THE QCD AXION

Selected Reading

DJEM, "Axion Cosmology", Ch. 2, App. A, B

Coleman, *Aspects of Symmetry*, Ch. 7.3

Sredinicki, *Quantum Field Theory,* Ch. 93, 94

Zee, *Quantum Field Theory in a Nutshell*, Ch. IV.7

The QCD "θ-term"

$$\mathcal{L}_\theta = \frac{\theta}{32\pi^2} \mathrm{Tr}\ G_{\mu\nu}\tilde{G}^{\mu\nu}$$

This term is a total derivative and classically does not affect the e.o.m's

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g f^{abc} A^b_\mu A^c_\nu$$

SU(3) index. Runs over the 8 gluons (generators)

Structure constants of the Lie alberga

$$\tilde{G}^{\mu\nu,a} = \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta}/2\sqrt{-g}$$

Indices raised with ε → this term does not depend on the spacetime metric, only its topology.

CP-violation

C = charge conjugation, P = parity

QFTs must obey CPT. Therefore P and T violation → CP violation.

Example, consider Electromagentism:

$$F_{\mu\nu}\tilde{F}^{\mu\nu} \propto \vec{E}\cdot\vec{B}$$

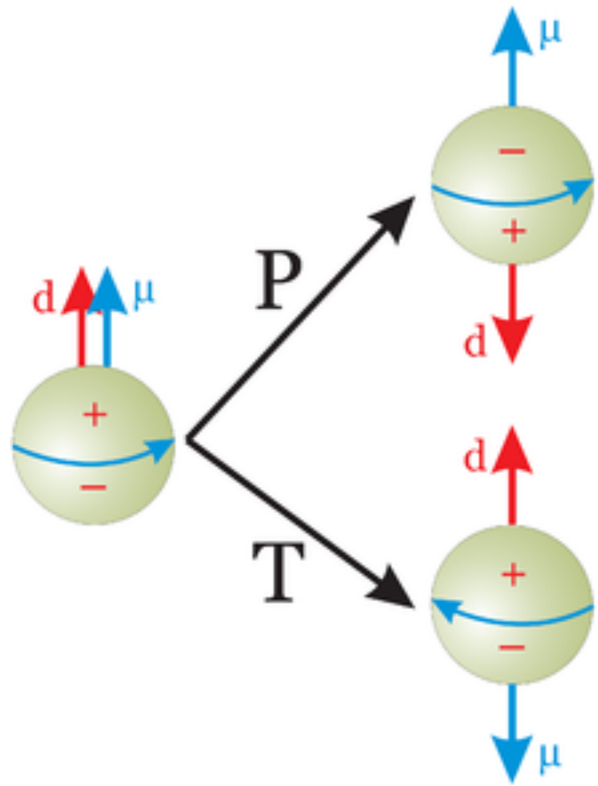
E is a proper vector → odd under P

B is caused by a current of charges → odd under T

→ This term violates P and T and thus CP.

