Primordial black holes from single-field inflation and their observational imprints

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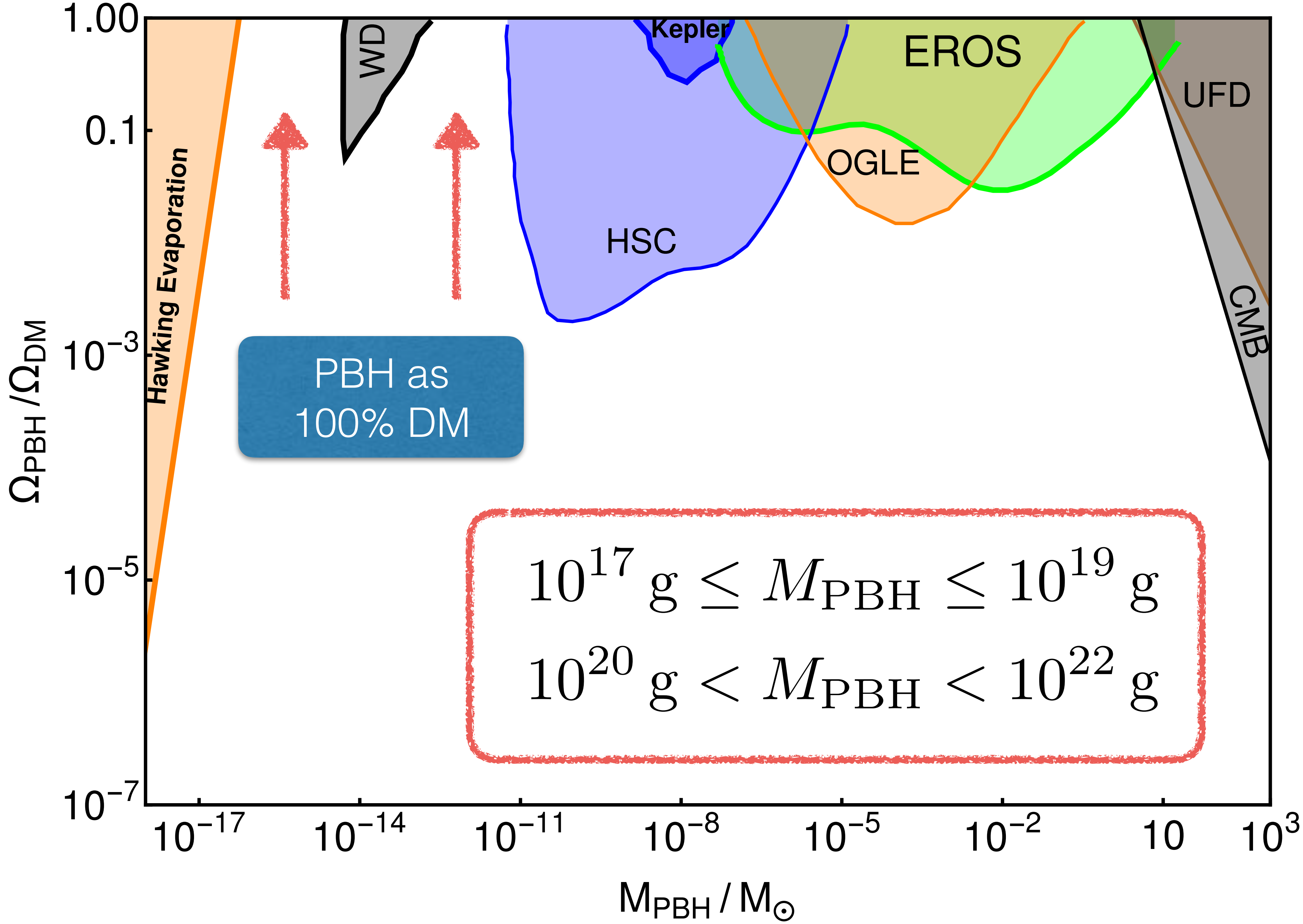
Outline of the talk

- Why Primordial Black Holes (PBH) ?
- PBH generation mechanisms
  - Single field inflation — inflection point models
- Primordial power spectra and PBH mass fraction — Press-Schechter formalism
- Observational imprints — induced GWs
- Ultralight PBHs with Advanced LIGO
- Conclusions

Why Primordial Black Holes (PBH) ?

- A novel and promising candidate for cold dark matter
- Interesting objects generated in the early universe e.g. during the inflationary phase
- Non-baryonic, non-relativistic and nearly collisionless
- No new physics required !
- LIGO detection of GWs from supermassive black holes — seeds from PBHs

