Recent advances on control theory of PDE systems

1. Poster presentations

The poster presentation will take place throughout the program, in between breaks.

(1)
- Théo Ali Gherdaoui (University of Rennes 1, Rennes)
- Title - Quatradic controlability of multi-input bilinear Schrödinger equations.
- Abstract - The purpose of the poster is to present the basis on the controllability of equation. After a small introduction, I will talk more particularly about my researches, and the article about the quadratic controllability, using the Magnus formula and moment problems, for ODE affine systems. Then, using spectral theory, we can extend theses results for PDE. This is the main difficulty of the paper.

(2)
- Vishal Neeraje (ICTS Bangalore)
- Title - Understanding state and parameter estimation using observers.
- Abstract - Though dynamical systems serve as models to a wide array of physical and engineering systems, one of the main challenges in developing such models is the estimation of parameters or coefficients in the model. Since parameters in a dynamical system can change both qualitative and quantitative aspects of a solution, their accurate estimation is of utmost importance. In our recent work, we investigate how one can estimate parameters for a finite dimensional dynamical system from partial observations of the state. Indeed we show, under certain assumptions, we can simultaneously achieve state and parameter estimation for a dynamical system by constructing a suitable observer for the same. We provide a convergence result for the linear case and numerical simulations as evidence of convergence for the nonlinear case. The idea is generic, easy to implement and applicable to a large class of systems.

(3)
- Sagar Gautam (IIT Roorkee)
- Title - Feedback stabilization of convective Brinkman-Forchheiemer extended Darcy equations.
- Abstract - In this article, the following controlled convective Brinkman-Forchheimer extended Darcy (CBFeD) system is considered in a d-dimensional torus:

$$\begin{aligned}&\frac{\partial \boldsymbol{y}}{\partial t}-\mu\Delta\boldsymbol{y}+(\boldsymbol{y}\cdot\nabla)\boldsymbol{y}+\alpha\boldsymbol{y}+\beta|\boldsymbol{y}|^{r-1}\boldsymbol{y}+\gamma|\boldsymbol{y}|^{q-1}\boldsymbol{y}+\nabla p=\boldsymbol{g}+\boldsymbol{u},\\&\nabla\cdot\boldsymbol{y}=0,\end{aligned}$$

where $d\in\{2,3\}$, $\mu,\alpha,\beta>0$, $\gamma\in\mathbb{R}$, $r,q\in[1,\infty)$ with $r>q\geq 1$. We prove the exponential stabilization of CBFeD system by finite- and infinite-dimensional feedback controllers. The solvability of the controlled problem is achieved by using the abstract theory of m-accretive operators and density arguments. As an application of the above solvability result, by using infinite-dimensional feedback controllers, we demonstrate exponential stability results such that the solution preserves an invariance condition for a given closed and convex set. By utilizing the unique continuation property of controllability for finite-dimensional systems, we construct a finite-dimensional feedback controller which exponentially stabilizes CBFeD system locally,

where the control is localized in a smaller subdomain. Furthermore, we establish the local exponential stability of CBFeD system via proportional controllers.

(4)
- Vaibhav Kumar Jena (TIFR CAM)
- Title - Control of wave equations via geometric Carleman estimates
- Abstract - The main question considered in control theory is the following: Given a system, is it possible to drive the system from any given initial state to any prescribed final state? One of the methods to solve control problems is to use Carleman estimates, which are weighted integral inequalities used to show unique continuation properties for PDEs. We will present some control results for wave equations, with time dependent lower order terms, obtained using suitable geometric Carleman estimates.

(5)
- Sidhartha Patnaik (Indian Institute of Space Science and Technology)
- Title - Analysis of the magnetization control problem for the 2D evolutionary Landau-Lifshitz-Gilbert equation
- Abstract - The optimal control of magnetization dynamics in a ferromagnetic sample at a microscopic scale is studied. The dynamics of this model are governed by the Landau-Lifshitz-Gilbert equation on a two-dimensional bounded domain with the external magnetic field (the control) applied through the effective field. We have studied the optimal control problem for this model in two ways. Firstly, we prove global existence and uniqueness of a regular solution in $\mathbb{S}^2$ (unit ball in $\mathbb{R}^3$) under a smallness condition on control and initial data. In the second way, we show the existence of a weak solution and establish a condition that any weak solution satisfying the condition $\nabla m \in L^4(0,T;L^4(\Omega))$ is a regular solution. The classical cost functional is modified by incorporating $L^4(0,T;L^4(\Omega))$-norm of ∇m so that a rigorous study of the optimal control problem is established. Then, we justified the existence of an optimal control and derived first-order necessary optimality conditions using an adjoint problem approach. For the latter case, we have established the continuous dependency and Fréchet differentiability of the control-to-state and control-to-costate operators and shown the Lipschitz continuity of their Fréchet derivatives. Using these postulates, we derived a local second-order sufficient optimality condition, and finally, we obtained another remarkable result on the global optimality condition.