Neutron EDM

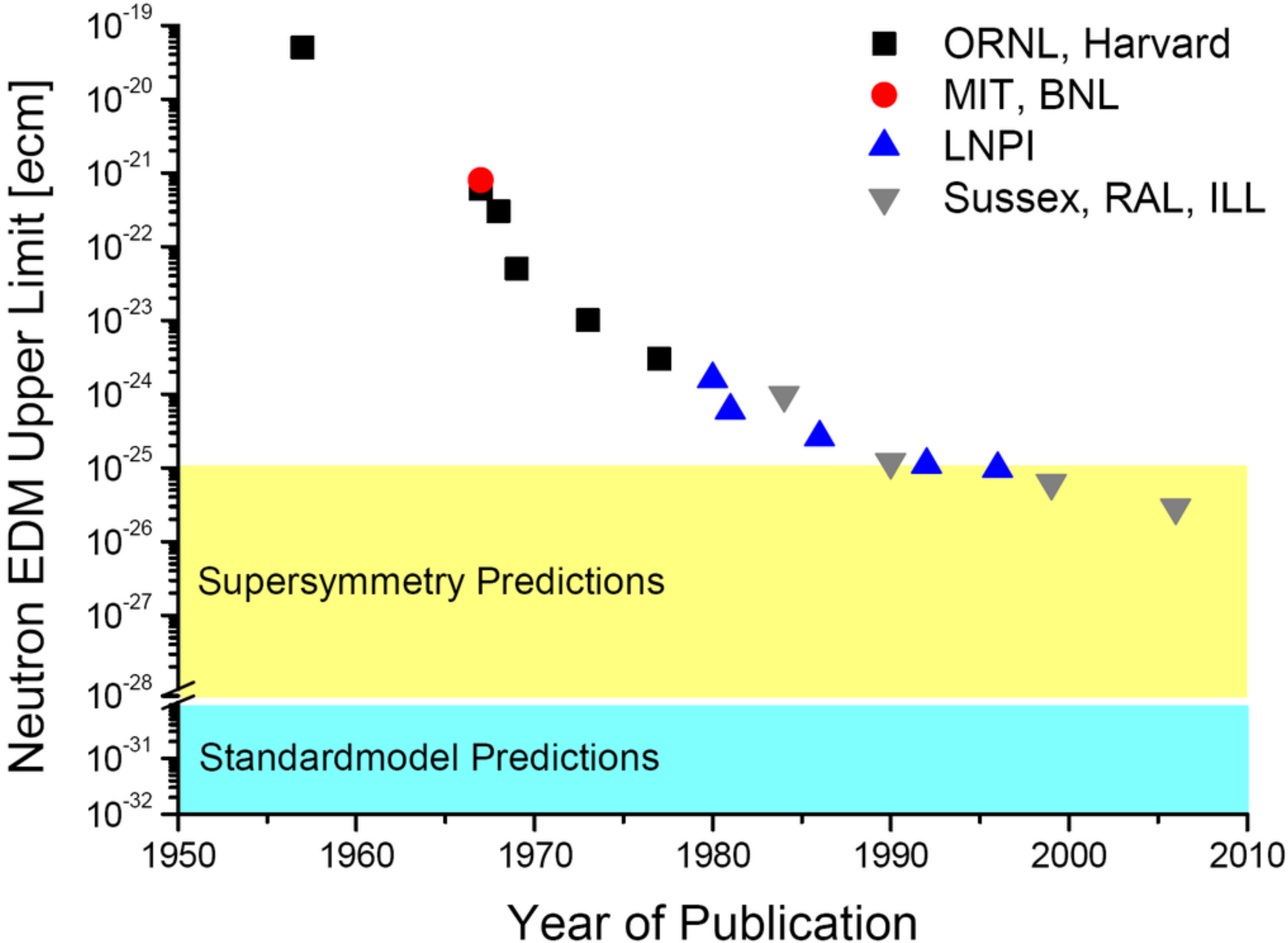
The neutron electric dipole moment violates P and T and thus CP.



In QFT operators in the theory generate all terms consistent with symmetry. Thus the θ term generates a neutron EDM.

$$d_n \approx 3.6 \times 10^{-16} \theta \; e \; \mathrm{cm}$$

Crewther et al (1979)



ORNL, Harvard
MIT, BNL
LNPI
Sussex, RAL, ILL
Supersymmetry Predictions
Standardmodel Predictions
Neutron EDM Upper Limit [ecm]
Year of Publication
10^-19
10^-20
10^-21
10^-22
10^-23
10^-24
10^-25
10^-26
10^-27
10^-28
10^-31
10^-32
1950
1960
1970
1980
1990
2000
2010

Wikipedia

The Strong-CP problem

θ has contributions from two separate parts of the Standard Model:

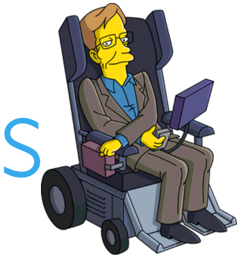
$$\theta = \tilde{\theta} + \arg\det M_u M_d$$

"Bare" QCD contribution

From quark mass matrix, i.e. SU(2) electroweak sector

EW sector violates CP in the CKM matrix (e.g. Kaon decays) → second term is non-zero → cancellation of SU(3) and SU(2) sector contributions at very high precision.

Fine tuning in theoretical Physics



$$\mathcal{L} = (\text{number}) \times (\text{operator})$$

=the value you measure for the effect.

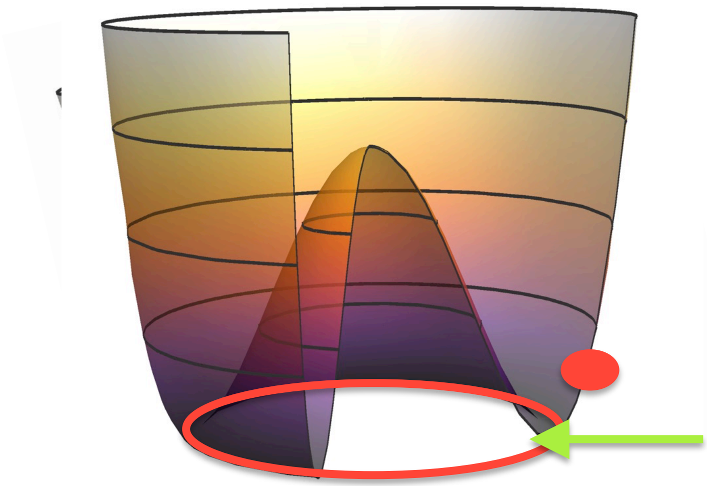
=physical effect. The neutron EDM.

$$(\text{number}) = \sum_{(\text{physics})} c_i$$

→ Contributions from different sources. Strong and weak nuclear forces.

The nEDM is measured consistent with zero. This implies a delicate cancellation at $\sim 10^{-10}$. We don't like coincidences.