PBH as DM — Current constraints



$$10^{17}\,\mathrm{g} \leq M_{\rm PBH} \leq 10^{19}\,\mathrm{g}$$

$$10^{20}\,\mathrm{g} < M_{\rm PBH} < 10^{22}\,\mathrm{g}$$

PBH formation — in a nutshell

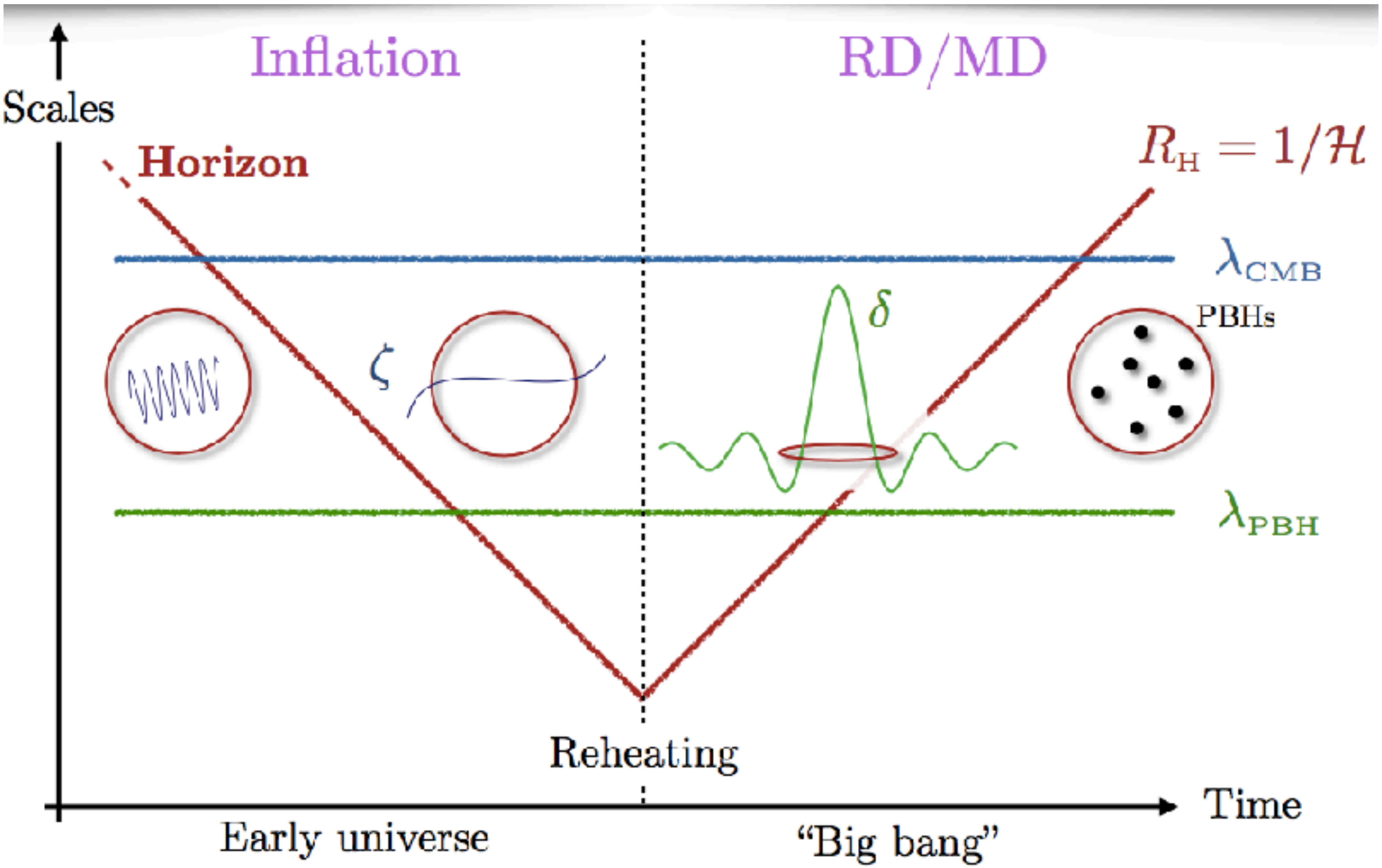


Fig. credit: G. Franciolini

PBH formation — in a nutshell

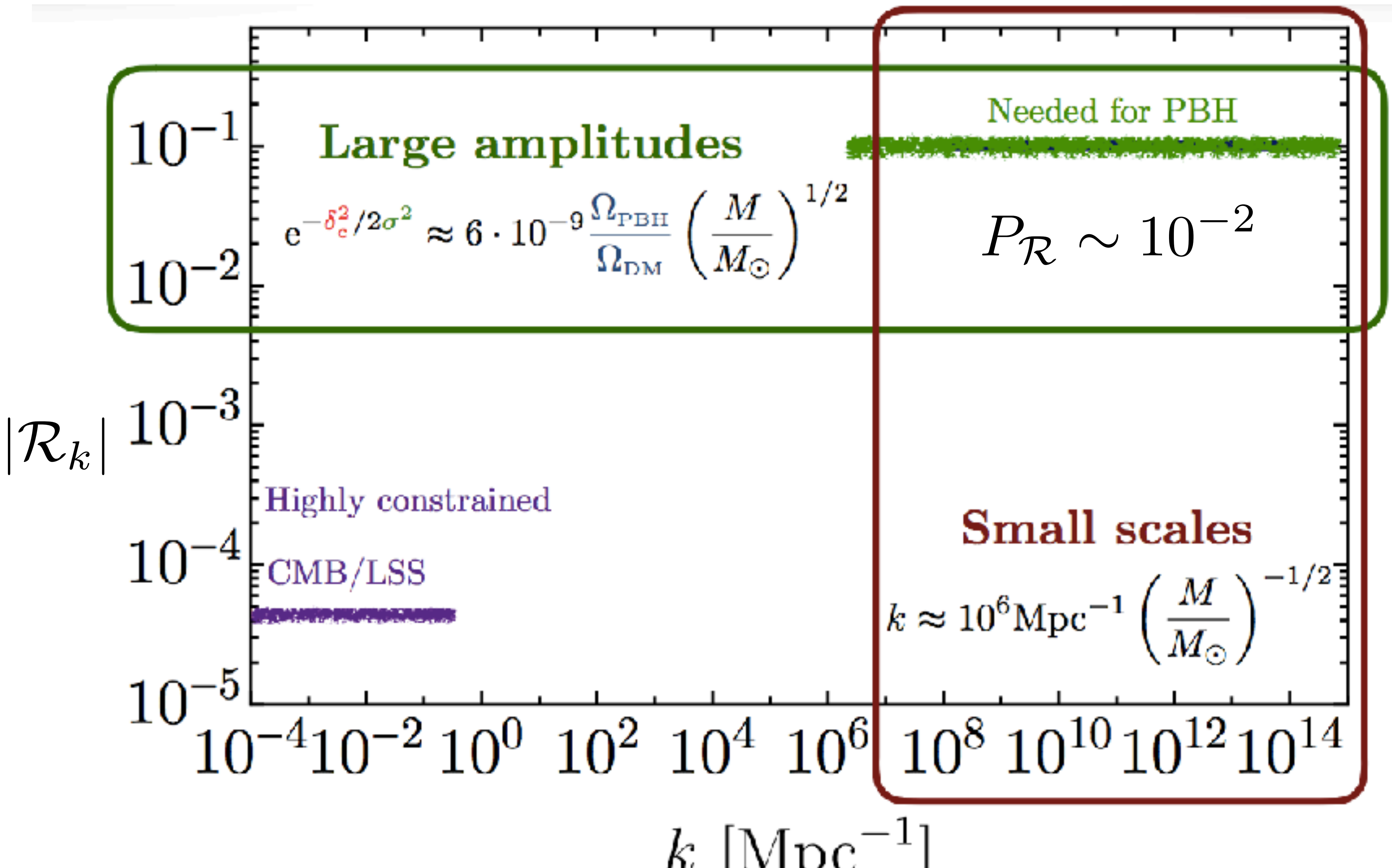
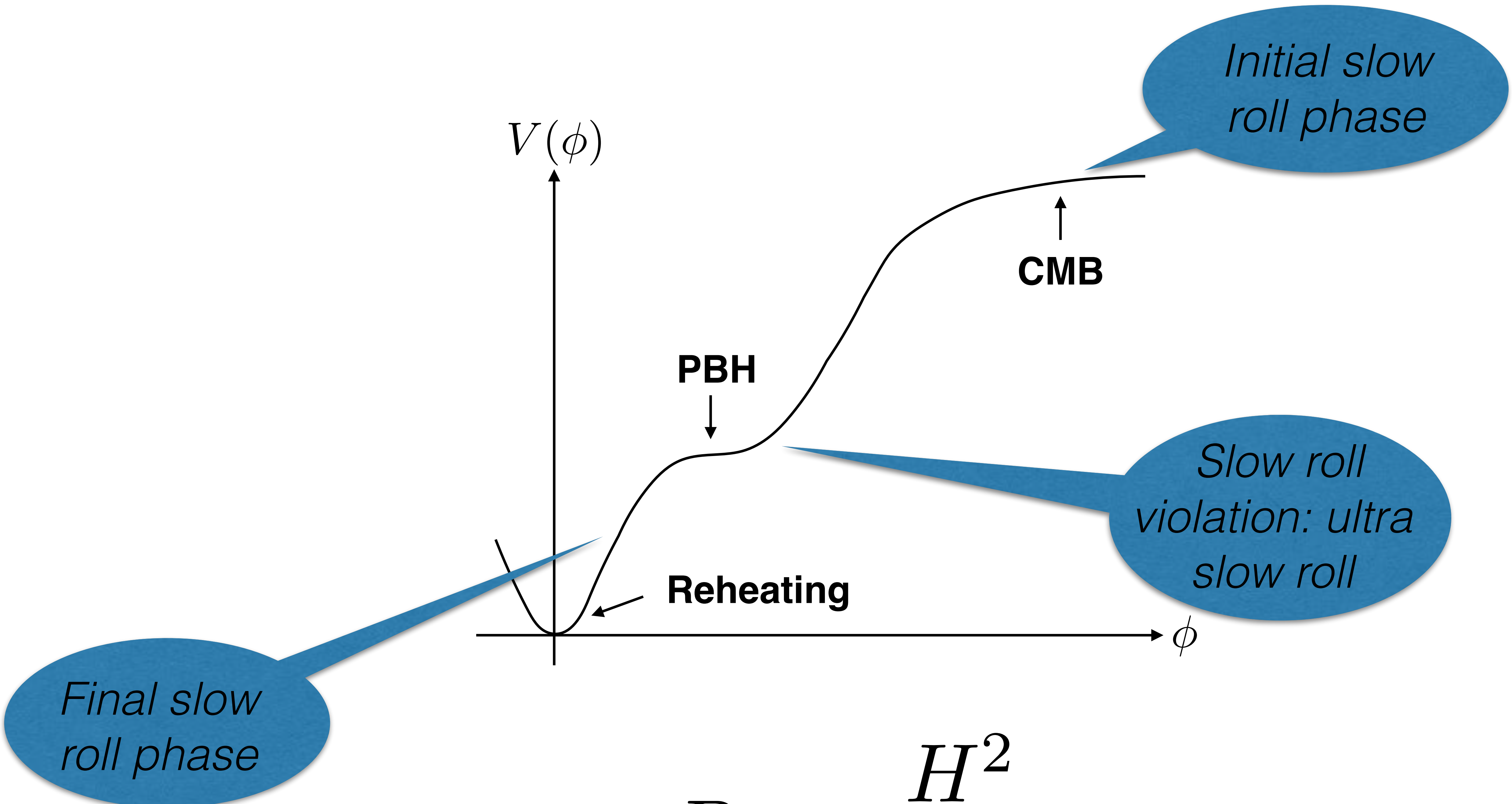


Fig. credit: G. Franciolini

PBH formation from inflation

- Single field inflation with polynomial inflection point models
- Inflation with running spectral index — but running is small !
- Preheating after inflation
- Hybrid inflation
- Inflating/axionlike curvaton
- Particle production during inflation
- Critical Higgs inflation, string inflation, thermal inflation….

Inflation — inflection point models



$$P_{\mathcal{R}} \sim \frac{H^2}{\epsilon}$$

An inflection point scenario

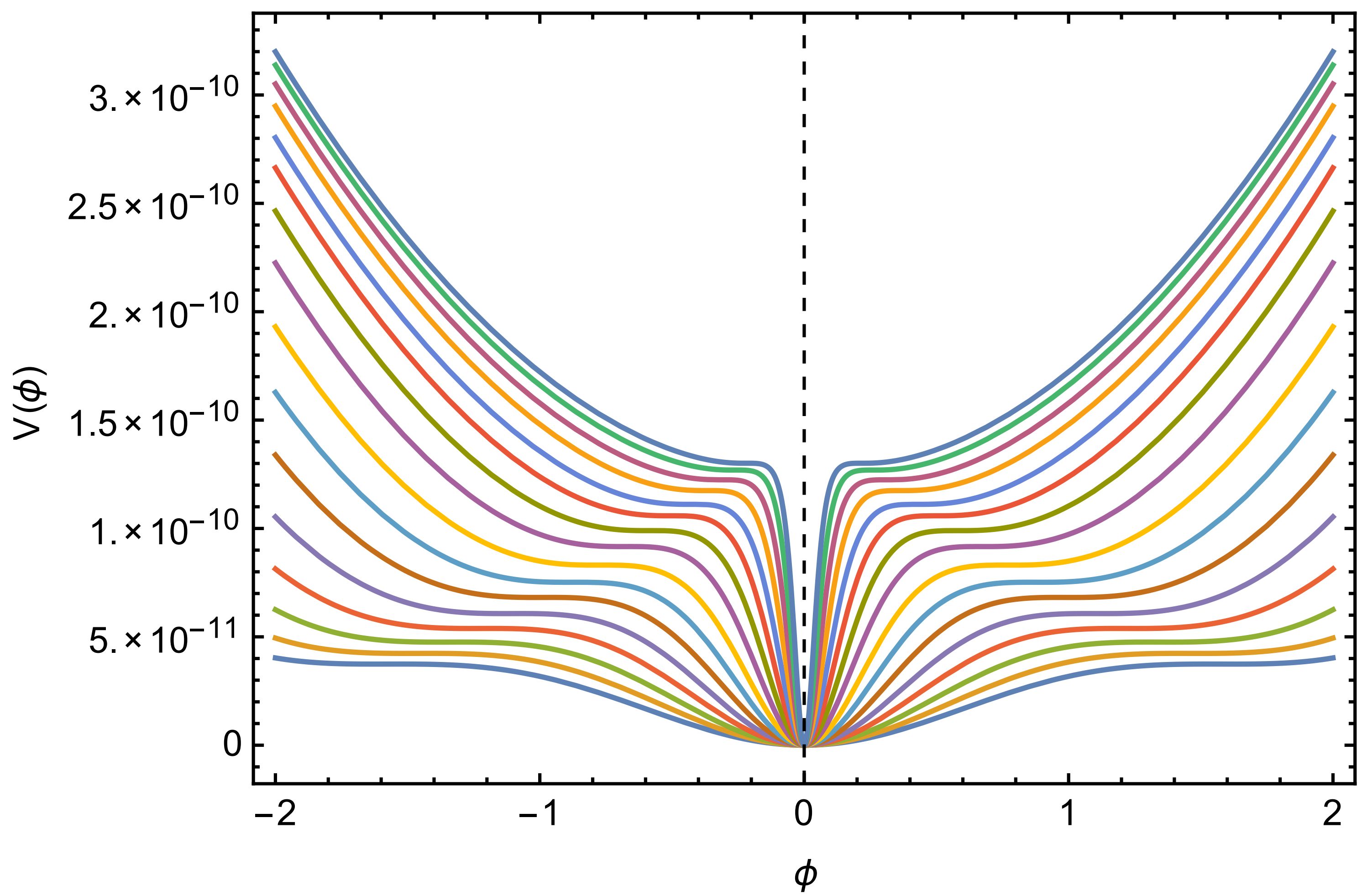
$$V(x) = V_0 \frac{ax^2 + bx^4 + cx^6}{(1 + dx^2)^2}, \qquad x = \phi/v$$

$$x \gg 1 : V(x) \simeq \frac{V_0 c}{d^2} x^2$$

$$x \ll 1 : V(x) \simeq V_0 a x^2$$

Quadratic for both large & small field values

$$r \sim 0.05$$



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Slow roll, ultra slow roll and all that…

