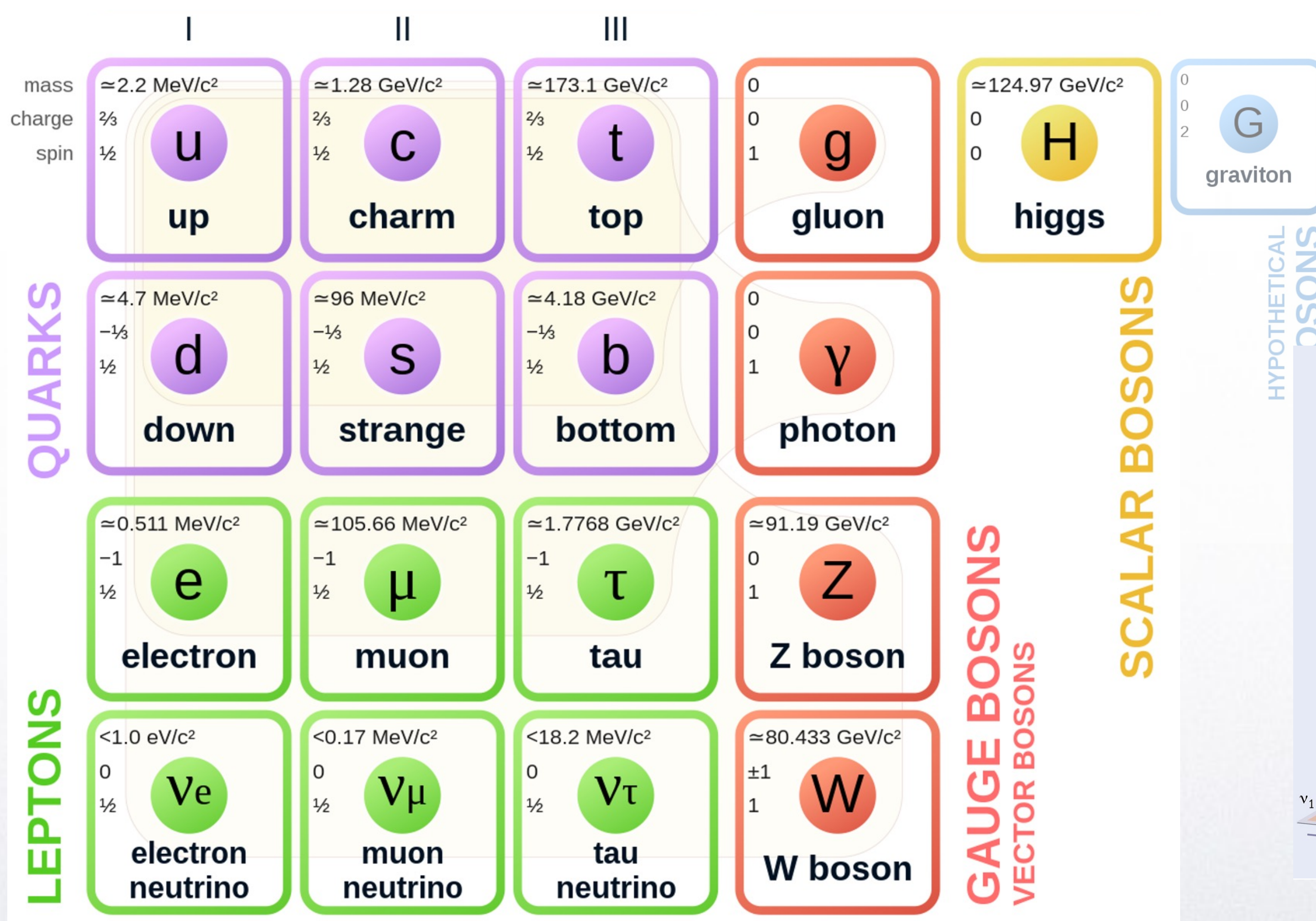


$m_{Higgs} < m_{top}$?

the Standard Model

elementary particles force carriers

Fermions Bosons

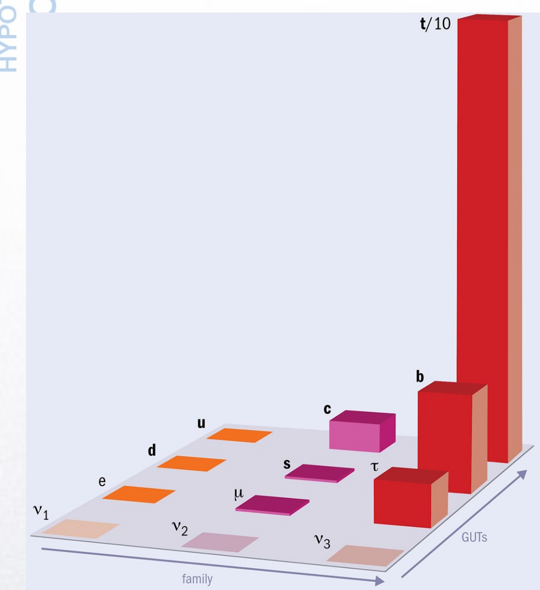


incomplete theory:

- quantum gravity?
- why 3 generations?
- hierarchy?
- origin of ν masses?
- matter-antimatter asymmetry?
- dark matter & energy?
- muon g-2 anomaly?
- why is Physics so difficult?

….

SM masses

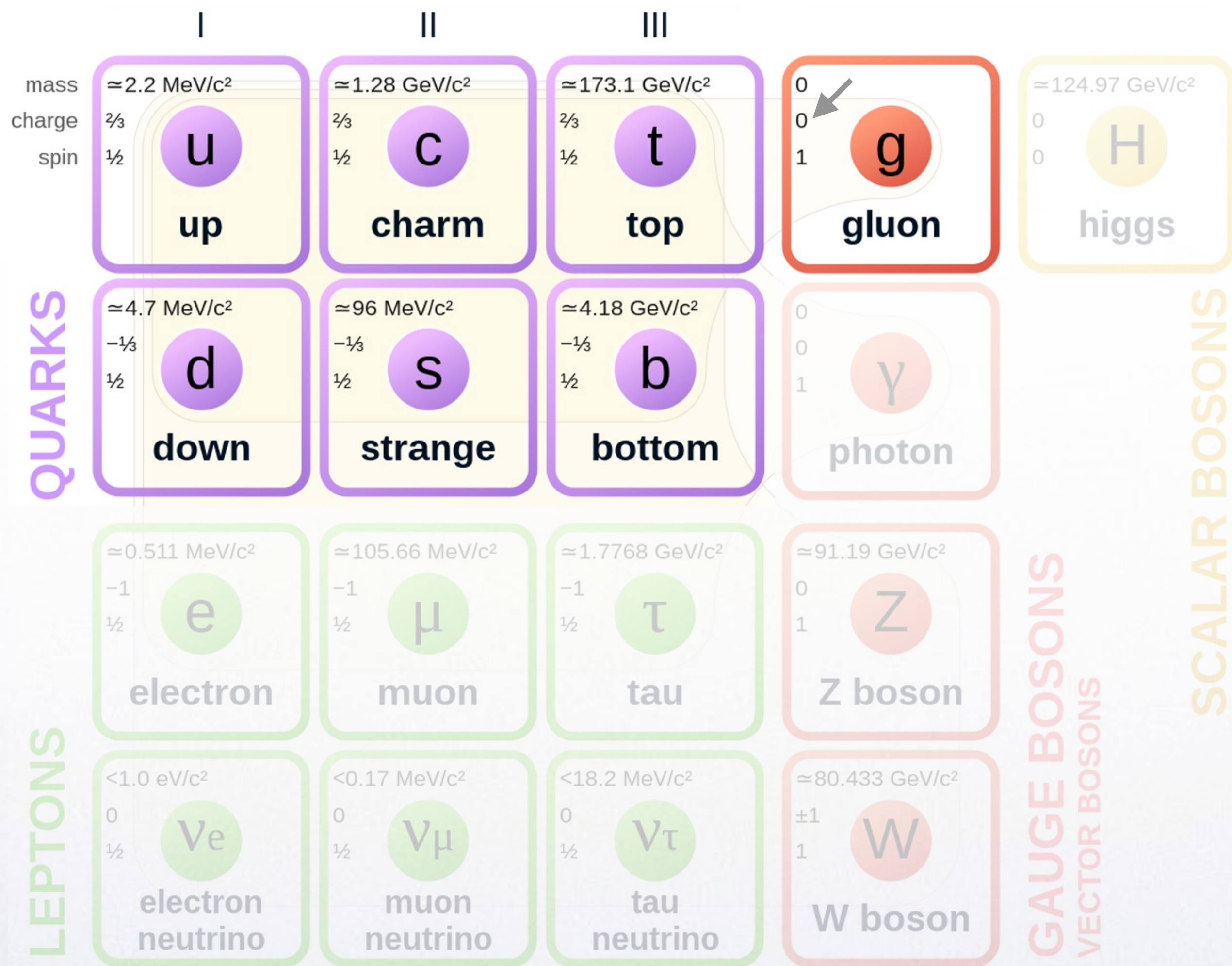


$m_{Higgs} < m_{top}$?

the Standard Model

elementary particles | force carriers

Fermions | Bosons



strong interactions

massless, neutral
but colored gluon

Theory of Strong Interactions: Quantum ChromoDynamics (QCD)

A renormalizable non-abelian gauge theory

The QCD Lagrangian

$$\mathscr{L}_{\mathrm{QCD}} = \bar{\psi}(x) \left[i\, \gamma^\mu D_\mu - m \right] \psi(x) - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2 + \mathscr{L}_{\mathrm{gauge-fixing}}$$

with $F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc}\, A^b_\mu\, A^c_\nu$ a, b, c color indices (others understood)

ψ = Dirac quark field (particle) A^μ = vector gluon field (force carrier)

Theory of Strong Interactions: Quantum ChromoDynamics (QCD)

A renormalizable non-abelian gauge theory

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ψ = Dirac quark field (particle) A^μ = vector gluon field (force carrier)

$D_\mu \equiv \partial_\mu - i\, g\, A^a_\mu\, t^a$ covariant derivative: makes $\mathscr{L}_{\rm QCD}$ locally gauge-invariant

identifies ψ - A interaction

$[t^a, t^b] = i f^{abc}\, t^c$ t = generators of gauge transformations

f = fine structure constant (fully antisymmetric in color indices)

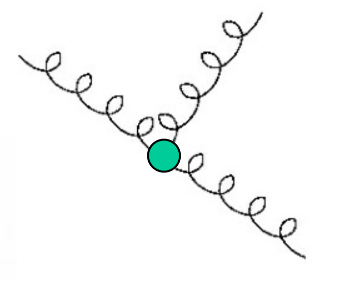
$\Rightarrow\; F^a_{\mu\nu} = -\, F^a_{\nu\mu}$

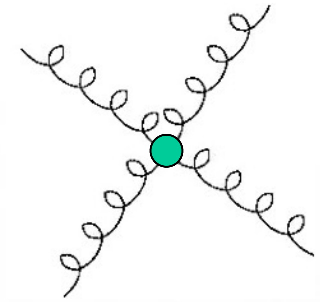
the “Maxwell” equations

$$\mathscr{L}_{\rm QCD} = \bar{\psi}(x) \left[i\,\gamma^\mu D_\mu - m \right] \psi(x) - \frac{1}{4} \left(F^a_{\mu\nu} \right)^2$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\, f^{abc}\, A^b_\mu\, A^c_\nu$$

gluons are colored
can self-interact





trilinear quadrilinear couplings

the “Maxwell” equations

$$\mathcal{L}_{\rm QCD} = \bar{\psi}(x)\,\left[i\,\gamma^\mu\, D_\mu - m \right]\,\psi(x) - \frac{1}{4}\,\left(F^a_{\mu\nu} \right)^2$$