Cleaning up the mess



Spontaneous symmetry breaking: a Goldstone Boson, θ, has a continuous "shift symmetry", i.e. the underlying rotation.

"instantons"

Trick: couple a Goldstone to the problematic operator.

$$(\text{number}) \to (\text{number}) + \theta \to \theta$$

Now "tilt the wine bottle": θ will now dynamically move to a fixed value = 0 by symmetry → no problematic nEDM.

Axion Ingredients

1. Shift symmetry $\theta \rightarrow \theta + c$
2. Coupling to SU(3) θ-term
3. Potential $V(\theta)$

1. Goldstone boson & SSB global U(1)
2. Loops of chiral fermions → anomaly
3. QCD non-perturbative effects

Spontaneous Symmetry Breaking

PQ mechanism achieved introducing a complex scalar:

$$\varphi = \chi e^{i\phi/f_a}$$

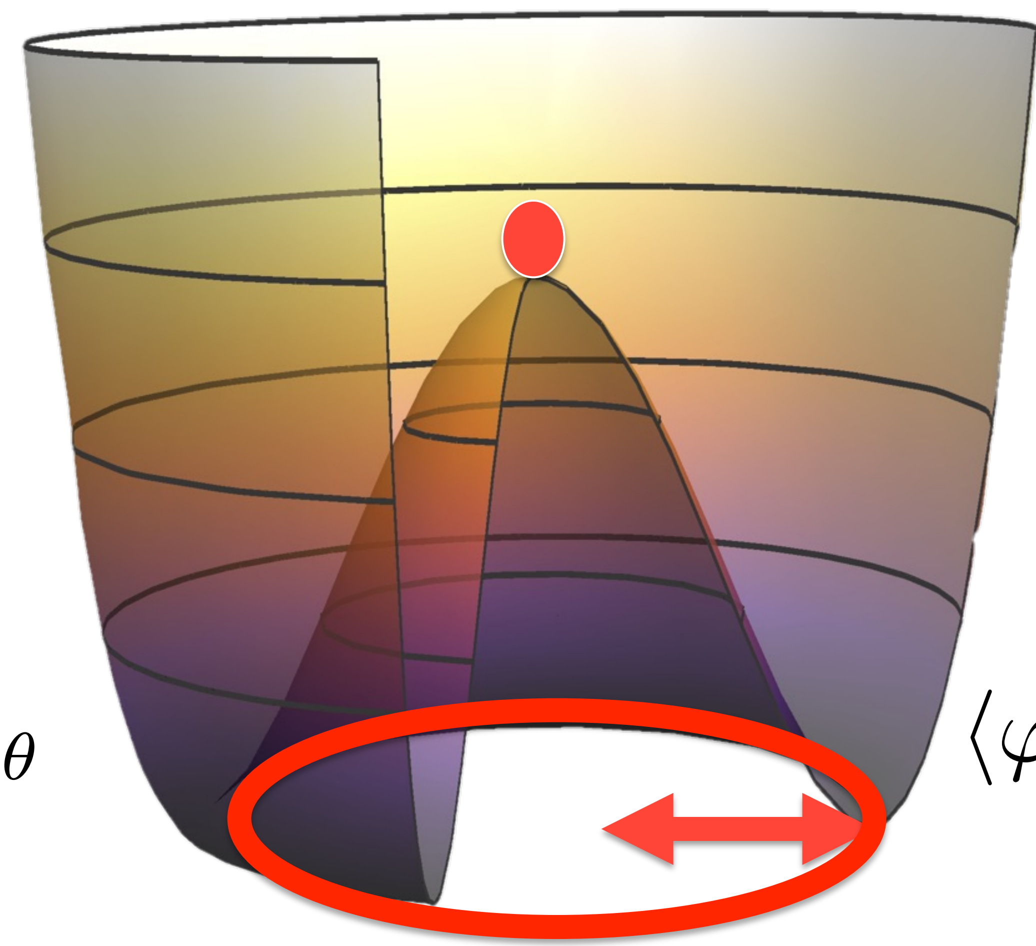
Global U(1) symmetry broken by scalar potential:

$$V(\varphi) = \lambda \left(|\varphi|^2 - \frac{f_a^2}{2} \right)^2$$

ϕ field is Goldstone boson. Shift-symmetry under U(1) α rot.:

$$\varphi \to \varphi e^{i\alpha}; \, \phi \to \phi + \mathrm{const.}$$

Spontaneous Symmetry Breaking



$T \gg f_a$

$\langle \varphi \rangle = 0$

$\varphi = \chi e^{i\theta}$

$T \ll f_a$

$\langle \varphi \rangle = f_a/\sqrt{2}$

$\theta = \phi/f_a$

The PQ Lagrangian

$$\mathcal{L} = -(\partial_\mu \varphi)(\partial^\mu \varphi^*) - V(\varphi) + y(\varphi^* \bar{\psi}_L \psi_R + \text{h.c.})$$

$$\psi = \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} \qquad \psi_{L,R} = P_{L,R}\psi = \frac{1}{2}(1 \pm \gamma_5)\psi$$