Background

$$H^2 = \frac{V(\phi)}{M_{\rm Pl}^2(3-\epsilon)},$$

$$\frac{d^2\phi}{dN^2} + (3-\epsilon)\frac{d\phi}{dN} + \frac{1}{H^2}V'(\phi) = 0,$$

SR: $$\frac{d\phi}{dN} + \frac{1}{3H^2}V'(\phi) \simeq 0,$$

USR: $$\frac{d^2\phi}{dN^2} + 3\frac{d\phi}{dN} \simeq 0,$$

$$\epsilon \sim \exp\left[-6(N-N_i)\right]$$

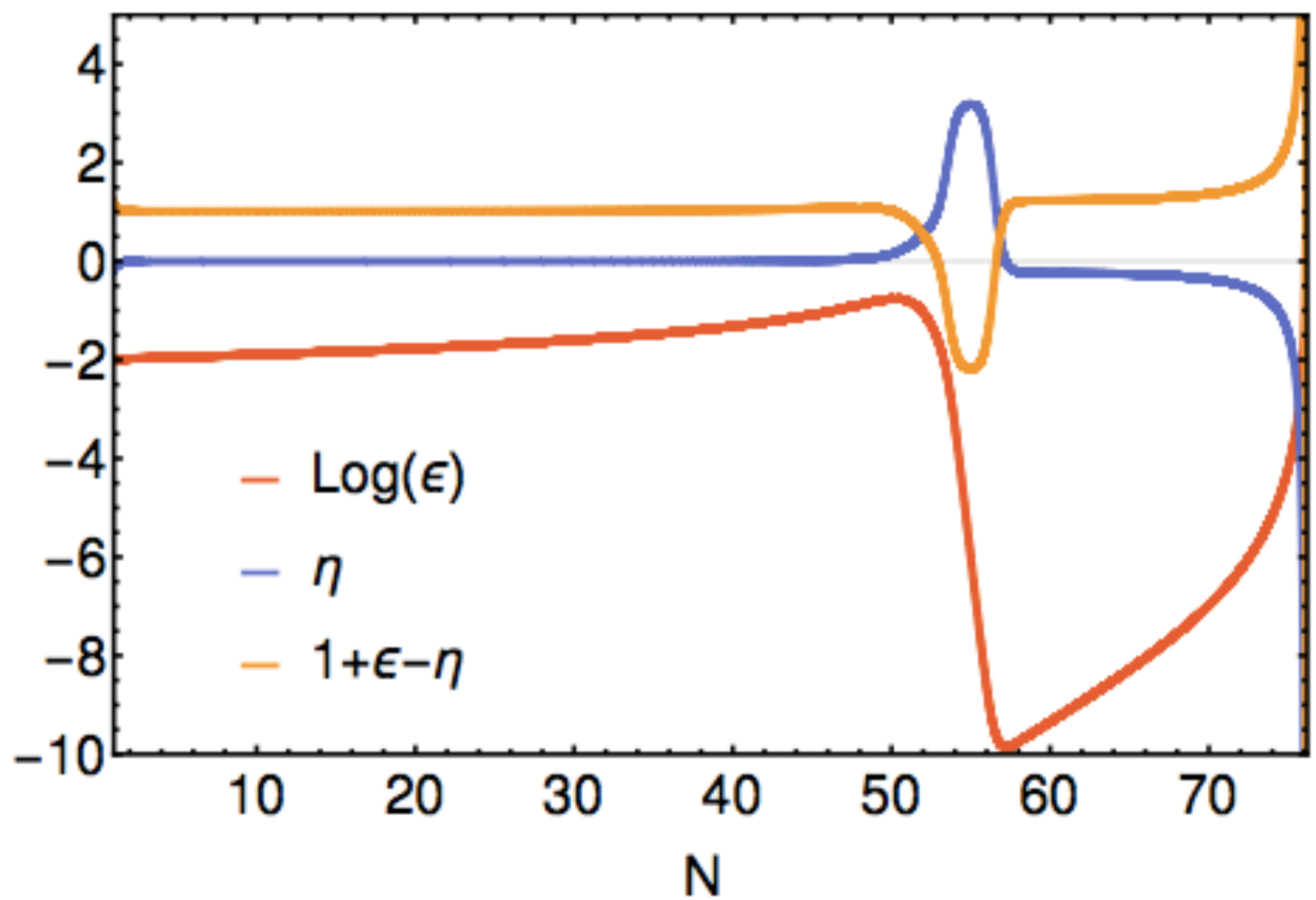
$$\eta \simeq \epsilon + (3-\epsilon) \sim 3$$

Curvature perturbations

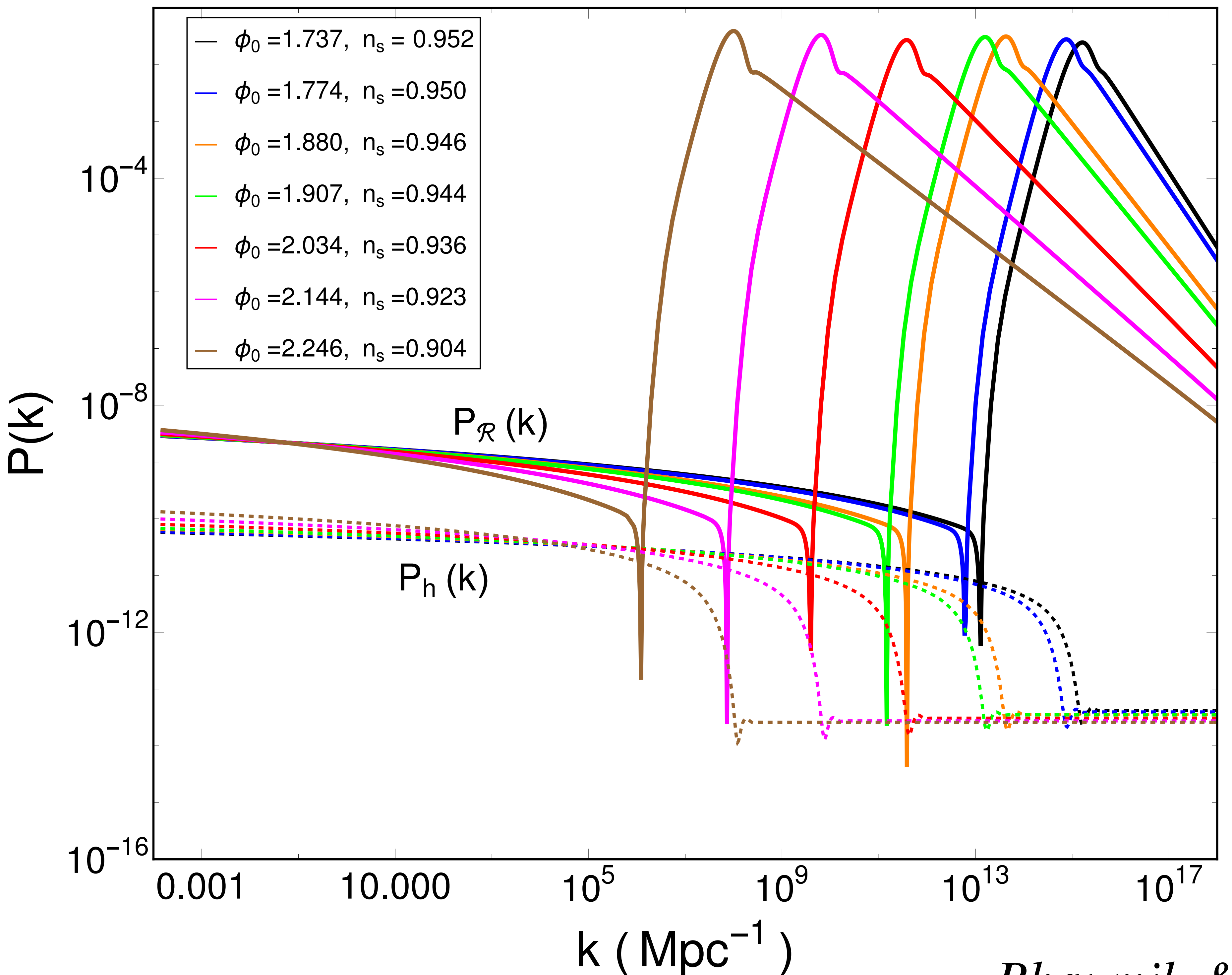
$$\mathcal{R}_k'' + 2\left(\frac{z'}{z}\right)\mathcal{R}_k' + k^2\mathcal{R}_k = 0.$$