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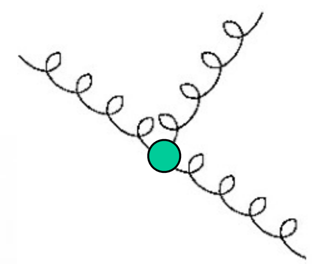
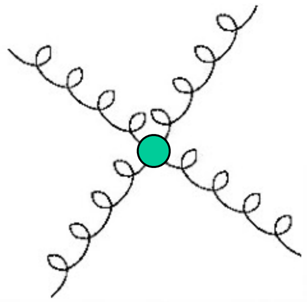


trilinear



quadrilinear

couplings

“Maxwell” equations for the vector field A

$$\frac{\partial \mathcal{L}_{\rm QCD}}{\partial A^a_\mu} = \partial_\nu \frac{\partial \mathcal{L}_{\rm QCD}}{\partial\, \partial_\nu A^a_\mu}$$

$$\partial_\mu F^{\mu\nu a} + g\, f^{abc}\, A^b_\mu\, F^{\mu\nu c} = -\, g\bar{\psi}\,\gamma^\nu\, t^a\,\psi$$

$$\partial_\mu F^{\mu\nu a} + g\, f^{abc}\, A_\mu^b\, F^{\mu\nu c} = -\, g\bar{\psi}\, \gamma^\nu\, t^a\, \psi$$

"Maxwell" equations for vector field A

take ν = 0 $\quad \partial_i \;\; F^{i\,0\,a} \;\; - g\, f^{abc}\, A_i^b\, F^{i\,0\,c} = -\, g\;\, \psi^\dagger\, \psi\, t^a$

$$\partial_\mu F^{\mu\nu a} + g\, f^{abc}\, A_\mu^b\, F^{\mu\nu c} = -\, g\bar{\psi}\,\gamma^\nu\, t^a\,\psi$$

“Maxwell” equations for vector field A

take ν = 0

$$\partial_i \underbrace{F^{i\,0\,a}} - g\, f^{abc}\, A_i^b\, F^{i\,0\,c} = -\, g\, \underbrace{\psi^\dagger\,\psi\, t^a}$$

color electric field $E_i^a = F^{0\,i\,a}$

density of color charge a ρ^a

$$\partial_i E_i^a - g\, f^{abc}\, A_i^b\, E_i^c = g\,\rho^a$$

the "Gauss" law

$$\partial_\mu F^{\mu\nu a} + g\, f^{abc}\, A^b_\mu\, F^{\mu\nu c} = -\, g\bar{\psi}\, \gamma^\nu\, t^a\, \psi$$

"Maxwell" equations for vector field A

take ν = 0 $\quad \partial_i \underbrace{F^{i0a}} - g\, f^{abc}\, A^b_i\, F^{i0c} = -\, g\, \underbrace{\psi^\dagger \psi\, t^a}$

color electric field $E^a_i = F^{0ia}$

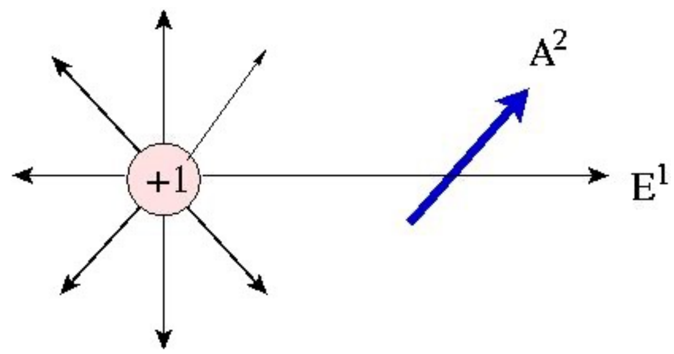
density of color charge a ρ^a

$$\partial_i E^a_i - g\, f^{abc}\, A^b_i\, E^c_i = g\, \rho^a$$

in Coulomb gauge $\nabla_i A^a_i = 0$, the Coulomb color potential generated by A^a_0 ; then

$$D_i E^a_i = g\, \rho^a$$

is the "Gauss" law for color charge a

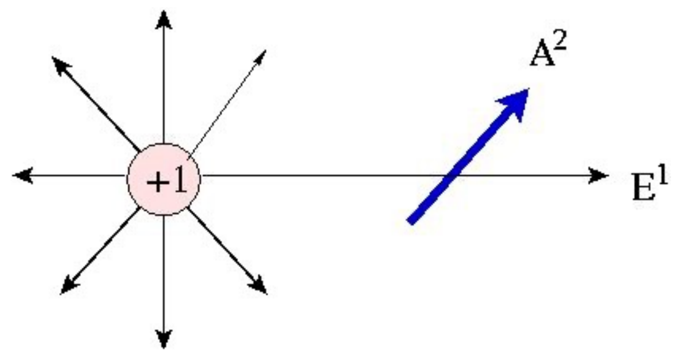


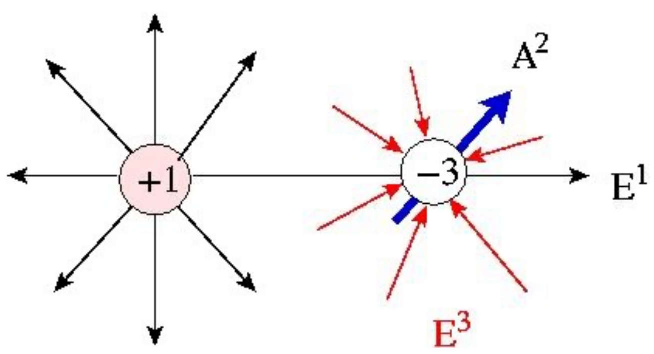
point-like color charge a=1
creates color electric field E_i^1

$$\partial_i E_i^1 = g\,\delta(\vec{x})\,\delta_{a\,1}$$

then vacuum fluctuation A_i^2
with color charge **2**

$$\partial_i\, E_i^a - g\, f^{abc}\, A_i^b\, E_i^c = g\,\rho^a$$





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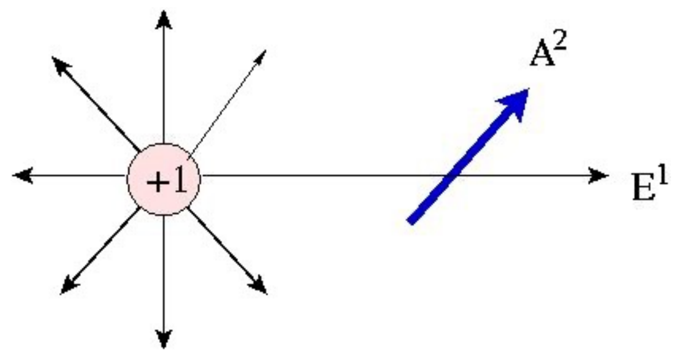
$$\partial_i E_i^3 = g\, f^{321}\, A_i^2\, E_i^1$$

$$= -\, g\, f^{123}\, A_i^2\, E_i^1$$

fluctuation A_i^2 and field E_i^1
create a "sink" of color
electric field with charge **3**

$$\partial_i E_i^a - g\, f^{abc}\, A_i^b\, E_i^c = g\,\rho^a$$

effect of the “Gauss” law



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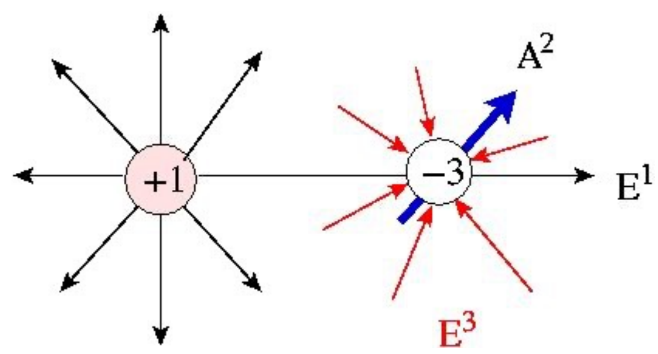

$$\partial_i\, E_i^3 = g\, f^{321}\, A_i^2\, E_i^1$$

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fluctuation A_i^2 and field E_i^1
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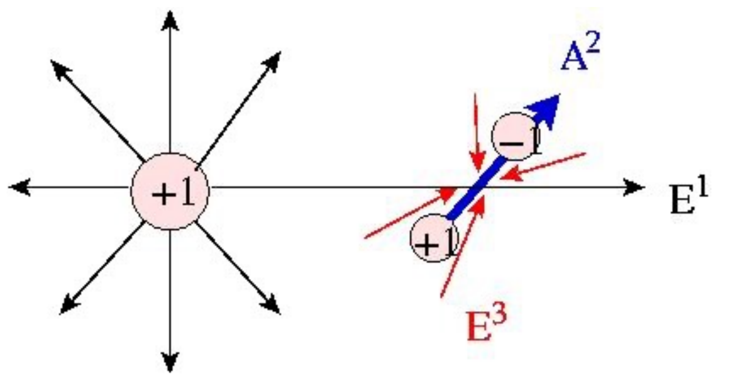


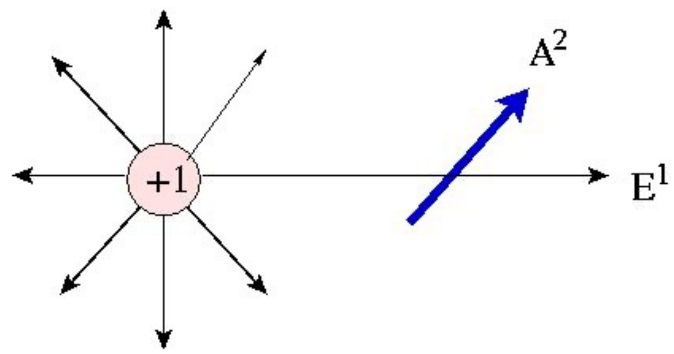
field E_i^3 contributes to field E_i^1

$$\partial_i E_i^1 = g\,\delta(\vec{x})\,\delta_{a\,1} + \underbrace{g\, f^{123}\, A_i^2\, E_i^3}$$

$> 0 \quad \vec{A}^2 \parallel \vec{E}^3$

$< 0 \quad \vec{A}^2 \parallel^{-1} \vec{E}^3$

creates gradient of field E_i^1
pointing toward charge a=1



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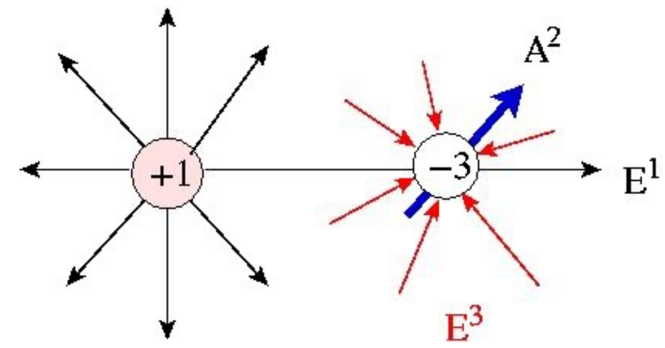