(6)
- Manish Kumar (IISER Kolkata)
- Title - Boundary controllability for coupled time-discrete heat equations.
- Abstract - The use of Carleman estimates in control theory is a well-explored tool to study the controllability of parabolic control systems. We derive a time-discrete Carleman-type inequality to study the notion of $\phi(\Delta t)$- null controllability for a time-discretized coupled heat equation with Kirchoff-type boundary conditions using one boundary control. Furthermore, we defined two functions using the discrete controls and solutions of the time discrete control system, which serve as approximations for the control and solution of the continuous control system. As a result, we were able to obtain the existing control result for the continuous system.

(7)
- Sakil Ahamed (IIT Kanpur)

- Title - Controllability of the linearized compressible Navier-Stokes system with Maxwell's law
- Abstract - Here, we discuss the control properties of the linearized compressible Navier-Stokes system with Maxwell's law around a constant steady state $(\rho_s, u_s, 0), \rho_s > 0, u_s > 0$ in the interval $(0, 2\pi)$ with periodic boundary data. We explore the exact controllability of the coupled system by means of a localized interior control acting in any of the equations when time is large enough. We prove the exact controllability of the system in the space $L^2(0, 2\pi) \times L^2(0, 2\pi) \times L^2(0, 2\pi)$ by proving an observability inequality with the help of an Ingham-type inequality. Next, we discuss some small-time lack of controllability of the concerned system for the case of localized interior control.

(8)
- Mohmedmunavvar Mubarak Bapu (IIT Kanpur)
- Title - Local exact controllability to the trajectories of a Gray-Scott system.
- Abstract - In this poster, we present the controllability of a Gray-Scott System. We discuss the null controllability of the linearized system around a trajectory using the Carleman inequality. Then, by applying the Kakutani fixed point theorem, the controllability to the trajectory of the nonlinear system is shown.

(9)
- Pranav Kumar (IISER Bhopal)
- Title - Local data inverse problem for the polyharmonic operator.
- Abstract - We study an inverse problem with local data for a linear polyharmonic operator with several lower order tensorial perturbations. We consider our domain to have an inaccessible portion of the boundary where neither the input can be prescribed nor the output can be measured. We prove the unique determination of all the tensorial coefficients of the operator from the knowledge of the Dirichlet and Neumann map on the accessible part of the boundary, under suitable geometric assumptions on the domain. This is based on a joint work with Dr. Sombuddha Bhattacharyya.

(10)
- Debanjit Mondal (IISER Kolkata)
- Title - Approximate controllability of Camassa Holm equation using a finite dimensional force.
- Abstract - The aim of this poster is to provide a brief introduction to the Agrachev-Sarychev approach for interior controllability of partial differential equations (PDEs).The focus will be on Camassa Holm equation with a control force acting directly only on a small number of Fourier modes. We will present our new result of global approximate controllability at any time.

(11)
- Sudipta Chattopadhyay (IIT Bombay)
- Title - Adaptive identification of SISO linear infinite-dimensional systems.
- Abstract - We propose an adaptive algorithm for identifying the unknown parameter (possibly vector-valued) in a linear stable single-input single-output infinite-dimensional system. We assume that the transfer function of the infinite-dimensional system can be expressed as a ratio of two infinite series in s (the Laplace variable). We also assume that certain identifiability conditions, which include a persistency of excitation condition, hold. For a fixed integer n, we propose an update law driven by real-time input-output data for estimating the first $n + 1$

coefficients in the numerator and the denominator of the transfer function. We show that the estimates for the transfer function coefficients generated by the update law are close to the true values at large times provided n is sufficiently large (the estimates converge to the true values as time and n tend to infinity). The unknown parameter can be reconstructed using the transfer function coefficient estimates obtained with n large and the algebraic expressions relating the transfer function coefficients to the unknown parameter. We also provide a numerical scheme for verifying the identifiability conditions and for choosing n sufficiently large so that the value of the reconstructed parameter is close to the true value. The class of systems to which our approach is applicable includes many partial differential equations with constant/spatially-varying coefficients and distributed/boundary input and output. We illustrate the efficacy of our approach using three examples: a delay system with four unknown scalars, a 1D heat equation with two unknown scalars and a 1D wave equation with an unknown linearly-varying (in space) coefficient.