$$\frac{z'}{z} = aH(1+\epsilon-\eta)$$

$$\mathcal{R}_k(\tau) \simeq C_1 + C_2\int\frac{d\tau}{z^2}$$



Primordial power spectra



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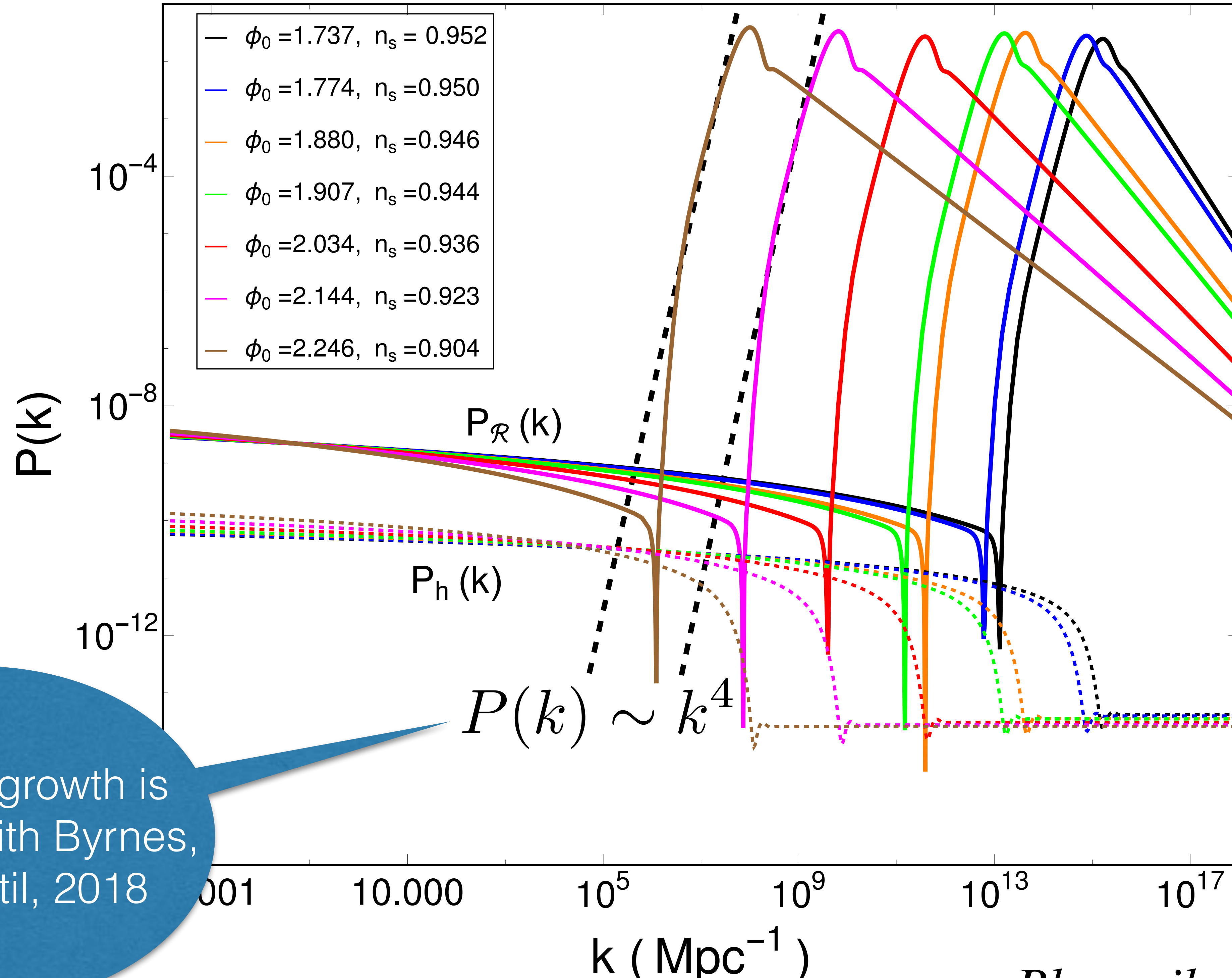
Primordial power spectra



This steep growth is consistent with Byrnes, Cole & Patil, 2018

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PBH mass fraction: Press-Schechter formalism

$$\delta(k,t) \simeq \frac{2(w+1)}{(3w+5)}\left(\frac{k}{aH}\right)^2 \mathcal{R}_k.$$

$$\sigma_\delta^2(R) \simeq \frac{16}{81}\int \frac{dk}{k}(kR)^4 P_{\mathcal{R}}(k) W^2(k,R).$$

$$\beta_f(M) \simeq \sqrt{\frac{1}{2\pi}}\frac{\sigma_\delta(M(R))}{\delta_c}\exp\left(-\frac{\delta_c^2}{2\sigma_\delta^2(M(R))}\right)$$

$$\beta(M) \equiv \frac{\rho_{\rm PBH}(M)}{\rho_{\rm tot}}.$$

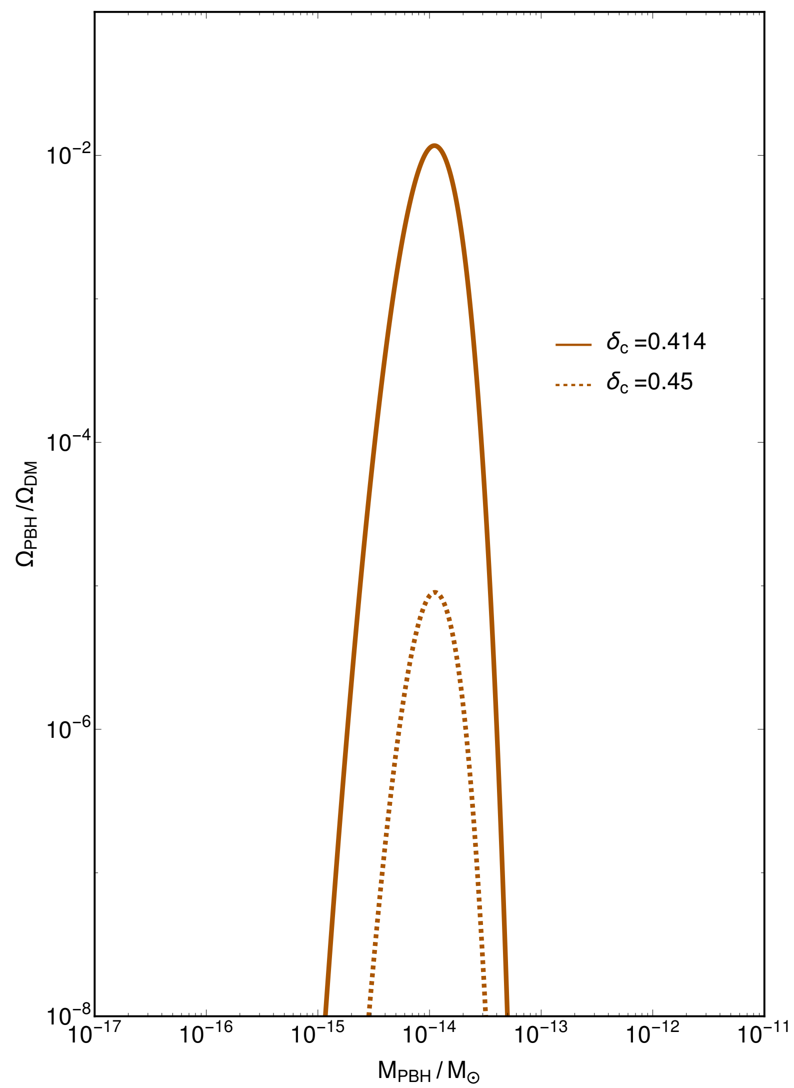
$$\beta_{\rm eq}(M) \simeq \beta_f(M)\left(\frac{a_{\rm eq}}{a_f}\right)$$

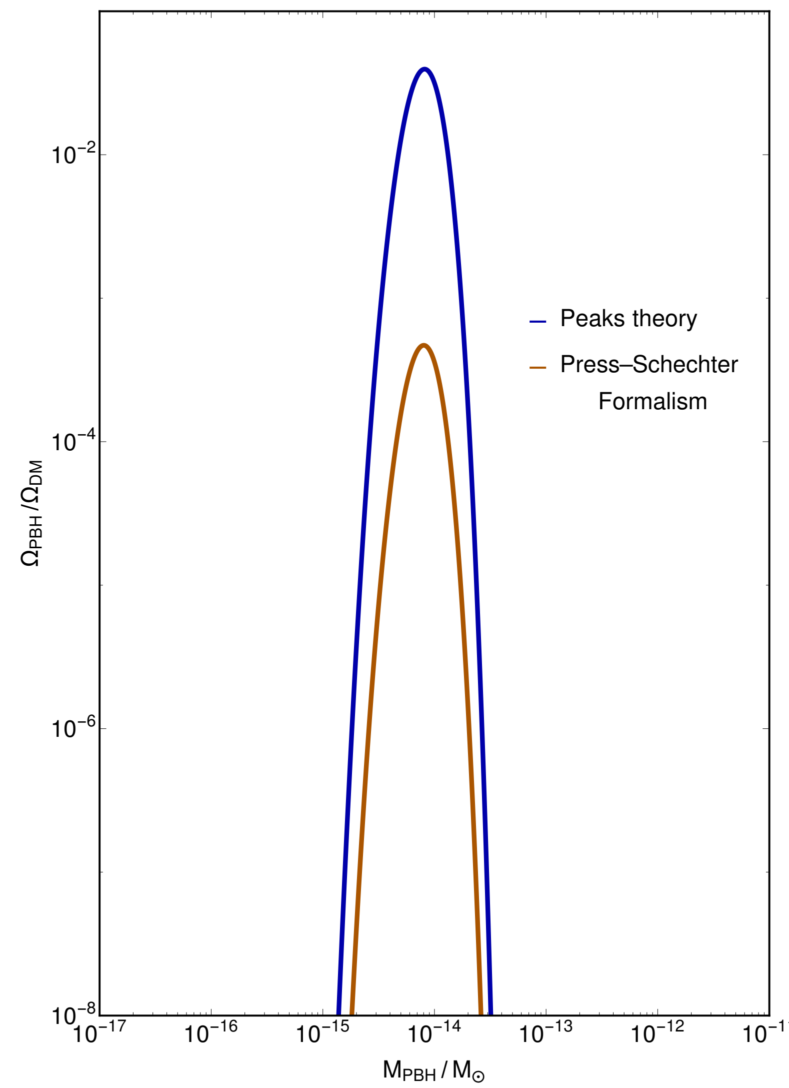
$$f_{\rm PBH}(M) \equiv \frac{\Omega_{\rm PBH}(M)}{\Omega_{\rm DM}} = \frac{\beta(M)}{8\times 10^{-16}}\left(\frac{\gamma}{0.2}\right)^{3/2}\left(\frac{g_*}{106.75}\right)^{-1/4}\left(\frac{M}{10^{18}\,{\rm g}}\right)^{-1/2}$$