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fluctuation A_i^2 and field E_i^1
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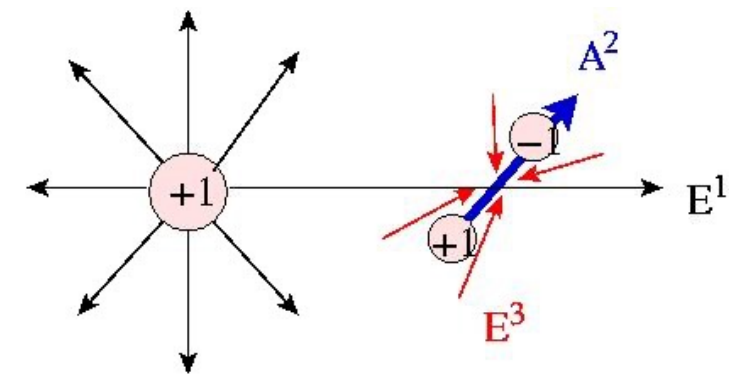

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creates gradient of field E_i^1
pointing toward charge a=1

$$\partial_i E_i^a - g\, f^{abc}\, A_i^b\, E_i^c = g\,\rho^a$$

getting away from source, the
color charge a=1 looks stronger!
antiscreening

running of coupling constant

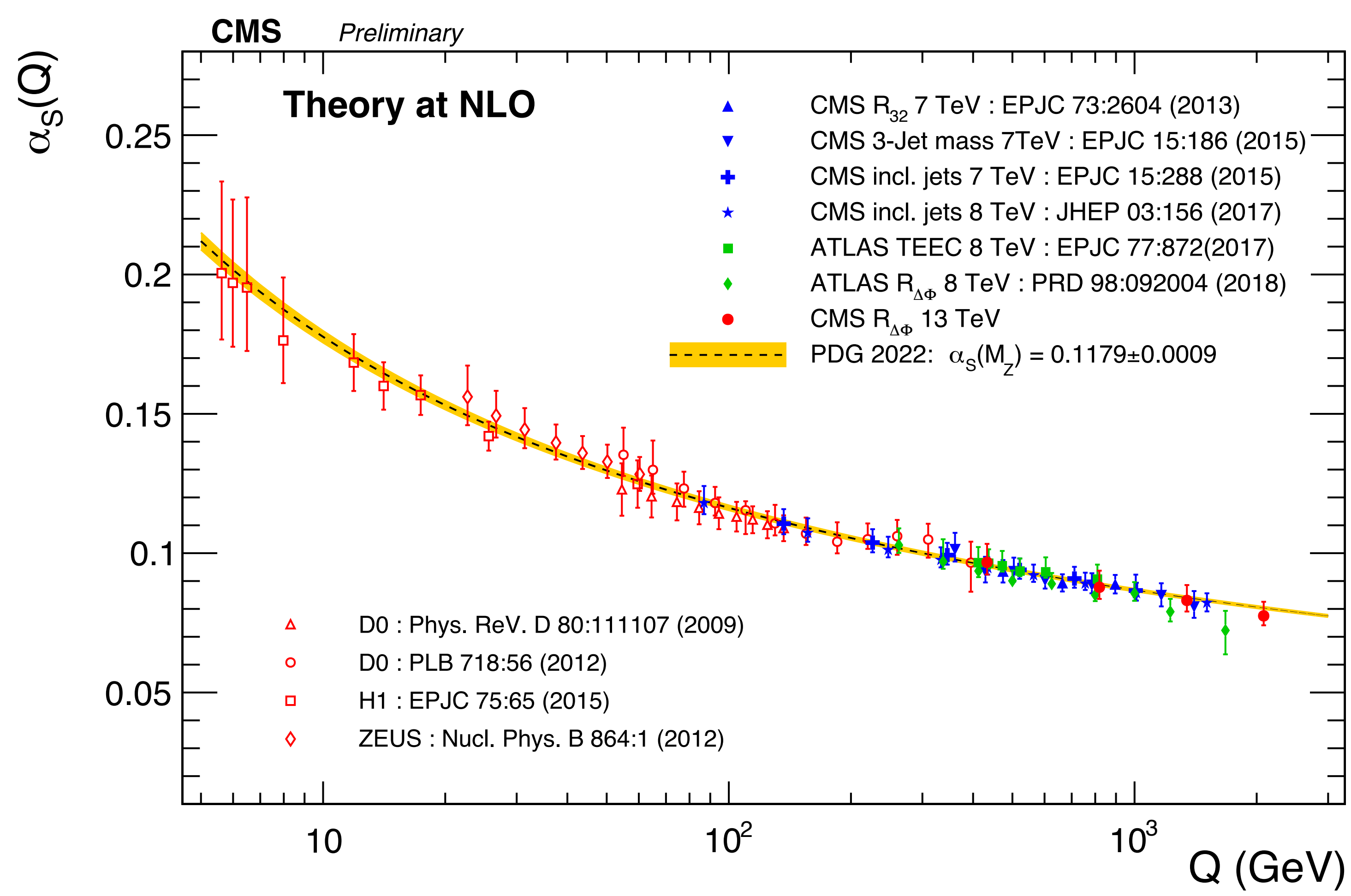
Available on the CERN CDS information server **CMS PAS SMP-22-005**

CMS Physics Analysis Summary

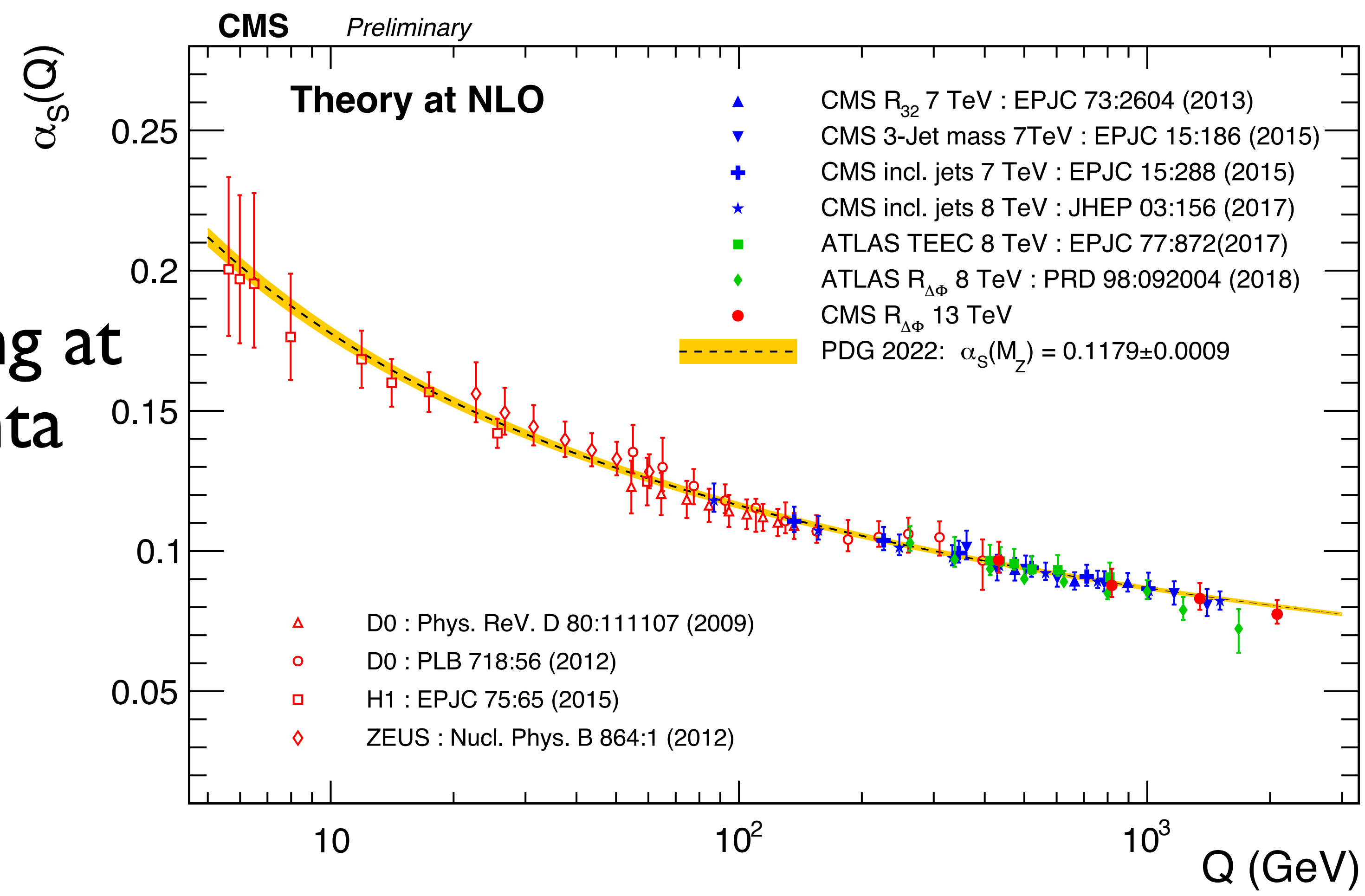
Contact: cms-pag-conveners-smp@cern.ch 2023/08/24

Measurement of azimuthal correlations among jets and determination of the strong coupling in pp collisions at $\sqrt{s} = 13$ TeV

The CMS Collaboration



antiscreening overwhelms screening at larger distances → smaller momenta



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QED **QCD**

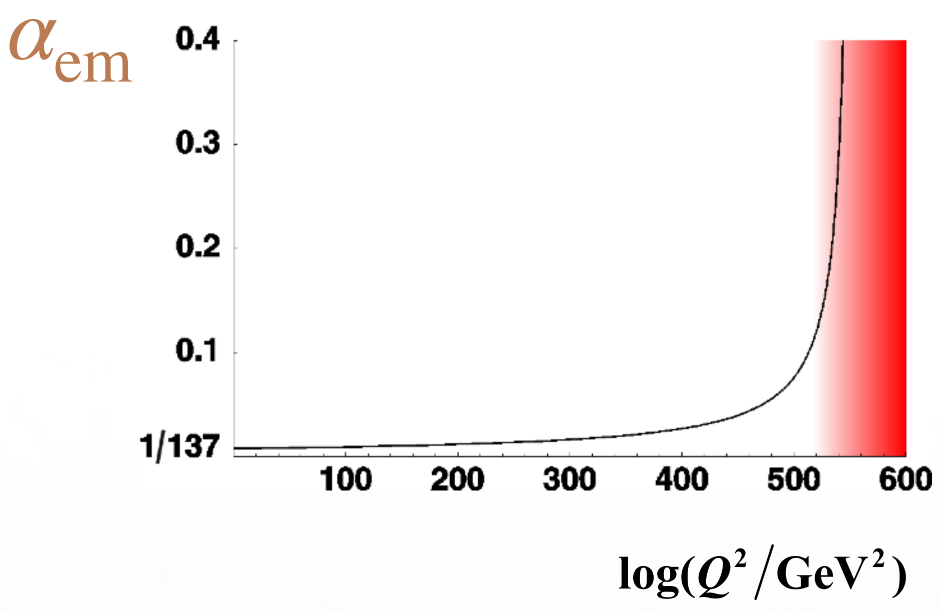
dimensionless coupling $g \rightarrow$ renormalizable field theory

invariance of physics from renormalisation scale $\mu_R \rightarrow$ Callan-Symanzik equations