L-fermion has charge Q, R-fermion has −Q, PQ-field has 2Q:

$$\varphi \to e^{i2Q\alpha}\varphi \qquad\qquad \psi \to e^{iQ\alpha\gamma_5}\psi$$

Global symmetry → conserved current (Noether's theorem):

$$\varphi = S e^{i\phi/f_a} \qquad j_\mu^{\mathrm{PQ}} = \partial_\mu \phi \qquad \partial^\mu j_\mu^{\mathrm{PQ}} = 0$$

Axion-Fermion Coupling

SSB: replace the PQ field with its vev:

$$\langle \varphi \rangle = \frac{f_a}{\sqrt{2}} e^{i\phi/f_a}$$

Promote the U(1) rotation α to the Goldstone field ϕ/f.

$$\psi \to \psi' = e^{iQ\gamma_5 \phi/f_a} \psi$$

Axion-fermion interaction arises from the rotated fermion kinetic term:

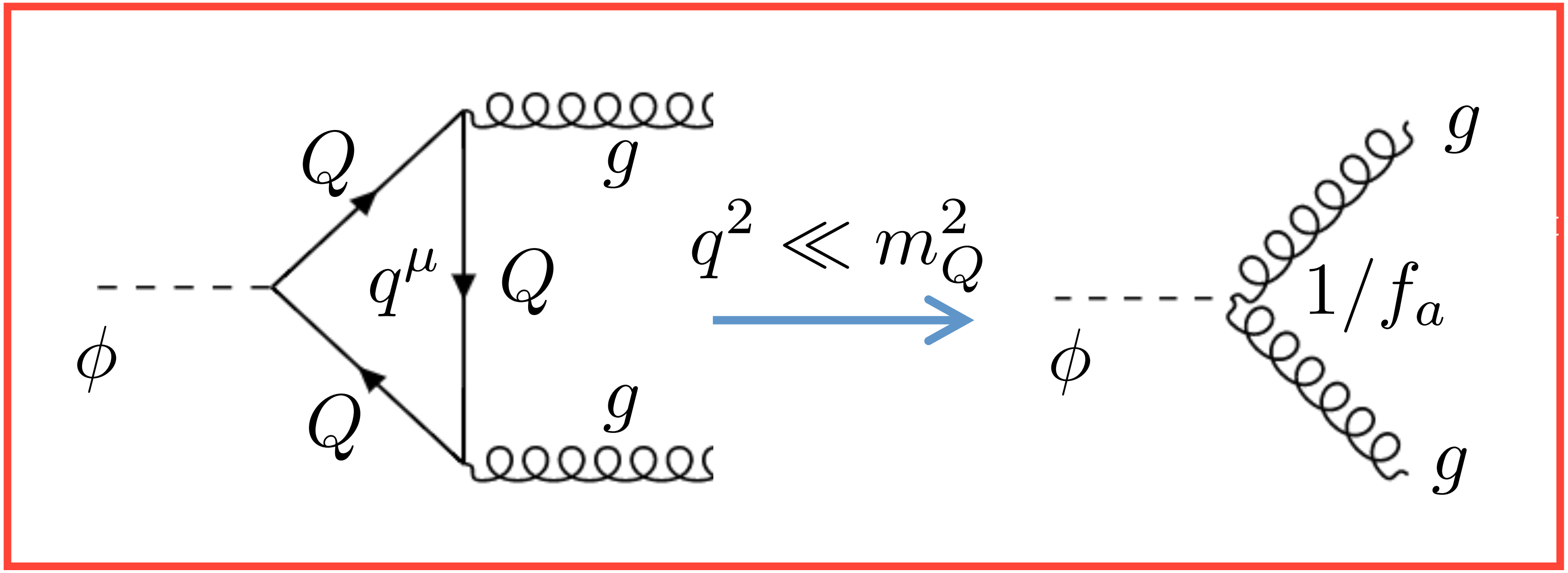
$$\mathcal{L}_{\rm kin} = i\bar{\psi}\gamma^\mu \partial_\mu \psi$$

$$= i\bar{\psi}'\gamma^\mu \partial_\mu \psi' + \frac{Q}{f_a}\partial_\mu \phi \; \bar{\psi}'\gamma^\mu \gamma_5 \psi$$

Manifestly invariant under $\phi \to \phi$ + const.

The Chiral Anomaly

Right and left handed fermions have opposite charges.
"Chiral anomaly" → invariance broken at loop level:



$$S \to S + \int d^4x \, \frac{\mathcal{C}}{32\pi^2} \frac{\phi}{f_a} \mathrm{Tr}\, G_{\mu\nu} \tilde{G}^{\mu\nu}$$

The Chiral Anomaly

(see Zee and Srednicki)

Exercise:

1) inspect the γ-matrix structure of the amplitude and show that it goes like $\epsilon^{\mu\nu\alpha\beta}\partial_\alpha A_\beta \partial_\mu A_\nu$

2) What is the group theory structure?