PBH Recipe

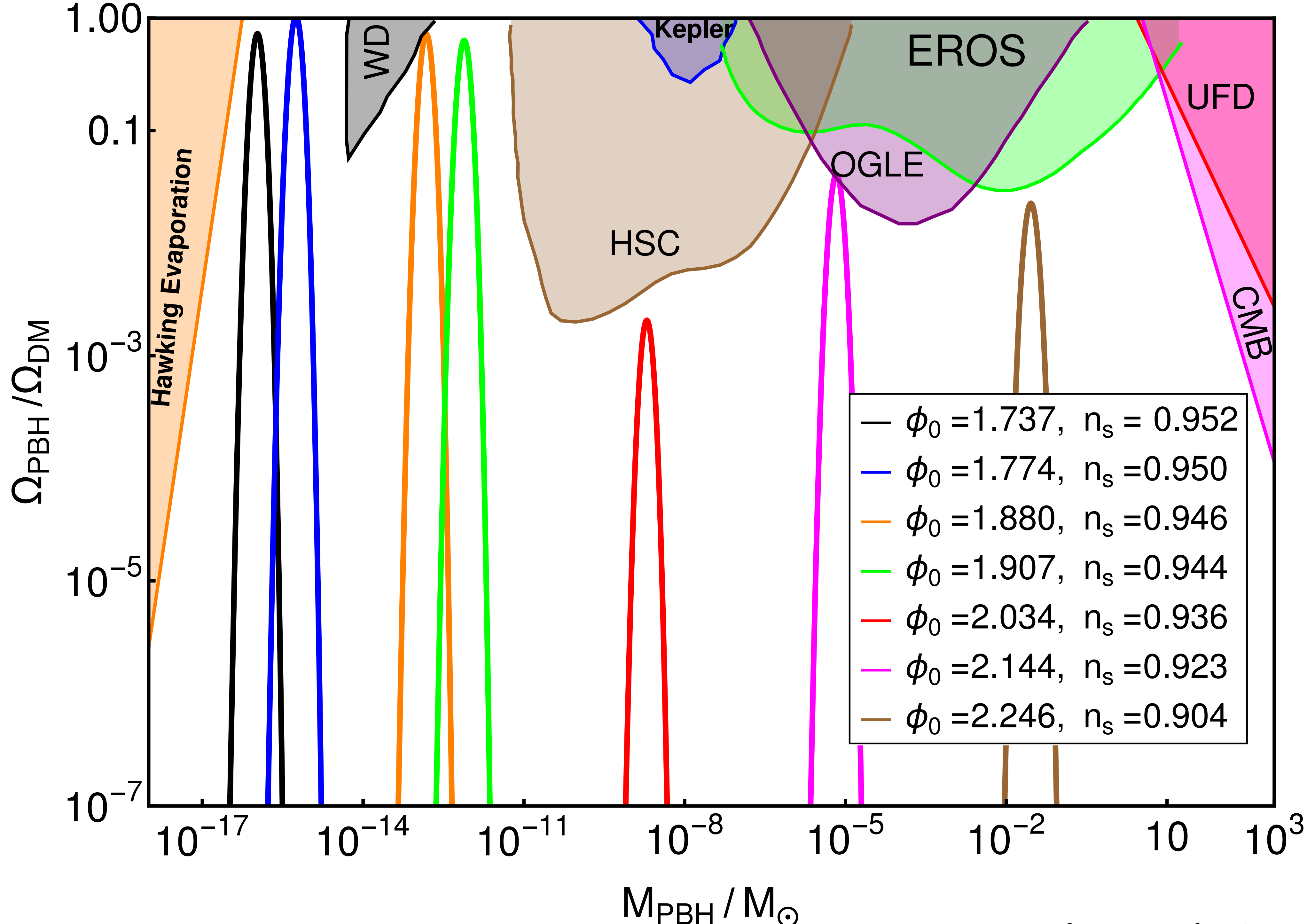
PBH mass fraction: Some uncertainties

- Peaks theory vs. Press-Schechter
- Choice of the window function
- Value of the critical density contrast





PBH mass fraction



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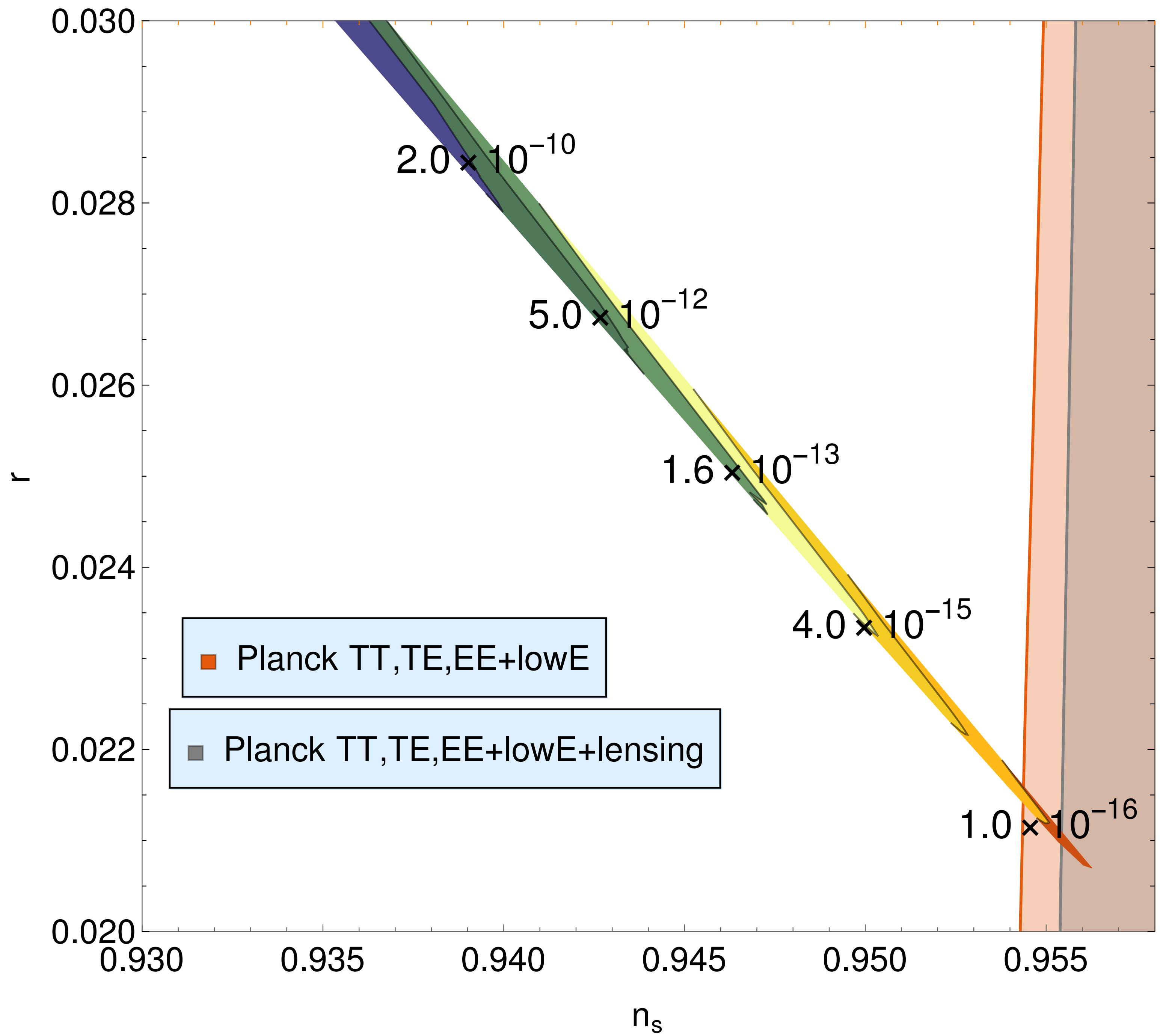
PBH mass contours: n_s vs. r