$$\frac{d\,g(t)}{d\,t} = \beta(g(t))$$

$$t = \log \frac{Q^2}{\mu_R^2}$$

QED **QCD**



dimensionless coupling g → renormalizable field theory

invariance of physics from renormalisation scale μ_R → Callan-Symanzik equations

$$g \to \alpha_{\rm em} = \frac{e^2}{4\pi}$$

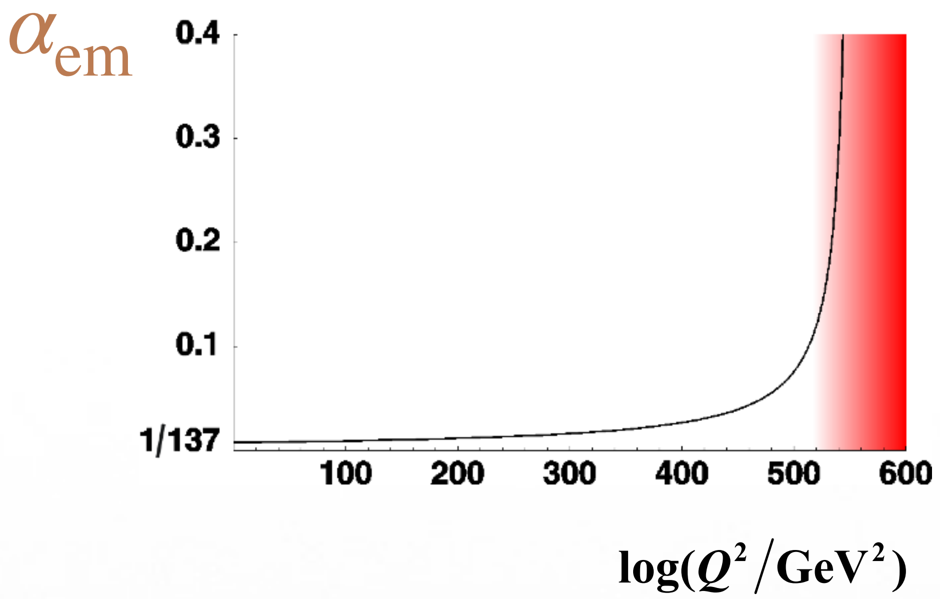
$$\beta_{\rm QED} = \frac{e^3}{12\pi^2} > 0$$

$$\frac{d\,g(t)}{d\,t} = \beta(g(t))$$

$$t = \log \frac{Q^2}{\mu_R^2}$$

running of coupling constant

QED



QCD

dimensionless coupling g → renormalizable field theory

invariance of physics from renormalisation scale μ_R → Callan-Symanzik equations

$$g \to \alpha_{em} = \frac{e^2}{4\pi}$$

$$\beta_{QED} = \frac{e^3}{12\pi^2} > 0$$

$$\frac{d\, g(t)}{d\, t} = \beta(g(t))$$

$$t = \log \frac{Q^2}{\mu_R^2}$$

$$g \to \alpha_s = \frac{g^2}{4\pi}$$

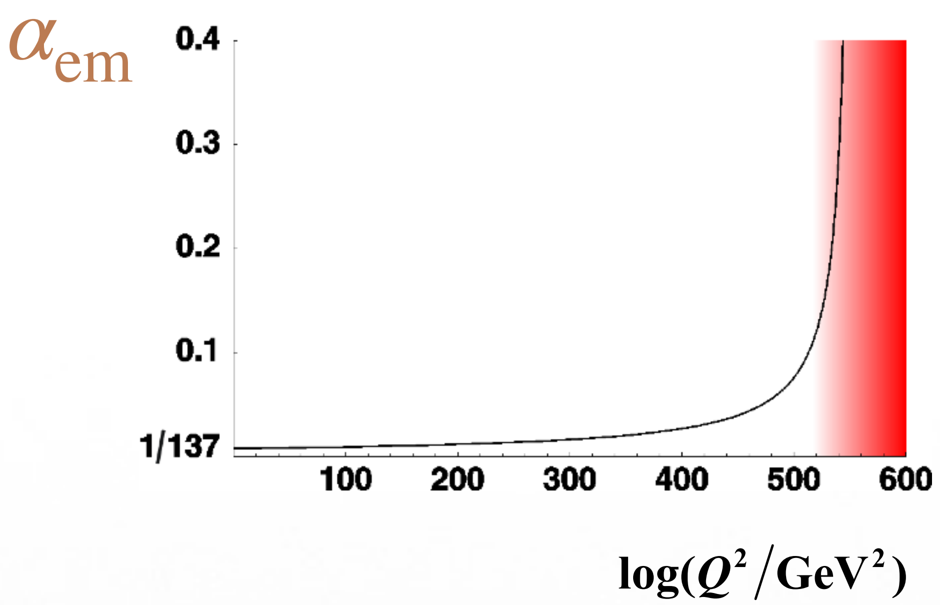
$$\beta_{QCD} = -\,\alpha_s^2\,(\beta_0 + \beta_1\,\alpha_s + \dots)$$

$$\beta_0 = \frac{1}{4\pi}\left(11 - \frac{2n_f}{3}\right)$$

$$\alpha_s(Q^2) = \frac{1}{\beta_0\,\log(Q^2/\Lambda^2)}\left(1 - \frac{\beta_1}{\beta_0}\,\frac{\log\log(Q^2/\Lambda^2)}{\log(Q^2/\Lambda^2)}\right)$$

QED

QCD



dimensionless coupling $g \to$ renormalizable field theory

invariance of physics from renormalisation scale $\mu_R \to$ Callan-Symanzik equations

$$\frac{d\,g(t)}{d\,t} = \beta(g(t))$$

$$t = \log \frac{Q^2}{\mu_R^2}$$

QED:

$$g \to \alpha_{\text{em}} = \frac{e^2}{4\pi}$$

$$\beta_{\text{QED}} = \frac{e^3}{12\pi^2} > 0$$

QCD:

$$g \to \alpha_s = \frac{g^2}{4\pi}$$

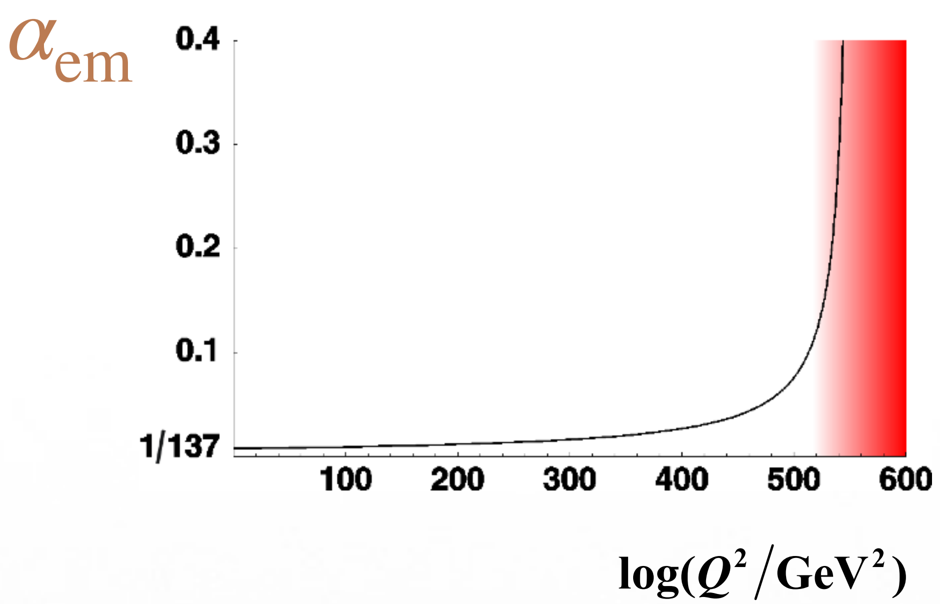
$$\beta_{\text{QCD}} = -\,\alpha_s^2\,(\beta_0 + \beta_1\,\alpha_s + \ldots)$$

< 0

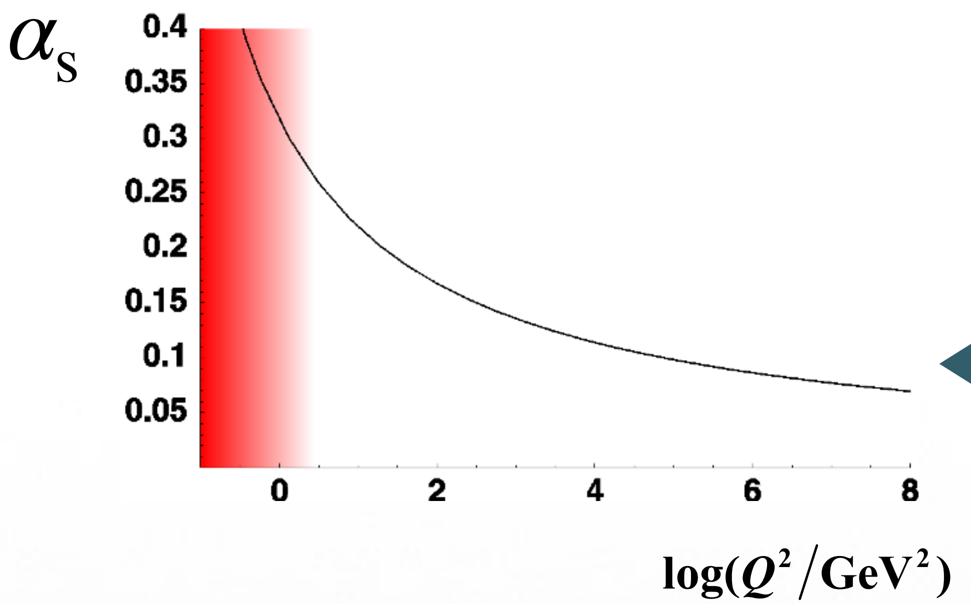
$$\beta_0 = \frac{1}{4\pi}\left(11 - \frac{2n_f}{3}\right)$$

$$\alpha_s(Q^2) = \frac{1}{\beta_0\,\log(Q^2/\Lambda^2)}\left(1 - \frac{\beta_1}{\beta_0}\,\frac{\log\log(Q^2/\Lambda^2)}{\log(Q^2/\Lambda^2)}\right)$$

QED



QCD



asymtotic freedom

exclusive feature of 4-dim renormalizable non-abelian gauge field theories

invariance of physics from renormalisation scale $\mu_R \to$ Callan-Symanzik equations

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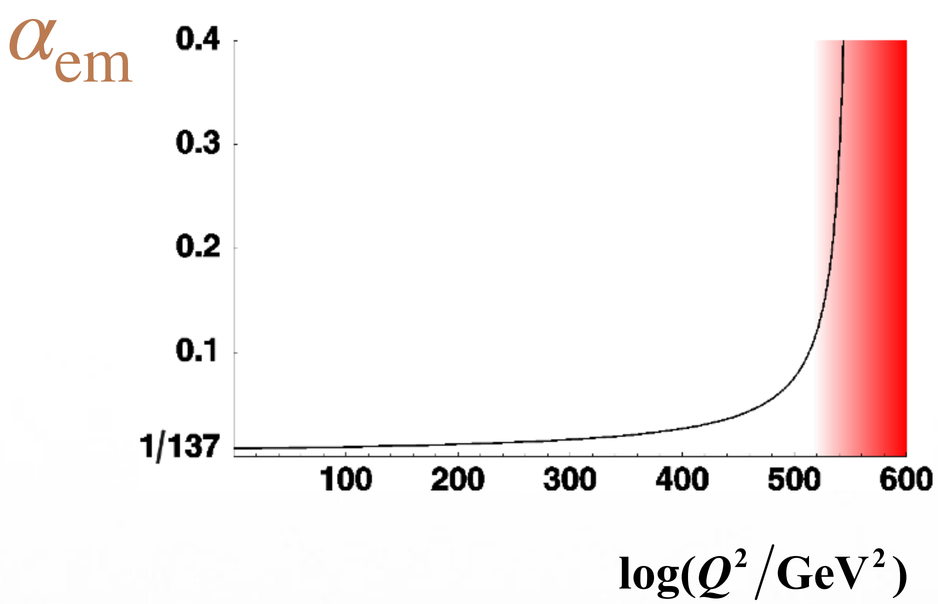