θ is a dynamical field

Accounting for the anomaly, the axion Lagrangian is now:

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi)^2 + \frac{Q}{f_a}\partial_\mu\phi \; j_5^\mu + \frac{1}{32\pi^2}\left(\mathcal{C}\frac{\phi}{f_a} + \theta\right) \mathrm{Tr}\; G_{\mu\nu}\tilde{G}^{\mu\nu}$$

Every term in the Largangian is invariant under shift in ϕ, except θ-term. Thus we make the replacement:

$$\phi \to \phi' = \phi + \frac{f_a}{\mathcal{C}}\theta$$

$$\mathcal{L}_\phi = \frac{1}{2}(\partial\phi')^2 + \frac{Q}{f_a}\partial_\mu\phi' \; j_5^\mu + \frac{1}{32\pi^2}\left(\mathcal{C}\frac{\phi'}{f_a}\right) \mathrm{Tr}\; G_{\mu\nu}\tilde{G}^{\mu\nu}$$

We see that the problematic q-term can be absorbed in a ϕ redefinition. Now we need some dynamics to set ϕ=0. Enter instantons…

QCD Axion Potential

Even without an axion, the vacuum energy can be shown to depend on θ, due to the "θ-vacua" of QCD (see Coleman).

The "Vafa-Witten theorem" guarantees that the minimum of the energy is at the CP-conserving value, i.e. θ=0 (+EW CP violation).

The role of the axion is to connect these different "super-selection sectors" and allow the energy to minimize by dynamics.

In the effective action, we include the energy dependence of the vacuum → potential for the axion:

$$\mathcal{L} \to \mathcal{L}_{\rm eff} = \mathcal{L} - E_{\rm vac} \equiv \mathcal{L} - V(\phi)$$

QCD Axion Potential

The potential must be periodic (since f is an angular co-ordinate):

$$V(\phi) = V(\phi + 2\pi f_a)$$

The appearance of the potential is a non-perturbative effect → the magnitude should depend on the strong-coupling scale of QCD:

$$V(\phi) = \chi(T)U(\phi/f_a)$$

← Function to be calculated

$$\chi(0) = \Lambda^4 \qquad \chi(\infty) = 0$$

↑ Scale to be calculated

↑ Asymptotic freedom

The function χ is the topological susceptibility, and needs calculating.

ChPT and the Axion Mass

Dimensional analysis:

$$\chi(0) \sim \Lambda_{\rm QCD}^4 \sim m_a^2 f_a^2 \Rightarrow m_a \sim 4 \times 10^{-5}\ {\rm eV} \frac{10^{12}\ {\rm GeV}}{f_a}$$

At zero temperature, use chiral perturbation theory (ms>>mu,md):

$$m_{a,0} = \frac{m_\pi f_\pi}{f_a} \frac{\sqrt{m_u m_d}}{m_u + m_d} = 5.9 \times 10^{-6}\ {\rm eV} \left(\frac{10^{12}\ {\rm GeV}}{f_a} \right)$$

$$U(\theta)_{T=0} = \frac{(m_u + m_d)^2}{m_u m_d} \left[1 - \frac{\sqrt{m_u^2 + m_d^2 + 2 m_u m_d \cos\theta}}{m_u + m_d} \right]$$

DILG and Finite-T

For T>>Λ_{QCD}, we have weak coupling → "dilute instanton gas". Potential is simple to compute from E(θ), e.g. Coleman:

$$U(\theta) = (1 - \cos\theta)$$

Taylor expand → write χ(T) from m_a(T):