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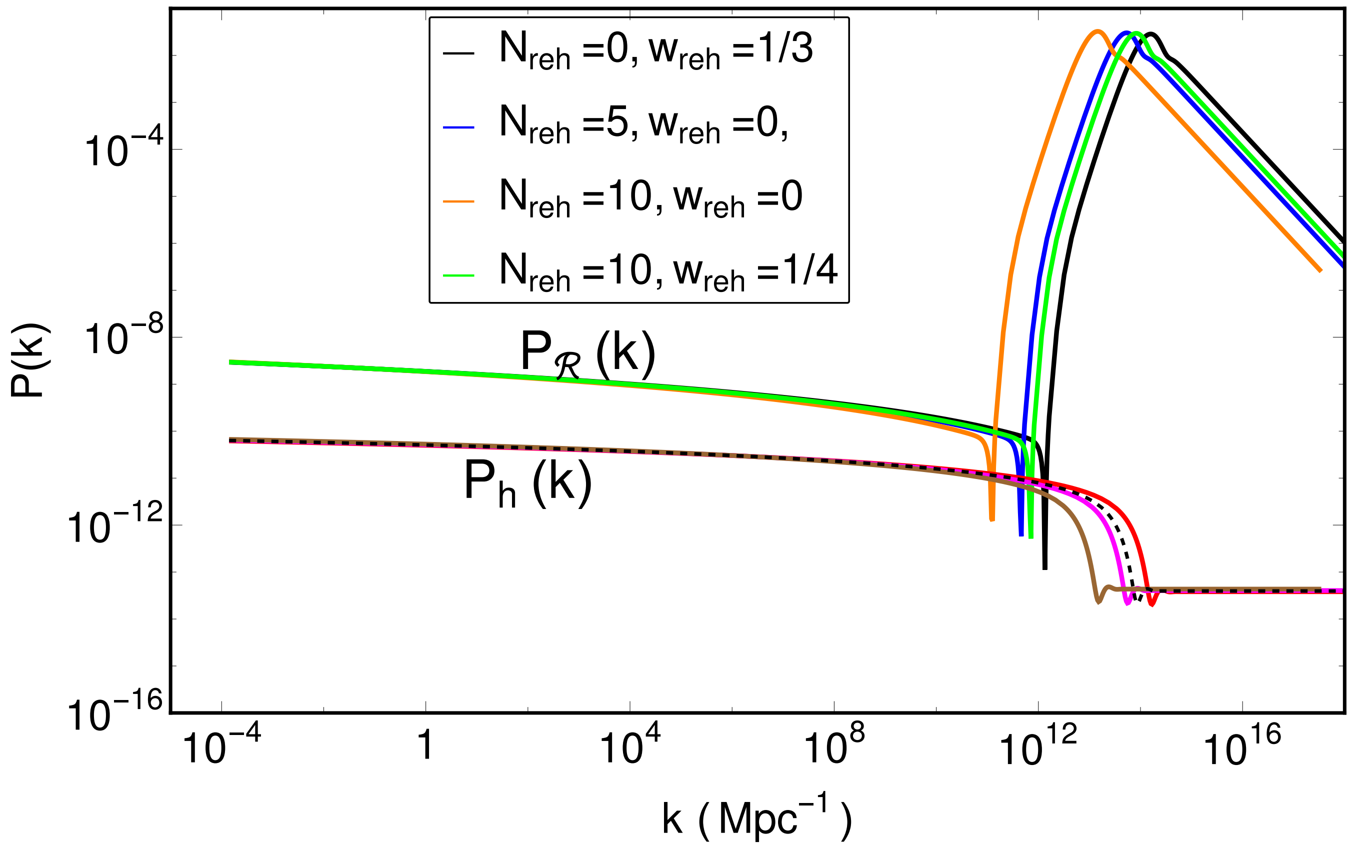
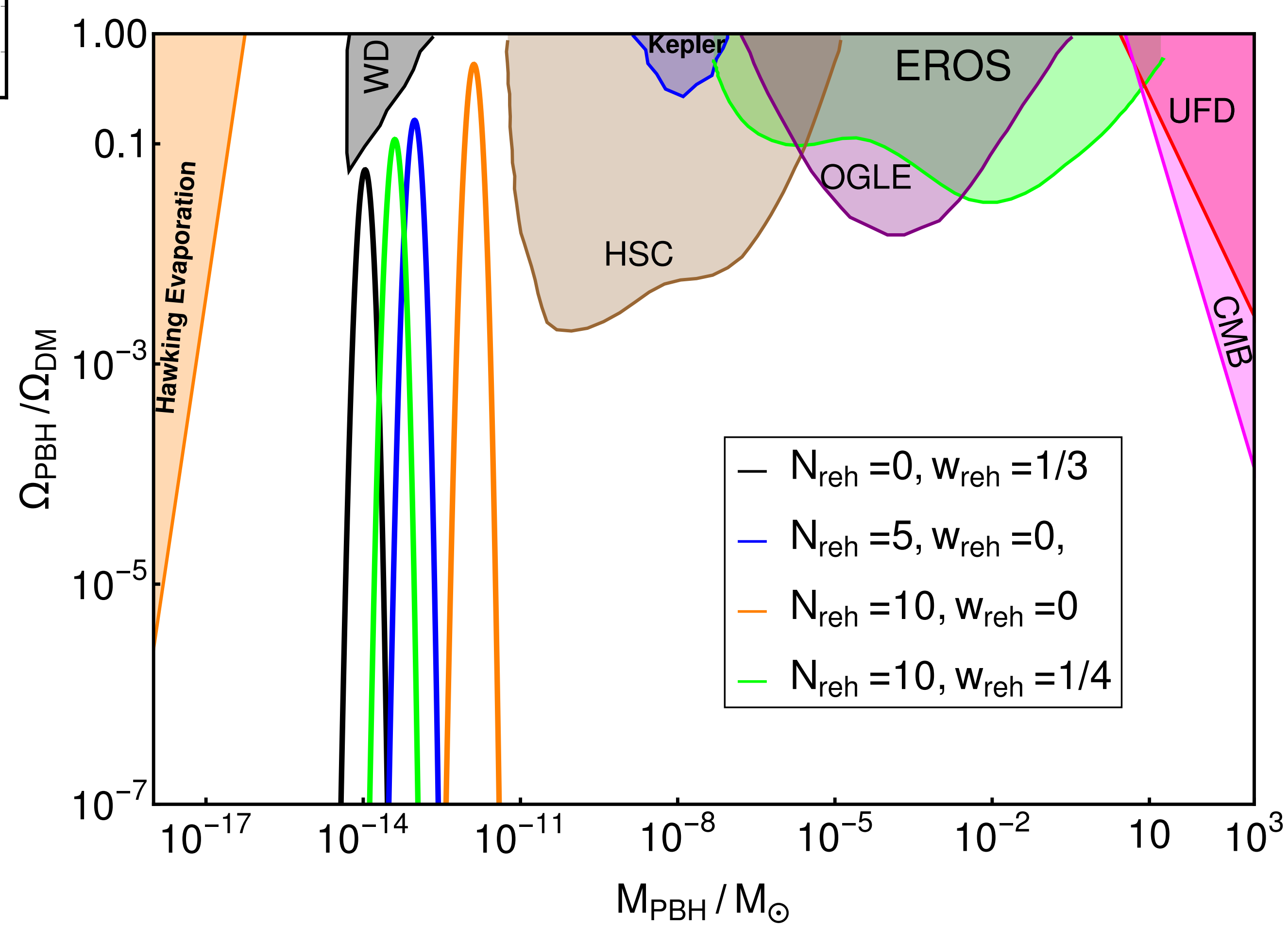
Effects of reheating





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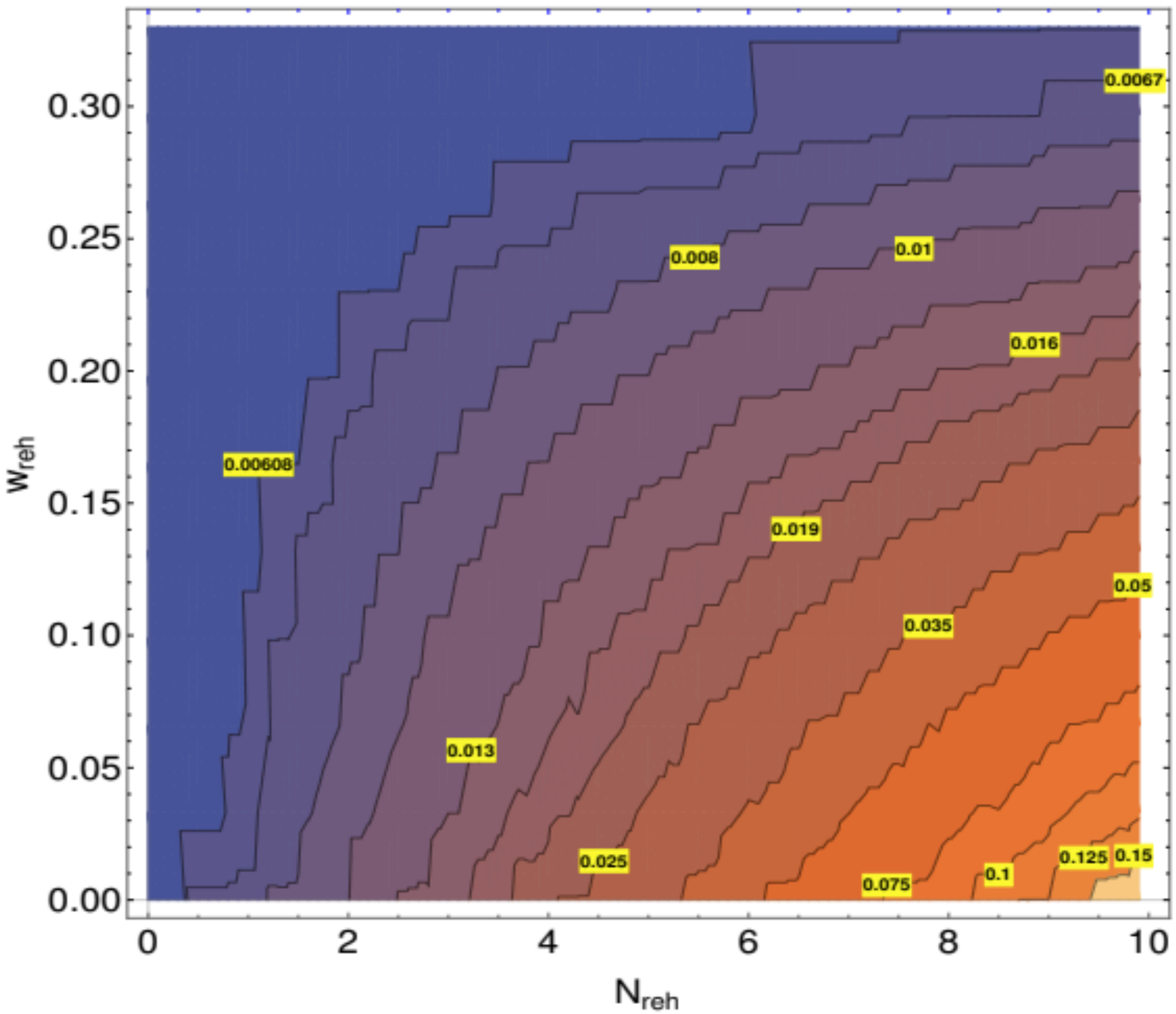

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PBH observational imprints: Induced secondary GWs