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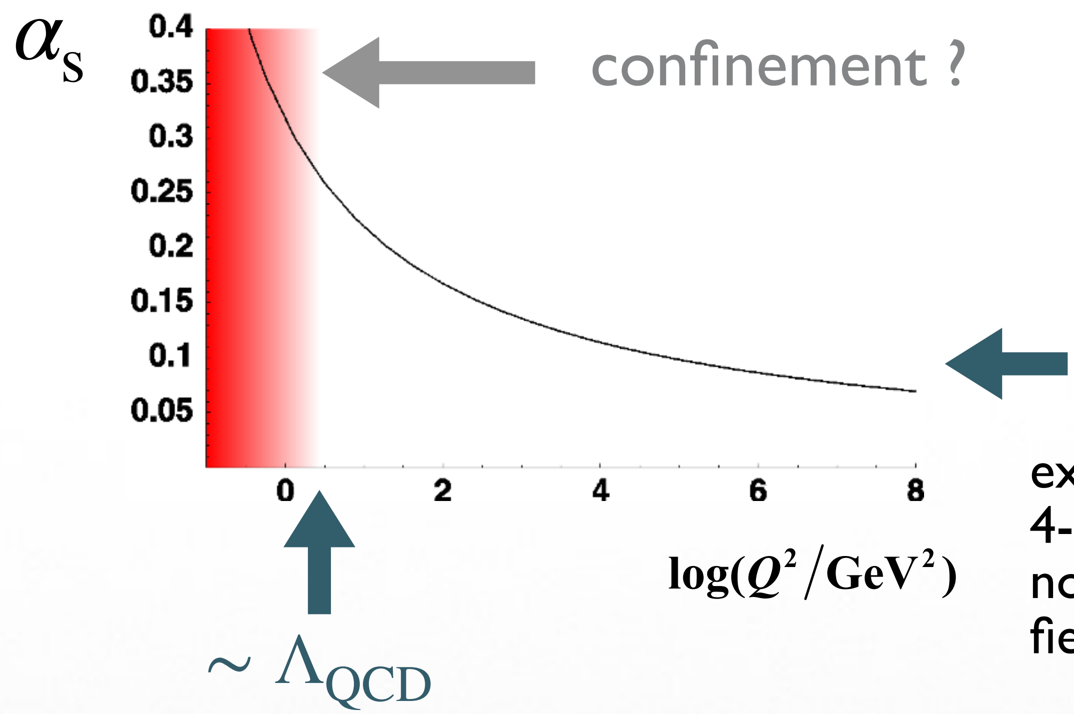

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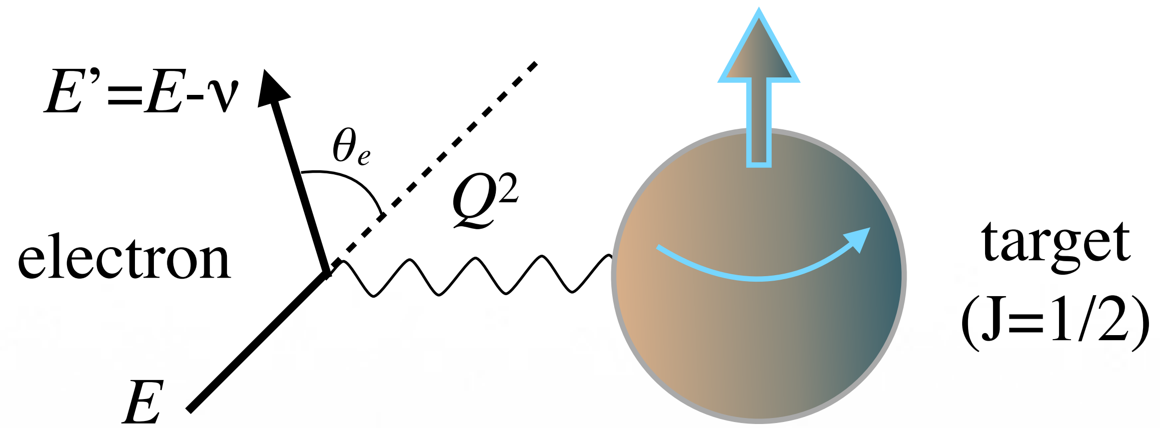
Outline

❷ Factorization Theorems,
evolution equations and all that

Example of probe - target interaction:
anelastic scattering of electron on a spin-$1/2$ target

$Q^2 = \mathbf{q}^2 - \nu^2$



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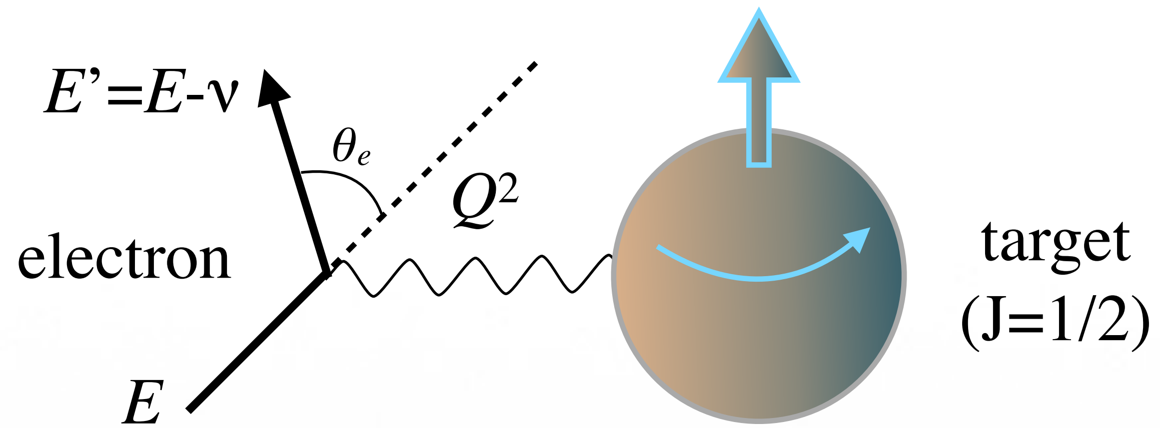

cross section differential in the solid angle Ω of the scattered electron:

Rosenbluth formula
$$\frac{d\sigma}{d\Omega} = \sigma_{\rm Mott} \left[W_2(\nu, Q^2) + 2\, W_1(\nu, Q^2)\, \tan^2 \frac{\theta_e}{2} \right]$$

Coulomb elastic scattering from pointlike charge
$$\sigma_{\rm Mott} = \frac{4Z^2\alpha^2}{Q^4} \frac{E'^3}{E} \cos^2 \frac{\theta_e}{2}$$

internal structure of target

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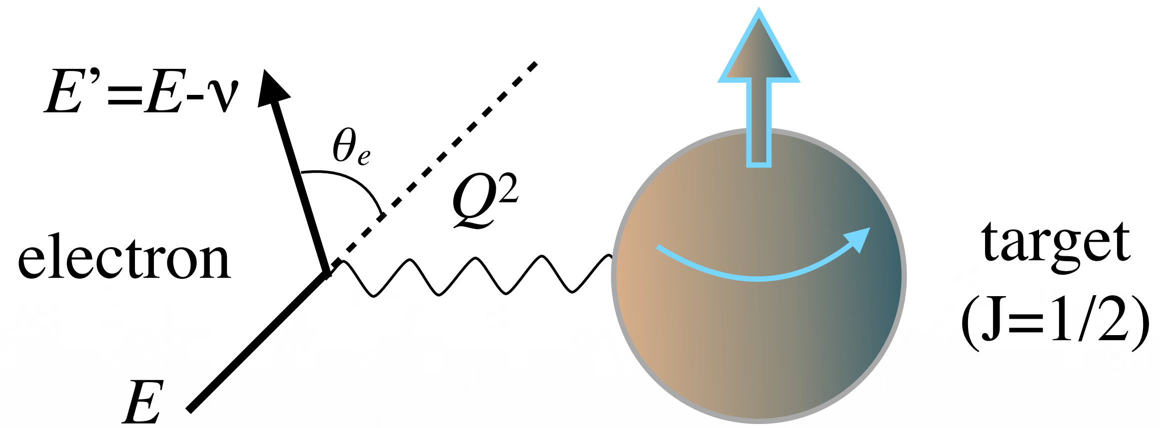

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Factorization between target structure and interaction with probe

Factorization Theorems

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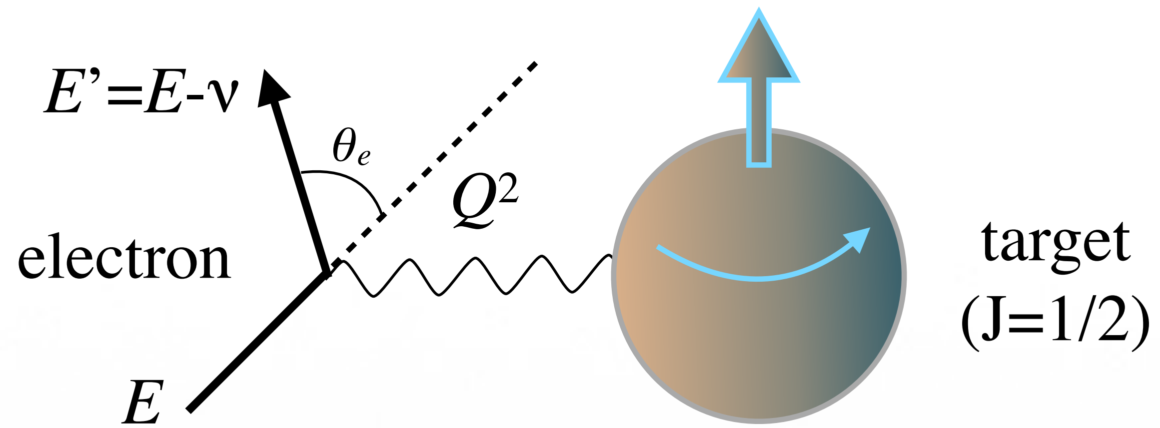
$$\nu = \frac{Q^2}{2M}$$