$$m_a(T) = m_{a,0}\left(\frac{T}{\Lambda_{\rm QCD}}\right)^{-n}$$

$$\chi(T) = m_a(T)^2 f_a^2$$

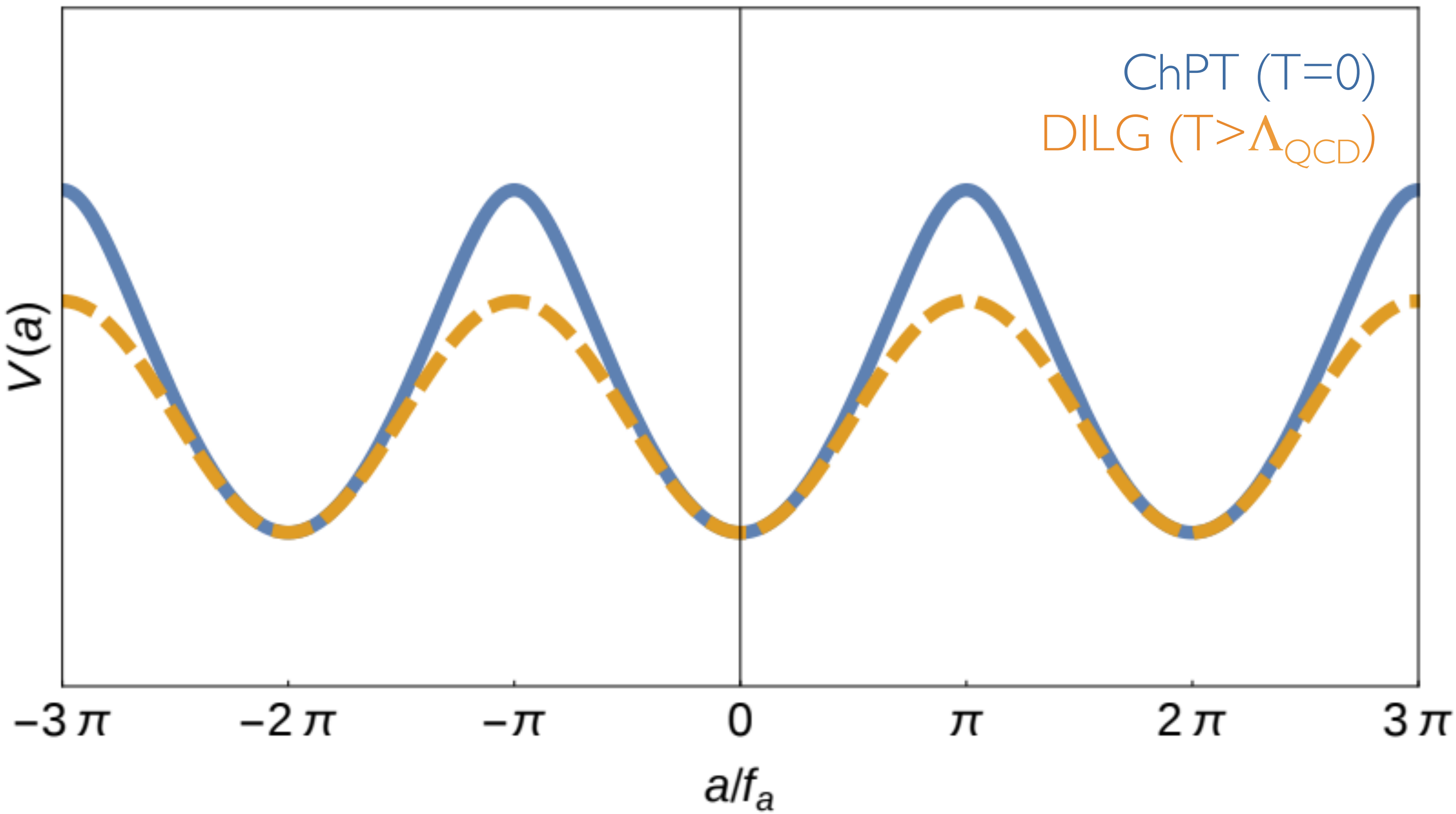
(+logarithmic correction, instanton liquid corrections etc. etc.)

$$n = (11N_c + N_f)/6 - 2$$

Gross et al (1974)