(12)
- Subrata Majumdar (IIT Bombay)
- Title - Stackelberg-Nash exact controllability for the Kawahara equation with Dirichlet-periodic mixed boundary conditions.
- Abstract - This work deals with a multi-objective control problem for the strongly dissipative fifth-order KdV equation, namely the Kawahara equation, by following a Stackelberg-Nash strategy. More precisely, we act on this equation with a hierarchy of three controls. The aim of the "leader" control is the local exact controllability to the trajectory property whereas the objective of two "follower" control is to keep the state close to a given trajectory. By solving first the optimal control problem associated with the follower controls, we are led to show the local exact controllability property of a system coupling a forward with a backward Kawahara. Our main result states that for an adequate weighted functional for the optimal control problem, this coupled system is locally exactly controllable to the trajectory. To show this result, we first study the adjoint system of the linearized system and obtain a weighted observability estimate by combining a new Carleman estimate and an adequate decomposition for the Kawahara system.

(13)
- Jiten Kumbhakar (IISER Kolkata)
- Title - Boundary controllability of linearized compressible Navier-Stokes equations in one dimension using one control.
- Abstract - This poster aims to summarize the boundary controllability results for the linearized compressible Navier-Stokes system in one dimension. This linearized system consists of a transport (density) equation coupled with a parabolic (velocity) equation (with first-order coupling). We consider all the possible cases where only one boundary control can act in the density or velocity component at one end of the domain or the difference between the values at both ends. In this setup, we state the existing results in the literature and some new results which we have obtained. We see that these controllability results are optimal concerning the regularity of initial states (in the velocity case) and time (in the density case). Finally, this poster ends with some future directions of research.

(14)
- Pardeep Kumar (IIT Roorkee)
- Title - Local exact controllability to the trajectories of the convective Brinkman-Forchheimer equations.
- Abstract - We discuss the local exact controllability to trajectories of the following convective Brinkman-Forchheimer (CBF) equations (or damped Navier-Stokes equations) defined in a bounded domain $\Omega \subset \mathbb{R}^d$ $(d = 2, 3)$ with smooth boundary:

$$\frac{\partial \boldsymbol{u}}{\partial t} - \mu \Delta \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\boldsymbol{u} + \alpha \boldsymbol{u} + \beta |\boldsymbol{u}|^2 \boldsymbol{u} + \nabla p = \boldsymbol{f} + \boldsymbol{\vartheta}, \quad \nabla \cdot \boldsymbol{u} = 0,$$

where the control $\boldsymbol{\vartheta}$ is distributed in a subdomain $\omega \subset \Omega$, and the parameters $\alpha, \beta, \mu > 0$ are constants. We first present global Carleman estimates and observability inequality for the adjoint problem of a linearized version of CBF equations by using a global Carleman estimate for the Stokes system. This allows us to obtain its null controllability at any time $T > 0$. We then use the inverse mapping theorem to deduce local results concerning the exact controllability to the trajectories of CBF equations.

(15)
- Samprita Das Roy (IISER Kolkata)
- Title - Exact controllability of coupled wave system with equal variable coupling coefficients using only one boundary control.
- Abstract - Control of coupled systems has been a topic of great interest in the last few decades, having several potential applications. The challenge is to control them with a lower number of controls. Here we want to study boundary control of coupled wave systems. This problem has been extensively studied in the last three decades, yet only a few controllability results for some special cases could be obtained. One such result obtained with variable coefficients and zero order coupling is presented here.