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Consider now the deep-inelastic regime: the basics of the parton model

$$Q^2 = -q^2 \to \infty \quad (\gg M)$$

$$x = \frac{Q^2}{2P \cdot q} \quad \text{fixed}$$

The Parton model

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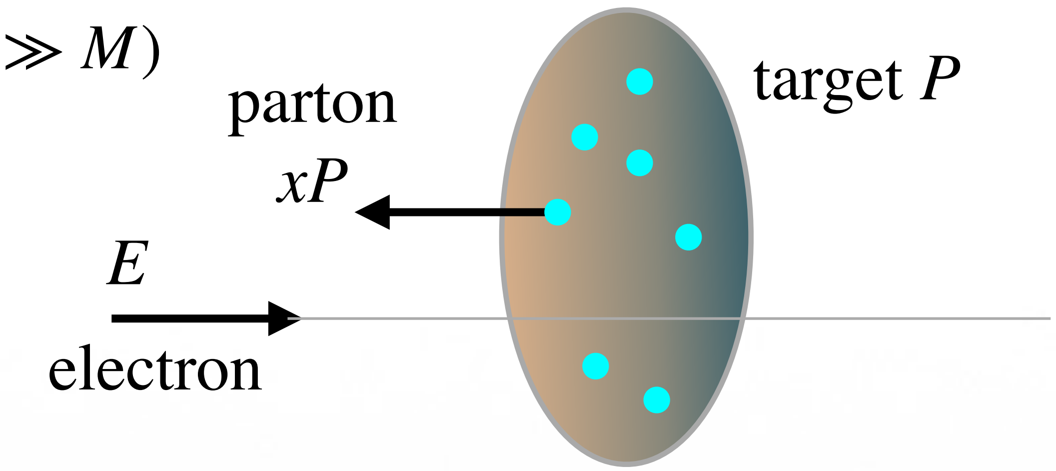

target = ensemble of partons carrying fraction $0 < x < 1$ of P moving collinear with target

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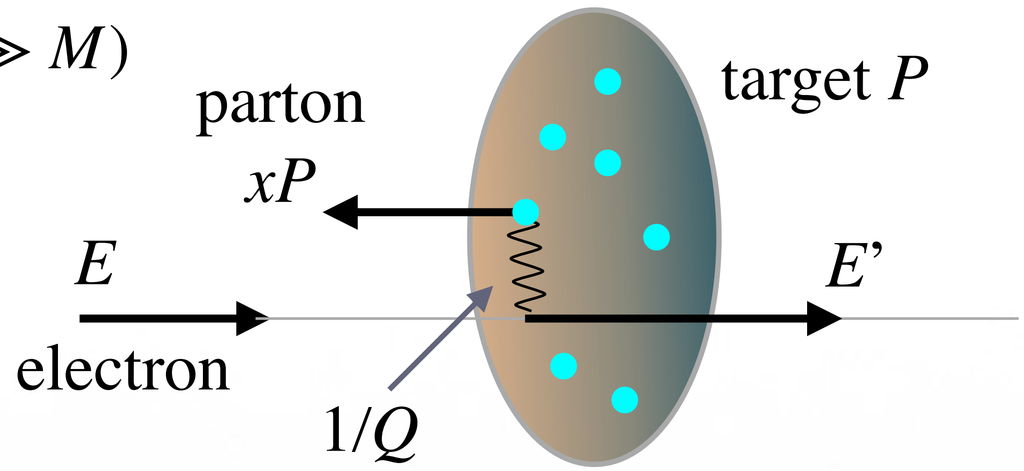

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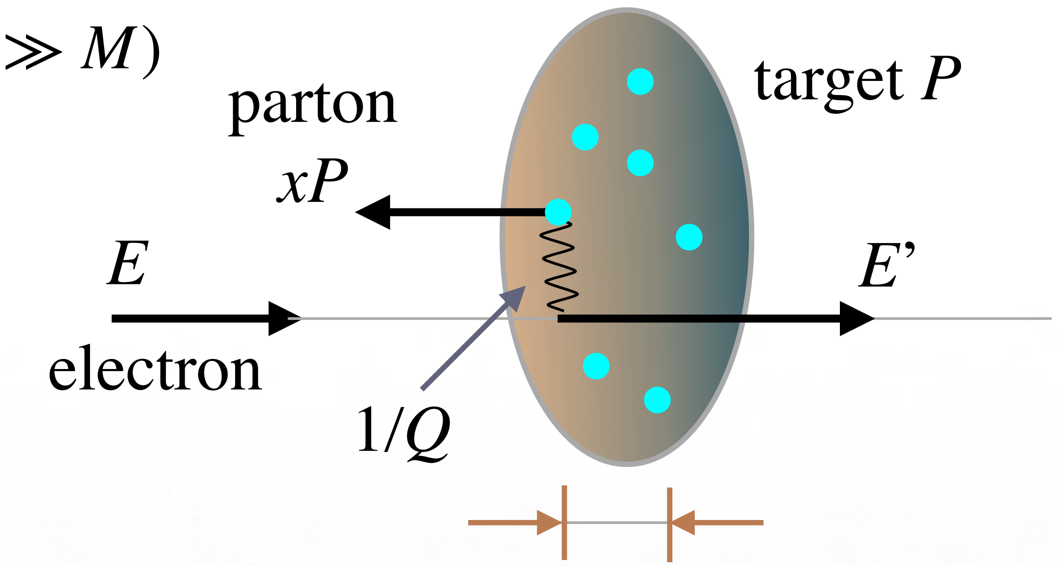

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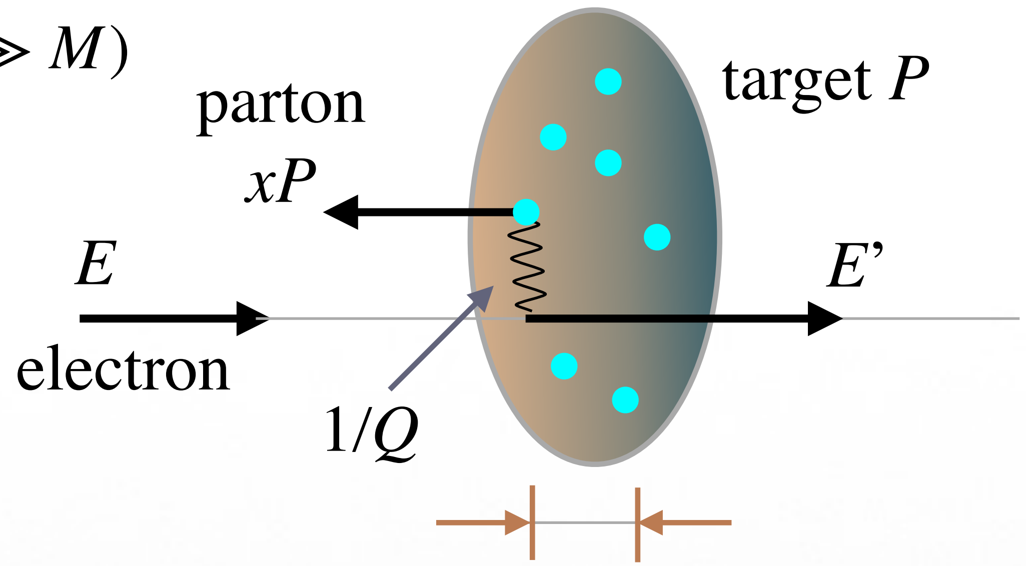

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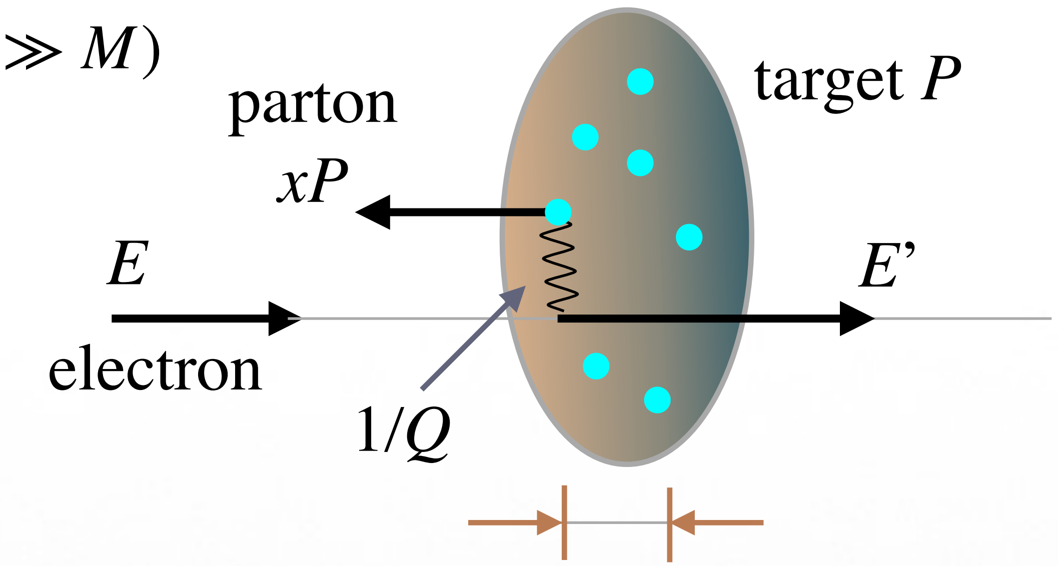

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- hard electron-parton scattering happens on much shorter time scale than hadronization of target remnants into the unobserved hadronic final state
- probability of finding another parton close to the struck parton $\sim$ $\dfrac{\text{area of hard scattering}}{\text{target impact surface}}$ $\quad \dfrac{1/Q^2}{\pi R^2} \xrightarrow{Q^2\to\infty} 0$

differential cross section:

$$\frac{d\sigma}{dx\,dQ^2} = \sum_f \left(\frac{d\hat{\sigma}}{dQ^2}\right)_f e_f^2\ \phi_f(x)$$

incoherent sum of hard electron-parton scatterings differential cross section:

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Factorization between target "structure" (parton density) and elementary interaction of partons with electron probe

From Parton model to QCD

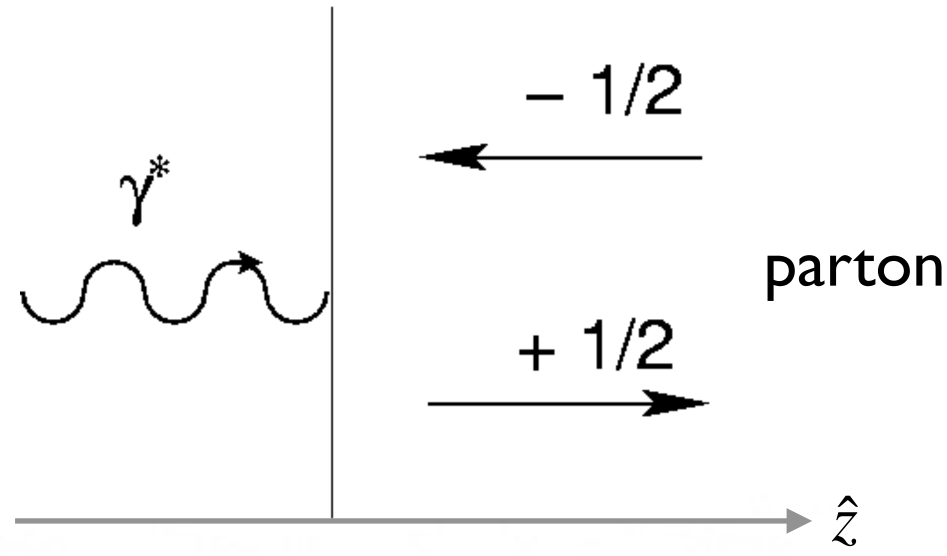
Parton model

Rosenbluth formula: 2 structure functions

Callan-Gross relation $F_2(x) = 2x\, F_1(x)$ (= spin-$1/2$ quarks absorb only T-polarized γ^*)