Induced secondary GWs

Tensor modes sourced by scalar perturbations

$$h''_{\mathbf{k}}(\tau) + 2\mathcal{H}h'_{\mathbf{k}}(\tau) + k^2 h_{\mathbf{k}}(\tau) = 4S_{\mathbf{k}}(\tau),$$

$$S_{\mathbf{k}} = \int \frac{\mathrm{d}^3 q}{(2\pi)^{3/2}} e^{\lambda}_{ij}(\mathbf{k}) q_i q_j \left[2\Phi_{\mathbf{q}}\Phi_{\mathbf{k}-\mathbf{q}} + \left(\mathcal{H}^{-1}\Phi'_{\mathbf{q}} + \Phi_{\mathbf{q}}\right)\left(\mathcal{H}^{-1}\Phi'_{\mathbf{k}-\mathbf{q}} + \Phi_{\mathbf{k}-\mathbf{q}}\right)\right].$$

$$\Phi_{\mathbf{k}}(\tau) = \frac{2}{3}\mathcal{T}(k\tau)\mathcal{R}(\mathbf{k}). \qquad \mathcal{T}(k\tau) = \frac{9}{(k\tau)^2}\left[\frac{\sqrt{3}}{k\tau}\sin\left(\frac{k\tau}{\sqrt{3}}\right) - \cos\left(\frac{k\tau}{\sqrt{3}}\right)\right].$$

$$\frac{k^3}{2\pi^2}\left\langle h^{\lambda}_{\mathbf{k}}(\tau)\, h^{\lambda'}_{\mathbf{k}'}(\tau)\right\rangle = \delta_{\lambda\lambda'}\delta^3(\mathbf{k}+\mathbf{k}')\mathcal{P}_h(\tau, k),$$

$$\Omega_{\mathrm{GW}}(\tau, k) \equiv \frac{1}{\rho_c}\frac{\mathrm{d}\,\rho_{\mathrm{GW}}}{\mathrm{d}\ln k} = \frac{\rho_{\mathrm{GW}}(\tau, k)}{\rho_{\mathrm{tot}}(\tau)} = \frac{1}{24}\left(\frac{k}{\mathcal{H}}\right)^2 \overline{\mathcal{P}_h(\tau, k)},$$

Induced secondary GWs

$$\Omega_{\rm GW}(\tau_0,k)\,h^2 \simeq 2.4\times 10^{-5}\left(\frac{\Omega_{r,0}\,h^2}{4.0\times 10^{-5}}\right)\left(\frac{k}{\mathcal{H}(\tau_f)}\right)^2 \int_{-\frac{1}{\sqrt{3}}}^{\frac{1}{\sqrt{3}}} {\rm d}d \int_{\frac{1}{\sqrt{3}}}^{\infty} {\rm d}s \left[\frac{(d^2-1/3)(s^2-1/3)}{s^2-d^2}\right]^2$$

$$\times\, \mathcal{P}_{\mathcal{R}}\left(\frac{k\sqrt{3}}{2}(s+d)\right)\mathcal{P}_{\mathcal{R}}\left(\frac{k\sqrt{3}}{2}(s-d)\right)\left[\mathcal{I}_c^2(d,s)+\mathcal{I}_s^2(d,s)\right].$$

$$\mathcal{I}_c(d,s) = -36\pi\frac{(s^2+d^2-2)^2}{(s^2-d^2)^3}\theta(s-1),$$

$$\mathcal{I}_s(d,s) = -36\frac{(s^2+d^2-2)}{(s^2-d^2)^2}\left[\frac{(s^2+d^2-2)}{(s^2-d^2)}\log\frac{(1-d^2)}{|s^2-1|}+2\right]$$

$$d=(u-v)/\sqrt{3}$$

$$s=(u+v)/\sqrt{3},$$

$$v=p/k,\; u=|\mathbf{k}-\mathbf{p}|/k$$

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The 'three' peaks

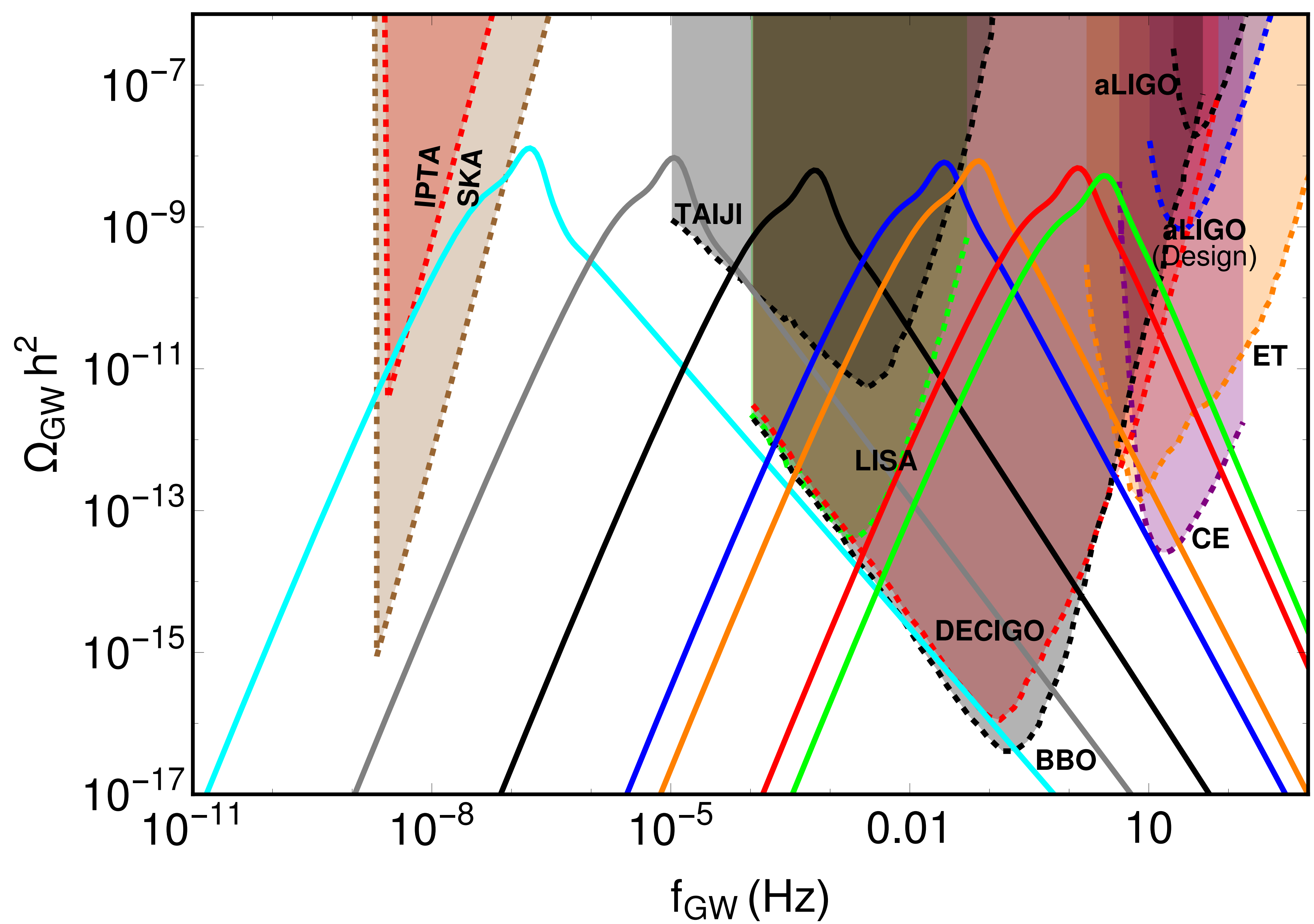
An interesting and useful relation between the 'three' peaks

$$\left(\frac{M_{\rm PBH}}{10^{17}\,{\rm g}}\right)^{-1/2} \simeq \frac{k}{2\times 10^{14}\,{\rm Mpc}^{-1}} = \frac{f}{0.3\,{\rm Hz}},$$

$f_{\rm PBH}$ $\qquad$ $P_{\mathcal{R}}$ $\qquad$ $\Omega_{\rm GW}$

- LISA: f ~ mHz → $k \sim 10^{12}\,{\rm Mpc}^{-1}$ → $M_{\rm PBH} \sim 10^{-12} M_\odot$
- Advanced LIGO: f ~ 30 Hz → $M_{\rm PBH} \sim 10^{13}\,{\rm g} \sim 10^{-20} M_\odot$