(16)
- Jaya Kumar Alageshan (IISC Bangalore)
- Title - Navigating Complexity: First Passage Time Analysis of Smart Micro Swimmers in Turbulent Flows.
- Abstract - Microscale robotic swimmers hold immense promise for applications in targeted drug delivery, environmental monitoring, and minimally invasive medical procedures. However, their efficient navigation through turbulent environments poses a significant challenge. In this study, we investigate first passage time (FPT) of smart micro-swimmers in turbulent flow, aiming to enhance our understanding of their navigation dynamics and optimize their performance. We ememploy pseudo-spectral method to simulate a 2D homogeneous isotropic turbulent flow with micro-swimmers under various control schemes. The swimmers can sense their local flow and are equipped with intelligent control mechanisms, allowing them to adapt and respond to changing flow patterns. Turbulence, characterized by complex and chaotic fluid motion, significantly influences the swimmers' trajectories and quantify the impact of turbulence on the FPT of smart-micro swimmers. Insights gained from this analysis contribute to developing adaptive control strategies that enhance the swimmers' ability to navigate efficiently through turbulent environments. We explore the interplay between swimming strategies, environmental conditions, and the inherent stochastic nature of turbulence. Results indicate that the smart-micro swimmers' FPT depends on the stochasticity of turbulent flows and the swimmers' adaptive

response mechanisms. Understanding these dynamics is crucial for optimizing the design and operation of microscale robotic swimmers in real-world applications.

(17)
- Keerthana Nanjundan (Anna University)
- Title - Fault estimation and fault tolerant control for networked based parabolic nonlinear PDE systems against cyber-attacks and disturbances
- Abstract - This study investigates the problem of fault estimation and fault tolerant control design for networked based parabolic-type nonlinear PDE systems in the presence of Neumann-type boundary conditions against cyber-attacks with faults and external disturbances. Primarily, Proportional integral based intermediate estimator is built to estimate the immeasurable states and faults occurring in the system. Moreover, the constructed estimator precisely facilitates for estimating the fault signals and system states in simultaneously. Besides this, the integral term in the proportional integral estimator offers greater design flexibility and higher resilience. Secondly, an intermediate estimator-based fault-tolerant control is devised by availing the information from the proportional integral-based fuzzy intermediate estimator, which aids in effectively compensating the faults arising in the system. Furthermore, by endowing Lyapunov stability theory, a set of sufficient conditions is developed in the frame of linear matrix inequalities to guarantee the augmented closed-loop system is stable. Subsequently, the explicit form of the desired proportional integral gain matrices is parameterized using the matrix inequality techniques.

(18)
- Manika Bag (IISER Thiruvananthapuram)
- Title - Long-time dynamics and optimal control of a diffuse interface model.
- Abstract - We consider a model for describing the evolution of an incompressible isothermal mixture comprising two immiscible fluids in a bounded domain in 2D. This is modeled by coupling the Cahn-Hilliard equation with the Navier-Stokes equation. This work has three parts.
    1. First, we study the well-posedness results for the Cahn-Hilliard-Navier-Stokes system (CHNS) with nonhomogeneous boundary condition for the velocity equation. We obtain the existence of global weak solutions and uniqueness of weak solutions in the two-dimensional bounded domain using semi Galerkin approximation. We further prove the existence of strong solutions.
    2. Second, we consider a suitable decay condition on the boundary data and prove that any global solution converges to a equilibrium as $t \to \infty$. For this we used Lojasiewicz-Simon approach.
    3. Finally, we study an optimal boundary control problem for this CHNS system. For that we take boundary data to act on part of the boundary. Then we formulate an optimal control problem to minimize a quadratic cost functional subject to controlled CHNS system where the control acts on the boundary of the Navier–Stokes equations.

(19)
- Greeshma K (IISER Thiruvananthapuram)
- Title - Optimal control problem of the non-local Cahn-Hilliard-Brinkman system with a singular potential.

- Abstract - The evolution of two incompressible, immiscible, isothermal fluids in a bounded domain and a porous media is described by the coupled Cahn-Hilliard-Brinkman(CHB) system. CHB system consists of the Cahn-Hilliard equation describing the dynamics of the relative concentration of fluids and the Brinkman equation for velocity. This work is concerning the optimal control problem for a two-dimensional non-local CHB system with a singular-type potential. The existence and regularity results are obtained by approximating the singular potential by a sequence of regular potentials and introducing a sequence of mobility terms to resolve the blow-up due to the singularity of the potential. Further, we prove the existence of a strong solution under higher regularity assumptions on the initial data and the uniqueness of the solution using the weak-strong uniqueness technique. By considering the external forcing term in the velocity equation as a control, we prove the existence of an optimal control for a tracking type cost functional. The differentiability properties of the control-to-state operator were studied to establish the first-order necessary optimality conditions. Moreover, the optimal control is characterised in terms of the adjoint variable.