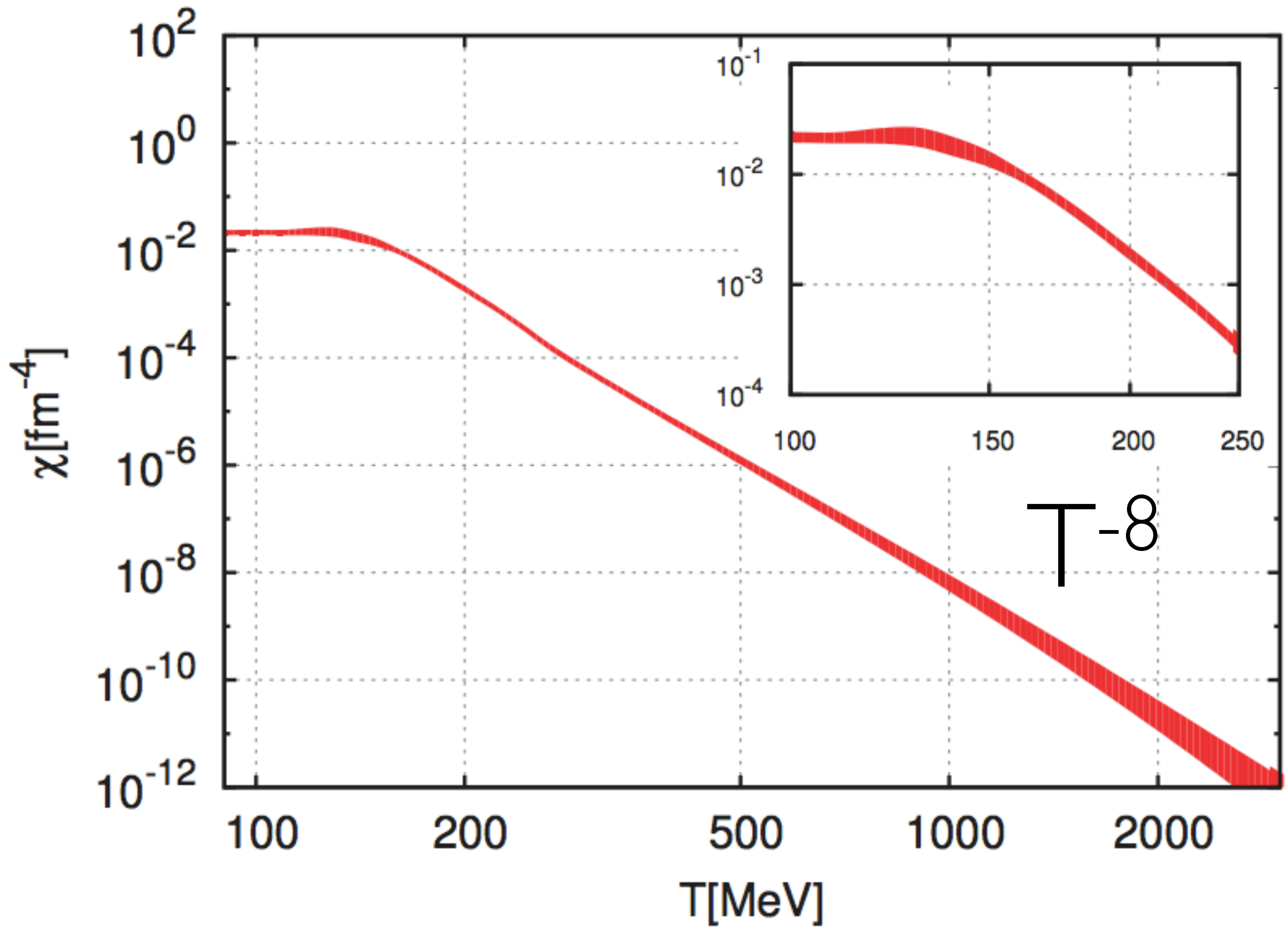
Nc = colours = 3. Nf = # of light fermions = 3 (T<mc~GeV) → n=4

QCD Axion Potential



di Cortona et al (2015)

Lattice QCD and χ(T)



$$\chi(0) = (75.6\ \mathrm{MeV})^4$$

Borsanyi et al (2016)

QCD Axion Models

Models differ by the choice of matter content charged under PQ.
→ Different anomaly co-efficients & different freedom in f_a.

PQWW:

Peccei & Quinn (1977); Weinberg (1978); Wilczek (1978)

Only new particle is PQ scalar, as 2nd Higgs doublet.
$\phi 1$ → u-type masses, $\phi 2$→ d-type masses
4 EM-neutral scalars: Higgs, Z-mass, Radial PQ, axion

$$f_a = v = 250 \text{ GeV}$$
$$m_a = 24 \text{ keV}$$

Excluded almost immediately (1970's) by beam-dump.

QCD Axion Models

Models differ by the choice of matter content charged under PQ.
→ Different anomaly co-efficients & different freedom in f_a.

KSVZ:

Kim (1979); Shifman, Vainshstein & Zakharov (1980)

One new Dirac quark charged under PQ and SM

$Q \in (R, 1, q)$ Canonical choice R=3, q=0 → C=1

$\mathcal{L}_{\rm int} = -\lambda \varphi \bar{Q}_L Q_R + \text{h.c.}$ Q has mass $O(f_a)$

"The hadronic axion": couplings to leptons are loop suppressed.

QCD Axion Models

Models differ by the choice of matter content charged under PQ.
→ Different anomaly co-efficients & different freedom in f_a.

DFSZ:

Dine, Fischler & Srednicki (1981); Zhitnitsky (1980)

Introduce two new scalars on top of Higgs:

$$H_1, H_2, \varphi$$

2 Higgs doublets charged under PQ

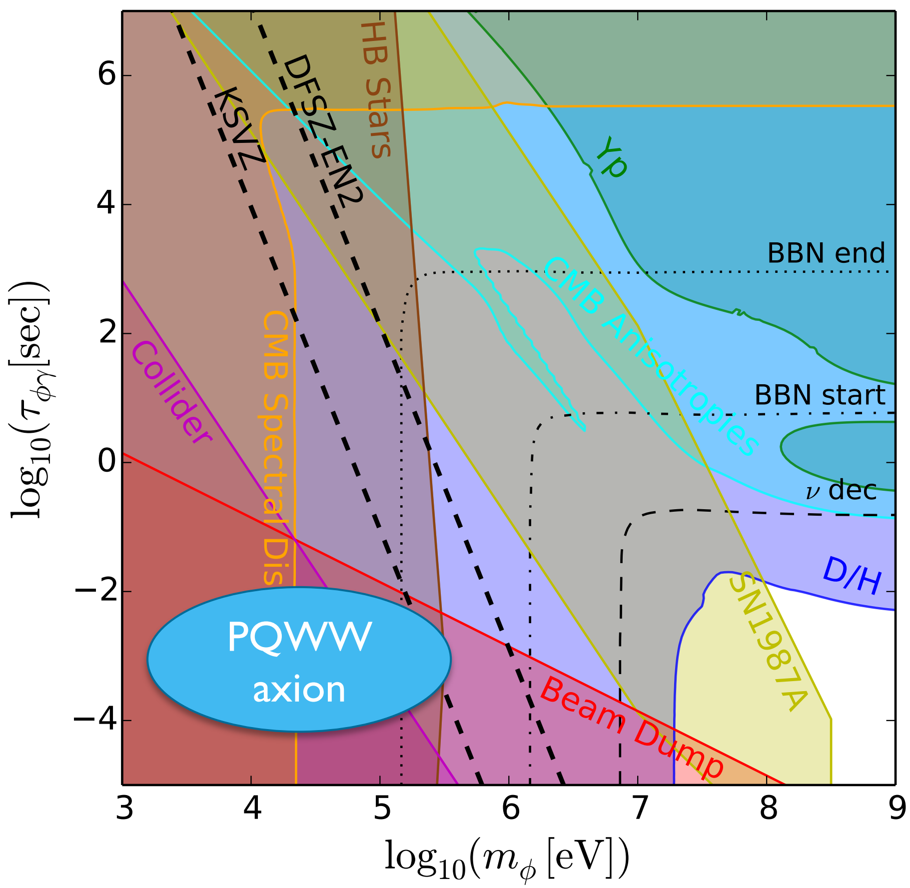
SM singlet PQ field

$$V_{\rm int} = \lambda \varphi^2 H_u H_d$$

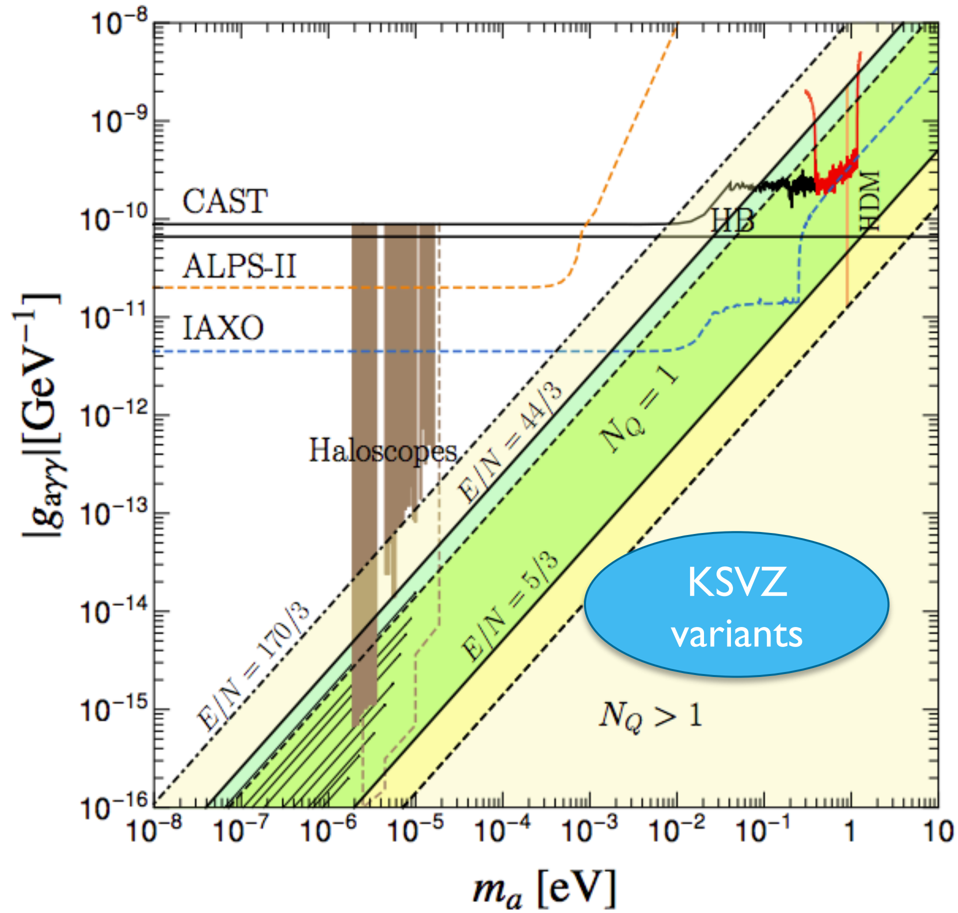
All SM fermions have PQ charge → C=6
SUSY like constraints on additional Higgs fields (H+, H-, A)

QCD Axion Constraints

(see tomorrow)



Millea et al (2015)



Di Luzio et al (2016)