Scattering in the Breit frame

c.m. frame with
no energy transfer



electromagnetic interaction conserves helicity along $\hat{z}$

Callan-Gross relation

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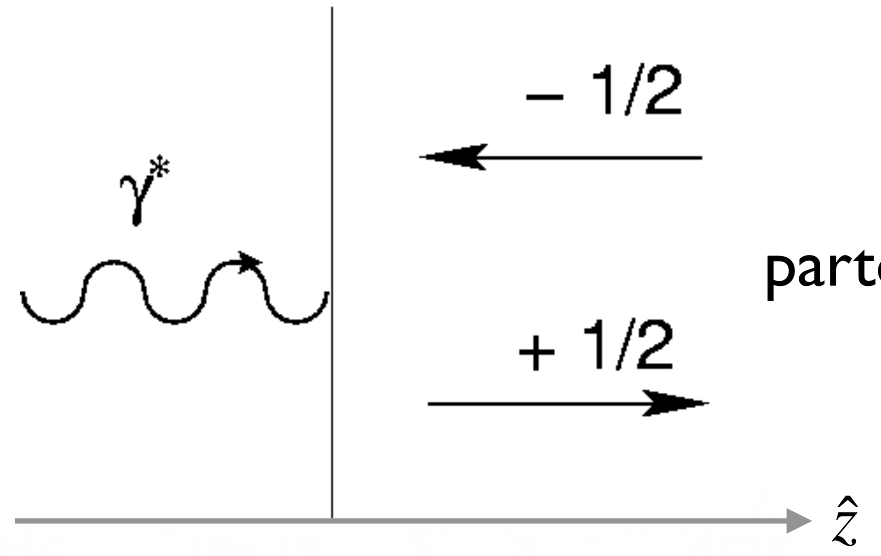
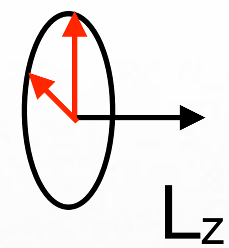


electromagnetic interaction conserves helicity along $\hat{z}$

transverse polarization of γ^* compensates the helicity variation Δh=+1 ✔



longitudinal polarization of γ^* does not ✘
$\Rightarrow F_L = F_2 - 2xF_1 = 0$

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Problems

- measured violations of sum rule $\sum_f \int_0^1 dx\;\, x\, \phi_f(x) \sim 0.5$

Momentum sum rule

SLAC-MIT measurement of DIS on proton and effective neutron targets (~ 1969-72)

$\int dx \approx$ discrete sum

$$\frac{1}{2}\int dx\,\left[F_2^p(x)+F_2^n(x)\right]=0.14\pm 0.005$$

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$$F_2^n(x) \approx \frac{1}{9}\, x\, \left[4\left(d(x) + \bar{d}(x)\right) + u(x) + \bar{u}(x)\right]$$

Then,

$$\frac{1}{2}\int_0^1 dx\, \left[F_2^p(x) + F_2^n(x)\right] = \frac{1}{2}\,\frac{5}{9}\int_0^1 dx\, x\, \left[u(x) + \bar{u}(x) + d(x) + \bar{d}(x)\right] \approx 0.28 \sum_f \int_0^1 dx\, x\, \phi_f(x)$$

Momentum sum rule

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compatible only if the (anti-)quarks carry 50% of momentum

later confirmed by Gargamelle (CERN, ~ 1975):

$$\frac{1}{2}\int dx\,\left[F_2^{\nu p}(x)+F_2^{\nu n}(x)\right]\underset{\nu n\leftrightarrow\bar{\nu}p}{=}\int dx\,x\,\left[u(x)+\bar{u}(x)+d(x)+\bar{d}(x)\right]=0.49\pm 0.7$$

From Parton model to QCD

Parton model

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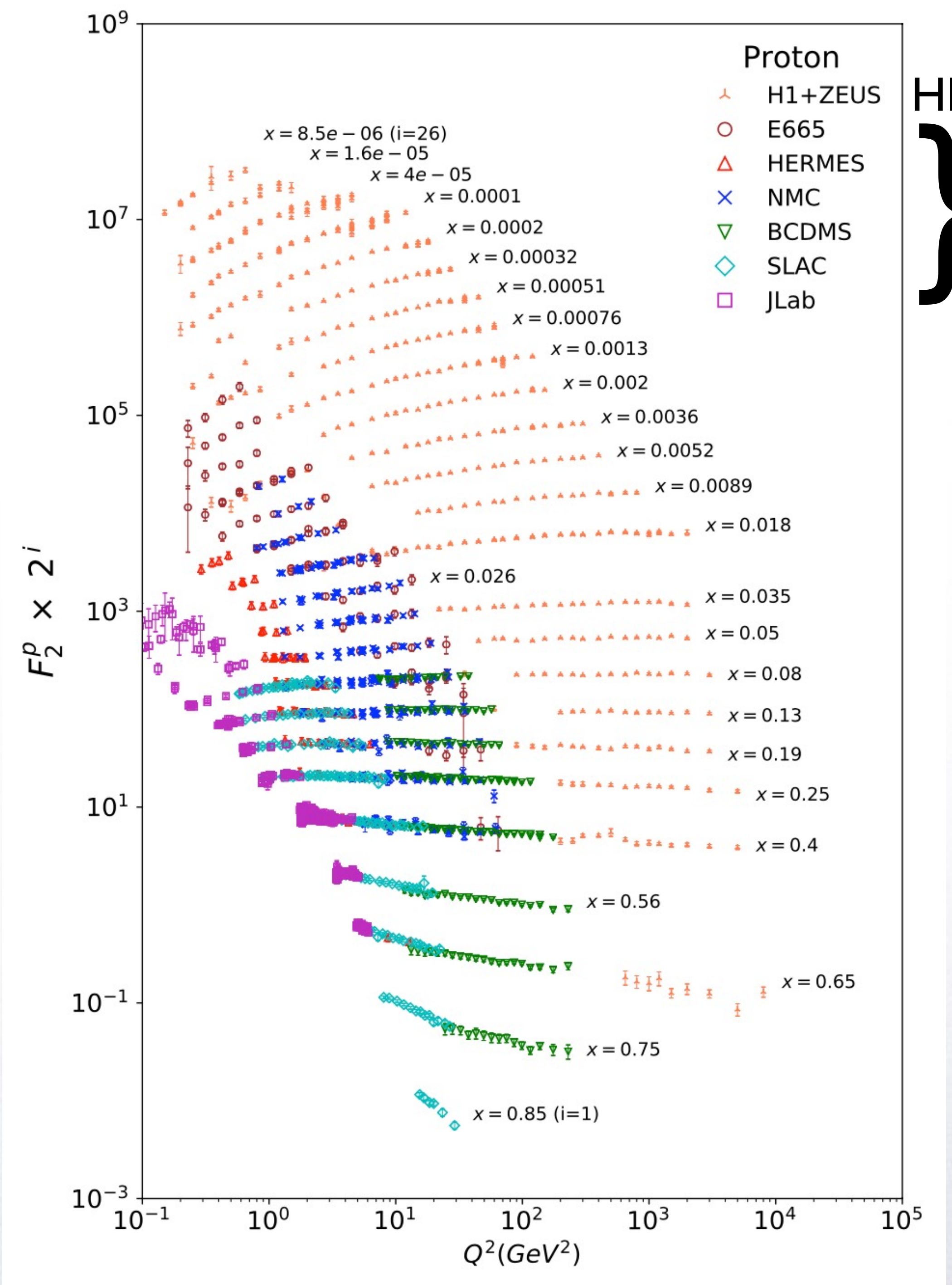
y = electron inelasticity $\sim \frac{\nu}{E}$

scaling: $F_2 \neq F_2(Q^2)$

Problems

- measured violations of sum rule $\sum_f \int_0^1 dx\;\, x\, \phi_f(x) \sim 0.5$
- observed scaling violations: F_2 depends also on Q^2
- breaking of Callan-Gross $F_L = F_2 - 2x\, F_1 \neq 0$

(with gluons, quarks absorb also L-polarized γ^*)



HERA

fixed target

From Parton model to QCD

Parton model

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$$F_i(x,Q^2) = \sum_f e_f^2 \int_x^1 \frac{d\xi}{\xi}\, d\hat{\sigma}_{i,f}\left(\alpha_s, \frac{x}{\xi}, \frac{Q^2}{\mu_F^2}\right)\, \phi_f(\alpha_s, \xi, \mu_F) \equiv \sum_f e_f^2\; d\hat{\sigma}_{i,f} \otimes \phi_f$$

usually $\mu_R^2 = \mu_F^2 = Q^2$

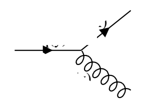
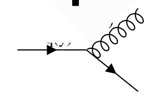
$$d\hat{\sigma}_{i,f} = d\hat{\sigma}_{i,f}^{(0)} + \frac{\alpha_s}{4\pi}\, d\hat{\sigma}_{i,f}^{(1)} + \dots$$

QCD collinear factorization theorem at scale μ_F, valid at all orders

Evolution equations

$$F_i(x,Q^2) = \sum_f e_f^2 \int_x^1 \frac{d\xi}{\xi}\, d\hat{\sigma}_{i,f}\left(\alpha_s, \frac{x}{\xi}, \frac{Q^2}{\mu_F^2}\right) \phi_f(\alpha_s,\xi,\mu_F) \equiv \sum_f e_f^2\, d\hat{\sigma}_{i,f} \otimes \phi_f$$

Physics does not depend on fictitious scale μ_F : DGLAP evolution equations

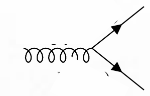
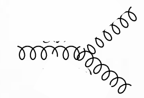



Describe how μ_F dependence of ϕ_f compensates the one of $d\hat{\sigma}_{i,f}$

$\phi_f \to$ quark q $$\frac{dq(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left[q \otimes P_{qq} + g \otimes P_{qg}\right]$$

$\phi_f \to$ gluon g $$\frac{dg(x,Q^2)}{d\log Q^2} = \frac{\alpha_s}{2\pi}\left[q \otimes P_{gq} + g \otimes P_{gg}\right]$$

splitting functions

Evolution equations

$$F_i(x,Q^2) = \sum_f e_f^2 \int_x^1 \frac{d\xi}{\xi}\, d\hat{\sigma}_{i,f}\left(\alpha_s, \frac{x}{\xi}, \frac{Q^2}{\mu_F^2}\right) \phi_f(\alpha_s, \xi, \mu_F) \equiv \sum_f e_f^2\, d\hat{\sigma}_{i,f} \otimes \phi_f$$