Induced secondary GWs



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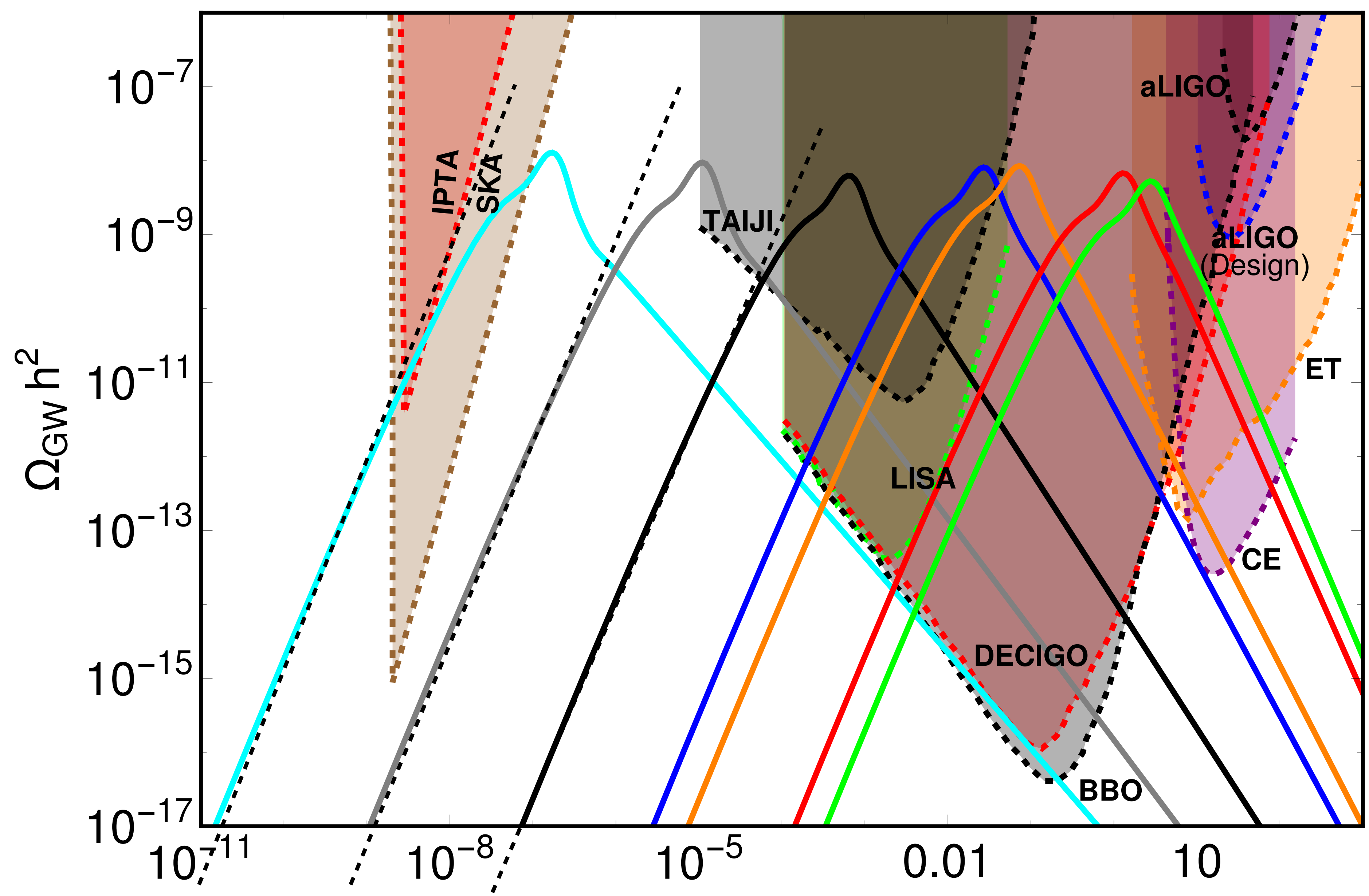
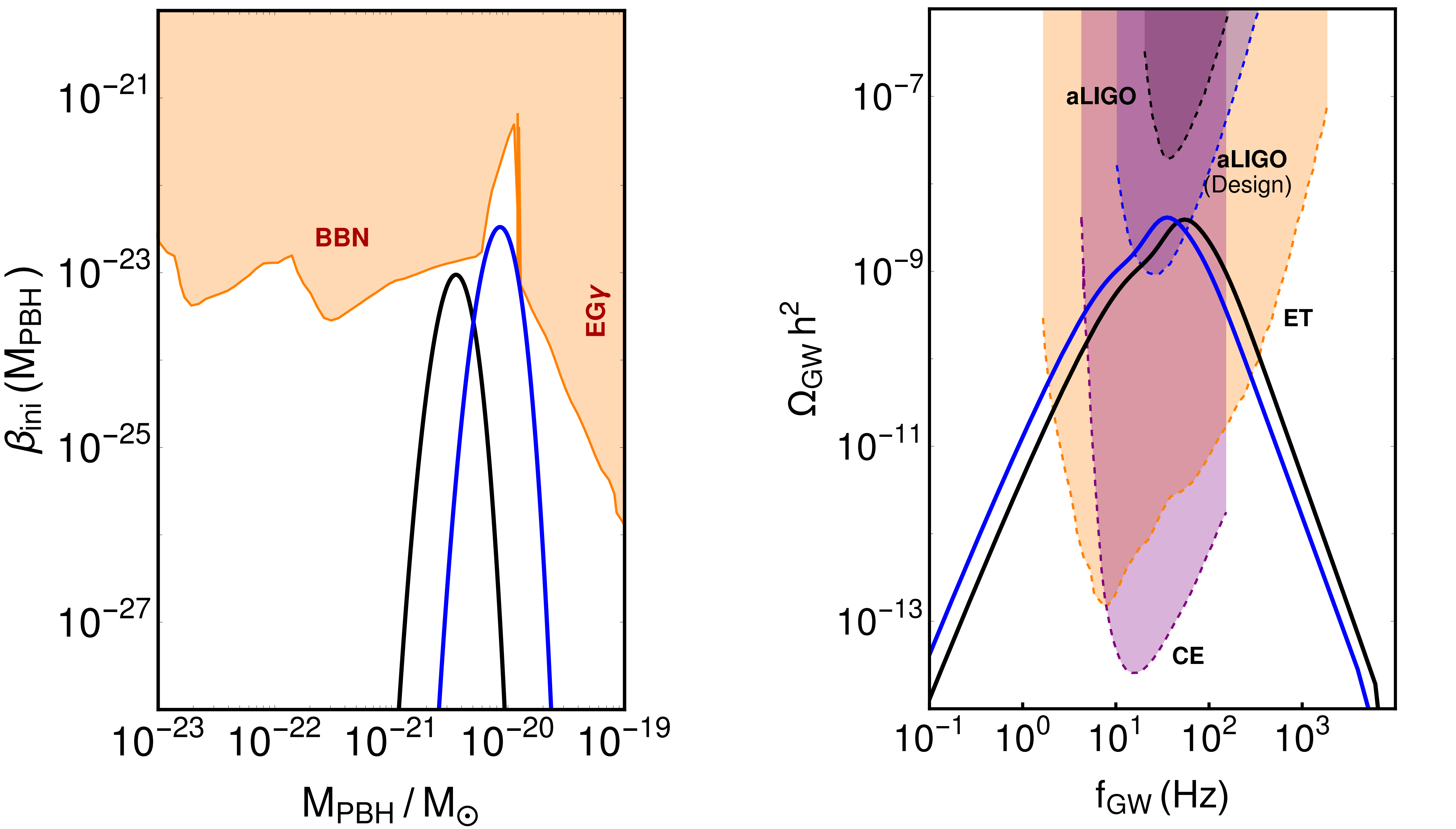
Induced secondary GWs



$$\Omega_{\rm GW} \sim f_{\rm GW}^3$$

- A universal IR scaling of GW spectrum, Cai, Pi & Sasaki, 2019
- IR scaling with log corrections, Yuan, Chen & Huang, 2019

Ultralight PBHs with Advanced LIGO



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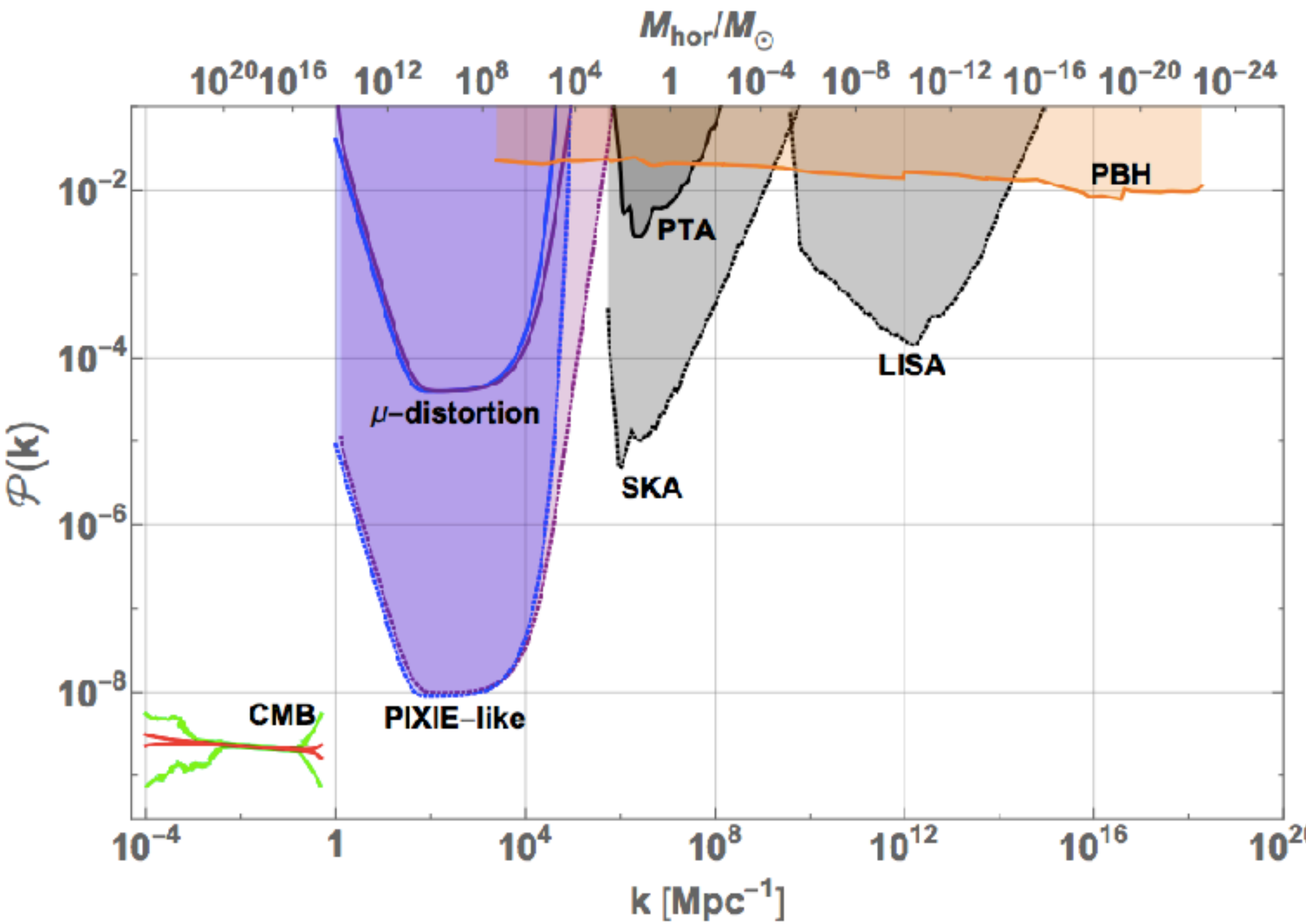
Future constraints on small scales



Byrnes, Cole & Patil, 2018

Conclusions

- PBHs are novel candidates for CDM in the universe
- Inflation can produce significant abundance of PBHs — inflection point models are most useful — model dependent results
- Slow roll approximation is not correct — full numerical computation of primordial spectra, PBH mass fraction and GW required
- Interesting observational implications — induced GWs on scales probed by LISA, DECIGO or BBO
- A testable prediction with LISA — non-observation of such GWs at LISA may rule out PBHs as DM
- Very useful probes of small scale dynamics during inflation
- PBH mass fraction — effects due to primordial NG and quantum duffusion during the ultra slow roll phase — non-trivial effects on model predictions

PBHs as Dark Matter candidates seem to have a very bright future ahead !

Thank you.