Physics does not depend on fictitious scale μ_F : DGLAP evolution equations

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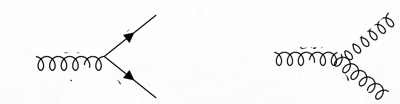
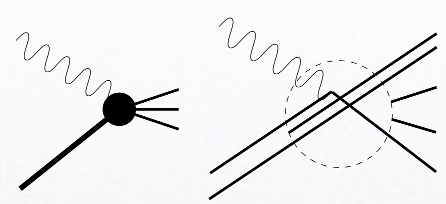
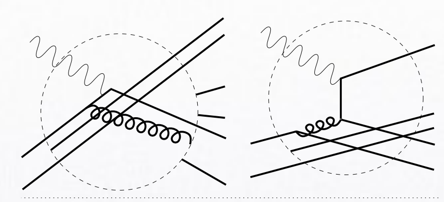


partons are part of "structure" in ϕ_f $< \mu_F^2 <$ partons are part of hard interaction $\to Q^2$



proton

valence structure



QCD Compton

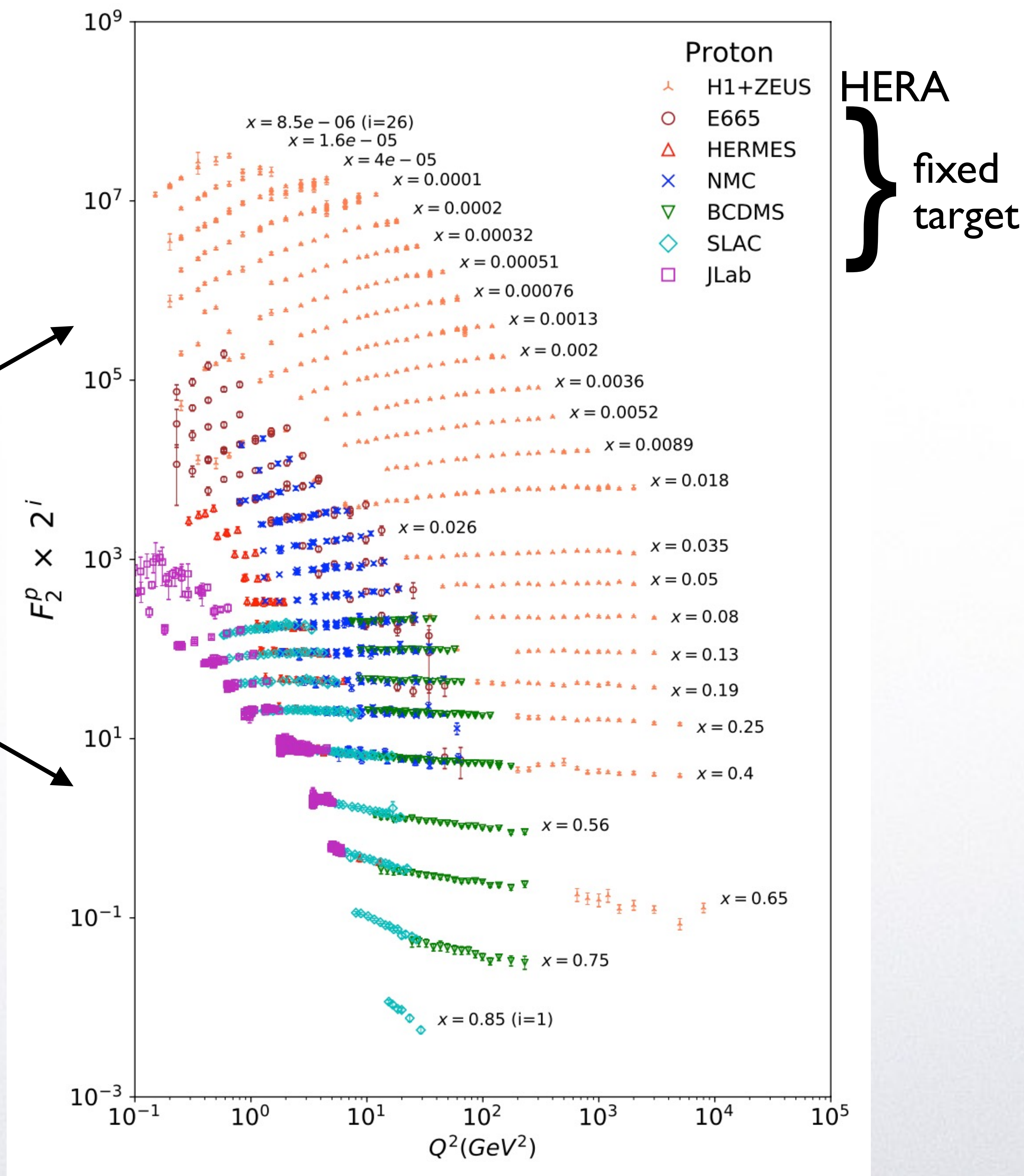
γ - gluon fusion

increasing resolving power $\mu_F^2 = Q^2$ → larger number of partons with lower x

For increasing Q^2, larger number of partons with lower x

F_2 grows with Q^2 at low x

F_2 drops with Q^2 at high x



Universality of partonic structure

DGLAP evolution equations describe how μ_F dependence of ϕ_f compensates the one of $d\hat{\sigma}_{i,f}$ to make observables F_i independent from μ_F

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partonic structure is independent of the process → "universal"

calculable hard interaction depends on the process

universality of Parton Distribution Functions (PDF)

perturbative "Wilson coefficients", calculable at given order in $(\alpha_s)^n$

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same target : ("structure")

process #1 at scale Q_1: $(d\sigma)_1 = (d\hat{\sigma}(Q_1))_1 \otimes \mathrm{PDF}(Q_1)$

evolution $Q_1 \to Q_2$ →

process #2 at scale Q_2: $(d\sigma)_2 = (d\hat{\sigma}(Q_2))_2 \otimes \mathrm{PDF}(Q_2)$

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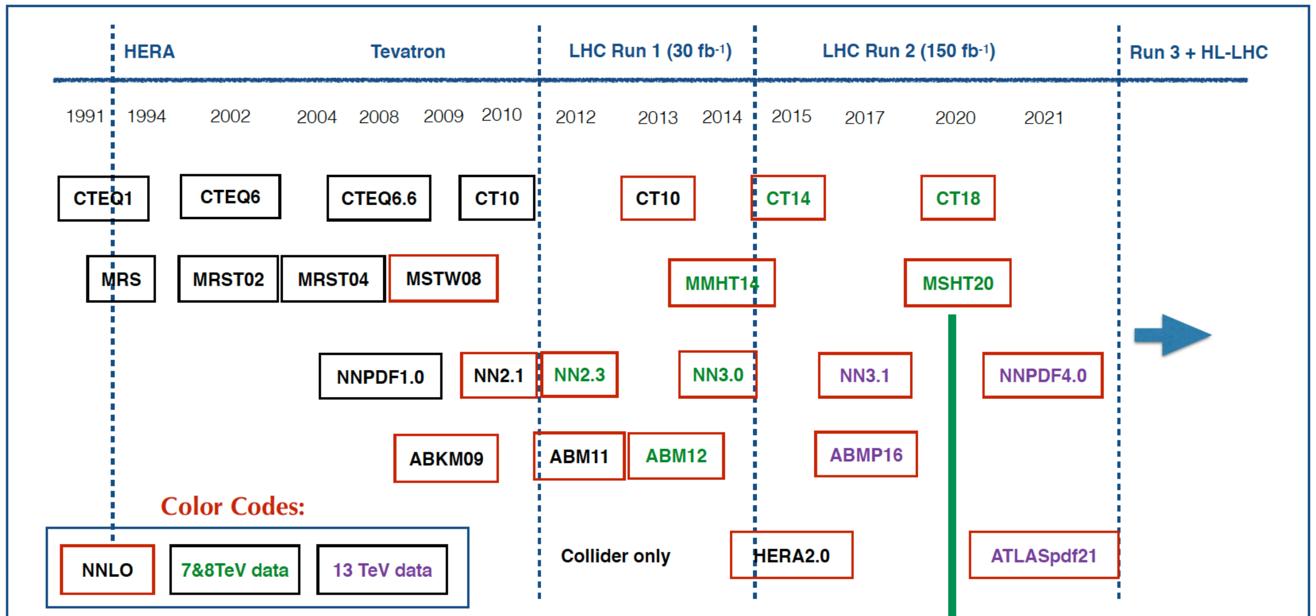
Fitting experimental data at different scales → extract PDF $\phi_f(x, Q^2)$

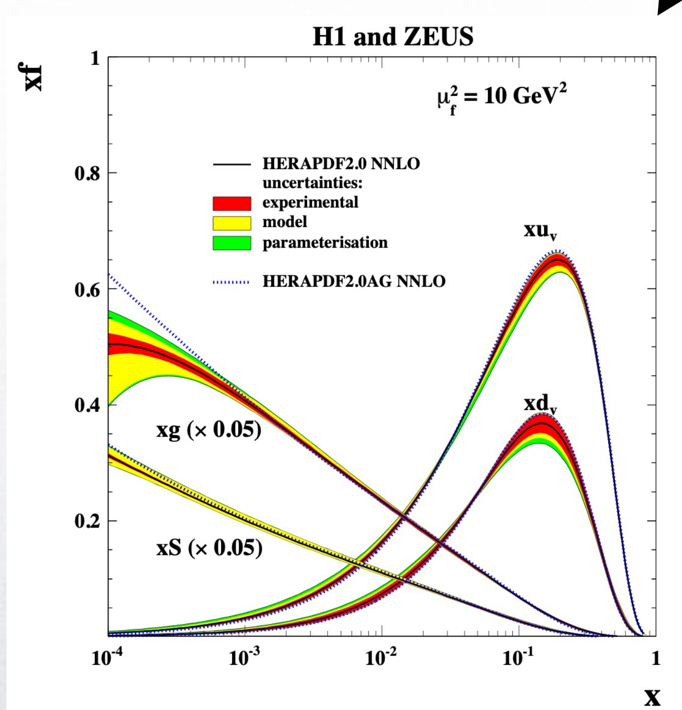
Parton Distribution Functions

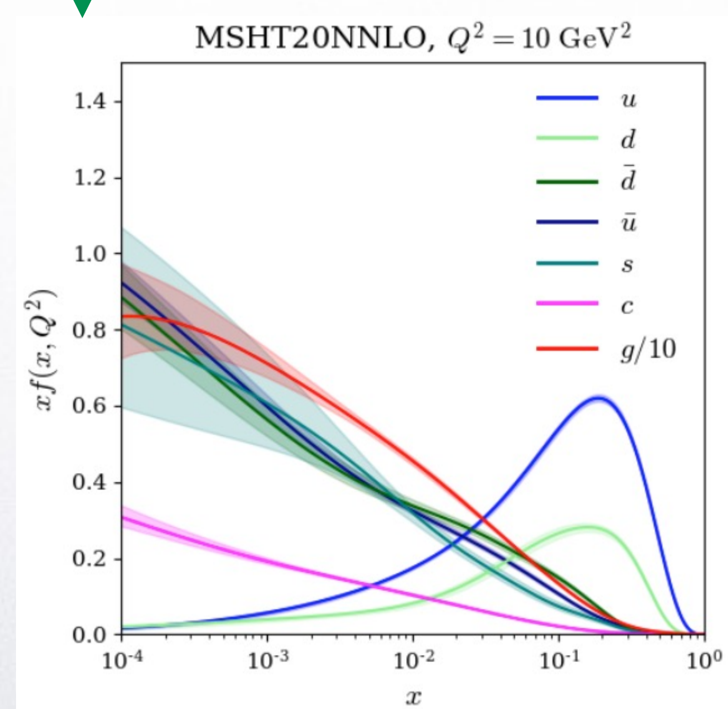


Many groups

NNLO accuracy is nowadays the standard

$$d\hat{\sigma}_{i,f} = d\hat{\sigma}^{(0)}_{i,f} + \frac{\alpha_s}{4\pi}\, d\hat{\sigma}^{(1)}_{i,f} + \left(\frac{\alpha_s}{4\pi}\right)^2 d\hat{\sigma}^{(1)}_{2,f} + \dots$$





but already approximate N^{3}LO results

arXiv:2207.04739